# Materials and Methods

To accurately map the template onto the target surface, the point correspondences and the appropriate transformation need to be found. However, both are closely related and one cannot be resolved without the other. For this, several approaches have been proposed, most of which assign correspondences using the *nearest-neighbor* method and then the respective transformation is applied. This process is performed in an iterative manner, making of point-matching a joint optimization problem where the distance between the template’s pointset and the respective correspondences on the target is minimized in every loop until coverage is reached.

## Explanation of process

Jonatan et. al. 2014, demonstrated that the best generic registration method is a combination of symmetrical weighted k-neighbor correspondences and a viscoelastic transformation model. This pair-wise registration algorithm can be summarized in five steps:

1. **Rough alignment**

An initial alignment of the surfaces is performed by positioning few landmarks on the target surface (e.g. only five markers are required for faces).

1. **Rigid registration**

The Iterative Closest Point (ICP) approach (Besl&McKay) is then used for a preliminary pose estimation and scaling of the template onto the target surface. It is based on a binary (rigid) nearest-neighbor relationship using an iterative least-square minimization, it relies on an adequate rough alignment.

1. **Non-Rigid registration - a) Finding correspondences**

From this stage, correspondences are found by using a combination of the pull-and-push forces (symmetrical correspondences) and the robust point matching (RPM) algorithm. The symmetrical correspondences are calculated by adding two affinity matrices: 1) from template nodes to target surfaces (push forces – the typical one-to-one correspondences calculation), and 2) from target nodes to template surfaces (pull forces). This ensures that the protrusions present at the target surface will be properly registered. Meanwhile, the RPM relaxes the binary correspondence by using the weighted k-nearest neighbor rule and the softassign. The later, introduces a fuzzy-correspondence (one-to-many) which is in turn controlled by a ‘temperature’ (*T*) parameter to the Gaussian used to calculate the weights (i.e. deterministic annealing). The higher the *T,* the broader the Gaussian and thereforethe bigger the fuzziness. *T* is lowered in every iteration until coverage is reached, making the surface behave as a fluid-like material. As a result, point-matching is improved gradually and continuously, guaranteeing a one-to-one correspondence and an accurate registration of the target surface.

1. **Non-Rigid registration - b) Pruning**

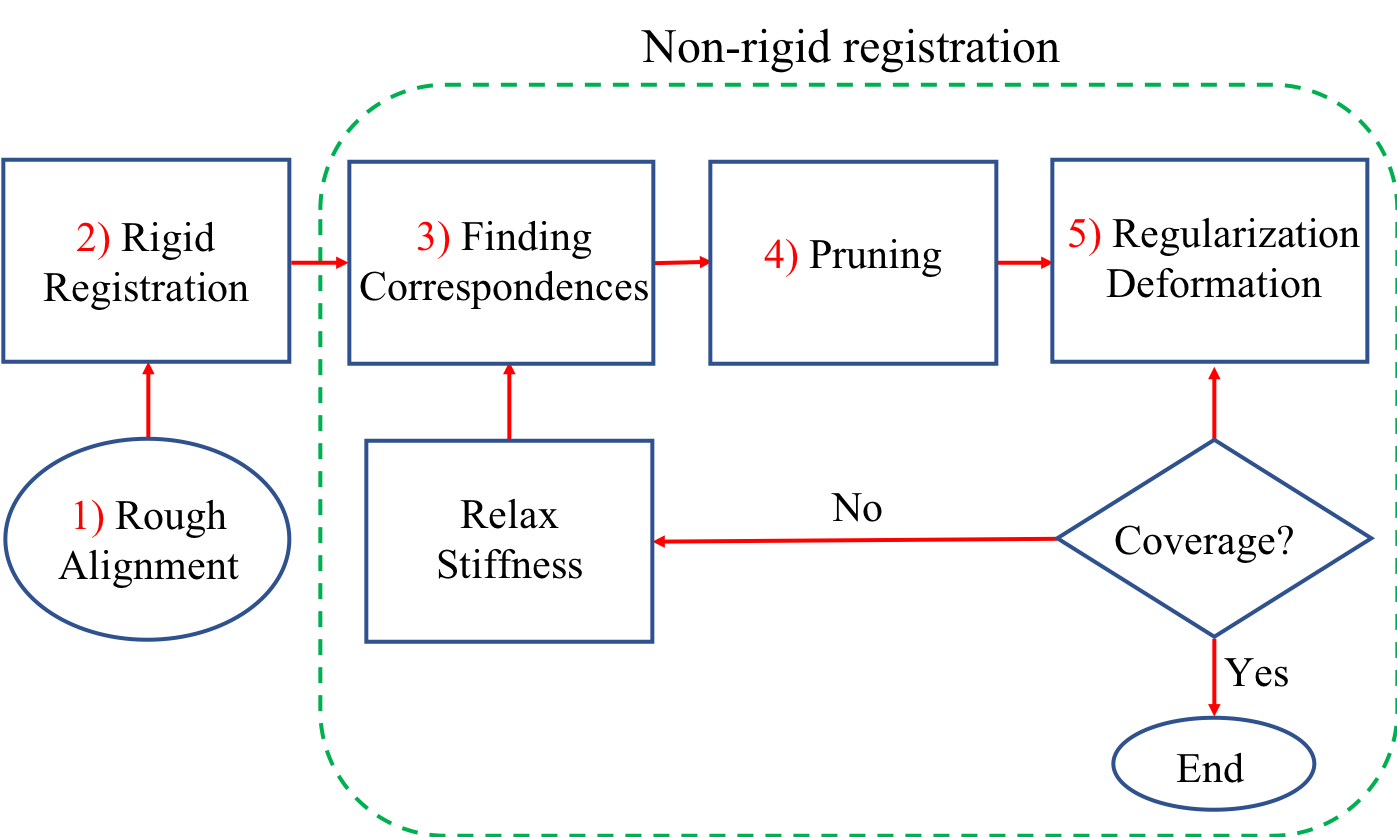
* After correspondences are found, borders and outliers need to be detected and its contribution to the transformation need to be removed. Following the statistically meaningful process described by Claes et. al. 2012, the inlier and outlier correspondences are flagged by assigning a confidence value to each pair of points. The inlier distances are assumed to form a Gaussian distribution, and any points falling out of +/- (κ) from it is considered to be an outlier. Then, the contributions of the points flagged as outliers are fixed and the confidence values of all the points are updated.

1. **Non-Rigid registration - c) Regularization and deformation**

Before the transformation is applied, a regularization of the energy function is required to ensure that points that lie close to each other move coherently. This regularization also includes the outliers, which do not contribute to the transformation but should be consistently transformed. In this way, a smooth displacement field is established. The smoothness of the transformation is parametrized by a Thin-plate Spline (TPS), which can be thought of as convolving the estimated deformation with a Gaussian. Thus, the energy function is minimized by running the algorithm iteratively and convolving it with a narrower Gaussian every time. Hence, making the global displacement more localized in every iteration. It can be considered as a rough simulation of the elastic model.

Finally, the transformation is applied to the already deformed surface. If coverage has not been reached, the correspondences are updated and the steps from 3) to 5) are repeated. Making the global transformation an incremental approach, hence, the viscous behavior of the algorithm.

A schematic of the registration algorithm is presented in fig. xx.

****