UNIVERSITY OF COPENHAGEN
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Advanced Microeconometrics

Project 1

Linear Panel Data and Production Technology

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1 Introduction

It is often assumed that production functions exhibit constant returns to scale (CRS). We examine whether this assumption can be empirically validated using data from manufacturing companies. We employ three estimation methods and only find the FD estimator consistent. The hypothesis of CRS is rejected on the basis of the FD results.

2 Data and model

The data stems from 441 French manufacturing firms for 1968-70. The variables are deflated sales, employment and adjusted capital stock, all in logs. We test for CRS in a Cobb Douglas production function: $F(K,L) = AK^{\beta_K}L^{\beta_L}$ which can be rewritten as:

$$y_{it} = \beta_K k_{it} + \beta_L l_{it} + v_{it}, \quad v_{it} = \ln A_{it} \tag{1}$$

where CRS implies that $\beta_K + \beta_L = 1$.

3 Theory

3.1 Estimators

3.1.1 Random Effects (RE)

RE estimation assumes the following form of the unobserved effects model (UEM):

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + v_{it}, \qquad t = 1, 2, ..., T \tag{2}$$

Under RE.1-2, the RE estimator is consistent conditional on $\mathbf{X}$ and asymptotically normal, but it does not exploit that we can split the composite errors, $v_{it} := c_i + u_{it}$, into a time-invariant (c_i) and a time-varying component (u_{it}). If we add Assumptions RE.3(a)-(b), not only are the usual RE standard errors and test statistics valid, but the RE estimator is also efficient in the class of estimators consistent under the condition $E(v_i|x_i) = \mathbf{0}$. The RE estimator uses the serial correlation in the v_{it} in a GLS transformation that eliminates this correlation by subtracting the mean of the basic UEM from the basic UEM for each t. Then, applying Pooled OLS (POLS) to the time-demeaned data, the RE estimator is obtained:

$$\hat{\beta}_{RE} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\mathbf{x}_{it}'\hat{\boldsymbol{\Omega}}^{-1}\mathbf{x}_{it}\right)^{-1}\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\mathbf{x}_{it}'\hat{\boldsymbol{\Omega}}^{-1}y_{it}\right) \tag{3}$$

where $\hat{\mathbf{\Omega}}$ is the estimated unconditional composite error variance.

3.1.2 First Differencing (FD)

A limitation of the RE estimator is that consistency hinges upon the $\mathbf{x}$ being uncorrelated with c_i. This is not the case for the FD and FE estimators, each of which transforms the data and eliminates c_i. For FD estimation, this involves taking the first difference of the data for each t. The basic UEM then becomes:

$$\Delta y_{it} = \Delta \mathbf{x}_{it}\boldsymbol{\beta} + \Delta u_{it}, \qquad t = 2, 3, ..., T \tag{4}$$

where $\Delta \mathbf{x}_{it} := \mathbf{x}_{it} - \mathbf{x}_{it-1}$ and similarly for Δy_{it} and Δu_{it}. Formally:

$$\hat{\beta}_{FE} = \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \Delta \mathbf{x}_{it}' \Delta \mathbf{x}_{it} \right)^{-1} \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \Delta \mathbf{x}_{it}' \Delta y_{it} \right) \tag{5}$$

For this estimator to be consistent and unbiased conditional on $\mathbf{X}$, FD.1-2 must hold. FD.3 is required for asymptotically normality. Although these assumptions allow for arbitrary correlation between c_i and x_{it}, they also rule out certain things. E.g., FD.1 rules out using lags of the dependent variable as regressors, and FD.2 excludes time-invariant variables among the regressors. Under FD.1-3, the FD estimator is the most efficient in the class of estimators using the strict exogeneity assumption FD.1. This assumption also implies that the u_{it} follow a random walk. If FD.3 does not hold, we use a robust variance matrix to compute robust standard errors:

$$\widehat{Avar}(\hat{\beta}_{FE}) = \left(\Delta \mathbf{X}' \Delta \mathbf{X} \right)^{-1} \left(\sum_{i=1}^{N} \Delta \mathbf{X}_i' \hat{\mathbf{e}}_i \hat{\mathbf{e}}_i' \Delta \mathbf{X}_i \right) \left(\Delta \mathbf{X}' \Delta \mathbf{X} \right)^{-1} \tag{6}$$

3.1.3 Fixed Effects (FE)

The FE estimator also allows for arbitrary correlation between c_i and the $\mathbf{x}_{it}$. Instead of first differencing, the data is time-demeaned before applying POLS:

$$\ddot{y}_{it} = \ddot{\mathbf{x}}_{it}\boldsymbol{\beta} + \ddot{u}_{it}, \qquad t = 1, 2, ..., T \tag{7}$$

where $\ddot{\mathbf{x}}_{it} = \mathbf{x}_{it} - \bar{\mathbf{x}}_i$, and similarly for $\ddot{y}_{it}$ and $\ddot{u}_{it}$. Applying POLS to the transformed data yields the FE estimator, formally defined as:

$$\hat{\beta}_{FE} = \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \ddot{\mathbf{x}}_{it}' \ddot{\mathbf{x}}_{it} \right)^{-1} \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \ddot{\mathbf{x}}_{it}' \ddot{y}_{it} \right) \tag{8}$$

The FE estimator is consistent and unbiased conditional on $\mathbf{X}$ under FE.1-2. Adding FE.3, it is also asymptotically normal and the most efficient in the class of estimators using the strict exogeneity assumption FE.1. Since FE.3 implies that the u_{it} are serially uncorrelated, the question of whether FD or FE is more efficient comes down to whether the u_{it} follow a random walk or are serially uncorrelated. Similar to FD.1-2, FE.1 rules out lags of the dependent variable among the regressors, and FE.2 implies that $\mathbf{x}_{it}$ cannot contain time-invariant variables. That the u_{it} are serially uncorrelated under FD.3 means that the $\ddot{u}_{it}$ are negatively serially correlated. One option is to use a robust variance matrix estimator and robust test statistics. See Table 1 for an overview of all estimator assumptions.

3.2 Inference and Model Selection

3.2.1 Wald Test

As we want to test the linear hypothesis of CRS, the Wald test is used. The test statistic for this test is:

$$W = (\mathbf{R}\hat{\beta} - r)'[\mathbf{R}\widehat{Avar(\hat{\beta})}\mathbf{R}']^{-1}(\mathbf{R}\hat{\beta} - r) \tag{9}$$

where $W \rightarrow_d \chi^2_Q$ under the null hypothesis, and Q is the number of restrictions. We reject the $H_{\mathbf{0}}$ at α significance level if $W < (1-\alpha)$-quantile of χ^2_Q. When testing for CRS, we impose one restriction, implying that:

$$\mathbf{R} = (1 \quad 1) \qquad r = 1$$

This leads us to the following hypotheses, for which we will use $\alpha = 5\%$ as our significance level:

$$H_{\mathbf{0}} : \mathbf{R}\begin{pmatrix}\beta_L \\ \beta_K\end{pmatrix} = \beta_L + \beta_K = 1 \qquad H_{\mathbf{A}} : \beta_L + \beta_K \neq 1$$

3.2.2 Hausman Test

To choose between the FE and RE estimator, we employ the Hausman test. The idea of the test is to check whether RE.1(b) fails, which indicates correlation between c_i and $\mathbf{x}$. If so, the RE estimator is inconsistent and the FE estimator is consistent, most likely resulting in statistically different estimates for the two. The test statistic for the test is:

$$H = (\hat{\beta}_{FE} - \hat{\beta}_{RE})'[\widehat{Avar(\hat{\beta}_{FE})}\widehat{Avar(\hat{\beta}_{RE})}]^{-1}(\hat{\beta}_{FE} - \hat{\beta}_{RE}) \tag{10}$$

where $H \rightarrow_d \chi^2_K$ under the null hypothesis, and K is the number of time-varying regressors (here, labor and capital). The null is that RE.1-3 and FE.2-3 hold, or alternatively:

$$H_{\mathbf{0}} : \mathrm{E}(\mathbf{x}_i' c) = 0 \qquad H_{\mathbf{A}} : \mathrm{E}(\mathbf{x}_i' c) \neq 0$$

3.2.3 Exogeneity Test

The exogeneity test is used to compare the FD and FE estimator. Following Wooldridge (2010), we test for exogeneity in the FD estimator, where the null hypothesis is $H_{\mathbf{0}} : \boldsymbol{\gamma} = 0$:

$$\Delta y_{it} = \Delta \mathbf{x}_{it} \boldsymbol{\beta} + \mathbf{w}_{it} \boldsymbol{\gamma} + u_{it} \qquad t = 2, ..., T$$

where $\mathbf{w}_{it}$ is a subset of $\mathbf{x}_{it}$, excluding time dummies. Under FD.1–3, the usual F statistic is asymptotically valid and used to evaluate the hypotheses. For the FE estimator, the following augmented regression is used where $\delta = 0$ under strict exogeneity:

$$y_{it} = \mathbf{x}_{it} \boldsymbol{\beta} + w_{i,t+1} \boldsymbol{\delta} + c_i + u_{it} \qquad t = 1, ..., T-1$$

where $\mathbf{w}_{i,t+1}$ is a subset of $\mathbf{x}_{i,t+1}$. The hypotheses are similar to the ones from the FD.

4 Analysis

We use the methods described in section 3.2 to estimate the model in equation (1). Table 2 shows the results of our estimation. We employ the Wald test to investigate the presence of CRS. All labor coefficients are found to be positive and significant, but the capital estimates are only significant for RE. Labor therefore seems to be the dominant production input. The Wald test for the FE/FD estimates suggests that we can reject the null of CRS in production, with all p-values being around 0. For RE it is not possible to reject the null hypothesis.

Next, we perform model selection procedures to choose between the various estimators for investigation of CRS in the production function. The result of our Hausman test (see Table 3) provides a p-value of 0, which implies that RE.1(b) is violated and that the RE estimator is inconsistent. This leads to a rejection of the hypothesis of CRS, as the null of CRS is both rejected for FD and FE. We then conduct an exogeneity test for the FE and FD estimators, which returns a p-value of respectively 0.00 and 0.17. Hence, we can reject the null of strict exogeneity for FE but not for FD. We therefore choose to use the FD results, as our tests do not lead to a conclusion of inconsistency for this estimator.

5 Discussion and Conclusion

Due to space limitations, we only discuss and conclude on the results for the FD estimator. Violation of FD.1 is of most concern, as we suspect that both l_{it} and k_{it} are correlated with the u_{it}. E.g., l_{it} might be correlated with time-varying factors such as labour strikes, and k_{it} might be correlated with time-varying factors such as supply-chain bottlenecks. Such correlation would render the FD estimator inconsistent. Despite this, we are not able to reject exogeneity of the regressors. Violation of FD.2 is less likely. Since neither l_{it} nor k_{it} are linear in time (as would be the case for a variable such as e.g. age), and there is no perfect collinearity between the regressors in our model (as would be the case if e.g. we had included the square of l_{it} and/or k_{it} as regressors). A crucial limitation of our assignment is that we have not considered the empirical validity of FD.3. This assumption states that the Δu_{it} are homoskedastic and serially uncorrelated, which implies that the u_{it} follow a random walk (i.e., substantial serial correlation). In our context, it is probably not appropriate to assume that the conditional variance and covariance of the Δu_{it} are constant. We would e.g. expect first differences of managerial talent to vary differently across entities with different levels of employment and capital stock. While we have not tested for this in our analysis, it would have been a good idea. We do however use robust standard errors and test statistics, which are valid even if FD.3 is violated. The sum of β_L and β_K is below 1 which indicates decreasing returns to scale (DRS). This could be due to rigidity in employment e.g. it is not easy to fire, and further that a new employee not necessarily can replace an experienced worker in terms of productivity. In addition, we do not find the estimated coefficient on capital to be statistically significant, which naturally is of concern. It should however be noted that a limitation of FD is that the data transformation reduces the variability in the regressors, which leads to less precision and thus makes it more difficult to establish statistical significance. This may explain the statistical insignificance of this coefficient. In conclusion, we reject the hypothesis of the production function exhibiting CRS, although the validity of the estimates is questionable.

References

Wooldridge, Jeffrey M (2010). *Econometric analysis of cross section and panel data*. MIT press.

Appendix

Table 1. Estimator Assumptions

Estimators	Assumptions	Effects
RE.1(a)	$\mathrm{E}(\mathbf{u}_{it}\|c_i, \mathbf{x}_i) = \mathbf{0}$	Strict exogeneity
RE.1(b)	$\mathrm{E}(c_i\|\mathbf{x}_i) = E(c_i) = \mathbf{0}$	Orthogonality
RE.2	$\text{rank}[\mathrm{E}(\mathbf{X}_i'\Omega^{-1}\mathbf{X}_i)] = K$	Full rank
RE.3(a)	$\mathrm{E}(\mathbf{u}_i\mathbf{u}_i'\|\mathbf{x}_i, c_i) = \sigma_u^2\mathbf{I}_T$	Homoskedasticity and no serial correlation
RE.3(b)	$\mathrm{E}(c_i^2\|\mathbf{x}_i) = \sigma_c^2$	Constant conditional variance
FE.1	$\mathrm{E}(\mathbf{u}_{it}\|\mathbf{x}_{it}, c_i) = \mathbf{0}$	Strict exogeneity
FE.2	$\text{rank } \mathrm{E}(\ddot{\mathbf{X}}_i'\ddot{\mathbf{X}}_i) = K$	Full rank
FE.3	$\mathrm{E}(\mathbf{u}_i\mathbf{u}_i'\|\mathbf{x}_i, c_i) = \sigma_u^2\mathbf{I}_T$	Homoskedasticity and no serial correlation
FD.1	$\mathrm{E}(\mathbf{u}_{it}\|\mathbf{x}_{it}, c_i) = \mathbf{0}$	Strict exogeneity
FD.2	$\text{rank}[\mathrm{E}(\mathbf{\Delta x}_{it}'\mathbf{\Delta x}_{it})] = K$	Full rank
FD.3	$\mathrm{E}(\mathbf{\Delta u}_{it}\mathbf{\Delta u}_{it}'\|\mathbf{x}_{it}, c_i) = \sigma_u^2\mathbf{I}_{T-1}$	Homoskedasticity and no serial correlation

Table 2: Estimators Results

Estimator	FE	FD	RE
$\hat{\beta}_L$	0.6004***	0.5509***	0.6912***
	(0.0497)	(0.0497)	(0.0490)
$\hat{\beta}_K$	0.0502	0.0381	0.2477***
	(0.0477)	(0.0493)	(0.0468)
R^2	0.28	0.22	0.80
Wald Test Statistics	38.64	46.66	1.52
Wald Test p-value	0.0000	0.0000	0.2176

Note: Robust standard-errors are in parenthesis.

Significance level: . 0.1 * 0.05 ** 0.01 *** 0.001.

Table 3: Tests

	Hausman Test	FE Exogeneity Test	FD Exogeneity Test
Test Statistic	95.16	10.32	1.85
p-value	0.0000	0.0013	0.1734
Violates	RE.1(b)	RE.1(a) and FE.1	-