

Introduction

Thomas H. Jørgensen

2024

Outline

- 1 This Course
- 2 Models
- 3 Programming in Python
- 4 Dynamic Programming

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(Associate Professor, University of Copenhagen)

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 - Dynamic programming in Python
- **Prerequisite:**
 - Introduction to Programming and Numerical Analysis

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- Plan for today:
 1. Course description
 2. Programming in Python
 3. Introduction to Dynamic Programming

Teaching Methods

- **Lectures:** Tuesday 12-15
 - ~2 hours lecture
 - ~1 hour of problem-solving (e.g. programming)

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 - I will provide code.
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- **Material:**

Web: <https://sites.google.com/view/householdbehavior>

GitHub:
<https://github.com/ThomasHJorgensen/HouseholdBehaviorCourse>

Course Plan (preliminary)

Introduction

1. Introduction
2. Dynamic Programming and Structural Estimation

Part 1: Labor Supply

3. Static and Dynamic labor supply
4. Dynamic labor supply and human capital
- .. *No lecture: Work on assignment / read papers*
5. Career costs of children
6. Household Labor Supply and Taxation
7. Household Labor Supply and Child-Related Transfers

Part 2: Family Formation and Dissolution

8. Models of Household Behavior
9. Divorce Law and Intra-Household Bargaining
10. Marriage and Divorce
11. Fertility and labor supply
12. Taxes, Transfers and Intra-Household Inequality

Outroduction

13. Children and Time Allocation

Assignments and Exam

- **Exam** (Portfolio):

1. 3 individual assignments.
+ peer feedback.
2. 48 (?) hour individual take-home exam.
Model formulation, code modification, simulations, economic interpretations.

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Model formulation, code modification, simulations, economic interpretations.
- 3 individual **assignments** (hand-in on Absalon)
 1. Based on our dynamic **labor supply** model
Deadline: March 15
 2. Based on our dynamic **bargaining** model
Deadline: April 26
 3. **Free:** Formulate a research question + model + data.
Deadline: May 10
Deadline for peer feedback: May 17
- All feedback can be used to improve assignments before exam date

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Models! What are they good for!?

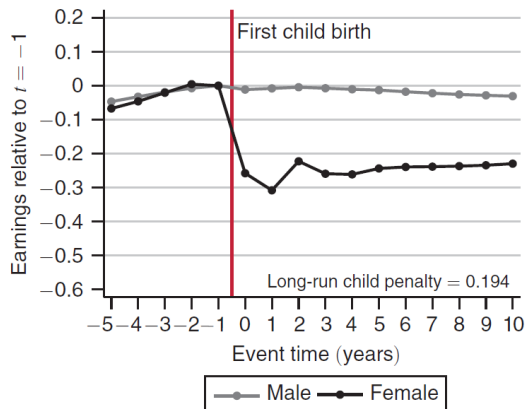
- We will use **empirical regularities as motivation**

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Example: “Child penalty” (Kleven, Landais and Sørensen, 2019)

Panel A. Earnings



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Example: “Career costs of children” (Adda, Dustmann and Stevens, 2017)

“We estimate a dynamic life cycle model of labor supply, fertility, and savings, incorporating occupational choices, with specific wage paths and skill atrophy that vary over the career. This allows us to understand the trade-off between occupational choice and desired fertility, as well as sorting both into the labor market and across occupations. We **quantify** the life cycle career costs associated with children, how they **decompose** into loss of skills during interruptions, lost earnings opportunities, and selection into more child-friendly occupations. We analyze the **long-run effects of policies** that encourage fertility and show that they are considerably smaller than short-run effects.”

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“All models are wrong, but some are useful”

George E.P. Box

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- Why ever use a model then?
- **The Lucas critique:** *Behavioral rules change with policy*
⇒ policy advice can not rely on estimated behavioral rules
(reduced-form estimates using existing variation)

⇒ we need to estimate *structural (deep) parameters*

"Invariance of parameters in an economic model is not, of course, a property which can be assured in advance, but it seems reasonable to hope that neither tastes nor technology vary systematically with variations in counter-cyclical policies." (Lucas, 1977)

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Implications of not-yet implemented policy reforms (e.g. child subsidies)

Responses to changes in the economic environment (e.g. wages)

Quantifying the importance of mechanisms (e.g. risk)

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→ Tight link between **research question** and model formulation.

Learning Objectives

See all at the Course page

- **Knowledge (two):**

Define, formulate and interpret *models* of household behaviour
Account for backwards induction and how to *solve* dynamic programming models

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Analyze *counterfactual* policy reform simulations from simple and more complex models of household behavior

- **Competences (two):**

Discuss and evaluate *research* on household behavior over the life cycle
Modify computer *code* to analyze small changes to simple models

How to read a research paper (in this course)

- Each lecture will be based on 1 mandatory (*) **research paper**
What is the main *research question*?
What is the (*empirical*) *motivation*?
What are the central *mechanisms in the model*?
What is the *simplest model* in which we could capture these?
Challenging: Research frontier.

How to read a research paper (in this course)

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What is the (*empirical*) *motivation*?

What are the central *mechanisms in the model*?

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Challenging: Research frontier.

- **How to read** a research paper in this course?

- **Focus** on the questions above

How do the questions interact and inform each other?

- Try **not to get stuck** in too many details!

(we can discuss some in class if you want)

- Research papers often include many **“bells and whistles”**

- Read **~30 min before** each lecture.

See reading-guide for each lecture

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→ We need numerical methods

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- **Purpose** of you coding:
Learn by implementing!
Appreciate bottlenecks and challenges
Better understanding of frontier research
Set you free...!

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- **Purpose** of you coding:
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Better understanding of frontier research
Set you free...!
- **Goal** is to keep things simple!
No fancy numerical tricks (at least in beginning)
Code should be “intuitive”
→ slow...
(might have to do more efficient code)

Programming in Python

- **Setup** *Visual Studio Code* as

Introduction to Programming and Numerical Analysis

<https://sites.google.com/view/numeconcph-introprog/>

- **Installation** guide:

<https://sites.google.com/view/numeconcph-introprog/guides/installation>

- **Packages** (all by Jeppe Druedahl):

EconModel

consav

See “01. Introduction to EconModel and consav.ipynb”

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- **Agents maximize** expected discounted sum of utility throughout life
Maximize wrt. $\{C_t\}_{t=1}^T$
Forward looking \rightarrow dynamic
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$$V_t(\mathcal{S}_t) = \max_{\mathcal{C}_t} U(\mathcal{C}_t, \mathcal{S}_t) + \beta \mathbf{E}[V_{t+1}(\mathcal{S}_{t+1}) | \mathcal{C}_t, \mathcal{S}_t]$$

s.t.

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- $\beta \mathbb{E}[V_{t+1}(\mathcal{S}_{t+1}) | \mathcal{C}_t, \mathcal{S}_t]$: expected discounted value of next-period
- $\mathcal{S}_{t+1} \sim F(\mathcal{C}_t, \mathcal{S}_t)$: transition density of states (fcn of \mathcal{C}_t !)
(there might be other constraints)

Backwards Induction

- Solved by **backwards induction**

1. Start with last/terminal period, T (no future)

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3. Continue backwards...

Introduction to Dynamic Programming (DP)

- **On a computer**, everything is discrete \rightarrow arrays + loops

Introduction to Dynamic Programming (DP)

- **On a computer**, everything is discrete \rightarrow arrays + loops
- ***Numerical*** Dynamic Programming
 - Backwards induction on arrays
 - Grids
 - Interpolation
 - Integration
- See “02. Consumption-Saving Model.ipynb”

Next Time

- **Next time:**

Dynamic programming with *uncertainty*
Structural *estimation*

- **Literature:**

Gourinchas and Parker (2002): “Consumption Over the Life Cycle”

- **Read** before lecture

- **Reading guide:**

Section 1: Introduction – *Key* (page 50 is not that important)

Section 2: Model – *Key*, we will discuss. Do not get stuck.

Section 3: Estimation method (SMM). *Key*, we will discuss.

Section 4: First stage calibrations. Skim fast.

Section 5: Data. Skim fast.

Section 6: Results. Focus on 6.1. Figures 5 and 7 are main results.

References I

- ADDA, J., C. DUSTMANN AND K. STEVENS (2017): “The Career Costs of Children,” *Journal of Political Economy*, 125(2), 293–337.
- GOURINCHAS, P.-O. AND J. A. PARKER (2002): “Consumption Over the Life Cycle,” *Econometrica*, 70(1), 47–89.
- KLEVEN, H. J., C. LANDAIS AND J. E. SØGAARD (2019): “Children and gender inequality: Evidence from Denmark,” *American Economic Journal: Applied Economics*, 11, 181–209.