Labor Supply: Static and Dynamic

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Outline

Plan

- Simple static and dynamic labor supply models
  - recap for some
  - brings us to same page
  - illustrate numerical approach with closed form checks
- Keane (2011, sections 1–5)
  - same notation as him

Motivation: Labor supply Elasticities are Important!

Figure: Estimated Elasticities. Hicks, Men (Keane, 2011).

TABLE 1
OPTIMAL TOP BRACKET TAX RATES FOR DIFFERENT LABOR SUPPLY ELASTICITIES

Labor supply elasticity (e)	Optimal top-bracket tax rate (τ) =1/(1+a*e)		
	$a = 1.50$	$a = 1.67$	$a = 2.0$
2.0	25%	23%	20%
1.0	40%	37%	33%
0.67	50%	47%	43%
0.5	57%	54%	50%
0.3	69%	67%	63%
0.2	77%	75%	71%
0.1	87%	86%	83%
0.0	100%	100%	100%

Note: These rates assume the government places essentially no value on giving extra income to the top earners.

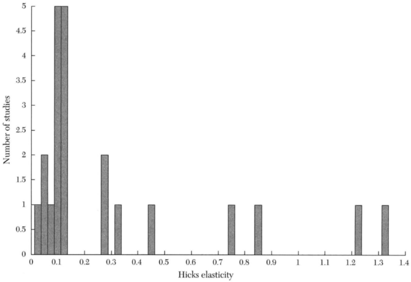

Figure 5. Distribution of Hicks Elasticity of Substitution Estimates

Note: The figure contains a frequency distribution of the twenty-two estimates of the Hicks elasticity of substitution discussed in this survey.

Outline

Static Setup

- Individuals maximize utility wrt. consumption and hours worked

$$\max_{C,h} U(C,h) = \frac{C^{1+\eta}}{1+\eta} - \beta \frac{h^{1+\gamma}}{1+\gamma}$$

where

- $\eta \leq 0$ is the CRRA coefficient
- $\gamma \geq 0$ is curvature in hours

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  where
  - $\eta \leq 0$ is the CRRA coefficient
  - $\gamma \geq 0$ is curvature in hours
- subject to the budget constraint

$$C = (1-\tau)wh + N$$

  where
  - τ is the (flat) marginal tax rate
  - w is the hourly wage rate
  - N is non-labor income

Static Solution

- Insert budget constraint

$$\max_h \frac{((1-\tau)wh+N)^{1+\eta}}{1+\eta} - \beta\frac{h^{1+\gamma}}{1+\gamma}$$

- First order condition (FOC)

$$\begin{aligned}\frac{\partial U}{\partial h} &= (1-\tau)w((1-\tau)wh+N)^{\eta} - \beta h^{\gamma} \\ &= 0\end{aligned}$$

such that the MRS is

$$(1-\tau)w = \frac{\beta h^{\gamma}}{((1-\tau)wh+N)^{\eta}} \tag{1}$$

Static Elasticities

- **No analytic solution** for optimal hours, $h^{\star}(w, N)$.
  We can solve numerically!
- **Elasticities** can be derived analytically (see extra slides)!
  Can compare with numerical!

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  Can compare with numerical!
- **Slutsky** equation:

$$\frac{\partial h}{\partial w} = \left.\frac{\partial h}{\partial w}\right|_u + h\frac{\partial h}{\partial N}$$

- **In elasticities** (% change from 1% change)

$$\frac{w}{h}\frac{\partial h}{\partial w} = \frac{w}{h}\left.\frac{\partial h}{\partial w}\right|_u + \frac{wh}{N}\frac{N}{h}\frac{\partial h}{\partial N}$$

$$\underbrace{e_M}_{\text{marshall}} = \underbrace{e_H}_{\text{hicks}} + \underbrace{\frac{wh}{N}e_I}_{\text{income effect}}$$

Static Elasticities

- Letting $S = \frac{(1-\tau)wh}{(1-\tau)wh+N}$, we have (see extra slides)

$$e_M = \frac{\partial \log h}{\partial \log w} = \frac{1+\eta S}{\gamma - \eta S}$$

$$e_H = \left.\frac{\partial \log h}{\partial \log w}\right|_u = \frac{1}{\gamma - \eta S}$$

$$ie = \frac{\eta S}{\gamma - \eta S} < 0$$

- Since $\eta \leq 0$:

$$e_H > e_M$$

that is, "ignoring income effects gives a larger response".

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  Simple setup
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1. **Solve** optimal labor supply, $h^{\star}(w, N)$
2. **Simulate baseline** labor supply for $w \rightarrow h_i(w, N)$
3. **Simulate alternative** with 1% higher wage $\rightarrow h_i(w(1+0.01), N)$
4. **Calculate** average pct change,

$$\frac{1}{n}\sum_{i=1}^{n}\frac{h_i(w(1+0.01), N) - h_i(w, N)}{h_i(w, N)} \times 100$$

- **Q**: Which of the elasticities should this equal?

Outline

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- 2-period model with **saving/borrowing**
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  - Perfect foresight + deterministic
  - Exogenous wages, w_1 and w_2
  - Same per-period utility as before
- Discounted utility is

$$U = U_1(C_1, h_1) + \rho U_2(C_2, h_2)$$

where

$$\begin{aligned} C_1 &= (1-\tau)w_1 h_1 + N_1 + b \\ C_2 &= (1-\tau)w_2 h_2 + N_2 - b(1+r) \end{aligned}$$

and $b = -[(1-\tau)w_1 h_1 + N_1 - C_1]$ is borrowing.

Dynamic Labor Supply

- We find optimal h_1, h_2 and b by maximizing utility
- First order conditions (FOCs)

$$\frac{\partial U}{\partial h_1} = [(1-\tau)w_1 h_1 + N_1 + b]^\eta w_1(1-\tau) - \beta h_1^\gamma = 0 \tag{2}$$

$$\frac{\partial U}{\partial h_2} = [(1-\tau)w_2 h_2 + N_2 - b(1+r)]^\eta w_2(1-\tau) - \beta h_2^\gamma = 0 \tag{3}$$

$$\begin{aligned}\frac{\partial U}{\partial b} &= [(1-\tau)w_1 h_1 + N_1 + b]^\eta \\ &\quad - \rho[(1-\tau)w_2 h_2 + N_2 - b(1+r)]^\eta (1+r) = 0\end{aligned} \tag{4}$$

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- Again, no closed-form solution for optimal labor supply
- We can find elasticities
  - and simulate them!
  - e_H and e_M basically the same as in static case (because no human capital)

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- In our setting (see extra slide)

$$e_F = \frac{1}{\gamma}$$

Summary: Elasticities

- **Combining**, we have

$$e_F > e_H > e_M$$

in this model.

  - **Frisch:** Can be simulated as an anticipated *transitory* increase in wage (income and thus wealth effects are small)
  - **Hicks:** Can be simulated as a unanticipated *permanent compensated* increase in wage
  - **Marshall:** Can be simulated as a unanticipated *permanent* increase in wage

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- If we can estimate $e_F = 1/\gamma$, we can **bound the policy-relevant elasticities**!
  - We can then bound the efficiency loss from labor income taxation.
  - And bound the optimal tax rate (see table from beginning).

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- **Next time:** If there is learning by doing (human capital accumulation) this relationship might not hold due to downward bias in e_F!

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such that for a grid of $\overrightarrow{b}$ we solve for

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- **First period:**

$$V_1 = \max_{C_1, h_1} U_1(C_1, h_1) + \rho V_2(b)$$

s.t.

$$b = -\left[(1-\tau)w_1 h_1 + N_1 - C_1\right]$$

Life-Cycle Model

- T periods, a_t is savings. No uncertainty (for now)
- Bellman Equation

$$V_t(a_t) = \max_{C_t, h_t} U(C_t, h_t) + \rho V_{t+1}(a_{t+1})$$

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$$a_{t+1} = (1+r)(a_t + (1-\tau_t)w_t h_t - C_t)$$

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- **Exogenous wages**:
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- **Endogenous wages** (through human capital, as next time):

$$w_t = w\,(1 + \alpha k_t)$$
$$k_{t+1} = k_t + h_t$$

  - Human capital, k_t, is a new state variable
  - Elasticities are different
  - In general no simple formula
  - *Can always simulate!*

Outline

Estimating Elasticities

- **Regressions** often like (pioneered by MaCurdy, 1981)

$$\log h_{it} = \alpha + e \log(w_{it}(1-\tau_t)) + \beta_I N_{it} + \varepsilon_{it}$$

- Controlling for non-labor income, $N \rightarrow e$ is **Marshall** elasticity.

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- Our **models can illuminate** potential problems (Keane, 2011, sec 4)
  1. Endogeneity of wages: tastes for work
  2. Endogeneity of wages: simultaneity
  3. Endogeneity of taxes (non-linear)
  4. Measurement error (downward bias)
  5. Wages only observed for workers (selection)
  6. Savings and non-labor earnings
  7. Human capital and other dynamics (next time)

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  5. Wages only observed for workers (selection)
  6. Savings and non-labor earnings
  7. Human capital and other dynamics (next time)
- Structural estimation can handle most of these

Estimated Elasticities, Men.

TABLE 6
SUMMARY OF ELASTICITY ESTIMATES FOR MALES

Authors of study	Year	Marshall	Hicks	Frisch
Static models				
Kosters	1969	−0.09	0.05	
Ashenfelter-Heckman	1973	−0.16	0.11	
Boskin	1973	−0.07	0.10	
Hall	1973	n/a	0.45	
Eight British studies[a]	1976–83	−0.16	0.13	
Eight NIT studies[a]	1977–84	0.03	0.13	
Burtless-Hausman	1978	0.00	0.07–0.13	
Wales-Woodland	1979	0.14	0.84	
Hausman	1981	0.00	0.74	
Blomquist	1983	0.08	0.11	
Blomquist-Hansson-Busewitz	1990	0.12	0.13	
MaCurdy-Green-Paarsch	1990	0.00	0.07	
Triest	1990	0.05	0.05	
Van Soest-Woittiez-Kapteyn	1990	0.19	0.28	
Ecklof-Sacklen	2000	0.05	0.27	
Blomquist-Ecklof-Newey	2001	0.08	0.09	
Dynamic models				
MaCurdy	1981	0.08[b]		0.15
MaCurdy	1983	0.70	1.22	6.25
Browning-Deaton-Irish	1985			0.09
Blundell-Walker	1986	−0.07	0.02	0.03
Altonji[c]	1986	−0.24	0.11	0.17
Altonji[d]	1986			0.31
Altug-Miller	1990			0.14
Angrist	1991			0.63
Ziliak-Kniesner	1999	0.12	0.13	0.16
Pistaferri	2003	0.51[b]		0.70
Imai-Keane	2004	0.40[e]	1.32[e]	0.30–2.75[f]
Ziliak-Kniesner	2005	−0.47	0.33	0.54
Aaronson-French	2009			0.16–0.61
Average		0.06	0.31	0.85

Notes: Where ranges are reported, mid-point is used to take means.

Estimated Elasticities, Women.

- Women have been viewed as more "complex"
  Literature started later
  When dynamic models was used more

TABLE 7
SUMMARY OF ELASTICITY ESTIMATES FOR WOMEN

Authors of study	Year	Marshall	Hicks	Frisch	Uncompensated (dynamic)	Tax response
Static, life-cycle and life-cycle consistent models						
Cogan	1981	0.89[a]				
Heckman-MaCurdy	1982			2.35		
Blundell-Walker	1986	−0.20	0.01	0.03		
Blundell-Duncan-Meghir	1998	0.17	0.20			
Kimmel-Kniesner	1998			3.05[b]		
Moffitt	1984				1.25	
Dynamic structural models						
Eckstein-Wolpin	1989				5.0	
Van der Klauuw	1996				3.6	
Francesconi	2002				5.6	
Keane-Wolpin	2010				2.8	
Difference-in-difference methods						
Eissa	1995, 1996a					0.77–1.60[b]

Notes:
[a] = Elasticity conditional on positive work hours.
[b] = Sum of elasticities on extensive and intensive margins.

Next Time

- **Next time:**
  Dynamic labor supply with learning by doing
  Human capital accumulation from working
  (Uncertainty?)

- **Literature:**
  Keane (2016): "Life-Cycle Labour Supply with Human Capital: Econometric and Behavioural Implications"
  - **Read** before lecture
  - **Reading guide:**
    Section 0: Introduction
    Section 1: Dynamic model. *Key section, main focus*.
    Section 2: Simulations of 2-period model. skim/drop.
    Section 3: Quantitative role of HC. Read fast, *focus on 3.2*.
    Section 4: Comparison with extensive margin. Read if time

References I

KEANE, M. P. (2011): "Labor Supply and Taxes: A Survey," *Journal of Economic Literature*, 49(4), 961–1075.

——— (2016): "Life-cycle Labour Supply with Human Capital: Econometric and Behavioural Implications," *The Economic Journal*, 126(592), 546–577.

MACURDY, T. E. (1981): "An Empirical Model of Labor Supply in a Life-Cycle Setting," *Journal of political Economy*, 89(6), 1059–1085.

Finding elasticities in static model

- Taking logs of eq. (1):

$$\underbrace{\log(1-\tau)+\log(w)}_{\text{LHS}} = \underbrace{\log(\beta)+\gamma\log(h)-\eta\log((1-\tau)wh+N)}_{\text{RHS}} \quad (5)$$

- Derivative wrt. $\log w$ gives (while keeping N fixed)

$$\frac{\partial \text{RHS}}{\partial \log w} = \gamma \underbrace{\frac{\partial \log h}{\partial \log w}}_{e_M} - \eta\left(\frac{\partial \log(\bullet)}{\partial w}\underbrace{\frac{\partial w}{\partial \log w}}_{w} + \frac{\partial \log(\bullet)}{\partial h}\underbrace{\frac{\partial h}{\partial \log h}}_{h}\underbrace{\frac{\partial \log h}{\partial \log w}}_{e_M}\right)$$

where $\log(\bullet) = \log((1-\tau)wh+N)$.

Marshall

- Letting $S = \frac{(1-\tau)wh}{(1-\tau)wh+N}$, we have that

$$\frac{\partial \text{RHS}}{\partial \log w} = \gamma e_M - \eta(S + S e_M)$$
$$\frac{\partial \text{LHS}}{\partial \log w} = 1$$

such that

$$1 = \gamma e_M - \eta(S + S e_M)$$
$$\Updownarrow$$
$$e_M = \frac{1+\eta S}{\gamma - \eta S}$$

Income effect

- Similarly, the income elasticity is

$$e_I = \frac{\partial \log h}{\partial \log N} = \frac{\eta(1-S)}{\gamma - \eta S}$$

- Found similarly, using that $1 - S = \frac{N}{(1-\tau)wh+N}$.
- The income effect is then

$$ie = \frac{wh(1-\tau)}{N} e_I = \frac{\eta S}{\gamma - \eta S}$$

where

$$ie < 0$$

because $\eta \leq 0$.

Hicks

- The Hicks, or "compensated" elasticity is

$$
\begin{aligned}
e_H &= \left.\frac{\partial \log h}{\partial \log w}\right|_u \\
&= e_M - ie \\
&= \frac{1+\eta S}{\gamma - \eta S} - \frac{\eta S}{\gamma - \eta S} \\
&= \frac{1}{\gamma - \eta S}
\end{aligned}
$$

  using the Slutsky equation.
- We have

$$e_H \geq e_M$$

Frisch Elasticity

- Deriving the Frisch via. the Lagrangian

$$\max_{h_1,h_2,C_1,C_2,b} U + \lambda_1[(1-\tau)w_1h_1 + N_1 + b - C_1] \\ + \lambda_2[(1-\tau)w_2h_2 + N_2 - b(1+r) - C_2]$$

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- FOC for hours in period 1

$$\frac{\partial U}{\partial h_1} = -\beta h_1^{\gamma} + \lambda_1(1-\tau)w_1 = 0$$

such that

$$\log h_1 = \frac{1}{\gamma}\log(\lambda_1) + \frac{1}{\gamma}\log((1-\tau)w_1) - \frac{1}{\gamma}\log(\beta)$$

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such that

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- and the partial derivative (fixed λ_1)

$$e_F = \left.\frac{\partial \log h_1}{\partial \log w_1}\right|_{\lambda_1} = \frac{1}{\gamma}.$$