Buffer-stock model

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Buffer-stock model (Intuitive)

$$v_t(M_t, P_t) = \max_{C_t} \left\{ \frac{C_t^{1-\rho}}{1-\rho} + \beta \mathbb{E}_t \left[v_{t+1}(M_{t+1}, P_{t+1}) \right] \right\} \qquad \text{(Bellman)}$$

s.t.

$$\begin{aligned}
A_t &= M_t - C_t && \text{(e.o.p. assets)} \\
M_{t+1} &= RA_t + Y_{t+1} && \text{(b.o.p. assets)} \\
Y_{t+1} &= P_{t+1}\xi_{t+1} && \text{(income)} \\
P_{t+1} &= GL_t \psi_{t+1} P_t && \text{(perm. part)} \\
\xi_{t+1} &= \begin{cases} \mu, \text{ with prob. } \pi \\ (\epsilon_{t+1} - \pi\mu)/(1-\pi) & \text{, else} \end{cases} && \text{(trans. shock)} \\
\epsilon_{t+1} &\sim \exp \mathcal{N}(-0.5\sigma_\xi^2, \sigma_\xi^2) && \\
\psi_{t+1} &\sim \exp \mathcal{N}(-0.5\sigma_\psi^2, \sigma_\psi^2) && \text{(perm. shock)} \\
A_t/P_t &\geq -\lambda && \text{(credit constr.)}
\end{aligned}$$

Buffer-stock model (Practical)

$$v_t(m_t) = \max_{c_t} \left\{ \frac{c_t^{1-\rho}}{1-\rho} + \beta \mathbb{E}_t \left[(GL_t\psi_{t+1})^{1-\rho} v_{t+1}(m_{t+1}) \right] \right\}$$

s.t.

$$a_t = m_t - c_t$$

$$m_{t+1} = \frac{1}{GL_t\psi_{t+1}} R a_t + \xi_{t+1}$$

$$\xi_{t+1} = \begin{cases} \mu \text{ with prob. } \pi \\ (\epsilon_{t+1} - \pi\mu)/(1-\pi) & \text{else} \end{cases}$$

$$\epsilon_{t+1} \sim \exp \mathcal{N}(-0.5\sigma_\xi^2, \sigma_\xi^2)$$

$$\psi_{t+1} \sim \exp \mathcal{N}(-0.5\sigma_\psi^2, \sigma_\psi^2)$$

$$a_t \geq -\lambda$$

Buffer-stock model (added retirement)

Debt must be repayable:

$$a_t \geq \max(-\lambda_t, -\Omega_t)$$

$$\lambda_t = \begin{cases} \lambda \text{ if } t < T_R \\ 0 \text{ if } t \geq T_R \end{cases}$$

$$\Omega_t = \text{Maximum guaranteed repayable debt at time t}$$

No uncertainty during retirement

$$\text{For } t >= T_R :$$

$$\psi_{t+1} = 1, \xi_{t+1} = 1$$

Estimation

- Look at Bertel's structural estimation slides for formulas
- Data is in absolute terms: e.g. C_t, M_t
  - Permanent income is a theoretical construct and is normally not observed
  - Here we simulate data for permanent income in order to estimate
- MLE
  - Notice that we need to observe all states and choices
- MSM
  - We don't need to observe states and choices in data
  - Model just has to be able to simulate moments

Euler equation

Bellman rewritten

$$v_t(m_t) = u(c_t^*) + \beta \mathbb{E}_t \left[(fac)^{1-\rho} v_{t+1}(m_{t+1}) \right], c_t^*(m_t) \text{ is optimal c}$$

FOC wrt. c

$$\frac{\partial v_t(m_t)}{\partial c_t^*} = \frac{\partial u(c_t^*)}{\partial c_t^*} + \beta \frac{\partial \mathbb{E}_t \left[(fac)^{1-\rho} v_{t+1}(m_{t+1}) \right]}{\partial m_{t+1}} \frac{\partial m_{t+1}}{\partial c_t^*}$$

$$\frac{\partial u(c_t^*)}{\partial c_t^*} = \beta \frac{\partial \mathbb{E}_t \left[(fac)^{1-\rho} v_{t+1}(m_{t+1}) \right]}{\partial m_{t+1}} \frac{R}{fac}$$

FOC wrt. m (applied envelope theorem, $\partial c_t^* / \partial m_t = 0$)

$$\frac{\partial v_t(m_t)}{\partial m_t} = \frac{\partial u(c_t^*)}{\partial m_t} + \beta \frac{\partial \mathbb{E}_t \left[(fac)^{1-\rho} v_{t+1}(m_{t+1}) \right]}{\partial m_{t+1}} \frac{\partial m_{t+1}}{\partial m_t}$$

$$\frac{\partial v_t(m_t)}{\partial m_t} = 0 + \beta \frac{\partial \mathbb{E}_t \left[(fac)^{1-\rho} v_{t+1}(m_{t+1}) \right]}{\partial m_{t+1}} \frac{R}{fac}$$

Euler equation

Note that from the previous equation, we have:

$$\frac{\partial v_t(m_t)}{\partial m_t} = \frac{\partial u(c_t^*)}{\partial c_t^*}$$
$$\frac{\partial v_{t+1}(m_{t+1})}{\partial m_{t+1}} = \frac{\partial u(c_{t+1}^*)}{\partial c_{t+1}^*}$$

Inserting this into FOC wrt. c, we have

$$\frac{\partial u(c_t^*)}{\partial c_t^*} = \beta \mathbb{E}_t \left[(fac)^{1-\rho} \frac{\partial u(c_{t+1}^*)}{\partial c_{t+1}^*} \right] \frac{R}{fac}$$
$$\frac{\partial u(c_t^*)}{\partial c_t^*} = \beta R \mathbb{E}_t \left[(fac)^{-\rho} \frac{\partial u(c_{t+1}^*)}{\partial c_{t+1}^*} \right]$$