# Minimum Entropy Models for Laser Line Extraction

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Abstract. Minimum entropy model can find the optimal gray space for laser line extraction. A global model named Minimum Entropy Deconvolution is established to search for the peaks which constitute the laser line. Not only does it reach a high accuracy, but also it retains the line smoothness, which the previous work often paid little attention to. Besides, this work could extract several laser lines. Experimental results show that the robust models and fast algorithms outperform the compared.

**Keywords:** Gray space; Minimum entropy blind deconvolution; Laser line extraction.

#### 1 Introduction

Laser line extraction is a decisive step that determines the accuracy of 3D reconstruction. Most researches focus on the peaks detection, then the laser line is composed of these peaks after further processing.

Fisher et al. [1] gave a comparison of the most common approaches and concluded that all of them displayed performance within the same range. Haug et al. [2] indicated that the center of mass produced the best results. Strobl et al. [3] presented a laser line extraction algorithm based on this approach by means of a color Look-Up table. In [4], proper observation and transition probability for HMM were defined while detecting laser stripes.

Normally, since the laser light is red, these aforementioned researches used the red component of image as a gray-level image to process. However, it may not show the distinct features of laser, and sometimes it is inappropriate to extract the laser line. For instance, it is difficult to distinguish the laser light from the white one in the background by using only red component. Some work reduced these interferences by using a color absorber to obtain the gray-level image. Although they illuminated good results, the requirement for optical filtering implies a line extraction algorithm was not robust [5–7]. H. Ta et al. [8] tried to extract the laser line in YCbCr and HSI spaces in light of past experience. The crucial point is that little research focused on how to select the most appropriate gray space to extract the laser line. And H. Ta et al. extracted laser line only

in low-level noises, as other work used local information and assumed the cross section of the laser line was a Gaussian distribution [1–3, 5, 9, 10]. Thus they needed not too many overexposed pixels, and they were only applied if the difference between the light intensity of laser and background was significant. Some approaches took advantage of global information for extraction to solve the problem [4,7]. Both methods are less sensitive to local interferences and robust to blurry images.

In this paper, a model based on minimum entropy is firstly built to find the optimal gray-level space for extraction. Then a minimum entropy deconvolution model is solved to reset the peaks to the center of laser stripe. At last, all the peaks that are selected along each row or column in the image constitute the laser line. The method uses global information for extraction and allows line gaps and strong noises. Although other work also searches for the peaks, they pay more emphasis on the accuracy, and this paper can obtain a robust and smooth result as well as high accuracy, especially in high noise level scenes.

### 2 Methodology

#### 2.1 Gray Space

Because linear transformation is continuous and does not cause gaps [11], this paper uses a linear transformation to obtain a gray-level image from original image. In a discrete color image  $f_{ij}$  (with a size of  $M \times N$ ), color of a pixel is given as corresponding tristimulus  $R_{ij}$  (red),  $G_{ij}$  (green), and  $B_{ij}$  (blue). Hence, the linear transformation defines as equation (1):

$$F_{ij} = \omega_r R_{ij} + \omega_q G_{ij} + \omega_b B_{ij} \tag{1}$$

where  $F_{ij}$  is the desired gray-level image,  $i=1,2,\ldots,M, j=1,2,\ldots,N, \omega_r, \omega_g$  and  $\omega_b \in R$ . In order to extract the laser line conveniently, the gray-level image determined by the transformation coefficients should highlight the characteristics of laser. Now an objective function need to be defined to search the optimal  $\omega_r$ ,  $\omega_g$  and  $\omega_b$ .

Owing to the tight focusing and strong brightness of beam, one of the most remarkable features of laser is that energy concentration is remarkably high, which makes the stripe show its waveform with a few spikes and distinguish it from the background in the image (Fig. 1, an example of laser profiles, which is obtained through showing all the image row vectors). In other words, the contrast between the laser and background is high. The objective function should retain and enhance this feature so that the laser line extraction will become much easier after transformation. Thus, the contrast can be used as the objective function. In Fig. 1, gray values of the pixels within the laser stripe are near to their averages. In these areas, the greater energy concentration is, the more striking contrast will be, and also a higher Kurtosis. They are equivalent to contrast and Kurtosis. Therefore, it is reasonable to define contrast as Kurtosis:

$$V = \frac{\mu_4}{\sigma^4} - 3 \tag{2}$$

where Kurtosis V is defined as the fourth moment around the mean  $\mu_4$  divided by the square of the variance  $\sigma^4$  of the probability distribution minus 3.

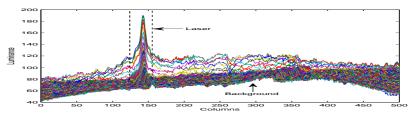


Fig. 1. Example of laser profiles

The transform result of the laser stripe is expected to be of very high Kurtosis while competing signals the background is of very low Kurtosis. A signal with high Kurtosis is great order, and vice versa. In the context of communication systems, disorder is synonymous to the concept of entropy. It is also declared that the greater the disorder of a signal is, the greater its entropy gets. Wiggins [12] originally presented the minimum entropy deconvolution technique (MED) based on this reason. He proposed to maximize a Varimax Norm function, which is equivalent to maximizing Kurtosis with assumed zero-mean. For this reason, the transformation model can be named as minimum entropy model. However, minimizing instead of maximizing equation (2) is appropriate for the signal with a kurtosis less than zero. Thereupon, an alternative choice of objective function which is differentiable everywhere can be defined as the square of Kurtosis. If the image is thought to process as a Multi-channel signal (with N segments and M elements per segment), the objective function can be written as

$$V = \left(\sum_{j=1}^{N} \frac{\sum_{i=1}^{M} (F_{ij} - \mu_j)^4}{\left(\sum_{i=1}^{M} (F_{ij} - \mu_j)^2\right)^2} - 3\right)^2$$
(3)

where  $\mu_i$  is the mean of column j in the transform image  $F_{ij}$ .

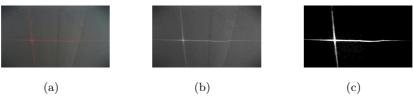
The solution of maximizing equation (3) is corresponding to equation (4)

$$\frac{\partial V}{\partial \omega_r} = 0, \frac{\partial V}{\partial \omega_g} = 0, \frac{\partial V}{\partial \omega_b} = 0 \tag{4}$$

Although it is difficult to solve equation (4), it could be approximately calculated the maxima value of referring to in [13]. An infinite set of color feature space is determined by the in continuous coefficients in equation (1). For convenience to obtain  $\omega_r$ ,  $\omega_g$  and  $\omega_b$ , they are discretized as integer and values range are limited from -2 and 2. It is feasible because the linear combinations sample the set of 1D color feature subspaces of 3D RGB space uniformly, and some common features are covered, such as R+G+B, R-B, and so on. Furthermore, it is also efficient to compute. Considering that the laser is red and R component of original image has

higher energy, narrow  $\omega_r \geq 0$ . That is  $\omega_r \in \{0, 1, 2\}$ ,  $\omega_g, \omega_b \in \{-2, -1, 0, 1, 2\}$ . Absolutely, it is not allowed if  $(\omega_r, \omega_g, \omega_b) = (0, 0, 0)$ . The optimal parameters can be computed by the traversal method.

In Fig. 2, the optimal coefficient vector of a typical image is  $(\omega_r, \omega_g, \omega_b) = (1, -1, 0)$ , and the gray-level space is R-G. A cross laser is used here. It also makes comparisons with R. Two advantages of R-G are obvious. On the one hand, it distinguishes the laser from the background. On the other hand, it has much smaller amount of data than raw image.



**Fig. 2.** The color feature transport result: (a) raw image, (b) the gray-level image of R, (c) the gray-level image of R-G.

Ideally, the horizontal component and the vertical component of a cross laser are comparatively independent, and their intensity is subject to Gaussian distribution. All the intensity maxima are right in the center of the stripes and the laser lines can be extracted precisely just by taking the peaks out.

However, it becomes not feasible in practical environments for the block speckle noise, energy diffusion, light saturation, light saturation crosstalk, natural light, and other factors [11]. These noise sources combine together, make the constructive and destructive interferences within the laser stripes, lead to the laser intensity no longer obey Gaussian distribution and some peaks are no longer in the center of stripes.

Some measures should be taken to focus energy on the center of laser stripes and reset the peaks back to the center. The image signal entropy will decrease with the energy focusing. At last, the signal with the minimum entropy will be composed of a series of narrowband pulses. From the point of view of signal processing, this process can be described as a problem of minimum entropy deconvolution.

#### 2.2 Minimum entropy deconvolution

Taking the input image as a Multi-channel signal in columns (with N segments and M elements per segment) or in rows (with M segments and N elements per segment). In the former case, the horizontal component of the laser stripe is largely restored, and the vertical component information is suppressed synchronously, and vice versa. Then the whole recovered information of the laser stripes can be obtained by executing the two operations separately. The model in columns to extract the horizontal laser line can be formulated as

$$B_{ij} = \sum_{k=1}^{L} W_k F_{i-k+1,j} \tag{5}$$

where  $\mu_{Bj}$  is the mean of column j of  $B_{ij}$ , k = 1, ..., L, L is the order of the filter, which has significant impact on the MED outputs. Experience shows that an efficiency and reliable length is 5% of the number of elements per segment. The objective function can be written as equation (3) with replacing  $F_{ij}$  as  $B_{ij}$ .

The MED searches for an optimum set of filter coefficients that recover the output signal with the maximum value of Kurtosis. For convenient, the causal filter is normalized. Differentiating V with respect to the  $W_k$  and equating to zero yields, an iteratively converging local-maximum solution can be derived as

$$\sum_{l=1}^{L} W_{l} \sum_{j=1}^{N} V_{j} U_{j}^{-1} \sum_{i=1}^{M} F_{i-l,j} F_{i-k,j} = \sum_{j=1}^{N} U_{j}^{-2} \sum_{i=1}^{M} (B_{ij} - \mu_{Bj})^{3} F_{i-k,j}$$
 (6)

where 
$$U_j = \sum_{i=1}^{M} (B_{ij} - \mu_{Bj})^2$$
,  $V_j = \frac{\sum_{i=1}^{M} (B_{ij} - \mu_{Bj})^4}{\left(\sum_{i=1}^{M} (B_{ij} - \mu_{Bj})^2\right)^2}$ , and  $W_k$  is iteratively

selected. The general procedure is as follows in Algorithm 1. Fig. 3 shows the comparison of energy concentration between input signal and signal is filtered by MED.

### Algorithm 1 The general procedure of MED

- 1. Assume the initial value of W. Here setting  $W = \begin{bmatrix} 11 \dots 1 \dots 11 \end{bmatrix}^T / \sqrt{L}$ .
- 2. Compute the output signal  $B_{ij}$  through Eq.(5).
- 3. Compute new filter coefficients by solving for W in Eq.(6).
- 4. Repeat from step 2 and 3 for a specified number of iterations or until the change in V between iterations is below a specified small value.

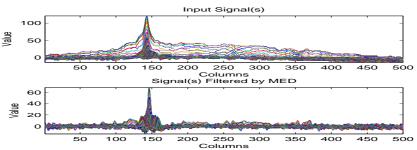


Fig. 3. The comparison of energy concentration between input signal and signal filtered by MED

#### 2.3 Laser Line Extraction

Extraction of the laser line can be completed by extracting the peaks of every column or every row, respectively. Furthermore, a group delay in rows compared

with the ideal curve has to be taken into account. This phase shift  $\Delta i = -P_W$  can be computed from 2D convolution theorem and equation (5), where  $P_w$  is the phase of  $W_k$ .

The model to extraction of the vertical laser line in rows has the same formula form and processing.

#### 2.4 Smoothness

This paper gives a definition of smoothness. A kind of understanding of smoothness is the measure of similarity between the extracted and true line. Consequently the smoothness should be the function about the error between the two data. The error is defined as  $\{e_i\}$ . Obviously, S should satisfy: 1)  $\forall i, e_i = C(C$  is a constant), the extracted line has the same form with the true line, the line is the most smooth, and S should be its minimum value, i.e. S=0; 2) the more even the distribution of  $\{e_i\}$  is, the smoother is the line, the letter S is, and vice versa. For example, the waveform of  $\{e_i\}$  is quickly waved, this line is very rough, and S is large, though the values of  $\{e_i\}$  may be very small. Therefore S is a function has the symmetrical form with 1D entropy (Fig. 4). It can be computed as

$$S = H + \log(num) \tag{7}$$

where  $H\left(p\right) = -\sum_{j=1}^{num} (n_j/num)\log_2\left(n_j/num\right)$  is the entropy of array  $\{e_i\}$ , num is quantization orders,  $n_j$  is the number of  $e_j$  after quantization.

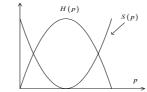


Fig. 4. The form of the smoothness function

# 3 Experimental Results

To evaluate the effectiveness of models, some experiments have been carried out to compare the performance with the proposed approaches [3, 6] which are known as Centre of Mass (CM), Linear Approximation (LA), Quadratic Approximation (QA), and Akima splines Approximation (AA) in different gray spaces (R & R-G, the gray-level images obtained by CCD using optical filters also are treated as R components). For comparing the better gray space further, the comparison experiments of MED are also used in R and R-G.

These experiments are concerned with a set of test images in outdoor scenes. This paper presents typical one with the size of  $500 \times 375$  pixels (Core 3.40 GHz CPU and a 4.0 GB RAM).

The extraction quality of the method in this paper shows similar results for all tested images (Table 1 and Fig. 5). As a whole, it is easy to see two points: on the one hand, the component R-G is better than R in all methods, and on the other hand, the laser line extraction model based MED is more valid, accurate, and smooth than other approaches or models.

It is worth noting that the data vertical laser line obtained in R is so tough that its consumed time is too long to afford in practice. Taking into account this factor, it no longer fits the data using LA, QA, AA and MED. Nonetheless, the error of MED overall is smaller than the former. And they are smoother than the fitting data obtained by LA, QA and AA in R-G. This phenomenon is determined by the more even distribution of the former error, although most of its elements are larger and they have similar offsets relative to the true data. It also suggests that the definition of smoothness is appropriate.

All the calculations are based on the full image. It can be seen that the speed of MED is fast enough to use to the reality. If the region of interest is selected, the speed will rise tenfold.

Laser line	Color space	Method Index	СМ	LA	QA	AA	MED
Horizontal laser line	R	Time $(ms)$	18.3	320.2	168.1	130.3	22.3
		Smoothness	5.2667	8.2495	7.9673	8.1243	1.4858
	R-G	Time $(ms)$	17.9	310.4	167.6	196.6	20.9
		Smoothness	5.1600	8.0299	7.9336	8.0243	1.4439
Vertical laser line	R	Time $(ms)$	18.8	$\infty$	$\infty$	$\infty$	20.6
		Smoothness	5.7304	5.8531	5.8531	5.8531	5.9231
	R-G	Time (ms)	17.6	120.2	147.2	166.6	19.9
		Smoothness	5.7282	7.6794	7.7102	7.6047	1.5669
The whole	R	Total time $(ms)$	35.6	$\infty$	$\infty$	$\infty$	41.1
Line	R-G	Total time $(ms)$	33.8	406.5	302.5	355.4	38.7

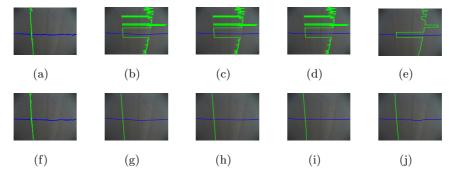
Table 1. Experimental result.

### 4 Conclusion

The paper proposes a fast, accurate and robust method to extract the laser line in high noise level environments. And it ensures smoothness of the extracted line, which others paid little attention to. Experimental results demonstrate that the models are promising. In the future, they are planned to apply in 3D reconstruction.

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<sup>&</sup>quot; $\infty$ " means that the time is too long to afford in practice, num = 500.



**Fig. 5.** The extraction result of the above methods in R and R-G: (a)-(e) CM, LA, QA, AA and MED in R, respectively, (f)-(j) CM, LA, QA, AA and MED in R-G, respectively.

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