Examples of single location, single population models using ordinary differential equations

Lecture 02

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The SLIRS models

Computing $\mathcal{R}_0$ more efficiently

An extension of KMK – SLIAR models

Computing the final size more efficiently

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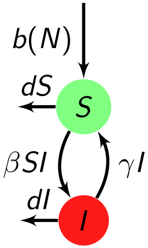
Simplifying the SIRS model



- We have already seen the epidemic KMK SIR model and the endemic SIRS model
- By making some simplifications, we can obtain the SIS model: assume the time spent in the R compartment goes to zero, i.e., $\nu \to \infty$

The main characteristics of the model are the same as the SIRS

$$\mathcal{R}_0 = \frac{\beta}{d+\gamma}$$

and determines whether we go to the DFE $(N^\star, 0)$ or to the endemic equilibrium

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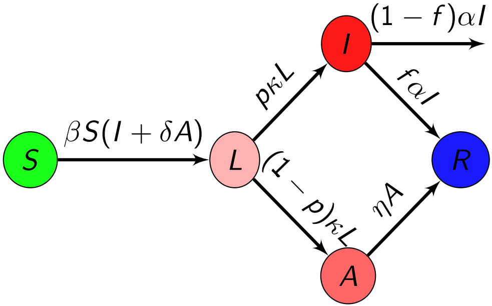
SIR is a little too simple for many diseases:

- No incubation period
- A lot of infectious diseases (in particular respiratory) have mild and less mild forms depending on the patient

$\Longrightarrow$ model with SIR but also L(atent) and (A)symptomatic individuals, in which I are now symptomatic individuals

Arino, Brauer, PvdD, Watmough & Wu. Simple models for containment of a pandemic (2006)



$(1-f)\alpha I$
I
$p\kappa L$
$f\alpha I$
$\beta S(I+\delta A)$
S
L
R
$(1-p)\kappa L$
ηA
A

Basic reproduction number

We find the basic reproduction number

$$\mathcal{R}_0 = S_0\beta\left(\frac{p}{\alpha} + \frac{\delta(1-p)}{\eta}\right) = \frac{S_0\beta\rho}{\alpha} \tag{1}$$

where

$$\rho = \alpha\left(\frac{p}{\alpha} + \frac{\delta(1-p)}{\eta}\right)$$

Final size relation

$$S_0(\ln S_0 - \ln S_\infty) = \mathcal{R}_0(S_0 - S_\infty) + \frac{\mathcal{R}_0 I_0}{\rho} \tag{2}$$

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