

# Examples of single location, single population models using ordinary differential equations

## Lecture 02

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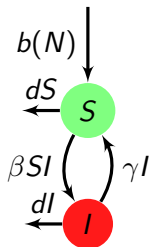
The SLIRS models

An extension of KMK – SLIAR models

## The SLIRS models

An extension of KMK – SLIAR models

# Simplifying the SIRS model



► We have already seen the epidemic KMK SIR model and the endemic SIRS model

► By making some simplifications, we can obtain the SIS model: assume the time spent in the  $R$  compartment goes to zero, i.e.,  $\nu \rightarrow \infty$

The main characteristics of the model are the same as the SIRS

$$\mathcal{R}_0 = \frac{\beta}{d + \gamma}$$

and determines whether we go to the DFE  $(N^*, 0)$  or to the endemic equilibrium

The SLIRS models

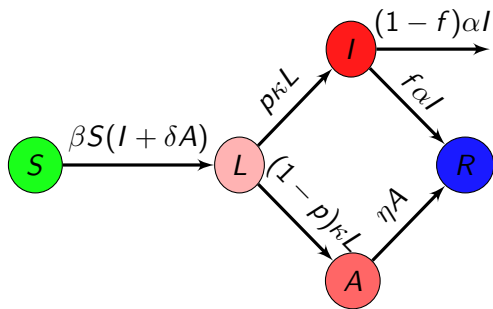
An extension of KMK – SLIAR models

SIR is a little too simple for many diseases:

- ▶ No incubation period
- ▶ A lot of infectious diseases (in particular respiratory) have mild and less mild forms depending on the patient

⇒ model with SIR but also L(atent) and (A)symptomatic individuals, in which I are now symptomatic individuals

Arino, Brauer, PvdD, Watmough & Wu. Simple models for containment of a pandemic (2006)



## Basic reproduction number

$$\mathcal{R}_0 = S_0 \beta \left( \frac{\rho}{\alpha} + \frac{\delta(1 - \rho)}{\eta} \right) = \frac{S_0 \beta \rho}{\alpha} \quad (1)$$

where

$$\rho = \alpha \left( \frac{\rho}{\alpha} + \frac{\delta(1 - \rho)}{\eta} \right)$$



## Final size relation

$$S_0(\ln S_0 - \ln S_\infty) = \mathcal{R}_0(S_0 - S_\infty) + \frac{\mathcal{R}_0 I_0}{\rho} \quad (2)$$