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Agent-based and network models

Lecture 06

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Incidence function versus force of infection

Two different forms for the rate of movement of S individuals from S to whatever infected compartment they end up in:

- $S' = -f(S, I, N)$ is an **incidence function**
- $S' = -\lambda(S, I, N)S$ is a **force of infection**

The two are of course essentially equivalent, the context tends to drive the form used. Advanced PDE models that consider for instance an age-of-infection structure need to integrate over $I(t, a)$ and thus often use force of infection, others are somewhat random..

Interactions – Infection

- Rate at which new cases appear per unit time is the *incidence function*
$$f(S, I, N)$$
- Depends of the number S of susceptible individuals, I of infectious individuals and, sometimes, of the total population N
- Incidence includes two main components
  - a denumeration of the number of contacts taking place
  - a description of the probability that such a contact, when it takes place, results in the transmission of the pathogen
- Choosing an appropriate function is hard and probably one of the flunkiest part of epidemic modelling

Two most frequently used functions

The two most frequently used incidence functions are **mass action incidence**

$$f(S, I, N) = \beta SI$$

and **standard** (or **proportional**) **incidence**

$$f(S, I, N) = \beta \frac{SI}{N}$$

In both cases, β is the *disease transmission coefficient*

Units of β

Recall that if $X(t)$ is the population in compartment X at time t, then X' has units number/time

In a differential equation, left and right hand side must have same units, so..

Mass action incidence

$$\beta SI \propto \beta \times \text{number} \times \text{number}$$

has units number/time if β has units 1/(number $\times$ time)

Standard incidence

$$\beta SI/N \propto \beta \times \text{number} \times \text{number}/\text{number} \propto \beta \times \text{number}$$

has units number/time if β has units 1/time

Mass action incidence

$$f(S, I, N) = \beta SI \tag{1}$$

- There is homogenous mixing of susceptible and infectious individuals
- Strong hypothesis: each individual potentially meets every other individual

In this case, one of the most widely accepted interpretations is that all susceptible individuals can come across all infectious individuals (hence the name, by analogy with gas dynamics in chemistry/physics)

When population is large, the hypothesis becomes unrealistic

Standard (proportional) incidence

The other form used frequently:

$$f(S, I, N) = \beta \frac{SI}{N} \tag{2}$$

Each susceptible individual meets a fraction of the infectious individuals

Or vice-versa! See, e.g., Hethcote, Qualitative analyses of communicable disease models, *Mathematical Biosciences* (1976)

Case of a larger population

Constant population $\implies$ (1) $\equiv$ (2)

When the total population is constant, a lot of incidence function are equivalent (to units)

Suppose $N(t) \equiv N_0$, then

$$\beta SI = \tilde{\beta}\frac{SI}{N} \iff \tilde{\beta} = N_0\beta$$

and if the right hand side is satisfied, then (1) and (2) identical

Keep in mind units are different, though

General incidence

$$f(S, I, N) = \beta S^q I^p \tag{3}$$

These functions were introduced with data fitting in mind: fitting to data, find the p, q best matching the available data

Incidence with refuge

The following implements a refuge effect; it assumes that a proportion $0 < q < 1$ of the population is truly susceptible, because of, e.g., spatial heterogenities

$$f(S, I, N) = \begin{cases} \beta I \left(N - \dfrac{I}{q} \right), & \text{if } I < qN \\ 0, & \text{if } I \geq qN \end{cases} \tag{4}$$