Mathematical Oncology

Non-dimensionalisation

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A simple case

We start by considering a simple **dimensional** problem

$$p = \alpha x + \beta \tag{1}$$

where $\alpha, \beta \in \mathbb{R}$ are constants and where x, p the independent and dependent variable respectively.

Assuming that the above equation refers to an actual problem involving x kilograms (kg) of a product and it price p in GBPs (£), it will be best if we write it in the following form

$$\boxed{[p\,£] = [\alpha\,£/\,\mathrm{kg}][x\,\mathrm{kg}] + [\beta\,£]} \tag{2}$$

which clearly highlights the role of each one of the variables and parameters as well as their dimensions.

The process of non-dimensionalisation relies on choosing **references** values $[p_{\mathrm{ref}}\,£]$ and $[x_{\mathrm{ref}}\,\mathrm{kg}]$ for the *dimensional* variables $[p\,£]$ and $[x\,\mathrm{kg}]$ respectively, and on **introducing** the corresponding **non-dimensional** variables $\tilde{p}$ and $\tilde{x}$ as follows

$$\tilde{p} = \frac{[p\,£]}{[p_{\mathrm{ref}}\,£]}, \quad \tilde{x} = \frac{[x\,\mathrm{kg}]}{[x_{\mathrm{ref}}\,\mathrm{kg}]} \tag{3}$$

or

$$[p\,£] = [p_{\mathrm{ref}}\,£]\tilde{p}, \quad [x\,\mathrm{kg}] = [x_{\mathrm{ref}}\,\mathrm{kg}]\tilde{x}.$$

Substituting in (2) we obtain

$$[p_{\mathrm{ref}}\,£]\tilde{p} = [\alpha\,£/\,\mathrm{kg}][x_{\mathrm{ref}}\,\mathrm{kg}]\tilde{x} + [\beta\,£]$$

or, after dividing by $[p_{\mathrm{ref}}\,£]$,

$$\tilde{p} = \frac{[\alpha\,£/\,\mathrm{kg}][x_{\mathrm{ref}}\,\mathrm{kg}]}{[p_{\mathrm{ref}}\,£]}\tilde{x} + \frac{[\beta\,£]}{[p_{\mathrm{ref}}\,£]}. \tag{4}$$

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We now **introduce** the **dimensionless** parameters

$$\tilde{\alpha} = \frac{[\alpha\,£/\,\mathrm{kg}][x_{\mathrm{ref}}\,\mathrm{kg}]}{[p_{\mathrm{ref}}\,£]}, \quad \tilde{\beta} = \frac{[\beta\,£]}{[p_{\mathrm{ref}}\,£]}$$

and (4) recasts into

$$\boxed{\tilde{p} = \tilde{\alpha}\tilde{x} + \tilde{\beta}.} \tag{5}$$

This is the **non-dimensional** version of the original dimensional equation (1) or (2).

Remark 1. *It should clear that (5) depends on the reference values $[p_{ref}\,£]$ and $[x_{ref}\,kg]$. For example,*

- *If a much-much-much larger reference value $[p_{ref}\,£]$ is chosen (while keeping the same $[x_{ref}\,kg]$) then it should be expected that the dimensionless coefficients $\tilde{\alpha}$ and $\tilde{\beta}$ will be much-much-much smaller; one could imagine that $\tilde{\alpha}$ and $\tilde{\beta}$ could be even assumed to be zero.*
- *If on the other hand, a much-much-much smaller value of $[x_{ref}\,kg]$ is chosen (while maintaining the same $[p_{ref}\,£]$), it should be expected that $\tilde{\alpha}$ would be much-much-much smaller; one could imagine that in this case only $\tilde{\alpha}$ could be assumed to be zero.*