UNIVERSITY OF MANITOBA

DATE: March 3, 2009 — Test 1
PAPER NO.: - — PAGE NO.: 1 of 2
DEPARTMENT & COURSE NO.: MATH 3820 — TIME: 120 minutes
EXAMINATION: Intro. Math. Modelling — EXAMINER: J. Arino

This is a 120 minutes exam, with 4 questions for a total of 50 marks. Lecture Notes are allowed. PLEASE SHOW YOUR WORK CLEARLY. A correct answer without explanation will not get full marks.

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1. (10 points) The dynamics of a population of birds (measured in thousand) is described by the equation
$$\frac{dP}{dt} = 4P(1 - 8P^3).$$
   (a) Find the equilibria.
   (b) Determine the local stability of each equilibrium.

2. (10 points) The population (measured in billions) of insects in generation t is described as follows
$$P_{t+1} = P_t e^{4(1-3P_t)}$$
   (a) Find all fixed points.
   (b) Determine the local stability of each fixed point.

3. (20 points) Assume that an insect population, $x(t)$, is controlled by a natural predator population, $y(t)$. We make the following assumptions:
   - In the absence of predators, the dynamics of the insects is governed by a logistic equation.
   - Preys and predators meet at a rate that is of mass action type.
   - When a contact takes place, the probability per contact that a prey dies is k_1. [Hint: think of mass action contact in an epidemic model.]
   - These contacts lead to an increase of the predator population with rate k_2.
   - The predators are subject to natural death at the per capita rate d.

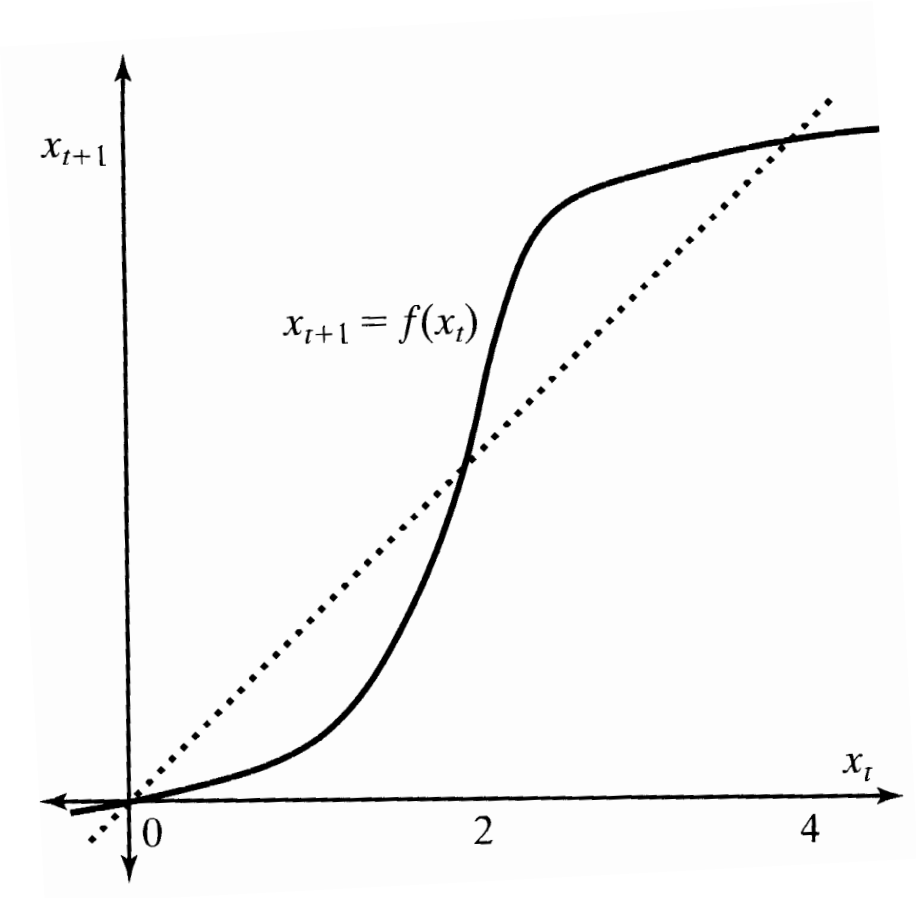
   (a) Write a model describing the interaction of the 2 populations.
   (b) Study the model you have written: is it well-posed, what are its equilibria (and when are they realistic), can you determine the stability of these equilibria?

(c) Assume that an insecticide is used to reduce the population of insects, but it is also toxic to the predators; hence, the poison kills both preys and predators at rates proportional to their respective populations. Modify your model from (a). [For bonus marks, you may want to see how this modifies the analysis you carried out in (b).]

4. (10 points) Consider the difference equation

$$x_{t+1} = f(x_t)$$

with graph shown in the figure below



(a) Find all fixed points.

(b) Determine the local stability of each fixed point. [Hint: what does the stability condition imply, in terms of the slope of f?]