Introduction to Mathematical Modelling

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DETERMINATION OF TIME OF DEATH

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INTRODUCTION

When a person dies, the coroner has to establish a time of death to be recorded on the death certificate. If the death is classified as homicide or suicide, the time of death can help a police investigation and lead to witnesses and suspects in order to solve the case. There are different methods used in the determination of the time of death. Many are in situ at the location of the found body, while others are ~~as well as those used~~ in the laboratory.

The objective of this essay is to examine some of the methods used by coroners to determine the time of death of a body.

ASSUMPTIONS

As with most exercises, assumptions have to be made in order to simplify the problem and therefore arriving at a manageable solution to a problem.

1) The human body temperature is assumed to be 37.0 $^{\circ}$C (Lena Wong, 1999) but this temperature actually is a range of values. The temperature of a human can range from 36.6-37.3 $^{\circ}$C (Lena Wong, 1999) depending upon the age of the human, metabolic rate, time of day, and part of the body the temperature is measurement. Since the core body temperature of a healthy, resting adult is fairly constant at 37.0 $^{\circ}$C, this is a reasonable assumption.

2) A second consideration is understanding the cooling of a body and the temperature of the environment. There is "natural" cooling where the body is cooling with the natural convection currents of the heat rising from it and also "forced" cooling where there exists external

convection currents (O'Sullivan, 1990). The coroner has to be aware if the environment temperature has remained constant or if there has been the addition of other cooling / heating forces involved. Most often the coroner would be unaware of any environmental temperature changes and therefore an average environment temperature would be recorded.

METHODS TO DETERMINE THE TIME OF DEATH

A) ALGOR MORTIS

There are rough estimates that are easily applied to determine the time of death of a body. One such method is algor mortis, the cooling of the body, which assumes that the body temperature was originally 37.0 °C and decreases at a constant rate of approximately 0.8 °C per hour according to Anoka (1999). This is a very rough estimate because there are many variables that are not included such as environment temperature and size of the body, such as surface area.

B) LIVOR MORTIS

Another rough estimate that is used is livor mortis, or the settling of blood in the lower regions of the body. Because the blood is not circulating throughout the body, gravity takes over and the blood settles in the lower parts of the body. The blood starts to settle in about ½ hour to 3 hours and has fully settled in about 8-12 hours (FSC Lecture, 1999). A coroner would press the skin in the lower parts of the body and if the skin blanches, the body died less than 12 hours ago. This is a very rough estimate of death as it depends more on the position on the body rather than on temperature of the body and environment but still is useful is determining the window of time of death.

C) RIGOR MORTIS

Another rough estimate of time of death is using rigor mortis, the chemical reaction that causes muscles to become rigid. The body requires ATP, adenosine triphosphate, for energy. At the time of death, the body does not produce any more ATP and although some cells and tissues continue to live after the body is dead, they eventually die due to lack of ATP and the increase of other ions in the cells. The muscles start to stiffen 3-4 hours after death with a peak rigidity after 12 hours and the decreases so that the body is limp in about 48 hours (Standford, 1999). To determine time of death, the ratio of ions and ATP is used but factors such a activity of the body before death and temperature of the body and environment can affect the ratio.

These three methods are not labor intensive by solving long mathematical equations but do involve scientific investigation. These methods are more subjective as the coroner uses tables of data to arrive at a window of time of death. The accuracy of the tables and if they vary from publication must be considered when relying on the results. Still most coroners use more than one estimate to arrive at the time of death when homicide is involved (Anoka, 1999).

D) NEWTON METHOD (1)

As defined in Nagle and Saff (1989), "Newton's law of cooling states that the rate of change in the temperature T of a body is proportional to the difference between the temperature of the medium M and the temperature of the body." Therefore, recording the temperature of the body when discovered and at one time later while the temperature of the environment, the medium, is kept constant should give an estimate of when the body died because after death, the temperature of the body starts to drop (Staerkeby, 1999). This gives the equation:

$$dT/dt = k\,(M - T)$$

where k is a constant with units $time^{-1}$ and M and T are the temperatures in degrees of the environment and body respectively taken at time t and dT/dt is the rate of change. Solving the differential equation gives:

$$M - T = Ce^{kt}$$

where C is a constant evaluated with the initial conditions of the time of death (t=0), normal body temperature and the temperature of the environment.

This equation gives a better estimate of time of death because it includes the temperature of the environment unlike the first three methods. However, this method does not take into account the surface area of the body and the estimate of time of death varies when the temperature is converted between degrees Centigrade and Fahrenheit as shown in Appendix 1. ?

Still this is only an approximate to the time of death as shown in the FSC Lecture (1999), even under ideal conditions this method can only estimate time of death to plus or minus three hours to the real time of death.

due to round-off in conversion

E) NEWTON'S METHOD (2)

The definition of Newton's law of cooling from O'Sullivan (1990), is "the law states that the rate of heat loss per unit area from a body is directly proportional to the temperature difference between the body and the surrounding fluid medium in contact with the body". The resulting equation is:

$$1/A \; dT/dt = h\,(T - M)$$

where M is the temperature of the fluid medium, T is the temperature of the body, h is the heat transfer coefficient and A is the surface area. With Q, the heat capacity of the system, the differential equation is:

a function

$$T - M = C\, e^{hAt/Q}$$

where C is a constant evaluated with the initial conditions of time of death (t=0) and normal body temperature and temperature of the environment, both in degrees Kelvin.

This method is better than the first Newton Method as it includes characteristics of the body, such as heat capacity and heat transfer coefficient. The resulting equation is more difficult to solve because of the addition of more variables including surface area of the body which would have to be estimated. Another variable to be included along side the heat transfer coefficient is the "layer of still fluid, the boundary layer" (O'Sullivan, 1990) on the surface of the body through which the heat has to pass before completely leaving the system. If this layer is uniform, which has to be assumed, then, the variable `h' is the ratio between the thickness of the boundary layer and the thermal conductivity of the fluid, which is beyond the objective of this essay.

F) NEWTON - STEFAN METHOD

Since the Newton methods rely on the conduction - convection processes between the body and it's medium, the addition of Stefan's equation takes in account of heat loss through radiation. The differential equation becomes:

$$1/A\ dt/dt = h(T - M) + \varepsilon\sigma(T^4 - M^4)$$

where the first part is the Newton's law of cooling and the second part is Stefan's radiative heat transfer (O'Sullivan, 1990). The equation includes ε, the emissivity of the body, which is some dimensionless number between 0 and 1 and is larger for darker surfaces and smaller for lighter surfaces (Jones and Childers, 1993) and σ is Stefan's constant ($5.67 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$). The temperatures are recorded in degrees Kelvin. Solving the differential equation is quite complicated but according to O'Sullivan (1990), if (T - M) is not too large, the equation is:

$$T - M = \frac{C\ e^{-\beta At / Q}}{\left[\frac{1+6\ \varepsilon\sigma\ M^2}{\beta\ (1 - e^{-\beta At / Q})}\right]}$$

where $\beta = h + 4\ \varepsilon\sigma\ M^3$. If T is close to M and M is constant, then the Newton Method (E) is obtained (Nagle and Saff, 1989).

As stated with the Newton Method (E), the more variables in the equation, the harder it is to solve and the more room for error as variables are estimated.

G) DULONG - PETIT METHOD

In the absence of forced cooling, the Dulong - Petit method tries to include the combined conductive - convection and radiation cooling with a power equation (O'Sullivan, 1990). The equation is given as:

$$1/A \, dT/dt = g\,(T - M)^{5/4}$$

The solved differential equation is:

$$T - M = (C^{-1/4} - hAt/4Q)^{-1/4}$$

where C is a constant calculated from the initial conditions, Q is the heat capacity of the system, h is the heat transfer coefficient and A is the surface area of the body. It is stated by O'Sullivan (1990), that this method is not very practical as it only works where natural cooling is allowed and all drafts are eliminated which is high unlikely to occur at a site of a dead body.

H) OTHER METHODS (NON-MATHEMATICAL)

According to the FSC Lecture (1999), much work can be done to estimate the time of death using mathematical models but because there is a great variance within the variables, the best way to establish the time of death of a murdered victim is through police investigation. Both of the Newton methods give a good estimate of time of death and are easily calculated and therefore give police a window of death time that is very useful in their homicide investigation.

CONCLUSION

Although there are different mathematical and non-mathematical ways to establish the time of death in homicide investigations, one has to keep in mind these are only estimates. Scientific method is very helpful but there is a fear of relying on instruments too much without realizing their limitations (Berry and Ishii, 1985). The use of technology for the sake of being modern is not always the route to take. Usually simple methods by use of simple models give the required information without creating a data overload. All methods in this essay have errors associated with them and so in theory, the data could be of top quality, but in a practice, the data still is only an estimate to the problem of the determination of time of death.

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APPENDIX 1

NEWTON'S METHOD USED TO ESTABLISH TIME OF DEATH

For example, given degrees in Fahrenheit, (Burghes and Borrie 1982), a body was discovered at 11:00 p.m. and the coroner arrives at 11:30 p.m. and records the temperature of the body at 94.6. One hour later, the body temperature is measured at 93.4 with the room temperature kept at 70.0. The estimate of time of death is calculated as:

Newton's Law of Cooling

$$dT/dt = k\,(M - T)$$

Solve the Differential Equation

$$M - T = Ce^{kt}$$

Use the initial conditions to solve for C:

t=0	time of death
T=98.6	normal body temperature

$$70.0 - 98.6 = C\,e^{k0}$$

$$C = -28.6$$

Use the conditions recorded by the coroner to calculate k and t:

$$70.0 - 94.6 = -28.6\,e^{kt}$$

$$kt = -0.15066$$

$$70.0-93.4 = -28.6e^{k(t+1)}$$

$$k(t+1) = -0.20067$$

$k = -0.050011$

$t = 3.01256$

So the body died 3 hours before 11:30 p.m. or at approximately 8:30 p.m. However, using the equivalent temperatures in Centigrade and solving to find the time of death results in a slightly different answer.

Use the initial conditions to solve for C:

t=0	time of death
T=37.0	normal body temperature

$$21.1 - 37.0 = C\,e^{k0}$$

$$C = -15.9$$

Use the conditions recorded by the coroner to calculate k and t:

$$21.1 - 34.8 = -15.9\,e^{kt}$$

$$kt = -0.14892$$

discrepancies due to round-off error

$$21.1 - 34.1 = -15.9\,e^{k(t+1)}$$

$$k(t+1) = -0.20137$$

$$k = -0.052447$$

$$t = 2.8395$$

Therefor the body died 2 hours and 50 minutes before 11:30 p.m. or at approximately 8:40 p.m.