An Introduction to Interest Rate Generators

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An Introduction to Interest Rate Generators

In this paper, I will answer a few questions pertaining to interest rate generators. One may ask the question as to the importance of predicting future interest rates. Predicting future interest rates is of great importance to any individual or company involved with loans or investments. To an individual, one possible use of an interest rate generator would be to predict possible returns on investments. This could be of even more importance if there is a possibility of retirement within the span of the investments. Calculating one's retirement nest egg and insuring that it is financially stable should be a concern to any investor. To a company, one possible use of an interest rate generator is to predict the financial standing of the company, not today, but a month or year from now. That is of great value to the company executives when determining the feasibility of long term projects. Numerous questions can be looked at, with this topic. I would like to talk about the calculation of interest rates, some general criteria for all interest rate generators and, finally, look at some specific interest rate generators in detail examining the strengths and weaknesses.

The first thing that I would like to discuss is how interest rates are calculated. I would not like to enter into the debate of whether or not charging interest on a loan or for an investment is valid. We live in a world where interest is collected. With that said, let us assume that all loans and investments are subject to some sort of interest.

There is no "magic" formula used to calculate an interest rate. Rather, interest rates are subjected to a supply versus demand system. Take, for example, the fictional Island of Berry, where no one leaves or enters. Let us say that on the Island of Berry, one person, Michael, is seeking a loan. If also on the island, there is another person, Sean, who has an excess of funds and is willing to loan some of his funds to Michael, our system is set. Sean and Michael could work out a feasible payment plan that is satisfactory to both. However, let us say another individual on the island, Corrie, is also seeking a loan. Sean may let Michael and Corrie outbid each other in an attempt to get the loan. This would increase the payment Sean will receive. On the other hand, what if Corrie was in a situation to give, rather than receive, a loan. We can see that our system can get very complicated very quickly.

Before developing a model to approximate future interest rates, one must first understand the various parameters that are essential to any interest rate. A large number of factors piece together in a complex way when calculating an interest rate. I would like to briefly mention a few of the major ones now:

Underlying "Pure" Rate of Interest

The underlying pure rate of interest is a theoretical base rate that all loans are subject to. This rate would be equal to the interest rate used on a stable (i.e. risk free) investment assuming no inflation. In Canada, as in the United States, the pure rate of interest is typically in the 2% to 3% range. On the Island of Berry, the pure rate of interest would be constant for all three scenarios that I have depicted.

Inflation

Inflation is defined to be the loss of purchasing power over time. There is definitively a correlation between the current rate of interest and the expected rate of inflation. Over time, both will tend to move in the same direction. On the Island of Berry, the inflation rate for the scenario of two borrowers versus one loaner would have the highest inflation. Conversely, the third scenario depicted with one borrower and two loaners would have the lowest inflation.

Risk and Uncertainty

Risk and uncertainty arise from various actions that are estimated and may vary over time. There are two main categories of risk; market risk, which refers to the risk involved with future price changes arising from changes in the interest rate, and credit risk, referring to possibility of defaults. In the Island of Berry scenarios, the possibility of market or credit risk would definitely be a factor for the interest rate in all three cases.

Time Period or Length of Investment

Strangely enough, the time period of an investment does affect the interest charged. Short term investments (less than 2 years) historically have lower interest rates than long term investments.

At this time, I would like to define a yield curve. A yield curve is a graph of the length of the investment versus the interest rate assigned. Historically, yield curves tend to have positive slopes. There are various theorems for this phenomenon; expectations theory (a higher percentage of people predict interest rates will rise rather than fall), liquidity preference theory (a higher percentage of people prefer access to their funds rather than having them tied up for a long period of time) and inflation premium theory (uncertainty of inflation for long term investments). Again, the length of the investment would be a factor in all three scenarios on the Island of Berry.

Quality of Information

Quality of information deals with the possibility of an "inefficient" market. In an "efficient" market, all buyers and sellers posses the same information. This should not be assumed true in all cases. An individual may spend more time and energy in research and thus have more information referring to a certain project. On the island, this may be a factor.

Legal Issues

Under the heading of legal issues falls a great deal of constraints. There are numerous legal restrictions and government policies that need to be taken into consideration for calculating interest rates. I will mention some of these later when discussing general criteria for any model. The inhabitants of the Island of Berry are no exception, and would have to take these concerns into consideration, as well.

Random Fluctuations and Unanticipated Phenomenons

Random fluctuations and unanticipated phenomenons include randomness of interest rates, the stabilizing effect (to be defined later) and acts of God or politics. These events should be assumed to occur at any given time and will definitely effect interest rates. Again, another issue for the people on the Island of Berry.

Again, these are some of the major factors concerning the calculation of interest rates. This list is not exhaustive. However, when dealing with a model to approximate interest rates, one may assume the effect of any other issue as negligible. Furthermore, the effects of some of the factors mentioned above may be assumed negligible.

There are numerous general criteria that all interest rate generators face.

I have already mentioned one of these and would like to go into more detail now. I have already mentioned the stabilizing effect under Random Fluctuations and Unanticipated Phenomenons. By the stabilizing effect, I am referring to the property for interest rates to tend to remain bounded for an indefinite time period. I will be discussing a few political reasons for this, later. As for physical reasons, one might assume interest rates to stay bounded as the markets "balance" themselves out. If the interest rates are unusually high, more people attempt to be loaning money (i.e. investing), with less borrowing. The supply versus demand system is now favoring the borrower. This will cause interest rates to drop. The converse is true for unusually low interest rate. Historically interest rates tend to be bounded above by 25% while bounded below by 3%.

Another general criterion for all interest rate generators is that the interest rates generated for short term should be more accurate than those generated for long term. Obviously, this is true. In order to be accurate for long-term interest rates, the model must first have been accurate for long term rates. This is of course if we ignore random fluctuations. Random fluctuations may affect a short-term rate, but then due to the stabilizing factor, have no effect on long-term rates.

As mentioned before, a yield curve is a graph of the length of the investment versus the interest rate assigned at a given time. Any interest rate generator should have parameters (perhaps defined by legal issues) that minimize the changes in the yield curve from one time period to the next. Again, ignoring random fluctuations may be sacrificed for this criterion.

As a final general criterion for all interest rate generators, I would like to mention some of the legal issues that must occur. Political actions, ranging from setting and enforcing limitations to changing extreme interest rates, can prevent hyperinflation. By hyperinflation, I am referring to extreme changes in inflation. It is in the governments' best wishes to prevent this from occurring as pandamonia has erupted in less stable economies where the governments have not inverted. Another example of possible legal issues that every generator must satisfy is regulation for reserves, a company's "rainy day" fund. An example of one of these regulations is New York Regulation 126, pertaining to any company performing business in New York State. New York Regulation 126 recommends that a company's reserves must be higher than each of the possible scenarios run under the New York 7. The New York 7 refers to seven parallel shifts. I will mention these later in the paper. Across Canada, as well as in other states, similar systems are in affect. Most companies have stricter guidelines to further protect their company from severe volatile interest rates.

Before introducing specific interest rate generators, I would like to define a few statistical terms. When describing an interest rate model, one should either use the term deterministic or stochastic.

A deterministic model is one where the underlying formulas are solely based on mathematical principles. We do not concern ourselves with unknown or uncontrolled variables, but rather with the data at hand. A stochastic model is one that handles nondeterministic situations. Also referred to as a probability model, a stochastic model takes into consideration uncontrolled parameters.

There are numerous pros and cons of looking at deterministic versus stochastic models. An obvious observation is that a stochastic model takes more variables into consideration. A deterministic model may ignore the possibility of one of those random fluctuations. The converse to this argument is that a deterministic model is easier to deal with. If one were to deal with a deterministic model, questions referring to dealing with calculating random events need not be looked at.

Another point for a stochastic model is that less data is required. Typically, stochastic models deal solely with the current interest rates. Generators that deal solely with the current interest rates are referred to as Marchov interest rate generators. Historical data need not be calculated nor stored. An excellent example of where this occurs is in the stock market. Traders normally do not concern themselves with what a stock did last week, but rather what it is doing today. Typically, deterministic models put emphasis on past interest rates.

To make a counterpoint for the use of the deterministic model, one could argue that the deterministic model has a better shape. Trends are common in interest rates, and ignoring past data points may cause errors in a stochastic model. A deterministic model will have a better understanding as far as "what is going on" and may be more suitable if an economy turns extremely volatile.

Another statistical term that I would like to define now is a random walk, also known as arithmetic Brownian motion. As its name implies, a random walk is just a random, aimless movement. A random walk can be described by the following equation:

$$W(t+1) = W(t) + e(t+1)$$

Where: $W(0) = W_0$

$e \sim \text{i.i.d. } N(0,1)$

For convenience, assume $t = 0$ to be the present. With $e(t)$ being a random variable from the normal distribution with mean zero and unit variance, the walk will be random. The probability of increasing is equally likely as the probability of decreasing. If interest rates are assumed to be unpredictable, interest rates will follow a random walk. Random walks occur in stochastic interest rate generators.

When looking at interest rate generators, one can divide them into two categories; parallel and nonparallel shifts.

A parallel shift interest rate generator is one where a set of additions is used to shift the yield curve from its previous position. An example of a deterministic parallel shift is the previously mentioned New York 7. The New York 7 are projected for 30 years and are assumed to remain constant after. They are defined as the following seven scenarios:

1) No change
2) Rising ½% per year for 10 years
3) Falling ½% per year for 10 years
4) Rising 1% per year for 5 years, falling 1% per year for the next 5 years
5) Falling 1% per year for 5 years, rising 1% per year for the next 5 years
6) Rise 3% for first year only
7) Fall 3% for first year only

While working this past year at the Great-West Life Assurance Company, I had the opportunity to look at their version of the New York 7. Great-West Life does not limit themselves to 7 possible interest rate scenarios, the New York minimum, but rather studies 20. By studying more varying scenarios, Great-West Life are protecting themselves should different situations occur.

Stochastic parallel shift interest rate generators are also looked at in practice. A random walk over a symmetric interval such as [-6,6] would produce such an event. However, violation of the bounds by any interest rate on the curve would cause that addition to be omitted.

Nonparallel shift interest rate generators change the shape of the yield curve as well as the rates at various levels. Most are based on a lognormal process, which I will talk about later. Nonparallel shift interest rate generators allow yield curves to rise or fall throughout the scenario. Random walks are usually used to generate the random numbers in nonparallel shift interest rate generators.

Due to the fact that nonparallel shift generators allow for changes to the shape of the yield curve, they are more sate after in practice. However, the simplicity of the parallel shift generator, does make it appealing, especially for reserve testing. However, due to the recent (the past decade) surges in computing power, nonparallel shift generators are used more frequently in practice as they provide for a more realistic approximation to the interest rate movements.

I would now like to mention a few of the nonparallel interest rate generators that are used in practice, stating pros and cons for each process.

<u>Lognormal Process</u>

A variable will have a lognormal distribution if the natural logarithm of the variable is normally distributed. A lognormal interest rate generator will satisfy the following equation:

$$\log(W(t+h)) = \log(W(t)) + uh + s(\sqrt{(h)})e_{t+h}$$

Where: $W(t)$ is the interest rate at time t

h is the time increment

u and s are constants

$e_{t+h} \sim$ i.i.d. $N(0,1)$

(For simplistic sake, assume $t = 0$ to be the present.)

Rearranging the above equation gives:

$$\Delta\log(W(t)) = uh + s(\sqrt{(h)})e_{t+h}$$

Which is equivalent to:.

$$\Delta[\log(W(t)) - ut] = s\sqrt{(h)})e_{t+h}$$

As e_{t+h} is assumed to be normally distributed, it implies that log(W(t)) is normal, with mean uh and variance s^2h. Therefore W(t) is lognormally distributed. To estimate the parameters u and s, one could use historical analysis. Lognormal process is the most popular of interest rate generators due to its speed and accuracy. It is a very complicated process, however, and does not work well in a very volatile, fluctuating economy.

Wiener Process

The Wiener process is a particular type of Marchov (only present values of the variable are considered), stochastic process. The Wiener process is sometimes referred to as Brownian motion. Given that W(t) is the variable as a function of t (time), W(t) will behave as a Wiener process if the following two properties are satisfied:

1) $\Delta W(t) = e\sqrt{(\Delta t)}$

   Where: $e \sim$ i.i.d. $N(0,1)$

2) $\Delta W(t)$ for any two short intervals of time Δt are independent

From the two properties above, we can conclude that $\Delta W(t)$ must have a normal distribution with mean 0 and variance Δt, and that W(t) is a Marchov process. Stock prices behave very similar to a Wiener process. Because the Wiener process is Marchov, only current interest rates are of concern.

Binomial Process

When using a binomial model for an interest rate generator, one is making the assumption that over each time period, the interest rate is either increasing or decreasing by set ratios. The increasing ratio need not be the inverse of the decreasing ratio. The probability that the interest rate will increase over a time period, rather than decrease, is chosen so that, when combined with the ratios, the interest rate movements experience a predetermined observed volatility. Binomial interest rate generators are rather simplistic, but not very effective in predicting interest rates. In addition, a large number of calculations (2^n) are needed to form an n-period projection. These calculations are very time consuming considering the other options available (Lognormal or Wiener).

With all of the models available, the lognormal process is the most widely used. It is a lot faster than the binomial and produces the best results for the three nonparallel shift interest rate generators that I have mentioned.

The development of interest rate generators is a relatively new topic in actuarial mathematics. Most of the models, especially those that are stochastic, are based heavily in computer programming. Due to this fact, these models were unattainable years ago. It has just been recently that the topic of interest rate generators has been discussed. Most of the literature, including the few texts and papers that I found, are very recent. The material that has been published has been so in the last decade. Large actuarial companies have studied interest rate generation for some time now, but are wary of publishing their findings due to the competitiveness of the market.

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