University of Manitoba
Math 38200 – Winter 2007

Assignment 1

Due Tuesday, February 20, 2007

Instructions

This assignment is due Tuesday, February 20. Note that I will accept files, so if you want to typeset or scan your answers, this is fine (pdf is preferred).

Try to be clear in your answers. If you use a result in the course notes or from another source, please indicate clearly the reference.

The Ricker model of growth of a single population takes the form

$$N_{t+1} = N_t e^{r\left(1-\frac{N_t}{K}\right)}, \tag{1}$$

with $r, K > 0$ and initial condition $N_0 > 0$. The aim of this assignment is to study the behavior of (1).

1. Define $x_t = N_t/K$, and show that the difference equation (1) then takes the form, as a function of the dimensionless variable x_t,

$$x_{t+1} = x_t e^{r(1-x_t)}, \tag{2}$$

2. Show that if $x_0 > 0$ (where $x_0 = N_0/K$), there holds that $x_t > 0$ for all t in (2).

3. Determine the fixed points of (2), as well as their stability as a function of the parameter r.

4. Show that (2) has no points of period 2 for $0 < r < 2$. [Hint: use Theorem 1 with $I = (0, \infty)$.]

5. Try to find 2-periodic points of (2) analytically (show your work). Then, do so using a numerical software. Under what conditions are these periodic points stable? Evaluate the stability of the points you found for a few sample values of r, using a numerical software.

6. Using numerical software, draw a bifurcation diagram for (2), for r varying in $(0, 5]$. What do you observe?

A useful result

Theorem 1. *Consider the function $f : I \to I$, and assume that f' is continuous on I. If $1 + f'(x) \neq 0$ for all $x \in I$, then $x_{t+1} = f(x_t)$ has no periodic point of period 2 in I.*