



University
of Manitoba

MATH 3610 – 01 – Introduction to mathematical modelling

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The University of Manitoba campuses are located on original lands of Anishinaabeg, Ininew, Anisininew, Dakota and Dene peoples, and on the National Homeland of the Red River Métis.

We respect the Treaties that were made on these territories, we acknowledge the harms and mistakes of the past, and we dedicate ourselves to move forward in partnership with Indigenous communities in a spirit of Reconciliation and collaboration.

Outline

Course description

Mathematical modelling

Math 3610 – Introduction to Mathematical Modelling

Lectures Tuesday and Thursday, 13:00–14:15 @ 415 MH

Office hours Unknown at present

Course objectives

The objective of the course is to introduce mathematical modelling, that is, the construction and analysis of mathematical models inspired by real life problems. The course will present several modelling techniques and the means to analyze the resulting systems.

Topics

Types of models (static, discrete time, continuous time, stochastic) with case studies chosen from population dynamics and other fields yet to be determined.

Evaluation

Assignments	20%
Midterm	15%
Modelling project	25%
Final examination	40%

Midterm and Final will be open book exams, calculators are not allowed

Project

- ▶ project subject must be decided before the end of February
- ▶ if you have a topic you are already working on, you are welcome to use it (but the report you produce must be specific to this course)

Mathematical modelling

- ▶ idealization of real-world problems
- ▶ try to help understand mechanisms
- ▶ never a completely accurate representation

Art vs math:

- ▶ a painting represents a model (reality)
- ▶ a mathematical model represents reality

Steps of the modelling process

- ▶ identify the most important processes governing the problem (theoretical assumptions)
- ▶ identify the state variables (quantities studied)
- ▶ identify the basic principles that govern the state variables (physical laws, interactions, ...)
- ▶ express mathematically these principles in terms of state variables (choice of formalism)
- ▶ make sure units are consistent

Steps of the modelling process (2)

Once a model is obtained

- ▶ identify and evaluate the values of parameters
- ▶ identify the type of mathematical techniques required for the analysis of the model
- ▶ conduct numerical simulations of the model
- ▶ validate the model: it must represent accurately the real process
- ▶ verify the model: it must reproduce known states of the real process

How to represent a problem

- ▶ static vs dynamic
- ▶ stochastic vs deterministic
- ▶ continuous vs discrete
- ▶ homogeneous vs detailed

Formalism

ODE, PDE, DDE, SDE, integral equations, integro-differential equations, Markov Chains, game theory, graph theory, cellular automata, L-systems . . .

Example: biological problems

- ▶ ecology (predator-prey system, populations in competition ...)
- ▶ etology
- ▶ epidemiology (propagation of infectious diseases)
- ▶ physiology (neuron, cardiac cells, muscular cells)
- ▶ immunology
- ▶ cell biology
- ▶ structural biology
- ▶ molecular biology
- ▶ genetics (spread of genes in a population)
- ▶ ...