January 8, 2007

Mathematical modeling

an idealization of the real-world problems and never a completely accurate representation

- ► Identify the most important processes governing the problem (Theoretical assumptions)
- Identify the state variables (quantities studied)
- Identify the basic principles that govern the state variables (Physical Laws, interactions, . . .)
- ► Express mathematically these principles in terms of state variables (Choice of the formalism)
- ▶ Identify and evaluate the values of parameters
- Make sure of the consistence of units

How to represent a problem

- Static vs dynamic
- Stochastic vs Deterministic
- Continous vs Discrete
- ► Homogeneous vs Detailed

Formalism

ODE, PDE, DDE, SDE, Integral equation, integro-differential equations, Markov Chains, Game theory, Graph theory, Cellular automata, L-systems . . .

Biological problems

- Ecology (Predator-Prey system, Populations in competition ...)
- Etology
- Epidemiology (Propagation of infectious diseases)
- Physiology (Neuron, cardiac cells, muscular cells)
- Immunology
- Cell biology
- Structural biology
- Molecular biology
- Genetics (Spread of genes in a population)
- **.**..

Lotka-Volterra Predator-Prey Model

Assumptions

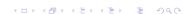
- Growth of prey population is exponential in absence of predators. Prey grow in an unlimited way when no predation.
- Predators depend on the presence of their prey to survive. Predators decline exponentially in absence of prey.
- The rate of predation depends on the likehood that a victim is encountered by a predator
- The rate of growth of the predator population is proportional to food intake.

Equations

- \triangleright x(t) biomass or population densities of the prey
- \triangleright y(t) biomass or population densities of the predators

$$\frac{dx}{dt} = ax - bxy$$

$$\frac{dy}{dt} = -cy + dxy$$



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Parameters

- ▶ a: net growth rate of the prey population when predators are absent a > 0, (1/time).
- c: net death rate of the predators in the absence of prey, c > 0, (1/time).
- ▶ b/d: efficiency of predation = efficiency of converting a unit of prey into a unit of predator mass.

