RUMOR DIFFUSION

6.337

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The spread of rumors is quite similar to the spread of disease. Rumor diffusion is spread through close contact and 'at a distance' contact, i.e., via telephones etc. The following model will describe the way a rumor travels through a fictitious group of towns in a country called Preston. There are a number of assumptions that must be made in order to proceed. The following are simplifying assumptions that help in avoiding any unnecessary complications.

The model is based on a set of towns whose only communication with each other is through a primitive telephone system. The telephone system only has the capacity to allow two individuals to talk from any two towns at once. We can further assume that each town only has one telephone. When it is said that an individual has 'heard the rumor', it means that an individual residing in a town that has heard the rumor, has since called an individual in another town that hasn't heard the rumor. The previous assumption does not allow any 'in person' rumor spreading, for we are assuming that the towns are far enough apart that they are isolated from each other. In addition we are not going to be concerned with the spread of the rumor within the town itself. When a telephone call occurs, the individual is equally likely to call each of the other N towns, and is not biased in any way to a particular town. In reality, when rumors are spread, they get more vivid and colorful as time passes, however we will assume that the rumor will stay exactly the same, when passed between individuals.

As mentioned previously the model described in this paper is largely stemmed from the model for the spread of simple epidemics. In a simple epidemic model there are various breakdowns of the population as follows:

S: The subpopulation composed of individuals who are uninfected and are susceptible to the infection.

E: The subpopulation composed of individuals who are infected, but who are not yet capable of spreading the infection.

I: The subpopulation composed of individuals who are infected and actively spreading the disease.

R: The subpopulation composed of individuals who are not susceptible or who have been infected and subsequently either cured or removed from possible contact with members of S.

The model for rumor diffusion begins here. However, we will do away with subpopulation E as it is of no relevance to our discussion. The subpopulations of rumor diffusion are as follows:

S: The set of towns who have not yet heard the rumor, but would spread it if they heard it.

I: The set of towns who have heard the rumor and are actively spreading it to other towns.

R: The set of towns who are not in either I or S. These are the set towns who have heard the rumor and are no longer interested.

There are a number of assumptions that must be made in order to model how towns move from one set to another:

If an individual from set I calls an individual from set S, then the individual from set S becomes part of the set I. Further, if and individual from set I calls an individual from set I or R, then the individual who placed the call becomes part of the set R, the called individual remains unchanged. The previous assumption states that when an individual who knows the rumor tells an individual who has already heard or who is no longer interested, the individual who made the call loses interest in the rumor. This is a parallel to the simple epidemic model, when an individual has been cured they are no longer susceptible to the disease. This is why the assumption that the rumor does not change was made for if we allowed the rumor to change, then it would be similar to the introduction of a new disease and would complicate the model tremendously. Therefore, once a town has become part of the R set, the interest in the rumor cannot be rekindled. We are

good

only interested in calls regarding the rumor, and as such calls between those sets who have not heard the rumor and those who do not care about the rumor do not affect the numbers in any set. Similarly, we are only concerned with calls originating in set I.

SUMMARY

Caller	Called	Outcome
I	S	Called becomes I
I	I or R	Caller becomes R
S or R	I	No change in either set
S	R	No change in either set
R	S	No change in either set

Once an individual is in set I, individuals in that set actively spread the rumor, until such time as they reach a individual in set I or R.

The spread of the rumor begins at time t_o. This is the time at which a rumor is introduced into one town. At some later time, that town calls another, and the spread of the rumor begins. The time t_k is the time of the last possible relevant phone call. If the population P(t) consists of all towns in Preston (the country where all towns are located) then P(t) can be written as follows

$P(t)=\{S(t),I(t),R(t)\}$

at time t_o, $P(t_o)=P_o=\{N,1,0\}$

$S_0 = N,\ I_0 = 1,\ R_0 = 0.$

At time t_1, when the first phone call is made, the individual in set I making the call has to reach an individual in set S, as there are no towns in set R at time t_o. Therefore,

$P_1=\{N-1, 2, 0)$, if we continue in this manner we will eventually arrive

at time t_{k-1}. The population at time t_{k-1} is

$P_{k-1} = \{S_{k-1}, I_{k-1}, R_{k-1}\}$, Notice at P_k there are two possibilities

1. $P_k = (S_{k-1} - 1, I_{k-1} + 1, R_{k-1})$

2. $P_k = (S_{k-1}, I_{k-1} - 1, R_{k-1} + 1)$,

The probability of getting an S town at time t_k is S_{k-1}/N and the probability of getting a I or R town at a time t_k is $(N-S_{k-1})/N$. We can write the above assertion as follows

Prob (S)=S_{k-1}/N

Prob (I or R) = $(N-S_{k-1})/N$

Therefore $S_k = S_{k-1}/N * (S_{k-1} - 1) + (N-S_{k-1})/N * S_{k-1}$

$$S_k = \frac{(S_{k-1})^2 - S_{k-1} + N S_{k-1} - (S_{k-1})^2}{N}$$

$$\mathbf{S_k = \frac{S_{k-1}(N-1)}{N}}$$ ✓ OK

explanation: like expected values

not nec. integer!

In a similar manner, we have

$I_k = (I_{k-1} + 1) * S_{k-1}/N + (I_{k-1} - 1)*(N-S_{k-1})/N$

$\mathbf{I_k = 2 S_{k-1}/N + I_{k-1} - 1}$ ✓ OK

From our initial assumptions, we know that

$S_k + I_k + R_k = N+1$

cannot be processed in parallel

We can use this equation to solve directly for R_k

$R_k = (N+1) - I_k - S_k$

$= N+1 - (2 S_{k-1}/N + I_{k-1} - 1) - (S_{k-1}(N-1)/N)$

$= N+1 - 2(N*S_{k-1}) - I_{k-1} + 1 - S_{k-1} + (S_{k-1})/N$

typo

$= N+2 - I_{k-1} + S_{k-1}[-2/N - 1 + 1/N]$

$\mathbf{R_k = N+2-I_{k-1} + S_{k-1}(-N-1)/N}$ ✓ OK

Therefore we now have:

$P_k = (S_{k-1}\ (N-1)/N,\ 2\ S_{k-1}/N + I_{k-1} - 1,\ N+2-I_{k-1} + S_{k-1}\ (-N-1)/N)$,

we would however like to have a non-recursive formula for S_k, I_k and R_k. Let us turn the discussion to that of S_k. Using our initial data and the above recursive formula we have:

$S_o = N$

$S_1 = S_o * (N-1)/N = (N-1)/N * N$

$S_2 = S_1 * (N-1)/N = [(N-1)/N]^2 * N$

$S_3 = S_2 * (N-1)/N = [(N-1)/N]^3 * N$

We can now see a pattern developing, this pattern leads us to the following non-recursive formula for S_k

no longer integer

$S_k = [(N-1)/N]^k * N$

brackets useful to avoid confusion!

Using what we now know for S_k, we have

$I_k = 2\ S_{k-1}/N + I_{k-1} - 1$

$I_k = 2\ [(N-1)/N]^{(k-1)} + I_{k-1} - 1$

The above formula is a difference equation for I_k. If we write the above formula as follows

$I_k - I_{k-1} = 2\ [(N-1)/N]^k - 1 - 1$ and write out the first few terms:

$I_1 - I_0 = 2 - 1$

$I_2 - I_1 = 2 * (N-1)/N - 1$

.

.

.

$I_p - I_{p-1} = 2[(N-1)/N]^{(p-1)} - 1$

If we then sum the above equations, many of the terms on the left hand side cancel, leaving us with:

$I_p - I_o = 2 * \sum_{k=0}^{P-1} [\, (N-1)/N \,]^k - p$

We know from our initial assumptions that $I_o = 1$, using this the above equation becomes

$I_p = 2 * \sum_{k=0}^{P-1} [\, (N-1)/N \,]^k - p + 1$, if we let $x = (N-1)/N$ then the summation in the I_p equation is

a geometric series, with the sum:

$\sum_{k=0}^{p-1} X^k = (1 - X^p)/(1 - X)$ using this in the equation for I_p we get

$(1 - X^p)/(1 - X) = (1 - [(N-1)/N]^p)/(1/N) = N[\, 1 - [(N-1)/N]^p \,]$

$I_p = 2N[\, 1 - [(N-1)/N]^p \,] - p + 1$

if $p = k$

$\mathbf{I_k = 2N[\, 1 - [(N-1)/N]^k \,] - k + 1}$

This formula is a non-recursive form of I_k.

Finding a non recursive formula for R_k is quite a simple task. Recall, from our initial assumptions that,

$S_k + I_k + R_k = N + 1$.

We have just derived the formula for S_k and I_k therefore, it is a simple matter of directly solving for R_k

$= N + 1 - I_k - S_k$

$= N + 1 - (2N[\, 1 - [(N-1)/N]^k \,] - k + 1) - ([(N-1)/N]^k * N)$

$\mathbf{R_k = -N + N[\, (N-1)/N]^k + k}$

Now that we know the formulas for S_k, I_k and R_k, we would like to see a graphical representation.

Given the following formula previously derived for S_k

$S_k = [(N-1)/N]^k * N$, it is easy to see that as k goes to infinity, $[(N-1)/N]^k$ goes to zero, i.e., lim

S_k =0 as k goes to infinity (see attached graphs) — for I_k, R_k.

What happens as k goes to infinity is not as clear. — asymptotically → 1

Given

I_k =2 N[1-[(N-1)/N]^k]-k+1 , as k increases [1-[(N-1)/N]^k] is increasing. However, for large values of k , [1-[(N-1)/N]^k] will still be increasing however, the -k in the equation will overpower the increasing portion and the function will then be decreasing. The values of I_k =2 N[1-[(N-1)/N]^k]-k+1 for small and large k as well as the resulting graph for N=100 is attached.

Not well described

We observe from the resulting graph I_k attains a maximum at some value for k.

may not be integer

To find the maximum value we set the first derivative ~~to zero~~:

d/dk [=2 N[1-[(N-1)/N]^k]-k+1]=0 Notation?

=d/dk[2N -2N(1-1/N)^k +1 -k]

=-2N(1-1/N)^k * ln (1-1/N) -1

-2N(1-1/N)^k = (1/ln(1-1/N))

(1-1/N)^k = 1/(-2N*ln(1-1/N))

$$\boxed{\frac{d\,a^u}{dx} = a^u \ln a \frac{du}{dx}}$$

k* ln (1-1/N) =ln(1/(-2N*ln(1-1/N)))

$\boxed{k=\ln(1/(-2N*\ln(1-1/N)))/\ln (1-1/N)}$ value of k (for given N) for which I_k attains a max

in the attached graph, N=100, if we substitute N=100 into the above formula for k we get

k=ln(1/(-200*ln(1-1/100)))/ln (1-1/100)

=69.46711651

This appears reasonable by looking at the graph. However, we must note that k must be an integer. Therefore, we may say that the maximum of I_k (for N=100) occurs at t_{69} or t_{70}.

The formula for R_k is as follows:

R_k = -N + N[(N-1)/N]^k +k

poorly explained again

$N[(N-1)/N]^k$ gets large as k gets large and since we never subtract k, this function is increasing, as we can see from the attached graph.

But $R_k \leq N+1$

We now have formulas and graphical representations that describe the number of towns in each of S_k, I_k and R_k for any given time t. We will now analyse how long the rumor will be in circulation. Using 'real world' experience, individuals lose interest in rumors over time and eventually arrive at a time where they are no longer discussed. The rumor will be considered dead when there are no more meaningful calls possible, or in other words, when the set I is zero. We would like to be able to find a method to determine how long the rumor stays in circulation.

empty

We define

$I_A >= 0$, the time t_A at which the rumor is either still in circulation or zero

$I_D < 0$, the time at which we may assume that the rumor is dead

$A < D$

poorly written

It is fairly obvious to realise that the larger the population, the longer that the rumor will be in circulation.

Therefore we define

$A = v*N$,

we will prove that v is some constant, but for the moments assume v(N).

We want to show that $I_A = 0$, therefore we plug A into the formula for I_k to arrive at

$0= 2N [1-(1-1/N)]^A)-A +1$

typo. error!

since A is equal to vN we write

$0= 2N [1-(1-1/N)]^vN -vN +1$ and solve for v

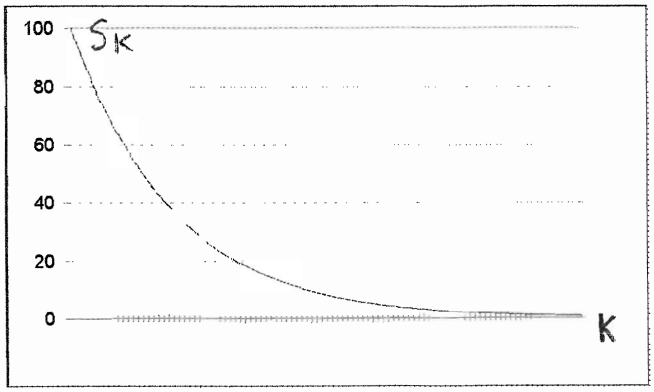
$v= 2-2[(1-1/N)^N]^v + 1/N$

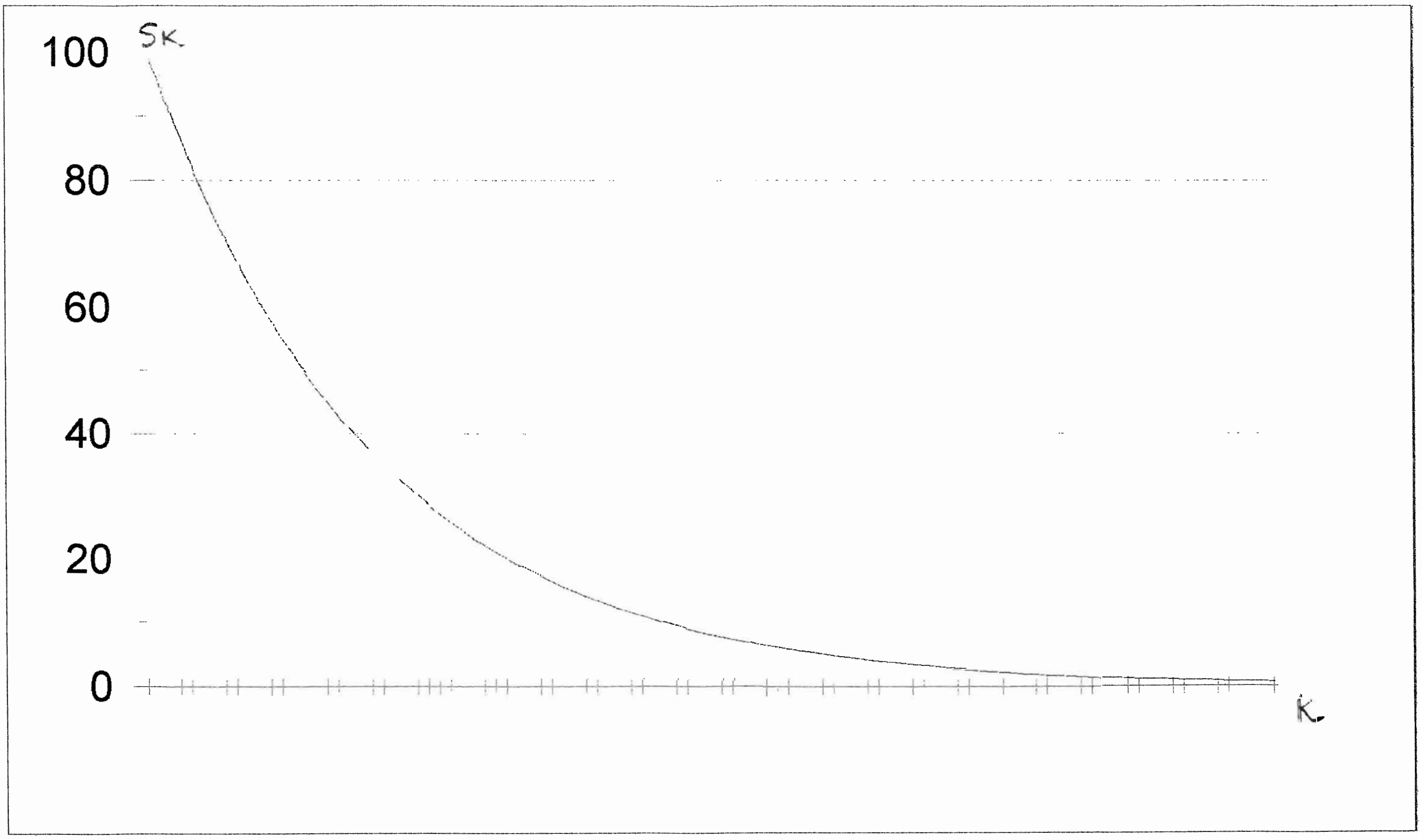
as N goes to infinity 1/N will go to zero leaving us with

This would be better as an appendix

For the graph Sk N=100
Sk= [(N-1)/N]^k *N

k	Sk
0	100
5	95.099
10	90.43821
15	86.00584
20	81.79069
25	77.78214
30	73.97004
35	70.34477
40	66.89718
45	63.61855
50	60.50061
55	57.53547
60	54.71566
65	52.03405
70	49.48387
75	47.05866
80	44.75232
85	42.55901
90	40.4732
95	38.48961
100	36.60323
105	34.80931
110	33.10331
115	31.48092
120	29.93804
125	28.47078
130	27.07543
135	25.74846
140	24.48653
145	23.28645
150	22.14518
155	21.05984
160	20.0277
165	19.04615
170	18.1127
175	17.22499
180	16.3808
185	15.57797
190	14.8145
195	14.08844
200	13.39797
205	12.74133
210	12.11688
215	11.52303
220	10.95829
225	10.42123
230	9.910482
235	9.424769
240	8.962862
245	8.523592





100
80
60
40
20
0
Sk
k

For a fixed N, the quantity
N=100
2N [1-(1-(1/N))^k)]-k-1
is increasing for small values of k, but decreasing for large values

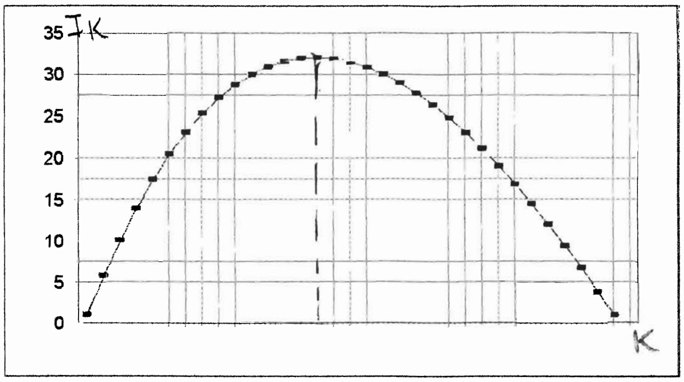
Isn't k an integer?

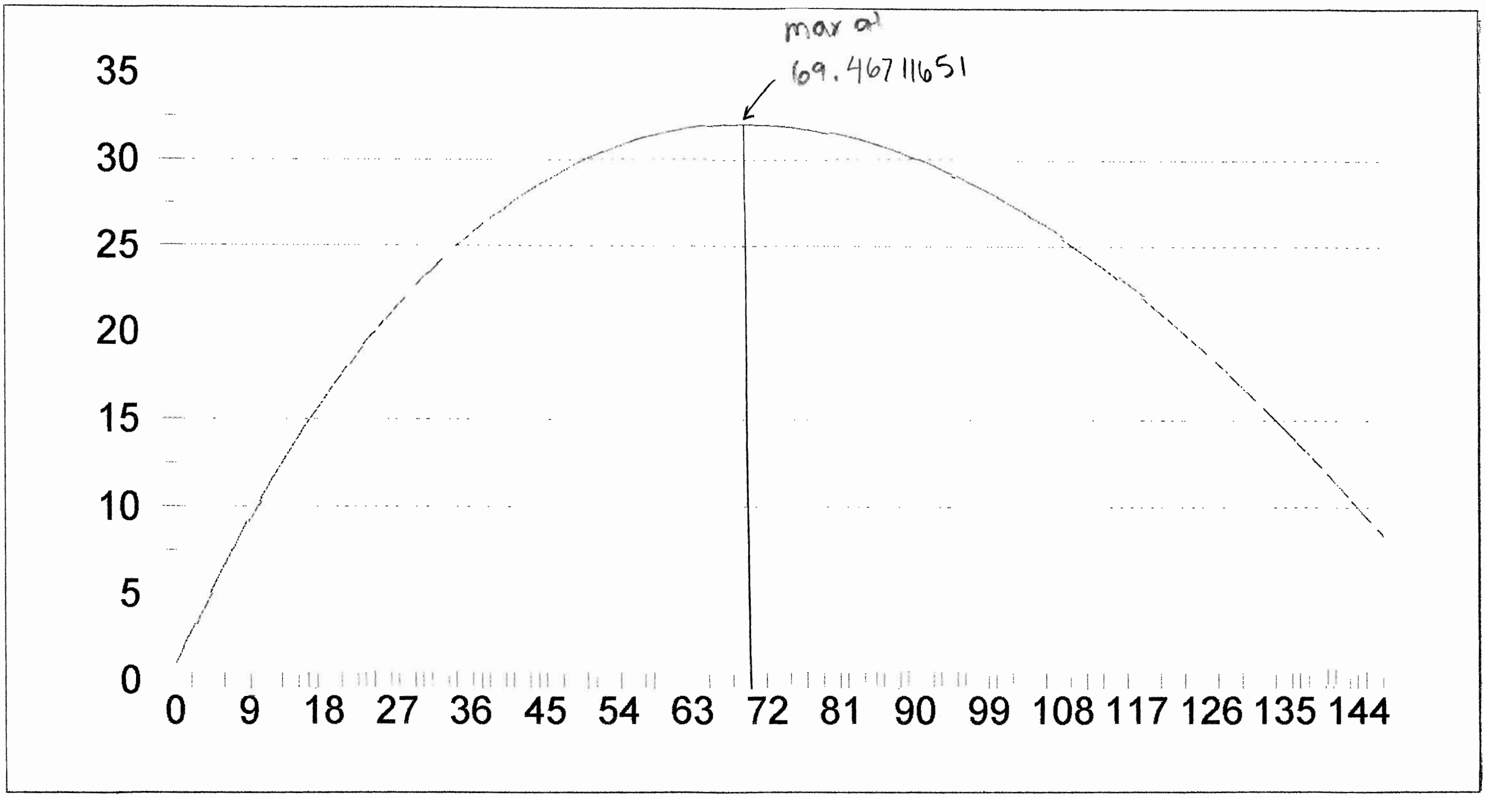
small k		large k	
0	1	100	27.79353
0.1	1.100906	101	27.5256
0.2	1.20161	102	27.25034
0.3	1.302112	103	26.96784
0.4	1.402413	104	26.67816
0.5	1.502513	105	26.38138
0.6	1.602411	106	26.07756
0.7	1.702109	107	25.76679
0.8	1.801606	108	25.44912
0.9	1.900903	109	25.12463
1	2	110	24.79338
1.1	2.098897	111	24.45545
1.2	2.197594	112	24.11089
1.3	2.296091	113	23.75979
1.4	2.394389	114	23.40219
1.5	2.492487	115	23.03817
1.6	2.590387	116	22.66778
1.7	2.688088	117	22.29111
1.8	2.78559	118	21.90819
1.9	2.882894	119	21.51911
2	2.98	120	21.12392
2.1	3.076908	121	20.72268
2.2	3.173618	122	20.31546
2.3	3.27013	123	19.9023
2.4	3.366445	124	19.48328
2.5	3.462563	125	19.05845
2.6	3.558483	126	18.62786
2.7	3.654207	127	18.19158
2.8	3.749734	128	17.74967
2.9	3.845065	129	17.30217
3	3.9402	130	16.84915
3.1	4.035139	131	16.39066
3.2	4.129881	132	15.92675
3.3	4.224429	133	15.45748
3.4	4.31878	134	14.98291
3.5	4.412937	135	14.50308
3.6	4.506898	136	14.01805
3.7	4.600665	137	13.52787
3.8	4.694237	138	13.03259
3.9	4.787615	139	12.53226
4	4.880798	140	12.02694
4.1	4.973787	141	11.51667
4.2	5.066583	142	11.0015
4.3	5.159184	143	10.48149
4.4	5.251593	144	9.956674
4.5	5.343808	145	9.427107
4.6	5.435829	146	8.892836
4.7	5.527658	147	8.353908

For the graph Ik N=100

Ik= 2N[1-[(N-1)/N]^k]-1+1

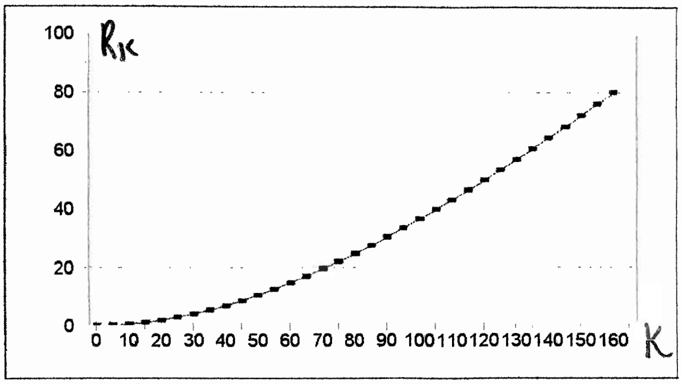
k	Ik
0	1
5	5.80199
10	10.12358
15	13.98833
20	17.41861
25	20.43573
30	23.05993
35	25.31046
40	27.20565
45	28.7629
50	29.99879
55	30.92905
60	31.56867
65	31.9319
70	32.03227
75	31.88267
80	31.49536
85	30.88198
90	30.05361
95	29.02078
100	27.79353
105	26.38138
110	24.79338
115	23.03817
120	21.12392
125	19.05845
130	16.84915
135	14.50308
140	12.02694
145	9.427107
150	6.709643
155	3.880311
160	0.944595

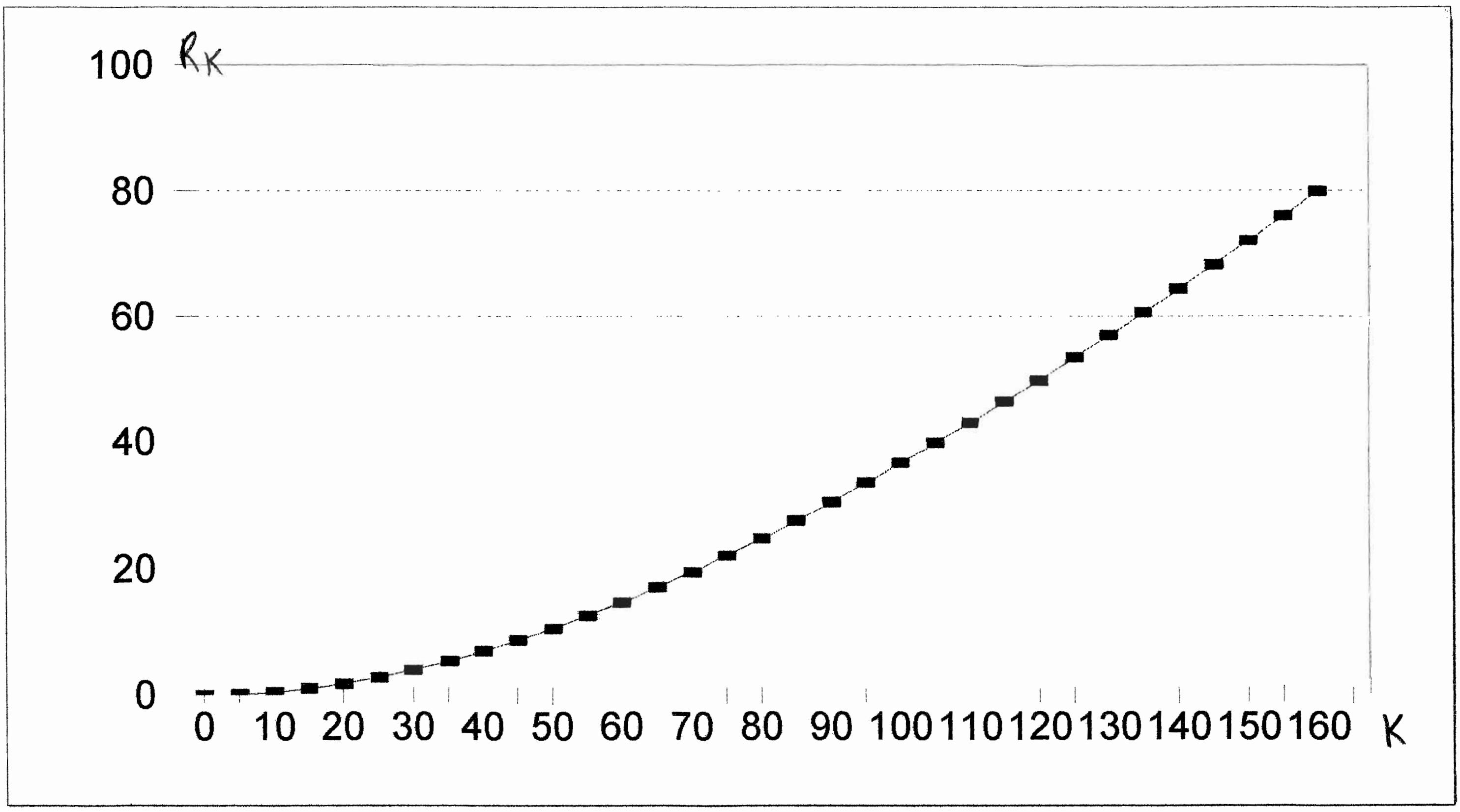



max at
69.46711651
35
30
25
20
15
10
5
0
0
9
18
27
36
45
54
63
72
81
90
99
108
117
126
135
144

For the graph Ik N=100
Rk

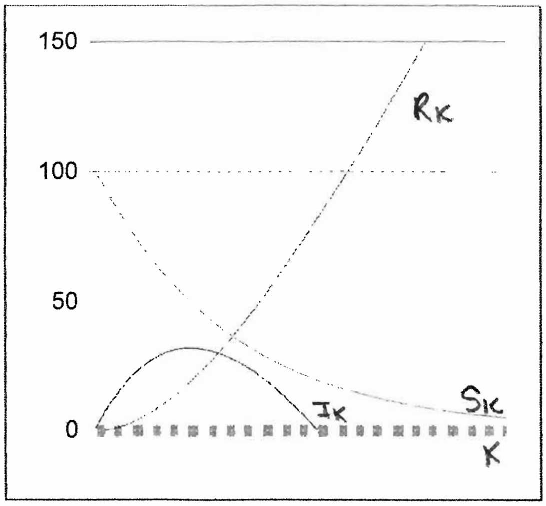
k	Rk
0	0
5	0.099005
10	0.438208
15	1.005835
20	1.790694
25	2.782136
30	3.970037
35	5.344769
40	6.897176
45	8.618549
50	10.50061
55	12.53547
60	14.71566
65	17.03405
70	19.48387
75	22.05866
80	24.75232
85	27.55901
90	30.4732
95	33.48961
100	36.60323
105	39.80931
110	43.10331
115	46.48092
120	49.93804
125	53.47078
130	57.07543
135	60.74846
140	64.48653
145	68.28645
150	72.14518
155	76.05984
160	80.0277

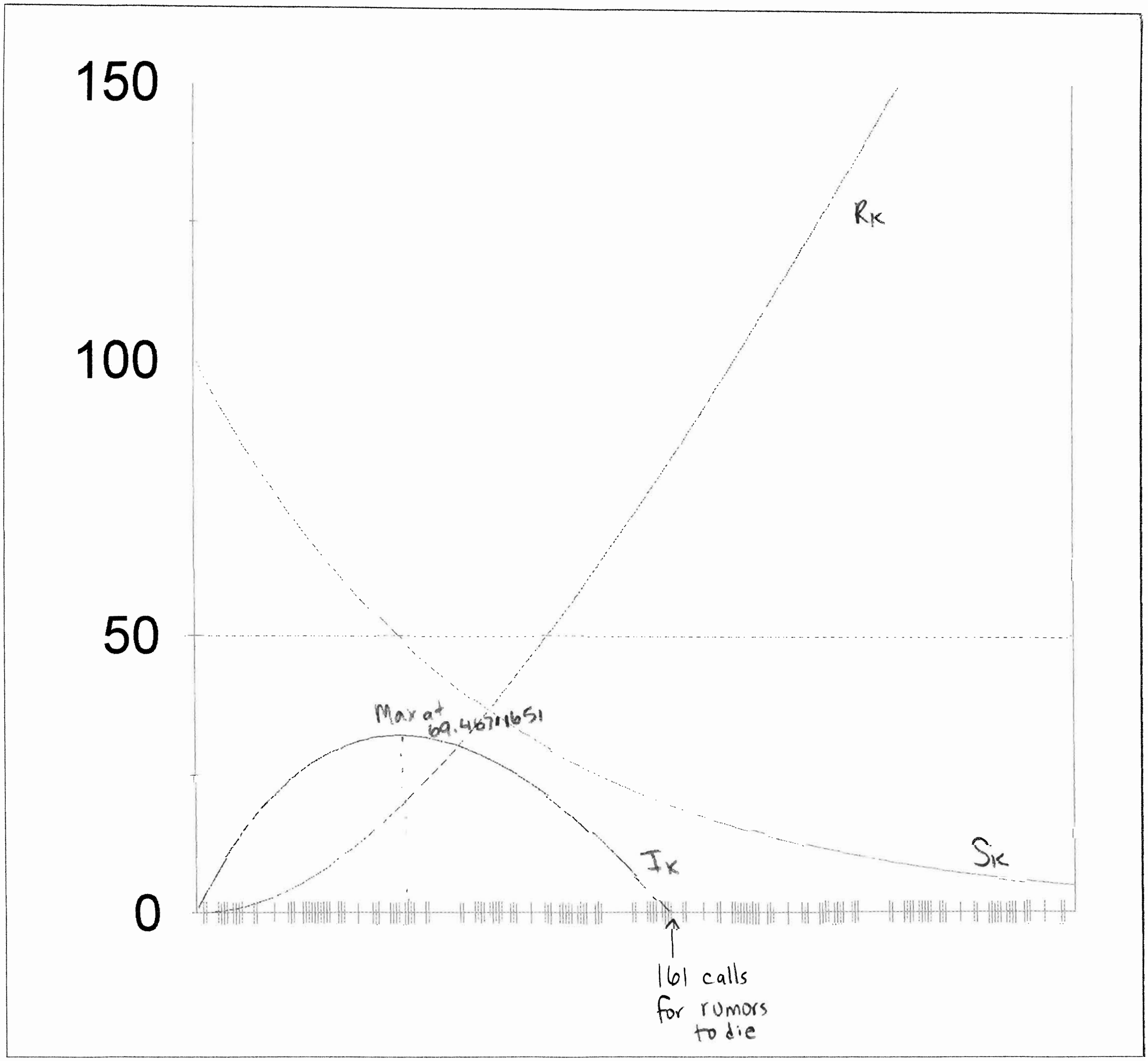




R_K
100
80
60
40
20
0
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160
K

k	Ik	Sk	Rk
0	1	100	0
1	2	99	0
2	2.98	98.01	0.01
3	3.9402	97.0299	0.0299
4	4.880798	96.0596	0.059601
5	5.80199	95.099	0.099005
6	6.70397	94.14801	0.148015
7	7.58693	93.20653	0.206535
8	8.451061	92.27447	0.274469
9	9.296551	91.35172	0.351725
10	10.12358	90.43821	0.438208
11	10.93235	89.53383	0.533825
12	11.72303	88.63849	0.638487
13	12.4958	87.7521	0.752102
14	13.25084	86.87458	0.874581
15	13.98833	86.00584	1.005835
16	14.70845	85.14578	1.145777
17	15.41136	84.29432	1.294319
18	16.09725	83.45138	1.451376
19	16.76628	82.61686	1.616862
20	17.41861	81.79069	1.790694
21	18.05443	80.97279	1.972787
22	18.67388	80.16306	2.163059
23	19.27714	79.36143	2.361428
24	19.86437	78.56781	2.567814
25	20.43573	77.78214	2.782136
26	20.99137	77.00431	3.004315
27	21.53146	76.23427	3.234271
28	22.05614	75.47193	3.471929
29	22.56558	74.71721	3.717209
30	23.05993	73.97004	3.970037
31	23.53933	73.23034	4.230337
32	24.00393	72.49803	4.498034
33	24.45389	71.77305	4.773053
34	24.88935	71.05532	5.055323
35	25.31046	70.34477	5.344769
36	25.71736	69.64132	5.641322
37	26.11018	68.94491	5.944909
38	26.48908	68.25546	6.25546
39	26.85419	67.5729	6.572905
40	27.20565	66.89718	6.897176
41	27.54359	66.2282	7.228204
42	27.86816	65.56592	7.565922
43	28.17947	64.91026	7.910263
44	28.47768	64.26116	8.26116
45	28.7629	63.61855	8.618549
46	29.03527	62.98236	8.982363
47	29.29492	62.35254	9.352539
48	29.54197	61.72901	9.729014
49	29.77655	61.11172	10.11172
50	29.99879	60.50061	10.50061
51	30.2088	59.8956	10.8956




150
100
50
0
R_K
Max at 69.4671651
I_K
S_K
161 calls for rumors to die

v= 2-2[(1-1/N)^N]^v sloppy No!

as N goes to infinity the value (1-1/N)^N approaches e^-1, therefore we write

v=2-2e^-v. This particular formula is a transcendental equation, and has the solution v*

v*=2-2e^-v*.

Therefore A is equal to v*N. If we make the following assumption

f(v*)=v*=2-2e^-v*, we may graph both of these functions on the same graph and where they

intersect if the solution v* (see attached graphs). From the graphs, we can approximate v* to be

1.5. Graphical methods are good for approximations but sometimes falter in accuracy. Therefore,

we will use Newtons' method to determine a more accurate v*.

Let g = v-f

d/dv g = 1-2e^-v

d^2 / dv^2 = 2 e^-v = h(v)

From the graph we determine that v should lie in (0, 5)

$v_{n+1} = v_n$ - (g)/(h)

$v_1 = 2$

v_2 = 2 - g(2)/h(2)

=2-{ [2-(2-2e^-2)]/ (1-2e^-2)}

=1.62887749

v_3 = 2 - g(1.62887749)/h(1.62887749)

=1.62887749 - { [1.62887749-{2-2e^-1.62887749}]/(1-2e^-1.62887749)}

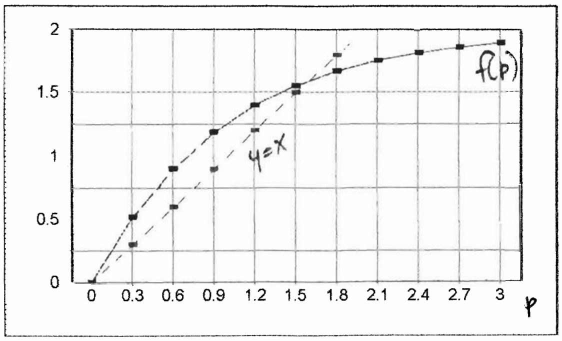
=1.59403016

v_4 = 2 - g(1.59403015)/h(1.59403016)

=1.59403016- [1.59403016-{2-2e^-1.59403016}]/(1-2e^-1.59403016)}

Finding the correct value of p

p	f(p)	y=x
0	0	0
0.3	0.518364	0.3
0.6	0.902377	0.6
0.9	1.186861	0.9
1.2	1.397612	1.2
1.5	1.55374	1.5
1.8	1.669402	1.8
2.1	1.755087	2.1
2.4	1.818564	2.4
2.7	1.865589	2.7
3	1.900426	3
3.3	1.926234	3.3
3.6	1.945353	3.6
3.9	1.959516	3.9
4.2	1.970009	4.2
4.5	1.977782	4.5
4.8	1.983541	4.8
5.1	1.987807	5.1
5.4	1.990967	5.4
5.7	1.993308	5.7
6	1.995042	6
6.3	1.996327	6.3
6.6	1.997279	6.6
6.9	1.997984	6.9
7.2	1.998507	7.2
7.5	1.998894	7.5
7.8	1.999181	7.8
8.1	1.999393	8.1



$=1.59362431$

Further iterations converge closer and closer to 1.594 (to three decimals) for large N

Newtons Method has given us a more accurate approximation to v*. We now know that g(v) is approximately equal to zero when v*= 1.594. We now can assume that for large values of N, the time t_A when the I set is equal to zero is proportional to 1.594 N. This result also is consistent with our assumption that v will be a constant.

If we return to our original notation, t_0 is the time when the rumor is first introduced. t_1 is the time of the fist phone call, t_2 is the time of the second phone call. Similarly t_A is the time of the Ath phone call, which is the last relevant phone call. We now know from the above analysis that A=1.594N. This result tells us that the rumor will die after there have been 1.594 N phone calls. In terms of time, the rumor dies at $t_{1.594N}$. We must note here that A must be an integer, therefore, we assume A to be the largest integer which is no larger than 1.594 N.

In addition to how long the rumor will be in circulation, there is the question of how many towns will not ever hear the rumor. This question is easily solved by finding out how may towns are in S when the rumor dies. We know from our above analysis that this occurs at t_A. Therefore, all we need is S_A.

A=1.594 N (for large N)

$S_A = S_{(v^*)N}$=N[(N-1)/N]^v*N=N{[(N-1)/N]^N}^v*

Again we assume N is large and that the value [(N-1)/N]^N goes to e^-1. Therefore, we can write

S_{v^*N}=Ne^-v*

=Ne^-1.594

=0.203 N

the above result tells us that when the $t = t_A$, the set I is empty and the set S contains 0.203 N.

This is the same as saying that approximately 20% of the initial population will have not yet heard the rumor when it dies.

Similarly, we would expect that if at time t_A, the set I is empty and the set S is 20% of N, then the set R would be about 80% of the initial N. A similar approach to that for finding S_A will be used to show that this is precisely so.

$R_A = R_{v*N} = N* \{[(N-1)/N]^{N}\}^{v*} - N + v*N$

if we assume that as N gets quite large $[(N-1)/N]^N$ will go to e^{-1}

$= Ne^{-v*} + N (v*-1)$

$= 0.203N + 0.594 N$

$= 0.797 N$

This is exactly what we had anticipated. At time t_A, the set I is empty, S is 20% the initial N and the set of towns who do not care about the rumor is 80% of the initial N.

There are many factors that could have been incorporated into this model, such as

I) more than one telephone per town

ii) rumor diffusion within the town itself in addition to intertown communication

iii) allowing the rumor to change

iv) close range communication in addition to telephone conversation

Rumor diffusion has the possibility to become very complicated. However, it describes what we would innately expect to occur with no surprising results. For instance, we would expect the number of people who haven't heard the rumor to decrease as t increases. In a similar fashion we expect the number of people telling the rumor to increase for a certain time and then begin to decrease as people lose interest. We also saw that the time it takes for the rumor to die is progressively longer as the initial population N gets larger. This can be seen from the attached graphs showing the I_k and S_k for varying values of N This model also described that not every town will hear the rumor, roughly 20% in our model, his however would possibly change given different initial assumptions. In a similar approach, the model told us that 80% of the initial population N, will have heard the rumor and no longer care about it. This result is quite typical of a ‘real world’ occurance of a rumor spreading through an office, some people keep to themselves and would never come upon such information, while the majority of us would be right in there spreading the juicy gossip. This model for rumor diffusion used many different ideas from a number of different models, more specifically the SIR model for simple epidemics. It is apparent that models are not always unique. They build on the discoveries and conclusions of previous models. It is truly fascinating however to use mathematics to describe such a social occurance as a rumor. This project has convinced me that Mathematical Modelling is indeed an art form and should not be overlooked as a truly remarkable descriptive tool.

Summary Table

N	M	Im	A
10	7	4.4	18
50	30	16	80
100	69	32	161
500	300	152	800
1000	750	306	1600

Where:

N The number of towns in set S initially

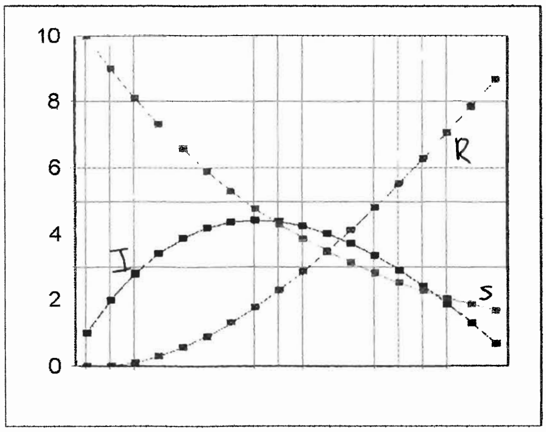
M The number of telephone calls made when the maximum number of towns telling the number

Im The highest number of towns telling the rumor at any given time

A The number of phone calls it takes for the rumor to die

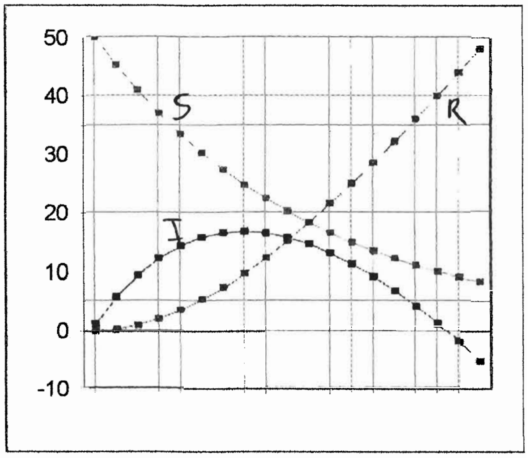
N=10

k	Ik	Sk	Rk
0	1	10	0
1	2	9	0
2	2.8	8.1	0.1
3	3.42	7.29	0.29
4	3.878	6.561	0.561
5	4.1902	5.9049	0.9049
6	4.37118	5.31441	1.31441
7	4.434062	4.782969	1.782969
8	4.390656	4.304672	2.304672
9	4.25159	3.874205	2.874205
10	4.026431	3.486784	3.486784
11	3.723788	3.138106	4.138106
12	3.351409	2.824295	4.824295
13	2.916268	2.541866	5.541866
14	2.424642	2.287679	6.287679
15	1.882177	2.058911	7.058911
16	1.29396	1.85302	7.85302
17	0.664564	1.667718	8.667718



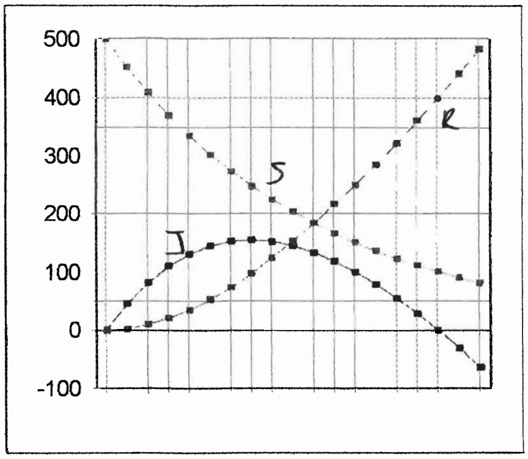
N=50

k	Ik	Sk	Rk
0	1	50	0
5	5.60792	45.19604	0.19604
10	9.292719	40.85364	0.85364
15	12.14309	36.92846	1.928455
20	14.2392	33.3804	3.380399
25	15.65353	30.17324	5.173236
30	16.45157	27.27422	7.274216
35	16.69254	24.65373	9.653731
40	16.42996	22.28502	12.28502
45	15.71221	20.14389	15.14389
50	14.58303	18.20848	18.20848
55	13.08195	16.45903	21.45903
60	11.24469	14.87766	24.87766
65	9.103553	13.44822	28.44822
70	6.687742	12.15613	32.15613
75	4.023644	10.98818	35.98818
80	1.135115	9.932443	39.93244
85	-1.95628	8.978141	43.97814
90	-5.23106	8.115529	48.11553



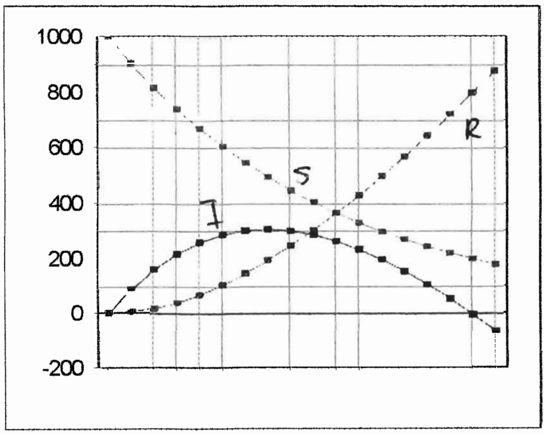
N=500

k	Ik	Sk	Rk
0	1	500	0
50	46.25318	452.3734	2.373409
100	82.4332	409.2834	9.283402
150	110.4043	370.2979	20.29786
200	130.9484	335.0258	35.02581
250	144.7729	303.1135	53.11353
300	152.518	274.241	74.241
350	154.7626	248.1187	98.11868
400	152.0308	224.4846	124.4846
450	144.7966	203.1017	153.1017
500	133.4887	183.7556	183.7556
550	118.4954	166.2523	216.2523
600	100.1675	150.4163	250.4163
650	78.82274	136.0886	286.0886
700	54.74849	123.1258	323.1258
750	28.20473	111.3976	361.3976
800	-0.57331	100.7867	400.7867
850	-31.3728	91.18641	441.1864
900	-64.0012	82.50061	482.5006



N=1000

k	Ik	Sk	Rk
0	1	1000	0
100	91.41571	904.7921	4.792147
200	163.7023	818.6488	18.64883
300	219.5859	740.707	40.70703
400	260.6282	670.1859	70.18591
500	288.2421	606.3789	106.3789
600	303.7062	548.6469	148.6469
700	308.1772	496.4114	196.4114
800	302.7017	449.1491	249.1491
900	288.2268	406.3866	306.3866
1000	265.6092	367.6954	367.6954
1100	235.6241	332.6879	432.6879
1200	198.9731	301.0134	501.0134
1300	156.2908	272.3546	572.3546
1400	108.1514	246.4243	646.4243
1500	55.07447	222.9628	722.9628
1600	-2.46992	201.735	801.735
1700	-64.0564	182.5282	882.5282



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