Modelling The Motion Of A Paraglider

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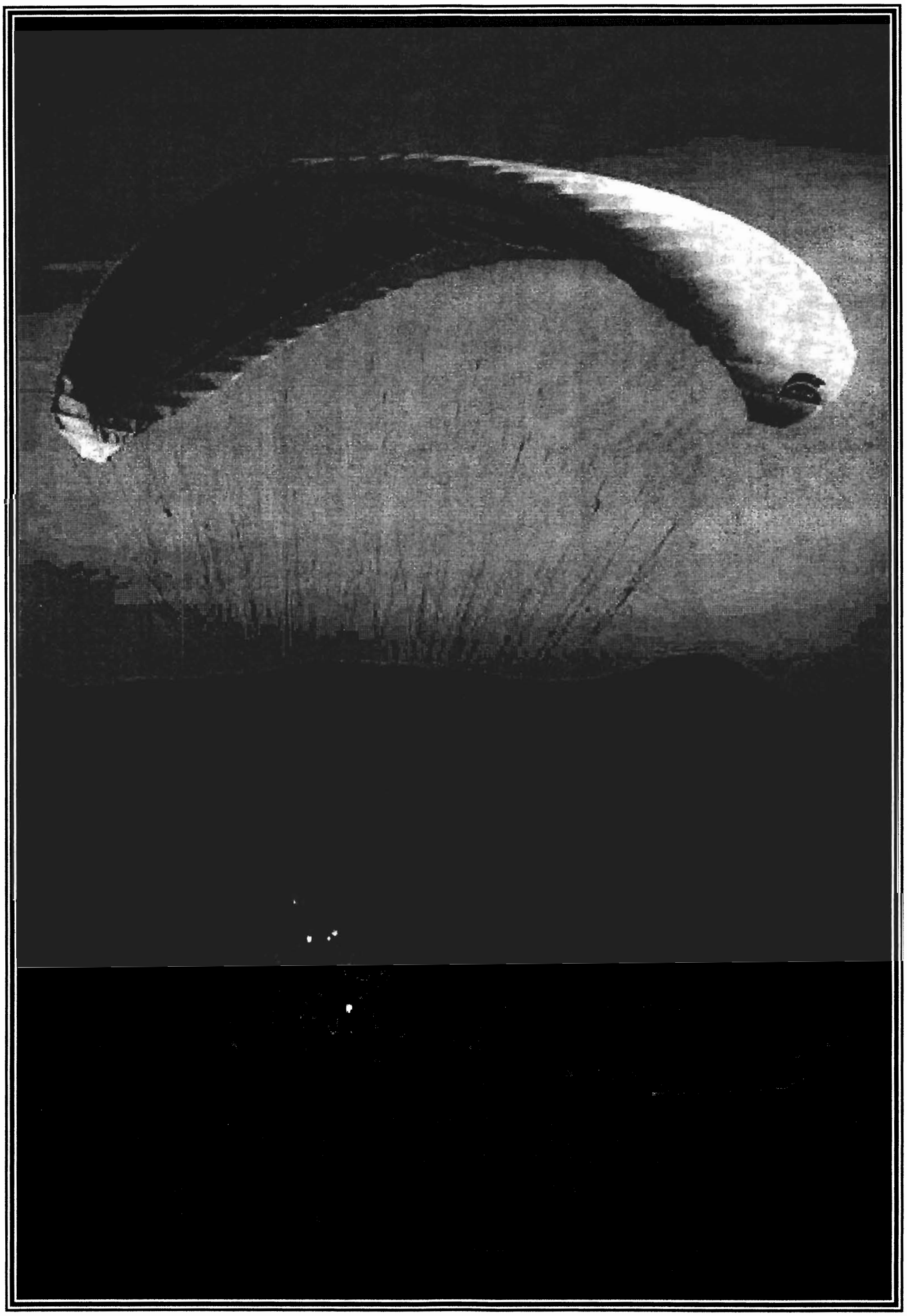

Introduction

The concept of the paraglider is simple, a textile wing, inflated by its own movement through the air, joined to a number of lines supporting the pilots seat. Paragliders are aircraft that fly by using exactly the same aerodynamic effects that keep jet planes in the air. By examining some of the key forces that cause paragliders to move through the air, I hope to develop a mathematical model that will describe this motion.

Problem

The situation I wish to model is the following: A piloted paraglider, which is initially some distance from the surface of the Earth, is launched horizontally; sometime later, the paraglider lands on the surface of the Earth.

Identification of Variables

The situation that was just introduced suggests a model that describes some kind of relationship between distance and time. In this case, the paraglider will travel for a time $\overset{T}{t}$, some horizontal distance $\overset{X}{x}$, and some vertical distance $\overset{Y}{y}$, before touching down on the Earth's surface. Since distance and time are familiar concepts that can be readily measured, I will seek to develop a model in terms of these quantities.

T, X, Y totals
intermediates
t, x, y

The variable x will be measured in meters from the horizontal point of launch, which I will denote x_0. It is assumed that forward motion is positive, and backward motion is negative.

The variable y will also be measured in meters from the vertical point of launch, which I will denote y_0. It is assumed that downward motion is negative, and upward motion is positive.

The variable t will be measured in seconds from the time of launch, which will be denoted t_0.

Mathematical Analysis

Now that I have identified the variables upon which my model will be based, I can proceed with the mathematical development of my model. During this analysis, new parameters will be introduced and explained as necessary.

I begin by restating my problem in a more mathematical context:

At time $t = t_0$ a paraglider is launched from a point (x_0, y_0) in the x-direction. At time $\overset{T}{\cancel{t}} - t_0$ the paraglider touches down on the surface of the Earth at the point $(\overset{X}{x - x_0}, \overset{Y}{y - y_0})$.

It is clear that x and y vary with time. To describe the motion of the paraglider for its entire flight, I need to measure where the paraglider will be at any given time during its flight. Therefore, the goal of my model will be to determine x and y as a function of time or simply, $x(t)$ and $y(t)$. Knowing $x(t)$ and $y(t)$ will allow me to predict the position of the paraglider at any time during its flight, hence describing its motion.

To develop a reasonably accurate model of a paragliders flight, I need to be aware that there are a number of different forces acting on the paraglider, which ultimately determine its motion. As in most mathematical models, we generally do not account for all the variables involved in the "real world" case. We attempt to keep the complexity of the mathematics to a minimum, at least in the early stages of the modelling process. This helps to ensure that we can produce a model that will mimic some of the behavior occurring in the "real world" case. The motion of a paraglider is no exception. In order to make this problem manageable, I need to make some basic assumptions about my model.

Assumptions

- The only forces that the paraglider will experience will be the force of gravity, a "drag" force and a "lift" force. These forces will be discussed in greater detail as they are encountered.

- The paraglider and pilot are assumed to ~~be~~ have a single, constant mass M. When I refer to the paraglider, I will be referring to the main wing along with the pilot, the harness, and all the other components. My goal is to treat the paraglider as a projectile with just three forces acting on it.

- Paragliders have the ability to change their shape, a process known as “braking”. In order to keep my model simple, I will assume that the shape of the paraglider is constant during its flight.

- My model will not be able to describe any meteorological phenomenon. For example, it will not be able to predict motion due to wind, thermals, or turbulence. still air

With these assumptions stated, I can now proceed in analyzing the motion of the paraglider.

Paraglider Motion Under Gravity Alone

As stated in my assumptions, I will be concerned with three key forces. The most fundamental of these forces is gravity. Gravity plays the biggest role in the motion of the paraglider. Since a paraglider has no engine, it relies on gravity to keep it moving. My first preliminary model will assume that gravity is the only force acting on the paraglider, like a projectile in a vacuum. This is the simplest

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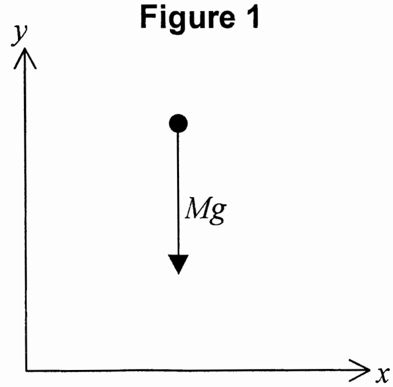
scenario whose model aims to describe the role gravity plays in the overall motion of the paraglider.

For motion in space, Newton's 2nd Law (F=ma) yields the following:

Figure 1



$$M\frac{d^2\mathbf{r}}{dt^2} = M\mathbf{g} \quad (1)$$

or

$$\frac{d^2\mathbf{r}}{dt^2} = -g\hat{\mathbf{j}} \quad (2)$$

where $\mathbf{r}$ is the position vector with respect to a fixed origin on the Earth's surface and g is the gravitational constant equal to 9.8 m/s^2.

If at $t_0 = 0$ the paraglider is travelling at a speed of v_0 at an angle α to the horizontal, then the initial velocity vector is:

$$\mathbf{v}_0 = v_0 cos(\alpha)\hat{\mathbf{i}} + v_0 sin(\alpha)\hat{\mathbf{j}} \quad (3)$$

where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors in the horizontal and vertical directions respectively.

The problem indicates that the paraglider is launched horizontally, therefore $\alpha = 0$ and (3) becomes:

$$\mathbf{v}_0 = v_0\hat{\mathbf{i}} \quad (4)$$

Rewriting (2) in component form I obtain:

$$\frac{d^2x}{dt^2}\hat{\mathbf{i}} + \frac{d^2y}{dt^2}\hat{\mathbf{j}} = -g\hat{\mathbf{j}} \qquad (5)$$

Therefore my over-simplified model becomes:

$$\frac{d^2x}{dt^2} = 0 \qquad x(0) = x_0 \text{ , } x'(0) = v_0$$

$$(6)$$

$$\frac{d^2y}{dt^2} = -g \qquad y(0) = y_0 \text{ , } y'(0) = 0$$

Solving for $x(t)$ I obtain:

$$\frac{dx}{dt} = x'(t) = c_1$$

$$x(t) = c_1 t + c_2$$
$$x'(0) = c_1 = v_0$$
$$x(0) = c_2 = x_0$$

$$x(t) = v_0 t + x_0 \qquad (7)$$

Solving for $y(t)$ I obtain:

$$\frac{dy}{dt} = y'(t) = -gt + c_1$$

$$y(t) = -\frac{1}{2}gt^2 + c_1 t + c_2$$
$$y'(0) = c_1 = 0$$
$$y(0) = c_2 = y_0$$

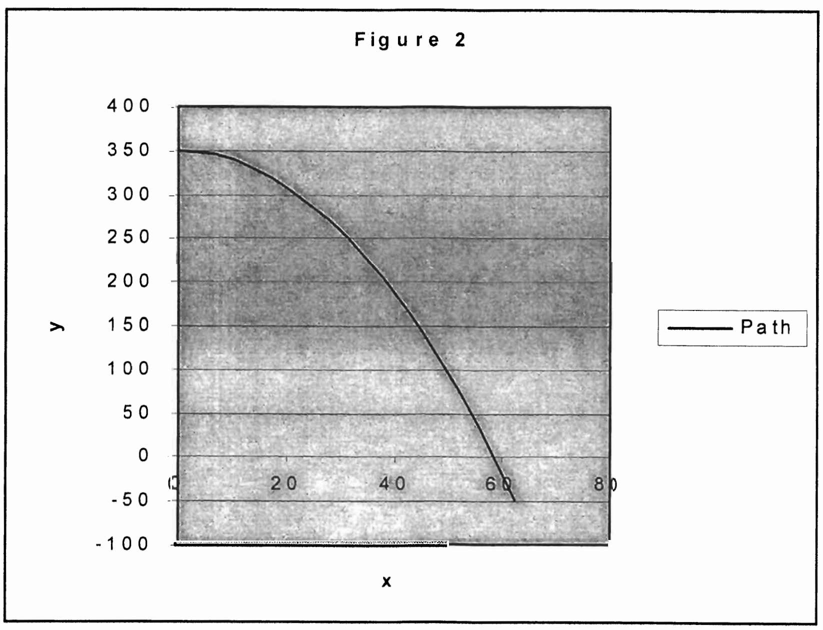
$$y(t) = -\frac{1}{2}gt^2 + y_0 \qquad (8)$$

(7) and (8) define a parametric solution to (6). To further analyze the paragliders gravitational motion, I will suggest a possible “real world” paragliding scenario, which will allow me to assign values to the parameters x_0, y_0 and v_0. From here forward I will assume that the launch site is located 350 meters above the surface of the Earth and the horizontal launch position x_0 is equal to zero. I will also assume an initial speed of 25 km/h or 6.94 m/s and the time of launch will always be equal to zero.

Applying the above information, (7) and (8) become:

$$\begin{aligned} x(t) &= 6.94t \\ y(t) &= -4.9t^2 + 350 \end{aligned} \qquad (9)$$

Figure 2 is a graph of y vs x which illustrates the paragliders path of decent.



We know that at the surface of the Earth $y = 0$. Knowing this, I can determine the total flight time T as follows:

$$T = \sqrt{\frac{350}{4.9}} \approx 8.45s$$

The horizontal landing point can now be computed:

$$x(T) = 6.94(8.45) \approx 58.64m$$

Therefore, in about 9 seconds, the paraglider descends 350 meters while moving forward about 60 meters.

While this model may do an excellent job at predicting a paragliders flight in a vacuum, it does not do a good job in the case of "real world" motion. However, this over-simplified model provides a foundation on which I can develop a more realistic model. It introduces Newton's 2nd Law of motion, major parameter values, as well as the mathematical notation that I will use as I proceed.

No lift

Motion With Air Resistance (Drag)

The motion that was previously modelled described motion under a constant gravitational force inside a vacuum. In order to develop a more realistic version of this model, I must take into account that on Earth, falling bodies must contend with the resistive force of the air through which they fall. For example, when a feather and a pin are dropped from equal heights, the pin hits the ground first, while the feather gently floats to the ground. This slowing down of the feather's motion is due to the retarding force of the air. This force acts in a direction that is opposite to the direction of speed and is called the "drag force". In several problems involving the motion of falling bodies, the force of drag is not linear with respect to the speed of the body, but usually assumes a more complicated form. Many experiments have recognized that the drag force $\vec{D}$ associated with aircraft, including paragliders, is directly proportional to the square of the speed of the aircraft at any given time. Therefore we express $\vec{D}$ as:

good

$\vec{D}$

vector

$$\vec{D} = -Kv^2\hat{\mathbf{v}} \qquad (10)$$

where $\hat{\mathbf{v}} = cos(\alpha)\hat{\mathbf{i}} + sin(\alpha)\hat{\mathbf{j}}$ is the vector in the direction of speed, and K is a constant that depends on the density of air ρ, the cross-sectional area of the body A, and a non-dimensional "drag coefficient" C_D. In fact, it has been determined that

$$K = \frac{\rho}{2}AC_D \qquad (11)$$

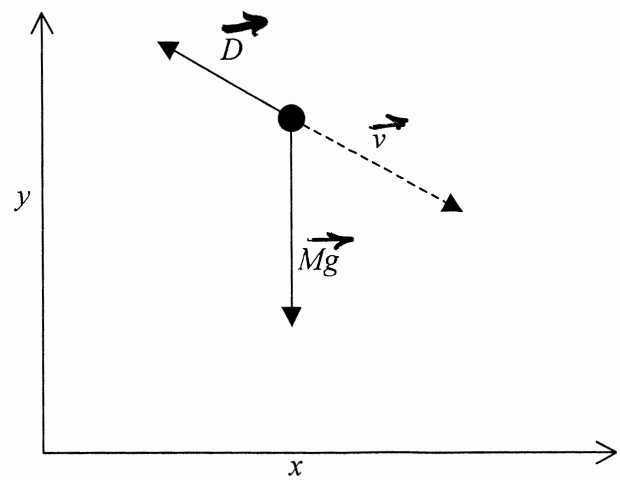
reference?

or

$$\vec{D} = -\frac{\rho}{2}AC_Dv^2\hat{\mathbf{v}} \qquad (12)$$

Now that the force of drag has been formally introduced, the following force diagram may be considered:

Figure 3



Newton's 2nd Law now suggests the following model:

$\vec{g} = -g\hat{j}$

$$M\frac{d^2\mathbf{r}}{dt^2} = M\mathbf{g} - Kv^2\hat{\mathbf{v}} \qquad (13)$$

or

$$\frac{d^2\mathbf{r}}{dt^2} = -g\hat{\mathbf{j}} - kv^2\hat{\mathbf{v}} \qquad (14)$$

where $k = \dfrac{K}{M}$

This new model looks similar to the gravitational model except for the second term, which takes into account the drag force due to air resistance.

Rewriting (14) in its component form I obtain:

$$\frac{d^2x}{dt^2} = -kv^2cos(\alpha)$$

$$\frac{d^2y}{dt^2} = -g - kv^2sin(\alpha) \qquad (15)$$

If I let $V_x = \frac{dx}{dt}$ denote the instantaneous velocity component in the x direction and $V_y = \frac{dy}{dt}$ denote the instantaneous velocity component in the y direction, then the speed v can be shown to be:

$$v = \left(V_x{}^2 + V_y{}^2\right)^{\frac{1}{2}} = \left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right)^{\frac{1}{2}} \qquad (16)$$

also, since $\frac{dx}{dt} = v[cos(\alpha)]$ and $\frac{dy}{dt} = v[sin(\alpha)]$ then $cos(\alpha)$ and $sin(\alpha)$ can be written as:

$$cos(\alpha) = \frac{dx}{dt}\left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right)^{-\frac{1}{2}}$$

$$sin(\alpha) = \frac{dy}{dt}\left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right)^{-\frac{1}{2}} \qquad (17)$$

Substituting (16) and (17) into (15) I obtain:

$$\frac{d^2x}{dt^2} = -k\frac{dx}{dt}\left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right)^{\frac{1}{2}}$$

$$\frac{d^2y}{dt^2} = -g - k\frac{dy}{dt}\left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right)^{\frac{1}{2}} \qquad (18)$$

Since the (18) consists of a system of coupled, highly non-linear differential equations, it would be difficult, if not impossible to solve using purely analytical techniques, as was achieved in the gravitational model. However, (18) can be rewritten as:

good

$$\frac{dV_x}{dt} = -kV_x\left(V_x^{\,2} + V_y^{\,2}\right)^{\frac{1}{2}}$$

$$\frac{dV_y}{dt} = -g - kV_y\left(V_x^{\,2} + V_y^{\,2}\right)^{\frac{1}{2}} \qquad (19)$$

which is a first order system of differential equations that can be solved using Runge-Kutta numerical methods. Once a solution to (19) is computed, it can be used to obtain an approximation to $x(t)$ and $y(t)$.

To solve (19) we first need a value for $k = \frac{K}{M} = \frac{\rho A}{2M}C_D$ as well as some initial conditions.

For a standard paraglider, the horizontal and vertical cross-sectional areas vary considerably. With respect to the horizontal plane we get $A \approx 28m^2$ and with respect to the vertical plane we get $A \approx 2m^2$. Also, it can be shown that $\rho \approx 1.17kgm^{-3}$, $C_D \approx 0.8$, and $M \approx 80kg$. Therefore, two k values must be computed. $k_x \approx 0.0117m^{-1}$ and $k_y \approx 0.1638m^{-1}$.

Since at $t = 0$ the horizontal velocity is 6.94 m/s and the vertical velocity is zero, then the initial conditions for (19) are $V_x(0) = 6.94$ and $V_y(0) = 0$. Therefore (19) becomes:

$$\frac{dV_x}{dt} = -0.0117V_x\left(V_x^{\,2} + V_y^{\,2}\right)^{\frac{1}{2}} \qquad V_x(0) = 6.94$$

$$\frac{dV_y}{dt} = -9.8 - 0.1638V_y\left(V_x^{\,2} + V_y^{\,2}\right)^{\frac{1}{2}} \qquad V_y(0) = 0 \qquad (20)$$

If a time stepsize of $h = 1$ second and number of subintervals $N = 48$ are chosen, then applying Runge-Kutta to (20) results in the following solution table which gives the instantaneous horizontal and vertical velocities for each second of flight:

explain method or give reference!

Figure 4

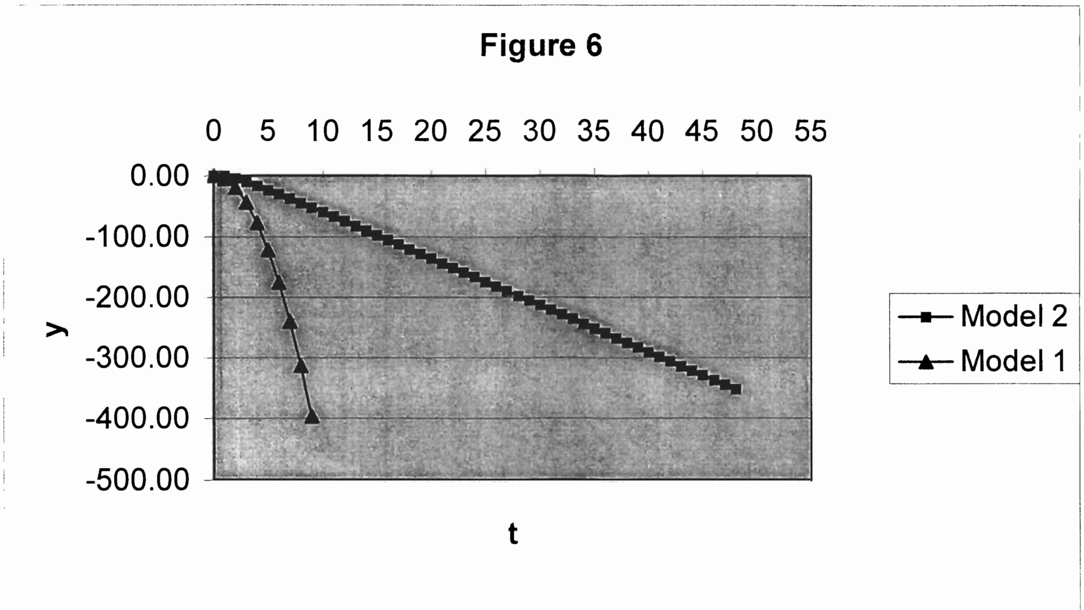
t	Vx	Vy
0	6.94	0.00
1	6.33	-4.74
2	5.72	-5.93
3	5.18	-6.49
4	4.69	-6.82
5	4.26	-7.03
6	3.87	-7.17
7	3.51	-7.28
8	3.20	-7.37
9	2.91	-7.43
10	2.65	-7.48
	....	
	....	
39	0 19	-7 73
40	0 17	-7 73
41	0 16	-7 73
42	0 14	-7 73
43	0.13	-7 73
44	0.12	-7 73
45	0.11	-7.73
46	0.10	-7.73
47	0.09	-7.73
48	0.08	-7 73

From figure 4, I can determine a table of x and y values for each second of flight. For example, at $t = 0$ the paraglider starts travelling at 6.94m/s in the x direction for 1 second. Therefore, a horizontal distance of 6.94 meters was traversed in that 1 second interval. Since each successive time interval is exactly one second, a table of approximated x and y values can be determined by simply adding successive velocities. This generates the following solution table which is a numerical solution to (18):

Figure 5

t	X	Y
0	0.00	0.00
1	6.94	0.00
2	13.27	-4.74
3	18.99	-10.67
4	24.17	-17 16
5	28.86	-23.98
6	33.12	-31.01
7	36.98	-38.18
8	40.50	-45.46
9	43.69	-52.83
10	46.60	-60.26
	....	
	....	
39	74.80	-283.09
40	74.99	-290.83
41	75.16	-298.56
42	75.32	-306.29
43	75.46	-314.03
44	75.60	-321.76
45	75.72	-329.50
46	75.83	-337.23
47	75.93	-344.97
48	76.02	-352.70

By plotting y vs t for model 1 and model 2 on the same graph, the effect of drag is clearly evident (figure 6).



Recall that in the gravitational model, the paraglider descended 350m in about 8.5s while traversing horizontally 60m (Not very realistic).

In the gravitational/drag model the paraglider descends 350m in 48s while traversing horizontally 76m. This appears to be a much more realistic scenario. In fact, if the paraglider did not posses any aerodynamic characteristics, which aim to maximize the flight time, then the gravitational/drag model would lend itself well to a simple parachute scenario. However, since the aerodynamics must be considered in the case of a paraglider, then I will proceed and introduce a third force into my model called "Lift".

The Effects of Lift

The shape of a paragliders wing is that of an airfoil (see figure 7) which is a term used in aerodynamic studies.

Figure 7

When a paraglider moves through the air, the airfoil shape of its wing forces the air that is moving over the top of the wing to take longer than the air that is moving under the wing. This effectively reduces the pressure on the upper surface of the wing, and results in a force called lift. The lift force L, acts in a direction that is perpendicular to the drag force and hence perpendicular to the direction of speed. Lift is the force that allows planes to fly. As a plane moves faster, more lift is generated. Paragliders work in much the same way. In fact, the relationship between lift and speed is similar to the relationship between drag and speed. Through studies in aerodynamics, the lift force has been found to be directly proportional to the square of the speed. The exact relationship is as follows:

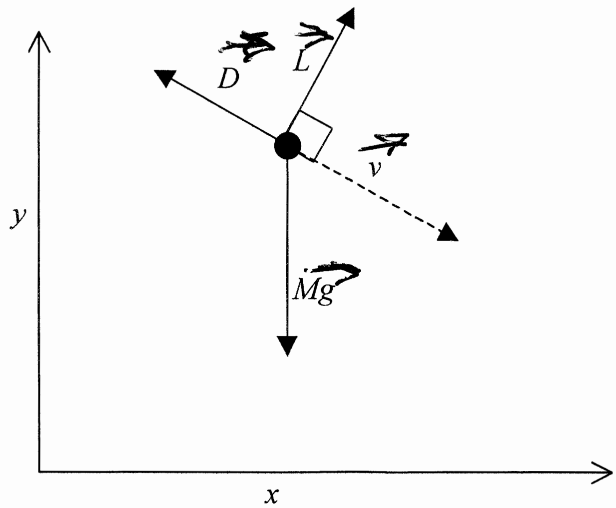
does it take longer or simply travel faster

reference

$$L = Jv^2\hat{\mathbf{n}} = \frac{1}{2}\rho A C_L v^2 \hat{\mathbf{n}} \qquad (21)$$

where C_L is the "lift coefficient" which depends on the angle ~~of attack~~ of the wing and $\hat{\mathbf{n}} = -sin(\alpha)\hat{\mathbf{i}} + cos(\alpha)\hat{\mathbf{j}}$ is a direction vector that is perpendicular to the direction of speed.

Figure 8



To incorporate lift into the model, Newton's Second Law is applied which results in the following model:

$$M\frac{d^2\mathbf{r}}{dt^2} = M\mathbf{g} - Kv^2\hat{\mathbf{v}} + Jv^2\hat{\mathbf{n}} \qquad (22)$$

or

$$\frac{d^2\mathbf{r}}{dt^2} = -g\hat{\mathbf{j}} - kv^2\hat{\mathbf{v}} + jv^2\hat{\mathbf{n}} \qquad (23)$$

where $k = \frac{K}{M}$ and $j = \frac{J}{M}$.

In component form (23) becomes:

$$\frac{d^2x}{dt^2} = -kv^2 cos(\alpha) - jv^2 sin(\alpha)$$

$$(24)$$

$$\frac{d^2y}{dt^2} = -g - kv^2 sin(\alpha) + jv^2 cos(\alpha)$$

Now substitute (16) and (17) into (24) to obtain:

$$\frac{d^2x}{dt^2} = -k\frac{dx}{dt}\left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right)^{\frac{1}{2}} - j\frac{dy}{dt}\left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right)^{\frac{1}{2}}$$

$$\frac{d^2y}{dt^2} = -g - k\frac{dy}{dt}\left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right)^{\frac{1}{2}} + j\frac{dx}{dt}\left(\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2\right)^{\frac{1}{2}} \tag{25}$$

To solve the above non-linear system, a transformation is applied to produce the following first order system that is solvable using Runge-Kutta methods:

$$\frac{dV_x}{dt} = -kV_x\left(V_x^2 + V_y^2\right)^{\frac{1}{2}} - jV_y\left(V_x^2 + V_y^2\right)^{\frac{1}{2}}$$

$$\frac{dV_y}{dt} = -g - kV_y\left(V_x^2 + V_y^2\right)^{\frac{1}{2}} + jV_x\left(V_x^2 + V_y^2\right)^{\frac{1}{2}} \tag{26}$$

To solve (26), the constant j must be determined. Recall that $j = \frac{\rho A}{2M}C_L$. The values for A, ρ and M have been previously determined. Therefore, only the "lift coefficient" needs to be determined. For the standard paraglider, $C_L \approx 1.0$. Since the values for A differ for horizontal and vertical cross-sections, two different j values need to be computed. Therefore $j_x \approx 0.01463m^{-1}$ and $j_y \approx 0.2048m^{-1}$. Using the same initial conditions for (26) that were used for the previous model, yields the following system:

$$\frac{dV_x}{dt} = -0.0117V_x\left(V_x^{\,2} + V_y^{\,2}\right)^{\frac{1}{2}} - 0.01463V_y\left(V_x^{\,2} + V_y^{\,2}\right)^{\frac{1}{2}} \qquad V_x(0) = 6.94$$

$$\frac{dV_y}{dt} = -9.8 - 0.1638V_y\left(V_x^{\,2} + V_y^{\,2}\right)^{\frac{1}{2}} + 0.2048V_x\left(V_x^{\,2} + V_y^{\,2}\right)^{\frac{1}{2}} \qquad V_y(0) = 0 \tag{27}$$

Using time stepsize $h = 1$ and number of sub-intervals $N = 97$, Runge-Kutta numerical techniques can be applied to (27) to obtain the following tabular solution:

Figure 9

t	Vx	Vy
0	6.94	0.00
1	6.43	-0.49
2	6.06	-1.32
3	5.78	-1.99
4	5.58	-2.46
5	5.42	-2.78
6	5.30	-3.01
7	5.20	-3.18
8	5.12	-3.30
9	5.06	-3.40
10	5.00	-3.48
	....	
	....	
87	4.77	-3.82
88	4.77	-3.82
89	4.77	-3.82
90	4.77	-3.82
91	4.77	-3.82
92	4.77	-3.82
93	4.77	-3.82
94	4.77	-3.82
95	4.77	-3.82
96	4.77	-3.82
97	4.77	-3.82

Using the same technique that was used for the previous model, I obtain the numerical solution for the instantaneous x and y positions for each second of flight:

Figure 10

t	x	y
0	0.00	0.00
1	6.94	0.00
2	13.37	-0.49
3	19.43	-1.81
4	25.21	-3.80
5	30 78	-6.26
6	36 20	-9 04
7	41.50	-12.04
8	46.70	-15 22
9	51.81	-18 52
10	56.87	-21 93
	....	
	....	
87	425.76	-314.09
88	430.54	-317.91
89	435.31	-321.73
90	440.08	-325.55
91	444.86	-329.36
92	449.63	-333.18
93	454.41	-337.00
94	459.18	-340.82
95	463.95	-344.64
96	468.73	-348.45
97	473.50	-352.27

To compare all three models, a plot of y vs. t is constructed for each model, and displayed on the same graph:

Figure 11

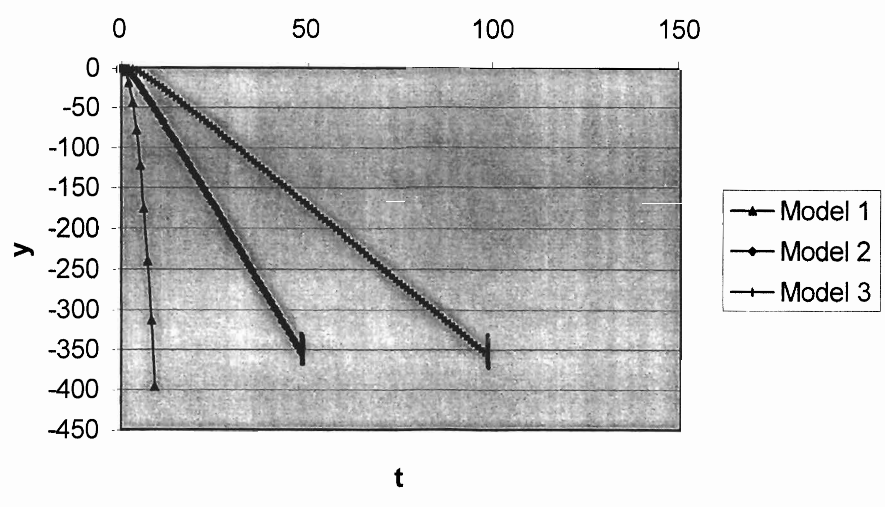
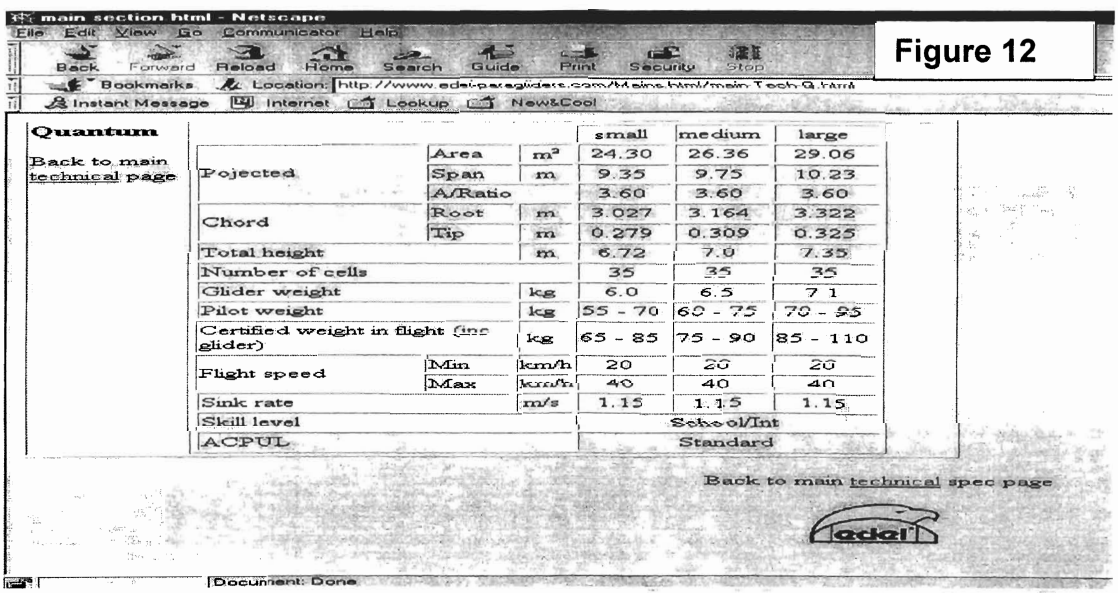


Figure 11 suggests that the three models are behaving as expected, at least relative to each other.

Recall that for the gravitational model, $T \approx 8.5s$ and $x \approx 59m$ at $y = -350m$. For the gravitational/drag model, $T \approx 48s$ and $x \approx 76m$ at $y = -350m$. Now for the gravitational/drag/lift model, $T \approx 97s$ and $x \approx 473m$ at $y = -350m$. Since the main objective of paragliding is to maximize flight time, the final model appears to be appropriate. However, without "real world" data to compare to, the accuracy of this model is in doubt. A test of my model against "real world" data is performed in the following example:

Example

Raw data in the sport of paragliding is not easy to obtain. However, a form of data can be readily obtained from paraglider manufacturer specifications. Here is an example of one paragliders specs that were obtained directly from the manufactures web site:



Quantum

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			small	medium	large
Pojected	Area	m²	24.30	26.36	29.06
	Span	m	9.35	9.75	10.23
	A/Ratio		3.60	3.60	3.60
Chord	Root	m	3.027	3.164	3.322
	Tip	m	0.279	0.309	0.325
Total height		m	6.72	7.0	7.35
Number of cells			35	35	35
Glider weight		kg	6.0	6.5	7 1
Pilot weight		kg	55 - 70	60 - 75	70 - 95
Certified weight in flight (inc glider)		kg	65 - 85	75 - 90	85 - 110
Flight speed	Min	km/h	20	20	20
	Max	km/h	40	40	40
Sink rate		m/s	1.15	1.15	1.15
Skill level			School/Int		
ACPUL			Standard		

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Figure 12

From figure 12, the "sink rate" can be obtained, which is the vertical decent rate and is equal to 1.15 m/s (3rd row from the bottom). Notice that 1.15m/s is the sink rate for all three sizes of glider. The main reason for this is that the ratio of the area to the mass is roughly equal for all three sizes. For this reason, I will not be concerned with changing any of my original constant values, and will proceed using previously computed data. From figure 9, the average vertical decent rate $\overline{V_y}$ can be computed by simply adding the V_y values for each second of flight and then dividing by N, which corresponds to the total flight time T. Therefore, the following result is obtained:

$$\overline{V_y} = \frac{\sum V_Y}{T} \approx 3.8m/s$$

Since the modelled sink rate is relatively close to the manufactures suggested sink rate, it alleviates some of the doubt regarding the predictive powers of the current model. However, the modelled sink rate is still about 2.5m/s off the manufacturer specs. This discrepancy could be caused by several factors. For example, manufactures tend to err on the side of optimism when listing specs; this makes their product more attractive. Also, C_L, C_D, and the vertical cross-sectional area, are roughly estimated. Small deviations in these values tend to have a significant impact on the resulting solution. Finally, my model is not perfect, which could also explain the difference.

Conclusion

My goal was to develop a mathematical model to describe the motion of a paraglider. After a detailed mathematical analysis, along with some testing, I managed to create a model that appears to serve its purpose to an acceptable degree. However, the assumptions that were made in the beginning, suggest some possible improvements to my model.

The fact that atmospheric phenomenon were assumed to be non-existent is highly unrealistic. A way to improve my model would be to somehow account for some of these phenomena. For example, I might try to model motion in the presence of a constant wind. I could also try to develop a model that describes motion in the presence of a "thermal". Thermals are atmospheric columns of warm rising air. When a paraglider encounters a thermal, it is forced upward which results in extra lift.

Another assumption that was made, restricted the paragliders angle of attack. This is also unrealistic. A major part of the motion of a paraglider, involves changing the angle that the main wing makes with the horizontal, which is called "braking". When braking is practiced, lift is generated which directly effects the motion. Incorporating a component into my model, which would allow for a variable angle of attack, would be considered an improvement.

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