Estimating the time of death

Newton's law of cooling

Introduction to Mathematical Modelling
Math 3820

<u>Assignment 2</u>

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Introduction

The objective of this project is to discuss, analyze and modify if possible one of the methods of estimating the time of death of a person, algor mortis. Agor mortis is the cooling of the body following death and it assumes that the temperature of a dead body decreases with constant rate until it matches the temperature of the surroundings and this method is based on the Newton's famous law "Newton's law of cooling".

Mathematical Modelling

Newton's law of cooling states that the rate of heat loss of a body is proportional to the difference in temperatures between the body and its surroundings, or environment.

Newton's law of cooling:

$$\frac{dT}{dt} = -k\,(T(t) - S)$$

Where $\frac{dT}{dt}$ is the rate of change of body temperature, t is time in hours, T(t) is the temperature of a body at time t in degree Celsius, k is some positive constant (experimental constant), and S is the ambient temperature also in degree Celsius.

Assumptions:

1. The range for the temperature of healthy human body is 36.1 to 37.8°C. For the simplicity of the model, the human body temperature is assumed to be 37°C. Since most of the books and people in the medical field say that 37°C is the average human body temperature, this assumption is reasonable.
2. The ambient temperature S is assumed to be constant for simplicity.

3. The ambient temperature S is assumed to be lower than the average human body temperature, 37°C. This is because if the ambient temperature is higher than the human body temperature then the body will not cool down, it will actually heat up. Also if the ambient temperature is constant at 37°C then it is almost impossible to estimate the time of death because by Newton's law of cooling, the body temperature will also be constant at 37°C. So we set $S < 37°C$.

Modification

1. The ambient temperature S was assumed to be constant for the model but in the real life situation, the ambient temperature might not be constant so what would happen if S was not constant. If we know the ambient temperature changes over time, the ambient temperature S can be represented as a function of time t. So let S(t) be the function represents the ambient temperature changes over time.

   Modified model:

   $$\frac{dT}{dt} = -k\,(T(t) - S(t))$$

   For example, let's say that the dead body was found on the street then to estimate the time of death, the investigator could get the weather data over time and calculate the function S(t) and then estimate t by using the model.
2. "Department of Forensic Medicine in University of Dundee" did the research on the estimating the time of death on the basis of post mortem findings and they found out that the greater the surface area of the body relative to its mass, the faster the temperature decreases. We assume that the ambient temperature S is constant again for simplicity.

(The 2nd modification will be done if the time permits and if I find enough data to set up the function)