URBAN GROWTH AND SPATIAL STRUCTURE: MATHEMATICAL MODELS AND EMPIRICAL EVIDENCE*

BRUCE E. NEWLING

IT IS well known that the growth of any complex structure is associated with changes in form. The phenomenon is familiar to the biologist, who relates the visible changes of form of a living organism to the differential growth of its parts; it is familiar also to the economist, who recognizes changes in the structure of a growing economy due to the differential growth of its several sectors. If we accept as a point of departure that geography, as both a physical and a social science, is primarily the study of the spatial structure of processes and the resultant spatial structure of phenomena, then the spatial structure of differential growth is an appropriate focus of attention for geographers; for growth and decay are, after all, the measurable expressions of the processes by which the observable spatial structure of any phenomenon at a given point in time is achieved. In this paper the spatial structure of a single spatial process is discussed, namely the differential growth of population density within cities. Certain empirical generalizations about the spatial variation of density in cities and changes in that spatial variation through time are set forth, from which are deduced the rules of intraurban allometric[1] growth and the density-growth rate relationship. In the manipulation of data within the mathematical framework thus constructed, two urban density constants are tentatively identified, one of which may be of significance in urban planning.

Mathematical Models of Intraurban Growth and Structure

Let us assume that population density varies with distance from the center of the city according to the equation

$$D_d = D_o e^{-bd} \tag{1}$$

* The author acknowledges with gratitude the help of Mr. Bruce Godwin, of the Center for Regional Economic Studies, University of Pittsburgh, who wrote the computer program for estimating city size from the density parameters.

[1] The adjective "allometric" refers to the systematic differential growth of parts within a complex structure. For a discussion of the concept see, for example, E. C. R. Reeve and Julian S. Huxley: Some Problems in the Study of Allometric Growth, *in* Essays on Growth and Form Presented to D'Arcy Wentworth Thompson (edited by W. E. Le Gros Clark and P. B. Medawar; Oxford, 1945), pp. 121–156.

➤ Dr. Newling is assistant professor of geography at Rutgers, The State University of New Jersey, New Brunswick.

where D_d is the population density at distance d from the center of the city, D_o is the density at the center of the city, e is the base of the natural logarithms, and b, the density gradient, is a natural logarithm measuring the rate of change of density with distance. The linear transformation of this function is

$$lnD_d = lnD_o - bd. \tag{2}$$

This rule was originally formulated by Bleicher[2] and was rediscovered some sixty years later by Clark.[3] Tanner[4] and Sherratt[5] have proposed an alternative model, that density declines exponentially as the square of distance; but Clark's rule appears to describe adequately the behavior of empirical data over a wide range of cases and is here adopted as a basic assumption.

It is also known from wide experience of historical series that the density gradient b falls in value through time as the city grows in population and areal extent. Let us assume that b is in fact a negative exponential function of time, such that

$$b_t = b_o e^{-ct} \tag{3}$$

where b_t is the density gradient at time t, b_o is the initial density gradient, e is the base of the natural logarithms, and the exponent c is a natural logarithm relating the change in the value of the density gradient to the passage of time.

From the two propositions that population density declines exponentially with distance from the center of the city and that the density gradient itself falls through time, we deduce the rule of intraurban allometric growth, namely that the rate of growth of density is a positive exponential function of distance from the center of the city, expressed by the equation

$$(1 + r_d) = (1 + r_o)e^{gd} \tag{4}$$

---

[2] Heinrich Bleicher: Statistische Beschreibung der Stadt Frankfurt am Main und ihrer Bevölkerung (Frankfurt am Main, 1892).

[3] Colin Clark: Urban Population Densities, *Journ. Royal Statist. Soc.*, Ser. A, Vol. 114, Part 4, 1951, pp. 490–496; *idem:* Transport—Maker and Breaker of Cities, *Town Planning Rev.*, Vol. 28, 1957–1958, pp. 237–250; *idem:* Urban Population Densities, *Bull. Inst. Internatl. de Statistique*, Vol. 36, Part 4, 1958, pp. 60–68; *idem:* The Location of Industries and Population, *Town Planning Rev.*, Vol. 35, 1964–1965, pp. 195–218.

[4] J. C. Tanner: Factors Affecting the Amount of Travel, *Road Research Tech. Paper No. 51*, Department of Scientific and Industrial Research, London, 1961.

[5] G. G. Sherratt: A Model for General Urban Growth, *in* Management Sciences, Models and Techniques: Proceedings of the Sixth International Meeting of the Institute [of Management Sciences] (edited by C. West Churchman and Michel Verhulst; 2 vols.; New York, 1960), Vol. 2, pp. 147–159.

where r_d is the percentage rate of growth at distance d, r_0 is the percentage rate of growth at the center of the city, e is the base of the natural logarithms, and g, the intraurban growth gradient, measures the rate of change of the rate of growth with distance from the center of the city.

Since both density and the rate of growth are functions of distance from the center of the city, we can express the rate of growth as a function of density with the following equation

$$(1 + r_D) = AD^{-k} \tag{5}$$

where r_D is the percentage rate of growth during a given period when the density at the beginning of the period is D, A is a constant, and the exponent k is the ratio of the intraurban growth gradient (g) to the population density gradient (b).

In equation (5), therefore, we have arrived deductively at a formal statement of the relationship between population density and the rate of growth. We should expect, other things being equal, that increasing density would have a depressive effect on the rate of growth, and equation (5) shows that the two variables are indeed inversely related.

Some other observations are appropriate with respect to the density–growth rate rule. First, we would have derived the same relationship if we had assumed that population density, and hence the rate of growth, were exponential functions of the square of distance: in other words, for present purposes it does not matter that we chose Clark's model of the spatial variation of population density instead of the Sherratt-Tanner model. Nor, indeed, does it matter that, for example, we have ignored the possibility of systematic spatial variation in density with distance laterally from the main radial routes: the density–growth rate rule holds regardless of location within the urban complex.

In the second place, it is important to note that equation (5) is not only a descriptive statement of the relationship between population density at the beginning of a period and the rate of growth which prevails during that period. If the density gradients are declining exponentially through time, as was assumed in equation (3), then equation (5) describes the variation in the rate of growth with variation in density in successive time periods; that is, it is a predictive statement also. This fact clearly has important ramifications in the field of planning.

If we assume that population density declines exponentially with distance from the center of the city and that the city is circular in shape, then, as Clark has shown, the total population within a given distance d from the center of the city can be conceived as the solid of revolution generated by the density curve about the vertical axis. Hence

$$P_d = \int_0^d D_o e^{-bx}(2\pi x)dx\,, \tag{6}$$

which is evaluated as

$$P_d = 2\pi D_o b^{-2}[1 - e^{-bd}(1 + bd)] \tag{7}$$

where P_d is the population residing within distance d from the center of the city, D_o is the central density, b is the density gradient, and e is the base of the natural logarithms. The term 2π is the radian measure of a complete revolution about a point, and for incompletely circular cities the radian measure will, of course, be less than 2π.

If we solve for d in equation (2) and substitute the solution in equation (7), the equation can be rewritten as

$$P_R = 2\pi D_o b^{-2}\,[1-(D_d/D_o)\,(1 + \ln D_o - \ln D_d)] \tag{8}$$

where P_R is the population residing in the urban region between the center of the city and that distance d at which the density is D_d.

We now demonstrate that equation (7) is useful for estimating the population size of the city of fixed radius (the central city proper), and that equation (8) is useful for estimating the population size of the urbanized region whose radius is changing but whose perimeter density may be assumed to be fixed.

Experiments with Urban Growth and Density Data

A. *Urban-Region Population Estimates*

Obviously it is difficult to measure the radius of the urban region, since to do so requires access to maps showing the extent of settlement, with adjustments made for irregularities in the perimeter caused by the digital extension of settlement along the major transportation lines. It is simpler to assume some constant perimeter density for all urban regions and to use this, together with the central density and density gradient parameters in equation (8), to make the total population estimates. Stewart[6] has suggested that the perimeter density of American urban regions is about 2000 persons per square mile, and this figure is accordingly employed with the parameter estimates developed by Muth[7] to calculate the population of each of forty-six urban regions in the United States.

---

[6] John Q. Stewart: Urban Population Densities, *Geogr. Rev.*, Vol. 43, 1953, pp. 575–576.

[7] Richard F. Muth: The Spatial Structure of the Housing Market, *Papers and Proc. Regional Science Assn.*, Vol. 7 (7th Annual Meeting), 1961 (Philadelphia, 1962), pp. 207–220.

TABLE I—DENSITY PARAMETERS, POPULATION ESTIMATES, AND SMA AND URBANIZED-AREA POPULATIONS FOR SELECTED UNITED STATES URBAN REGIONS, 1950
(Population in millions)

URBAN REGION	DENSITY PARAMETERS[a]		POPULATION ESTIMATES		ACTUAL POPULATION[b]	
	Central[c]	Gradient[d]	I[e]	II[f]	SMA	UA
1 Akron, Ohio	38	.84	0.34	0.27	0.41	0.37
2 Atlanta, Ga.	22	.48	0.60	0.41	0.67	0.51
3 Baltimore, Md.	69	.52	1.6	1.4	1.3	1.2
4 Birmingham, Ala.	9.4	.20	1.5	0.68	0.56	0.45
5 Boston, Mass.[g]	78	.30	2.7	2.4	2.9	2.2
6 Buffalo, N. Y.[g]	29	.19	2.5	1.9	1.1	0.80
7 Chicago, Ill.[h]	60	.18	6.2	5.3	5.5	4.9
8 Cincinnati, Ohio[g]	120	.69	0.79	0.72	0.90	0.81
9 Cleveland, Ohio[g]	22	.13	4.1	2.8	1.5	1.4
10 Columbus, Ohio	10	.19	1.7	0.83	0.50	0.44
11 Dallas, Tex.	26	.48	0.71	0.51	0.61	0.54
12 Dayton, Ohio	18	.32	1.1	0.71	0.46	0.35
13 Denver, Colo.	17	.33	0.98	0.62	0.56	0.50
14 Detroit, Mich.[g]	19	.098	6.2	4.1	3.0	2.7
15 Flint, Mich.	26	.73	0.31	0.22	0.27	0.20
16 Fort Worth, Tex.	17	.42	0.61	0.38	0.36	0.32
17 Houston, Tex.	14	.28	1.1	0.65	0.81	0.70
18 Indianapolis, Ind.	9.2	.18	1.8	0.80	0.55	0.50
19 Kansas City, Mo.	13	.26	1.2	0.67	0.81	0.70
20 Los Angeles, Calif.[g]	14	.078	7.2	4.2	4.4	4.0
21 Louisville, Ky.[g]	29	.47	0.41	0.31	0.58	0.47
22 Memphis, Tenn.	14	.22	1.8	1.1	0.48	0.41
23 Miami, Fla.[g]	14	.24	0.76	0.44	0.50	0.46
24 Milwaukee, Wis.[g]	61	.44	0.99	0.85	0.87	0.83
25 Nashville, Tenn.	9.3	.071	12.0	5.3	0.32	0.26
26 New Haven, Conn.	46	.99	0.29	0.24	0.26	0.24
27 New Orleans, La.	35	.41	1.3	1.0	0.69	0.66
28 Oklahoma City, Okla.	16	.43	0.54	0.33	0.33	0.28
29 Omaha, Nebr.	18	.38	0.78	0.50	0.37	0.31
30 Philadelphia, Pa.	86	.40	3.4	3.0	3.7	2.9
31 Pittsburgh, Pa.	17	.091	13.0	8.1	2.2	1.5
32 Portland, Oreg.	11	.16	2.7	1.4	0.70	0.51
33 Providence, R. I.	14	.41	0.52	0.30	0.68	0.58
34 Richmond, Va.	41	.82	0.38	0.31	0.33	0.26
35 Rochester, N. Y.	43	.64	0.66	0.53	0.49	0.41
36 Sacramento, Calif.	15	.36	0.73	0.43	0.28	0.21
37 St. Louis, Mo.[g]	47	.28	1.9	1.6	1.7	1.4
38 San Diego, Calif.	18	.39	0.74	0.48	0.56	0.43
39 San Jose, Calif.	21	.46	0.62	0.42	0.29	0.18
40 Seattle, Wash.[g]	25	.31	0.82	0.59	0.73	0.62
41 Spokane, Wash.[g]	5.9	.34	0.32	0.094	0.22	0.18
42 Syracuse, N. Y.	48	.92	0.36	0.29	0.34	0.27
43 Toledo, Ohio	6.1	.20	0.46	0.29	0.40	0.36
44 Utica, N. Y.	51	1.2	0.22	0.19	0.28	0.12
45 Washington, D. C.	20	.27	1.7	1.2	1.3	1.3
46 Wichita, Kans.	19	.53	0.42	0.28	0.22	0.19

[a] Muth, *op. cit.* [see text footnote 7 above], Table 1.

[b] Donald J. Bogue: Population Growth in Standard Metropolitan Areas, 1900–1950 (Scripps Foundation for Research in Population Problems, Washington, D. C., 1953).

[c] Thousands per square mile.

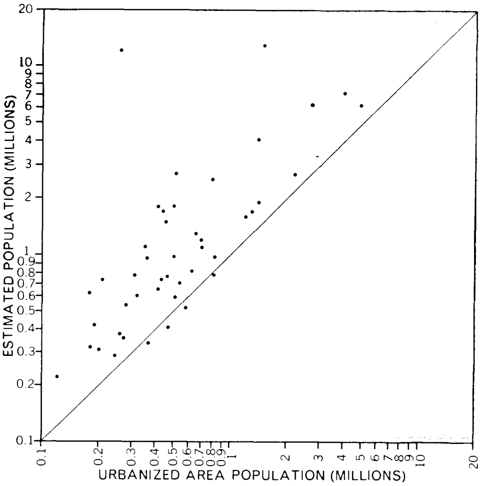
[d] Rate of change of density per mile, expressed as a natural logarithm.

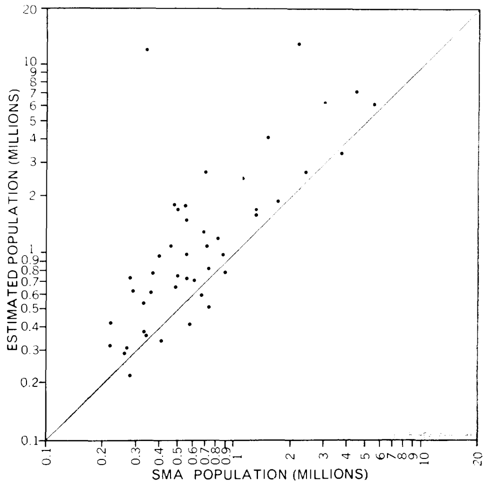
[e] Estimated from Muth's density parameters, and assuming a perimeter density of zero, with adjustments made for those urban regions which occupy less than a full circle.

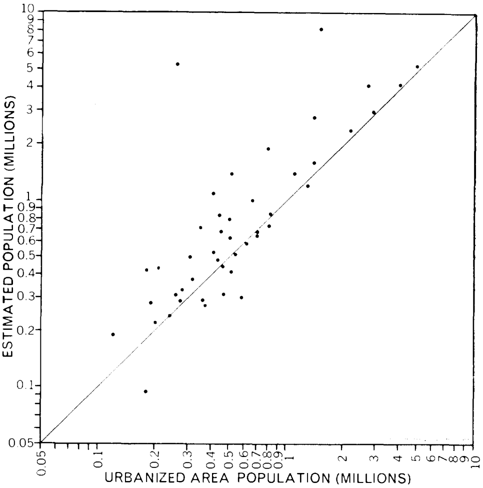
[f] Estimated from Muth's density parameters, and assuming a perimeter density of 2000 persons per square mile, with adjustments made for those urban regions which occupy less than a full circle.

[g] The urban region is assumed to occupy 50 percent of a full circle.

[h] The urban region is assumed to occupy 53.5 percent of a full circle.







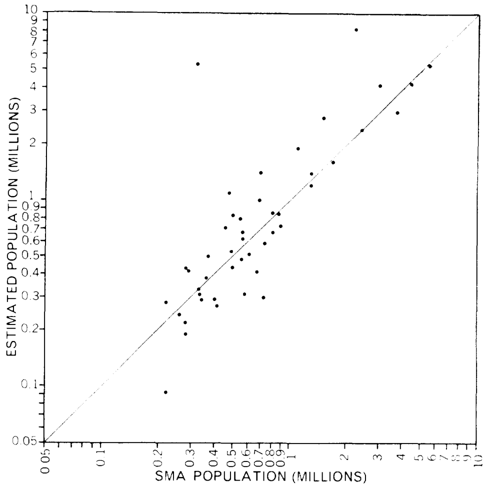


FIG. 1—Populations of forty-six urbanized areas in the United States, 1950, compared with population estimates based on Muth's density parameters and an assumed perimeter density of zero.

FIG. 2—Populations of forty-six SMA's in the United States, 1950, compared with population estimates based on Muth's density parameters and an assumed perimeter density of zero.

FIG. 3—Populations of forty-six urbanized areas in the United States, 1950, compared with population estimates based on Muth's density parameters and an assumed perimeter density of 2000 per square mile.

FIG. 4—Populations of forty-six SMA's in the United States, 1950, compared with population estimates based on Muth's density parameters and an assumed perimeter density of 2000 per square mile.

Table I presents the estimates so obtained, together with estimates based on an assumed perimeter density of zero, and the actual population of each SMA and urbanized area in 1950 for purposes of comparison. These data, also presented graphically in Figures 1–4, demonstrate the efficiency of Muth's parameter estimates and pari passu the validity of Stewart's perimeter density constant. With respect to both the SMA and the urbanized area, we conclude

TABLE II—CENTRAL DENSITY AND DENSITY GRADIENT PARAMETERS FOR CHICAGO, 1860–1950

YEAR	CENTRAL DENSITY[a]	DENSITY GRADIENT[b]	YEAR	CENTRAL DENSITY[a]	DENSITY GRADIENT[b]
1860	30.0	.917	1910	100.0	.369
1870	70.8	.877	1920	73.0	.251
1880	96.6	.781	1930	72.8	.215
1890	86.3	.508	1940	71.1	.210
1900	100.0	.415	1950	63.7	.182

Source: Halliman H. Winsborough: A Comparative Study of Urban Population Densities (unpublished Ph.D. dissertation, Department of Sociology, The University of Chicago, 1961).

[a] Thousands per square mile.

[b] The density gradient values are expressed as natural logarithms and measure the rate of change of density per mile. These values are the reciprocals of Winsborough's g values.

TABLE III—ESTIMATED POPULATION OF THE CHICAGO URBAN REGION AND ACTUAL POPULATION OF THE SMA AND THE URBANIZED AREA, 1900–1950
(*In millions*)

YEAR	POPULATION ESTIMATES		ACTUAL POPULATION	
	I[a]	II[b]	SMA[c]	Urbanized area[d]
1900	1.95	1.76	2.09	1.89
1910	2.47	2.23	2.75	2.53
1920	3.89	3.40	3.52	3.29
1930	5.31	4.64	4.68	4.43
1940	5.42	4.72	4.83	4.52
1950	6.43	5.53	5.50	5.05

[a] Estimated from Winsborough's density parameters (see Table II above), assuming a perimeter density of zero, and assuming that the Chicago urban region occupies 53.5 percent of a full circle.

[b] Estimated from Winsborough's density parameters, assuming a perimeter density of 2000 persons per square mile, and assuming that the Chicago urban region occupies 53.5 percent of a full circle.

[c] Bogue, *op. cit.* [see footnote *b* in Table I above].

[d] *Ibid.* These figures are the sum of the central-city population and the population of the urban ring.

that an assumed perimeter density of 2000 per square mile provides an estimate superior to the estimate obtained with an assumed perimeter density of zero; and that it provides a better estimate of the population of the SMA than of the urbanized area.

There is evidence that a perimeter density of 2000 per square mile is also relevant historically. Table II presents estimates of the density parameters calculated by Winsborough for Chicago from 1860 to 1950; and Table III gives population estimates for the Chicago urban region from 1900 to 1950 based on these parameters. Again, two estimates are generated, one assuming a zero perimeter density and the other assuming a perimeter density of 2000 persons per square mile.

The data show that the estimates for 1900 and 1910 are low, perhaps because for those early years the data for the Chicago SMA and urbanized area are inflated by a peripheral rural population not functionally a part of the Chicago urban region. From 1920 on, however, the perimeter density estimate of 2000 persons per square mile again overestimates the population of the urbanized area, though it provides good estimates of the SMA. The zero perimeter density estimate also repeats a familiar pattern, consistently overestimating the population size both of the urbanized area and of the SMA.

B. *Central-City Population Estimates*

Since the central city has, in contrast with the urban region, a constant radius, it is clearly easier to generate estimates of the total population from the radius in conjunction with the density parameters in equation (7). Table IV presents such estimates for the city of Chicago, with a constant site factor and a single adjustment made in the radius for the major annexations that occurred between 1880 and 1890. The table reveals how closely the estimates agree with the actual population for the years listed and attest to the reliability of Winsborough's parameter estimates.

C. *Intraurban Growth and the Critical Density*

Figure 5 presents a historical series of density curves for Kingston, Jamaica, for which the parameters are listed in Table V. The striking feature of the diagram is the intersection of the density curves for 1911, 1943, and 1960 at an almost identical radius (one thousand yards) and density (30,000 persons per square mile).

A circle with a radius of one thousand yards covers an area of one square mile and coincides broadly with the area of the present-day central business district (Fig. 6). We may surmise, therefore, that in the competition between land uses in twentieth-century Kingston commercial use is displacing residential use on the valuable centrally located land where residential density has historically exceeded 30,000 persons per square mile. Below this critical density, on the other hand, population growth is positive, and presumably residential use is in general more important than commercial use in the competition for available space.

In cities such as Pittsburgh topographic controls and the development of the city through the coalescence of several primary settlement nodes have produced a highly complicated spatial structure that is not adequately described in terms of simple linear distance from the center of the city. This is amply demonstrated when density and growth rate data (Table VI) are related to distance from the city center, as in Figures 7 and 8. In both diagrams it is clear that distance has little explanatory power.

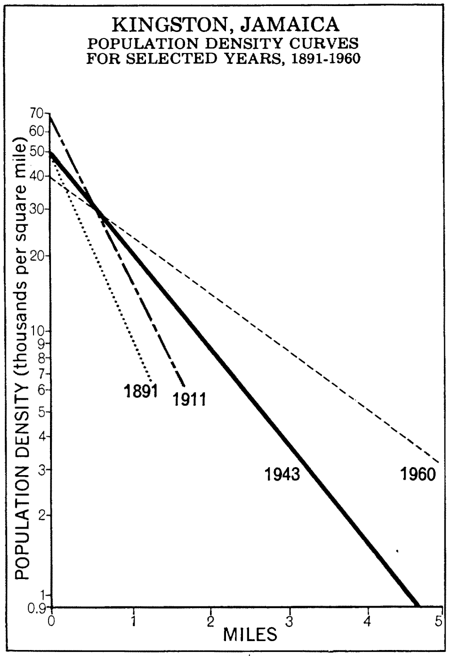


FIG. 5

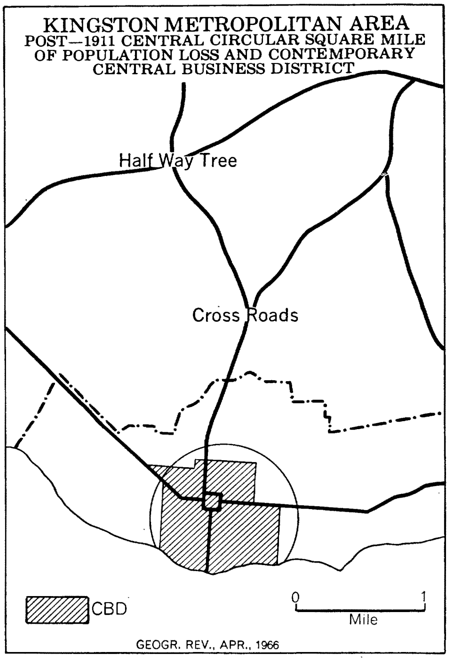


FIG. 6

TABLE IV—ACTUAL AND ESTIMATED POPULATION OF THE CITY OF CHICAGO, 1860–1950
(In millions)

YEAR	POPULATION Actual[a]	POPULATION Estimated[b]	YEAR	POPULATION Actual[a]	POPULATION Estimated[b]
1860	0.112	0.113	1910	2.19	2.18
1870	0.299	0.292	1920	2.70	2.78
1880	0.503	0.480	1930	3.38	3.36
1890	1.10	1.08	1940	3.40	3.36
1900	1.70	1.79	1950	3.62	3.50

[a] U. S. Bureau of the Census.

[b] Estimated from Winsborough's density parameters (see Table II above), with assumed radii of five miles from 1860 to 1880 and ten miles from 1890 to 1950, and assuming that the site of the city occupies 53.5 percent of a full circle.

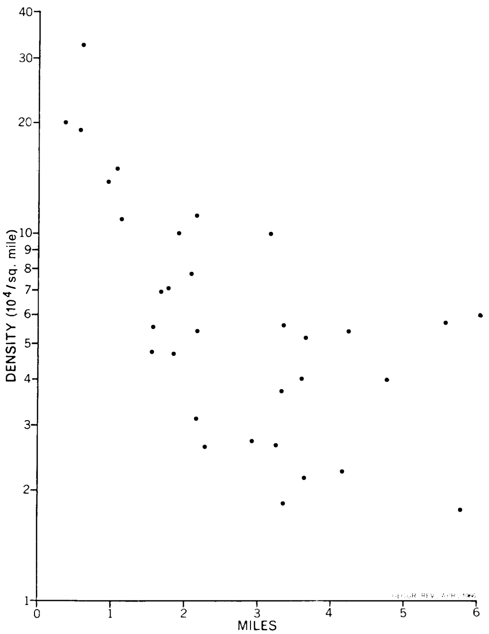
TABLE V—CENTRAL DENSITY AND DENSITY GRADIENT PARAMETERS FOR KINGSTON, JAMAICA, 1891–1960

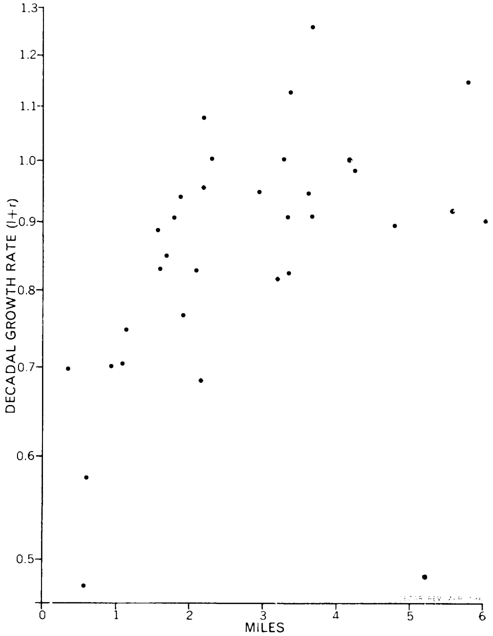
YEAR	CENTRAL DENSITY[a]	DENSITY GRADIENT[b]	YEAR	CENTRAL DENSITY[a]	DENSITY GRADIENT[b]
1891	49.1	1.635	1943	49.8	0.876
1911	68.4	1.443	1960	40.7	0.526

Source: Bruce E. Newling: The Growth and Spatial Structure of Kingston, Jamaica (unpublished Ph.D. dissertation, Department of Geography, Northwestern University, 1962).

[a] Thousands per square mile.

[b] The density gradient values are expressed as natural logarithms and measure the rate of change of density per mile.





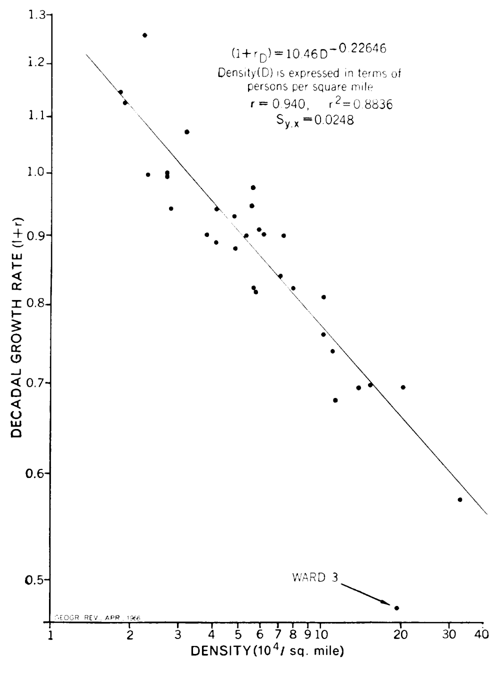


FIG. 7 (upper left)—Relationship between population density on residential land in 1950 and distance from the center of Pittsburgh, by wards.

FIG. 8 (upper right)—Relationship between the rate of population change 1950–1960 and distance from the center of Pittsburgh, by wards.

FIG. 9 (lower left)—Relationship between population density on residential land in 1950 and the rate of population change 1950–1960 in Pittsburgh, by wards. Ward 3 is excluded from the calculation of the regression equation.

Yet the basic logic of the urban spatial system can be identified if we dispense with distance and simply relate the rate of growth to density, as in Figure 9, according to the density-growth rate rule expressed formally in

TABLE VI—DISTANCE FROM THE CITY CENTER, NET RESIDENTIAL DENSITY IN 1950, AND POPULATION GROWTH RATE ($1 + r$) 1950–1960 FOR THE WARDS OF THE CITY OF PITTSBURGH

WARD	DISTANCE (*miles*)[a]	DENSITY 1950[b]	GROWTH RATE 1950–1960[c]
1	0.35	199,770	.6962
2	0.60	327,290	.5745
3	0.57	191,686	.4770
4	2.09	78,195	.8243
5	1.77	71,405	.9018
6	2.15	112,512	.6810
7	3.60	40,410	.9428
8	3.34	56,819	.8210
9	3.18	100,429	.8131
10	4.23	54,842	.9794
11	4.77	40,480	.8896
12	5.58	57,798	.9150
13	6.02	60,736	.9045
14	4.15	22,758	.9997
15	3.65	52,314	.9036
16	2.17	54,624	.9495
17	1.12	100,005	.7419
18	1.55	47,437	.8839
19	2.18	31,405	1.0744
20	2.28	26,438	1.0036
21	1.91	100,237	.7617
22	1.09	150,298	.7004
23	0.94	137,325	.6969
24	1.58	55,962	.8242
25	1.66	69,664	.8425
26	2.92	27,437	.9432
27	3.31	37,261	.9030
28	3.35	18,515	1.1261
29	3.26	26,733	.9972
30	1.85	47,277	.9347
31	5.78	17,965	1.1469
32	3.62	21,843	1.2671

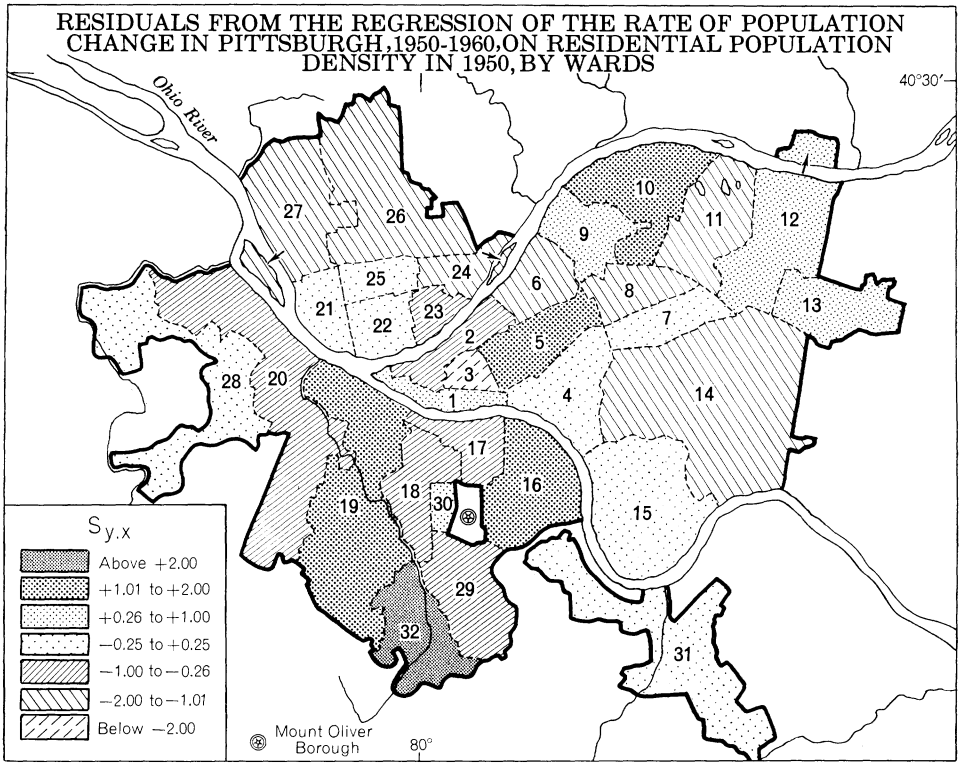
[a] Airline distance from Mellon Square to the center of the ward.

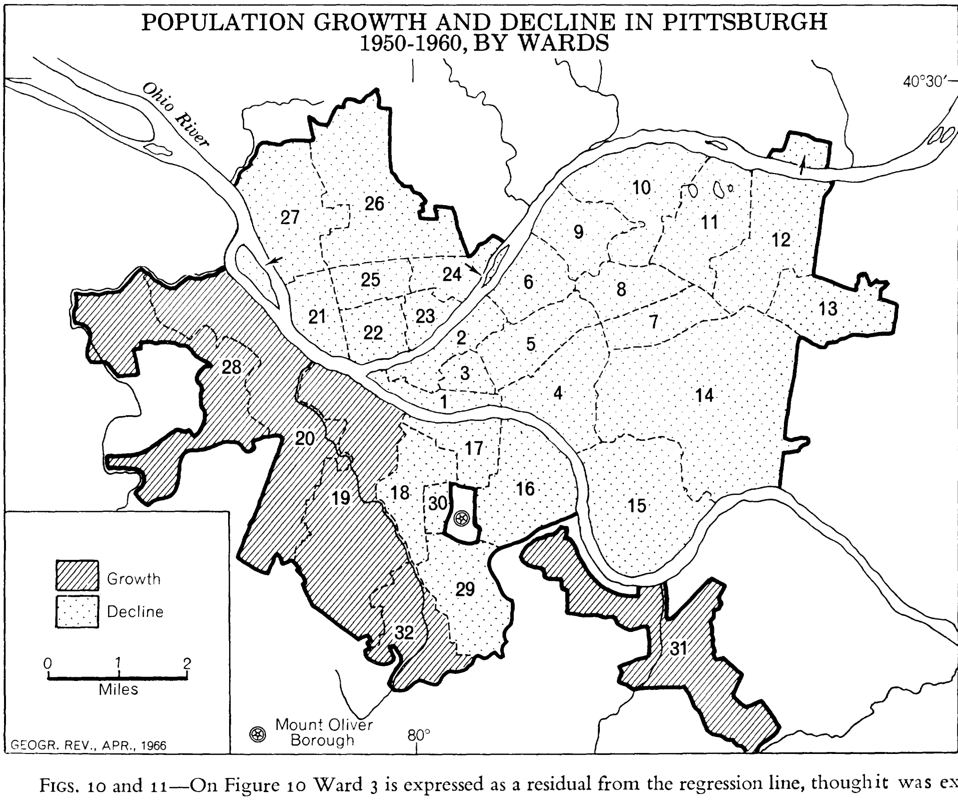
[b] Persons per square mile of residential land. These figures are estimates based on the United States census and the net acreage of land in residential use in 1952 according to the land-use survey conducted by the Pittsburgh City Planning Department. The change in residential acreage between 1950 and 1952 is assumed to have been negligible.

[c] The decadal growth rate ($1 + r$) is formed by the ratio of the 1960 population to the 1950 population.

equation (5). We can now identify a relationship so strong that with thirty-one observations[8] the coefficient of determination (r^2) indicates that more than 88 percent of the variation in the growth rate among Pittsburgh's wards is associated with variation in density among the wards. We conclude that the unexplained variation in the data (12 percent) has no special significance in terms of the spatial structure of the city: the spatial distribution of residuals from the regression line (Fig. 10) appears to be entirely random and unrelated to the spatial pattern of any other social or economic variable.

[8] Ward 3 was excluded because it was cleared in the urban renewal program.





FIGS. 10 and 11—On Figure 10 Ward 3 is expressed as a residual from the regression line, though it was excluded in the calculation of the regression equation.

By setting r equal to zero in the regression equation

$$(1 + r_D) = 10.46D^{-.22646} \tag{9}$$

we solve for density D to obtain 32,000 per square mile as the critical density above which growth is negative and below which growth is positive. The extraordinary correspondence of this value with the critical density obtained for Kingston is noteworthy, though one hesitates to generalize from only two results. One obvious difference between the two situations is that in Kingston the area of negative growth coincides with the central business district, whereas in Pittsburgh (Fig. 11) it is widely spread through the city and is associated with urban renewal activities, the decay of old neighborhoods, and racial change, in addition to the expansion of the commercial areas.

The inverse relationship between population density and the rate of growth, the identification of a critical density, and the observation that negative growth, occurring as it does above the critical density, is not solely attributable to competition between commercial and residential use of land all lead one to speculate that perhaps there is indeed some optimum urban population density to exceed which inevitably incurs social costs. We may surmise that certain events in the history of the city will cause this optimum to be exceeded (for example, heavy immigration without a commensurate expansion of housing and the supply of social overhead capital), with deleterious consequences for the affected areas (such as blight, crime and delinquency, and other social pathological conditions), and an eventual decrease in their population. If these speculations have any validity, then it is clear that the identification of such critical densities in cities is important in the field of planning.