The Humber Tunnel Authority:
A Recommended Vehicle Speed and Separation Distance to Alleviate Congested Tunnel Traffic Flow

Submitted to:
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6.337 Mathematical Modelling

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Introduction:

The Humber Tunnel Authority completed a tunnel in the late 70's, which linked the English towns of Hull and North Lincolnshire. North Lincolnshire at that time was a thriving expanding town, and to alleviate traffic congestion between the two towns a tunnel was suggested. The tunnel was expected to expedite the flow of commerce and people between the two towns. A small tunnel was built in order to save money and future traffic flow problems were not expected. Today there are major traffic flow problems during peak hours of use. I will propose constraints on speed and separation distance for the Humber Tunnel Authority to enforce, thereby optimizing the traffic flow at such peak hours.

The Tunnel Proposals:

The first tunnel design suggested was to build two tunnels, each with two lanes. This two tunnel, four-lane proposal turned out to be very expensive, but the original contractor suggested that this high cost could be justified. The contractor indicated that indeed such an expensive public work would not meet its traffic flow capacity until well into the new millenium, but future generations would benefit from such forethought. This proposal was however voted down. The new tunnel design agreed upon was a single tunnel with two lanes, one lane for each opposing lane of traffic. The cost of this venture was well below the cost of the original proposal. With regards to the worry of traffic flow problems in the future, the committee simply could not justify the added expenditure based upon large volume traffic flow problems predicted for the future. The new tunnel was supposed to be able to accommodate such future contingencies and its flow capacity was agreed to be sufficient.

Problem Solution:

Since a smaller tunnel was built, a solution for optimizing traffic flow for today's traffic volume must be examined. At the time of this problem there was little traffic congestion, but since that time work and settlement patterns have changed which have inevitably lead to serious traffic hold-ups at both ends of the tunnel during the early morning and evening rush hour.

The important concept that I will start with is the flow rate of the traffic within a tunnel.

The flow rate is defined as the number of cars passing a fixed point in a unit time interval. The flow rate will depend on a number of factors, such as:

a) traffic speed

b) separation distance between cars, and

c) length of cars.

I will assume a uniform stream of traffic, moving with speed v(mph), with an average separation distance D(ft) between vehicles and an average car length b(ft). Where I will choose b, the length of a vehicle, to be a constant 12 feet.

Very short but British cars are smaller

The velocity is given by:

$$v \text{ miles/hour} = v*5280 \text{ ft} / 60*60\text{seconds}$$

$$= v*528/360 \text{ ft/sec}$$

$$v(\text{mph}) = 22v/15 \text{ feet per second} \qquad (1)$$

Now consider a fixed point, A, and suppose the back of a car has just passed this point (fig. 1). The time taken for the next car in the stream to pass point A is given by:

$$\text{Time taken by each car} = \text{Dist. yet to travel / Speed}$$

$$= (D + b) / (22v/15) \qquad (2)$$

Now the flow rate f is the number of cars passing A in 1 second:

$$\text{Flow rate} = 1 / \text{Time taken by each car}$$

$$= 1 / (D + b) / (22v/15)$$

$$F = (22v/15) / (D + b) \qquad (3)$$

Thus the flow rate depends on both speed v and the separation distance D.

Fig.1 Distance yet to travel for a car passing a fixed point A

```
Vehicle          Vehicle        Vehicle
-----------------------A---------------
------{----D---}{--b--}----------------
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D = an average separation distance between vehicles (ft)

b = average vehicle length of 12 feet

A = fixed point

The manager of the Humber Tunnel Control Authority has two possible controls: the speed of the vehicle v and the separation distance between vehicles D.

The speed v clearly affects the distance D. The British Highway Code gives the following information in

Table 1, which suggests that the shortest stopping distance Ds is made up of two parts:

reaction distance.

a) the thinking distance Dt (which is the distance travelled while the driver moves his/her foot from the accelerator to the brake pedal)

b) the braking distance Db (which is the distance travelled while braking from speed v to speed 0)

From the Highway Code data, it is clear that the thinking distance is modelled by the following:

could be explained more fully

$$Dt = v \text{ (feet)} \qquad (4)$$

,since both the speed and thinking distance have the same numeric values.

Table 1: Shortest Stopping Distance (British Highway Code)

Speed(mph)	30	50	70
Thinking Distance(ft)	30	50	70
Braking Distance(ft)	45	125	245
Overall Stopping distance(ft)	75	175	315

From the data in Table 1 it is not however clear what the formula for the braking distance is, it does not appear to be a linear relationship. I will assume a power law relationship of the form:

$Db(v) = k \cdot v^{\alpha}$ (where k and alpha are positive constants) (5)

I will calculate the constants k and alpha using the method of least squares. See fig.2 for the calculations.

After calculating the least squares for (5) I have found that the equation describing the braking distance is as follows:

Braking distance = Db = 1/20 v^2 (feet) (6)

Where Alpha = 2

K = 0.05 = 1/20

(Note:see Appendix 1 for an alternative derivation of Db)

The graph of (6) is as follows and is clearly quadratic:

Graph 1: The braking distance vs. the velocity

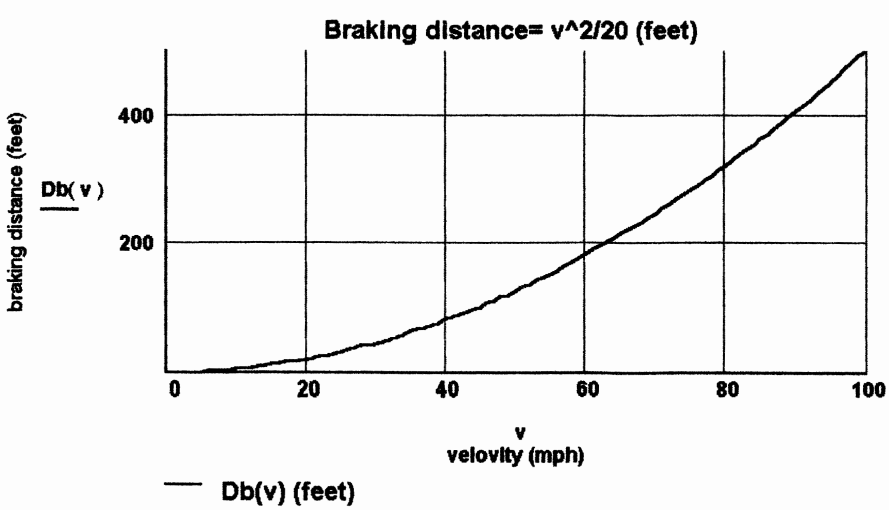


Fig.2 Least squares analysis for the braking distance

Data (see Table 1)	
V	D_b
$V_1 = 30$	$D_{b1} = 45$
$V_2 = 50$	$D_{b2} = 125$
$V_3 = 70$	$D_{b3} = 245$

$D_b(v) = k\,v^{\alpha}$

$\ln D_b(v) = \ln k v^{\alpha}$

$\ln D_b(v) = \ln v^{\alpha} + \ln k$

$\left\{\begin{array}{l} D_b(v) \text{ - is the braking distance.} \\ k, \alpha \text{ - positive constants} \\ v \text{ - velocity} \end{array}\right.$

$\ln D_b(v) = \bar{D} \quad \rightarrow \quad D_b(v) = e^{\bar{D}}$

$\ln k = \bar{k} \quad \rightarrow \quad k = e^{\bar{k}}$

$\ln v = \bar{v} \quad \rightarrow \quad v = e^{\bar{v}}$

$(A^T A)^{-1} \cdot (A^T Y) = \begin{pmatrix} \alpha \\ \bar{k} \end{pmatrix}$

$\bar{D} = \alpha \bar{v} + \bar{k}$

$\bar{D} = \ln D_b(v)$

$\alpha = 2.0000$

$\bar{k} = -2.9957 \rightarrow k = e^{\bar{k}}$
$= 0.49$
≈ 0.5

$\bar{D}(v_i) = \alpha \bar{v}_i + \bar{k}$

$\bar{D}_i = \ln D_b(v)$

$S(a,b) = \sum_{i=1}^{n} \left[\bar{D}(\bar{v}_i) - \bar{D}_i \right]^2$

$= \sum_{i=1}^{n} \left[\alpha \bar{v}_i + \bar{k} - \bar{D}_i \right]^2$

$\alpha = 2$

$k = 0.5$

$$\boxed{\therefore D_b(v) = 0.5\,v^2 = \frac{1}{20} v^2}$$

$$\left\{\begin{array}{l} \dfrac{\partial S}{\partial \alpha} = \sum_{i=1}^{n} \bar{v}_i^{\,2}\alpha + \sum_{i=1}^{n} \bar{v}_i \bar{k} = \sum_{i=1}^{n} \bar{v}_i \bar{D} = 0 \\ \dfrac{\partial S}{\partial \bar{k}} = \sum_{i=1}^{n} \bar{v}_i \alpha + \sum_{i=1}^{n} \bar{v}_i^{\,0} \bar{k} = \sum_{i=1}^{n} \bar{v}_i^{\,0} \bar{D} = 0 \end{array}\right.$$

$$\begin{bmatrix} \sum_{i=1}^{n} \bar{v}_i^{\,2} & \sum_{i=1}^{n} \bar{v}_i \\ \sum_{i=1}^{n} \bar{v}_i & \sum_{i=1}^{n} \bar{v}_i^{\,0} \end{bmatrix} \begin{bmatrix} \alpha \\ \bar{k} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \bar{v}_i \bar{D} \\ \sum_{i=1}^{n} \bar{v}_i^{\,0} \bar{D} \end{bmatrix}$$

$$\begin{bmatrix} \sum_{i=1}^{n} (\ln v_i)^2 & \sum_{i=1}^{n} \ln v_i \\ \sum_{i=1}^{n} \ln v_i & \sum_{i=1}^{n} n \end{bmatrix} \begin{bmatrix} \alpha \\ \bar{k} = \ln k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} \ln v_i \ln D_b \\ \sum_{i=1}^{n} \ln D_b \end{bmatrix}$$

$$A^T A \bar{x} = A^T Y$$

$$\begin{bmatrix} v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 & 1 \\ v_2 & 1 \\ v_3 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \bar{k} \end{bmatrix} = \begin{bmatrix} v_1 & 1 \\ v_2 & 1 \\ v_3 & 1 \end{bmatrix} \begin{bmatrix} \bar{D}_{b1} \\ \bar{D}_{b2} \\ \bar{D}_{b3} \end{bmatrix}$$

$$\begin{bmatrix} \ln v_1 & \ln v_2 & \ln v_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \ln v_1 & 1 \\ \ln v_2 & 1 \\ \ln v_3 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \bar{k} = \ln k \\ k = e^{\bar{k}} \end{bmatrix} = \begin{bmatrix} \ln v_1 & 1 \\ \ln v_2 & 1 \\ \ln v_3 & 1 \end{bmatrix} \begin{bmatrix} \ln D_{b1} \\ \ln D_{b2} \\ \ln D_{b3} \end{bmatrix}$$

$$\begin{array}{ccc} V & D_b & \\ \text{Velocity} & \text{Braking Distance} & \text{Predicted Braking Distance.} \end{array}$$

$$x := \begin{pmatrix} 30 \\ 50 \\ 70 \end{pmatrix} \qquad y := \begin{pmatrix} 45 \\ 125 \\ 245 \end{pmatrix} \qquad Q = \begin{pmatrix} 45.00000000000215 \\ 125.0000000000077 \\ 245.0000000000177 \end{pmatrix}$$

$$\begin{array}{cc} \ln \text{velocity} & \ln \text{Braking Distance.} \end{array}$$

$$xx = \begin{pmatrix} 3.401197381662156 \\ 3.912023005428146 \\ 4.248495242049359 \end{pmatrix} \qquad yy = \begin{pmatrix} 3.80666248977032 \\ 4.828313737302302 \\ 5.501258210544727 \end{pmatrix}$$

$$\begin{array}{cc} \text{Predicted } \ln D_b & \text{Residuals} \end{array}$$

$$A \cdot X = \begin{pmatrix} 3.806662489770368 \\ 4.828313737302363 \\ 5.501258210544799 \end{pmatrix} \qquad d = \begin{pmatrix} -0.000000000000048 \\ -0.000000000000061 \\ -0.000000000000072 \end{pmatrix}$$

$$A^T \cdot A = \begin{pmatrix} 44.92177944574061 & 11.56171562913966 \\ 11.56171562913966 & 3 \end{pmatrix} = \begin{pmatrix} \sum \ln v_i^2 & \sum \ln v_i \\ \sum \ln v_i & n \end{pmatrix}$$

$$A^T \cdot yy = \begin{pmatrix} 55.20775424361395 \\ 14.13623443761735 \end{pmatrix}$$

$$\left(A^T \cdot A\right)^{-1} \cdot \left(A^T \cdot yy\right) = \begin{pmatrix} 2.000000000000028 \\ -2.99573227355404 \end{pmatrix} = \begin{pmatrix} \sum \ln v_i \ln D_b \\ \sum \ln D_b \end{pmatrix} = \begin{pmatrix} \alpha \\ \bar{k} \end{pmatrix}$$

$$\left[\left(A^T \cdot A\right)^{-1} \cdot \left(A^T \cdot yy\right)\right]_2 = -2.99573227355404 = \bar{k}$$

$$e^{\left[\left(A^T \cdot A\right)^{-1} \cdot \left(A^T \cdot yy\right)\right]_2} = 0.049999999999998 = K$$

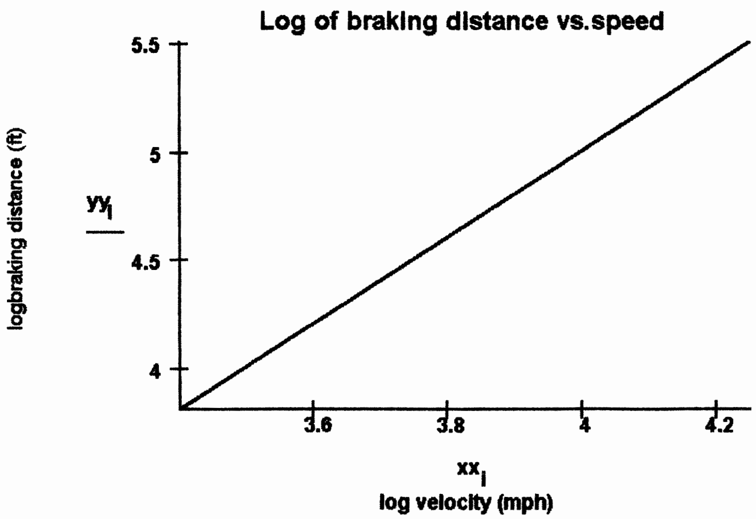
$$\alpha = 2$$

$$K = 0.05 = \frac{1}{20}$$

$$\therefore \; D_b(v) = \frac{1}{20} v^2$$

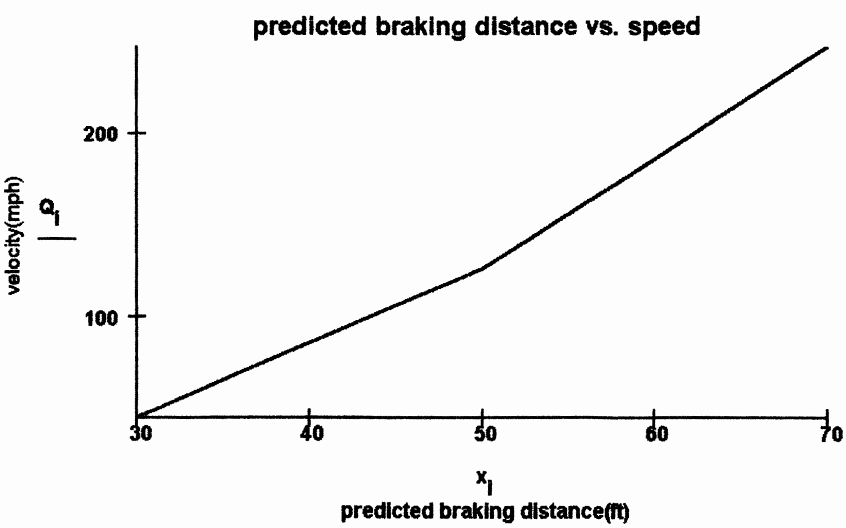
The graph of equation (5) using natural logarithms to make it linear and using data from Table 1 follows:

Graph 2: Natural logarithm of braking distance vs. speed



The graph of the predicted least squares equation Db(v)=1/20v^2 is linear and therefore the equation holds. See Graph 3.

Graph 3: The least squares predicted braking distance vs.speed



I now return to the problem of maximizing the flow rate given in equation (3). The form of the separation distance D as recommended by the Highway Code is

$$D = Dt + Db \quad (7)$$

,the sum of the thinking and the braking distances. In my experience however, I have found that more aggressive driving is generally the case and the separation distance is just the thinking distance,

$$D = Dt \quad (8)$$

I will examine both of these extreme cases and them determine a case somewhere between these two extremes in order to determine a optimal separation distance and speed for tunnel traffic in the tunnel.

Case 1: D=Dt

In this case from (3) $F = 22v/15(D+b)$,

Where: F= flow rate

v= speed (mph)

D= separation distance (ft)

= Dt , but from (4) Dt= v (ft) so

= v

b=12 ft (average car length)

For Case 1 we therefore get the following flow rate:

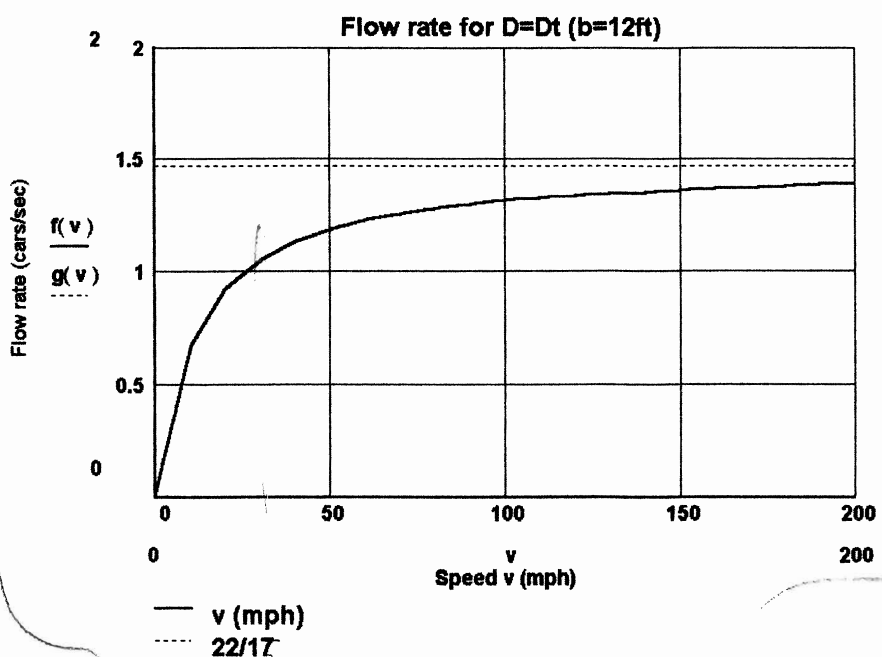
$$f(v) := \frac{22 \cdot v}{15 \cdot (v + 12)} \qquad (9)$$

limiting value, not max.

The graph of (9) (see Graph 4) suggests that there is a maximum value for the function. The ~~maximum~~ value occurs at 22/15. As the velocity approaches infinity, f (v) approaches 22/15. This seems to suggest that the policy of most drivers is the faster the better. The Humber Tunnel Authority would most definitely not approve this extreme case of excessive speed. From the graph it is also apparent that the flow rate only increases very slowly above 30 mph.

?

Graph 4: Flow rate for separation distance equaling the thinking distance.



Case 2: D = Dt + Db

In this case from (3) F = 22v/15(D+b),

Where: F= flow rate

v= speed (mph)

D= separation distance (ft)

= Dt + Db

[but from (4) Dt= v (ft)]

[(and from (6) Db= 1/20*v^2 (ft)]

= v + 1/20*v^2

b=12 ft (average car length)

For Case 2 we therefore get the following flow rate:

$$f(v) := \frac{22 \cdot v}{15 \cdot \left(v + \frac{v^2}{20} + b\right)} \qquad (10)$$

The graph of (10)(see Graph 5) suggests that there is a maximum value at a finite value of v, given by the solution of df(v)/dv = 0. For the analysis of the first derivative and maximum value of the function (10) see Fig.3.

In Fig.3 it was calculated that the maximum velocity is,

$$vmax := \sqrt{(20) \cdot b} \qquad (11)$$

Fig.3 Analysis of Case 2 flow-rate, derivative and maximum value.

$$F(v) = \frac{22v}{15(D+b)} = \frac{22v}{15(D_t + D_b + b)} = \frac{22v}{15(v + \frac{v^2}{20} + b)}$$

$F(v)$ = flow rate.
v = velocity (mph)
D = separation distance (ft)
$= D_t + D_b$ from (7)
D_t = v (mph) thinking distance from (4)
D_b = $\frac{v^2}{20}$ (mph) braking distance from (6)
b = average car length = 12 feet

$f = 22v \qquad g = 15(v + \frac{v^2}{20} + b)$

$f' = 22 \qquad g' = 15(1 + \frac{2v}{20})$

$= 15(1 + \frac{v}{10})$

$$F'(v) = \frac{f'g - fg'}{g^2}$$

$$= \frac{22 \cdot 15(v + \frac{v^2}{20} + b) - 22v \cdot 15(1 + \frac{v}{10})}{[15(v + \frac{v^2}{20} + b)]^2}$$

The maximum value has a solution when $F'(v) = 0$

$$F'(v) = 0 = \frac{22 \cdot 15(v + \frac{v^2}{20} + b) - 22v \cdot 15(1 + \frac{v}{10})}{[15(v + \frac{v^2}{20} + b)]^2}$$

$$0 = \frac{v^2}{20} + v + b - v(\frac{v}{10} + 1)$$

$$\frac{v^2}{20} + v + b = v(\frac{v}{10} + 1)$$

$$\frac{v^2}{20} + \cancel{v} + b = \frac{v^2}{10} + \cancel{v}$$

$$b = \frac{2v^2}{20} - \frac{v^2}{20}$$

$$b = \frac{v^2}{20}$$

$$v^2 = 20 \cdot b$$

$$V_{max} = \sqrt{20 \cdot b} \text{ (mph)}$$

$$= \sqrt{20 \cdot 12}$$

$$= \sqrt{240}$$

V_{max}	$= 15.492$ (mph)
$F(V_{max})$	$= .575$

The Corresponding Separation Distance for V_{max} is:

$$D(V_{max}) = D_t(V_{max}) + D_b(V_{max})$$

$$= V_{max} + \frac{V_{max}^2}{20}$$

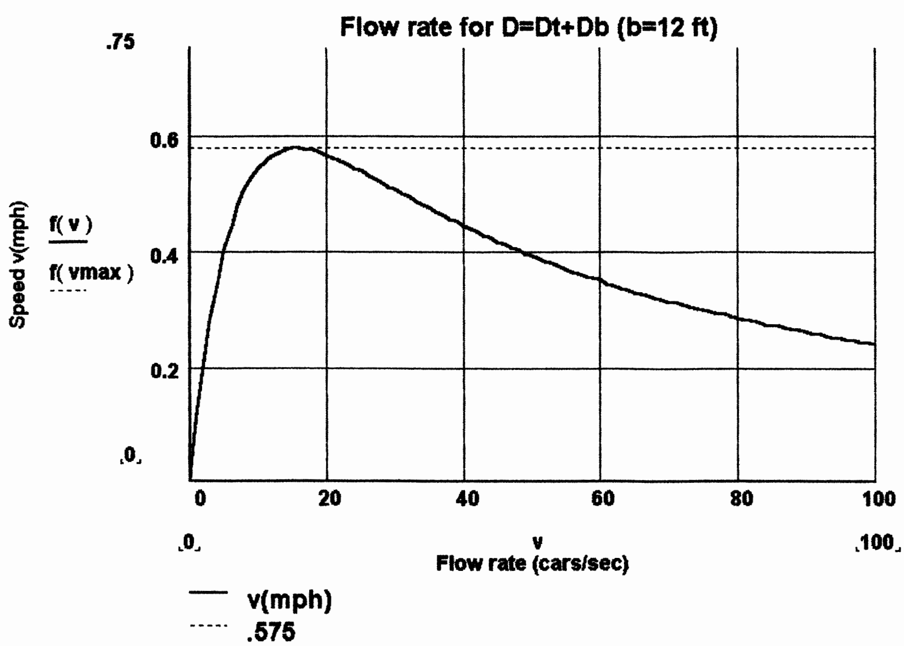
$$= 15.492 + \frac{15.492^2}{20}$$

$$\boxed{D(V_{max}) = 27.492 \text{ (feet)}}$$

Where vmax was calculated to be 15.492 mph and the correspnding separation distance to vmax was D(vmax)= 27.492 ft. The flow-rate at vmax is .575(see Graph 5)

Graph 5: Behavior of function (10) case 2 flow rate,

$$f(v) := \frac{22 \cdot v}{15 \cdot \left(v + \frac{v^2}{20} + b\right)}$$



Once again it appears as though case 2 is also not very realistic. Most drivers, except those who only frequent the roads on Sundays, will tend to drive faster than 16 mph and closer than 28 feet!

Can we give an interpretation to α?

measures the significance of braking distance!

Case 3: D=Dt + α*Db

For this case I will look for a situation somewhere between the two extremes of Case 1 and Case 2. For this case a parameter α is introduced which has a range 0 less than α less than 1.

The separation distance now becomes:

$$D = Dt + \alpha Db \tag{12}$$

The flow rate with the parameter now becomes the following:

$$f(v) := \frac{22 \cdot v}{15 \cdot \left(v + \frac{\alpha v^2}{20} + b\right)} \tag{13}$$

See Fig.4 for the analysis of (13). From Fig.4 vmax and D(vmax) were both calculated.

note: both decrease as α↑ so how do we choose?

$$Vmax(\alpha) = sqrt(240/\alpha) \tag{14}$$

$$D(Vmax(\alpha)) = sqrt(240/\alpha) + 12 \tag{15}$$

A table of values for vmax and D(vmax) for values of α Between 0 and 1 will be examined to see if this information yields an optimal speed and separation distance, thereby solving the problem. See Table 2.

how do we choose this.

From Table 2 a possible solution exists in the range of α from about 0.5 to 0.75. For this parameter the velocity is around 20 mph and the separation distance is

Fig.4 Analysis of Case 3 flow-rate incorporating a range parameter, derivative and maximum value.

α = parameter , $(0 < \alpha < 1)$

D = separation distance (ft)

$= Dt + Db$ from (7)

$= Dt + \alpha Db$ (12)

$Dt = v$ (mph) thinking distance from (4)

$Db = \frac{v^2}{20}$ (mph) braking distance from (6)

b = 12 feet , average car length

$$F(v) = \frac{22v}{15(D+b)} = \frac{22v}{15\left(v + \alpha\frac{v^2}{20} + b\right)}$$

$f = 22v$ $\quad f' = 22$

$g = 15\left(v + \alpha\frac{v^2}{20} + b\right)$

$g' = 15\left(1 + \frac{2\alpha v}{20}\right) = 15\left(1 + \frac{\alpha v}{10}\right)$

$$F'(v) = \frac{f'g - fg'}{g^2} = \frac{22 \cdot 15\left(v + \frac{\alpha v^2}{20} + b\right) - 22v \cdot 15\left(1 + \frac{\alpha v}{10}\right)}{\left[15\left(v + \frac{\alpha v^2}{20} + b\right)\right]^2}$$

The maximum value has a solution when $F'(v) = 0$

$$F'(v) = 0 = \frac{22 \cdot 15\left(v + \frac{\alpha v^2}{20} + b\right) - 22v\, 15\left(1 + \frac{\alpha v}{10}\right)}{\left[15\left(v + \frac{\alpha v^2}{20} + b\right)\right]^2}$$

$$0 = 22 \cdot 15\left[\left(v + \frac{\alpha v^2}{20} + b\right) - v\left(1 + \frac{\alpha v}{10}\right)\right]$$

$$v + \frac{\alpha v^2}{20} + b = v\left(1 + \frac{\alpha v}{10}\right)$$

$$v + \frac{\alpha v^2}{20} + b = v + \frac{\alpha v^2}{10}$$

$$\frac{\alpha v^2}{20} + b = \frac{2\alpha v^2}{20}$$

$$b = \frac{\alpha v^2}{20}$$

$$v^2 = \frac{20b}{\alpha}$$

$$\boxed{V_{max} = \sqrt{\frac{20b}{\alpha}} \text{ mph}}$$ (b = 12 feet, average car length)

$$V_{max} = \sqrt{\frac{20 \cdot 12}{\alpha}}$$

$$\boxed{V_{max} = \sqrt{\frac{240}{\alpha}} \text{ mph}}$$ after substituting for b (14)

The corresponding separation distance is:

$$D = D_t + \alpha D_b$$

$$D = V + \alpha \frac{V^2}{20}$$

$$D(V_{max}) = V_{max} + \alpha \frac{V_{max}^2}{20}$$

$$= \sqrt{\frac{20 \cdot b}{\alpha}} + \frac{\alpha \left(\sqrt{\frac{20 \cdot b}{\alpha}}\right)^2}{20}$$

$$= \sqrt{\frac{20 \cdot b}{\alpha}} + \frac{\cancel{\alpha}\, \frac{20 \cdot b}{\cancel{\alpha}}}{20}$$

$$= \sqrt{\frac{20 \cdot b}{\alpha}} + \frac{\cancel{20} \cdot b}{\cancel{20}}$$

$$\boxed{D(V_{max}) = \sqrt{\frac{20 \cdot b}{\alpha}} + b \text{ ft.}}$$ (b = 12 feet, average car length)

$$= \sqrt{\frac{20 \cdot 12}{\alpha}} + 12$$

$$\boxed{D(V_{max}) = \sqrt{\frac{240}{\alpha}} + 12 \text{ feet}}$$ after substituting for b (15)

Table 2: Dependance of optimal speed and separation distance based on parameter α $(0<\alpha<1)$

Optimal Speed v(mph)

Corresponding separation distance D(feet)

$$v(\alpha) := \sqrt{\frac{20 \cdot b}{\alpha}} \qquad D(\alpha) := v(\alpha) + b$$

	α	$v(\alpha)$	$D(\alpha)$
	0.05	69.282	81.282
	0.1	48.99	60.99
	0.15	40	52
	0.2	34.641	46.641
	0.25	30.984	42.984
	0.3	28.284	40.284
	0.35	26.186	38.186
	0.4	24.495	36.495
	0.45	23.094	35.094
	0.5	21.909	33.909
Possible	0.55	20.889	32.889
	0.6	20	32
Solution	0.65	19.215	31.215
	0.7	18.516	30.516
	0.75	17.889	29.889
	0.8	17.321	29.321
	0.85	16.803	28.803
	0.9	16.33	28.33

10 yds

about 30 feet(11 yards). These values also seem reasonable for congested rush hour traffic.

Conclusion:

The Traffic Manager of the Humber Tunnel Autority could therefore maximize the flow-rate by displaying a notice board at the entrance to the tunnel. This sign could read,

In Congested Traffic

Travel at 20 mph

Separation Distance 11 yards

An important point to note is that as well as specifying the optimal speed, the recommended separation distance must also be given, otherwise operating conditions may be far from optimal.

When modelling the solution for the maximum traffic flow velocity and separation distance between cars, I examined three cases. Two cases gave extreme solutions to the model, one extreme model produced an excessive speed solution, while the other model produced an unrealistic slow speed solution. A middle ground model was finally determined however.

From the Case 1 solution where D=Dt I found that this model produced an excessive speed solution. This model would indeed increase the traffic flow through the tunnel but because of safety concerns for motorists, this model solution would not be practical. This model did reveal a very interesting solution however. From the graph of the flow rate it was evident that the flow rate only increased very slowly above 30mph. This model therefore gave me an indication that the speed I was looking for was somewhere less than 30mph.

OK! (I had not realized this before!)

From the Case 2 solution where D=Dt+Db I found that this model yielded my minimum limiting values. It was found that the maximum velocity for this model was about 15mph and that the motorist following distance was about 28ft. This model therefore gave me values which I knew my final solution should be greater than. Most people travel faster than 15mph and follow closer than 28ft.

From the Case 3 solution where D=Dt+αDb I finally found results which were less than the extreme maximum solution and greater than the extreme minimum solution. For this case, the model solution existed in a parameter range of 0.5 and 0.75. Which gave the optimum model solution of 20mph at a separation distance of 11 yards.

The sensitivity of this solution could be improved by integrating different sized vehicles into the model. For my solution I assumed that every vehicle was 12ft long, but this is certainly not the case in realistic terms. I also assumed a uniform stream of traffic, moving with speed v(mph). Traffic, in my experience is never uniform, this model could therefore be improved by somehow allowing for non-uniform flow (ie.PDE solutions?). I also assumed that the distance between each vehicle was an average separation distance. The model could again be improved by allowing for separation distances based on real data and not on a simple average. To do a thorough analysis of this model it seems that some real life data based on real vehicles in the tunnel would be necessary.

I learnt much valuable knowledge from working on this project. I never realized how complicated the analysis of even really simple mathematical models could get. I also now see what you meant by clearly stating all assumptions made in the model. The assumptions are where the potential improvements of the model exist. This traffic flow model made simple assumptions and yet yielded a complicated solution which gave me valuable insight into the problem I was examining. I guess this is where you start when analyzing a very complicated system. You first make simple assumptions and obtain your results. Once these results have been analyzed the model can further be refined with much more insight rather than simple educated guesses.

Appendix 1: Alternate derivation of Db.

For this alternate derivation of Db I start by assuming that the maximum braking force is $\frac{2}{3}mg$ (where m = mass of the car) and the distance travelled is denoted by x. This gives the following.

$$\frac{d^2x}{dt^2} = -\frac{2}{3}g$$

Integrating gives

$$\frac{dx}{dt} = -\frac{2}{3}gt + v$$

Integrating again gives

$$x = -\frac{1}{3}gt^2 + vt$$

When $\frac{dx}{dt} = 0$ we have $x = Db$ giving,

$$0 = -\frac{2}{3}gt + v$$

$$v = \frac{2}{3}gt$$

$$\frac{3}{2}v = gt$$

$$\boxed{t = \frac{3}{2}\frac{v}{g}}$$

Thus $x = -\frac{1}{3}gt^2 + vt$

$$Db = -\frac{1}{3}gt^2 + vt$$

$$= -\frac{1}{3}g\left(\frac{3v}{2g}\right)^2 + v\left(\frac{3v}{2g}\right)$$

$$= -\frac{1}{3}g\frac{9v^2}{4g^2} + \frac{3v^2}{2g}$$

$$= -\frac{1}{3}\frac{9v^2}{4g} + \frac{3v^2}{2g}$$

$$= -\frac{3v^2}{4g} + \frac{3v^2}{2g}$$

$$= -\frac{3v^2}{4g} + \frac{6v^2}{4g}$$

$$\boxed{Db = \frac{3v^2}{4g}}$$

To obtain D_b in feet, we must write the speed as $\frac{22v}{15}$ (with v in mph) and g as $32.2\frac{ft}{s^2}$. This gives.

$$Db = \frac{3(v\,^{22}/_{15})^2}{4(32.2)}$$

$$\boxed{Db = \frac{v^2}{19.96}\ ft}$$

or approximated by.

$$\boxed{Db = \frac{v^2}{20}\ ft.}$$

The important factor here is the v^2 dependence rather than the exact numerical value. This is why the modelling has not been precise

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