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Math 3820
Dr. Arino
Assignment 2

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Good. Some explanations needed as to how equations governing the dynamics are obtained from your hypotheses ... (see me for help, if needed).

The problem that the model is formulated for is the coexistence and exclusion of species over a region of similar habitable patches. Many environments have a patchy, island-like pattern of occurrence. Since the balance of local extinction and colonization would leave some patches unoccupied even without competitors, species may coexist even when all the patches are the same. It is generally assumed that species that occur on s a small fraction of the available patches will have little effect on each other because of their co-occurrence would be exceedingly rare event. Regional competition coefficients are found when species affect the local extinction or migration rates of each other. Rare species can regulate each other and even exclude other species completely. There is evidence of competition between rare species.

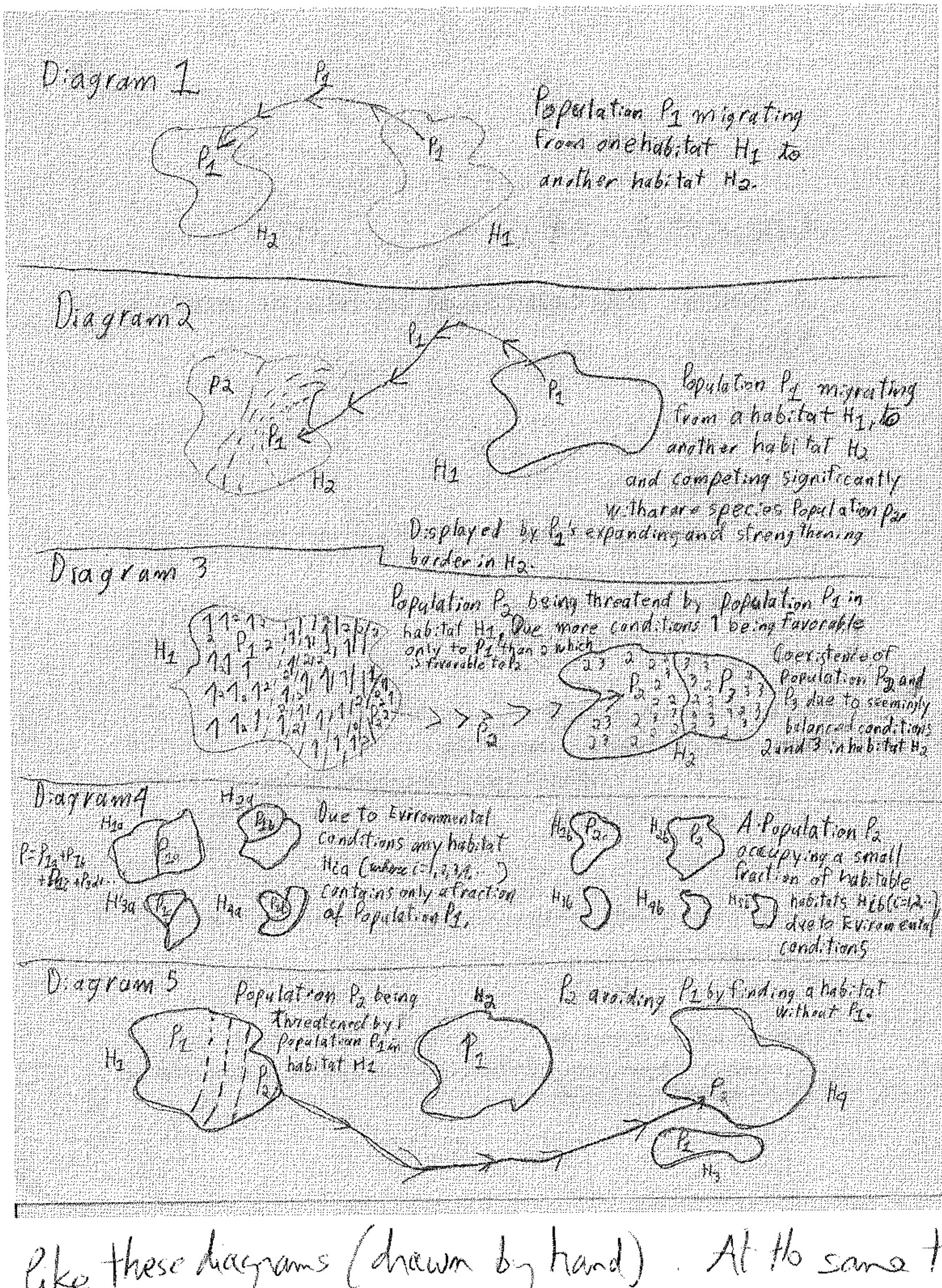
An immigration-extinction model developed by R. Levins can be used for predicting the number of islands, or island-like habitats, occupied by a species and it can allow for competition to affect either the immigration or extinction rate. In contrast to traditional competition theory, the focus will be on changes in the number of populations of a species, rather than on the sizes of the local populations.

First I'll present the migration-extinction model for a species in the absence of competitors (Diagram 1). Then the possibility of significant competitive effects on a rare species by other species (Diagram 2), for example the rare species being driven to extinction. Next will be the conditions for coexistence of two species in a habitat where two populations competition isn't significantly affecting the other nor causing extinction (Diagram 3). Consequently the effect of environment on coexistence (Diagram 4) in the situation where a small fraction of a population inhabits any habitat and where the population inhabits a small fraction of habitats. Lastly mechanisms for the avoidance of competition (Diagram 5).

A group of predaceous ants recorded by R.E. Gregg in Colorado, all of which are rare, showed a lower than expected microhabitat overlap. R.H. MacArthur and E. R. Pianka predicted reduced micro-habitat overlap for "searching" predators competing for the same prey; the data from R. Levins and D. Culver seemed to confirm R.H. MacArthur and E. R. Pianka's optimization model. The problem is not why the ants reduced competition by the optimization proposed by R.H. MacArthur and E. R. Pianka, but rather how there could be any significant competition to reduce.

Call me old fashioned, but in a written work, I prefer if abreviations like those are not used...

In your report, the papers you mention will have to be "properly" cited.



I quite like these diagrams (drawn by hand). At the same time, in your report, it would be better to generate them using some software, it will look cleaner and make printing easier...

In this immigration-extinction model I'll consider a number of local populations, total number of sites, extinction rate per population, and the rate of migration from one given site to another given site. The migration rate from one patch to any other patch is assumed to be the same. Making one species common and another with similar biology rare can occur with small difference in rate of migration and extinction rate. While a standard explanation is that species are rare, because suitable habitats are rare, which may be true for many situations, this is not necessarily the case. So I'll look for competition between initially rare species.

I'll look for the conditions where invasion of a species is impossible and then assume one rare species can exclude another rare species from a region. Insufficient time for a species to reinvade an area where another closely related species occurs is the usual explanation for allopatry, particularly for rare species. However some arguments will indicate that competitive exclusion may be important.

Since migration depends primarily on the distance between patches, whereas the extinction rate depends primarily upon local conditions, I'll look at the effect on coexistence of increasing extinction rates while the migration rate is held constant. Then decrease the migration rate while the extinction rates are held constant. ← good!

? Two possible evolutionary responses are by a species faced with competition acting extinction rate are assumed to be, to avoid patches where the second species is present and reduce the extinction rate. However a species may not be able to reduce its extinction rate so increasing the probability of avoiding a patch where the second species is present may be the only strategy. The rate of extinction at any particular site can be related to stability of the local community matrix, at least in a qualitative way.

A reasonable interpretation of a migration rate is that it is the rate of "successful" migration. However "successful" is difficult to define, but the essential point is that a model allows for the effect of residence on a site by another species. One possible definition of successful migration is where the send or spore reproduces at least once. It will be assumed that in the analysis of extinction competition, a species can avoid patches where a second species is present or evolve to reduce successful migration rate. ?

The immigration-extinction model for a species without competitors given by, letting N be the number of local populations, T the total number of sites, x the extinction rate per population and m' the rate of migration from one given site to another. Then

$$\frac{dN}{dt} = m'N(T - N) - \underbrace{XN}_{= x?}. \quad (1)$$

Explain this equation better.

Letting $p = N/T$ and $m = m'T$ makes the equation easier to manage.

$$\frac{dp}{dt} = mp(1 - p) - xp \quad (2)$$

p is easy to interpret, so you should do that.

At equilibrium:

$$\hat{p} = 1 - x/m, \tag{3}$$

which is the proportion of sites occupied in the absence of competitors.

Now for the model for the effect of competition on extinction rate, letting x_1 be the extinction rate when the second species is present and x_0 when the second species is absent, Then

$$\frac{dp}{dt} = mp(1 - p) - p[x_1q + x_0(1 - q)], \tag{4}$$

once again, you should explain the mechanisms leading to this equation.

where q is the proportion of sites occupied by the second species.

At equilibrium:

$$\hat{p} = 1 - x_0/m - [(x_1 - x_0)/m]q. \tag{5}$$

Letting θ be the probability that species p avoids a patch where species q is present. Then Eqn. (4) becomes

$$\frac{dp}{dt} = mp(1 - p) - p[x_1(q - \theta q) + x_0(1 - q + \theta q)]. \tag{6}$$

At equilibrium:

$$\hat{p} = 1 - x_0/m - [(x_1 - x_0)/m](q - \theta q). \tag{7}$$

Letting m_1 be the migration rate when the second species is present and m_0 when the second species is absent, the rate of change of p is

$$\frac{dp}{dt} = p(1 - p)[m_1q + m_0(1 - q)] - xp. \tag{8}$$

At equilibrium:

$$\hat{p} = 1 - [x/(qm_1 + m_0(1 - q))]. \tag{9}$$

A species can avoid patches where q is present, or evolve to reduce m_1, which gives the equation by analogy to (6):

$$\frac{dp}{dt} = p(1 - p)[m_1(q - \theta q) + m_0(1 - q + \theta q)] - xp.$$