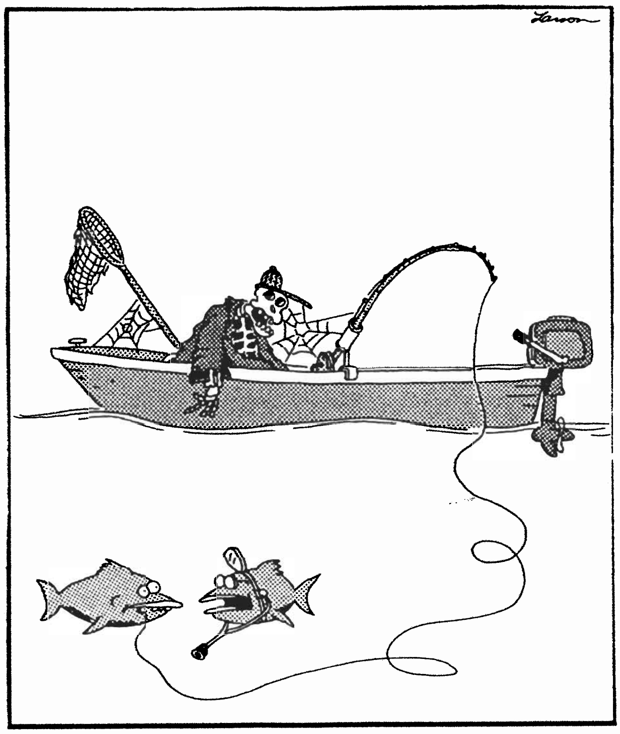
Analysis and Applications of the Dissolved Oxygen Deficit Resulting from Wastewater Treatment Discharge



"Well, first the bad news — you're definitely hooked."

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6.337 Mathematical Modeling

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This goal of this paper is to formulate a model that will trace the dissolved oxygen deficit in a stream where a wastewater effluent has been discharged. H.W. Streeter and E.B. Phelps (1925) were the first created a model to trace dissolved oxygen concentrations. This paper will show how they might have formulated their model and will also extend its use to practical purposes. There are numerous variables to consider, but first it would be beneficial to familiarize ourselves with the final effluent discharged from a wastewater treatment plant and its effect on a stream.

When raw sewage first enters the wastewater treatment plant it contains high amounts of biodegradable organic matter. If untreated this can have a hazardous effect on a water body. The major impact of large quantities of biodegradable organic matter is with respect to the oxygen content of the water body. The biodegradable organics will be degraded in the stream. This process requires significant amounts of dissolved oxygen. This means that there will be less oxygen available in the stream. This can have hazardous effects on the aquatic life of the water body, such as fish, as they require sufficient amounts of dissolved oxygen to survive (Rich, 1980). Another side effect of high amounts of biodegradable organic matter in a water body is the foul odor that they produce. A wastewater treatment plant is able to significantly reduce the amounts of biodegradable organics, but it does not totally eliminate them. Small quantities are still added to the waterbody where they deplete the amount of dissolved oxygen. This effect is not permanent as the water body has the ability to recover on its own over time. It does this by absorbing oxygen from the atmosphere and from photosynthesis (Rich, 1980). The absorption of oxygen from the atmosphere is referred to as reaeration. The rate of reaeration is dependent on factors such as the depth, width, and surface area of the stream (Thomas, 1972). Green plants and algae in the water body expel oxygen when they intake carbon dioxide. The rate of photosynthesis is also effected by the turbidity of the water body. An increase in turbidity makes it more difficult for the sunlight to reach the organism in the water body. A wastewater discharge increases the turbidity of the water body and therefore decreases the rate of photosynthesis and the rate at which organisms provide dissolved oxygen to the water body.

The model that will be formulated will calculate the dissolved oxygen deficit of the water body. The dissolved oxygen deficit is the amount of oxygen that must be recovered for the stream to become saturated with dissolved exceeds!

see p12

oxygen. It is important to be able to trace the dissolved oxygen deficit in a stream or river because if the deficit drops below a certain point it can become fatal to the aquatic life in the water body and hazardous to those who consume it. We will begin to formulate our model by looking at the dissolved oxygen demand of the water body.

Dissolved Oxygen Demand

The oxygen demand of the stream is dependent on the biodegradable content added by the discharge of wastewater. The biodegradable content of the stream is measured by it's biochemical oxygen demand, otherwise known as its BOD. It has been determined experimentally that the BOD profile resembles that of a first order decay process (Thomas, 1972). In other words, as the biodegradable organic content is degraded, less and less dissolved oxygen is demanded from the stream. This can be represented by a differential equation. The rate of change of BOD in the stream as flow time changes is proportional to the BOD in the stream. This is represented by the differential equation (1).

$$dL/dt = -k_1 L \qquad (1)$$

where dL/dt is the rate of change in the BOD,
L is the BOD,
k_1 is a constant and deoxygenation parameter. It varies depending on the ability of the stream to break down the biodegradeable organic matter.

This equation can be solved using the technique of separation.

$$dL/L = -k_1 dt$$

Integrate both sides to get:

$$\ln|L| = -k_1 t + C$$,where C is a constant of integration

$$\ln L = -k_1 t + C$$

The absolute value signs are not necessary as we are only interested in positive values of BOD. Exponentiate both sides to get:

$L(t) = Fe^{-k_1 t}$,where $F = e^C$

If we let the initial condition be $L(0) = L_0$ we get :

$L(0) = Fe^{-k_1(0)} = L_0$

This can be solved for F:

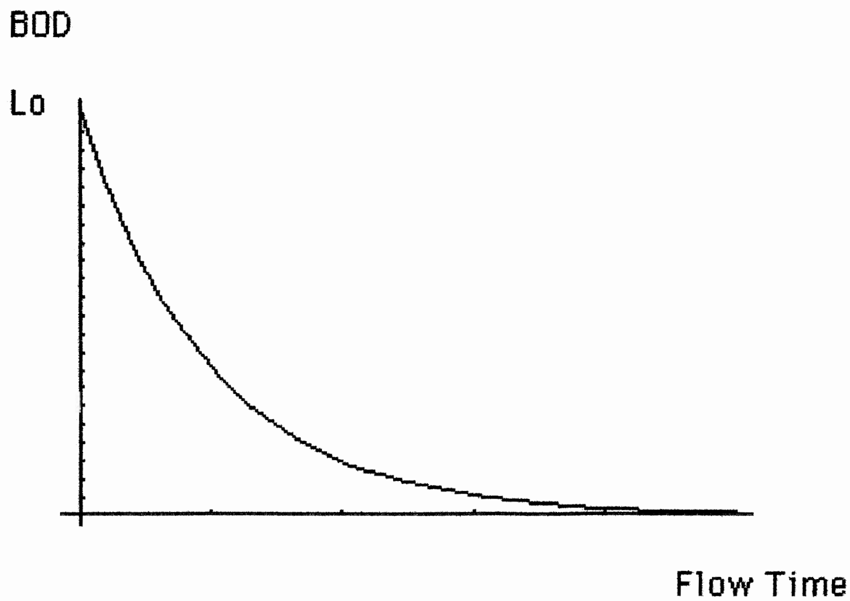
$F = L_0$

We can now get a functional representative of BOD over time:

$$L(t) = L_0 e^{-k_1 t} \tag{2}$$

A graph of equation (2) can be seen below in Figure 1. The steepness of the curve depends on the deoxygenation parameter k_1.

Figure 1: The change in the BOD over time.



Now that we have a function for the dissolved oxygen demand of the stream we must derive a function for the oxygen supply of the stream.

Dissolved Oxygen Supply

The oxygen supply of a water body is primarily dependent on oxygen absorption from the atmosphere and from the photosynthesis of green plants and algae in the water. To simplify the model we will assume that photosynthesis has no additive effect on the amount of dissolved oxygen in the stream. This is like saying that all the oxygen expelled by the plants and algae is used by the other aquatic organisms in the stream.

assumption OK

The process of absorption from the atmosphere replenishes the dissolved oxygen in a stream and is known as reaeration. We will begin by letting the amount of dissolved oxygen absorbed from the atmosphere as a function of time be denoted be C. The absorption of oxygen into the stream from the atmosphere is dependent on the amount of dissolved oxygen that the stream can hold. This is referred to as the saturation capacity of dissolved oxygen in the stream. We will let this be denoted by C_s. Different streams also have the capacity to absorb different amounts of oxygen and the rate they do so is dependent on the ratio of surface area of the stream to volume of the stream. We will let these constant factors be represented by K_f and A/V respectively. All of these factors play a role in the amount of oxygen absorbed back into the stream. To simplify we will let a new constant, k_2 become equal to $K_f A/V$. This is new constant k_2 is known as the reaeration coefficient.

2?

We will assume that the rate that dissolved oxygen is absorbed into the stream as flow time increases is proportional to the difference between the dissolved oxygen saturation capacity of the stream and the amount of dissolved oxygen in the stream. This is represented by the differential equation (3).

$$dC/dt = k_2(C_s - C) \qquad (3)$$

This can be rearranged to get:

$$dC/dt + k_2C = k_2C_s \qquad (4)$$

This equation can be solved using the integrating factor method.

$$e^{\int k_2 dt} = e^{k_2 t} \tag{5}$$

Multiply equation (4) by equation (5) to get:

$e^{k_2 t}dC/dt + e^{k_2 t}k_2 C = e^{k_2 t}k_2 C_s$ or:

$d(e^{k_2 t}C)/dt = e^{k_2 t}k_2 C_s$

Integrate both sides with respect to t to get:

$e^{k_2 t}C = C_s e^{k_2 t} + F$, where F is a constant of integration.

If we let $C(0) = C_0$, we can solve for F:

$F = Co - Cs$

Substitute this in for F to get:

$e^{k_2 t}C = C_s e^{k_2 t} + C_0 - C_s$

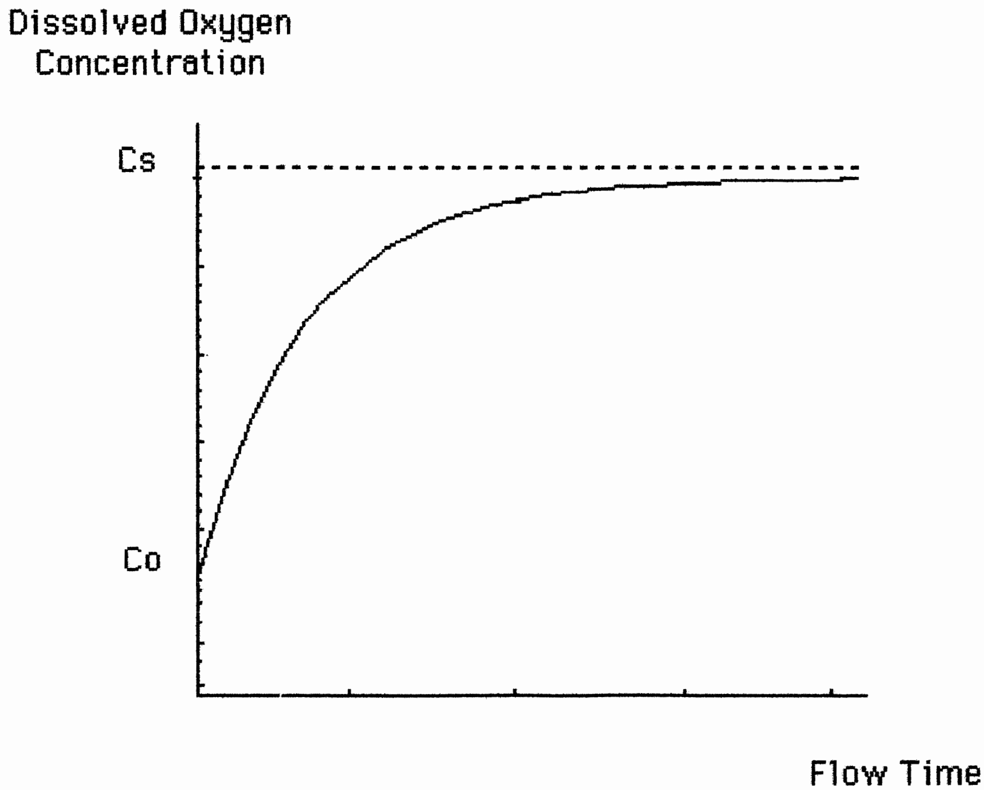
Divide this equation through by $e^{k_2 t}$ to get:

$$C(t) = C_s + C_0 e^{-k_2 t} - C_s e^{-k_2 t} \tag{6}$$

This function traces the amount of dissolved oxygen in a stream over time.

A graph of the equation (6) can be seen in Figure 2.:

Figure 2: The change in the concentration of dissolved oxygen over time.



We can also observe the relationship between these two factors. An equation can be obtained because both models contain the flow time variable. If we divide equation (1) by equation (3) we get the following equation:

(only in a special way & hence it means the elimination - autonomous)

$dL/dC = -k_1 L / (k_2(Cs - C))$

This equation is sep~~e~~arable.

$-dL/(k_1 L) = dC/(k_2(C_s - C))$

Integrate both sides:

$-(1/k_1)\int dL / L = (1/k_2)\int dC / (C_s - C) + G$,where G is a constant of integreation

This can be rearranged to get:

$\ln L = (k_1/k_2)(\ln(C_s - C)) + H$,where $H = k_1 G$

Exponentiate both sides to get:

$L = J((C_s - C)^{k_1/k_2})$,where $J = e^H$

$L(C_0) = L0$, so we can solve for J:

$L0 = J((Cs - C_0)^{k_1/k_2})$

This can be substituted back in for J to get:

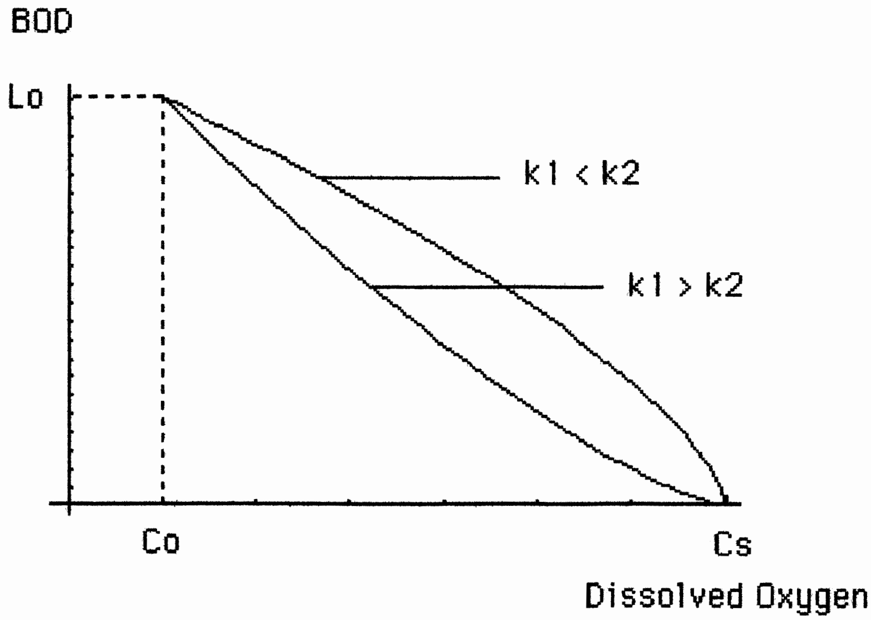
$$L(C) = L0(((C_s - C) / (C_s - C_0))^{k_1/k_2}) \tag{7}$$

A graph of equation (7) can be seen in figure 3. It shows that when the dissolved oxygen concentration in the stream is low the biochemical oxygen demand to break down the biodegradable organic matter is high. It is also consistent with figure 2 as C can only exist inbetween C_0 and C_s. When the BOD decreases the dissolved oxygen concentration of the lake is increased. As the dissolved oxygen concentration goes to its saturation capacity, C_s the BOD goes to zero. The relationship is concave up when $k_1 > k_2$ and concave down when $k_1 < k_2$.

$k_1 = k_2$?

see p10

Figure 3 : The effect of changing BOD on the concentration of dissolved oxygen in a stream



So far we have only looked at dissolved oxygen concentrations. We will now shift our focus to study the dissolved oxygen deficit of the stream. This is the model that was developed originally by Streeter and Phelps (1925). The dissolved deficit is the amount of dissolved oxygen that must be reabsorbed back into the stream to bring it back to its saturation level. Therefore the dissolved oxygen deficit at any given time is represented by the difference in the saturation capacity of dissolved oxygen and the amount of dissolved oxygen at any given time. If we let D be the dissolved oxygen deficit then we can get:

$D = C_s - C$

From this we can get :

$dD/dt = -dC/dt = -k_2(C_s - C)$ or:

$$dD/dt = -k_2 D \qquad (7)$$

?

This function represents the rate of change in the dissolved oxygen deficit over time. It is a negative relation because over time the amount of dissolved oxygen in the stream increases and therefore the dissolved oxygen deficit decreases. This function does not take into account the effect that the biodegradable organic matter from the wastewater has on the rate of change of dissolved oxygen. Given by

Previously we found that the amount of biodegradable organic matter in the stream is measured by the BOD of the stream. The rate of change of the BOD in a stream was previously found to be ~~equal to~~ equation (1). We know that biodegradable organics use up dissolved oxygen and will therefore increase the dissolved oxygen deficit. Due to this we can change the sign on the right hand side of equation (1) and add it to equation (7) to get:

$$dD/dt = k_1 L - k_2 D$$

We now have a model that takes into account the effect of biodegradable organic matter from a wastewater treatment plant and the reaeration effect of the oxygen absorption from the atmosphere. This model can be integrated to obtain what Harold Thomas (1972) in <u>Models for Managing Regional Water Quality</u> refers to as the Street-Phelps "oxygen-sag" function.
We will now attempt to solve it.

First, we can assume that the initial conditions are $D(0) = D_0$ and $L(0) = L_0$. We can immediately substitute equation (2) for L (as this is L(t) with the assumed initial condition) to get:

$$dD/dt = k_1 L_0 e^{-k_1 t} - k_2 D$$

This can be rearranged to the form:

$$dD/dt + k_2 D = k_1 L_0 e^{-k_1 t} \qquad (8)$$

We can now use the integrating factor method to sole the differential equation.

$e^{\int k_2 dt} = e^{k_2 t}$,multiply equation (8) through by this to get:

$e^{k_2 t} dD/dt + e^{k_2 t} k_2 D = k_1 L_0 e^{(k_2 - k_1)t}$ or:

Do we need to consider the case $k_2 = k_1$ or not?

$d(e^{k_2 t} D)/dt = k_1 L_0 e^{(k_2 - k_1)t}$

$k_2 \neq k_1$

Integrate both sides with respect to t to get:

$e^{k_2 t} D = \int k_1 L_0 e^{(k_2 - k_1)t} dt$ or:

$e^{k_2 t} D = k_1 L_0/(k_2 - k_1) e^{(k_2 - k_1)t} + C$, where C is a constant of integration.

We can solve for C by imposing the initial condition $D(0) = D_0$ to get:

$D_0 = k_1 L_0/(k_2 - k_1) + C$ or:

$C = D_0 - k_1 L_0/(k_2 - k_1)$

We can substitute this back into the original equation to get:

$e^{k_2 t} D = k_1 L_0/(k_2 - k_1) e^{(k_2 - k_1)t} + D_0 - k_1 L_0/(k_2 - k_1)$

Divide through by $e^{k_2 t}$ to get:

$D(t) = k_1 L_0/(k_2 - k_1) e^{-k_1 t} + D_0 e^{-k_2 t} - k_1 L_0/(k_2 - k_1) e^{-k_2 t}$ or:

$$D(t) = k_1 L_0/(k_2 - k_1)(e^{-k_1 t} - e^{-k_2 t}) + D_0 e^{-k_2 t} \qquad (9)$$

This function traces the dissolved oxygen deficit over time.

The graphs of equation (9) can be seen in figure 3 and figure 4. When the the three functions are plotted on figure 4 the relationship between BOD, the dissolved oxygen concentration, and the dissolved oxygen deficit can be seen. The deficit increases to a maximum as long as BOD exceeds the dissolved oxygen being supplied by reaeration. At the maximum the BOD is equal to the dissolved oxygen supplied by reaeration. The deficit continues to

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decreases as the BOD goes to zero and the dissolved oxygen in the stream goes to the saturation capacity of the stream.

Figure 3 : The change in the dissolved oxygen deficit over time.

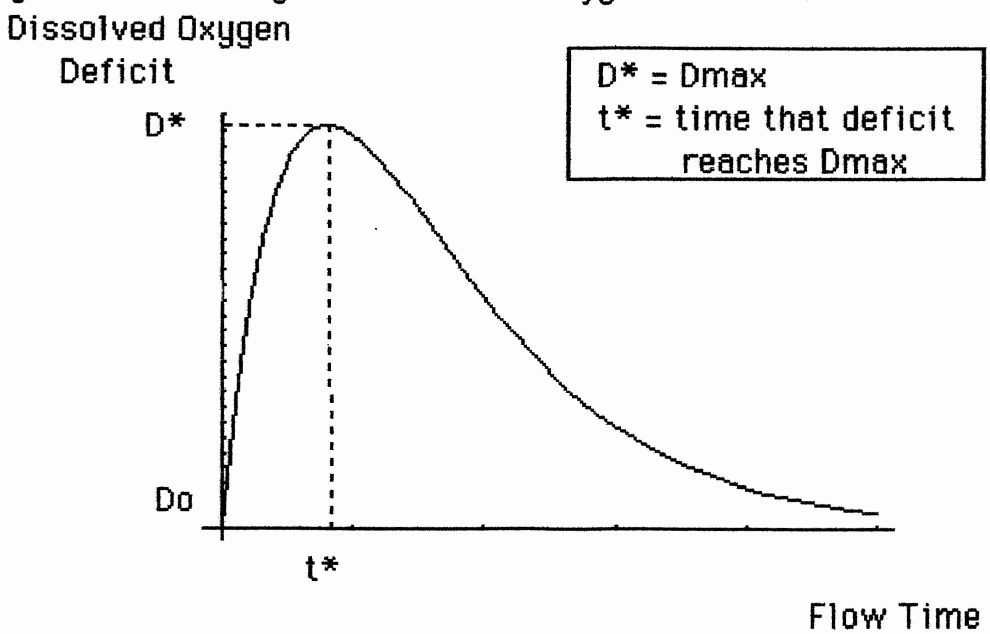
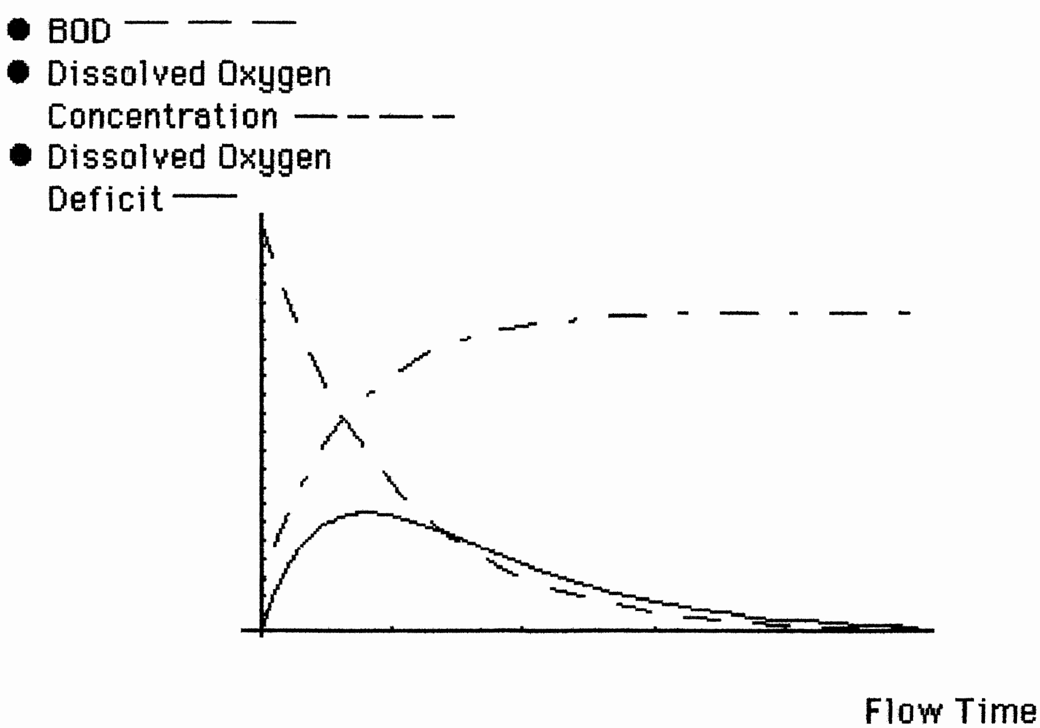


Figure 4 : This is the plot of BOD, the dissolved dxygen concentration, and the dissolved oxygen deficit all plotted versus flow time.



The most important feature of the dissolved oxygen deficit curve is the maximum. This point is critical to health of the stream. As discussed earlier the aquatic life in the stream needs a certain amount of dissolved oxygen to survive. If the dissolved oxygen deficit increases past a certain point it can be fatal to the aquatic life in the stream. ~~This~~ maximum dissolved deficit can be calculated by setting the first derivative equal to zero as seen below.

$$dD/dt = 0 = k_1 L_0 e^{-k_1 t} - k_2 D \tag{10}$$

We can then solve equation (10) for D to get:

$$D = (k_1 L_0 e^{-k_1 t})/k_2$$

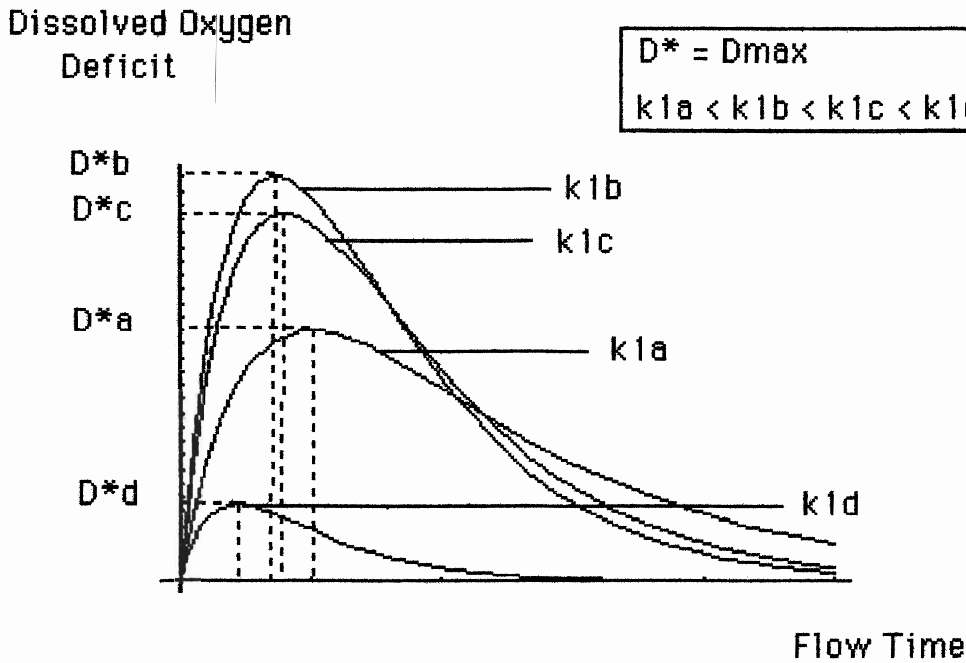
maximum

We will refer to this dissolved oxygen deficit as D_{max}.

$$D_{max} = (k_1 L_0 e^{-k_1 t})/k_2$$

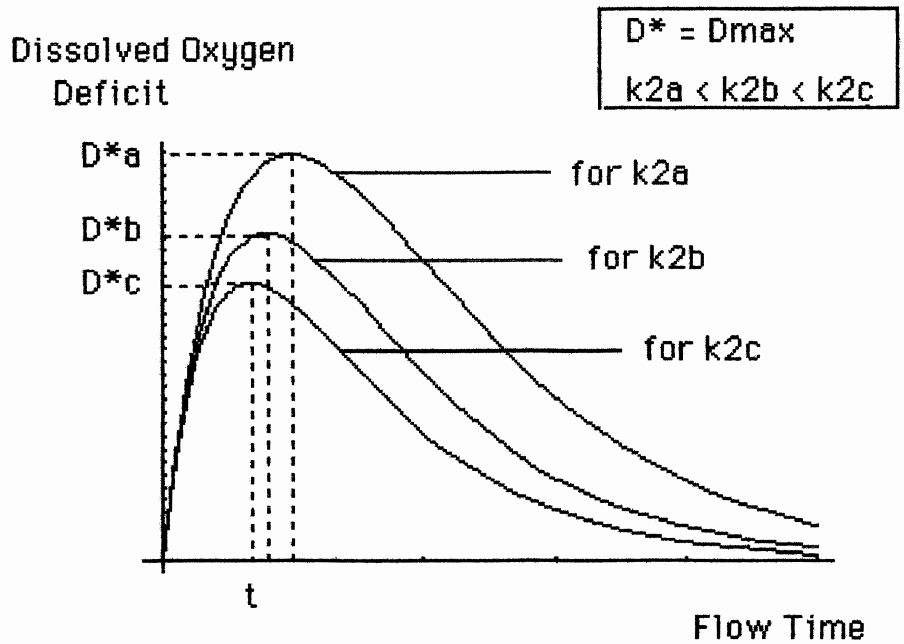
Along any stretch in the stream D_{max} depends on k_1, k_2, and L_0. k_1 and k_2 vary for different streams and can be calculated if two data points along the stream are found through experimentation. If we have values for k_1 and k_2 that are fixed for the stream, then the D_{max} can be controlled by controlling the initial amount of biodegradable organic material. This is an important aspect to consider when building a wastewater treatment plant. If the stream that the final effluent will be discharged into has high values for k_1 and k_2 then the treatment plant will not have to be as successful at reducing the amounts of biodegradable organic matter. On the other hand if the stream does not have a high capacity to recover on its own then the treatment facility will have to more successful at reducing the biodegradable organic matter. The graph in figure 6 demonstrates how deoxygenation parameters produce different dissolved oxygen deficits. It can be seen in the graph that the dissolved oxygen deficit does not decrease every time that the deoxygenation parameter increases. This is because the dissolved oxygen deficit function is dependent on the difference between k_2 and k_1. As the difference decreases the deficit increases. The curve with k_{1b} has the smallest difference between k_2 and k_1 and it has the largest maximum deficit. The smallest maximum deficit is obtained when k_1 is large and the difference between k_2 and k_1 is also large. This is the curve with k_{1d}.

Figure 6 : The effect of changing the deoxygenation parameter in the dissolved oxygen deficit function.



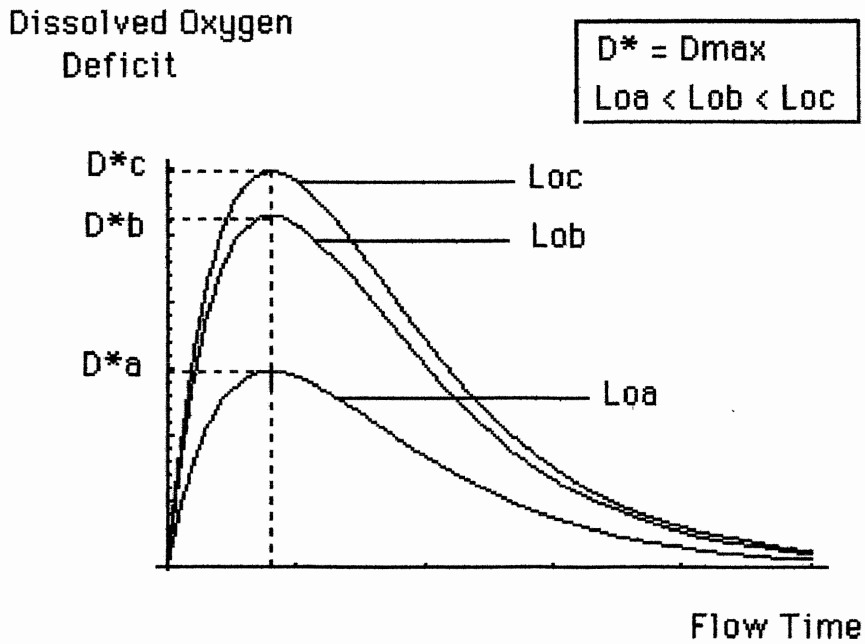
The relationship between k_2 and the dissolved oxygen deficit is straight forward. The larger the reaeration parameter the smaller the maximum deficit. This pattern can be seen in the figure 7.

Figure 7 : The effect of changing the reaeration parameter in the dissolved oxygen deficit function.



The opposite is true for the relationship between the discharge of biodegradable organic matter and the dissolved oxygen deficit. The smaller the initial biodegradable organics are contained in the final effluent from the treatment plant the smaller the maximum deficit will be. This pattern can be seen in figure 8.

Figure 8 : The effect of changing the initial amount of biodegradable organic matter in the dissolved oxygen deficit function.



It is also helpful for practical purposes to determine the flow time for the function to reach Dmax. This knowledge is beneficial when siting a wastewater treatment plant. If the technology of the treatment facility is limited the facility can only achieve a predetermined reduction in biodegradable organic matter. Therefore the maximum dissolved oxygen deficit cannot be altered. The stream into which the effluent is discharged could have many other purposes. For instance there could be a campsite situated on the stream. The popularity of the campsite might hinge on it being a good fishing spot or that the stream is a good source for drinking water. It would then be beneficial to site the treatment plant far enough up stream so that the maximum dissolved oxygen deficit does not occur arround the campsite.

The flow time for the maximum dissolved oxygen deficit can be calculated by substituting equation (10) into equation (8) and solving for t.

$$(k_1 L_0 e^{-k_1 t})/k2 = k_1 L_0/(k_2 - k_1)(e^{-k1t} - e^{-k2t}) + D_0 e^{-k2t}$$

Divide through by $e^{-k_1 t}$ to get:

$(k_1L_0)/k_2 = k_1Lo/(k_2 - k_1) - k_1L_0/(k_2 - k_1)e^{(k_1 - k_2)t} + D_0e^{(k_1 - k_2)t}$

The constant term on the right hand side can be moved to the left to give:

$(k_1L_0)/k_2 - k_1L_0/(k_2 - k_1) = - k_1L_0/(k_2 - k_1)e^{(k_1 - k_2)t} + D_0e^{(k_1 - k_2)t}$

$e^{(k_1 - k_2)t}$ can be factored out of the equation on the right to give:

$(k_1L_0)/k_2 - k_1L_0/(k_2 - k_1) = [D_0 - k_1L_0/(k_2 - k1)]e^{(k_1 - k_2)t}$

This can then be reduced to:

$[(k_1L_0)/k_2 - k_1L_0/(k_2 - k_1)] / [D_0 - k_1L_0/(k_2 - k_1)] = e^{(k_1 - k_2)t}$

If we then take the logarithm of both sides we get:

$\ln[(k_1L_0)/k_2 - k_1L_0/(k_2 - k_1)] / [D_0 - k_1L_0/(k_2 - k_1)] = (k_1 - k_2)t$

Divide through by $k_1 - k_2$ to get:

$[1/ (k_1 - k_2)] \ln[(k_1L_0)/k_2 - k_1L_0/(k_2 - k_1)] / [D_0 - k_1L_0/(k_2 - k_1)] = t$ or:

$t = [1/ (k_1 - k_2)] \ln[(k_1L_0)/k_2 - k_1L_0/(k_2 - k_1)] / [D_0 - k_1L_0/(k_2 - k_1)]$

This is the point were the dissolved oxygen deficit is at its maximum.

Although there are many practical applications of this model one must remember that the model is a simplification of real biological setting. The model does not take into account variations in temperature. Temperature can have a significant effect on the reaeration of dissolved oxygen. It was found that temperature increases in the stream have an effect on the model parameters. It was found that a temperature increase of 5 degrees in the stream causes a 28 percent increase in k_1 and a 16 percent increase in k_2 (Thomas, 1972). good

Another downside to the model is that every river is different and tests must be performed to get data to calculate k_1 and k_2. Rivers and streams are also prone to changes in flow time. The flow time is also not always constant through out the length of the stream. This could lead to difficulties in interpreting real life data. Although it is not within the scope of this paper it would be beneficial to use a probabilistic variation of this model as opposed to the deterministic model that was used.

Although this model is simplistic and has many assumptions it can act as a good estimating tool. One could use this model both when considering the best location for a wastewater treatment facility and for determining the acceptable level of biodegradable organic matter in the final effluent as has been demonstrated in this paper. The unpredictable nature of natural systems must be considered when applying this model and for this reason the results that it produces should only used as rough estimates as opposed to cold hard facts.

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