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Mathematical modeling

an idealization of the real-world problems and never a completely accurate representation

- Identify the most important processes governing the problem (Theoretical assumptions)
- Identify the state variables (quantities studied)
- Identify the basic principles that govern the state variables (Physical Laws, interactions, ...)
- Express mathematically these principles in terms of state variables (Choice of the formalism)
- Identify and evaluate the values of parameters
- Make sure of the consistence of units

How to represent a problem

- Static vs dynamic
- Stochastic vs Deterministic
- Continous vs Discrete
- Homogeneous vs Detailed

Formalism

ODE, PDE, DDE, SDE, Integral equation, integro-differential equations, Markov Chains, Game theory, Graph theory, Cellular automata, L-systems . . .

Biological problems

- Ecology (Predator-Prey system, Populations in competition ...)
- Etology
- Epidemiology (Propagation of infectious diseases)
- Physiology (Neuron, cardiac cells, muscular cells)
- Immunology
- Cell biology
- Structural biology
- Molecular biology
- Genetics (Spread of genes in a population)
- ...

Lotka-Volterra Predator-Prey Model

Assumptions

1. Growth of prey population is exponential in absence of predators. Prey grow in an unlimited way when no predation.
2. Predators depend on the presence of their prey to survive. Predators decline exponentially in absence of prey.
3. The rate of predation depends on the likehood that a victim is encountered by a predator
4. The rate of growth of the predator population is proportional to food intake.

Equations

- $x(t)$ biomass or population densities of the prey
- $y(t)$ biomass or population densities of the predators

$$\frac{dx}{dt} = ax - bxy$$
$$\frac{dy}{dt} = -cy + dxy$$

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Parameters

- a: net growth rate of the prey population when predators are absent $a > 0$, $(1/time)$.
- c: net death rate of the predators in the absence of prey, $c > 0$, $(1/time)$.
- b/d: efficiency of predation = efficiency of converting a unit of prey into a unit of predator mass.