Mathematical Modeling: Call Volume Distribution for a Major Call Centre

This paper is meant to be a discussion of some possible mathematical models for the probability distribution of individual events, and also for the trends in time for these events. The Purolator Courier call centre in Moncton, NB was kind enough to send me call volume data for the last month, including daily calls. In analyzing this data I will attempt to answer two main questions.

1) Does the normal distribution do a suitable job of describing the distribution of calls that come in on a given half hour interval of a day?
2) What function best fits the curve plotting number of calls vs time intervals?

How well does the standard normal statistical distribution describe the occurrence of real life events? To answer this question I used data for call volume from an international courier call centre. For each day I compiled the number of calls in each half hour interval for all working hours. All data used was for weekdays (weekend call volume was substantially lower), and I assumed the average and standard deviations for these intervals to be constant (ie we can expect the same distribution regardless of the day). The distribution of calls for each of the half hour intervals I expect to be approximately normal and I will use these averages to attempt an approximation of a function, perhaps a polynomial, to the data using least squares analysis. Reasonable bounds for each of these values will likely give similar functions, shifted above and below the average function. We will use least squares to estimate model parameters and find the best –fitting function in each of these areas. Using the standard deviation we can find a 95% confidence interval (bounds two standard deviations on either side of the approximating function) where we have a 0.95 probability that the number of calls will lie in a certain range.

Individual Call Volumes for Half Hour Intervals

1.) Normal Distribution

Table 1 shows the raw data for call volume by i+1 intervals (the last interval is the first interval of the next day, which we want to be the same as the first interval of the first day.) Also calculated are average and standard deviation bounds. Figures 1 through 5 show a histogram of probabilities vs number of calls, generated in JMPin, for five evenly spaced sample time intervals: 12:00-12:30AM, 4:00 – 4:30 AM, 8:00-8:30AM, 12:00 – 12:30PM, 4:00 to 4:30 PM and 8:00 – 8:30 PM. The distributions vary widely in their averages, standard deviations and the ratios between these two amounts. The histograms look roughly bell-shaped (i.e. "normal"), but they seem to be not quite symmetric, but rather skewed to one side.

Normal distributions are usually used to describe continuously varying data (as opposed to number of calls, which is discrete), but for sufficiently large volumes of calls (such as occur in the middle of the day) discrete data approaches the continuous distribution. Data must also then be rounded to the nearest integer. There is no hard and fast rule as to what constitutes "sufficiently large", but it must be sufficient that three standard deviations away from the mean does not encompass an impossible negative number of calls. In Table 1 we see that this does in fact occur a number of times in the early morning hours, so for simplicity we will exclude the hours from 12:00 AM to 7:00AM (thereby eliminating Figures 1 and 2)

We can determine whether the normal distribution assumption is justified by using least squares. The normal probability distribution function is given by

$$P_i(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

where μ = theoretical mean (approximated by sample mean)
σ = standard deviation from mean
x = some number of calls $(0, 1, 2, \ldots 1600)$
$P_i(x)$ = probability of x calls occurring in the ith interval

which is in the form

$$P_i(x) = a e^{-b^2(x-\mu)^2}$$

where $a = 1/\sqrt{2\pi\sigma}$
$b = 1/2\sigma^2$

?

Taking the natural logarithm of both sides we find a linear function, which we can approximate with least squares using the following transformation:

$$\ln P_i(x) = \ln(a) - b(x-\mu)^2$$

I used the statistical package JMPin to approximate   and   for four sample time periods i=17, 25, 33, 41 (12:00-12:30AM, 4:00 – 4:30 AM, 8:00-8:30AM, 12:00 – 12:30PM, 4:00 to 4:30 PM and 8:00 – 8:30 PM.). For each I determined the average and standard deviation, from which can be determined. Since we have only 21 data points, and the data range is for up to hundreds of calls in some cases, we obviously cannot determine probabilities of single call volumes, so we group the data using a bar graph known as a histogram, generated for each sample point. (see Figure __). The normal probability functions Pi(t) above was also plotted for the four given sample points. The distributions vary widely in their averages, standard deviations and the ratios between these two amounts. The histograms look roughly bell-shaped (i.e. "normal"), but they seem to be not quite symmetric, but rather skewed to one side.

2.) Poisson Distribution

The excluded range of time intervals could probably be described more accurately by a Poisson distribution (the equation below)

$$P_i(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

with a logarithmic transformation as shown below.

$$\ln P_i(x) = x \ln \lambda - \lambda - \ln(x!)$$

Where $P_i(x)$ = probability of exactly x calls in the ith time period
λ = average number of calls in the ith time period
x = discrete random variable for no. of calls (x can take on values 0,1,2,3…)

For example, when i=6,

we can find the probability of say x=0 calls occurring as follows.

$$P_6(0) = \frac{9^0 e^{-9}}{0!} = 0.0001234 \text{, or about } .01\%$$

The Poisson distribution is used to describe the probability of a certain type of discrete event occurring within a given (often time) interval. It is usually used when the average (lambda) of the variable to be described is relatively small. (It is not symmetric and does not allow the variable x to take on values less than zero, whereas the normal distribution has no such bound.) The disadvantage to the Poisson function is that it does not lend itself to any apparent transformation of variables for least squares approximations, so we will have to omit its analysis.

Average Call Volume over a 24 Hour Day

Graph __ plots the average number of calls Purolator received in each half hour interval during a 21 day period. Chart __ shows half hourly call volume for each day of a 21 day period, along with the average and standard deviation of the number of calls over each time interval, and this data is plotted in Graph __. The original data given was for 31 days (chart __), but the number of calls received on weekends was substantially lower than the numbers received on weekdays, so these days were excluded. There appear to be two peaks, one at 12:30 (26^{th} interval) and one at 3:30 (32^{nd} interval) before the close of many businesses at 4:00 or 5:00 PM, as well as the last stretch of time to schedule a pickup to be delivered by 9 AM the next business day. Immediately following the first peak calls stbilize or drop off during lunchtime and pick up again (no pun intended) thereafter. As can be seen from the sample individual days' call volume (Graphs __ through __), the graph of average daily calls is quite indicative of the trends in the number of calls throughout a typical day. Peaks generally occur in same time intervals, with similar numbers of calls, with the 2 lunch hour time periods spanning 1:00 to 2:00 PM consistently receiving lower numbers of calls.

The daily call volume looks as though it may be approximated by some polynomial. I used the statistical program JMPin to estimate model parameters for polynomials of degrees 4, 5 and 6 and to plot the fitted polynomials against the points.

Possible Models:

1) Polynomial of degree 4: (p. 16)

Standard error

2) Polynomial of degree 5: (p. 17)

Standard error

3) Polynomial of degree 6: (p. 18)

Standard error

(from Figure 3)

However, none of these polynomials appear to capture the 12:30 to 1:00 PM local maximum nor the subsequent lunch hour slowdown. (JMPin will not plot polynomials with degree higher than 6.) The most noticeable flaw in the model is that none of polynomials flatten out to account for the late night/early morning hours when there are virtually no calls, and instead, generate maxima or minima where there are none. Excluding the points numbered 1 through 15 (from 12:00 AM to 7:00AM from the fit yields the results in Table 2, which are not significantly different from the first set of polynomials. This can be immediately seen from the modified polynomial graphs and a comparison of model parameters for polynomials of like degree.

On closer inspection, it appears that ~~two~~ different polynomials may do a more satisfactory job of approximating the distribution, each with its own maximum. I divided the range of i into i=15 to 27 and i=27 to 49. (See results in Figures __ to __). The best-fitting polynomials were both of degree 4, and in the case of the second polynomial the degree 5 approximation was exactly the same as for degree 4. Predicted and residual values are shown in Table__. Residual sum of squares is quite high for both the one polynomial and the two polynomial approximations, however, there is an improvement of about 500% (from 326976 to 64 048). Although perhaps a crude judge of fit, residual sum of squares is one of the only quantitative basis of fit we have to go on, although we could have summed the absolute values of residuals as well.

Another alternative could be to temporarily ignore the lunch hour interval completely or approximate it by . By eliminating points 27, 28, and 29 we eliminate all extrema except for the absolute maximum at 3:30. We can then attempt once more to find the best fitting polynomial. (Note that predictions made for the three missing points would have to be made on some other basis, but the results may have little statistical meaning.)

Yet another possibility would be to use more than two functions to approximate different sections of the graph. Without the lunch hour points, the section from points 21 to 32 (10:00 AM to 5:30 PM) can be approximated by a straight line, and we could approximate the other two pieces either with straight lines of different slopes, or if there is some pattern in the residuals, this may indicate that a polynomial or some other curve would give a more suitable approximation.

Conclusions:

Purolator's call volume data may well be normally distributed over each i time intervals, but we would likely need more data in order to be certain. One must note that results may be slightly different due to the fact that call volume data is discrete. A Poisson distribution would likely be more accurate in this regard, however, with so few data points there is not likely a significant difference, although the slightly skewed distribution may also point to the Poisson.

Overall, polynomials did a fair approximation of the time distributed data. Two separate approximations using polynomials of degree 4 appeared to do a reasonable job of describing the data, given the fact that our averages used in the plot were over only 21 data points. If we were to take another random sample of a 21 day interval, the results could be at least anywhere within the 95% confidence interval, so it might not do us any good to try to fit these points more accurately. However, if we wished to do so, a spline function which combines multiple polynomials or other similar functions, would likely do nicely. JMPin's spline with lambda=0.01 provided a very close fit, as I recall, but I have not included it here as I was unable to interpret how its approximation worked.

References:

Levin, Richard I., Statistics for Management, 3rd Edition, Prentice-Hall Canada Ltd., 1978

Lee, Wayne, Experimental Design and Analysis, Freeman & Co., 1975

Advice from:
T.G. Berry
M. McDermid
P.W. Aitch~~e~~ison

Submitted by: Delene Matula

T1/1

Table 1

Daily Call Volume Data

	Time								
i	1	2	3	4	5	6	7	8	9
Hour	0:00	0:30	1:00	1:30	2:00	2:30	3:00	3:30	4:00
Day									
1	30	20	15	10	6	2	5	2	2
2	52	45	21	19	12	10	8	4	3
3	47	34	28	26	15	8	11	3	4
4	55	40	28	21	13	7	2	2	4
5	53	34	25	21	8	9	3	7	5
6	35	28	19	13	6	6	5	4	3
7	39	33	30	21	12	8	8	5	3
8	51	33	24	20	17	10	14	3	3
9	43	39	24	18	17	12	5	7	4
10	49	32	28	22	23	10	11	4	2
11	29	23	20	16	12	10	5	5	4
12	56	31	18	10	11	12	7	3	2
13	53	46	32	17	15	8	3	4	4
14	52	26	24	23	26	8	7	9	4
15	51	35	13	17	15	12	6	6	4
16	31	16	12	14	5	12	7	5	3
17	50	36	19	14	15	7	4	1	0
18	60	43	22	20	14	11	6	4	4
19	62	41	29	20	14	11	6	6	2
20	49	31	29	20	16	11	7	9	5
21	44	30	32	21	15	10	7	7	4
Average	47.190	33.143	23.429	18.238	13.667	9.238	6.524	4.762	3.286
Std Dev	9.287	7.643	5.844	4.070	4.960	2.408	2.788	2.136	1.161
Ratio Avg to Std Dev	5.081	4.337	4.009	4.481	2.755	3.836	2.340	2.229	2.831
3 Standard Deviations Above	75.052	56.070	40.960	30.447	28.547	16.463	14.888	11.170	6.767
Maximum Actual Value	62.000	46.000	32.000	26.000	26.000	12.000	14.000	9.000	5.000
1 Standard Deviation Above	56.478	40.785	29.272	22.308	18.627	11.647	9.312	6.898	4.446
Average	47.190	33.143	23.429	18.238	13.667	9.238	6.524	4.762	3.286
1 Standard Deviation Below	37.903	25.500	17.585	14.168	8.707	6.830	3.736	2.626	2.125
Minimum Actual Value	29.000	16.000	12.000	10.000	5.000	2.000	2.000	1.000	0.000
3 Standard Deviations Below	19.329	10.215	5.897	6.029	-1.214	2.013	-1.840	-1.646	-0.196

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10	11	12	13	14	15	16	17	18	19	20	21
4:30	5:00	5:30	6:00	6:30	7:00	7:30	8:00	8:30	9:00	9:30	10:00
2	2	1	1	9	29	68	195	393	487	653	928
2	1	5	3	11	23	56	143	354	477	640	965
3	3	2	5	3	21	67	172	328	447	655	968
1	2	1	5	9	15	67	116	349	412	630	932
4	3	1	3	14	28	67	159	321	393	638	913
3	5	1	5	7	32	76	186	397	479	757	926
7	2	4	3	7	31	82	172	337	488	666	939
3	2	6	11	10	25	63	143	332	457	706	953
2	3	3	2	9	24	59	160	347	486	732	990
4	6	2	0 -		15	59	135	329	420	623	923
3	1	3	5	9	28	68	184	444	510	749	976
3	3	3	3	12	22	76	111	291	511	723	976
0	1	2	6	7	20	73	161	366	482	726	912
4	4	3	4	15	24	69	156	382	437	656	914
3	2	2	7	11	26	75	143	365	430	678	876
0	0	6	2	17	36	83	162	460	525	736	1055
3	1	1	8	8	30	77	178	339	502	715	983
3	1	1	5	10	30	68	177	394	461	795	1047
5	1	2	2	14	27	67	192	409	519	796	1106
3	2	1	9	13	28	85	161	418	531	756	1045
2	5	3	3	7	28	72	201	405	514	718	967
2.857	2.381	2.524	4.381	9.619	25.810	70.333	162.238	369.524	474.667	702.286	966.381
1.521	1.527	1.562	2.645	3.305	5.142	7.655	23.734	42.228	38.628	52.149	55.578
1.879	1.559	1.616	1.656	2.911	5.019	9.188	6.836	8.751	12.288	13.467	17.388
7.420	6.961	7.210	12.317	19.533	41.235	93.299	233.441	496.208	590.551	858.733	1133.115
7.000	6.000	6.000	11.000	17.000	36.000	85.000	201.000	460.000	531.000	796.000	1106.000
4.378	3.908	4.086	7.026	12.924	30.951	77.989	185.973	411.752	513.295	754.435	1021.959
2.857	2.381	2.524	4.381	9.619	25.810	70.333	162.238	369.524	474.667	702.286	966.381
1.336	0.854	0.962	1.736	6.314	20.668	62.678	138.504	327.296	436.039	650.137	910.803
0.000	0.000	1.000	0.000	3.000	15.000	56.000	111.000	291.000	393.000	623.000	876.000
-1.705	-2.199	-2.162	-3.555	-0.295	10.384	47.368	91.035	242.840	358.782	545.838	799.647

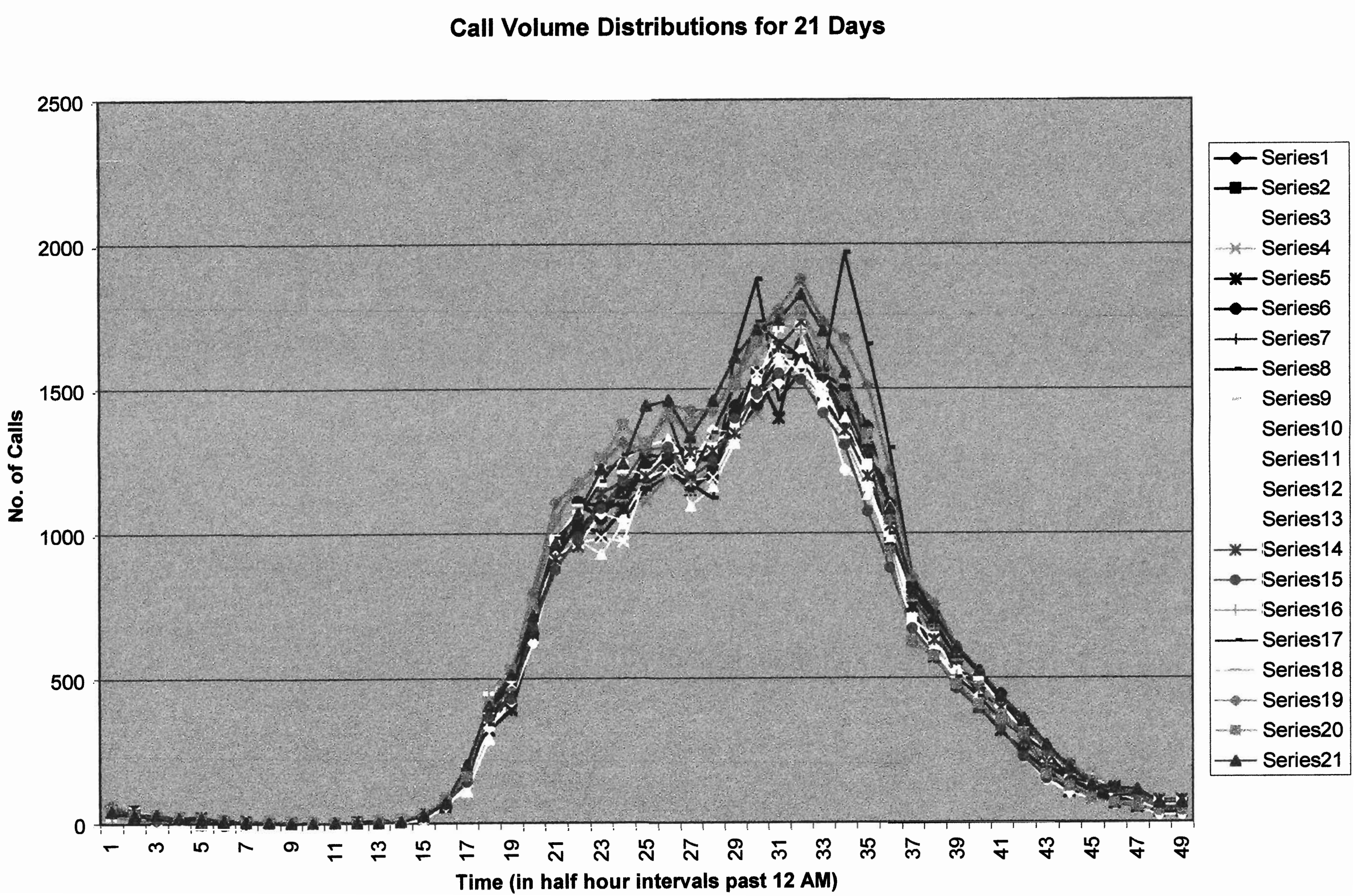
T1/3

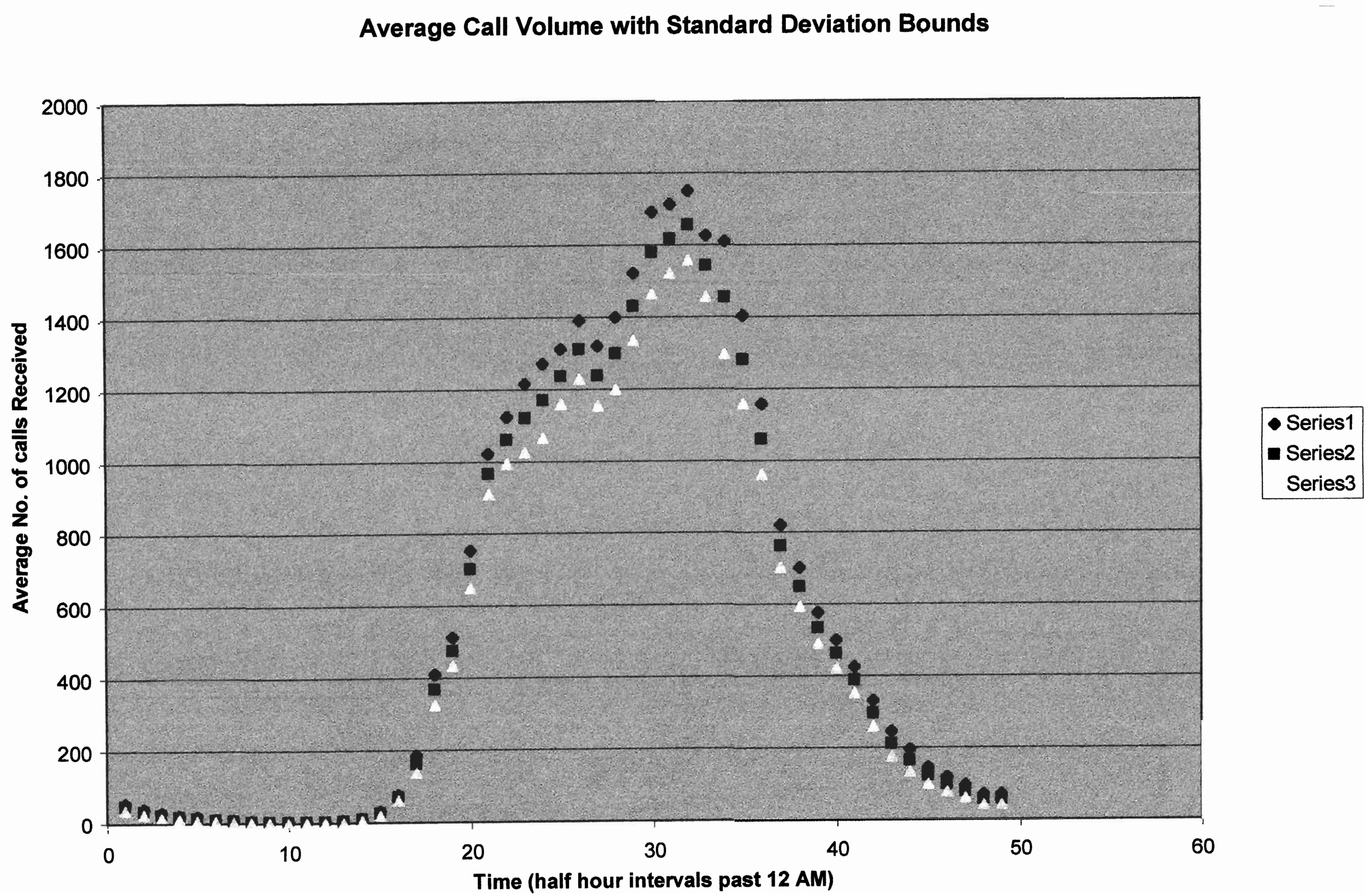
22	23	24	25	26	27	28	29	30	31
10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30	15:00
1108	1097	1104	1238	1272	1261	1253	1443	1558	1630
1015	1084	1108	1278	1258	1169	1202	1348	1442	1522
978	929	1035	1192	1296	1096	1163	1313	1491	1583
1048	1027	1098	1112	1201	1175	1227	1346	1412	1539
1040	995	1086	1172	1231	1152	1215	1393	1556	1398
1109	1051	1172	1188	1315	1278	1254	1416	1484	1604
1021	1089	1152	1150	1261	1291	1302	1367	1553	1606
1096	1063	1052	1152	1207	1174	1121	1631	1878	1461
1091	1130	1113	1166	1207	1138	1519	1399	1611	1596
966	1067	1048	1198	1305	1232	1359	1351	1469	1520
1085	1236	1226	1203	1296	1274	1306	1371	1517	1714
1074	1184	1258	1305	1326	1244	1293	1344	1530	1606
969	987	973	1195	1224	1184	1195	1339	1559	1629
957	1140	1183	1197	1275	1282	1286	1346	1496	1646
989	1088	1180	1292	1299	1195	1251	1399	1482	1554
1158	1215	1263	1317	1421	1179	1345	1546	1595	1723
1057	1181	1271	1294	1411	1194	1348	1509	1730	1662
1086	1255	1295	1280	1414	1333	1319	1491	1722	1735
1173	1245	1314	1284	1445	1428	1426	1533	1703	1773
1171	1257	1373	1310	1401	1383	1442	1514	1663	1741
1068	1224	1242	1443	1461	1334	1460	1609	1705	1738
1059.952	1121.143	1168.857	1236.476	1310.762	1237.905	1299.333	1428.952	1578.857	1618.095
64.294	94.435	102.953	75.891	81.053	82.858	99.706	92.852	113.883	96.330
16.486	11.872	11.353	16.293	16.172	14.940	13.032	15.390	13.864	16.797
1252.833	1404.449	1477.715	1464.150	1553.921	1486.480	1598.451	1707.508	1920.505	1907.084
1173.000	1257.000	1373.000	1443.000	1461.000	1428.000	1519.000	1631.000	1878.000	1773.000
1124.246	1215.578	1271.810	1312.367	1391.815	1320.763	1399.039	1521.804	1692.740	1714.425
1059.952	1121.143	1168.857	1236.476	1310.762	1237.905	1299.333	1428.952	1578.857	1618.095
995.659	1026.708	1065.904	1160.585	1229.709	1155.046	1199.627	1336.100	1464.974	1521.765
957.000	929.000	973.000	1112.000	1201.000	1096.000	1121.000	1313.000	1412.000	1398.000
867.072	837.837	859.999	1008.802	1067.603	989.330	1000.216	1150.397	1237.209	1329.106

T1/4

32	33	34	35	36	37	38	39	40	41	42
15:30	16:00	16:30	17:00	17:30	18:00	18:30	19:00	19:30	20:00	20:30
1623	1549	1504	1369	1101	807	623	553	464	444	306
1602	1549	1408	1229	961	809	655	559	453	416	323
1569	1439	1307	1133	987	726	589	518	460	410	298
1547	1480	1471	1310	1070	754	642	547	460	398	276
1710	1541	1377	1236	1089	746	571	477	397	316	240
1596	1465	1351	1292	1100	809	695	558	441	401	292
1570	1531	1417	1308	1098	782	675	476	487	392	286
1562	1559	1971	1653	1294	844	722	600	514	427	352
1690	1604	1516	1286	1168	771	711	578	488	404	281
1632	1475	1219	1164	942	677	597	496	439	329	223
1713	1425	1344	1235	1007	703	639	509	500	408	307
1588	1462	1401	1220	1097	700	580	531	501	409	313
1548	1509	1350	1165	984	788	626	531	412	392	284
1731	1556	1356	1199	1027	788	632	562	465	396	284
1532	1415	1307	1076	881	670	579	464	393	324	227
1705	1555	1444	1310	1044	805	682	507	469	357	320
1614	1543	1461	1263	998	798	698	501	433	353	296
1743	1675	1620	1318	1164	764	666	558	480	383	307
1880	1731	1673	1509	1217	845	756	607	524	430	353
1771	1618	1470	1347	936	622	574	475	406	355	303
1827	1701	1557	1290	1087	809	716	603	524	434	360
1654.905	1542.000	1453.524	1281.524	1059.619	762.714	648.952	533.810	462.381	389.429	296.714
95.823	84.899	156.486	122.505	97.563	58.337	53.911	42.888	38.769	36.046	35.667
17.270	18.163	9.289	10.461	10.861	13.074	12.037	12.447	11.927	10.804	8.319
1942.374	1796.696	1922.981	1649.039	1352.309	937.724	810.686	662.474	578.686	497.566	403.714
1880.000	1731.000	1971.000	1653.000	1294.000	845.000	756.000	607.000	524.000	444.000	360.000
1750.728	1626.899	1610.010	1404.029	1157.182	821.051	702.864	576.698	501.149	425.474	332.381
1654.905	1542.000	1453.524	1281.524	1059.619	762.714	648.952	533.810	462.381	389.429	296.714
1559.082	1457.101	1297.038	1159.019	962.056	704.378	595.041	490.921	423.612	353.383	261.048
1532.000	1415.000	1219.000	1076.000	881.000	622.000	571.000	464.000	393.000	316.000	223.000
1367.435	1287.304	984.067	914.009	766.929	587.705	487.219	405.145	346.075	281.291	189.714

43	44	45	46	47	48
21:00	21:30	22:00	22:30	23:00	23:30
200	178	135	100	87	71
248	198	116	111	72	63
199	193	143	115	71	42
190	157	115	85	88	67
179	104	99	65	52	36
195	148	105	79	76	66
212	190	139	103	74	63
216	180	130	116	100	78
204	216	123	106	84	59
144	102	81	63	50	26
249	154	126	98	70	62
225	180	146	120	94	67
253	176	123	125	79	71
205	159	130	122	83	78
152	124	88	66	50	40
195	174	125	103	83	49
238	152	135	87	97	53
267	167	179	107	104	55
240	199	141	112	90	47
165	122	81	67	54	40
268	189	127	120	113	68
211.619	164.857	123.190	98.571	79.571	57.190
34.613	30.515	22.704	19.992	17.391	14.083
6.114	5.402	5.426	4.931	4.576	4.061
315.457	256.402	191.304	158.547	131.743	99.441
268.000	216.000	179.000	125.000	113.000	78.000
246.232	195.372	145.895	118.563	96.962	71.274
211.619	164.857	123.190	98.571	79.571	57.190
177.006	134.342	100.486	78.580	62.181	43.107
144.000	102.000	81.000	63.000	50.000	26.000
107.781	73.312	55.077	38.596	27.399	14.940


Call Volume Distributions for 21 Days
No. of Calls
2500
2000
1500
1000
500
0
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
41
43
45
47
49
Time (in half hour intervals past 12 AM)
Series1
Series2
Series3
Series4
Series5
Series6
Series7
Series8
Series9
Series10
Series11
Series12
Series13
Series14
Series15
Series16
Series17
Series18
Series19
Series20
Series21


Average Call Volume with Standard Deviation Bounds
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
10
20
30
40
50
60
Average No. of calls Received
Time (half hour intervals past 12 AM)
Series1
Series2
Series3

Graph 1

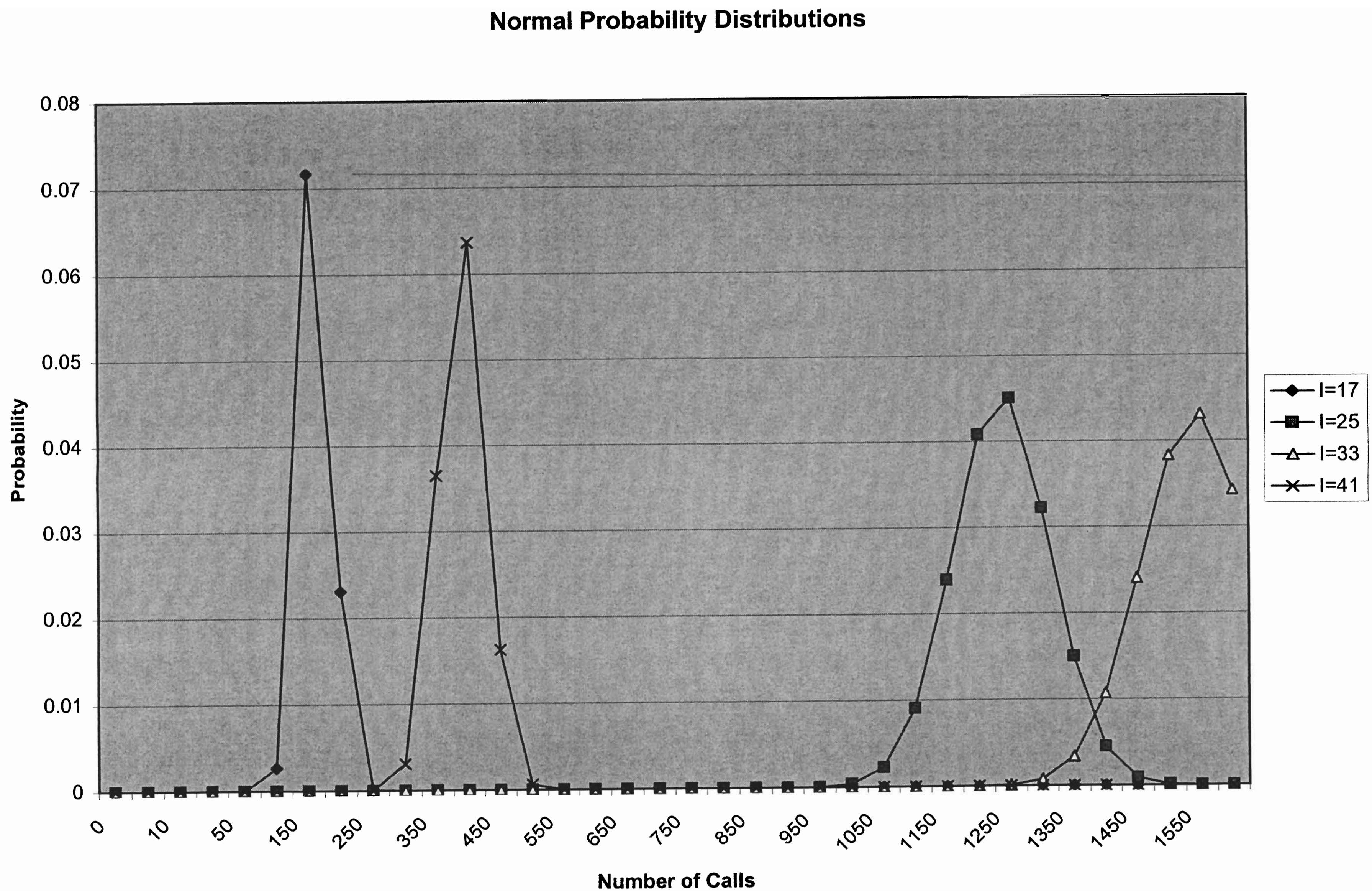
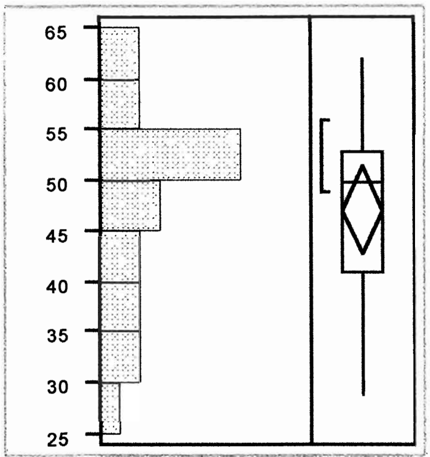

Normal Probability Distributions
Probability
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
10
50
150
250
350
450
550
650
750
850
950
1050
1150
1250
1350
1450
1550
Number of Calls
I=17
I=25
I=33
I=41

Figure 1

Sample Distributions at 4 hour intervals

Column 1



Quantiles

maximum	100.0%	62.000
	99.5%	62.000
	97.5%	62.000
	90.0%	59.200
quartile	75.0%	53.000
median	50.0%	50.000
quartile	25.0%	41.000
	10.0%	30.200
	2.5%	29.000
	0.5%	29.000
minimum	0.0%	29.000

Moments

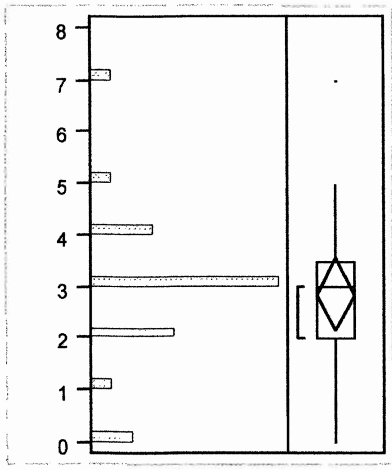
Mean	47.19048
Std Dev	9.51640
Std Error Mean	2.07665
Upper 95% Mean	51.52226
Lower 95% Mean	42.85869
N	21.00000
Sum Weights	21.00000

Test for Normality

Shapiro-Wilk W Test

W	Prob<W
0.920142	0.0840

Column 9



Quantiles

maximum	100.0%	7.0000
	99.5%	7.0000
	97.5%	7.0000
	90.0%	4.8000
quartile	75.0%	3.5000
median	50.0%	3.0000
quartile	25.0%	2.0000
	10.0%	0.2000
	2.5%	0.0000
	0.5%	0.0000
minimum	0.0%	0.0000

Moments

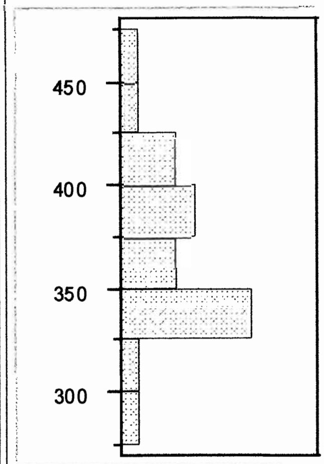
Mean	2.85714
Std Dev	1.55839
Std Error Mean	0.34007
Upper 95% Mean	3.56651
Lower 95% Mean	2.14778
N	21.00000
Sum Weights	21.00000

Test for Normality

Shapiro-Wilk W Test

W	Prob<W
0.906721	0.0443

Column 17



Quantiles

maximum	100.0%
	99.5%
	97.5%
	90.0%
quartile	75.0%
median	50.0%
quartile	25.0%
	10.0%
	2.5%
	0.5%
minimum	0.0%

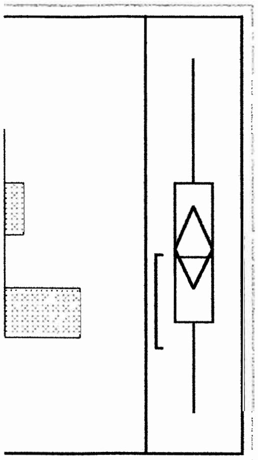
Moments

Mean
Std Dev
Std Error Mean
Upper 95% Mean
Lower 95% Mean
N
Sum Weights

Test for Normality

Shapiro-Wilk W Test

W	Prob<W
0.970477	0.7269



	100.0%	460.00
	99.5%	460.00
	97.5%	460.00
	90.0%	438.80
	75.0%	401.00
	50.0%	365.00
	25.0%	334.50
	10.0%	322.40
	2.5%	291.00
	0.5%	291.00
	0.0%	291.00

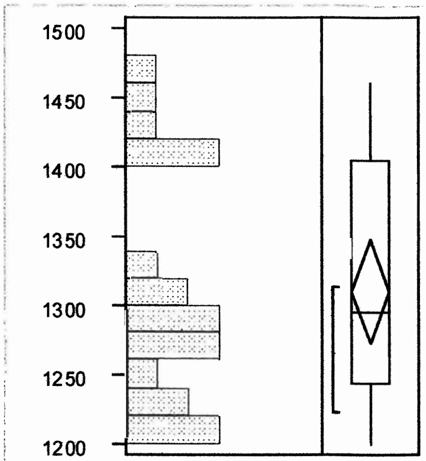
	369.5238
	43.2708
	9.4425
ı	389.2203
ı	349.8273
	21.0000
	21.0000

y

est

W	Prob<W
	0.7269

Column 25



Quantiles

maximum	100.0%	1461.0
	99.5%	1461.0
	97.5%	1461.0
	90.0%	1440.2
quartile	75.0%	1406.0
median	50.0%	1296.0
quartile	25.0%	1244.5
	10.0%	1207.0
	2.5%	1201.0
	0.5%	1201.0
minimum	0.0%	1201.0

Moments

Mean	1310.762
Std Dev	83.055
Std Error Mean	18.124
Upper 95% Mean	1348.568
Lower 95% Mean	1272.956
N	21.000
Sum Weights	21.000

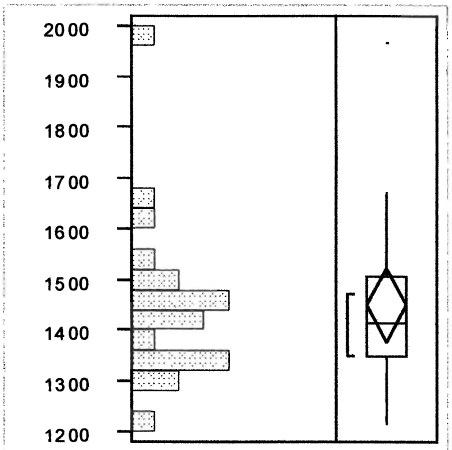
Test for Normality

Shapiro-Wilk WTest

W	Prob<W
0.915707	0.0680

F 3/3

Column 33



Quantiles

maximum	100.0%	1971.0
	99.5%	1971.0
	97.5%	1971.0
	90.0%	1662.4
quartile	75.0%	1510.0
median	50.0%	1417.0
quartile	25.0%	1350.5
	10.0%	1307.0
	2.5%	1219.0
	0.5%	1219.0
minimum	0.0%	1219.0

Moments

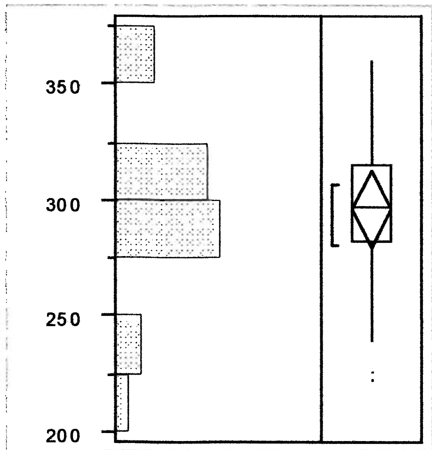
Mean	1453.524
Std Dev	160.350
Std Error Mean	34.991
Upper 95% Mean	1526.514
Lower 95% Mean	1380.534
N	21.000
Sum Weights	21.000
Sum	30524.000
Variance	25712.162
Skewness	1.754
Kurtosis	4.602
CV	11.032

Test for Normality

Shapiro-Wilk W Test

W	Prob<W
0.862444	0.0057

Column 41



Quantiles

maximum	100.0%	360.00
	99.5%	360.00
	97.5%	360.00
	90.0%	352.80
quartile	75.0%	316.50
median	50.0%	298.00
quartile	25.0%	282.50
	10.0%	229.60
	2.5%	223.00
	0.5%	223.00
minimum	0.0%	223.00

Moments

Mean	296.7143
Std Dev	36.5474
Std Error Mean	7.9753
Upper 95% Mean	313.3503
Lower 95% Mean	280.0782
N	21.0000
Sum Weights	21.0000
Sum	6231.0000
Variance	1335.7143
Skewness	-0.3406
Kurtosis	0.2807
CV	12.3174

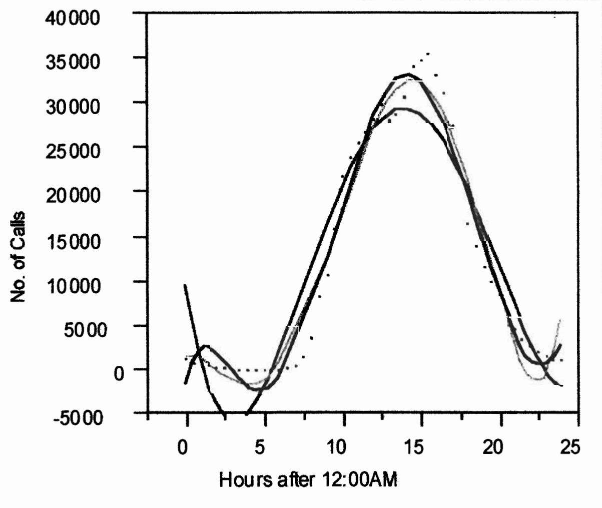
Test for Normality

Shapiro-Wilk W Test

W	Prob<W
0.940747	0.2215

Graph 2

No. of Calls By Hours after 12:00AM



Polynomial Fit degree=4

No. of Calls = 9484.89 û 10915.9 Hours after 12:00AM + 2370.57 Hours after 12:00AM^2 û 142.319 Hours after 12:00AM^3 + 2.57014 Hours after 12:00A

Summary of Fit

RSquare	0.895852
RSquare Adj	0.886384
Root Mean Square Error	4318.005
Mean of Response	12166.43
Observations (or Sum Wgts)	49

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	7056735163	1.7642e9	94.6188
Error	44	820387221	18645164	Prob>F
C Total	48	7877122384		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	9484.8863	2742.232	3.46	0.0012
Hours after 12:00AM	-10915.87	1617.943	-6.75	<.0001
Hours after 12:00AM^2	2370.5695	278.2834	8.52	<.0001
Hours after 12:00AM^3	-142.3191	17.50814	-8.13	<.0001
Hours after 12:00AM^4	2.5701356	0.361793	7.10	<.0001

Polynomial Fit degree=5

No. of Calls = 1582.92 + 616.531 Hours after 12:00AM û 1132.19 Hours after 12:00AM^2 + 252.691 Hours after 12:00AM^3 û 16.0368 Hours after 12:00A

Summary of Fit

RSquare	0.961068
RSquare Adj	0.956542
Root Mean Square Error	2670.548

Table 2

Predicted and Residual Values for 3 Polynomial Approximations.

i	Hrs Past 12 AM	Average Daily Calls	Predicted Values 1	Residual Values 1	Predicted Values 2	Residual Values 2	Predicted Values 3	Residual Values 3
15	7	25.81	-82.1604	107.9704	31.51962	-5.70962	-26423	.
16	7.5	70.333	124.0767	-53.7437	56.7459	13.5871	-21715.1	.
17	8	162.238	287.0486	-124.811	171.2129	-8.9749	-17595	.
18	8.5	369.524	424.9854	-55.4614	339.3439	30.18006	-14014.6	.
19	9	474.667	550.509	-75.842	531.3459	-56.6789	-10928.1	.
20	9.5	702.286	671.5567	30.72925	723.2091	-20.9231	-8291.47	.
21	10	966.381	792.2394	174.1416	896.7075	69.67347	-6062.58	.
22	10.5	1059.952	913.6329	146.3191	1039.399	20.55337	-4201.37	.
23	11	1121.143	1034.505	86.63803	1144.623	-23.4804	-2669.68	.
24	11.5	1168.857	1151.975	16.88248	1211.506	-42.6493	-1431.35	.
25	12	1236.476	1262.106	-25.6303	1244.956	-8.47955	-452.118	.
26	12.5	1310.762	1360.439	-49.6769	1255.663	55.0994	300.2824	.
27	13	1237.905	1442.447	-204.542	1260.103	-22.1976	856.1801	.
28	13.5	1299.333	1503.938	-204.605	1280.534	.	1243.946	55.38705
29	14	1428.952	1541.383	-112.431	1345	.	1489.996	-61.0441
30	14.5	1578.857	1552.178	26.67901	1487.325	.	1618.792	-39.935
31	15	1618.095	1534.848	83.24729	1747.119	.	1652.84	-34.7453
32	15.5	1654.905	1489.174	165.731	2169.774	.	1612.693	42.21212
33	16	1542	1416.264	125.7359	2806.468	.	1516.947	25.05296
34	16.5	1453.524	1318.551	134.9728	3714.159	.	1382.245	71.2788
35	17	1281.524	1199.729	81.79491	4955.592	.	1223.275	58.24895
36	17.5	1059.619	1064.621	-5.00206	6599.292	.	1052.77	6.849428
37	18	762.714	918.9828	-156.269	8719.572	.	881.507	-118.793
38	18.5	648.952	769.2392	-120.287	11396.52	.	718.3108	-69.3588
39	19	533.81	622.1558	-88.3458	14716.03	.	570.0497	-36.2397
40	19.5	462.381	484.4432	-22.0622	18769.74	.	441.6378	20.74321
41	20	389.429	362.297	27.13205	23655.11	.	336.0342	53.39476
42	20.5	296.714	260.87	35.84396	29475.36	.	254.2436	42.47041
43	21	211.619	183.6807	27.93834	36339.51	.	195.3156	16.30336
44	21.5	164.857	131.9533	32.90368	44362.35	.	156.3454	8.511596
45	22	123.19	103.8943	19.29571	53664.46	.	132.4732	-9.28319
46	22.5	98.571	93.90098	4.670018	64372.2	.	116.8845	-18.3135
47	23	79.571	91.70553	-12.1345	76617.72	.	100.8103	-21.2393
48	23.5	57.19	81.45231	-24.2623	90538.95	.	73.52644	-16.3364
49	24	47.19	40.70958	6.480421	106279.6	.	22.35439	24.83561
Residual Sum of Squares				326976		16106.57		47941.54
Combined Residual Sum of Squares 2, 3								64048.11

Notes:

1.)Data points for i=1 to 14 (12 AM to 7 AM) were excluded from all analyses, as no polynomial up to degree 6 came close to this flat portion of the graph, as can be seen in graph __.

2.) Predicted and Residual values 1 were determined from the best-fitting polynomial of degree 6 plotted using all other data pts.

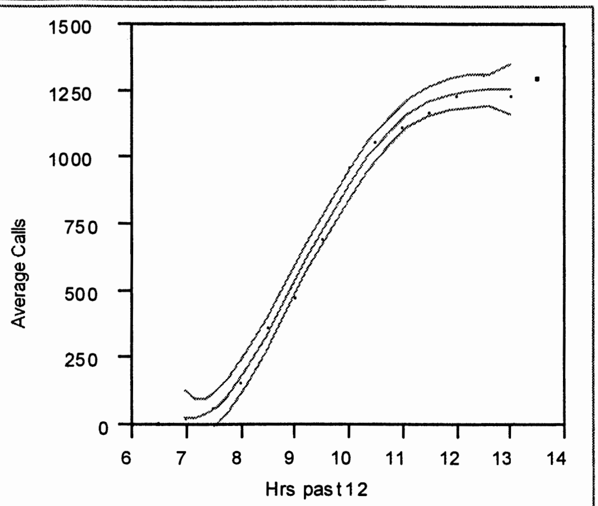
3.) Predicted and Residual Values 2 were determined from the best fitting polynomial of degree 4 for I=15 to 27 (7:00 AM to 1:30PM)

3.) Predicted and Residual Values 3 were determined from the best fitting polynomial of degree 4 for I=27 to 49 (1:30 PM to 12:30AM)

Figure 3

F3/1

Average Calls By Hrs past 12



Polynomial Fit degree=4

Polynomial Fit degree=4

Average Calls = 42736.4 û 17672.1 Hrs past 12 + 2632.87 Hrs past 12^2 û 166.962 Hrs past 12^3 + 3.85569 Hrs past 12^4

Summary of Fit

RSquare	0.994244
RSquare Adj	0.991366
Root Mean Square Error	44.87006
Mean of Response	762.0257
Observations (or Sum Wgts)	13

Analysis of Variance

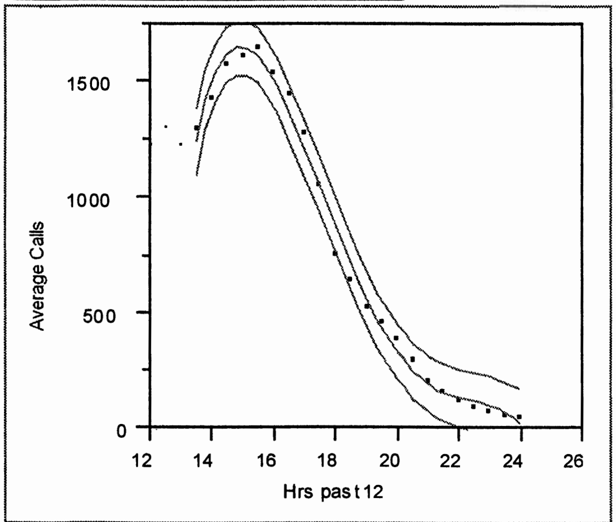
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	2781966.3	695492	345.4448
Error	8	16106.6	2013	Prob>F
C Total	12	2798072.9		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|	Lower 95%	Upper 95%
Intercept	42736.378	14841.9	2.88	0.0205	8510.5609	76962.195
Hrs past 12	-17672.12	6181.849	-2.86	0.0212	-31927.63	-3416.615
Hrs past 12^2	2632.8661	951.9151	2.77	0.0245	437.72462	4828.0075
Hrs past 12^3	-166.9619	64.25564	-2.60	0.0317	-315.1371	-18.78675
Hrs past 12^4	3.8556894	1.605173	2.40	0.0430	0.1541178	7.5572611

F3/2

Average Calls By Hrs past 12



—— Polynomial Fit degree=4

Polynomial Fit degree=4

Average Calls = -185531 + 39639.1 Hrs past 12 û 3081.81 Hrs past 12^2 + 104.289 Hrs past 12^3 û 1.30316 Hrs past 12^4

Summary of Fit

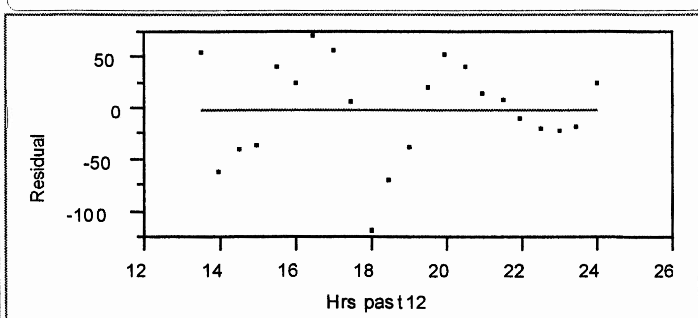
RSquare	0.993906
RSquare Adj	0.992472
Root Mean Square Error	53.10452
Mean of Response	763.318
Observations (or Sum Wgts)	22

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	7819150.9	1954788	693.1650
Error	17	47941.5	2820	Prob>F
C Total	21	7867092.4		<.0001

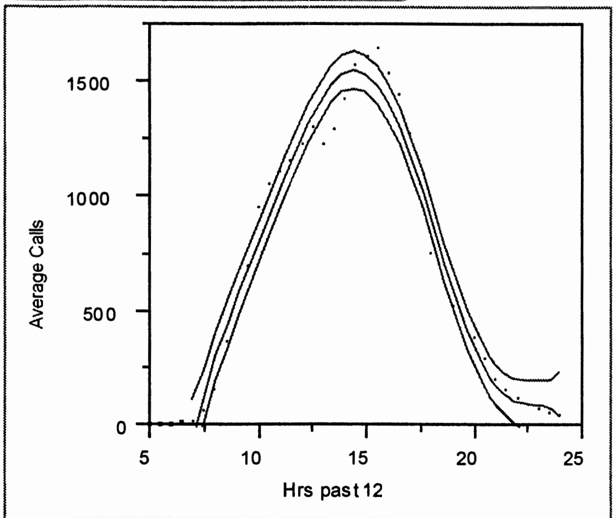
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-185530.6	19426.07	-9.55	<.0001
Hrs past 12	39639.125	4283.379	9.25	<.0001
Hrs past 12^2	-3081.807	350.1075	-8.80	<.0001
Hrs past 12^3	104.28909	12.57602	8.29	<.0001
Hrs past 12^4	-1.303161	0.16757	-7.78	<.0001



F3/3

Average Calls By Hrs past 12



Polynomial Fit degree=6

Polynomial Fit degree=6

Average Calls = -4.08e4 + 18990 Hrs past 12 û 3634.54 Hrs past 12^2 + 362.977 Hrs past 12^3 û 19.5645 Hrs past 12^4 + 0.53649 Hrs past 12^5 û 0.00586 Hrs past 12^6

Summary of Fit

RSquare	0.969342
RSquare Adj	0.962772
Root Mean Square Error	108.0635
Mean of Response	762.838
Observations (or Sum Wgts)	35

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	6	10338203	1723034	147.5489
Error	28	326976	11678	Prob>F
C Total	34	10665179		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	-40773.82	19663.01	-2.07	0.0474
Hrs past 12	18990.044	8883.047	2.14	0.0414
Hrs past 12^2	-3634.544	1614.435	-2.25	0.0324
Hrs past 12^3	362.97709	151.3454	2.40	0.0234
Hrs past 12^4	-19.56451	7.734701	-2.53	0.0173
Hrs past 12^5	0.5364925	0.204791	2.62	0.0141
Hrs past 12^6	-0.005861	0.0022	-2.66	0.0126