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AGE, PERIOD, AND COHORT EFFECTS IN DEMOGRAPHY: A REVIEW

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Age, Period, and Cohort Sources of Variation in Demography

Narrowly defined, demography deals with the measurement of vital events (birth, death, and marriage) and migration, studies the factors that influence the rate at which those events occur, and, to a lesser extent, investigates the consequences of the patterns of these events. In this paper, we adopt this narrow definition of the field and consider only the first three events so that by limiting the scope of our review, we can examine these selected topics in detail.

In one way or another, demography has concerned itself with the measurement of age, period, and cohort effects for well over a century. The first social information to become available in time series with age detail was the number of deaths and births, information that dates back to the eighteenth century in Sweden and to the nineteenth century in most countries of Western Europe. These early statistics established with absolute clarity that vital rates varied enormously with age, and one of the earliest analytic problems was how to "control" this variation in order to elucidate differences in rates between populations or over time. The early solutions to this problem were to develop summary measures that were independent of age composition--life expectancy, gross reproduction rate, and net reproduction rate--or to "standardize" rates by artificially imposing one population's age structure on another population. Comparisons based on these measures contain assumptions, almost always implicit, about the age, period, and cohort effects that underlie the observed data.

These simple algebraic techniques are still central in demographic curricula, and the measures have acquired a significance beyond their original

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purpose. These well-entrenched measures created a useful common lexicon for demography. However, they were developed before the era of high-speed computers and sophisticated multivariate techniques, and in a sense their currency may have made demographers less receptive than some other social scientists to newer developments.

Another reason for the demographer's somewhat different perspective is that many of the standard measures can be calculated from either period age-specific or cohort age-specific information. In the past, period-based calculations led demographers to conclusions and predictions that later experience proved wrong. As a result, the relationship of a period-specific measure to the remaining life experience of cohorts active in that period has become as compelling a question for demographers as is the delineation of age, period, and cohort effects.

The phrase "age, period, and cohort effects" is probably an unfortunate one. Ages, periods, and cohorts do not have either direct or indirect effects on demographic or social phenomena. Age is a surrogate--probably a very good one in most applications--for aging or more generally for physiological states, amount of exposure to certain social influences, or exposure to social norms. However, it is clear that individuals age physiologically and socially at different rates.[1] To understand the sources of variation in vital rates, it is more satisfactory wherever possible to measure the underlying variables for which age is a proxy. Indicators of period and cohort are much further removed from variables that presumably influence vital processes. "Period" is a poor proxy for some set of contemporaneous influences, and "cohort" is an equally poor proxy for influences in the past. Measured "effects" of periods and cohorts are thus measures of our ignorance: in particular, of whether the factors about which we are ignorant are more or less randomly distributed along chronologically measurable dimensions. When these factors can themselves be directly measured, there is no reason to probe for period or cohort effects. When the factors cannot be identified explicitly, it would seem that analysts probing for age, period, and cohort effects are, more precisely, partitioning observed variation to ages, periods, and cohorts.

Demographers' efforts to examine age, period, and cohort sources of variation have frequently taken the form of postulating mathematical models of age variation in vital events. The influence of periods and cohorts, where measured at all, is registered by allowing one or more parameters in the model to vary in intensity from period to period or cohort to cohort. There are three sound reasons for preoccupation with age models. First, because age is closely related to physiological state, biological models and reasoning can be used to provide predictions or theories of the way that age should be related to vital events. The biological theories, though of course inexact, nevertheless provide a firmer and more precise basis for prediction than do the primarily social theories that would relate to period or cohort. Second and more important, age is quite simply the most important source of variation in vital rates. At present, death rates vary among countries of the world by at most a factor of eight, and birth rates by at most a factor of five. On the other hand, death rates are more than one hundred times higher in the United States at ages 70-74 than at ages 10-14.[2] Proportionate age variation in fertility rates is even greater since many ages have zero rates. A third reason for emphasis on age models is that age data are subject to great error in most of the world, so that models of age variation, fitted to reported age data, can be used to "discipline" and correct reported figures.

Thus, no one would dispute the importance of age as an influence on demographic processes. Period effects also are evident from casual inspection of time series on fertility, mortality, marriage, and divorce. All (or many) ages tend to have similar patterns of rise and fall in vital rates from period to period, and the fact that the fluctuations often correspond to changes in social conditions that are expected to influence rates (e.g., wartime, changes in health technology, and economic recession) helps to confirm the importance of these influences even to the most skeptical observer. The real question about the applicability of age-period-cohort analysis to demography relates to the substantive importance of cohort influences. Do age patterns, for individuals who have the timing of an event (such as their birth or marriage) in common, respond only to period influences, so that the experience of a cohort can be described completely by age effects and the effects of the periods its members live through, or do they respond to additional, cohort influences as well? And, if so, how does the cohort influence manifest itself?

A related question is whether or not period effects are age-specific. If they are, the experience of a cohort is largely determined by those period influences that coincide with ages of greater or lesser susceptibility.

Demographers also have another, largely pragmatic and non-theoretical reason for considering period and cohort effects, which is their need for forecasting future trends in vital rates. If, especially over the short term, either period or cohort effects predominate, then one avenue for providing projections is to model the observed trend in these effects and continue it into the future.

Before considering in detail empirical examples attempting to identify the importance of cohort influences in demography, it is useful to review various theories of these influences that have structured the applications.

Cohort Theories in Demography

The Conventional Linear Model

Much of the relevant demographic work is based on the usual assumption that, either in the original or a transformed scale, the dependent variable is a linear function of age, period, and cohort effects; only the resolution of the problem of overidentification varies. The linear model is appropriate when a cohort has some unique susceptibility to the phenomenon in question, a susceptibility that is established by the time of initial observation and that manifests itself as a constant through the periods and ages of observation. The model appears in the classic work of Greenberg, Wright, and Sheps (1950) on syphilis and is also implicit in some of the earliest work on generation approaches to the projection of mortality (see especially Derrick, 1927, and Kermack, McKendrick, and McKinlay, 1934a,b). The model can be written as

$$f(r_{apc}) = W + W_a + W_p + W_c \qquad (1)$$

where r_{apc} is the age-period-cohort specific rate, f( ) is a simple transformation (e.g., linear, logarithm, or logit), W is an overall constant, and W_a, W_p, and W_c are the effects for age group a, period p, and cohort c, respectively. The logit transformation is usually selected for demographic applications since it is the natural one to adopt for rates. The problems of estimating the parameters of this model are hardly unique to demographic studies and are treated elsewhere (e.g., Fienberg and Mason, 1979). One approach to these problems is suggested in the Appendix.

Here we note only that it is possible and sensible to view a constant-effect cohort model as fitting a particular model to the set of two-way interactions in the age-period dimensions. This can be visualized as follows for a 4 x 4 table where the age and period effects are indicated along the margins and the cohort effects appear as constants along the diagonals in the body of the table. Thus, if e_{ij} is an element of the table of residuals, $e_{ij} = c_k$ for i,j such that $k = j - i + 4$. The c_k are fitted to the residuals on the transformed scale under the age and period model.

	p_1	p_2	p_3	p_4	Time
a_1	c_4	c_5	c_6	c_7	
a_2	c_3	c_4	c_5	c_6	
a_3	c_2	c_3	c_4	c_5	
a_4	c_1	c_2	c_3	c_4	
Age					

Cohort-Inversion Models

An alternative model is conceptually far more common in demographic work, although rarely formally estimated. The basic and intuitively reasonable idea behind what will be referred to as cohort-inversion models is that cohorts experiencing particularly hard or good times early in life will respond inversely later in life. There are at least three types of cohort-inversion models. The first derives from an assumption that the cohort is not homogeneous with respect to the occurrence of the marker event.

1. Heterogeneous susceptibility

For mortality, this type of model corresponds to the theory of "impaired lives", which can be summarized as follows: a cohort is viewed as starting life with a distribution of susceptibility to mortality among its members; unfavorable health conditions early in life will eliminate the more susceptible and thus reduce cohort mortality later in life. This hypothesis was recently invoked in attempts to "explain" the slowdown or reversal of continued mortality decline in the 40 to 70 age range in many Western countries, by suggesting that the substantial mortality reductions that had occurred in childhood had led to the survival into adulthood of many more susceptible people, who succumbed more rapidly to the increased risks in later life. The resumption of substantial mortality declines in the United States and elsewhere has cast doubts upon the validity of these explanations, although the thesis is clearly a tenable one deserving further scrutiny (Vaupel, Manton, and Stallard, 1979). A similar hypothesis is plausible for divorce, with vulnerable marriages being weeded out from a cohort more rapidly when conditions are unfavorable in early marriage or when divorce is more socially acceptable (Preston and McDonald, 1979). Such a theory could be extended to migration from place of birth, where the more mobile in a cohort perhaps respond more readily to period-induced changes in underlying propensities to migrate. The transition from birth order i to birth order i + 1 could be subject to the impaired-lives phenomenon through underlying differences in fecundity, fertility control, and motivations, which are subject to period influences leading to a cohort-inversion phenomenon. In each case, for nonrenewable cohort processes, heterogeneity of response within the cohort may lead to the cohort-inversion phenomenon.

2. Fixed and changing target models

A second type of cohort-inversion model has most often been suggested in fertility analyses. It is argued that individual cohorts of women have in mind some completed fertility target and that the timing of the achievement of this target is affected by period influences. For example, adverse conditions early in reproductive life that cause postponement of births may lead to an increase, or "catch-up" phenomenon, later in the childbearing period. Such theories have been used to account for the substantial rises in period total fertility after each World War and even for the rises in England and Wales during the years 1942 and 1943. Hajnal (1950a,b) explicitly set out to discover whether postponement was sufficient (although not necessary) to have brought about the rises in England and Wales and concluded that it was for 1942-1943 but that it could not entirely account for the peak in 1947.

Among the most consistent proponents of this cohort-target theory of fertility has been Ryder (1973, 1978). He proposes (1978) that, "couples are conceptualized as having both quantitative and temporal intentions with respect to childbearing. They intend a particular number of births and at a particular tempo...." In a long series of articles (e.g., 1963, 1964, 1965a, 1965b, 1969, 1971, 1973, 1980), Ryder has argued that it is essential (or at least advisable) to analyze fertility for cohorts, especially if determinants are being studied, and that both the quantum and timing of fertility have to be interpreted as cohort variables, with timing often being distorted by period phenomena. Ryder (1953) and Lee (1977) have gone some way toward formalizing this type of model, including, in the latter, an elaboration of the impact of a constant target.

However, Westoff and Ryder (1977) conclude that the target intentions are not immutable. In their study, many women changed their reported intentions about family size over a five-year period. One interpretation "would be that the respondents failed to anticipate the extent to which times would be unpropitious for childbearing, that they made the understandable but frequently invalid assumption that the future would resemble the present--the same kind of forecasting error that demographers have often made. Perhaps the answers to questions about intentions are implicitly conditional: 'This is how I think I will behave if things stay the way they are now, but, if they don't I may change my mind.'" Recently, both Lee (1980) and Butz and Ward (1979) have moved toward formalization of this third type of cohort-inversion model, which incorporates the idea of changing targets. In their models, which will be reviewed more fully later in this paper, all couples are trying to attain a particular desired completed family size at each point in time, but this target changes from period to period.

Because models of this type are not usually treated within the general linear model framework, it seems useful to do so here.

For a renewable event, such as fertility, the ultimate or target total fertility for a particular cohort can be denoted by $F_w{}^c$, where w is the highest age considered, and the births achieved by age a by $F_a{}^c$. For a nonrenewable event (e.g., first marriage), the target can be defined as the proportion who will ever have the event, denoted by $1 - \underline{l}_w{}^c$ (using life table notation with $\underline{l}_0{}^c = 1$). The proportion of the cohort who have had the event by age a is denoted by $1 - \underline{l}_a{}^c$. Models can be formed either in terms of the number of events remaining per person, $E_a{}^c$, given by $F_w{}^c - F_a{}^c$ or $\underline{l}_a{}^c - \underline{l}_w{}^c$, or the proportion remaining, $R_a{}^c$, given by $(F_w{}^c - F_a{}^c)/F_w{}^c$ or $(\underline{l}_a{}^c - \underline{l}_w{}^c)/(1 - \underline{l}_w{}^c)$.

Then the extension of the linear model (1) to include fixed targets introduces an inventory-control mechanism of a sort, i.e.,

$$f'(r_{apc}) = v + v_a + v_p + v_c\ g'(E_a{}^c)$$

or (2)

$$f''(r_{apc}) = u + u_a + u_p + u_c\ g''(R_a{}^c)$$

where f′ and f″ are transformations of the rates and g′ and g″ are transformations of the measure of distance from the target. The logit transformation, defined only for proportions, is not appropriate for g′ but has certain advantages over a linear or logarithmic transformation for the other functions. Both the logarithmic and the logit formulations eliminate the possibility of estimating negative rates but would still be subject to the usual normalization constraints.

Since models of this type have received attention only rather recently, it is not surprising that a number of problems remain to be resolved. First, it should be noted that the value of F_w^c or $\underline{l}_w^c$ in (2) is regarded as known, so that some estimate is required if incomplete cohorts are not to be excluded from the analysis. Second, if it is assumed that the target fertility for a cohort is equal to its observed completed fertility, there are fitting problems. An alternative, less awkward formulation would allow fertility to fall short of or overshoot the target due to period-specific influences and would treat both F_w^c and $\underline{l}_w^c$ as unknown. Another problem is encountered if the early part of a cohort′s experience is not observed. For example, if birth registration begins when cohort c is aged a, none of its fertility prior to age a can be observed. The detection of a cohort-inversion phenomenon then becomes difficult, if not impossible.

The specification of a cohort-inversion model with period-specific targets is a straightforward extension of (2) but introduces problems with some possible formulations that cannot handle achieved cohort fertility greater than the period target. It is clearly only a first approximation for such models to assume homogeneity of the population with respect to targets, so that one direction for future work is the introduction of heterogeneity.

Several competing models have been suggested to attempt to capture the so-called cohort-inversion model. Because the theoretical underpinnings seem plausible, empirical tests of these alternatives and others not proposed here against real data are being carried out (e.g., Gilks and Hobcraft, 1980) for a few cases, and more tests would be valuable.

Continuously-Accumulating Cohort Effects

Cohort effects occur whenever the past history of individuals exerts an influence on their current behavior in a way that is not fully captured by an age variable. If only events that occur prior to the initial observation influence cohort behavior, then the linear model is appropriate. However, cohorts are continuously exposed to influences that affect their biological susceptibilities and social propensities. Obvious examples are wars and epidemics that may break out in the middle of a cohort′s life and leave an imprint on all subsequent behavior. If these disturbances affect all cohorts then alive in similar fashion, they can best be treated in the form of lagged period effects. But if, as seems more likely, their imprint is differentiated by age and becomes embodied in cohorts differentially, then a more complex form of cohort analysis is required. This is the version of cohort theory promoted by Ryder (1965a) in his classic paper.

> The case for the cohort as a temporal unit in the analysis of social change rests on a set of primitive notions: persons of age a in time t are those who were age a - 1 in time t - 1; transformations of the social world modify people of different ages in different ways; _the effects of these transformations are persistent._[3] In this way a cohort meaning is implanted in the age-time specification. Two broad orientations for theory and research flow from this position: first, the study of intra-cohort development throughout the life cycle; second, the study of comparative cohort careers, i.e., intercohort temporal differentiation in the various parameters that may be used to characterize these aggregate histories.

Few examples have appeared in which cohort behavior at a specified point in life is explicitly treated as a function of lagged effects of earlier period-age interactions, the persistent transformations described by Ryder. However, we would be remiss not to recognize this potentially important approach to cohort analysis, even though the procedures for investigation are undeveloped. In fact, there is some question whether or not such a model is analytically, as contrasted with conceptually, feasible.

Age Models in Demography

As already mentioned in the introduction, demographic models have been concerned primarily with age variation of rates either for time periods or for cohorts but very rarely with explicit consideration of an age-period-cohort framework. However, these models, which have fewer free parameters than the number of age effects to be estimated in a full age-period-cohort model, can be used to remove the identification problem and thus allow estimation of the period and cohort parameters without constraints other than normalization.

Empirical Models

Many of the age models in mortality analysis are formed by using vectors of age-specific rates for a particular period in various countries and estimating statistical relations among the rates in order to represent normal changes in age-specific rates as mortality levels change. Other models have estimated the relationship of the rates to some summary measure (e.g., expectation of life at age 10). The earliest example of such models was the Breslau life table of Halley in 1693 (see Smith and Keyfitz, 1977), which was subsequently used for the calculation of annuities (see also Pearson, 1978). Model life tables in use today are based on period mortality data from a wide range of countries and have summarized age variations through regression models on the infant mortality rate (United Nations, 1955, 1956; Gabriel and Ronen, 1958), on expectation of life at age 10 (Coale and Demeny, 1966), or on various other measures (Ledermann, 1969). Principal components models have also been employed (Ledermann and Breas, 1959; Le Bras, 1979; Hogan and McNeil, 1979).

Most population specialists appear to believe that cohort mortality effects are sufficiently minor that they need not be incorporated into models of mortality relations. It is certainly true that their incorporation would be quite difficult and perhaps not worth the cost. But there are indications that some exploration along these lines is desirable. For example, Figures 1 and 2 plot historic female death rates at ages 55-59 for Sweden and for Scotland against, respectively, cohort and period estimates of life expectancy. It is clear that relations between death rates and the cohort measures are far more regular than those involving period measures. There is less scatter about the cohort relationship, particularly for Sweden, and more impressively, Swedish

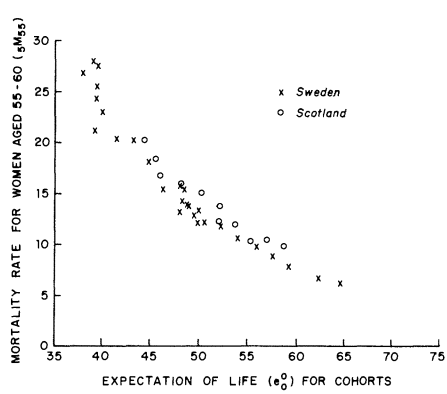

Figure 1: Relationship between expectation of life for cohorts and mortality rate for women aged 55-60: females, Sweden and Scotland.

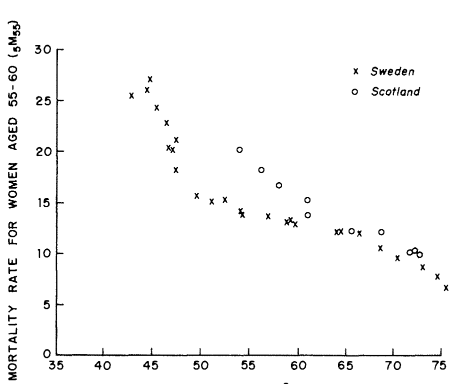

Figure 2: Relationship between expectation of life for periods and mortality rate for women aged 55-60: females, Sweden and Scotland.

and Scottish relations appear to be quite similar when plotted on a cohort basis but are quite different for periods. Early Swedish and Scottish data were placed in different families of model life tables by Coale and Demeny (1966), but it may not have been necessary to draw this distinction had cohort data been used. That the basic cohort relation is quite different from the period relation is evident from the sharper slope of the cluster of points in Figure 1. It also appears that use of cohort data helps to normalize relations between male and female age-specific mortality and hence may provide a firmer foundation for model construction that relates mortality of the sexes.

In part, period rates are employed in the development of age models of mortality because they are so much more readily available than cohort data. However, there is now a great deal of information awaiting application of sound age-period-cohort analysis (see Case et al., 1962; Vallin, 1973; Netherlands, Centraal Bureau voor de Statistiek, 1975; Bolander, 1970; and U.S. National Center for Health Statistics, 1972). The evidence is that there is still much to be learned from these data.

In the same empirical tradition, Henry (1961) described an age pattern of fertility in the absence of deliberate control that he called "natural fertility" and that is thought to give an indication of the variation with age of fecundity or capacity to bear children.

Mathematical Models

Another major tradition in modeling age patterns of demographic phenomena involves more explicit mathematical models fitted to age-specific data. Often, the form of these curves is derived from some theoretical basis. The curve-fitting approach could be regarded as starting with Cardano in 1570 and De Moivre in 1725 and perhaps including the famous life table of Graunt in 1662 (see Smith and Keyfitz, 1977; Pearson, 1978). The earliest laws of mortality that gained wide acceptance were due to Gompertz in 1825 and Makeham in 1867 (see Smith and Keyfitz, 1977). In many different forms, this descriptive tradition persists to the present day in actuarial work (cf. Benjamin and Haycocks, 1970, Chapter 14). Other mortality models do have a quasi-theoretical basis (cf. Beard, 1971; Strehler, 1962; Brillinger, 1961). Hernes (1972) proposed a mathematical model of marriage with a theoretical basis for cohorts.

In fertility studies, there have been many attempts to fit various mathematical curves, without a behavioral or theoretical interpretation, to age-specific data. Functions employed include polynomials, beta distributions, Hadwiger functions, Wicksell or gamma distributions and log-normal distributions, and Gompertz curves fitted to cumulated fertility (see Duchêne and Gillet-de Stefano, 1974, for a useful review and comparison of all but the Gompertz, for which see Farid, 1973, and Brass, 1974b).

Relational Models

A recent and promising approach to modeling age patterns of demographic rates combines the empirical and mathematical procedures. Age patterns present in a particular population or cohort are viewed as a simple statistical transformation of a "standard" age pattern that is empirically specified. The analyst must then solve for the parameters that best transform the standard into the observed set of rates in the population or cohort. This procedure has the advantage that it incorporates regularities in age patterns that can only be replicated mathematically by many-parameter models; also, the parameters of the transformation usually have a straightforward substantive interpretation. The procedure also provides more flexibility to age patterns than is normally contained in the empirical models, since these are usually one-parameter models of how age patterns change as levels change.

The first demographic application of relational models was by William Brass (1971, 1974a). He took as a standard one of the U.N. model life tables and demonstrated empirically that many observed life tables could be reproduced closely by a two-parameter transformation of the logit of the survivorship function in that standard. Later work has increased the number of parameters (Zaba, 1979) and has synthesized Brass's model into a principal components framework (Le Bras, 1979; Hogan and McNeil, 1979).

In a similar vein, Coale (1971; Coale and Trussell, 1974) extracted a single standard from Henry's age patterns of natural fertility and proposed a two-parameter transformation of that standard in order to model observed age curves of marital fertility. One parameter is interpretable as the level of natural fertility, and the other is related to the extent of anti-natal practices. Brass (1974b) demonstrated how the Gompertz mathematical model of fertility could be converted to relational form. Coale has also developed a relational model of nuptiality in which a Swedish "standard" is adopted and three interpretable parameters are employed (Coale, 1971; Coale and McNeil, 1972; Ewbank, 1974). This model differs from the others cited in that one of the three parameters transforms the age scale itself. Rogers, Raquillet, and Castro (1978) propose a model of the age pattern of migration that derives from the work of Coale and associates on marriage and fertility.

Relational models have considerable promise for age-period-cohort analysis. For example, the parameters that transform the standard to produce an observed matrix of age-period rates can be assumed to be linear functions of corresponding parameters in periods and cohorts. In this fashion, the identification problem is readily solved. However, several of the models are based on transformations of cumulative functions (survivorship, proportion ever marrying). In these cases, first differences of the standards would have to be employed, which may result in more complex expressions.

One problem in all of these models is that cohort effects are not explicitly controlled in the models that are used to describe normal period-age variation in mortality. If such effects are important, then the model age patterns developed will depend on the distribution of cohort effects in the populations yielding information for the standard. If the distribution is similar to that in a particular population being compared with the standard, no cohort effects may appear even though they are present. If the distributions differ between the standard and the actual population, age or cohort disturbances may appear even when they are absent. There is, therefore, some doubt about the use of these models to overcome identification problems in estimating age-period-cohort models. However, these doubts are probably less serious than those raised by the commonly used procedures for fitting such models, which often involve point fitting or use of moments and only rarely use more efficient procedures such as least squares or maximum likelihood.

At this stage it is useful to consider demographic work that has attempted to identify, sometimes loosely, period and cohort effects. The statistical methods used to identify these effects are, in many cases, inadequate by modern standards. Our purpose here is not to criticize or even review the procedures in detail, but to consider the range of problems examined and the models proposed for them. In so doing, it is convenient to group the studies according to the phenomenon they examine.

Mortality

Apparently the first reorganization of a time series of age-specific mortality rates in such a way as to distinguish the set of rates pertaining to persons born in the same year or period was that of Derrick (1927), who argued that cohorts provided a more consistent basis for projecting mortality than did period rates. This conclusion was based on a graphical examination of the logarithms of age-specific death rates for England and Wales from 1841 to 1925, omitting the experience of World War I, which indicated that the ratio of mortality for one cohort to that of another cohort was approximately constant for all ages above 10. This is essentially a log-linear model with age and cohort effects, which can be written $\underline{\ln}(r_{apc}) = w + w_a + w_c$. Presumably the modern analyst would infer from the dropping of the experience of World War I that a proper model should also include period effects. Although Derrick's fitting procedures and methods of analysis would not meet modern standards, his graphical procedures correspond closely with modern approaches to displaying the presence or absence of interactions (e.g., Duncan and McRae, 1979, p. 75).

Convincing demonstration of the power of a cohort approach to data was left to three Scottish actuaries. Kermack, McKendrick, and McKinlay (1934a,b) computed the ratio of English death rates in 10-year age intervals in the decades from 1851-1860 to 1921-1930 to age-specific rates for 1841-1850. They then noted "a general tendency for numbers of approximately the same magnitude to be arranged diagonally in the Tables...it is now to be noted that a diagonal line in the diagram represents the course of a group of people all born in a particular year" (p. 698). They reached similar but somewhat less convincing conclusions for Scotland using 1855-1864 as a base period and for Sweden using 1750-1759 rates as a base. The authors were aware that the apparent regularity could be a fortuitous result of a series of independent conditions affecting successively older ages (i.e., age-period interaction) but considered this combination of events to be very unlikely. Instead, they argued that the health of a cohort was principally determined by environmental conditions encountered in its first 15 years of life.

A second remarkable feature of the study was the authors' observation that death rates at ages 0-4 did not show the earliest improvement but improved only when the cohort changes reached well into the ages of maternity. They argued that early childhood mortality was closely linked to the health and physique of mothers. The mechanisms that they postulated have received ample verification only in the past decade, largely on the basis of individual-level studies in developing countries. The combination of infection and diarrheal disease in early years of life has been shown to have a very important and enduring effect on indices of growth and development; furthermore, mothers who are unusually small tend to have lower birth weight babies who are at much higher risk of death (Mata, 1977). Preston and van de Walle (1978) pursued an identical approach for the analysis of nineteenth-century data for three urban departments of France and found very similar results. Relative mortality levels tended to be arrayed diagonally rather than vertically, and infant improvements awaited maternal improvements. It was also found that the initiation of mortality improvements in a department tended to coincide with improvements in water supply and sewage disposal.

It should be noted that both studies were successful in demonstrating cohort effects only because mortality data were available before the process of mortality decline had begun.[4] In this case, early periods and cohorts maintained very similar age patterns of mortality, and it was reasonable to choose an early period age pattern as standard. If data had been available only after the decline had reached all cohorts and subsequent mortality levels had been measured relative to an already-deformed standard, then periods and cohorts alike could have shown a steady pace of mortality change. Distinguishing period and cohort effects would then have required more sophisticated statistical procedures and would have been crucially dependent on the choice of standard, whether represented explicitly by a mathematical law or implicitly by a functional form chosen for estimation purposes.

Only one year after Kermack, McKendrick, and McKinlay's article, a more elaborate examination of period and cohort effects appeared. Cramér and Wold (1935) fitted Makeham curves to both period and generation mortality experience for Sweden.[5] Their primary interest was in forecasting mortality rates. They did not consider the question of whether the period or cohort curves fit the observed mortality rates better. Instead, they went on to model the individual parameter values over time and to compare goodness of fit statistics for the period parameter values with those for cohorts. For example, they fitted logistic curves to values of $\underline{\ln} b_3$ for successive cohorts and, separately, to values of $\underline{\ln} b_3$ calculated for successive periods. They concluded that there was little or no difference in fit between the cohort and period approaches.

Cramér and Wold made a substantial contribution to another aspect of the age-period-cohort problem by demonstrating, in an appendix, that it is impossible to define a general Makeham surface for mortality rates in both the period and cohort dimensions. This problem of the interrelationship of generation and period mortality recently received attention from Cox and Scott (1977) and is closely related to recent work on bivariate exponential distributions in the analysis of competing risks (see David and Moeschberger, 1978, Chapter 4).

The relevance of cohort analysis to public health issues was first publicized in a posthumous paper on tuberculosis by Frost (1939), although the idea goes back to Andvord (1921, 1930), who believed that infection rates in childhood largely determined cohort experience. Frost's demonstration was principally graphical. He showed that age-specific death rates from tuberculosis in Massachusetts between 1880 and 1930 appeared to be more regular (similar from set to set) for cohorts than for periods. In particular, the highest age-specific rate for cohorts always occurred at ages 20-29, while for periods there was more variability. This variability was not particularly convincing support for cohort effects, however, since the period peak for males rose by only 30 years of age in the 60-year period reviewed, rather than in a one-to-one correspondence; furthermore, the period peak was constant for females at ages 10-29 throughout the period of observation.[6] The notion that tuberculosis was more responsive to influences early in the life of a cohort than to current conditions gained credibility because of the potentially long latency of the disease. But Frost's main statistical contribution seems to have been the demonstration that age patterns of mortality from tuberculosis look quite different for cohorts than for periods. Disentangling cohort from period effects in his data would be quite difficult because the period data begin well after the time when declines in mortality from tuberculosis had begun. There is no clearly appropriate standard to use for distinguishing between the effects; all periods and cohorts experience successively reduced mortality in his tables.

Frost's brief demonstration was usefully followed up by Springett (1950), among others, who examined tuberculosis experience in other geographic areas and concluded that "one assumption will explain most of the observed results, viz. that the relative age distribution of mortality is constant in cohorts rather than in years" (p. 392). However, he also demonstrated that the age pattern of tuberculosis mortality, whether studied by cohort or period, showed considerable variation. It tended to be broad and flat in densely populated urban regions and among males and to be sharply peaked in sparsely populated regions and among females. These differences have not been explained convincingly; they clearly caution against using rigid mathematical "laws" of age variation in tuberculosis.

Sacher (1957, 1960, 1977) fitted a just-identified log-linear age-period-cohort model to tuberculosis data from Connecticut. His results were obtained by assuming that there was no cohort trend for six decades, an assumption that is perhaps unwise and opens his analysis to serious questions. Unfortunately the sources of his data are also not clearly specified.

The pioneering application of sound statistical procedures to age-period-cohort analysis of morbidity and mortality was by Greenberg, Wright, and Sheps (1950). The authors specified the first properly identified age-period-cohort model, adopting a parameterization of age effects through a beta distribution. All three types of effect were found to be statistically significant using an analysis of variance framework and F-tests, rather than today's more usual chi-square tests. Barrett's studies of cancer of the cervix, heart, and bladder (1973, 1978a,b) employed similar approaches to identify the models.

Another disease that has received considerable attention from a cohort perspective is lung cancer. Case (1956a) pointed out that cohort mortality from lung cancer invariably rose with age in England and Wales, whereas period rates formed an inverted U-shape, with a peak around age 60-65. Again the demonstration was graphical, and again the importance of cohort effects rested on the plausibility of biological mechanisms rather than on statistical tests. Indeed, the period rates were no less regular in appearance than cohort rates. But almost all other forms of cancer mortality rise with age. The most obvious explanation of lung cancer's peculiarity, which was supported by additional evidence then available, was that successively younger cohorts were smoking cigarettes more heavily, thus achieving higher death rates at each successive age than older cohorts. In a second paper, Case (1956b) displayed graphs of cohort mortality from cancer at 21 different sites for England and Wales using data for 1911-1954. It is interesting to note that the graph for uterine cancer, another site frequently thought to display cohort effects related to childbearing patterns, is nearly the reverse of that for lung cancer. For uterine cancer, cohort age patterns tend to peak around age 65 to 70, while period patterns trend sharply upwards with age (see p. 186). There are some reasons to expect uterine cancer mortality to decline with age, so that once again the cohort rates may be providing a truer picture of physiological processes. But the point is simply that, without introducing such external information (or reasoning), it would not be possible to distinguish effectively between cohort and period effects. A purely statistical test that gave dominance to cohort effects in lung cancer would surely award it to period effects in uterine cancer.

Beard (1963) specified a full age-period-cohort log-linear model for death rates due to cancer of the lung and was probably the first author to make the linear identification constraint explicit for such a model. His fitting procedure seems to make use of quadratic contrasts, which are properly defined for each dimension. Two assumptions on linear scaling are then adopted. First, the cohort effects are chosen to approximate the proportion of smokers in the population, which they do reasonably well except for the extreme cohorts, although the fit is not presented for each cohort individually. It is then claimed that the period effects should approximate the average cigarette consumption some 10 to 15 years earlier, and again the fit of the reparameterized model to available data is moderately good. This approach of making the arbitrary linear constraint take a value that allows one of the effects to be assimilated to some external reference distribution seems eminently sensible, although it cannot be used to suggest causation, as Beard acknowledges. (A similar tactic might profitably be used in fertility models by making the structure of the age effects correspond as closely as possible to natural fertility.)

Thus, in mortality analyses, it seems clear that in many cases cohort effects are biologically plausible and have been demonstrated in a variety of ways that may not be statistically rigorous but are nevertheless convincing.

Nuptiality and Divorce

Very few simultaneous analyses of period and cohort effects on marriage patterns exist. Ewbank (1974) approached the problem by first estimating a five-parameter age curve for data on the single-year Swedish cohorts (male and female) of 1872-1929. In a second stage, he examined the residuals for what he refers to as "partial period effects". A similar strategy was used by Goldman (1980) to examine anomalies in male mortality in the Far East by analyzing residuals in period data from the model age patterns for cohort effects and by Vasantkumar (1979) to study mortality among black male Americans.

Another analysis of recent Swedish marriage data (for cohorts up to 1959) focused much more on estimation of trends in period and cohort effects so that the nuptiality pattern could be projected for recent cohorts. Thurston (1977) employed the "constant effects" log-linear model. To identify her model, Thurston assumed that the age pattern was constant over time and chose to match a Coale two-parameter nuptiality model (for those who ever marry) as closely as possible. The effects extracted for periods and cohorts provide an illuminating illustration of the problems of projecting trends under conditions of rapid change.

No explicit models of age-period-cohort effects on divorce have come to our attention. Wunsch (1979) and Preston and McDonald (1979) come to opposite conclusions about the relative regularity of period and cohort effects in their studies, respectively, of recent trends in Europe and of U.S. trends over the century from the late 1860s to the late 1960s. The need for further empirical work on divorce is clear.

Fertility

There has been relatively little analytic work explicitly directed at the separation of age, period, and cohort effects in fertility. Several authors argue that the greater stability of cohort completed fertility over time compared with period total fertility is an indication of a tendency for real groups of women to even out their experience so as to achieve notional reproductive targets or norms. This tendency corresponds with the cohort-inversion viewpoint elaborated earlier and has been used to argue that population projections should be based upon cohort cumulative fertility (cf. Akers, 1965). The U.S. Bureau of the Census (cf. 1977) now prepares its projections from cohort fertility information. However, the critical question of whether the future volume of fertility for cohorts of reproductive age can best be estimated from period or cohort data has not yet been answered satisfactorily.

To demonstrate the existence of cohort effects requires either explicit age-period-cohort models or at least a demonstration that cohort completed fertility is different from a weighted average of period total fertilities (weighted by a constant age pattern of fertility). This point first seems to have been made by Ryder (1953) in an unpublished paper and by Lee (1974a) and first examined statistically by Brass (1974b).

Ryder (1953) analyzed Swedish age-specific fertility rates for 200 years, divided into 40 quinquennial periods, between 1751 and 1950 for quinquennial groups of women aged 15-49. He noted that the period total fertility rate (TFR) was more variable than the cohort TFR, with "long stretches during which the two series remain apart, despite the fact that the components of both are derived from the same table of age-specific fertility." He suggested that the cohort series would tend to be smoother than the period series because the cohort TFR is drawn from seven successive periods and is thus a type of moving average. The cohort TFR for the women who are 15-19 in period j is given by

$$\text{Cohort TFR} = 5 \sum_{a=1}^{7} r_{a,\, j+a-1} \qquad (3)$$

where r_{ap} is the fertility of women in age group a in period p. There are, however, other measures that are averages taken over seven periods and seven ages, in fact (7!-1) or 5,039 of them. If we consider the 7 x 7 matrix (r_{ap}) for the first seven periods, a total fertility rate averaged over the seven periods can be defined so that one value is selected from each row and each column of (r_{ap}). Ryder singled out the trohoc (or reversed cohort) for special attention. The trohoc TFR starting in period j is given by

$$\text{Trohoc TFR} = 5 \sum_{a=1}^{7} r_{a,\, j+7-a} \qquad (4)$$

In terms of the models discussed thus far, it is obvious that if an additive model (with no cohort effect) is appropriate, i.e., if

$$r_{ap} = W + W_a + W_p, \tag{5}$$

then so long as the r_{ap} are selected so that all seven age effects and seven period effects go into the sum, the 5,040 definitions of the moving average should have the same expected value and all time series should exhibit the same variability.

Ryder calculated a complex index of variability. Its definition need not concern us here; however, the relative values are of interest:

	Index of variation
period TFR	7,488
cohort TFR	223
trohoc TFR	233
average of 5,040 permutations	770

Clearly, the moving averages across periods are less variable than the period TFR. The cohort TFR, while showing considerably less than average variation, differs little from the trohoc. Ryder pointed out that both of these averages were heavily weighted by fertility in adjacent time periods for the adjacent age groups with the highest fertility rates, so that the age-weighting scheme is quite similar. Thus, the similarity of cohort and trohoc variability provides little evidence of a powerful cohort effect or a cohort-inversion phenomenon.

The analysis of U.S. fertility using additive models has led different authors to opposite conclusions about the existence or importance of cohort effects. A number of these studies have employed the cohort fertility tables for single years of age and single-year periods issued recently by the U.S. National Center for Health Statistics (Heuser, 1976).

Pullum (1980) modeled data for U.S. white females for the period 1917-1973 from this source using a logit-linear approach. He considered four models, three with each possible pair of age, period, or cohort main effects, and the fourth with all three main effects. Two indicators of fit were provided, the first a chi-squared statistic and the second an index of dissimilarity, which can be interpreted as the minimum proportion of cases that would have to be shifted for perfect agreement between the model and the table. As he did not have the exposures to risk available, all calculations were based on a nominal thousand woman-years per cell. The models were fitted to 11-year runs of the data, although 21-year runs were employed in some cases. In general the fits were remarkably good, although longer periods were not considered "because of the evidence of increasing interactions between age and period which cannot be described as cohort effects." Pullum measured the impact of adding a cohort effect to an age-period model by the difference in the chi-squared statistics for the age-period and the age-period-cohort model divided by the difference in their respective degrees of freedom; the impact of adding period effects to an age-cohort model was measured in similar fashion. Adding the period dimension had greater impact per degree of freedom (and often absolutely) than adding the cohort dimension, although inclusion of both sometimes led to noticeable further improvement. To check whether formulation of a true set of complete cohort rates changed the outcome, Pullum assembled the 22 complete birth cohorts and again tried the various models. Again, period was a more successful explanatory variable than cohort. His conclusion was that, for 11- or even 21-year periods, a simple main-effects age-period model was quite adequate and preferable to an age-cohort one. Both this negative finding and the fact that real cohorts of women bear children for 30-year periods suggest that constant-effect cohort analysis of fertility in developed countries may not be promising; most cohort fertility theories (as opposed to models), of course, do not postulate constant-effect models.

Pullum also suggested that such models could usefully be applied to order-specific fertility rates. Isaac et al. (1979) undertook the analysis of birth probabilities by race, age, order, and year using data from the Heuser (1976) volume updated through 1976. Parameters were estimated for three models: an additive model including age, period, and cohort effects and two multiplicative (log-linear) models positing age and period effects and including or omitting the cohort dimension. Each birth order (0-4) was analyzed separately by race using ordinary least squares regression procedures. No mention is made of how the overidentification problem was handled. The authors found that the multiplicative models in general explained more variance and argued that the cohort effects could be eliminated "with little or no empirical loss" because addition of the cohort dimension to an age-period model hardly increased the explanatory power.

Sanderson (1976) calculated his own set of age- and order-specific birth probabilities beginning with the cohort of women born in 1900 and continuing through calendar year 1966. His investigation was directed toward an examination of the Easterlin hypothesis (cf. 1973, 1978) regarding cohort fertility. He also assumed a log-linear model and estimated age, period, and cohort effects for each order using an unconventional and dubious method that arbitrarily apportioned any trend in the birth probabilities equally to the current year and cohort components. His period effect plots closely resemble those prepared by Isaacs et al. In general, Sanderson found cyclical variation in the cohort components, especially for the lower birth orders. The amplitude was greatest for the first and second births, and the cyclical movements were not necessarily in phase with one another.

Page (1977) also concluded that an adequate description of fertility rates can be achieved without introducing cohort effects into her model. This work is of especial interest for substantive and methodological reasons. Page was able to model both the age and duration effects, reaching conclusions that offer considerable insight into patterns of changing fertility. However, Gilks (1979) later examined the same data and models, adjusting the data to allow for changes in definitions over time and using somewhat different estimation procedures. This led him to qualify some of the original conclusions. Because these studies clearly illustrate the lack of agreement on estimation procedures for such analyses, a rather detailed discussion is offered here.

Page took period fertility rates by age and marriage duration for Sweden, England and Wales, and Australia. For each period (and separately by country), she fitted models with age and duration main effects, adopting various "robust" or "resistant" fitting procedures of the type proposed by Tukey (1977). The model used is, effectively,

$$\underline{\ln}(r_{apd}) = u_p + u_{ap} + u_{dp} + e_{apd} \tag{6}$$

which is a version of the model

$$\underline{\ln}(r_{apd}) = w + w_p + w_a + w_{ap} + w_d + w_{pd} + e'_{apd} \tag{7}$$

with no marginal constraints on the period dimension. Models (6) and (7) can be reparameterized one to another, but care is required in interpreting the estimates. The fitting procedure uses data and model-determined weights. Although the Tukey-type procedures are intuitively attractive, since each model changes the weights for a given cell, model discrimination on statistical criteria is virtually impossible. Page in fact adopted a method of assessing overall goodness-of-fit that gives all non-zero cells equal weights, ignoring those used in the fitting procedure itself. She then calculated the proportion of variance in the rates accounted for by the model. By this criterion, the fits are very good indeed.

Page then examined the structure of the effects. She found that the age effects (the u_{ap}'s) were approximately constant over time and corresponded closely in shape to the age pattern of natural fertility (Henry, 1961). There was some age-period interaction, in that the slope of the age effects became greater for more recent periods.

Page went on to study the structure of the duration effects (the u_{dp}'s). They showed a marked change over time, moving from a concave to a more linear pattern, indicating interactions. Page suggested that the interaction might be at least partly due to year of marriage effects. She therefore normalized the duration effects for each time period so that the effect was zero for duration two and then combined the new terms, u'_{pd}, for marriage cohorts rather than time periods. These normalized (within marriage cohort) effects declined fairly linearly for each cohort above duration two, although with differing slopes for each cohort. Since fertility at duration zero was always substantially higher than the values implied by the straight line, Page adopted an ad hoc adjustment of adding 0.3 to the predicted value at this duration so that the model became

$$\underline{\ln}(r_{apd}) = u'_p + u'_a + (d-2)u'_{(p-d)} + 0.3b \qquad (8)$$

where u'_p is the redefined period effect, u'_a is the natural fertility set of age effects, $u'_{(p-d)}$ is now a marriage cohort effect, and b is one when duration is zero and zero otherwise. This is a considerable simplification of the earlier model and still appears to fit the Swedish experience tolerably well. Similar results were reported for England and Wales.

It is worth quoting Page's concluding remarks (1977, pp. 103-4): "Demographers have become increasingly accustomed to thinking of fertility in terms of cohort experience; in this framework, period effects tend to be viewed as contributing irregularities to cohort patterns. Here, by twisting the data kaleidoscope a different way, we have focused on the existence of period regularities. Our data show that, at any given time, all birth and marriage cohorts react, in some sense, as a single unit to whatever factors determine the general level of fertility at that time....It is as if each cohort were characterized by a latent exponential decline in its fertility as it passes through marriage, interrupted only by period effects to which all cohorts respond by proportionately the same amount."

Gilks (1979) both adjusted the data for England and Wales and used different procedures for estimating the parameters of similar models. For example, he tested goodness-of-fit by using the weighted sum of squares of the residuals of the fitted models, with weights defined by a biweight function over all the residuals. He found the age patterns more variable over time and less like the pattern of natural fertility. The respecified (within marriage cohort) duration effects were approximately linear for later cohorts, although generally slightly convex. Perhaps most importantly, with his methods of analysis, England and Wales showed considerable effects of World War II on fertility patterns and levels that did not show up in the original analysis. During this period the duration effects (normalized and reconsidered for marriage cohorts) became quite nonlinear. Gilks also found severe problems with the rearrangement of durational effects for periods into ones for marriage cohorts and concluded that these data require models that include marriage cohort effects explicitly, with the usual caveats about confounding of linear parts of main effects.

Both Page and Gilks noted considerable patterning of their residuals in each period table. Particularly poor fits are seen at the early durations, mainly because much of the fertility shortly after marriage results from premarital conceptions. For young ages at marriage, this excess fertility in the first eight months of marriage clearly causes consequential waves in the fertility rates by duration, through the removal of a high proportion from exposure to risk for the next year or so.

Thus, despite methodological and substantive disagreements, both of these investigators suggest that fertility responds not only to age and period effects, but to marriage duration, marriage cohort, and age at marriage. Their work indicates that models considerably more complex than those specified thus far may be required for adequate analysis of fertility. In the remaining studies in this review, complexity is introduced either by specifying nonlinear cohort effects or by adding additional variables.

Lee (1974b, 1977, 1980) has moved work on age, period, and cohort effects closer to theories of fertility. In the first paper (1974b), he examined several possible models of homeostatic fertility control, including a period and a cohort formulation. In the former case, age-specific fertility rates were a function of the size of the labor force; in the latter, they depended only on the relevant cohort size. The implications of each of these models for the autocorrelation function of fertility (the correlation of the birth rate at time t with that at time t - m, for values of m in years) were examined. Lee found that when fertility responded either to relative cohort size or period labor force size, there would be fluctuations in fertility of about 40 years, rather than the generational-length fluctuations expected when relatively constant age-specific rates were applied to an irregular initial age distribution. Fluctuations consistent with controlled fertility were not observed for preindustrial European populations; however, U.S. fertility exhibited cycles similar to those predicted by the cohort or period control models. Lee concluded that the latter model was more plausible since the estimated elasticity in the TFR estimated for the cohort model was very close to -1, which would dampen any disturbance and lead to a prediction of a constant number of births regardless of cohort size.

As Lee correctly acknowledged, cycles that are compatible with control may be induced by factors not considered in his model. Nevertheless, his work is suggestive that control mechanisms operate primarily through period effects if at all. At least he is able to demonstrate nonexistence of a high degree of control operating through cohorts. All these conclusions require further qualification since simple (and linear) models were posited for the control process.

Lee (1977) went on to develop a stock-adjustment or cohort-inversion model by distinguishing between terminators (who desire no more children) and non-terminators, although he presumed that both groups always have the same achieved fertility. This assumption seems unlikely to be true and makes his model indistinguishable in many respects from one that ignores the distinction. Even so, his model is more attractive conceptually for this distinction. In its simplest form and ignoring a nine-month or one-year lag, Lee's model can be written as

$$r_{apc} = b_a(T_p - F_a{}^c) \quad \ldots \qquad (9)$$

where $F_a{}^c$ is the cumulated fertility of the cohort c that is age a in period p, and b_a can be interpreted as the rate of adjustment at age a or the proportion of desired additional births achieved in each year and is further defined as the "ratio of the fertility rate of those wanting additional births to the number of additional births wanted by those wanting more".[7] T_p is the total fertility desired. It may change over time and is approximated by desired (or expected) completed fertility as measured by questions on reproductive intentions. In this paper, Lee also used a rearranged version of equation (9) to estimate T_p.

He found that if, for a particular marriage cohort, the target is constant over time and b_a is constant over age, the model implies that marital fertility declines exponentially with age. Interestingly, this model therefore corresponds closely to Page's (1977) parameterization of marriage cohort contributions to fertility rates. Lee also presented some evidence that b_a increases slightly with age and suggested a linear change, which leads to a model of the form used by Jain and Hermalin (1971) for fertility within a cohort.

A somewhat different view on the subject of period and cohort influences on fertility was taken by Ryder (1980) in a recent paper in which he attempted to decompose period total fertility rates into components attributable to changes in the quantum and tempo (or volume and timing) of cohort fertility. Employing the Heuser (1976) data updated through 1975, Ryder first projected completed fertility for birth cohorts through 1950, on the assumption that age-parity-specific birth rates remain fixed at their 1975 level. He then calculated an _index of distributional distortion_, defined, for a given period, as the sum over cohorts of the proportion of its total fertility that each cohort experiences in that period. Regardless of level of cohort fertility, the index would equal unity if each cohort had the same age distribution (or tempo) of fertility. This index was proposed as a period-specific measure of change in cohort tempo that can have both long-term and short-term components. Ryder related the long-term changes to alterations in the cohort mean age of childbearing. After adjusting for this type of change, he treated the residuals as short-term deviations. For the United States, the three major fluctuations were associated with wars--World Wars I and II and the Vietnam War.

Ryder also defined a period-specific measure of changes in the quantum of cohort fertility as the period total fertility rate divided by the index of distributional distortion. The trends in these measures led him to conclude that changes in quantum of cohort fertility explain most of the variation in period rates during the time period 1922-1936; changes in tempo (i.e., the index of distributional distortion) have been dominant since then. Most of the baby boom, according to this analysis, would have occurred without any changes in quantum of cohort fertility and depended scarcely at all on numbers of unwanted children.

Butz and Ward (1979) independently partitioned the period TFR into exactly the same two components. Their timing index is Ryder's index of distributional distortion (or tempo of fertility), and their average completed fertility is Ryder's measure of change in the quantum of cohort fertility. Their calculations differ only in the method of estimating completed fertility for cohorts still of reproductive age.

The main thrust of the Butz and Ward paper is the development of a new theory about why these indices change. The authors hypothesize that the timing index depends on the way in which couples make decisions about when to have their next child and that a couple makes the decision by gauging the pace of its current births not against actual completed fertility but against the "completed fertility it expects at the time the birth decision is made". They describe the timing index based on known completed fertility as an _ex post_ timing index and one based on the completed fertility expected at time t as the _ex ante_ timing index for time t.

Their _ex ante_ measures were derived from two quite different procedures for estimating expected completed fertility. The first depends on an economic model in which fertility for the next year is determined by fertility in the current year, current age of the woman, and predictions for the next year of women's employment and wages and husbands' income. Values of male income and female wages and employment ratios were projected independently and entered into the model to yield estimates of fertility rates used to calculate the _ex ante_ timing index. The second version of the _ex ante_ timing index is based on expectations data collected by the Current Population Surveys. The levels and movements of both indices are quite similar. Butz and Ward, like Ryder, concluded that for recent time periods "most of the year-to-year change in period fertility rates is due purely to the altered timing of births." They also suggested that, to a great extent, current fertility is determined by the couple's assessment of its total expected fertility and its projected economic situation.

Lee (1980) extended the analysis of the model proposed in his 1977 paper (and given in equation (9)). He assumed that T_p, the desired completed family size in period p, was the same for all cohorts and examined the implications of time trends in T_p for period total fertility. He found that if T_p is increasing, then the period total fertility rate will be higher than T_p because the rate of childbearing must increase at every age to compensate for lower fertility at the younger ages. Similarly, if T_p is declining, the period total fertility rate will be lower than T_p since couples will have attained a higher proportion of T_p by any given age.

If target fertility fluctuates, then the period TFR rate will fluctuate also, but its turning points may precede those of T_p. The counter-intuitive reason for this finding is that the period TFR depends upon the gap between current cumulated fertility and target fertility, which is the additional desired fertility. As the rate of increase in T_p slows, this gap narrows, so that the period TFR begins to fall when the rise in T_p begins to slow down. From this theoretical framework, it also can be predicted that the fertility of older women will fluctuate proportionately most, that of young women least.[8]

This simplified model does not preclude negative fertility in terms of declining desired completed fertility, as Lee acknowledges. Nor does it fit real data especially well. His model can be rearranged to give

$$F_a{}^c = T_p - r_{apc}/b,$$

where a is here used to represent duration of marriage. It is clear that a linear regression of this form does not fit 1974 data for England and Wales at all well. For practical fitting purposes, alternative approaches through log-linear or logit relations seem more sensible. Despite these criticisms, the Lee model is an extremely useful abstraction for understanding the theory of fertility.

Lee also has begun to develop models in which negative fertility is precluded. He finds that irreversibility of fertility causes an asymmetric response to rising and falling levels of target fertility that cannot be ignored without producing large errors in estimates of final cohort fertility.

Models that incorporate factors other than age, period, and cohort are considerably more difficult to apply. Suppose one was interested in modeling the behavior of the fertility rate r_{apcdmy}, where the subscripts represent, respectively, age, period, cohort, marriage duration, age at marriage, and year of marriage. Solution of the identification problem in a linear model is obviously much more demanding than in the simple age-period-cohort case. Just as age and period uniquely identify cohort membership, so do age at marriage and duration of marriage uniquely identify the marriage cohort. Furthermore, certain sets of three of the six indices are sufficient to identify the remaining three. For example, age, period, and year of marriage identify not only cohort but also duration of marriage and age at marriage. There are substantial difficulties in fitting models of this type, although one approach is outlined in the Appendix. Furthermore, the analyst must address several new practical problems: for example, how to deal with marital disruption (perhaps by eliminating all non-marital rates even though they would contribute to age, period, and cohort rates, or by defining duration as pertaining to years since most recent marriage); and how to handle the many zero cells that will emerge in the high duration-low age categories. Presumably because of the complexity of these problems, there has not been, to our knowledge, any attempt to estimate the full-blown model.

Brass (1974b) used two models that included some of these effects to examine possible differences between a weighted average of period total fertilities and actual completed cohort fertility. The first, which is implicitly given in his paper, was, in the notation used here, essentially a log-linear model of period and duration main effects for each age at marriage group:

$$\underline{\ln}(r_{pdm}) = u + u_p + u_d + u_m + u_{pd} + u_{dm} \; (+e_{pdm}) \qquad (10)$$

where all six linear effects are implicitly included in the model and u_{dm} incorporates all the age main effects. Addition over the duration of marriage diagonals gave expected values of cohort completed fertility. Brass's second model was a hybrid one and not very clearly specified. For each age at marriage group, a log-linear model like (10), with period and duration main effects, was fitted. The residuals on the natural scale (constrained to sum to zero) were then modeled by fitting straight lines. If the expected value under model (10) is denoted by r'_{pdm}, this seems to imply that

$$r_{pdm}/r'_{pdm} = y_{dm} + z_{dm}p + e'_{pdm} \qquad (11)$$

with $\sum_p (y_{dm} + z_{dm}p) = 0$ for all d, m. This adaptation was intended to allow for changes in the durational pattern of fertility over time and must pick up some of the duration-period interaction. Given the mixed form of the model, it is hard to say how much of the part of the interaction so fitted would correspond to marriage cohort effects, however defined.

Brass concluded from this second model that

> The notable feature of the comparisons is the broad general agreement between the series of observed and expected rates, although the individual differences are considerable relative to the range of variation....There is no indication of a cohort effect of the kind which would arise from 'making-up' or 'making-down' of births. The cohorts which were married in early 1940's and experienced the low war-time fertilities ended up with rather fewer children than the averaging of time-periods would give; those subject to the higher

> early 1930's rates at the beginning of marriage exceeded the expectations. These deviations would suggest the possibility that the time-period fertility level early in marriage might persist for a cohort rather than the reverse. But, on average, the women married in 1945-48 when fertility was high ended with families in good agreement with the model levels and those from the low initial rate cohorts of 1951-55 needed to exceed them. The overwhelming impression is that the cohort completed family sizes reveal no significant feature which distinguishes them from time averages.

A few comments are in order. The standard error of the actual completed fertility of cohorts married in 1931-1951 is, at .0549, noticeably less than the standard error predicted under either model (.0714 for the first and .0679 for the second), suggesting greater cohort stability than expected. In addition, Brass's detailed results (such as he presents) show some evidence of a cohort-inversion effect. In particular, the war cohorts, which experienced lower fertility initially, show signs of overcompensation during the immediate postwar period but subsequently have lower than expected rates. This finding is consistent with the view that some women who had delayed childbearing would not make up postponed births either through infecundity or because they substituted other activities for childbearing (cf. Rindfuss and Bumpass, 1978). The cohort-inversion models proposed earlier in this paper (with the possible exception of the period-target model) may not be capable of capturing this type of effect. Brass's interpretation may be correct, but there is at least a prima facie case for more careful model specification and a suggestion that a cohort effect may exist. Again the problems of model specification and estimation need to be solved before conclusive results can be obtained.

Gilks and Hobcraft (1980) have begun investigating explicit models containing (some or all of) age, period, cohort, duration, age at marriage, and year of marriage effects employing data for England and Wales. They have concluded from their preliminary analyses that cohort-inversion formulations fit the data no better than conventional fixed-effects age-period-cohort models and that better fitting models can be found that only incorporate age and period effects, at least for the period 1938-1975. Their work with the more elaborate data set, which adds the age at marriage, duration of marriage, and time of marriage dimensions, is still in progress. It has proved somewhat inconclusive thus far, owing to substantial difficulties encountered in finding a model capable of capturing the range of variability in patterns for this period.

Their work again illustrates our conclusion from this review that nearly the same data sets, analyzed on the basis of differing theoretical considerations about the nature of cohort reactions, yield quite different results from those achieved by statistical models that are not derived from consideration of the underlying phenomenon being studied.

Summary

We have reviewed the state of the art of age-period-cohort analysis for demographic dependent variables. Major examples of such analyses have appeared both in mortality and fertility studies. In the area of mortality, the conventional approach to such analysis appears to be well suited to a wide range of applications and often yields plausible and useful results. There are several reasons for its suitability. First, early childhood experience (for subsequent periods) does seem quite important in many major disease and death processes, so that cohorts are legitimately viewed as acquiring early on a certain fixed susceptibility. Second, data sometimes stretch back far enough that stationary "standards" of age patterns can be developed empirically and applied to later experience. Third, logarithmic or logistic transformations do seem to linearize comparisons of age schedules or mortality quite effectively, so that standard statistical procedures are appropriate. Nevertheless, there are many instances when application of age-period-cohort analysis is anything but routine and where external constraints are required, typically in the form of theoretically based and mathematically expressed age patterns of mortality, in order to distinguish effectively between period and cohort effects. A set of models of age patterns of mortality that are based on cohort as well as period experience could be constructed and might find useful application.

The conventional approach is much less suitable for the analysis of fertility, except perhaps in natural fertility populations where physiological mechanisms play a dominant role. Once goal-directed behavior is introduced, it is important to base any empirical examination on theories or assumptions about how such goals are formulated and pursued. Only if one is prepared to accept the assumption that all of the pertinent goals and strategies are formulated before the initiation of childbearing and remain unaffected by subsequent events would conventional analysis suffice. This assumption is untenable for modern developed populations, and the forms of analysis appropriate to age-period-cohort investigations of fertility will have to develop hand in hand with the theories of reproductive behavior.

Notes

[1] An interesting discussion of the many influences reflected in the age variable in studies of fertility may be found in Rindfuss and Bumpass (1978).

[2] Figures for 1966 show death rates at ages 70-74 of 35.0/1,000 for females and 61.8/1,000 for males, compared with 0.31/1,000 and 0.5/1,000 at ages 10-14 (Keyfitz and Flieger, 1968).

[3] Our underlining.

[4] The studies were also dependent, though less critically so, on the assumption that mortality change would appear in the form of equi-proportionate declines in death rates at all ages. The index numbers of relative mortality levels would not have tended to cluster diagonally were this assumption seriously violated. Nevertheless, the fact that mortality declines were initiated in a cohort-specific rather than period-specific way would still have been clearly registered in the data even if proportionality had not pertained.

[5] The Makeham curve uses three parameters to describe age-specific mortality rates, m_a, as

$$m_a = b_1 + b_2 b_3^{a}.$$

[6] From data for ages 5-59 in Table 1 of Frost (1939).

[7] However, this interpretation is only sensible if all women are perceived as wanting additional births, and thus it implies a homogeneity of both achieved and desired fertility or at least of desired additional fertility. As mentioned previously, this homogeneity assumption is implicit in the cohort-inversion models proposed so far but could be reduced or overcome by introducing parity into the models.

[8] Yule, in a remarkable paper in 1906, introduced what was effectively a cohort-inversion model for marriage. He observed that wars in France, Prussia, and Denmark had led to substantial postponement of marriages with subsequent "catching-up" afterwards. The swings in vital rates induced by wars and pestilence were documented at least as early as Sussmilch (1761), and Malthus (1798) argued that these were induced by period changes in trade, missing the concept of postponement. Yule, following Hooker (1898), went on to argue that

> those who postpone marriages in one year only survive to marry, or possibly marry, in the next; the divergence of the marriage rate in any one year from normal depends, therefore, not on the postponements caused by unfavourable factors in that year alone, but _on the difference between the postponements of that year and the postponements of the year before_. If the postponements are increasing from one year to the next, the marriage-rate will be below normal, but not necessarily falling; the marriage-rate will only fall if the volume of postponements is increasing at an increasing rate, i.e. if the unfavourable factors are _accelerating_. But applying this idea to the case of oscillations, we see that the marriage-rate ought to attain its maximum not when the favourable factors are at a maximum and the unfavourable at a minimum or shortly after, but when the favourable factors are increasing and the unfavourable decreasing most rapidly.

Yule gave evidence that no such phenomenon appeared to occur for marriage in more normal times, with cycles in marriage frequency in England and Wales having the same periodicity and phase as other indices. He also suggested that the postponement argument was even less applicable to the analysis of birth rates in normal times and that wars had a lesser effect on births than marriages. His work constitutes an important precursor of the recent research on period targets for fertility and is more subtle than most recent arguments within a "cohort-inversion" framework.

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Appendix

An Approach to Hierarchical Fitting of Linear or Polynomial Age-Period-Cohort Models

Recent work (e.g., Fienberg and Mason, 1979) on age-period-cohort models has focused heavily on the identification problems when a linear model for the rates (or for a transformation such as logarithm or logit) is proposed. In addition to the usual analysis of variance type constraints on the sums of the age, period, and cohort effects, there is the additional problem caused by the confounding of linear effects in each of the three variables. In many ways the simplest approach to this problem seems to be to work with the modified model (for the expected value)

$$f(r_{apc}) = W + (b_1 - t_1)a + (b_2 + t_1)p + (b_3 - t_1)c + {}_2R_{A-1}(a) + {}_2R_{P-1}(p) + {}_2R_{C-1}(c) \qquad (1.1)$$

where ${}_2R_{A-1}(a)$ denotes a polynomial in a with terms included for all powers of a between 2 and A-1, with A denoting the number of age groups included in the analysis; ${}_2R_{P-1}(p)$ and ${}_2R_{C-1}(c)$ are defined similarly; and t_1 is an arbitrary constant, which can always be set equal to one of b_1, b_2, or b_3 to remove one of the linear terms and identify the model. Following the spirit of Goodman's notation (cf. Bishop, Fienberg, and Holland, 1975) for fitting contingency tables through marginal configurations, it is possible to define L_A as the linear part due to age and ${}_2R_{A-1}$ as the remaining part due to the polynomial defined above. Then the conventional age-effects model, denoted by S_A, is equivalent to $[L_A, {}_2R_{A-1}]$, and $[S_A, S_P]$ is equivalent to $[L_A, {}_2R_{A-1}, L_P, {}_2R_{P-1}, L_C]$, since a model consisting of any two of the linear effects actually includes all three. This formulation makes the problem clear and allows hierarchical fitting of models. Provided the confounding of linear effects is recognized and no attempt is made to attach meaning to the linear effect parameters, the results are interpretable. Following the concept of marginality expounded by Nelder (1977), it seems reasonable to restrict such hierarchical fitting to those cases where the relevant linear term is always included before the corresponding polynomial, as the polynomial terms are defined relative to the linear and grand mean terms. Consideration would then be restricted to two or three types of hierarchy, namely,

$$\begin{tikzcd}
L_I \arrow[r] & S_I \arrow[dr] & & & \\
& & S_I, L_J, (L_K) \arrow[r] & S_I, S_J, (L_K) \arrow[r] & S_I, S_J, {}_2R_{K-1}, (L_K) \\
\text{and } L_I \arrow[r] & L_I, L_J, (L_K) \arrow[ur] \arrow[dr] & & & \\
& & S_J, L_I, (L_K) \arrow[uur] & &
\end{tikzcd}$$

where I can be any one of A, P, or C; J one of the remaining two; and K the final one, giving six possible ways of achieving each of the hierarchies above. (The term L_K is bracketed to indicate its redundancy and non-explicit fitting--an approach that is similar to the fitting of a two-way margin implying fitting on the one-way margins in normal hierarchical models for contingency tables.) The hierarchy could be completed by inclusion of the saturated model S_{IJ} where S_{IJ} is used to denote a model that has the IJ interaction present in addition to the I and J effects, and model selection can be carried out on the usual criteria using chi-squared statistics as the basis of choice (cf. Fienberg, 1977).

The problems become even more complex when other factors are added to the models. For example, a model for fertility rates may be proposed in which

$$f(r_{apcdmy}) = W + W_a + W_p + W_c + W_d + W_m + W_y + \ldots. \qquad (1.2)$$

where the subscripts represent the usual age, period, and cohort influences and, additionally, duration of marriage, age at marriage, and year of marriage, respectively. There are now three separate linear identification problems, which can be highlighted as before by breaking up the model into linear and remaining polynomial terms:

$$\begin{aligned} f(r_{apcdmy}) = {} & w + (b_1 + t_1 - t_2)a + (b_2 + t_2 + t_3)p \\ & + (b_3 - t_1 - t_3)c + (b_4 - t_1)d + (b_5 - t_2)m + (b_6 - t_3)m \\ & + {}_2R_{A-1}(a) + {}_2R_{(P-1)}(p) + {}_2R_{C-1}(c) + {}_2R_{D-1}(d) \\ & + {}_2R_{M-1}(m) + {}_2R_{Y-1}(y)\ldots. \qquad (1.3) \end{aligned}$$

where b_1 to b_6 are the linear effects and t_1, t_2, and t_3 are arbitrary constants that can always be used to force up to three of the effects to zero. It is a simple matter to go on to elaborate hierarchies of models as was done earlier for the simpler case.

Perhaps more interesting is to consider how all these effects might be discovered informally in a conventional log- or logit-linear model framework.

For the simple age-period-cohort case it suffices to examine either the pattern of residuals after fitting a two main-effect model or the pattern of interaction terms from the saturated model. Consistent positive or negative values along each of the diagonals defining the third dimension would indicate the value of trying a constant-effect age-period-cohort model; regular tendencies for a string of positive (or negative) values to be followed by a string of negative (or positive) values along each cohort diagonal would suggest a cohort-inversion approach, although care is required due to the non-normalization across cohorts. In this more complex situation it is simpler to examine parameter estimates from a model specifying three two-way interactions than to study the residuals from a three main-effect model for each of the three possible two-way margins (note that the three variables have to be chosen to span the entire table space). There are two reasons for this: there will be substantially fewer values to examine and, more importantly, the parameter estimates are controlled for covariation across the other two-way margins, whereas the residuals would not be so controlled and would be harder to interpret. A simpler approach to examination of residuals might be to fit the model with two of the two-way margins and display residuals for the third two-way combination: this would control for covariation but would still require examination of more values. An alternative approach is to launch into a full six-effect model from the outset, but in view of the triple identification problem this requires care, although ultimately such models need testing.