A BRIEF ANALYSIS OF NEWTON'S LAW OF COOLING

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Sir Issac Newton (1643-1727), a well respected mathematician and physicist, dominated and revolutionized his field in the 17^{th} century. He was an Englishman who graduated from university without any particular distinction and was reportedly an absent-minded professor (what other kind). The bachelor spent a great deal of time studying theology and biblical chronology, but seemed to have a knack for math and physics. His book *Naturalis Principia Mathematica,* usually known just as *Principia,* is considered one of the greatest single contributions in the history of science (Clapham, 1996). He is responsible for the essentials of calculus, the theory of mechanics, the law of gravity, the theory of planetary motion, the binomial series, Newton's method in numerical analysis, and many important results in the theory of equations. However, we are not here to discuss the man and his many accomplishments, rather one of his physical laws, the law of cooling. The second law of thermodynamics says that heat cannot of itself pass from a colder to a hotter body. This is obvious, however the rate at which the heat passes from the hotter body to the cooler is what is interesting to Newton and therefore to us.

After experimental data was analyzed, a rather interesting conclusion was reached about the relationship between the temperature of an object and the surrounding environment. The temperature of an object changes at a rate proportional to the difference between the temperature of the object and its ambient temperature. This is Newton's law of cooling. If $T(t)$ is the temperature of the object at time t, and $A(t)$ is the ambient temperature (also a function of time), then the rate of temperature change of the object with time (dT/dt) is:

$$\frac{dT}{dt} = -k(T(t) - A(t)) \tag{1}$$

where k is a positive constant (k>0). Removing the minus sign and taking k< 0 is valid, but convention dictates k as a positive real number. The minus sign in front of the (positive) k is important, since if the object is warmer than its surroundings (T>A), it will become cooler with time. That is, dT/dt < 0 when T-A > 0.

good

It is often reasonable to assume that the ambient temperature is relatively constant over the given amount of time. Although it requires great attention for it to be completely constant, typically the fluctuations in A(t) are insignificant and can be ignored. To solve the differential equation with A as a constant, first separate it:

good

$$(T - A)^{-1}\, dT = -k\, dt$$

then integrate: $\ln|T - A| = -kt + D$

where D is a constant from the integration. Taking the exponential of both sides gives:

$$T - A = Ce^{-kt}$$

where $C=e^{D}$, that is: $T(t) = A + Ce^{-kt}$

This is a very useful form, as we can usually determine the constant C easily. If the initial temperature of the object (time t=0) is known, then let $T(0)=T_0$. Then $T_0=A+Ce^{-k(0)}$ and since $e^0=1$ we have $T_0= A + C$, or $C=T_0 - A$. This gives the convenient equation:

$$T(t) = A + (T_0 - A)\, e^{-kt}$$

To find the rate of exchange of heat between the environment and the object (k) a second reading of the object's temperature is required. Denote this by $T(t_1)=T_1$ then $T_1 = A + (T_0 - A)\,e^{-kt}$ so we have that $(T_1 - A)/(T_0 - A) = e^{-kt}$ or taking logs:

$$\ln \frac{T_1 - A}{T_0 - A} = -kt_1 \quad \text{and solving for k gives:} \quad k = \frac{-1}{t_1} \ln \frac{T_1 - A}{T_0 - A}$$

giving

$$\boxed{T(t) = A + (T_0 - A)e^{-\left(-\frac{1}{t_1}\ln\frac{T_1-A}{T_0-A}\right)t} \qquad (2)}$$

To take a simple example, consider dropping a small metal ball in a bucket of ice water. If the water is at 0ºC and the ball is at 35ºC, and five minutes later the ball is at 23ºC and the temperature of the water is unchanged. This means that A=0, $T_0=35$, and $T_1 = 23$, using the formula for k, we find that k = 0.83970769 and so according to equation (2) we have : $T(t)= 35\, e^{-0.86970769\, t}$. From this we can predict the temperature of the ball one half hour after it was placed in the ice water (assuming that the water has been kept at 0ºC) that is $T(30) = 35\, e^{-0.86970769 * 30} = 2.81855682 \approx 3^0\text{C}$. Notice that as $t \to \infty$, the temperature of the ball approaches 0ºC (since as $a \to \infty$, $e^{-a} \to 0$). This is what we would expect, since if the ball were to get colder than the ambient temperature, it would be giving heat off into a environment that was warmer than itself. The second law of thermodynamics says that a colder body cannot pass heat onto a hotter one, so this would not be possible. This means that is eventually the ball becomes the same temperature as the ice water. This is true of any example of Newton's law of cooling with A as a constant.

sentence structure

If the ambient temperature is not a constant, but a function of time, then the differential equation is a bit more complicated to solve. It is, however, possible to put in the form: $dT/dt + kT(t) = kA(t)$. To solve, simply use an intergrating factor

a linear d.e.

$e^{\int k dt} = e^{kt}$. [A] Multiplying both sides of the equation gives $d/dt\,(e^{kt}\,T(t)) = ke^{kt}\,A(t)$.

Integrating will result in $e^{kt}\,T = k\int e^{kt}\,A(t)\,dt$ $+C$ or just, $T(t)=ke^{-kt}\int e^{kt}\,A(t)\,dt$. $+Ce^{-kt}$

Notice if we take the ambient temperature to be constant, that is, $A(t)=A$ then we have $e^{kt}\,T = Ae^{kt} + C$ solving, $T(t) = A + Ce^{-kt}$ which is consistent with what we had before.

Consider that a man is in cold water, with his head above water and wearing a life jacket so that he will not drown. Assuming the water is cooling exponentially, we can use Newton's law to predict how long he will be expected to survive the effects of hypothermia. First a brief introduction to hypothermia, the condition in when the body's core temperature is below 35.6°C. The core temperature of a human, that is, the temperature of the body's deep tissues (such as those found in the thorax, cranium, and abdomen), is normally 37°C. The surface temperature, however, will often range from 20°C to 40°C. (Koxier & Erb, 1995) Heart rate and blood pressure decrease during mild hypothermia, which is from 35°C to 28°C. Breathing becomes slower and more shallow, and from 30°C to 28°C the victim becomes unconscious. During deep hypothermia (18°C-15°C) the action of the heart, the flow of the blood, and the electrical activity of the brain stop completely. (Boyde, 1990) Common symptoms include shivering, hypotension, decrease in organ function, lack of muscle coordination, disorientation, and drowsiness progressing to a coma. Since we are assuming he is wearing a life jacket, any lack of coordination

use non-homogeneous

see page 7

[A]The reader should have a basic knowledge of methods used to solve ordinary linear differential equations, including the use of integrating factors.

and disorientation should not affect him too much, and even if comatose his head will not slip under water.

Since the water is cooling exponentially, we can represent this by $A(t) = \eta e^{-Mt}$ and as we've previously seen, η is the initial temperature of the water, that is, when he is first immersed. If the water is at A_1 at some later time t_1, then we can solve for the constant M: $M= (-t)^{-1} \ln |A_1/\eta|$. This, with equation (1), gives: $dT/dt =-k (T - \eta e^{-Mt})$. Solving this using the method earlier discussed gives

$$e^{kt} T(t) = k \eta \int e^{(k-M)t}\, dt \qquad \text{or} \qquad T(t) = \frac{k\eta}{k-M} e^{-Mt} + De^{-kt}$$

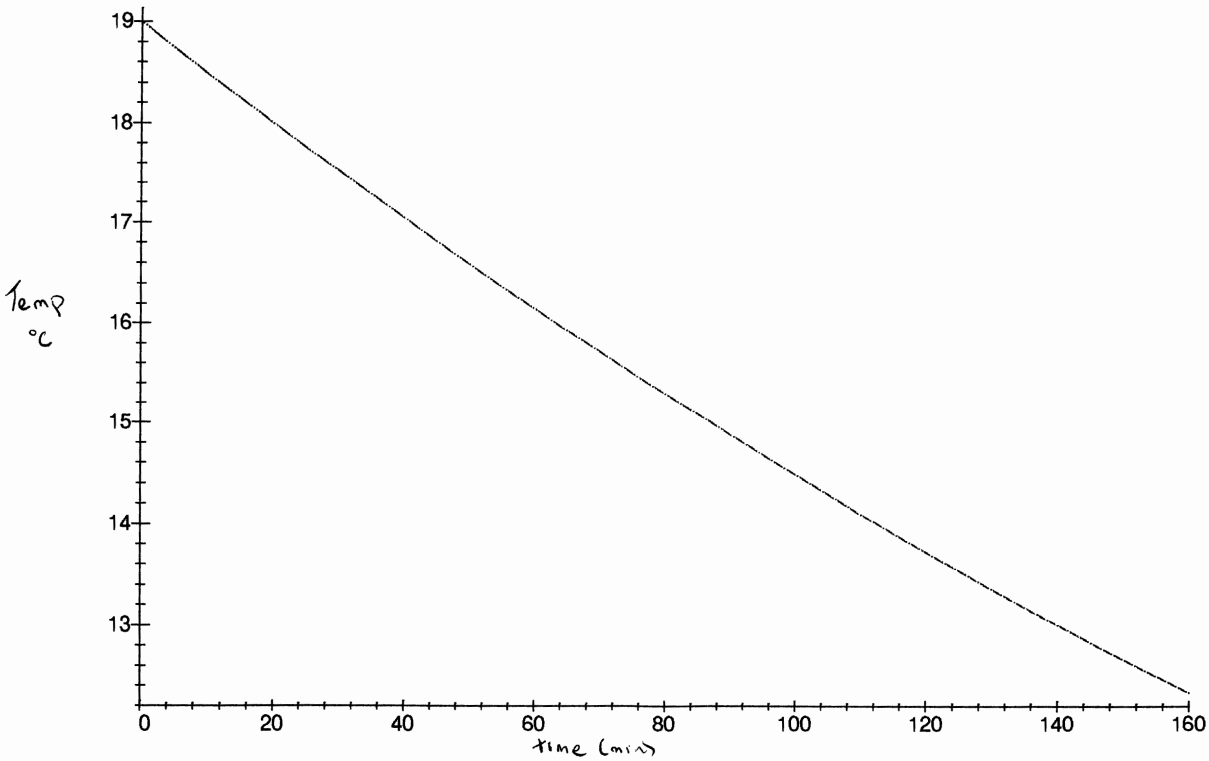
To give an accurate numerical example is rather difficult, as it would involve doing experiments, but some generalized values can be used to illustrate the process. Suppose the initial temperature of the water is 19°C and after twenty minutes it is 18°C. SEE GRAPH #1 It seems reasonable to assume that the victim entered the water at approximately 37°C, as this is normal core temperature, though we do not know the circumstances surrounding the incident (perhaps he was cold before entering). One minute after entering, the victim's temperature has dropped to 36.5°C.

From this information we have that $\eta=19$, and $M= (-20)^{-1} \ln(18/19) = .002703$.

$$\text{So } \frac{dT}{dt} = -k (T - 19e^{-0.002703\, t})$$

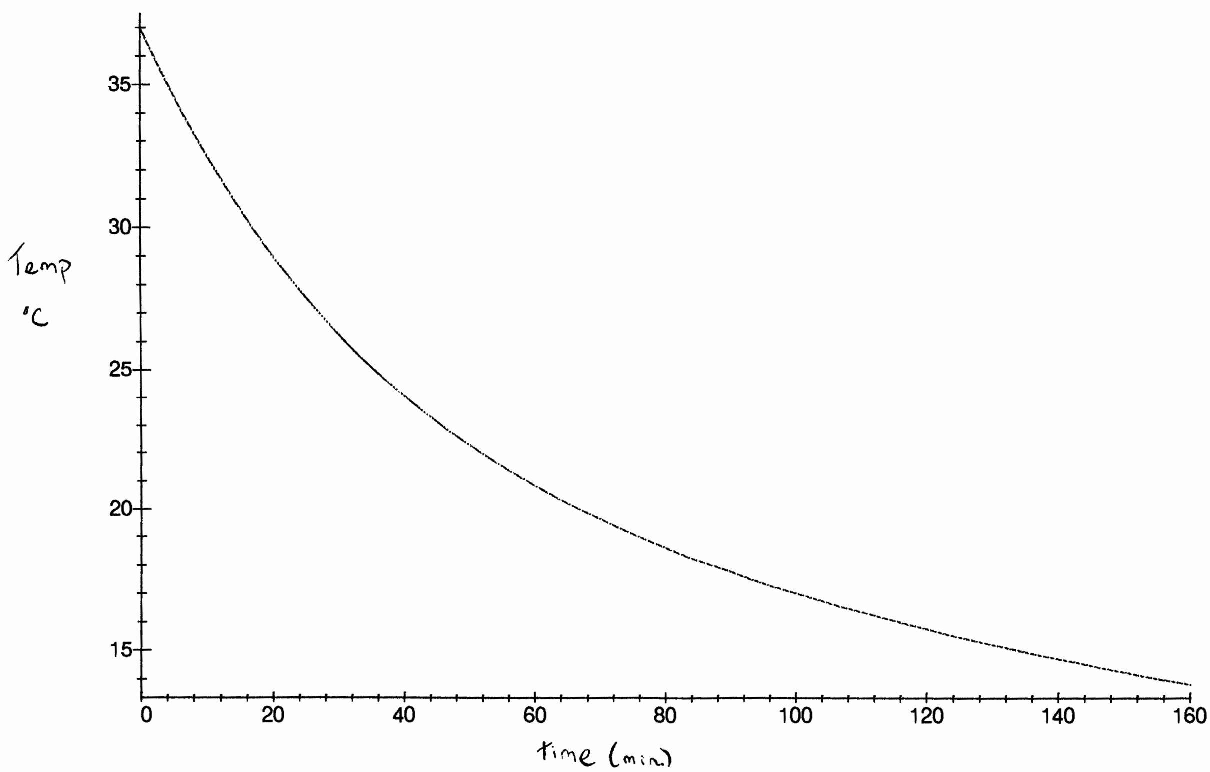
We know that $T(0)= 37 = (19k)/(k-M) + D$, or $D= 37 - (19k)/(k-M)$ We also know that $T(1) = 36.5 = [19k\, e^{-M} + (37 - 19k\, e^{-k}]/[k-M]$ Solving we get $k=0.0282307$ and hence $D=15.988185$. SEE GRAPH #2 Using these results, we find that after ten minutes the body is at 32°C, and the victim is very close to losing consciousness. The

WATER_TEMPERATURE



(1)

VICTIM_BODY_TEMPERATURE



(2)

victim will be at 20°C after an hour, and about 15°C after two hours, when his heart will stop.

Here we have ignored any insulation the victim may have, and the fact that the water is probably not at a consistent temperature at all depths. The human body is also not homogeneous, so technically Newton's law of cooling is not completely accurate. Given that no-one rescues him, and he is far enough away from shore so that he cannot save himself, we have assumed that his fate is inevitable. It is unreasonable to assume that the victim will remain motionless, and we are also saying that no other injuries are affecting the victim. Moving would enable him to keep warm for some time, but he may tire, and the motion will probably be detrimental to his situation. Newton's law of cooling may also not be applicable to a short amount of time. It is likely that there will be a drastic temperature loss during the first few minutes, and again later when he is low on energy. Many problems arrise, but in the general case, the approximation given by Newton's law is reasonable.

good!

The following example of Newton's law of cooling shows how it can be used to compare temperatures of two objects, and could in fact be altered for more. Two men, Bob and Chris, pour themselves a cup of coffee from the same pot, into the same type of cup. Each takes a creamer from the fridge, sits down and puts his cup on the table. Chris adds the cream to his coffee right away and gives it a quick stir. Bob, however, does not add his cream. A few minutes later, neither has had any coffee and they run out of things to say. Bob takes the cream, which has been on the table, and adds it to his coffee. Whose cup of coffee is hotter?

Assume the room temperature is relatively constant, represented by A. If the temperature of the creamless coffee as a function of time is denoted by B(t), and the cream as C(t), then consider the initial temperatures of to be $B(0)=B_0$ and $C(0)=C_o$ respectively. Denote the amount of coffee to be λmL and the amount of cream to be ζmL. (these are just greek letters, they have no pre-assigned value). Let the coffee whose cream is added first (Chris's cup) be denoted by $T_F(t)$ as a function of time, and the coffee and cream added later (Bob's coffee) be known as $T_S(t)$. If the coffee and the cream obey newton's law of cooling, we can use equation (2) we find that :

$C(t)=A + (C_0 - A)e^{-Kt}$ where $K=(-t_1)^{-1} \ln[(C(t_1) -A) / (C_0 - A)]$

$B(t)=A + (B_0 - A)e^{-Lt}$ where $L=(-t_1)^{-1} \ln[(B(t_1) -A) / (B_0 - A)]$

Where $B(t_1)$ is the temperature of the coffee taken at a later time (ie after t=0) similarly for C.

To find the equation to describe the coffee with cream, we must first determine the amount of heat initially in the cup. Since we know the amounts of coffee and cream, we can ascertain the initial volume weighted temperature of Chris's drink. This is given by $T_F(0)=(\lambda B_0 +\zeta C_0)(\zeta+\lambda)^{-1}$ that is, the volumes of coffee and cream multiplied by their initial temperatures, divided by the total volume. We can then assume that this cup of coffee obeys Newton's law of cooling. This is reasonable since we said that the coffee and cream obeyed the law independently of each other. So using equation (2) we get

$T_F(t) = A + (T_F(0) - A)e^{-Mt}$ where $M=(-t)^{-1} \ln [(T_F(t_1) - A) / (T_F(0) - A)]$

This means that at time t * the temperature of Chris's coffee can be expressed as:

$$T_F(t^*) = A + \left[\frac{(\lambda B_0 + \zeta C_0)}{(\zeta+\lambda)} - A \right] e^{-Mt^*}$$

Using the same method we can find the volume weighted temperature of Bob's beverage at the instant he adds the cream. that is, $T_S(t^*) = [\ \lambda B(t^*) + \zeta C(t^*)\ ] \ /\ [\lambda+\zeta]$ where ζ and λ are the same as in Chris's case, since they took the same amount of coffee and cream. To find out whose drink is hotter, we just need to compare their weighted temperatures at the time Bob adds his cream. (Since after that time they begin to drink, and it is a different question, as different volumes are present) So taking $T_S(t^*) - T_F(t^*)$ we get:

$$\frac{\lambda[A + (B_0 - A)\, e^{-Lt^*}\,] + \zeta[A + (C_0 - A)\, e^{-Kt^*}\,]}{(\zeta+\lambda)} \quad - \quad A + \left[\frac{(\lambda B_0 + \zeta C_0)}{(\zeta+\lambda)} - A \right] e^{-Mt^*}$$

which simplifies to:

$$\frac{A[\,\lambda(e^{-Mt^*} - e^{-Lt^*}) + \zeta(e^{-Mt^*} - e^{-Kt^*})] + \lambda B_0(e^{-Lt^*} - e^{-Mt^*}) + \zeta C_0(e^{-Kt^*} - e^{-Mt^*})}{(\zeta + \lambda)}$$

where A, B_0, C_0, K, L, M, λ , and ζ are all positive constants.

Now this is a very difficult equation to analyse if the values of K, L, and M are unknown and/or different. If, however, we were to assume that they are all equal then it becomes quite elementary. Take K = L = M then:

$$T_S(t^*) - T_F(t^*) = \frac{A[\,\lambda(e^{-Kt^*} - e^{-Kt^*}) + \zeta(e^{-Kt^*} - e^{-Kt^*})] + \lambda B_0(e^{-Kt^*} - e^{-Kt^*}) + \zeta C_0(e^{-Kt^*} - e^{-Kt^*})}{(\zeta + \lambda)}$$

and this simplifies to $[e^{-Kt^*} - e^{-Kt^*}\,]\ [A(\lambda+\zeta) + \lambda B_0 + C_0\zeta]\ (\zeta+\lambda)^{-1}$ which is just zero. This just means that there is no detectable difference in the temperature of the coffee regardless of the time when the cream is added.

This is a nice result, but it is necessary to realize that there were some significant assumptions made initally to obtain it. Although it seems reasonable to say that the rates of heat transfer for the three types of liquid are the same, it is not necessarily true. There are minute differences between them since the cream is usually a thicker liquid than the coffee. Also the containers holding the liquids are different, so that means we are ignoring any insulation provided by the containers. However, it does seem reasonable to say that if the rates of heat exchange are the same for the black coffee and the cream, then the coffee with cream will also have the same rate.

good

We have also assumed that the coffees and creams were exactly the same temperature when we began, and that each had the same amount. Since they came from the same pot of coffee they should have approximately the same initial temperature. The amounts are difficult to keep constant between the two, but again we need to allow for slight differences in data. If either of the cups were placed near some sort of heat source (or colder area), or if the cream were held in a warm hand, it would affect the rate of heat transfer. Had Bob or Chris stirred their coffee for any significant amount of time, they would have chanced increasing the temperature of the liquid slightly. The stirring utensil may have an effect, for example, using a cold metal spoon the drink might have cooled somewhat. Ignoring all the minor possible modificatons, and assuming the rates of heat transfer for all three liquids are the same, then mathematically there will be no difference in the temperatures at time t^*. And since we can't detect it, it is doubtful that Bob or Chris will notice a difference.

If Bob had kept his cream in the fridge, we would expect a difference between the two beverages. Keeping the cream cold is equivaltent to keeping it at its initial temperature, that is, $C(t) = C_0$. Now this does not affect the temperature of Chris's coffee, only Bob's. We find that $T_S(t^*) = [\lambda B(t^*) + \zeta C_0] [\lambda+\zeta]^{-1}$ which is

$T_S(t^*) = \{ \lambda[A + (B_0 - A) e^{-Kt^*}] + \zeta C_0 \} [\lambda+\zeta]^{-1}$. So we get

$$T_S(t^*) - T_F(t^*) = \frac{\lambda[A + (B_0 - A) e^{-Kt^*}] + \zeta C_0}{(\zeta+\lambda)} - A + [\frac{(\lambda B_0 + \zeta C_0)}{(\zeta+\lambda)} -A] e^{-Mt^*}$$

which simplifies to $\frac{(1-e^{-Kt^*}) \zeta(C_0 - A) + \lambda B_0 (e^{-Kt^*} - e^{-Mt^*})}{(\lambda+\zeta)}$

So to compare them, we will again say that K=M, that is, the rate of cooling of coffee with cream is equal to that of black coffee. This gives:

$$T_S(t^*) - T_F(t^*) = \frac{(1-e^{-Kt^*}) \zeta(C_0 - A)}{(\lambda+\zeta)}$$

Notice what happens if $t^* =0$, that is, if Bob adds his cream at the same time Chris does. We find this equation becomes zero, which is what we would intuitively believe, since neither coffee had a chance to cool. Also, if K=M=0 then the difference is again zero, since neither cup loses any heat. As $t \to\infty$, regardless of the values of K and M, the difference becomes $[\zeta(C_0 - A)] [\lambda+\zeta]^{-1}$ giving the range for the difference:

$$0 \leq \frac{(1-e^{-Mt^*}) \zeta(C_0 - A) + \lambda B_0 (e^{-Kt^*} - e^{-Mt^*})}{(\lambda+\zeta)} \leq \frac{\zeta(C_0 - A)}{(\lambda+\zeta)}$$

which means the difference between the two beverages is no more than the amount of cream times the difference between cream and ambient temperatures divided by the amount of liquid in the cup.

Newton's law of cooling can also be used in perhaps a rather unexpected area.

An important consideration when investigating a possible murder is when it occured. Newton's law can be used to approximate the time of death, given that a corpse is found at time t=0, and that its temperature was T_0. It is necessary to assume that at the time of death (t_D) the body was at the normal core temperature, 37°C. This method does not pertain to a situation when a person was burnt, or frozen to death (see example on hypothermia). Any plausible modifications for these situations would be very complex, and probably not the optimal way to determine the time of death. Applying Newton's Law of cooling gives, with A and T as functions of time, $dT/dt = -k\ (T-A)$. This type of problem is best explained with a numerical example.

Suppose at noon Saturday a body is found at 30°C in a room kept at around 20°C. From information provided by the coroner's office, the rate of cooling of a body is usually $0.5207hr^{-1}$ under those conditions. To simplify some future calculations, we will put this in terms of minutes, that is, $k= 0.5207/60 = 0.0086783min^{-1}$.

good

When the temperature is not constant, then as before, the problem becomes more difficult. If a body was discovered in a room whose temperature is controlled by a thermostat, it is posssible to represent the room's temperature as a function of time. We will assume that the thermostat is set at 20°C , and the furnace starts when the temperature drops to 18°C and stops when it reaches 21°C. It would be a waste of energy for the furnace to work constantly to keep the room at 20°C, so this seems reasonable. The temperature of the room will probably experience exponential growth when the furnance is running, and the cooling of the room is less rapid. If the amount of time the furnance is on (t_1) and off (t_2) are known, it seems reasonable to approximate the function as follows:

$$A(t)=\begin{cases} ae^{L(t-t_0)} & \text{for} \quad t_0 \le t \le t_0+t_1 \\ d-ce^{L(t-(t_0+t_1))} & \text{for} \quad t_0+t_1 \le t \le t_0+t_1+t_2 \end{cases}$$

where a is the temperature when the furnace comes on, in our case 18°C. We can find the values of d, c, and L by noting that $A(t_0+t_1) = b = ae^{Lt_1}$

so $\qquad L = (t_1)^{-1} \ln (b/a)$

where b is the temperature when the furnace turns off.

If we assume that when the furnace comes on again it is at the initial temperature (a) then $A(t_0+t_1+t_2) = a = d - ce^{Lt_2}$ and since $A(t_0+t_1) = b = d - c$, we find that

$$c=(a-b)\,(1-e^{Lt_2})^{-1} \qquad \text{and} \qquad d= b + c.$$

Now this is the function for the first cycle of the furnace, and this repeats itself every $t_0 + t_1 + t_2$ minutes. SEE GRAPH #3 for a representation of the first cycle, with the cycle length being 16 minutes.

Using Newton's law of cooling (equation 1) we find that $dT/dt = -k\,(T-A(t))$ or

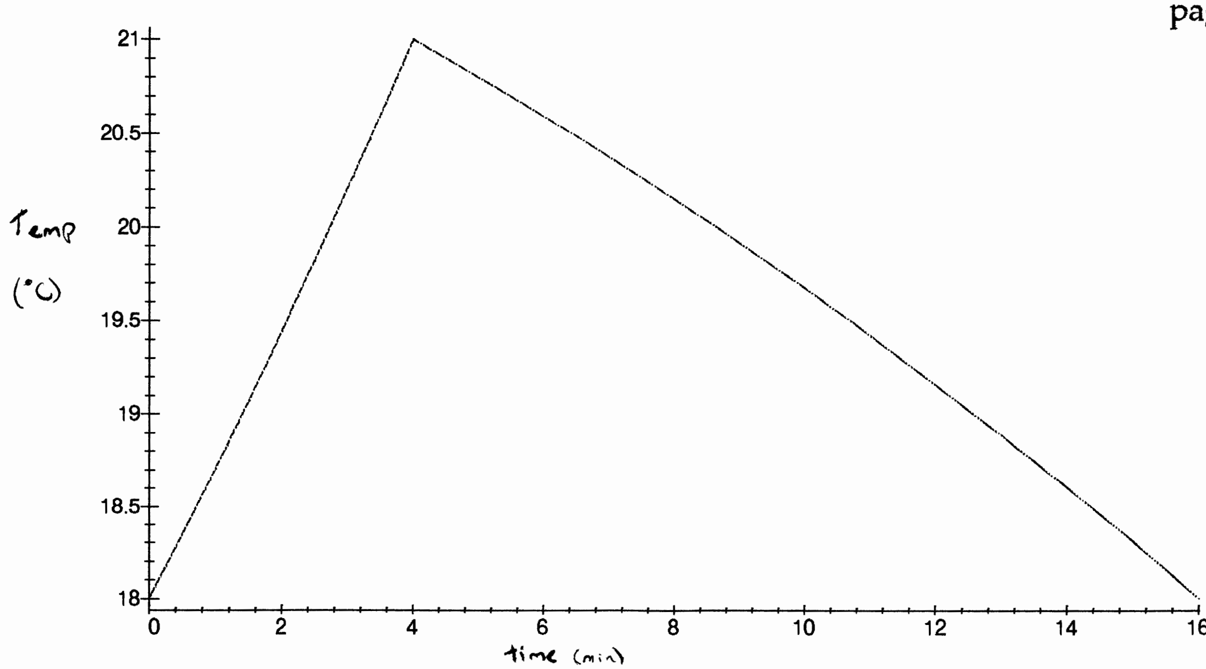
$$\frac{dT}{dt}=\begin{cases} -kT+kae^{L(t-t_0)} & t_0 \le t \le t_0+t_1 \\ -kT+kd-kce^{-L(t-t_0-t_1)} & t_0+t_1 \le t \le t_0+t_1+t_2 \end{cases} \qquad (3)$$

Let $T_1(t)$ be for the $t_0 \le t \le t_0+t_1$ so we have, $dT_1/dt = -kT_1 + kae^{L(t-t_0)}$ with the initial condition $T_1(t_0) = S_1$. Using the integrating factor $\mu=e^{\int k\,dt} = e^{kt}$ we find that :

$$T_1(t)=\frac{kae^{L(t-t_0)}}{k+L}+De^{-kt} \qquad (4)$$

where $D= e^{kt}\,[\,S_1 - (ka)/(k+L)\,]$.

ENVIRONMENT



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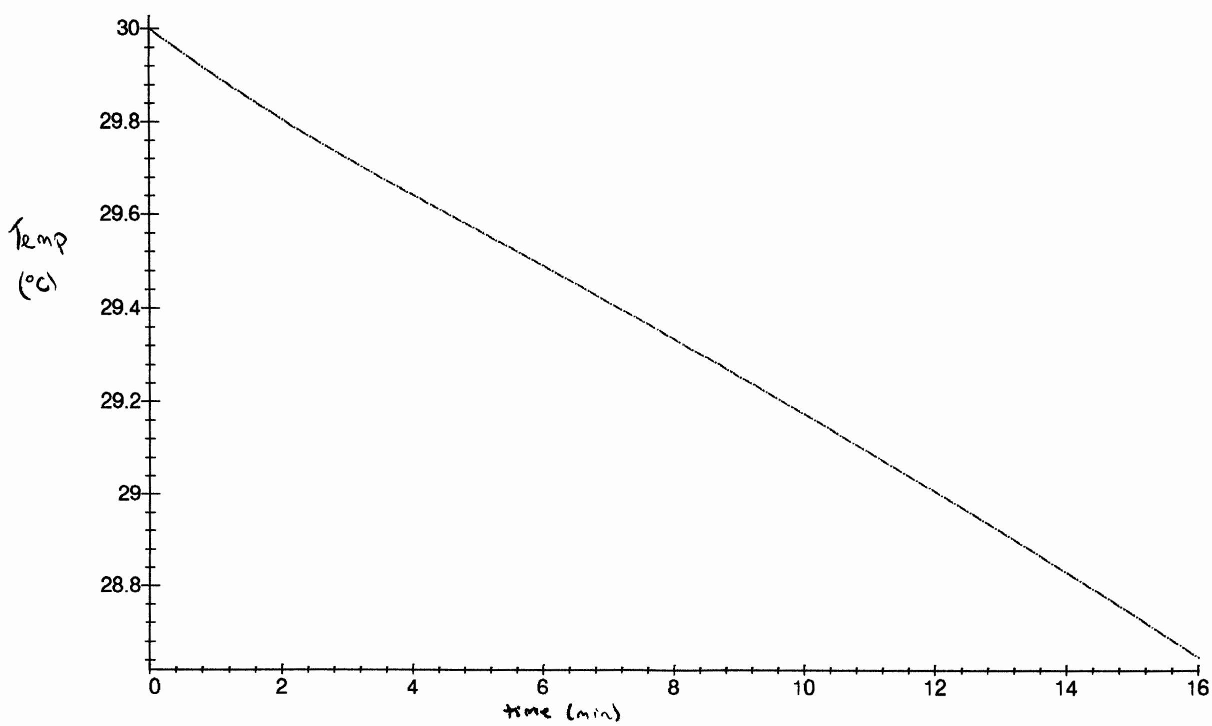
BODY



(4)

Assume that the value at time t_0+t_1 is the same for both $T_1(t)$ and $T_2(t)$. That is $T_1(t_0+t_1)=T_2(t_0+t_1) = S_2$, since the body's temperature will not change if time does not. So if $T_2(t)$ is for $t_0+t_1 \le t \le t_0+t_1+t_2$ using μ as before, and the initial condition $T_2(t_0+t_1) = S_2$ to get:

$$T_2(t)=d-\frac{kce^{L(t-t_0+t_1)}}{k+L}+Me^{-kt} \tag{5}$$

where $M= e^{k(t_0+t_1)}[S_2 - d + (kc)(k+L)^{-1}]$

If the furnace starts at 18°C, and takes four minutes to reach 21°C, when it turns off, then twelve minutes later it has again reached 18°C, so it starts all over again, in our formula for A we have $t_0 = 0$, $t_1=4$, $t_2=12$. We can find that c=5.1024, and d=26.1024 and L=.0385 As noted before, $k=0.0086783\text{min}^{-1}$.

For the first cycle of the furnace, we solve for D and M, with the previously discussed requirements. This gives D=26.6889462 and M=4.63500304. so we have

$$T(t)=\begin{cases} 0.156209994\dfrac{e^{0.0385t}}{0.047178333}+26.68894620e^{-0.008678333t}, & t_0\le t\le t_0+t_1 \\ 26.1024-\dfrac{0.044280326e^{-0.0385(t-4)}}{0.047178333}+4.635003035e^{-0.008678333t}, & t_0+t_1\le t\le t_0+t_1+t_2 \end{cases}$$

and T(16) = 28.64675498°C. SEE GRAPH #4 (page 14) We use this for the initial condition for the next cycle, from time 16 to 32 minutes after the body's temperature is first measured.

Denote the body's temperature during the second cycle as $T^*(t)$, where

$T^*(t)= T_1^*(t)$ for $t_0 + t_1 + t_2 \le t \le 2t_0 + 2t_1 + t_2$ and

$T^*(t)= T_2^*(t)$ for $2t_0 + 2t_1 + t_2 \le t \le 2t_0 + 2t_1 + 2t_2$.

The same values are used for k, L, a, c, and d as before, and we find that $D^*=29.10959509$ and $M^*=3.770589548$ and the values are plugged into equations (4) and (5). With these we estimate the temperature of the corpse after two cycles of the furnace, that is, 32 minutes after. $T^*(32)=27.468948°C$. We continue as before, solving for $T^{**}(t)$ on the interval $32 \leq t \leq 48$. This gives $D^{**}=31.8908131$ and $M^{**}=2.77742676$. After 48 minutes, the corpse is at approximately 26.4438412°C. SEE GRAPH #5

We could continue like this indefinitely, but what we are really interrested in is the time of death, so we want to go backwards, not fowards as we were. If we consider the ambient temperature to be relatively constant, then we find that

$T(t) = A - (T_0 - A)\, e^{-kt}$ with $k = (-t_1)^{-1} \ln[(T(t_1) - A) / (T_0 - A)]$

(where $T(t_1)$ is the temperature at a later time.)

and substituting in the normal body temperature, we can solve for t_d

$T(t_D) = 37 = A - (T_0 - A)e^{-kt_D}$ or $t_D = (k)^{-1} \ln[(T_D - A)/(T_0 - A)]$

This means that the cooling of the corpse can be approximated by the equation:

$T(t)= 20+10e^{-kt}$ see equation (2)

Assuming the temperature of the body was 37°C at the time of death, we have

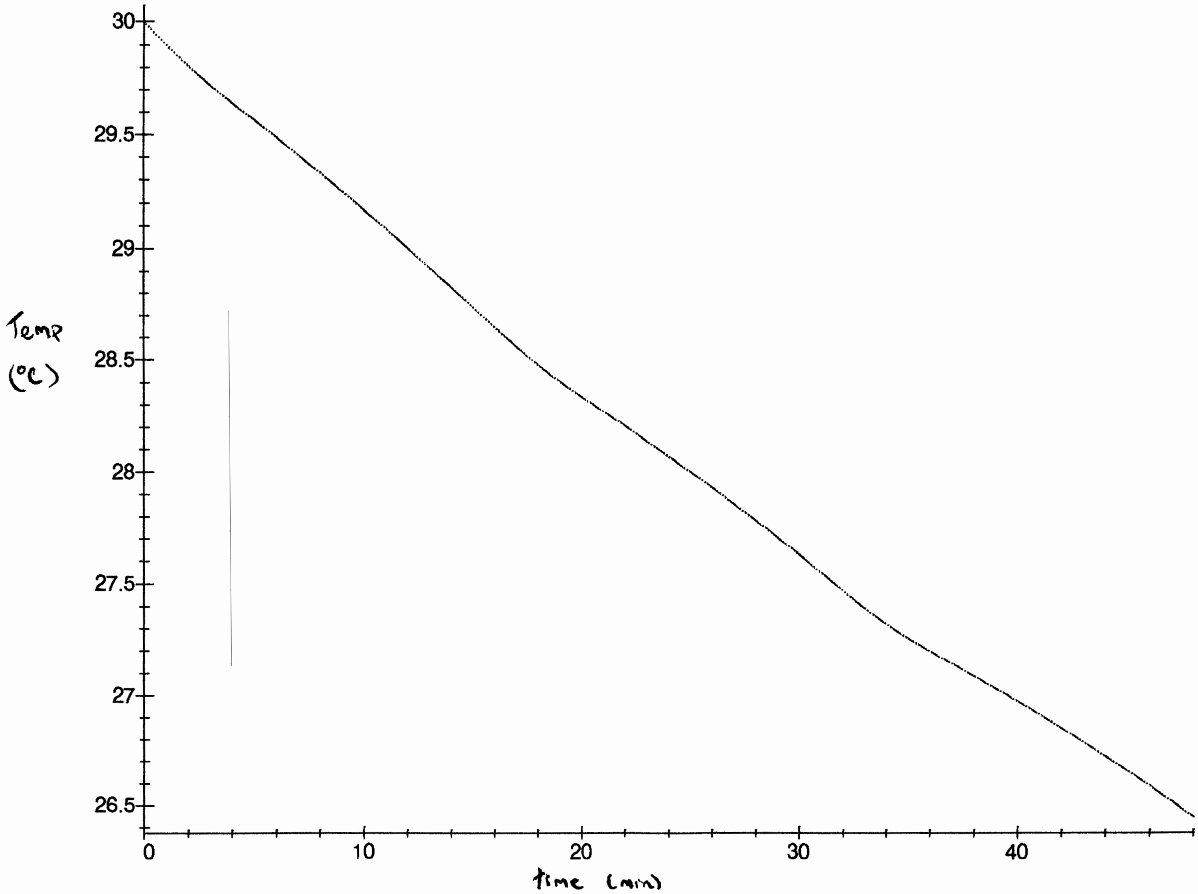
$T(t_D)=37 = 20 + 10e^{-kt_D}$

as before we have $k=0.0086783\text{min}^{-1}$, so we can solve for t_D :

$t_D = (-k)^{-1} \ln\{(37-20)/(10)\} \approx -61.144$ minutes.

This means that the time of death was approximately 1 hour before noon, or around 11 a.m.. Note that since the time of death is before the time when the corpse was found (t_0), the value of t_D will be negative. SEE GRAPH #6

WITH_HEATING_CONSIDERED



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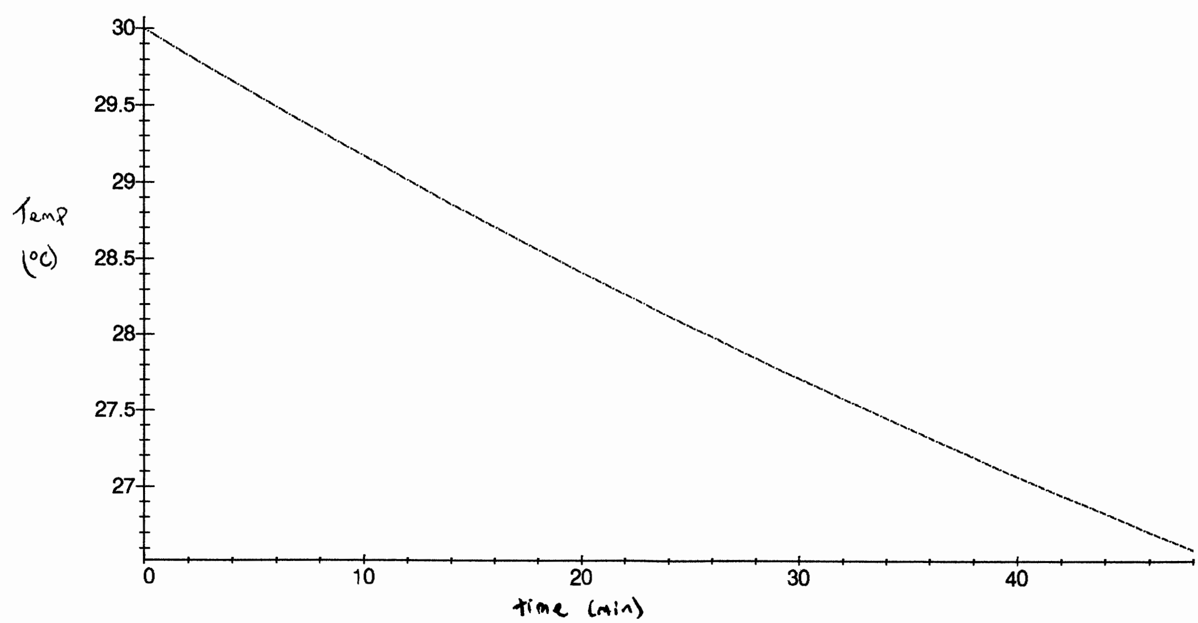
AMBIENT_TEMPERATURE_CONSTANT

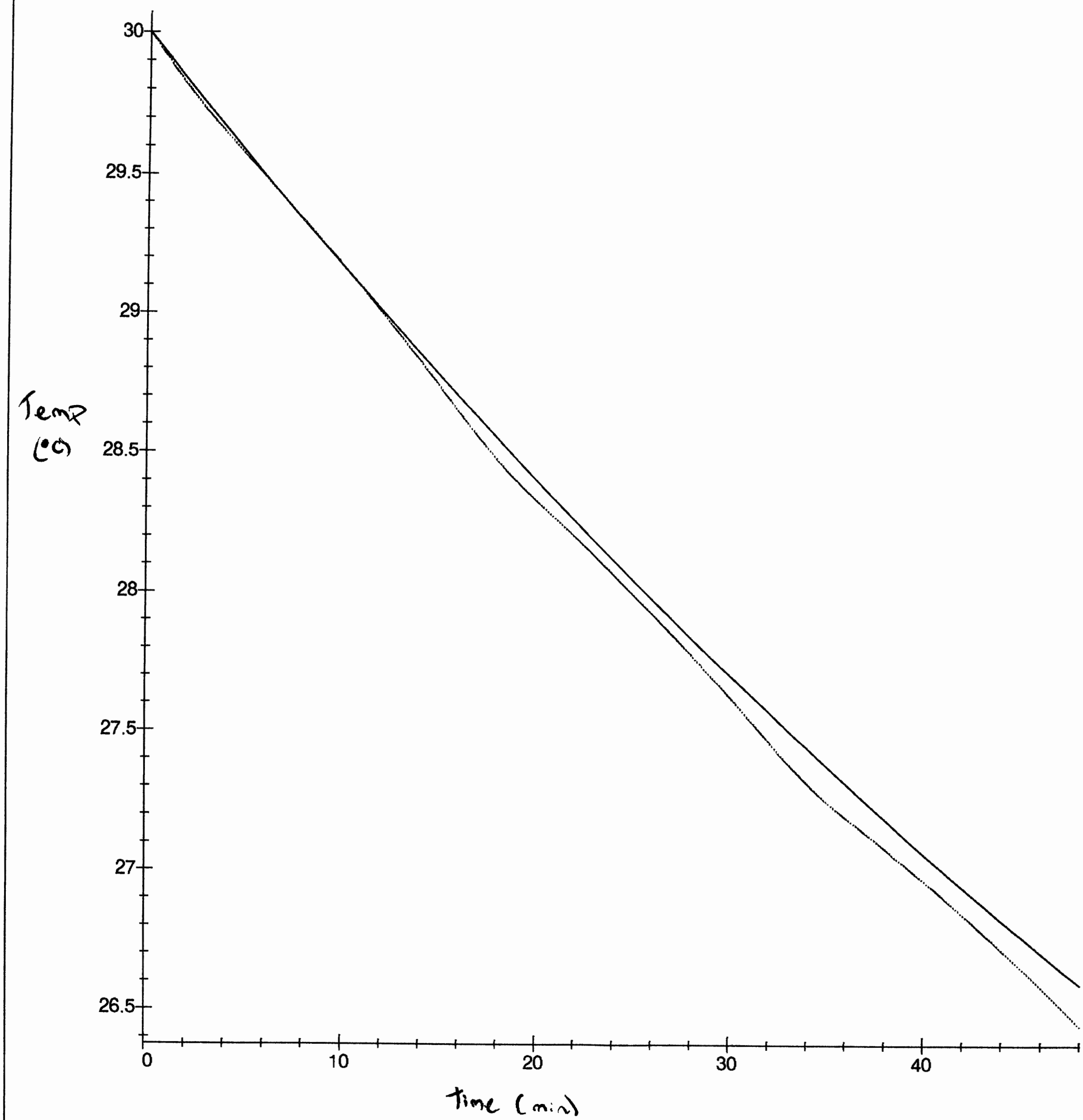


(6)

Looking at the graph for the corpse's temperature for the first 48 minutes after the temperature was taken, SEE GRAPH #7, we see that it is basically the same for both when the ambient temperature is a constant and when it is the given function of time. In fact, the difference between the two is at most 0.14926785°C when t=48. As time goes on, the accuracy of the first model will decrease, so the estimate of time of death will be larger, but not drastically.

Of significant importance is to realize that if the body's temperature was 20°C when it was discovered, it is impossible to use Newton's law of cooling for a precise time of death. However, it is possible to say that the body began to cool before a certain time, that is, a minimum amount of time that has passed since the person was killed. If we use the same k as before, and say that the body was found at 20.01°C (to the highest accuracy of thermometer used) we can estimate the minimum time dead. Proceeding as above, we find that $t_D = (-k)^{-1} \ln[\,(37\text{-}20)/(20.01\text{-}20)] = -857$. This means that a person found at 20°C was killed at least 14 hours before being found.

Again, various assumptions have been made about the situation in order for Newton's law to be applied. First, the law requires that the object be homogeneous, and as mentioned before, the human body is not, but since we are considering the core temperature this can be overlooked. Recall that if a small room has more than one person in it, the temperature will be noticably different than if just one was present. For this reason, the room should be large enough not to be affected by the body's temperature. If, for example, the corpse was found in a closet, the results would not be valid. The clothing, size and previous health of the individual are not



(7)

being considered. For someone who was in a parka with high blood pressure, the cooling and initial temperatures might be different than in our example. Realistically, those interrested in determining the time of death are not concerned with the exact time. Usually they require a interval of an hour or two in order to continue their investigation. Also other pieces of information are often available, such as the victim was seen a certain amount of time before his corpse was found.

It should be evident that Newton's law of cooling cannot be directly used for various real-life problems. However, it is a good approximator for general cases of many different situations. Since one is frequently interrested in an approximation of data, Newton's law is sufficent, and will often give light to many interresting questions.

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