University of Manitoba

A Logistic Predator-Prey Model

Submitted to:

Dr. T. G. Berry
6.337 Introduction to Mathematical Modelling

By:

Sarah H. Dean
6306217

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Introduction:

This paper will discuss the effects of the introduction of a logistic growth term into the prey species equation of the Lotka-Volterra Predator-Prey model.

The Lotka-Volterra model comprises the following basic assumptions:

- the predator species feeds only on the prey species
- the prey has only one predator
- the prey has an ample food source

This model also makes the assumption that in the absence of the predator species the prey species experiences Malthusian growth and that in the absence of the prey species, the predator experiences Malthusian decay. In addition to the Malthusian growth and decay terms, the model contains competition terms for each species. These are incorporated by noting that interaction between predator and prey benefits the predator species and is detrimental to the prey species. Thus, the Lotka-Volterra model is given by the following system of differential equations:

$$\frac{dx}{dt} = x(l-ny)$$

$$\frac{dy}{dt} = y(mx-k)$$

$$\textit{where} \quad k, l, m, n > 0$$

<u>Note</u>: $x(t)$ = Number of prey at time t

$y(t)$ = Number of predators at time t

This system has equilibrium points at (0,0) and at (k/m, l/n). We also find, through local stability analysis and solution of the system after elimination of t, that the trajectories are closed and approximately elliptical.

Although this model is "nice" in that it is relatively easy to study and the system of differential equations can be solved, it is only useful when the predator and prey populations fluctuate and are essentially periodic.

This paper will present a modified model in which it is assumed that in the absence of the predator species the prey species experiences Logistic rather than Malthusian growth. This Logistic Predator-Prey model maintains all the other assumptions of the Lotka-Volterra model. It is hoped this model will improve upon the Lotka-Volterra model.

The Logistic Predator-Prey Model:

If it is assumed that the prey species experiences Logistic growth in the absence of the predator species, the Lotka-Volterra model becomes:

$$\frac{dx}{dt} = lx(1-\frac{x}{C}) - nxy$$

$$\frac{dy}{dt} = mxy - ky$$

$$where \quad k, l, m, n > 0; \quad C > 0$$

Note: C is defined as the carrying capacity of the prey species.

We will use phase plane analysis to find potential trajectories of the system. This requires finding the nullclines for both species as well as the equilibrium points for the system.

Nullclines:

for the prey:

$$\frac{dx}{dt} = 0 \quad \Rightarrow \quad x(l(1-\frac{x}{C}) - ny) = 0$$

$$\Rightarrow \quad x = 0 \ ; \ y = \frac{l}{n}(1-\frac{x}{C})$$

for the predator:

$$\frac{dy}{dt}=0 \;\Rightarrow\; y(mx-k)=0$$

$$\Rightarrow\; y=0\;;\; x=\frac{k}{m}$$

<u>Equilibrium points</u>:

There are five possibilities:

(1) $x=0\;,\;\; y=0$

(2) $x=0\;,\;\; x=\frac{k}{m}$

(3) $y=\frac{1}{n}(1-\frac{x}{C})\;,\;\; y=0$

(4) $y=\frac{1}{n}(1-\frac{x}{C})\;,\;\; x=\frac{k}{m}$

(5) $x=C\;,\;\; y=0$

(5) actually comes from (3)

The fifth point is the point at which the line y = 1/n(1 - (x/C)) intersects the x-axis.

Clearly, (2) and (3) are inconsistent results. Therefore there are three equilibrium points. However, the equilibrium point (0,0) is uninteresting as it means that if the system starts with a zero population for both the predator and the prey, it will stay there.

As we wish to sketch phase plane diagrams for this system, we must find the directional fields ~~about~~ on either side of the nullclines x = k/m and y = (l/n)(l-(x/C)). We obtain the following information:

$$x > \frac{k}{m} \Rightarrow \frac{dy}{dt} > 0 \qquad y > \frac{1}{n}\left(1 - \frac{x}{C}\right) \Rightarrow \frac{dx}{dt} < 0$$

$$x < \frac{k}{m} \Rightarrow \frac{dy}{dt} < 0 \qquad y < \frac{1}{n}\left(1 - \frac{x}{C}\right) \Rightarrow \frac{dx}{dt} > 0$$

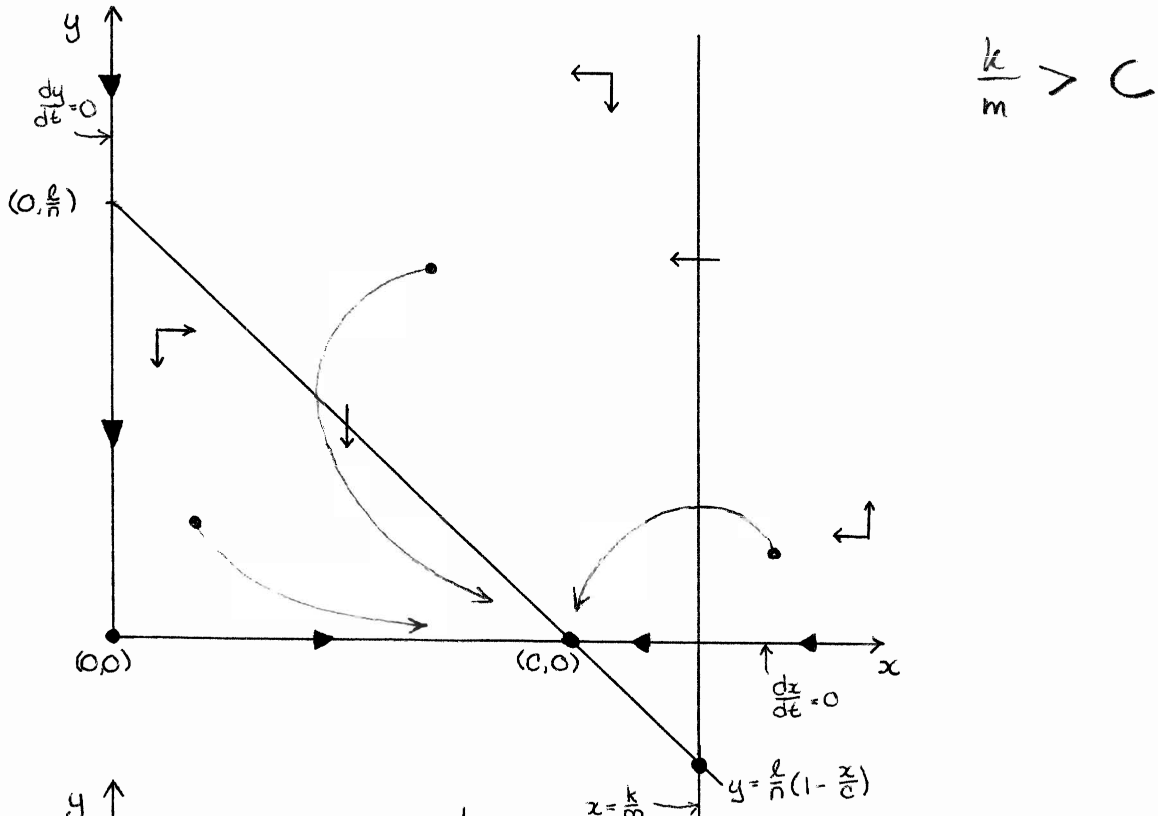
assuming $x \geq 0$ & $y \geq 0$

<u>Phase Plane Diagrams</u>:

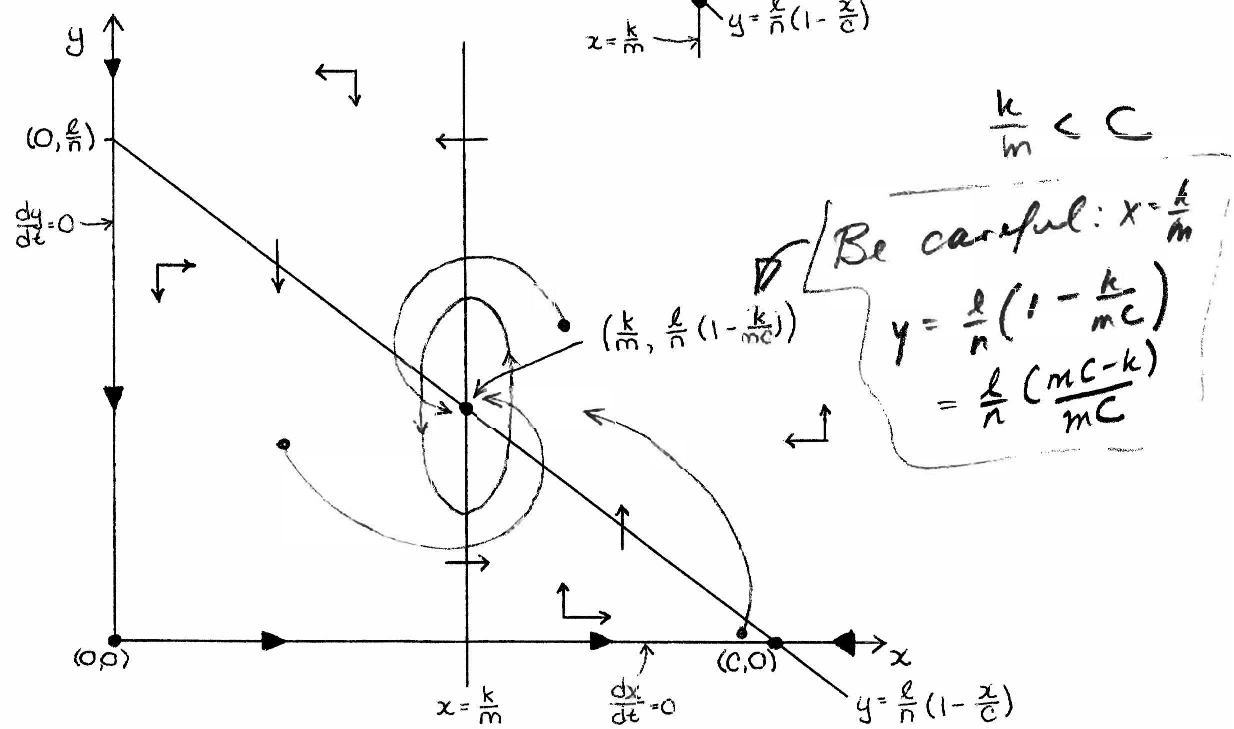
There are two possible phase plane diagrams for this system as the nullcline x = k/m, which is a vertical line, may intersect the nullcline y = l/n(l-(x/C)), which is a line of slope -l/nC, such that the point of intersection is above or below the x-axis. Obviously if this point is below the x-axis it is no longer a realistic equilibrium point as the predator population can never ~~obtain~~ attain a negative value.

We obtain the following diagrams:

(1)



(2)



Analysis:

(1) This phase plane diagram has two points of equilibrium:

a) (0,0) is an unstable equilibrium point

b) (C,0) is a stable equilibrium point

The diagram suggests that the system reaches or at least approaches the equilibrium point (C,0). This means that the prey population approaches its carrying capacity and the predator population approaches extinction.

(2) This phase plane diagram has three points of equilibrium:

a) (0,0) is again an unstable equilibrium point

b) (C,0) is now an unstable equilibrium point. The system will only reach this equilibrium if it starts there.

c) $(k/m, 1/n(1-(k/mC)))$ is a stable equilibrium point. The phase plane diagram suggest that this stability may be approached in the following two ways:

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i) If the stability is periodic, we will see closed orbits and we expect these orbits to be essentially elliptical.

ii) If this point is asymptotically stable, we expect trajectories to spiral in towards the equilibrium point.

As the phase plane diagrams do not give enough information to distinguish which trajectories actually occur, we will do some local stability analysis to try and determine their behaviour as t increases.

N.B.: The first phase plane diagram, in which the equilibrium point (C,0) is stable, will not be treated any further. Although it is not definite which trajectory the populations will follow, we know that the populations will approach (C,0). Therefore, further analysis, although possibly giving better information about exact trajectories, is not really needed. We will conjecture that in this first case the prey will approach its carrying capacity and the predator its extinction and we will not concern ourselves with the time taken for this process to occur.

In order to determine how populations behave around the equilibrium point (k/m , l/n(1-(k/mC))) we will use a linearized version of the predator-prey system.

Let,

$$u = x - \frac{k}{m} \qquad \Rightarrow \qquad x = u + \frac{k}{m}$$

$$v = y - \frac{l}{n}\left(1 - \frac{k}{mC}\right) \quad \Rightarrow \quad y = v + \frac{l}{n}\left(1 - \frac{k}{mC}\right)$$

Recall:

$$\frac{dx}{dt} = xl - x^2\frac{l}{C} - nxy$$

$$\frac{dy}{dt} = mxy - ky$$

Therefore,

$$\frac{du}{dt}=\frac{dx}{dt}=(u+\frac{k}{m})l-(u+\frac{k}{m})^2\frac{l}{C}-n(u+\frac{k}{m})(v+\frac{l}{n}(1-\frac{k}{mC}))$$

$$\frac{dv}{dt}=\frac{dy}{dt}=m(u+\frac{k}{m})(v+\frac{l}{n}(1-\frac{k}{mC}))-k(v+\frac{l}{n}(1-\frac{k}{mC}))$$

That is,

$$\frac{du}{dt} = -\frac{l}{C}u^2-\frac{lk}{mC}u-nuv-\frac{nk}{m}v$$

$$\frac{dv}{dt} = muv+\frac{l}{n}(m-\frac{k}{C})u$$

Now, if trajectories begin close to the equilibrium point and remain close, then u and v are small. Thus, terms with u^2, uv, or v^2 are considered negligible. With this assumption, we obtain the following linearized system:

$$\frac{du}{dt} = -\frac{lk}{mC}u-\frac{kn}{m}v$$

$$\frac{dv}{dt} = \frac{l}{n}(m-\frac{k}{C})u$$

Solving, we find:

$$\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \underbrace{\begin{pmatrix} -\frac{lk}{mC} & -\frac{kn}{m} \\ \frac{l}{nC}(mC-k) & 0 \end{pmatrix}}_{\mathbf{A}} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\det(\mathbf{A}-\lambda\mathbf{I}) = 0 \Rightarrow \begin{vmatrix} -\frac{lk}{mC}-\lambda & -\frac{kn}{m} \\ \frac{l}{nC}(mC-k) & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda = -\frac{lk}{2mC} \pm \frac{\sqrt{lk}}{2mC}\sqrt{lk-4mC(mC-k)}$$

The form of the solutions to the linearized system depends on the sign of $lk-4mC(mC-k)$ (or $lk-4(mC)^2+4kmC$)

(1) If $lk + 4kmC < 4(mC)^2$ then $lk + 4kmC - 4(mC)^2 < 0$ and so the eigenvalues are

$$\lambda = -\frac{lk}{2mC} \pm \frac{\sqrt{lk}}{2mC}\sqrt{4mC(mC-k)-lk}\; i$$

The associated eigenvectors have the form: $C_1 = \mathbf{a} + i\mathbf{b}$

$C_2 = \mathbf{a} - i\mathbf{b}$

Therefore, the solutions have the form:

$$U = a\cos\beta t - b\sin\beta t$$

$$V = b\cos\beta t + a\sin\beta t$$

$$\text{where}\quad \beta = \frac{\sqrt{lk}}{2mC}\sqrt{4mC(mC-k)-lk}$$

Solution of

$$(\mathbf{A}-\lambda\mathbf{I})\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\text{gives}\quad a=\begin{pmatrix} 1 \\ \frac{-l}{2nC} \end{pmatrix};\quad b=\begin{pmatrix} 0 \\ \frac{m}{kn}\beta \end{pmatrix}$$

Substituting into our solutions U and V we find:

$$U = \begin{pmatrix} \cos\beta t \\ -\frac{l}{2nC}\cos\beta t - \frac{m}{kn}\beta\sin\beta t \end{pmatrix} \qquad V = \begin{pmatrix} \sin\beta t \\ \frac{m}{kn}\beta\cos\beta t - \frac{l}{2nC}\sin\beta t \end{pmatrix}$$

Thus, $\quad u = \cos\beta t + \sin\beta t$

$$v = \left(\frac{m}{kn}\beta - \frac{l}{2nC}\right)\cos\beta t - \left(\frac{m}{kn} + \frac{l}{2nC}\right)\sin\beta t$$

The form of these solutions suggests that the trajectories of the predator-prey system (for $lk + 4kmC < 4(mC)^2$) are periodic (closed).

(2) If 1k + 4kmC > 4(mC)^2 then 1k + 4kmC − 4(mC)^2 > 0 and the eigenvalues are real and are given by:

$$\lambda = -\frac{1k}{2mC} \pm \frac{\sqrt{1k}}{2mC}\sqrt{1k+4mC(k-mC)}$$

Solutions will have the form:

$$\begin{pmatrix} u \\ v \end{pmatrix} = Ce^{(-\frac{1k}{2mC}+\alpha)t} + C'e^{(-\frac{1k}{2mC}-\alpha)t}$$

$$where \quad \alpha = \frac{\sqrt{1k}}{2mC}\sqrt{1k+4mC(k-mC)}$$

Solving as before for the eigenvectors, we find

$$C = \begin{pmatrix} 1 \\ -\frac{1}{2nC} - \frac{m}{kn}\alpha \end{pmatrix} \qquad C' = \begin{pmatrix} 1 \\ -\frac{1}{2nC} + \frac{m}{kn}\alpha \end{pmatrix}$$

Therefore the solutions are

$$u = e^{(-\frac{1k}{2mC}+\alpha)t} + e^{(-\frac{1k}{2mC}-\alpha)t}$$

$$v = (-\frac{1}{2nC} - \frac{m}{kn}\alpha)\, e^{(-\frac{1k}{2mC}+\alpha)t} + (-\frac{1}{2nC} + \frac{m}{kn}\alpha)\, e^{(-\frac{1k}{2mC}-\alpha)t}$$

In the solution for v, the coefficient of $e^{(-(1k/2mC)+\alpha)t}$ is smaller than that of $e^{(-(1k/2mC)-\alpha)t}$. This suggests that trajectories spiral in towards the equilibrium point. The coefficients in the solution for u are the same, suggesting that trajectories are closed in this case.

These two should not be separated!

From these two cases we can conjecture that trajectories of this system are either closed or spiral in towards the equilibrium point (k/m , l/n(l-k/mC)). We will try to find further support for these conjectures by eliminating t in the linearized system. We obtain,

$$\frac{du}{dv} = \frac{-\frac{lk}{mC}u-\frac{kn}{m}v}{\frac{l}{n}(m-\frac{k}{C})u}$$

$$\Rightarrow \quad \frac{du}{dv} = A(1+\frac{nC}{l}\frac{v}{u})$$

$$\text{where} \quad A=\frac{-kn}{m(mC-k)}$$

In order to solve this homogeneous differential equation, we must let u = pv which gives us,

$$\int\frac{lp}{Alp+AnC-lp^2}\,dp = \int\frac{1}{v}\,dv$$

Integration, using Mathematica to solve the left hand integrand and replacing p by u/v gives,

$$-\sqrt{\frac{Al}{Al+4Cn}}\,Arctanh\left(\sqrt{\frac{l}{A^2l+4ACn}}\,(A-\frac{2u}{v})\right)-\frac{\ln(ACn+\frac{Alu}{v}-\frac{ul^2}{v})}{2}=\ln|v|+\zeta$$

We have not been able to find a simplification of this equation. Thus, the elimination of t in the linearized system yields an equation which does not enable us to strengthen our conjectures.

We would like to find the exact trajectories for this model. The way to do this is to eliminate t in the original system (the system in x and y) and to solve for the trajectories. Elimination of t yields the following equation:

$$\frac{dy}{dx} = \frac{y(mx-k)}{lx-\frac{l}{C}x^2-nxy}$$

This equation cannot be solved explicitly. Thus, although the introduction of a logistic growth term in the prey species equation seems a small alteration, it renders the equation complicated enough to make it impossible to solve.

As we cannot solve for the trajectories explicitly, we will try using numerical methods to see how well a logistic model fits real-world problems. The data to be used comes primarily from an article entitled Population Modelling:II Cycles in Nature by Ian Huntley and John McDonald. However, the only data given in this article is a graph of the varying populations of the snowshoe hare (prey) and the lynx (predator) which is impossible to read precisely. Therefore the data we are using is already very approximate. Populations were read off the graph for every second year starting from 1846 and finishing at 1932 (Note: Not all data points were used). The numerical method used is the Euler method. As it is preferable to take a smaller time interval when calculating with this method, three averages were taken between each of the populations read off the graph simply by taking the middle point between two "known" values to be their sum divided by 2 and then calculating the mid-points between this average and the "knowns" on either side.

Recall: The Euler method proceeds as follows:

$$\frac{dy}{dx} = f(x) \quad \rightarrow \quad \frac{\Delta y}{\Delta x} = f(x)$$

$$\rightarrow \quad \frac{y(x_{i+1}) - y(x_i)}{\Delta x} = f(x_i)$$

$$\rightarrow \quad y_{i+1} = y_i + f(x_i)\,\Delta x$$

In the logistic predator-prey model the two Euler method equations to apply are:

$$x_{i+1} = (1 + l\Delta t - n\Delta t y_i)\,x_i - \frac{l\Delta t}{C}x_i^2$$

$$y_{i+1} = y_i + y_i\Delta t\,(mx_i - k)$$

These equations were iterated using MathCad and some results are appended. The results were not very promising. The actual population variation was very different from that found using this numerical method and the percentage error between the two was often very high. The graphs for the predator population (appended) have a similar form except that the graph made from numerical analysis results has a much flatter curve which suggests our model doesn't give enough importance to fluctuations in the prey species. The graphs for the prey species seem quite different although the main difference is just that the graph of data obtained numerically has a smoother first crest and its second crest is slower in coming than on the graph of real-life data. One good point is that the prey species does show considerable fluctuation in numbers.

This suggests that the logistic predator-prey model may not be very adept at predicting or modelling real life phenomena precisely. However it is quite an improvement over the Lotka-

Volterra model in that it offers more possibilities for population dynamics which is more realistic than simple closed curves as predicted by the Lotka-Volterra system.

Conclusions:

The introduction of a Logistic growth term into the Lotka Volterra Predator-prey model produces the following model:

$$\frac{dx}{dt} = lx(1-\frac{x}{C}) - nxy$$

$$\frac{dy}{dt} = y(mx-k)$$

Through the use of phase plane analysis and study and solution of the linearized model, we have found that this system (provided we do not start with zero populations of predator and prey) will probably stabilize, in time, and that it will do this in one of the following ways:

i) Regardless of the starting point, the prey population will approach its carrying capacity and the predator its extinction.

ii) The system will spiral towards the equilibrium point (k/m,l/n(l-k/mC)).

iii) The system will be periodically stable having approximately elliptical orbits about the equilibrium point (k/m,l/n(l-k/mC)).

As the logistic model gives a differential equation dy/dx which cannot be solved explicitly, we endeavoured to support our conjectures through the use of numerical methods. We used the Euler method and although we were able to find graphs using our results which ressembled graphs of actual data we cannot say that these graphs were a "good" fit. Of course, other numerical methods may give results which are more useful in substantiating our conjectures about the population dynamics of the Logistic model.

In conclusion we can say that the Logistic Predator-Prey Model seems to be an improvement on the Lotka-Volterra Model, as the two phase plane diagrams possible both suggest stabilization. One problem with the model is that if the intersection between the two nullclines is below the x-axis, the phase plane diagram suggests that the only equilibrium is the point (C,0). The interpretation of this is that no matter what the starting point of the system, the prey species will approach its carrying capacity and the predator species will approach extinction. The other case for the intersection gives more realistic results as the point (C,0) becomes an unstable equilibrium point.

Thus, it seems that the Logistic predator-prey system does offer some improvement to the Lotka-Volterra model but it is difficult to provide substantial evidence of this due to the necessity of using numerical methods to find actual trajectories predicted by the model. However, the goal of this paper was to study this modification of the Lotka-Volterra model and although we cannot prove our conjectures concerning the form of the trajectories, the overall possible dynamics of the system seem to indicate that the Logistic Predator-Prey model is an improvement over the Lotka-Volterra model.

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LOGISTIC MODEL (PREDATOR)

$i := 1 \,..27$

$y_1 := 21000$

$t := 0.5$

$k := 0.001$

$m := -0.000004076$

Modification of these values might give better results

$x_1 := 25000$

$x_2 := 32000$

$x_3 := 39000$

$x_4 := 46000$

$x_5 := 53000$

$x_6 := 59750$

$x_7 := 66500$

$x_8 := 73250$

$x_9 := 80000$

$x_{10} := 77500$

$x_{11} := 75000$

$x_{12} := 72500$

$x_{13} := 70000$

$x_{14} := 75500$

$x_{15} := 81000$

$x_{16} := 86500$

$x_{17} := 92000$

$x_{18} := 76500$

$x_{19} := 61000$

$x_{20} := 45500$

$x_{21} := 30000$

$x_{22} := 26625$

$x_{23} := 23250$

$x_{24} := 19875$

$x_{25} := 16500$

$x_{26} := 14625$

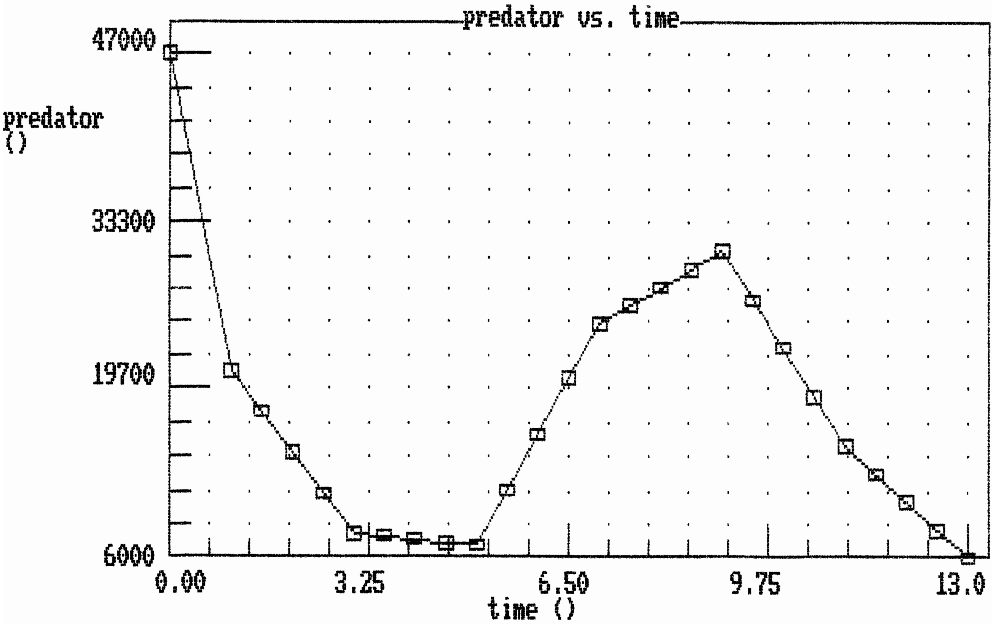
$x_{27} := 12750$

$$\boxed{y_{i+1} := y_1 + y_i \cdot t \cdot \left[m \cdot x_i - k \right]}$$

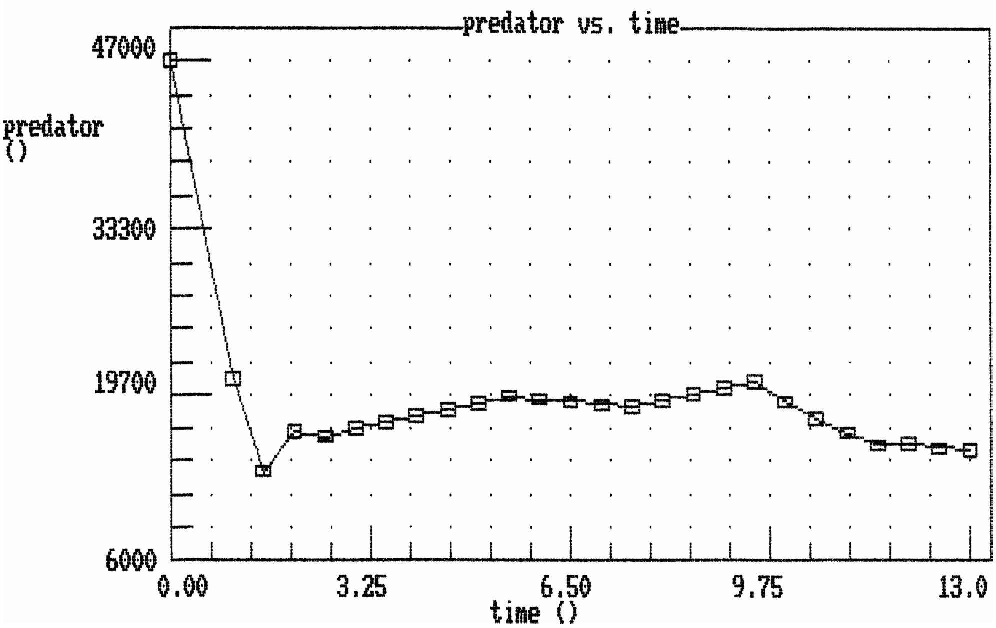
y_i
$2.1 \cdot 10^{4}$
$1.992 \cdot 10^{4}$
$1.969 \cdot 10^{4}$
$1.943 \cdot 10^{4}$
$1.917 \cdot 10^{4}$
$1.892 \cdot 10^{4}$
$1.869 \cdot 10^{4}$
$1.846 \cdot 10^{4}$
$1.824 \cdot 10^{4}$
$1.802 \cdot 10^{4}$
$1.815 \cdot 10^{4}$
$1.822 \cdot 10^{4}$
$1.83 \cdot 10^{4}$

$1.838 \cdot 10^{4}$
$1.816 \cdot 10^{4}$
$1.799 \cdot 10^{4}$
$1.782 \cdot 10^{4}$
$1.765 \cdot 10^{4}$
$1.824 \cdot 10^{4}$
$1.872 \cdot 10^{4}$
$1.925 \cdot 10^{4}$
$1.981 \cdot 10^{4}$
$1.991 \cdot 10^{4}$
$2.005 \cdot 10^{4}$
$2.018 \cdot 10^{4}$
$2.031 \cdot 10^{4}$
$2.038 \cdot 10^{4}$

GRAPH USING GIVEN DATA



GRAPH USING EULER METHOD RESULTS



LOGISTIC MODEL (PREY)

$i := 1 \,..25$

$x_1 := 25000$

$t := 0.5$

$l := 2$

$n := 0.000044043$

modification of these values might give better results

$C := 150000$

Note: This is a hypothetical carrying capacity

$y_1 := 21000$

$y_2 := 17750$

$y_3 := 14500$

$y_4 := 11250$

$y_5 := 8000$

$y_6 := 7750$

$y_7 := 7500$

$y_8 := 7250$

$y_9 := 7000$

$y_{10} := 11500$

$y_{11} := 16000$

$y_{12} := 20500$

$y_{13} := 25000$

$y_{14} := 26500$

$y_{15} := 28000$

$y_{16} := 29500$

$y_{17} := 31000$

$y_{18} := 27000$

$y_{19} := 23000$

$y_{20} := 19000$

$y_{21} := 15000$

$y_{22} := 12750$

$y_{23} := 10500$

$y_{24} := 8250$

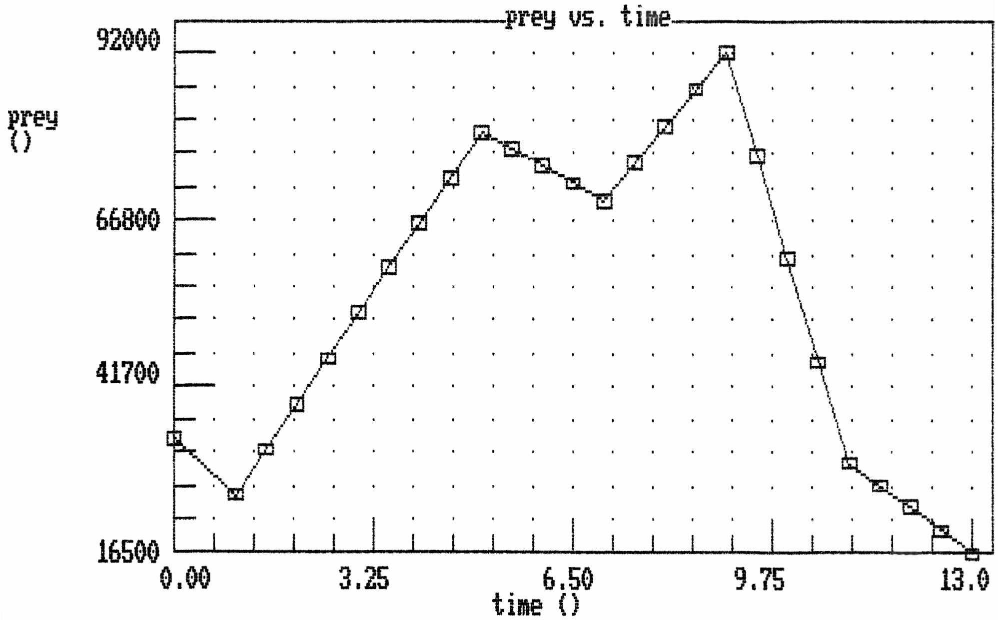
$y_{25} := 6000$

$$\boxed{x_{i+1} := \left[1 + l \cdot t - n \cdot t \cdot y_i\right] \cdot x_i - \left[l \cdot \frac{t}{C}\right] \cdot {x_i}^2}$$

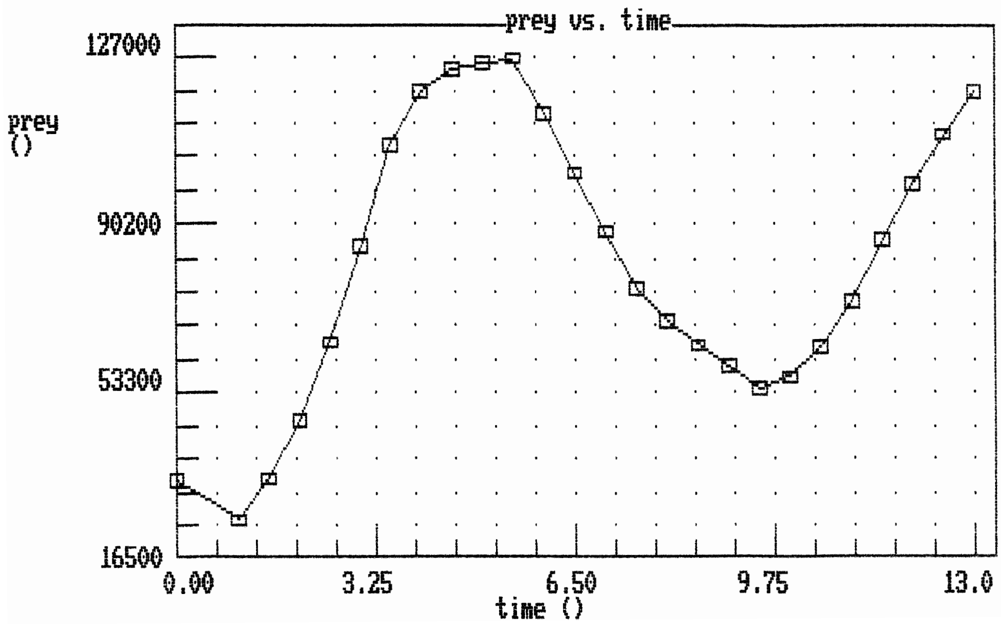
x_i
$2.5 \cdot 10^4$
$3.427 \cdot 10^4$
$4.732 \cdot 10^4$
$6.46 \cdot 10^4$
$8.537 \cdot 10^4$
$1.071 \cdot 10^5$
$1.195 \cdot 10^5$
$1.241 \cdot 10^5$
$1.257 \cdot 10^5$
$1.267 \cdot 10^5$
$1.143 \cdot 10^5$
$1.012 \cdot 10^5$
$8.844 \cdot 10^4$
$7.605 \cdot 10^4$

$6.916 \cdot 10^{4}$
$6.379 \cdot 10^{4}$
$5.901 \cdot 10^{4}$
$5.452 \cdot 10^{4}$
$5.681 \cdot 10^{4}$
$6.333 \cdot 10^{4}$
$7.342 \cdot 10^{4}$
$8.665 \cdot 10^{4}$
$9.892 \cdot 10^{4}$
$1.097 \cdot 10^{5}$
$1.193 \cdot 10^{5}$

GRAPH USING GIVEN DATA



GRAPH USING EULER METHOD RESULTS



LOTKA - VOLTERRA MODEL (PREY)

The Student Edition of MathCAD 2.0 For Educational Use Only

$i := 1 \,..27$

$y_1 := 21000$

$t := 0.5$

$k := 0.8$

$m := -0.000009064$

$K := 323264611.8$

$x_1 := 25000$

$x_2 := 32000$

$x_3 := 39000$

$x_4 := 46000$

$x_5 := 53000$

$x_6 := 59750$

$x_7 := 66500$

$x_8 := 73250$

$x_9 := 80000$

$x_{10} := 77500$

$x_{11} := 75000$

$x_{12} := 72500$

$x_{13} := 70000$

$x_{14} := 75500$

$x_{15} := 81000$

$x_{16} := 86500$

$x_{17} := 92000$

$x_{18} := 76500$

$x_{19} := 61000$

$x_{20} := 45500$

$x_{21} := 30000$

$x_{22} := 26625$

$x_{23} := 23250$

$x_{24} := 19875$

$x_{25} := 16500$

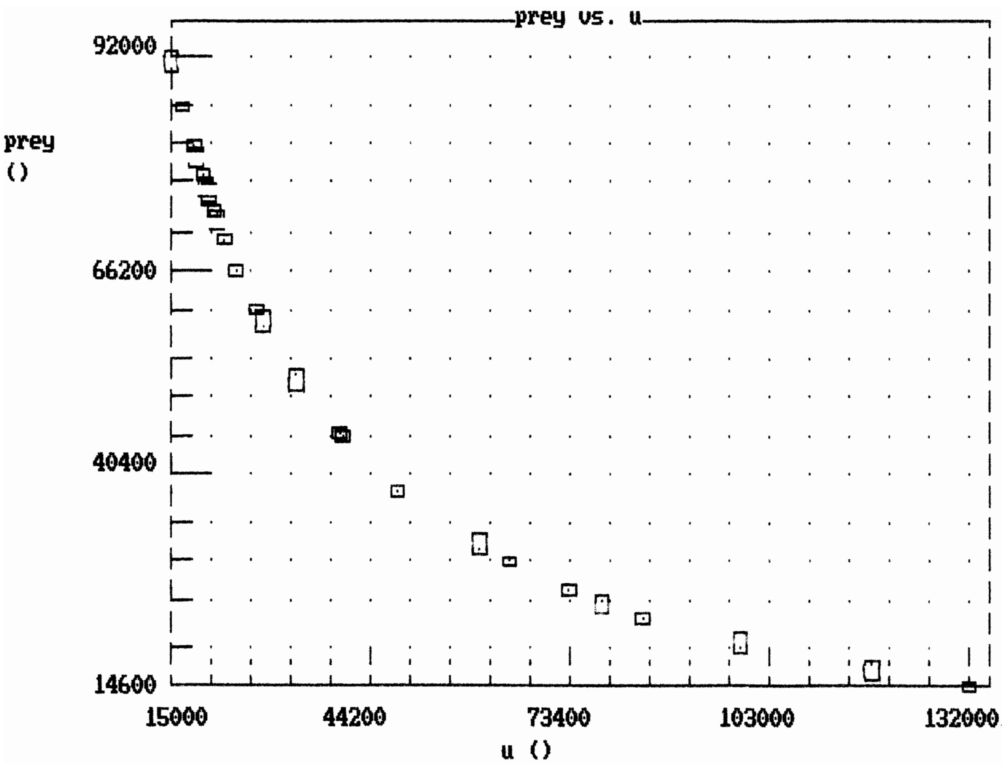
$x_{26} := 14625$

$x_{27} := 12750$

$$u_i := K \cdot \exp\left[m \cdot x_i\right] \cdot x_i^{-k}$$

u_i
$7.813 \cdot 10^4$
$6.018 \cdot 10^4$
$4.822 \cdot 10^4$
$3.965 \cdot 10^4$
$3.323 \cdot 10^4$
$2.84 \cdot 10^4$
$2.452 \cdot 10^4$
$2.135 \cdot 10^4$
$1.871 \cdot 10^4$
$1.964 \cdot 10^4$
$2.062 \cdot 10^4$
$\cdot 10^4$

$2.167 \cdot 10^{4}$
$2.28 \cdot 10^{4}$
$2.042 \cdot 10^{4}$
$1.836 \cdot 10^{4}$
$1.657 \cdot 10^{4}$
$1.501 \cdot 10^{4}$
$2.002 \cdot 10^{4}$
$2.762 \cdot 10^{4}$
$4.018 \cdot 10^{4}$
$6.453 \cdot 10^{4}$
$7.32 \cdot 10^{4}$
$8.412 \cdot 10^{4}$
$9.833 \cdot 10^{4}$
$1.177 \cdot 10^{5}$
$1.318 \cdot 10^{5}$



prey vs. u
92000
66200
40400
14600
prey
()
15000
44200
73400
103000
132000
u ()

LOTKA - VOLTERRA MODEL (PREDATOR)

$i := 1 .. 25$

$x_1 := 25000$

$t := 0.5$

$l := 1.2$

$n := 0.000040969$

$y_1 := 21000$

$y_2 := 17750$

$y_3 := 14500$

$y_4 := 11250$

$y_5 := 8000$

$y_6 := 7750$

$y_7 := 7500$

$y_8 := 7250$

$y_9 := 7000$

$y_{10} := 11500$

$y_{11} := 16000$

$y_{12} := 20500$

$y_{13} := 25000$

$y_{14} := 26500$

$y_{15} := 28000$

$y_{16} := 29500$

$y_{17} := 31000$

$y_{18} := 27000$

$y_{19} := 23000$

$y_{20} := 19000$

$y_{21} := 15000$

$y_{22} := 12750$

$y_{23} := 10500$

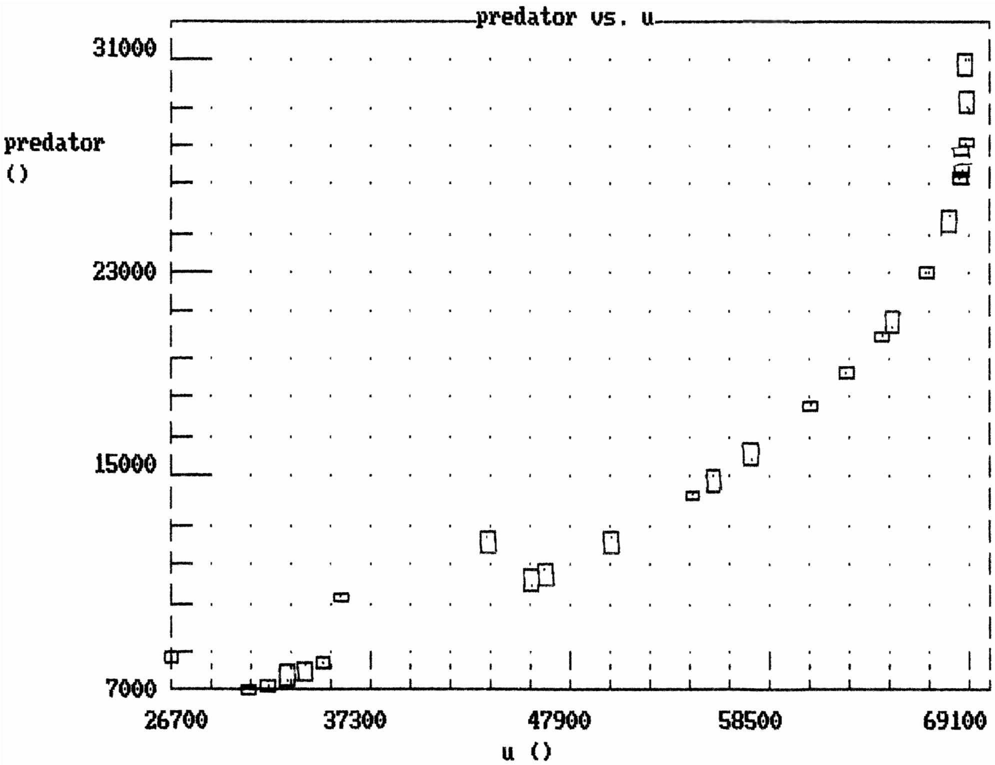
$y_{24} := 8250$

$y_{25} := 6000$

i i i

u_i

u_i
$6.502 \cdot 10^{4}$
$6.07 \cdot 10^{4}$
$5.441 \cdot 10^{4}$
$4.584 \cdot 10^{4}$
$3.478 \cdot 10^{4}$
$3.383 \cdot 10^{4}$
$3.286 \cdot 10^{4}$
$3.187 \cdot 10^{4}$
$3.087 \cdot 10^{4}$
$4.658 \cdot 10^{4}$
$5.758 \cdot 10^{4}$
$6.447 \cdot 10^{4}$
$6.803 \cdot 10^{4}$
$6.861 \cdot 10^{4}$
$6.893 \cdot 10^{4}$
$6.901 \cdot 10^{4}$
$6.888 \cdot 10^{4}$
$6.874 \cdot 10^{4}$
$6.681 \cdot 10^{4}$
$6.258 \cdot 10^{4}$
$5.552 \cdot 10^{4}$
$5.009 \cdot 10^{4}$
$4.351 \cdot 10^{4}$
$3.572 \cdot 10^{4}$
$2.673 \cdot 10^{4}$


predator vs. u
predator
()
31000
23000
15000
7000
26700
37300
47900
58500
69100
u ()