UNIVERSITY OF MANITOBA

DATE: March 7, 2007 — Test 1
PAPER NO.: - — PAGE NO.: 1 of 3
DEPARTMENT & COURSE NO.: MATH 3530 — TIME: 90 minutes
EXAMINATION: Mathematical Problems in Biology — EXAMINER: S. Portet

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NAME: _____

STUDENT NUMBER: _____ SEAT NUMBER: _____

SIGNATURE: _____

This is a 90 minute exam. Lecture Notes are allowed.

This exam has 5 questions.

PLEASE SHOW YOUR WORK CLEARLY. A correct answer without explanation will not get full marks.

Question	Points	Score
1	15	
2	10	
3	8	
4	17	
5	10	
Total:	60	

1. (15 points) Consider a species that lives for two years and may reproduce at the end of year one or year two.

   - the mean number of offspring that 0-year olds have the following year is 1
   - the mean number of offspring that 1-year olds have the following year is 4
   - the probability that a 0-year old survives to be a 1-year old is 0.1.

   (a) Set up the Leslie model. (Draw the life-cycle diagram, define variables, write the model equations)

   (b) Assume that the initial population consists of ten 0-year olds and six 1-year olds. How many 0-year olds and how many 1-year olds will there be one year later.

   (c) Find the stable age distribution. Does the population increase or decrease over time?

2. (10 points) The dynamics of a particular population (measured in thousand) of birds is described as follows

$$\frac{dP}{dt} = 4P(1 - 8P^3)$$

   (a) Find the equilibria.

   (b) Determine the local stability of each equilibrium.

3. (8 points) Show that

$$x_{t+1} = \frac{ax_t}{b + x_t}, \quad a, b > 0 \quad x_t > 0$$

   has no $2-$cycle.

4. (17 points) Show that the two-dimensional system

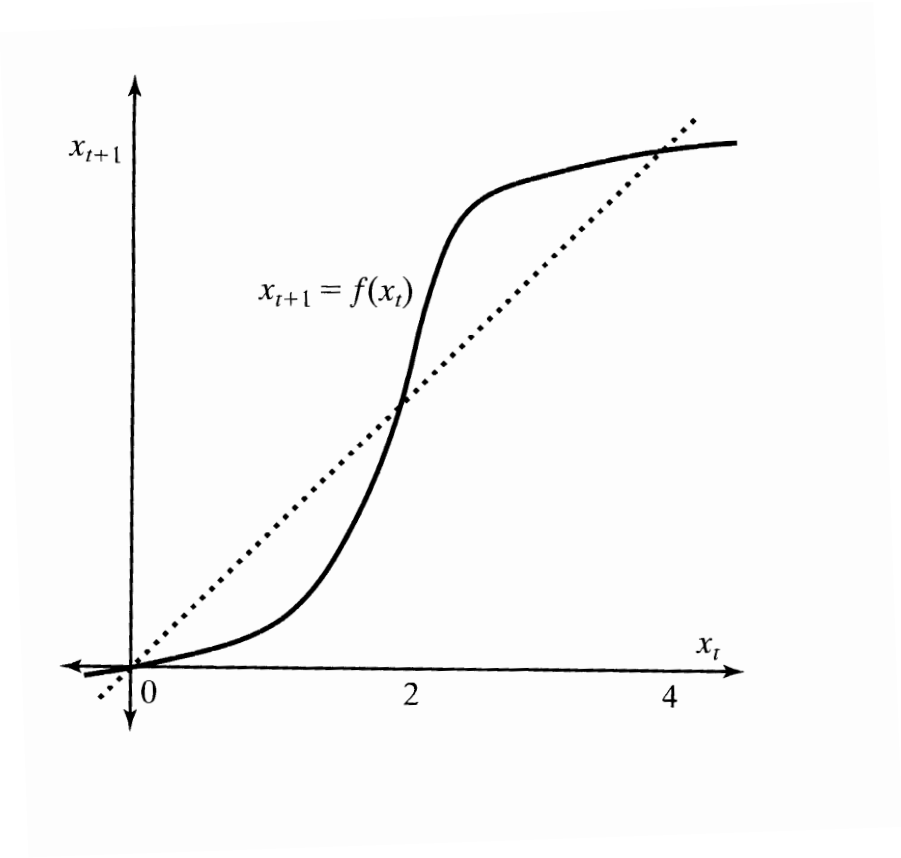
$$\begin{aligned} x_{t+1} &= x_t(1 + x_t + y_t)/3 \\ y_{t+1} &= y_t(1 - x_t + y_t)/2 \end{aligned}$$

   has four fixed points, but only one that is locally stable.

5. (10 points) Consider the difference equation that represents the size of a population in generation $t + 1$

$$x_{t+1} = f(x_t),$$

   its graph is shown in the Figure below



(a) Find all equilibria.

(b) Determine the local stability of each equilibrium.