January 10, 2007

Cell division

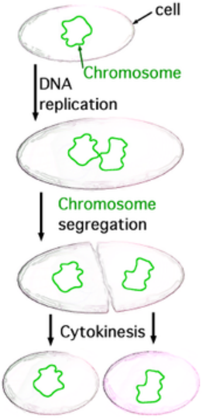
Definition

Cell division is the process by which a cell, called the parent cell, divides into two cells, called daughter cells. Cell division is usually a small segment of a larger cell cycle.

- Prokaryotic cells: binary fission
- Eukaryotic cells: mitosis+cytokinesis

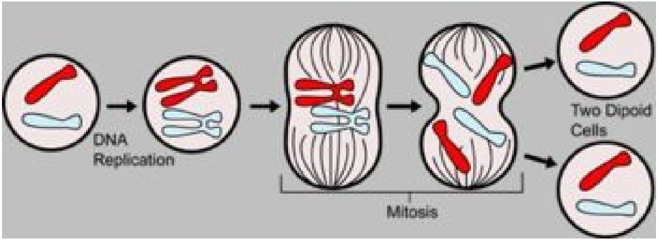
Binary fission

The prokaryotic chromosome is a single DNA molecule that first replicates, then attaches each copy to a different part of the cell membrane. When the cell begins to pull apart, the replicate and original chromosomes are separated. Following cell splitting (cytokinesis), there are then two cells of identical genetic composition (except for the rare chance of a spontaneous mutation).



Mitosis+Citokinesis

Mitosis is the process by which a cell separates its duplicated genome into two identical halves. It is generally followed immediately by cytokinesis which divides the cytoplasm and cell membrane. This results in two identical daughter cells with a roughly equal distribution of organelles and other cellular components. Mitosis and cytokinesis together is defined as the mitotic (M) phase of the cell cycle, the division of the mother cell into two daughter cells, each the genetic equivalent of the parent cell.



Example 1

E. coli are able to divide every 20 minutes under optimal conditions. Describe the temporal evolution of a colony of E. coli.

Example 2

Assume that adult females of a species produce offspring at a fixed period of time each year. A proportion of the offspring (juveniles) survives to adulthood, reproduces, and dies (nonoverlapping of generations). Let

- j_t number of juveniles in years t
- a_t number of adult females in year t
- p number of juveniles that survive in year t
- f number of offspring produced per female
- r ratio of females to adults.

Example 3

A drug is administred once every four hours. Let D_n be the amount of the drug in the blood system at the n^{th} interval. The body eliminates a certain fraction p of the drug during each time interval. If the amount administred is D_0, find D_n and $\lim_{n\to} D_n$.

Example 4

Plants produce seeds at the end of their growth season (August), after which they die. A fraction of these seeds survive the winter, and some of these germinate at the beginning of the season (May), giving rise to the new generation of plants. The fraction that germinates depends on the age of the seeds.
Additional assumption: Seeds older than two years are no longer viable

- γ number of seeds produced per plant in August
- σ fraction of seeds that survive a given winter
- α fraction of one-year-old seeds that germinate in May
- β fraction of two-year-old seeds that germinate in May

Difference equations

Definition

A difference equation of order k has the form

$$f(x_{t+k}, x_{t+k-1}, \ldots, x_{t+1}, x_t, t) = 0 \quad t = 0, 1, \ldots,$$

where f is a real-valued function of the real variable x_t through x_{t+k} and t.

Order

The order of a difference equation is the difference between the largest and the smallest arguments k appearing in it.

Autonomous - Nonautonomous

The difference equation is called autonomous if f does not depend explicitly on t and it is called nonautonomous otherwise.

Difference equations

$$x_{t+k} + a_1 x_{t+k-1} + a_2 x_{t+k-2} + \cdots + a_{k-1} x_{t+1} = b_t \quad t = 0, 1, \ldots$$

Linear - Nonlinear

If the coefficients a_j, $j = 1, \ldots, k$ are constant or depend on t but **do not depend on the state variables**, then the difference equation is said to be linear; otherwise, it is to be nonlinear.

Homogeneous - Nonhomogeneous

If the difference equation is linear and $b_t = 0$ for all t, then it is said to be homogeneous; otherwise, it is said to be nonhomogeneous.

Solution of difference equation

Definition

A solution of the difference equation

$$f(x_{t+k}, x_{t+k-1}, \ldots, x_{t+1}, x_t, t) = 0 \quad t = 0, 1, \ldots,$$

is a function x_t, $t = 0, 1, 2, \ldots$ such that when substituted into the equation makes it a true statement.

First-order linear homogeneous difference equation

$$x_{t+1} = ax_t$$

If x_0 (initial value) is known, the solution is unique and is

$$x_t = a^t x_0$$

- $0 < a < 1$ then $\lim_{t\to\infty} x_t = 0$
- $a = 1$ then $x_t = x_0 \ \forall t$
- $a > 1$ then $\lim_{t\to\infty} x_t = +\infty$

In general

- $|a| < 1$ x_t converges to 0.
- $a = 1$ x_t is constant.
- $|a| > 1$ x_t diverges (either approaches infinity or oscillates).

First-order linear homogeneous difference equation

$$x_{t+1} = a(t)x_t \quad t = 0, 1, 2, \dots$$

If x_0 (initial value) is known, the solution is unique and is

$$x_t = \left[\prod_{i=0}^{t-1} a(i)\right] x_0$$

First-order linear nonhomogeneous difference equation

$$x_{t+1} = a(t)x_t + b(t) \quad t = 0, 1, 2, \dots$$

If x_0 (initial value) is known, the solution is unique and is

$$x_t = \left[\prod_{i=0}^{t-1} a(i)\right] x_0 + b(t-1) + \sum_{i=0}^{t-2} \left[\prod_{r=i+1}^{t-1} a(r)\right] b(i)$$

- $x_{t+1} = ax_t + b(t) \quad x(0) = x_0$, then

$$x_t = a^t x_0 + \sum_{i=0}^{t-1} a^{t-i-1} b(i)$$

- $x_{t+1} = ax_t + b \quad x(0) = x_0$, then

$$x_t = \begin{cases} a^t x_0 + b\left[\frac{a^t - 1}{a-1}\right] & a \neq 1 \\ x_0 + bt & a = 1 \end{cases}$$