UNIVERSITY OF MANITOBA

DATE: March 3, 2009   Test 1
PAPER NO.: -   PAGE NO.: 1 of 2
DEPARTMENT & COURSE NO.: MATH 3820   TIME: 120 minutes
EXAMINATION: Intro. Math. Modelling   EXAMINER: J. Arino

This is a 120 minutes exam, with 4 questions for a total of 50 marks. Lecture Notes are allowed. PLEASE SHOW YOUR WORK CLEARLY. A correct answer without explanation will not get full marks.

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1. (10 points) The dynamics of a population of birds (measured in thousand) is described by the equation
$$\frac{dP}{dt} = 4P(1 - 8P^3).$$

   (a) Find the equilibria.

   (b) Determine the local stability of each equilibrium.

2. (10 points) The population (measured in billions) of insects in generation t is described as follows
$$P_{t+1} = P_t e^{4(1-3P_t)}$$

   (a) Find all fixed points.

   (b) Determine the local stability of each fixed point.

3. (20 points) Assume that an insect population, $x(t)$, is controlled by a natural predator population, $y(t)$. We make the following assumptions:

   - In the absence of predators, the dynamics of the insects is governed by a logistic equation.

   - Preys and predators meet at a rate that is of mass action type.

   - When a contact takes place, the probability per contact that a prey dies is k_1. [Hint: think of mass action contact in an epidemic model.]

   - These contacts lead to an increase of the predator population with rate k_2.

   - The predators are subject to natural death at the per capita rate d.

   (a) Write a model describing the interaction of the 2 populations.

   (b) Study the model you have written: is it well-posed, what are its equilibria (and when are they realistic), can you determine the stability of these equilibria?

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(c) Assume that an insecticide is used to reduce the population of insects, but it is also toxic to the predators; hence, the poison kills both preys and predators at rates proportional to their respective populations. Modify your model from (a). [For bonus marks, you may want to see how this modifies the analysis you carried out in (b).]

4. (10 points) Consider the difference equation

$$x_{t+1} = f(x_t)$$

with graph shown in the figure below
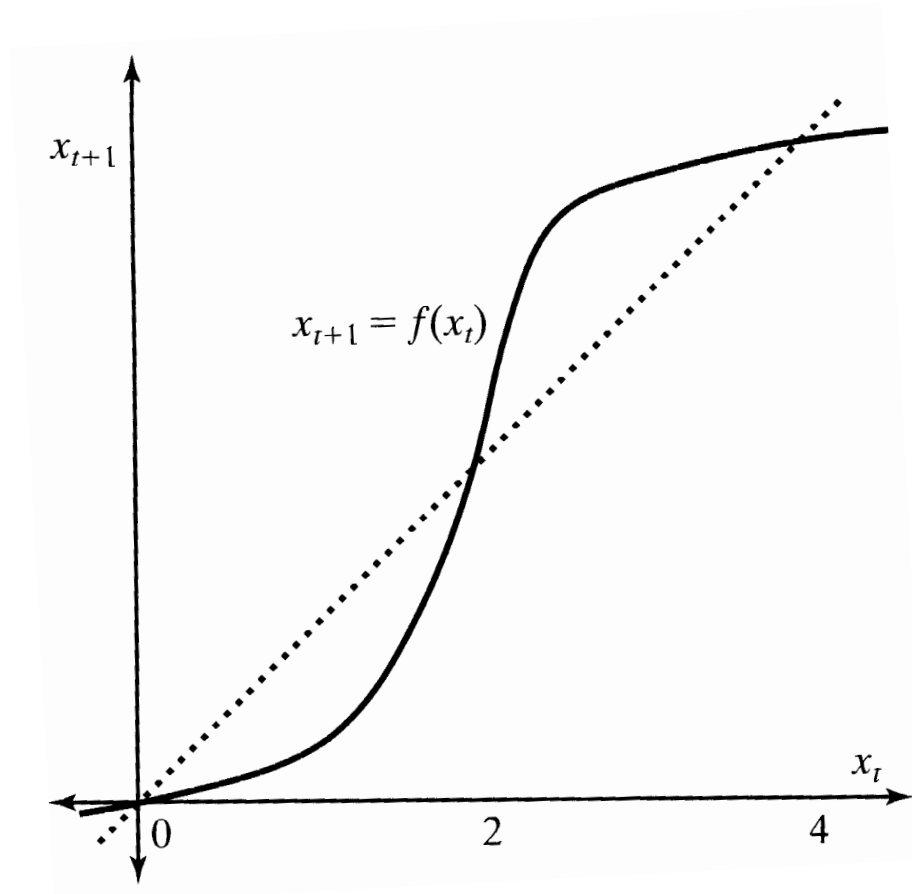
(a) Find all fixed points.

(b) Determine the local stability of each fixed point. [Hint: what does the stability condition imply, in terms of the slope of f?]