

A second-order linear, homogeneous differential equation with constant coefficients has two solutions $\{x_1, x_2\}$ which may take one of three forms:

$$\{e^{\lambda_1 t}, e^{\lambda_2 t}\}, \{e^{\lambda_1 t}, te^{\lambda_1 t}\}, \text{ or } \{e^{ut}\cos(vt), e^{ut}\sin(vt)\}.$$

In each case, compute the Wronskian and show that the solutions are linearly independent; the Wronskian does not equal zero.



For the following differential equations, find the equilibria; then graph the phase line diagrams. Use the phase line diagrams to determine the stability of the equilibria.

(a)
$$dx/dt = \sin(x)\cos(x)$$

(b)
$$dx/dt = x(a-x)(x-b)^2$$
, $0 < a < b$

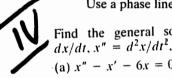
$$(c) dx/dt = x(e^x - x - 2)$$

A spatially implicit model for the proportion of islands occupied by a species was modeled by Levins (1969, 1970) and Hanski (1999). The model takes the following form:

$$\frac{dp}{dt} = (m+cp)(1-p) - ep = f(p),$$

where p(t) is the fraction of occupied islands at time $t, m \ge 0, c > 0$, and e > 0. The constants m, c, and e are the rates of immigration from the mainland, colonization, and extinction, respectively.

- (a) Suppose there is no immigration from the mainland, m = 0, and the colonization rate is greater than the extinction rate, c > e. In Levins original model, m = 0. For this model, find the nonnegative equilibria and determine their stability. What happens if the colonization rate is less than the extinction rate, c < e?
- (b) Suppose there is immigration from the mainland, m > 0. For this model, show that there exists a unique positive equilibrium which is asymptotically stable for all initial values $0 \le p(0) \le 1$. Note that f(p) is quadratic in p. Use a phase line diagram.



Find the general solution to the following differential equations, x' = dx/dt, $x'' = d^2x/dt^2$, and $x''' = d^3x/dt^3$.

(a)
$$x'' - x' - 6x = 0$$

(b)
$$x'' - 4x' + 5x = 0$$

(c)
$$x''' - 5x'' + 3x' + 9x = 0$$

(d)
$$x''' + 16x' = 0$$

(e)
$$x'' + 2ax' + (a^2 + b^2)x = 0$$
, $a, b \neq 0$

Use an integrating factor to find the unique solution to the following initial

(a)
$$\frac{dx}{dt} - 3t^2x = 4te^{-t^3}$$
, $x(0) = 1$

(b)
$$\frac{dx}{dt} + \frac{2}{t}x = 2t + 5$$
, $x(1) = 1$

Growth of a population is modeled by the following differential equation:

$$\frac{dN}{dt}=\frac{\alpha n_2+\beta n_1}{N}-(\alpha+\beta),$$

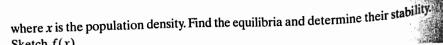
where α , β , n_1 , and n_2 are positive constants.

(a) Find the equilibrium solution for this model; then draw a phase line diagram.

(b) If N(0) > 0, find $\lim_{t\to\infty} N(t)$.

A mathematical model for the growth of a population is

$$\frac{dx}{dt} = \frac{2x^2}{1+x^4} - x = f(x), \quad x(0) \ge 0,$$



Classify the following ordinary differential equations by determining whether they are linear, what their order is, whether they are homogeneous, and whether their coefficients are constant. (f) $\frac{dy}{dt} = \frac{1}{1+y}$. (a) $(\sin x)y'' + \cos x = 0$.

(d) $\frac{d}{dt}(y^2 + 2y) = y$.

 $\frac{dy}{dt} = xy$.

Consider the equation

The differential equation

has the general solution

$$\begin{aligned}
\frac{dt}{dt} & 1 + y \\
\mathbf{(g)} & \frac{dy}{dx} = \frac{1}{1 + x}.
\end{aligned}$$

(b)
$$y'' + y^2 = 2y'$$
.
(c) $\frac{d^3y}{dt^3} + \frac{2dy}{dt} = \sin y$.

Find the steady states of the following systems of equations, and determine the

 $\frac{dy}{dx} = xy - y.$

Show that $x_1(t) = e^{3t}$ and $x_2(t) = e^{-t}$ are two solutions. Show that $x(t) = c_1x_1(t) + c_2x_2(t)$ is also a solution.

 $\frac{d^2x}{dx^2} - 2\frac{dx}{dx} - 3x = 0.$

 $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0$

 $x(t) = c_1 e^{-t} + c_2 e^{-2t}$ If we are told that, when t = 0, x(0) = 1 and its derivative x'(0) = 1, we can

$$\frac{dy}{dx}$$

$$\frac{dy}{dx}$$

$$\frac{dx}{dx}$$

$$\frac{dx}{d^5y}$$

$$\frac{dx}{d^5y}$$

$$\frac{dx}{dx} = 1 + x^{-1}$$
(h) $\frac{d^5y}{dx^5} = x^6 + 5x + 6$.

$$\frac{dx}{d^5}$$

$$\frac{d}{dx}$$

$$\frac{dy}{dx}$$

$$\frac{dy}{dx}$$
 d^{5}

$$\frac{d}{dz}$$

$$\frac{dy}{dz}$$

$$\frac{dy}{dx}$$

$$\frac{dy}{dx}$$

(i) $t \frac{dy}{dt} + ty = 1$.

$$\frac{d}{d}$$

$$\frac{dy}{dx}$$

$$\frac{d}{d}$$

(e) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y = e^t + e^{-t}$.

Jacobian of the system for these steady states:

(b) $\frac{dx}{dt} = y - xy$, (d) $\frac{dx}{dt} = x - xy$,

determine c_1 and c_2 by solving the equations

(a) $\frac{dx}{dt} = x^2 - y^2$, (c) $\frac{dx}{dt} = x - x^2 - xy$,

 $\frac{dy}{dt} = x(1-y). \qquad \frac{dy}{dt} = y(1-y).$