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THE FRENCH AEROSPACE LAB

Parametric optimization of an acoustic liner

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ST5 – Contrôle de la pollution acoustique et électromagnétique
CentraleSupélec, 7 novembre 2022

Reducing aircraft noise

france bleu

Toulouse Changer

Infos Sports Culture Vie quotidienne

Transports

Aéroport Toulouse-Blagnac : les plaintes pour nuisances sonores se multiplient

Lundi 24 février 2020 à 4:46 - Mis à jour le lundi 24 février 2020 à 11:13 - Par Alexandre Berthaud, Clémence Fulleda, France Bleu Occitanie, France Bleu

Toulouse

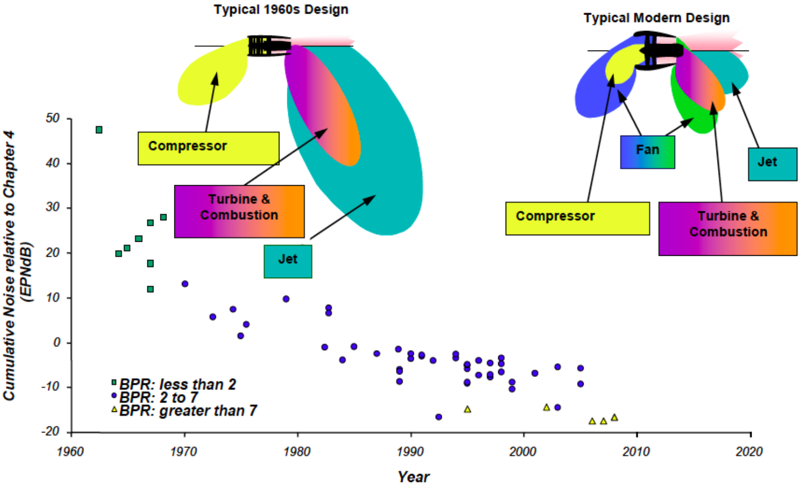
En à peine deux mois, le CCNAAT (collectif contre les nuisances aériennes dans l'agglomération toulousaine) a recensé près de 5.000 plaintes de riverains sur son site internet. Cette forte augmentation serait liée à la mise en place de nouveaux couloirs aériens à Toulouse-Blagnac.

Reducing aircraft noise

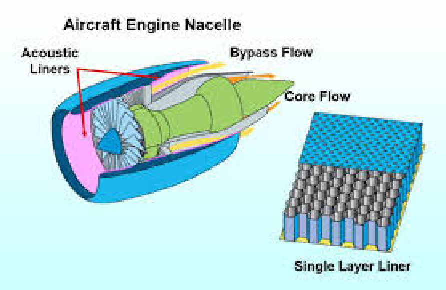
La moitié des Français considère le bruit des transports comme la principale source de nuisances sonores. Des recherches se poursuivent donc activement pour permettre de réduire le bruit des réacteurs des avions au sol et en vol.

6,5 millions d'euros : c'est le montant total des amendes infligées par l'ACNUSA (Autorité de contrôle

Reducing aircraft noise

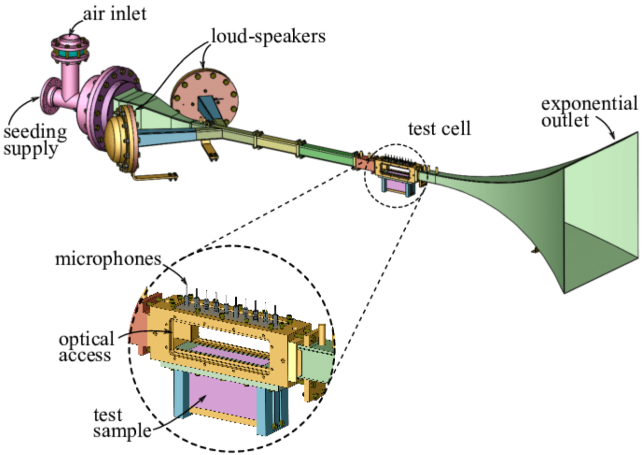


Reducing aircraft noise



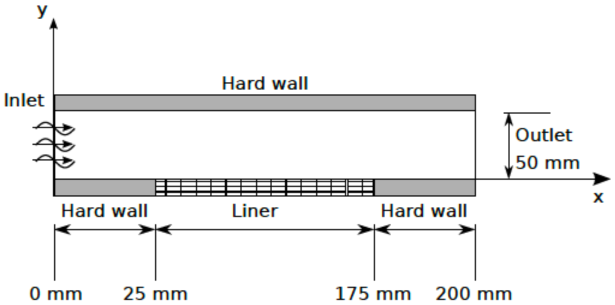
Use of liners in nacelle aircraft engine to reduce fan, turbine and combustion noises
ACARE, Clean Sky 2, ACNUSA ...

Acoustic liners in ducts



ONERA's aero-thermo-acoustic test bench

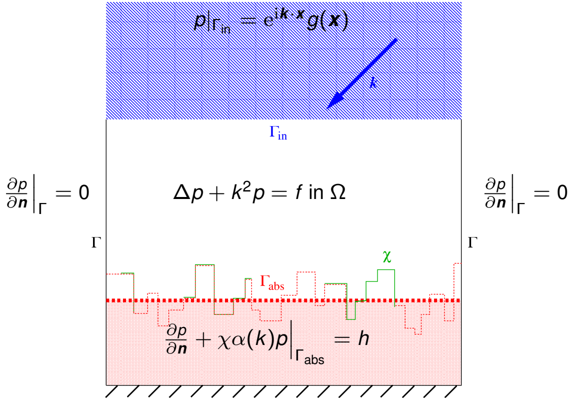
Acoustic liners in ducts



ONERA's aero-thermo-acoustic test bench

J. Primus, F. Simon, E. Piot, AIAA Paper #2011-2869 (2011)
O. Léon, E. Piot, D. Sebbane, F. Simon, *Exp. Fluids* **58**, 72 (2017)

Model problem



Model problem for acoustical liners with $\partial\Omega = \Gamma_{\text{in}} \cup \Gamma_{\text{abs}} \cup \Gamma$

- Γ_{in} inflow condition (*e.g.* incident plane wave with vanishing Mach number $M_0 = 0$);
- Γ hard-wall ($Z \to \infty$);
- Γ_{abs} acoustic liner with absorption coefficient $\alpha \in \mathbb{C}$.

Optimization problem: Setting

- The characteristic function χ gives the distribution of absorbing layer on the boundary Γ_{abs}:

$$\chi(\boldsymbol{x}) = \begin{cases} 1 & \text{if there is porous material at } \boldsymbol{x} \\ 0 & \text{if there is no porous material at } \boldsymbol{x} \,, \end{cases}$$

  with the condition that $0 < \beta = \int_{\Gamma_{\text{abs}}} \chi dS < \int_{\Gamma_{\text{abs}}} dS$.
- The goal is to find χ such that the acoustic disturbance $J(\chi) := \int_\Omega |p_\chi|^2 d\boldsymbol{x}$ is minimal, when p_χ is the solution of:

$$\begin{cases} \Delta p + k^2 p = 0 & \text{in } \Omega \,, \\ p = g & \text{on } \Gamma_{\text{in}} \,, \\ \partial_{\boldsymbol{n}} p = 0 & \text{on } \Gamma \,, \\ \partial_{\boldsymbol{n}} p + \alpha \chi p = 0 & \text{on } \Gamma_{\text{abs}} \,. \end{cases} \qquad (P_\chi)$$

- Eligible admissible set:

$$U_{\text{ad}}(\beta) = \left\{ \chi \in L^\infty(\Gamma_{\text{abs}});\ \forall \boldsymbol{x} \in \Gamma_{\text{abs}},\ \chi(\boldsymbol{x}) \in \{0,1\},\ \int_{\Gamma_{\text{abs}}} \chi dS = \beta \right\}.$$

- Relaxation method: we rather seek χ in $U^*_{\text{ad}}(\beta) \supset U_{\text{ad}}(\beta)$ with

$$U^*_{\text{ad}}(\beta) = \left\{ \chi \in L^\infty(\Gamma_{\text{abs}});\ \forall \boldsymbol{x} \in \Gamma_{\text{abs}},\ \chi(\boldsymbol{x}) \in [0,1],\ \int_{\Gamma_{\text{abs}}} \chi dS = \beta \right\},$$

  and it can be shown that $\min_{\chi \in U^*_{\text{ad}}(\beta)} J(\chi)$ has at least one solution (Th. 7.4.4 page 106).

Optimization problem: $J'(\chi)$

- Fréchet derivative of the cost function $J(\chi)$: the linear form $J'(\chi) : L^\infty(\Gamma_{\text{abs}}) \to L^1(\Gamma_{\text{abs}})$ such that

$$J(\chi+\chi_0) = J(\chi) + \langle J'(\chi), \chi_0 \rangle + \mathrm{o}(\chi_0)\,, \qquad \lim_{\|\chi_0\|_\infty \to 0} \frac{|\mathrm{o}(\chi_0)|}{\|\chi_0\|_\infty} = 0\,.$$

- **Theorem**: the cost function $J(\chi) = \int_\Omega |p_\chi|^2 d\boldsymbol{x}$ is derivable in the sense of Fréchet and

$$\boxed{J'(\chi) = -\mathrm{Re}\{\alpha p_\chi q_\chi\}}\,,$$

where q_χ is the solution of the adjoint problem (P'_χ) considering $J(\chi)$ as the objective function

$$\begin{cases} \Delta q + k^2 q = -2\overline{p}_\chi & \text{in } \Omega\,, \\ q = 0 & \text{on } \Gamma_{\text{in}}\,, \\ \partial_{\boldsymbol{n}} q = 0 & \text{on } \Gamma\,, \\ \partial_{\boldsymbol{n}} q + \alpha\chi q = 0 & \text{on } \Gamma_{\text{abs}}\,. \end{cases} \qquad (P'_\chi)$$

- **Outlook**: ambient flow $\varrho_0(\boldsymbol{x}), p_0(\boldsymbol{x}), \boldsymbol{v}_0(\boldsymbol{x})$, uncertainty quantification...

Optimization problem: Gradient descent

- Gradient descent algorithm:

$$\chi^{(k+1)} = \mathcal{P}_{\ell_k}\left[\chi^{(k)} - \mu_k J'(\chi^{(k)})\right] \,,$$

  where $\mu_k > 0$ is the learning rate, which is iteratively updated:
  - If $J(\chi^{(k+1)}) < J(\chi^{(k)})$, then $\mu_{k+1} = \mu_k + \epsilon$ for the next iteration;
  - If $J(\chi^{(k+1)}) \geq J(\chi^{(k)})$, then $\chi^{(k+1)}$ is re-evaluated by dividing μ_k by 2.
- To ensure that $\chi^{(k+1)} \in [0, 1]$, we use the projector:

$$\mathcal{P}_\ell[\chi] = \max\left(0, \min\left(\chi + \ell, 1\right)\right) \,,$$

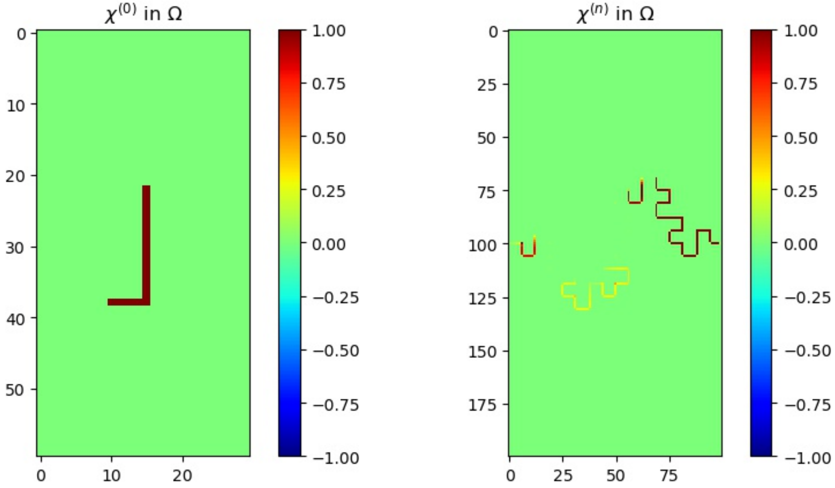
  where ℓ plays the role of a Lagrange multiplier to ensure that the constraint $\int_{\Gamma_{\text{abs}}} \chi dS = \beta$ is fulfilled; it is updated iteratively by dichotomy.
- The algorithm is ended after K iterations, or whenever $\|\chi^{(k+1)} - \chi^{(k)}\|_2$ for instance is arbitrarily small.

G. Allaire, *Conception Optimale des Structures*. Springer-Verlag, Berlin (2007)

Optimization problem: Algorithm

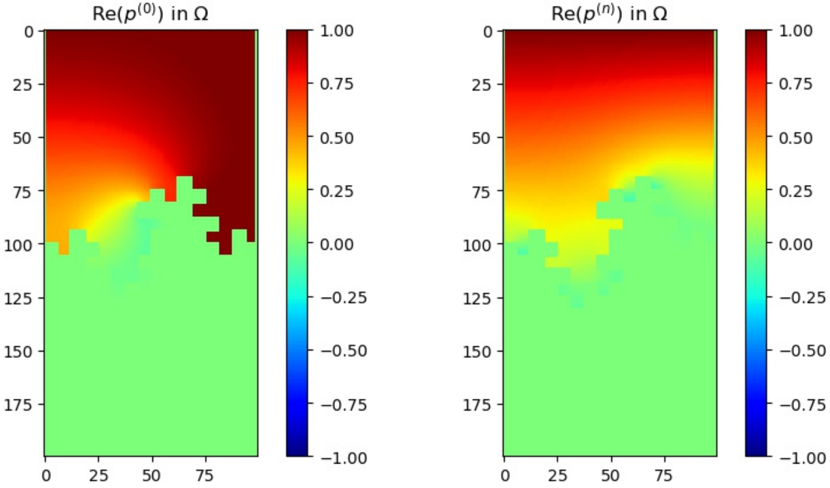
$\chi^{(0)} \in U_{\text{ad}}(\beta)$; $\mu = \mu^{(0)}$;
for $k = 0 : K$ **do**
    compute $p^{(k)}$; $q^{(k)}$; $J(\chi^{(k)})$; $J'(\chi^{(k)})$;
    $E = J(\chi^{(k)})$;
    while $E \geq J(\chi^{(k)})$ & $\mu > \epsilon_0$ **do**
        $\ell = 0$;
        $\chi^{(k+1)} = \mathcal{P}_\ell \left[\chi^{(k)} - \mu J'(\chi^{(k)})\right]$;
        while $|\int_{\Gamma_{\text{abs}}} \chi^{(k+1)} dS - \beta| \geq \epsilon_1$ **do**
            if $\int_{\Gamma_{\text{abs}}} \chi^{(k+1)} dS \geq \beta$ **then**
                $\ell \leftarrow \ell - \epsilon_2$;
            else
                $\ell \leftarrow \ell + \epsilon_2$;
            end if
            $\chi^{(k+1)} = \mathcal{P}_\ell \left[\chi^{(k)} - \mu J'(\chi^{(k)})\right]$;
        end while
        compute $p^{(k+1)}$; $J(\chi^{(k+1)})$;
        $E = J(\chi^{(k+1)})$;
        if $E < J(\chi^{(k)})$ **then**
            $\mu \leftarrow \mu + \epsilon_3$;
        else
            $\mu \leftarrow \mu/2$;
        end if
    end while
end for

Optimization problem



$k = 1.0$

Optimization problem



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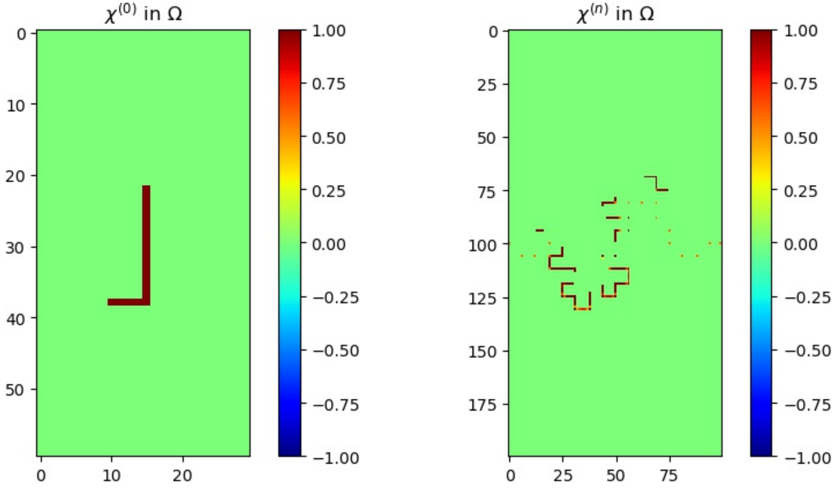
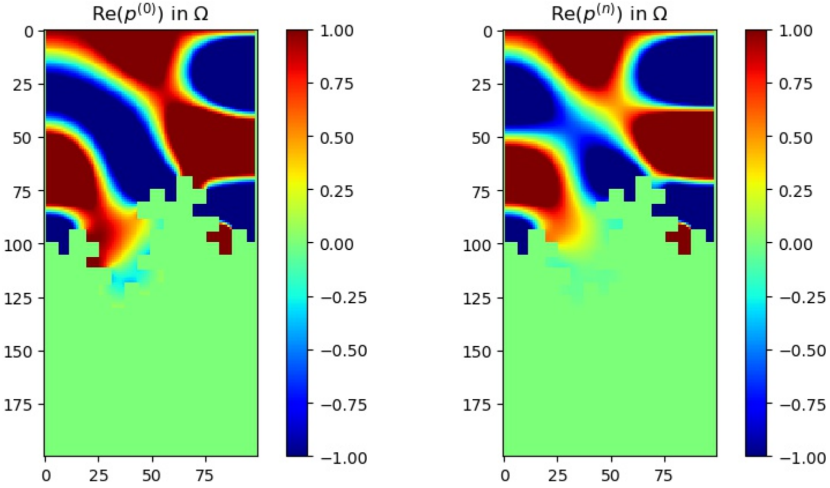
Optimization problem



$k = 10.0$

Optimization problem



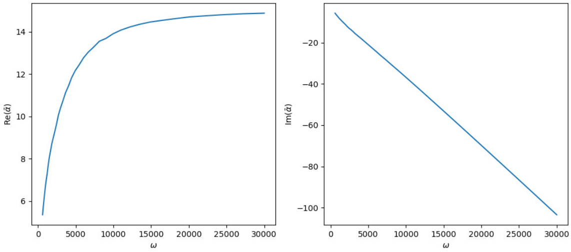
$k = 10.0$

Absorption

- The complex-valued, regular function $\omega \mapsto \alpha(\omega)$ has a strictly positive real part $\mathrm{Re}\{\alpha\} > 0$ and strictly negative imaginary part $\mathrm{Im}\{\alpha\} < 0$;
- The condition $\frac{\partial p}{\partial \boldsymbol{n}} + (\mathrm{Re}\{\alpha\} - \mathrm{i}|\mathrm{Im}\{\alpha\}|)p = 0$ is satisfied by the wave:

$$\mathrm{e}^{-\alpha \boldsymbol{n}\cdot\boldsymbol{x}} = \mathrm{e}^{-\mathrm{Re}\{\alpha\}\boldsymbol{n}\cdot\boldsymbol{x}}\mathrm{e}^{\mathrm{i}|\mathrm{Im}\{\alpha\}|\boldsymbol{n}\cdot\boldsymbol{x}},$$

which is a propagating plane wave $\times$ an exponentially decreasing amplitude where dissipation grows with $\mathrm{Re}\{\alpha\}$.



Real and imaginary parts of α for melamine foam

F. Magoulès, T. P. K. Nguyen, P. Omnes, A. Rozanova-Pierrat, *SIAM J. Control Optim.* **59**(1), 561 (2021)

Absorption vs. Impedance

- The pressure $\widehat{p}(\boldsymbol{x},\omega)$ and particle velocity $\widehat{\boldsymbol{v}}(\boldsymbol{x},\omega)$ of a fluid in a porous material with motionless, rigid skeleton satisfy (constitutive equation and linearized Darcy's law in the frequency domain with a $\mathrm{e}^{-\mathrm{i}\omega t}$ dependence):

$$\frac{\mathrm{i}\omega}{K_{\mathrm{eq}}(\omega)}\widehat{p} = \boldsymbol{\nabla}\cdot\widehat{\boldsymbol{v}}\,,\quad \mathrm{i}\omega\varrho_{\mathrm{eq}}(\omega)\widehat{\boldsymbol{v}} = \boldsymbol{\nabla}\widehat{p}\,,$$

  where $K_0 = \varrho_0 c_0^2$ and $\varrho_{\mathrm{eq}}(\omega) = \varrho_0\widehat{\alpha}(\omega)$, $K_{\mathrm{eq}}(\omega) = K_0/\widehat{\beta}(\omega)$.
- Dynamic tortuosity accounting for viscous and inertial effects in the frequency domain:

$$\widehat{\alpha}(\omega) = \alpha_h + \frac{\eta\phi}{\varrho_0 k_0}\widehat{A}(\omega)\,,$$

  where $t \mapsto A(t)$ a time-dependent dimensionless viscosity coefficient accounting for viscous effects; η pore fluid dynamic viscosity; ϕ porosity; α_h tortuosity (the ratio of the microscopic mean-square velocity of fluid particles relative to the skeleton with respect to their macroscopic quadratic velocity); k_0 intrinsic permeability depending on the geometry of the pore network solely.

Absorption vs. Impedance

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  where $K_0 = \varrho_0 c_0^2$ and $\varrho_{\mathrm{eq}}(\omega) = \varrho_0\widehat{\alpha}(\omega)$, $K_{\mathrm{eq}}(\omega) = K_0/\widehat{\beta}(\omega)$.
- Dynamic compressibility, or bulk modulus, accounting for thermal effects:

$$\widehat{\beta}(\omega) = \gamma(1+\widehat{B}(\omega))\,,$$

  where $t\mapsto B(t)$ a time-dependent compressibility coefficient (in s^{-1}) accounting for thermal effects of the fluid phase; γ ratio of specific heats.

Absorption vs. Impedance

- For a homogeneous equivalent fluid layer of porous material of thickness L, the transfer matrix $\boldsymbol{T}$ for plane acoustic waves travelling in the direction of the (inward) normal $\boldsymbol{n}$ to the layer, such that:

$$\begin{pmatrix} \widehat{p}(\boldsymbol{x},\omega) \\ \widehat{\boldsymbol{v}}(\boldsymbol{x},\omega)\cdot\boldsymbol{n} \end{pmatrix}_{\boldsymbol{x}\cdot\boldsymbol{n}=0} = \boldsymbol{T}(\omega) \begin{pmatrix} \widehat{p}(\boldsymbol{x},\omega) \\ \widehat{\boldsymbol{v}}(\boldsymbol{x},\omega)\cdot\boldsymbol{n} \end{pmatrix}_{\boldsymbol{x}\cdot\boldsymbol{n}=L} ,$$

is:

$$\boldsymbol{T}(\omega) = \begin{bmatrix} \cos(k_{\mathrm{eq}}(\omega)L) & -\mathrm{i}Z_{\mathrm{eq}}(\omega)\sin(k_{\mathrm{eq}}(\omega)L) \\ \frac{1}{\mathrm{i}Z_{\mathrm{eq}}(\omega)}\sin(k_{\mathrm{eq}}(\omega)L) & \cos(k_{\mathrm{eq}}(\omega)L) \end{bmatrix} ,$$

with the equivalent frequency-dependent speed of sound c_{eq}, wave number k_{eq}, and characteristic impedance Z_{eq}:

$$c_{\mathrm{eq}}(\omega) = \sqrt{\frac{K_{\mathrm{eq}}(\omega)}{\varrho_{\mathrm{eq}}(\omega)}} , \quad k_{\mathrm{eq}}(\omega) = \frac{\omega}{c_{\mathrm{eq}}(\omega)} , \quad Z_{\mathrm{eq}}(\omega) = \varrho_{\mathrm{eq}}(\omega)c_{\mathrm{eq}}(\omega) .$$

Absorption vs. Impedance

- Boundary impedance of the layer of thickness L such that $Z = \frac{\widehat{p}}{\widehat{\boldsymbol{v}}\cdot\boldsymbol{n}}|_{\boldsymbol{x}\cdot\boldsymbol{n}=L}$:

$$Z(\omega) = -\frac{T_{22}(\omega)}{\phi T_{21}(\omega)}\,,$$

where ϕ porosity of the last layer in contact with the air–typically a perforated plate.

- Hence one identifies:

$$\alpha(\omega) = -\mathrm{i}\omega\frac{\varrho_{\mathrm{eq}}(\omega)}{Z(\omega)} = -\omega\varrho_{\mathrm{eq}}(\omega)\frac{X(\omega)+\mathrm{i}R(\omega)}{|Z(\omega)|^2}\,.$$