Submanifold reconstruction using Coxeter-Freudhental-Kuhn triangulations

Contact : Jean-Daniel Boissonnat jean-daniel.boissonnat@inria.fr tel. +33 6 72 81 72 11

Location : DataShape Project Team, INRIA Sophia Antipolis, France

The subject of the internship is to compute a piecewise (PL) approximation of an m-dimensional submanifold $\mathcal{M}$ embedded in $\mathbb{R}^d$ that is only known through a finite set of points $P \in \mathbb{R}^d$ lying close to $\mathcal{M}$. This is a very important problem with numerous applications in Scientific Computing, Machine Learning, Graphics, Robotics, and even in pure mathematics. A case of particular interest is when m is much smaller than d.

The problem is well understood when $m = 2$ and $d = 3$: this is the well studied problem of surface reconstruction. In higher dimensions, the problem is much less understood. Although a few methods are known, they are mostly of theoretical interest and there is currently no robust code that works in cases of practical interest [1, 2].

We propose a new approach based on Coxeter-Freudhental-Kuhn triangulations. These triangulations have been used with great success to sample and mesh isomanifolds, i.e. manifolds defined as the zero set of a multivariate multivalued function $f : \mathbb{R}^d \to \mathbb{R}^{d-m}$ [3]. The goal of this project is to combine this algorithm with the Moving Least-Squares (MLS) approach [4]. From the data set P, a smooth submanifold is interpolated using the MLS approach and used by the oracle of the manifold tracing algorithm.

The project consists in exploring in detail the approach, proving its mathematical correctness and algorithmic efficiency (we aim at a complexity that depends mostly on the intrinsic dimension m). The algorithm will be implemented in C++ as a module of the GUDHI library (http://gudhi.gforge.inria.fr/). It will be tested on data coming from several fields such as molecular chemistry, material science and pure mathematics.

We expect results of unprecedented quality within the duration of the internship.

This internship will give the opportunity to learn about many fascinating areas (high dimensional computational geometry, computational topology, geometry processing) and software (CGAL, GUDHI).

The expected output of the internship includes papers reporting on the theoretical advances and C++ code to be integrated in the GUDHI library.

The internship will be coadvised by Jean-Daniel Boissonnat (INRIA Sophia Antipolis), Mathijs Wintraecken (IST Austria, Vienna) and Pierre Alliez (INRIA Sophia Antipolis). Aurélien Alvarez (ENS Lyon) is also associated to the project.

References

[1] Jean-Daniel Boissonnat, Frédéric Chazal, and Mariette Yvinec. *Geometric and Topological Inference*. Cambridge Texts in Applied Mathematics. Cambridge University Press, 2018.

[2] Jean-Daniel Boissonnat and Arijit Ghosh. Manifold reconstruction using tangential Delaunay complexes. *Discrete & Computational Geometry*, 51(1):221–267, 2014.

[3] Jean-Daniel Boissonnat, Siargey Kachanovich, and Mathijs Wintraecken. Sampling and Meshing Submanifolds in High Dimension. Preprint https://hal.inria.fr/hal-02386169, November 2019.

[4] Barak Sober. *Structuring High Dimensional Data: A Moving Least-Squares Projective Approach to Analyze Manifold Data*. PhD thesis, Tel Aviv University, 2018.