Parameter estimation: We aim at a maximum-likelihood estimation of the parameters of the distribution

$$\phi(x) = Ce^{\frac{1}{2}(\Theta_X+\Theta_Y)x^2-\Theta_X x} x^{N_X+Ai-1}\left(1-x\right)^{N_Y+A-Ai-1},$$

where C is determined by $\int_0^1 \phi(x)dx = 1$. This integral becomes numerically unstable for large parameter values, so we re-parameterize the distribution, defining $\hat{\nu}, \hat{s}, \hat{\theta}, \hat{\psi}, \hat{\chi}$ by

$$\Theta_X = \hat{s}\hat{\theta}\hat{\psi},\ \Theta_Y = \hat{s}\hat{\theta}(1-\hat{\psi}),\ N_X-1 = \hat{s}(1-\hat{\theta})\hat{\nu}(1-\hat{\xi}),\ N_Y-1 = \hat{s}(1-\hat{\theta})(1-\hat{\nu})(1-\hat{\xi}),\ A = \hat{s}\hat{\theta}\hat{\xi},$$

where $\hat{s}, \hat{\theta}, \hat{\nu}, \hat{\xi} \in [0, 1]$ and $\hat{s} > 0$, with the restriction $\hat{s}(1-\hat{\theta})\hat{\nu}(1-\hat{\xi}) > 1$ and $\hat{s}(1-\hat{\theta})(1-\hat{\nu})(1-\hat{\xi}) > 1$. Therewith, the distribution becomes

$$\begin{aligned}\phi(x) = \hat{C}\exp\Bigg[\hat{s}\Big\{\hat{\theta}\left(\frac{x^2}{2} - \hat{\psi}x\right) + \ln(x)\Big((1-\hat{\theta})\hat{\nu}(1-\hat{\xi}) + \hat{\theta}\hat{\xi}i\Big) \\ + \ln(1-x)\Big((1-\hat{\theta})(1-\hat{\nu})(1-\hat{\xi}) + \hat{\theta}\hat{\xi}(1-i)\Big) + B\Big\}\Bigg]\end{aligned}$$

Here, B is a constant that can be chosen in dependency on the data at hand. In practice, it is used to avoid an exonent that has a very large absolute number. The constant $\hat{C}$ is, as before, determined by the fact that the integral is 1.