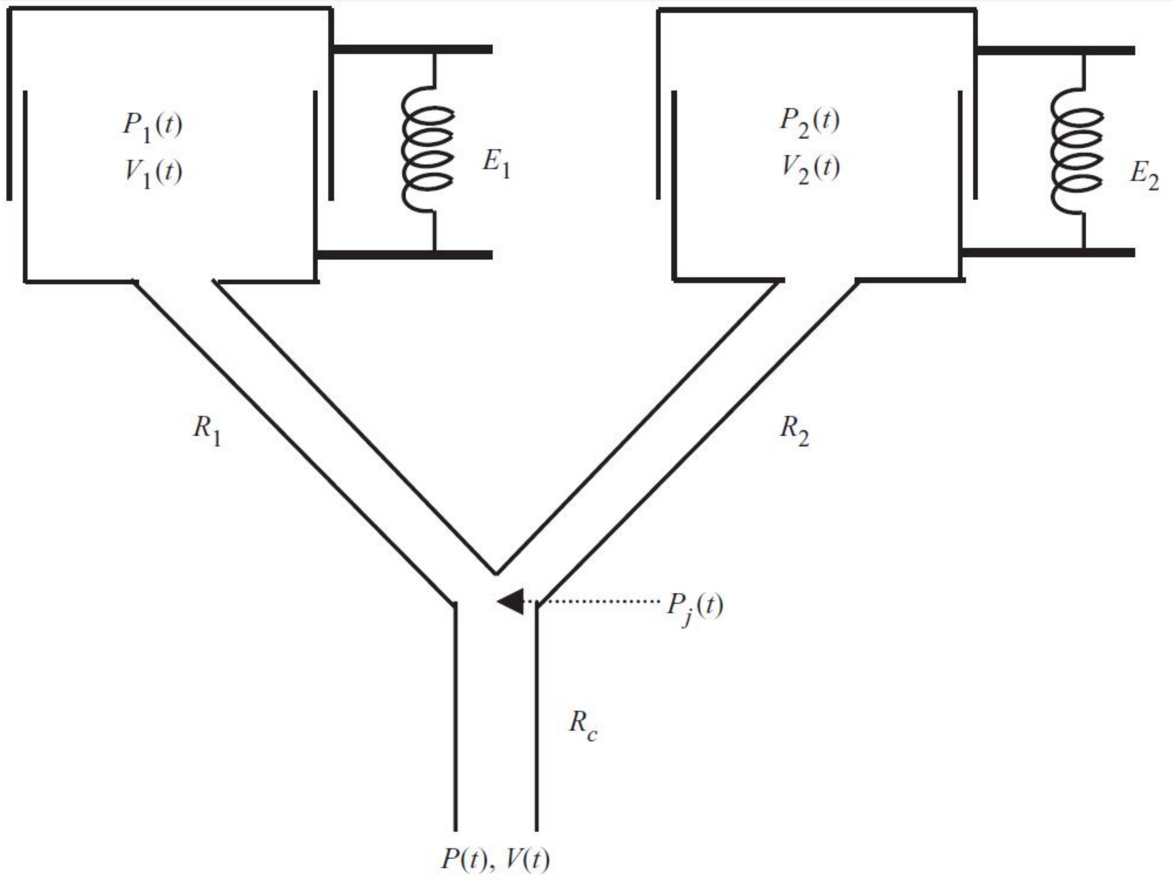
# Parallel model (1)

- Two compartments representing two different alveolar regions connected to the airway opening



# Parallel model (2)

- For each alveolar compartment, we have:

$$P_1(t) = E_1 V_1(t)$$
$$P_2(t) = E_2 V_2(t)$$

- If $P_j$ is the pressure at the junction of the airways:

$$P_j(t) - P_1(t) = P_j(t) - E_1 V_1(t) = R_1 \dot{V}_1(t)$$
$$P_j(t) - P_2(t) = P_j(t) - E_2 V_2(t) = R_2 \dot{V}_2(t)$$

- For the common airway:

$$P(t) - P_j(t) = R_c\left[\dot{V}_1(t) + \dot{V}_2(t)\right]$$

# Parallel model (3)

$$P(t) = E_1 V_1(t) + (R_1 + R_c)\dot{V}_1(t) + R_c \dot{V}_2(t)$$
$$P(t) = E_2 V_2(t) + (R_2 + R_c)\dot{V}_2(t) + R_c \dot{V}_1(t)$$

- Differentiating these two equations gives:

$$\dot{P}(t) = E_1 \dot{V}_1(t) + (R_1 + R_c)\ddot{V}_1(t) + R_c \ddot{V}_2(t)$$
$$\dot{P}(t) = E_2 \dot{V}_2(t) + (R_2 + R_c)\ddot{V}_2(t) + R_c \ddot{V}_1(t)$$

- Eliminating $\ddot{V}_2(t)$ from the last two equations:

$$R_2 \dot{P}(t) = (R_2 + R_c) E_1 \dot{V}_1(t) + (R_1 R_2 + R_1 R_c + R_2 R_c)\ddot{V}_1(t) - R_c E_2 \dot{V}_2(t)$$

- Then replacing $\dot{V}_2(t)$ with its expression from

$$P(t) = E_1 V_1(t) + (R_1 + R_c)\dot{V}_1(t) + R_c \dot{V}_2(t)$$

$$R_2 \dot{P}(t) + E_2 P(t)$$
$$= [R_1 R_2 + R_c(R_1 + R_2)]\ddot{V}_1(t) + [(R_2 + R_c)E_1 + (R_1 + R_c)E_2]\dot{V}_1(t) + E_1 E_2 V_1(t)$$

# Parallel model (4)

- Finally, the symetry of the model allows to write the same equation for compartment 2 by interchanging subscripts 1 and 2
- Adding the resulting equations and noting that $V(t) = V_1(t) + V_2(t)$:

$$
\begin{aligned}
&(\mathrm{R}_1 + \mathrm{R}_2)\,\dot{\mathrm{P}}(\mathrm{t}) + (\mathrm{E}_1 + \mathrm{E}_2)P(t) \\
&= [R_1R_2 + R_c(\mathrm{R}_1 + \mathrm{R}_2)]\ddot{V}(\mathrm{t}) + [(R_2 + R_c)E_1 + (\mathrm{R}_1 + \mathrm{R}_\mathrm{c})E_2]\dot{V}(\mathrm{t}) \\
&+ E_1E_2V(\mathrm{t})
\end{aligned}
$$

- The equation of motion is thus a second order linear differential equation.
- This process can be repeated for models with more compartments
- BUT this is mathematically more and more difficult…