MECA0010 “Uncertainty quantification and stochastic modeling” Assignment 2

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- This assignment is a numerical project that involves the application of methods from uncertainty quantification (stochastic modeling, Monte Carlo simulation, stochastic sensitivity analysis, ...) to an application of your interest.
- You may work on this assignment either alone or in a small group of two or three students, provided that it is the same group of students as for the first assignment. If you work in a small group of students, only a single report must be turned in.
- At the latest on Wednesday 13 November, send Nicolas (n.leclere@uliege.be) an email with the composition of your group.
- **Step 1: Problem definition and stochastic modeling:** Please define a problem of uncertainty quantification relevant to an application of your interest. Choose a model for which an implementation in a computer code is already available to you or which you can easily implement yourself (python, matlab, C++, ...). The model should have certain (scalar-valued) parameters that can be considered as input parameters and a (scalar-valued) prediction that can be considered as a quantity of interest. There should be more than one input parameter.

  Throughout this project, the input parameters will be considered uncertain, and interest will be directed towards quantifying and understanding how uncertainty in the input parameters induces uncertainty in the quantity of interest. Thus, the model that you choose should lend itself to such a question of uncertainty quantification being asked and being interesting.

  You could consider taking a model relevant to another course or integrated project (Bloc 1) or your final-year project (TFE, Bloc 2). The model can be as simple as the beam model from lecture 4, but it can also be a more complex simulation, such as a finite element model.

  Define and/or describe the model, the model parameters that you consider to be the uncertain input parameters, and the model prediction that you consider to be the quantity of interest. Propose for each uncertain input parameter a probability distribution that meaningfully describes uncertainty in this uncertain input parameter.
- **Step 2: Monte Carlo simulation:** Please implement a Monte Carlo simulation to assess the impact of the uncertainty in the input parameters on the quantity of interest. Implement a sampling method to generate samples from the probability distributions that you assigned to the uncertain input parameters. Evaluate the model for them to determine the corresponding samples of the quantity of interest. Approximate statistical descriptors of the uncertainty quantity of interest, such as its mean, its variance, its coefficient of variation, and its probability density function (kernel density estimation). Assess the convergence of your results with respect to the number of Monte Carlo samples.
- **Step 3: Stochastic sensitivity analysis:** Please implement a stochastic sensitivity analysis to assess the significance of each uncertain input parameter in inducing uncertainty in the quantity of interest. Use the Monte Carlo method to estimate the sensitivity indices. Assess the convergence of your results with respect to the number of Monte Carlo samples.
- **Comments on the numerical implementation**: You may proceed by coding everything yourself from scratch (in Matlab, Python, C++, ...). Or you may use an existing functionalities of an existing Matlab/Python/C++ toolbox, such as the MIT Uncertainty quantification library (https://mituq.bitbucket.io/source/_site/index.html), the UQLab

library (https://www.uqlab.com), the equadratures toolbox (https://equadratures.org), or machine learning frameworks such as PyTorch, Tensorflow, or scikit-learn. If you use existing functionalities of an existing Matlab/Python/C++ toolbox, please discuss in your report the functionalities that you used and how you used them.

- Document your work in a report with a length of between 5 and 7 pages. Submit your report, along with your code, to the instructors (maarten.arnst@uliege.be, n.leclere@uliege.be) at the latest on Friday 20 December.

Good luck!