Static Analysis by Abstract Interpretation and Decision Procedures

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Advisors:
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January 24^{th} 2013

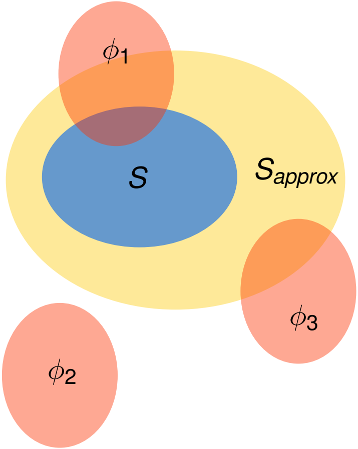
Summary

1. Introduction
2. Weakness of the Standard Approach
3. Contributions

Static Analysis

- Discover properties on programs
- Find program invariants, bugs...
- Allow compile-time optimizations
- blablabla

Over-approximation



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  S is unsafe w.r.t ϕ_1 and the analyser emits an alarm.

S	:	set of reachable states	(not computable)
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S_{approx}	:	over-approximation of S	(computable)

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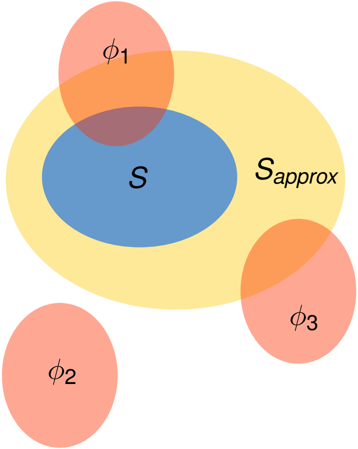

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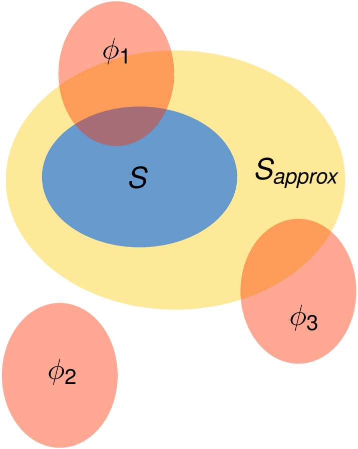

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Abstract Interpretation

- Compute an increasing sequence of the set X of reachable states of the program.
- Fixpoint computation : X grows until $F(X) \subseteq X$.

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x = 0;
while (x < 100) {
    x++;
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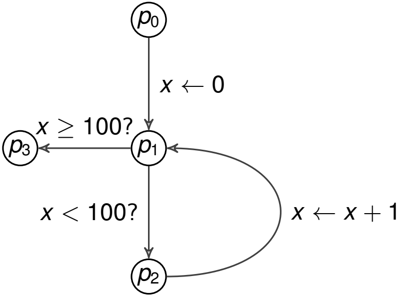

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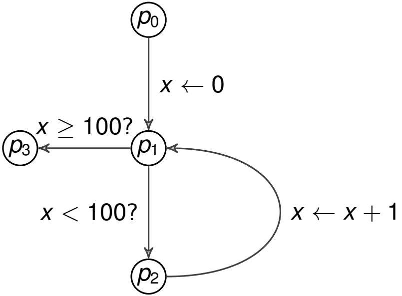

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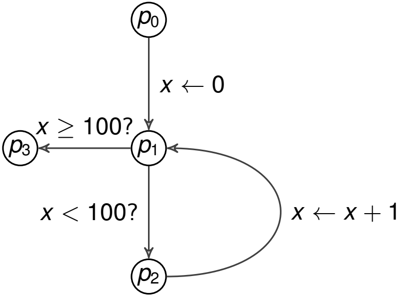

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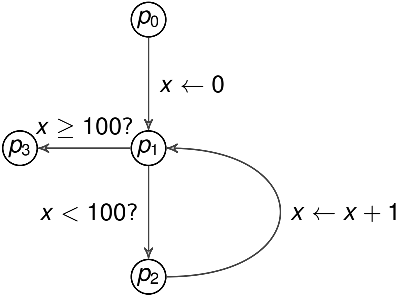

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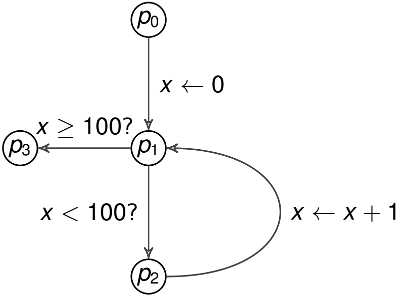

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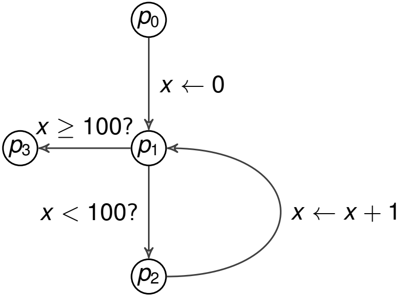

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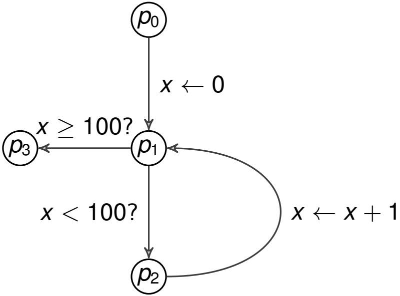

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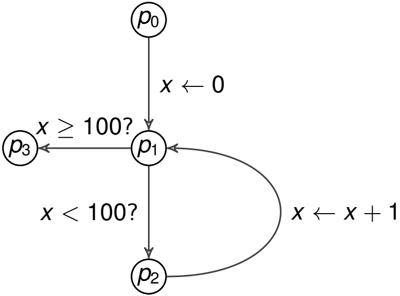

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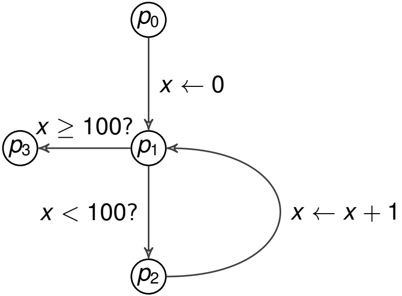

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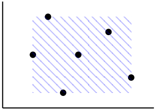
Abstract Interpretation

Cousot & Cousot 1977

Abstract domain to represent the set of possible states:

- Intervals



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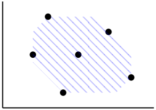

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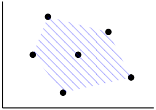

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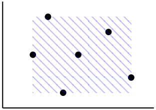

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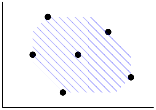

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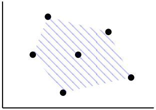

$\Rightarrow$ Over-approximation of the set of states

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Ph.D topic

Improve precision of Abstract Interpretation, by combining it with Decision Procedures (SMT-solving).

Summary

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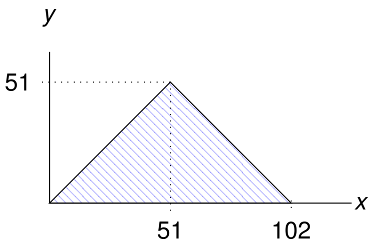
3 Contributions

Example of Standard Abstract Interpretation

```
x = 0;
y = 0;
while (true) {
        if (x <= 50)
                y++;
        else
                y--;

        if (y < 0) break;
        x++;
}
```



- x and y incremented during 51 iterations
- x incremented and y decremented during 51 iterations

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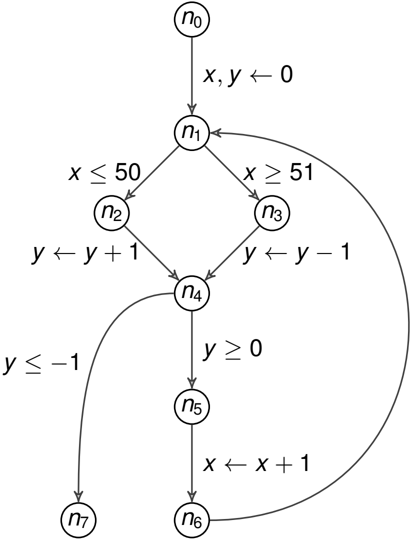

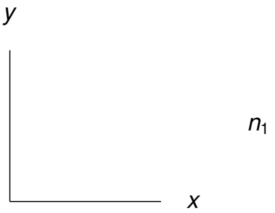

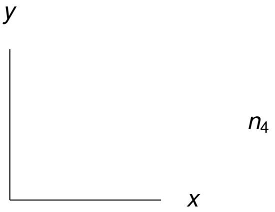

Ascending iterations

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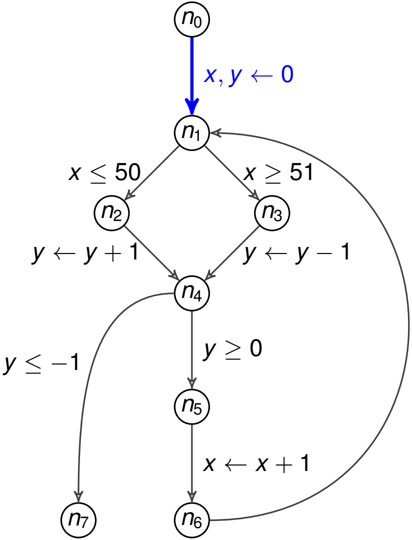

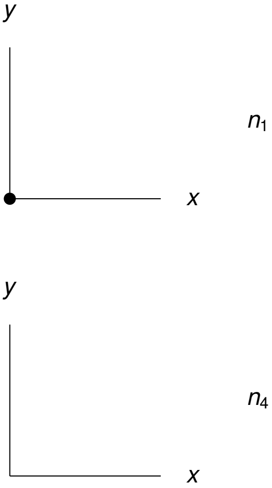

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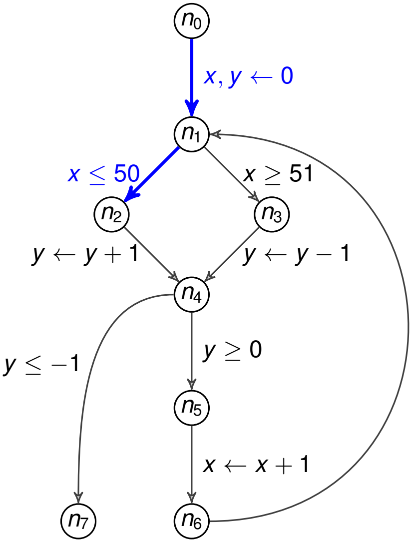

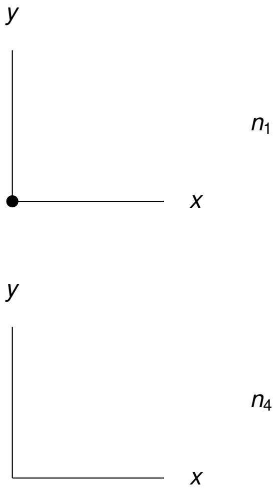

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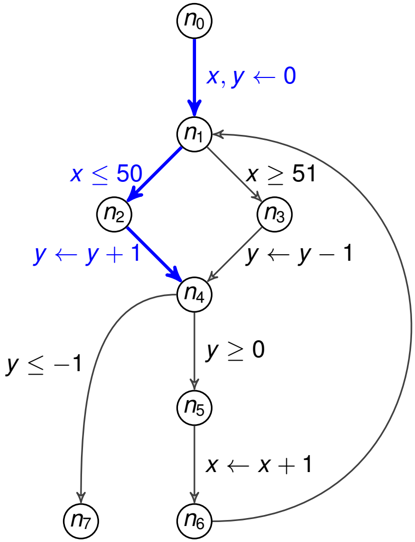

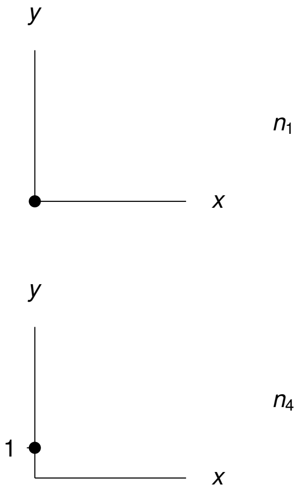

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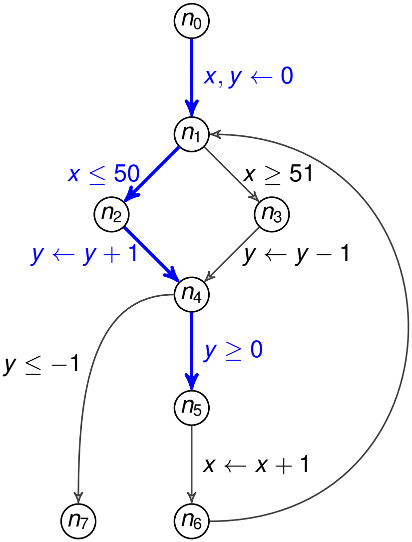

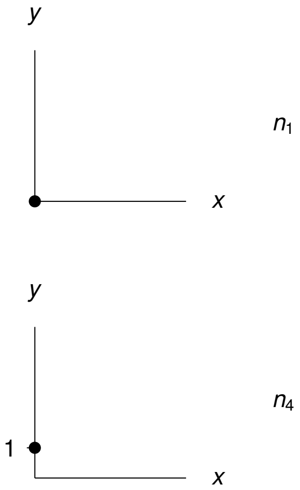

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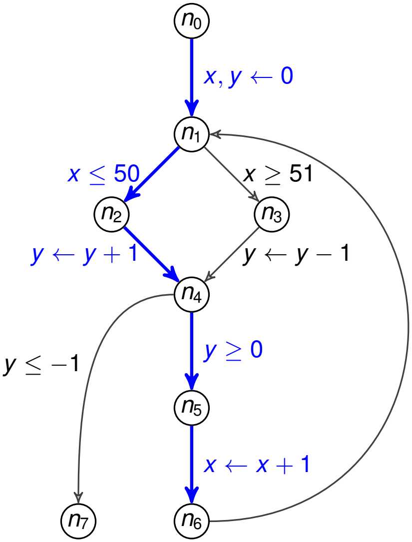

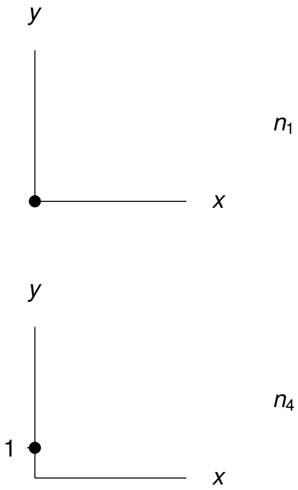

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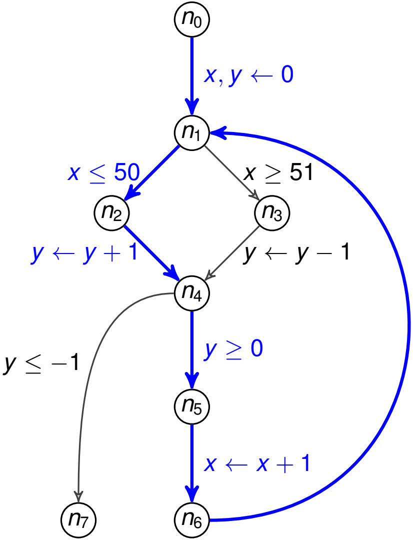

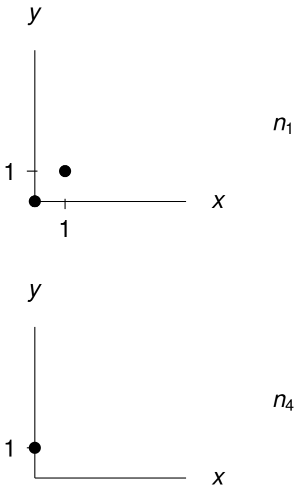

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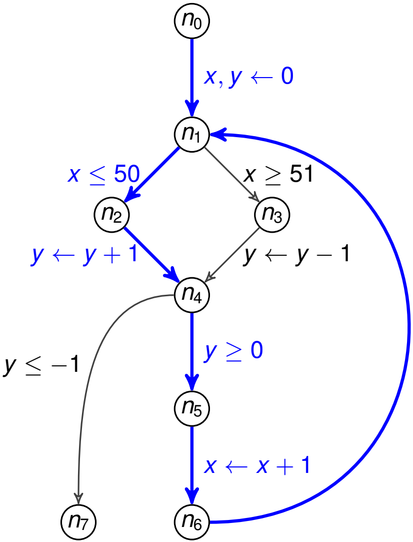

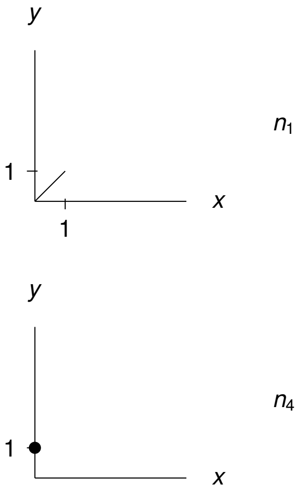

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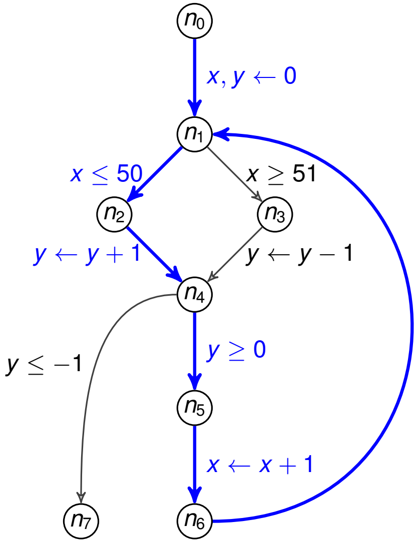

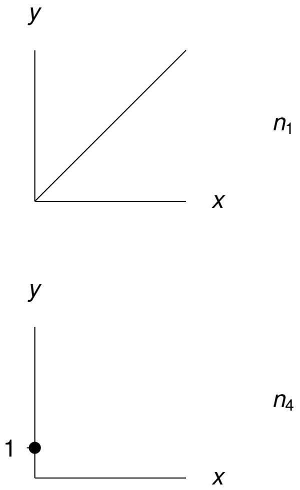

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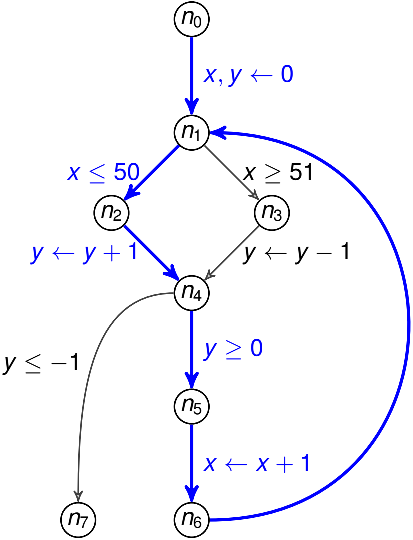

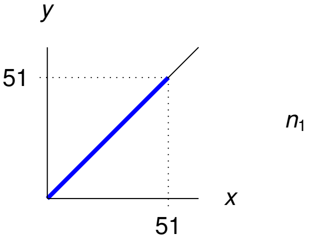

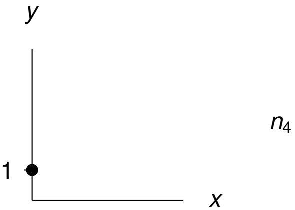

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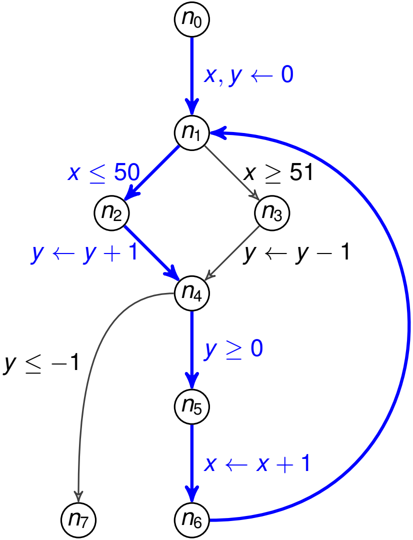

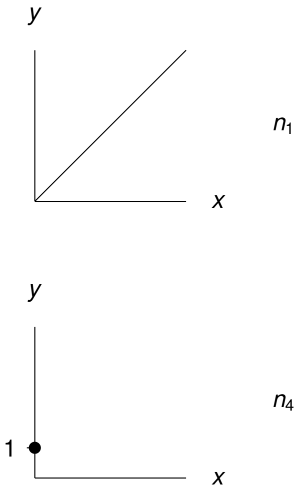

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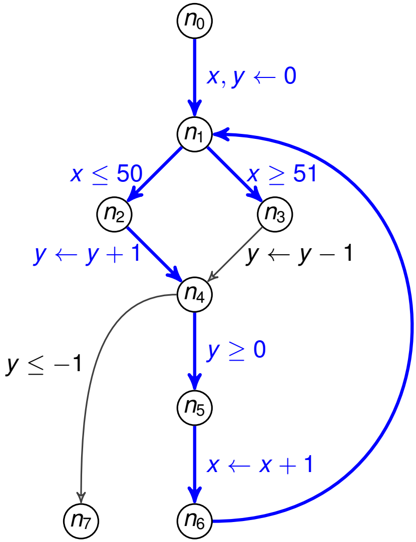

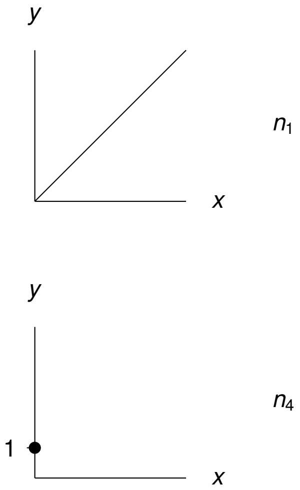

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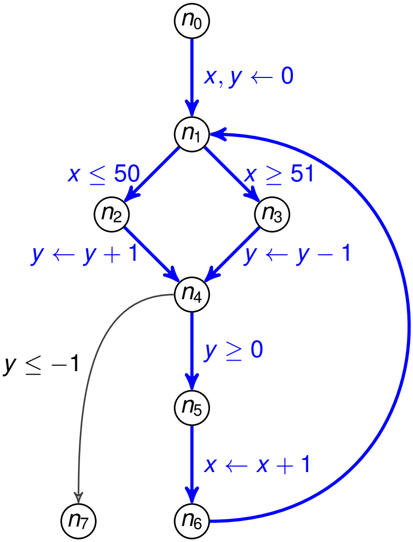

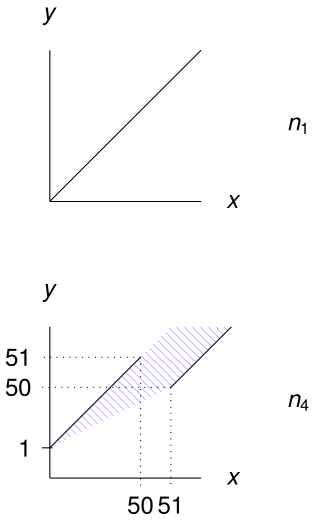

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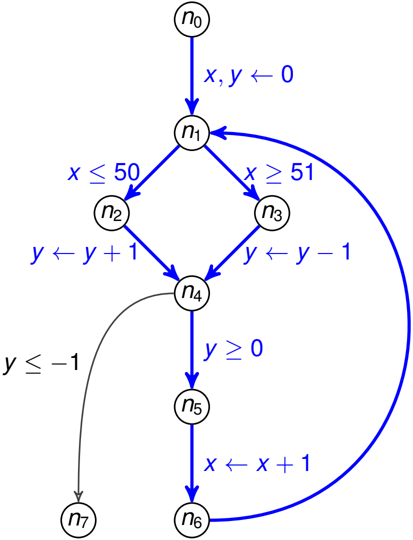

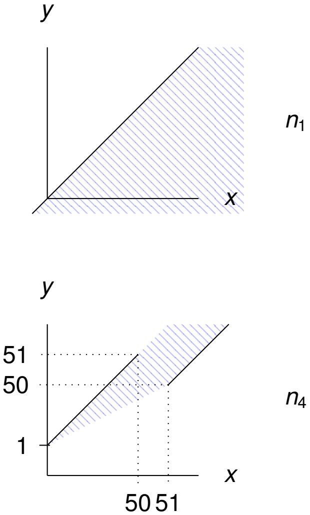

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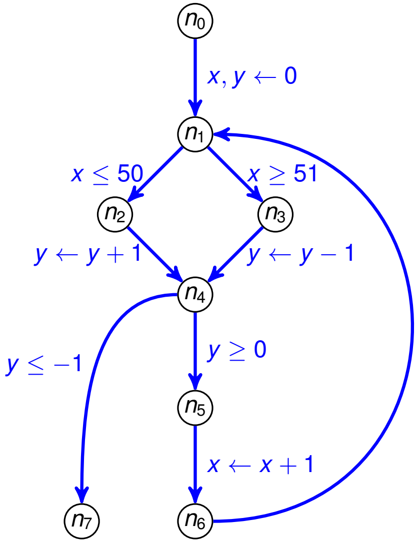
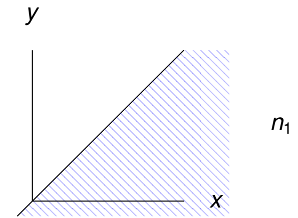
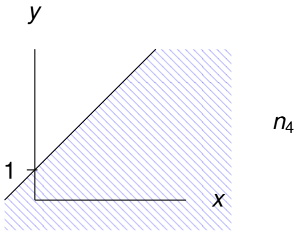
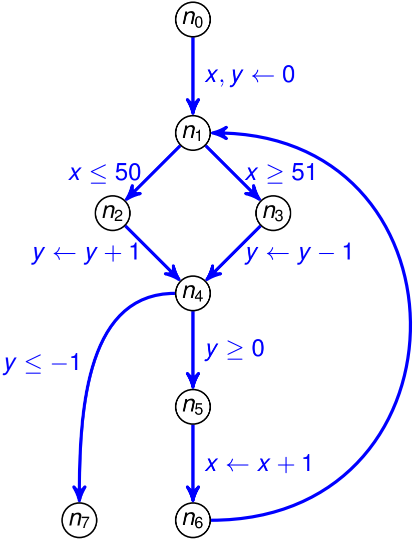


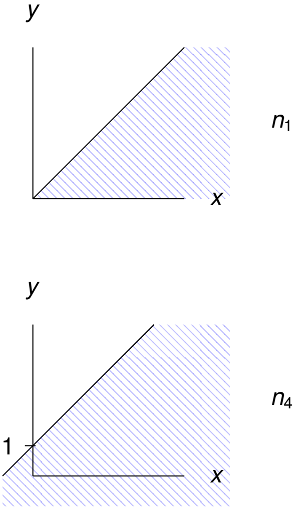




Ascending iterations

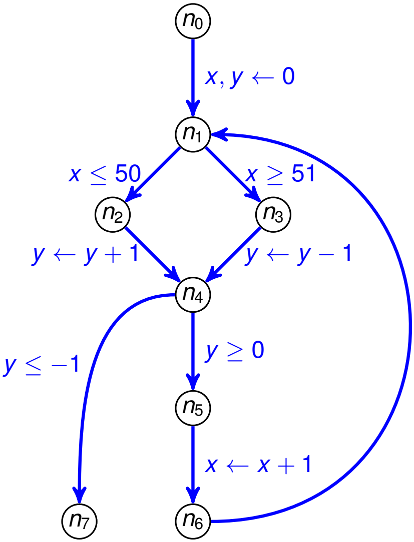
Example of Standard Abstract Interpretation

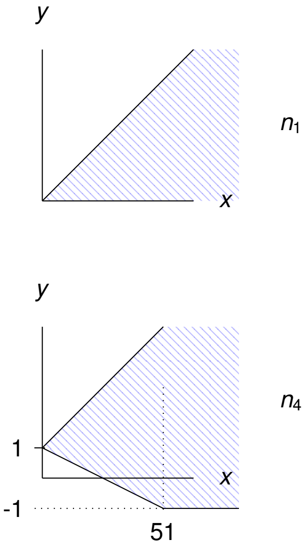




Descending iterations

Example of Standard Abstract Interpretation





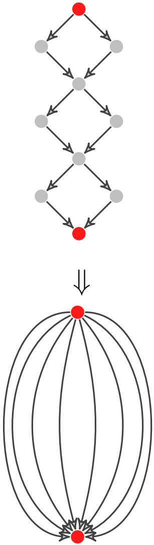
Descending iterations

Summary

Improving precision of Abstract Interpretation

Principle :

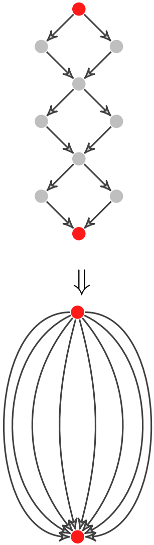
- Avoid as much as possible *Least Upper Bound* operations
- In practice, it consists in distinguishing every paths inside loops



Path Focusing

D. Monniaux & L. Gonnord - SAS 2011

- Take a set P_R of nodes (loop headers)
- Distinguish all the paths between 2 nodes of P_R
- Compute the fixpoint iterations on the resulting “expansed” graph



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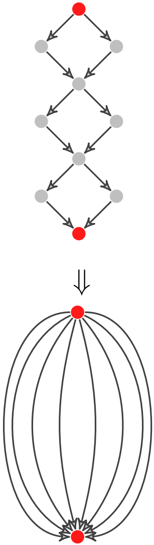
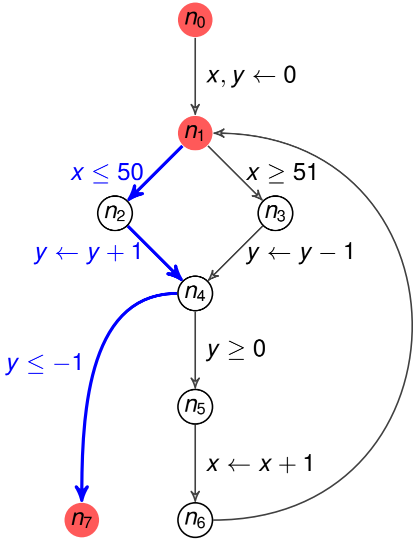
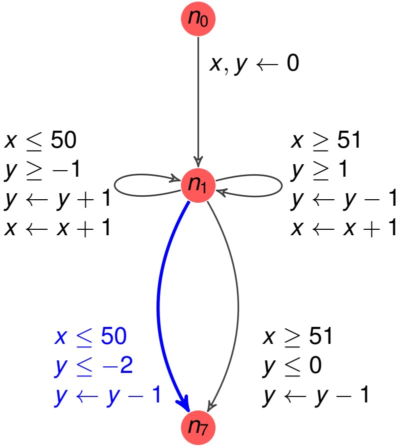


Exponential number of paths $\Rightarrow$

- We don't construct this graph explicitly
- We use SMT-solving to find interesting paths

Example : Expanded Graph





“Interesting” paths

- Abstract Interpretation : we update an invariant candidate X until it becomes an inductive invariant.
- The only “interesting” paths are those that make this invariant computation progress.

Finding Paths Using SMT-solving

We encode the semantics of the control flow graph into an SMT formula :

- 1 boolean predicate per control point and transition
- semantics of the transitions are coded w.r.t a certain theory (e.g Linear Integer Arithmetic, . . .)

We then find interesting paths using SMT queries :

"Does there exist a path starting in the invariant candidate X, that arrives in a state outside the candidate X ?"

PAGAI Static Analyser

PAGAI is a prototype of static analyser implementing state-of-the-art techniques, including our techniques using SMT.

- LLVM IR as input
- Apron Library for the abstract domains
- SMT-lib 2 interface, Microsoft Z3

In TAPAS'12:
PAGAI: a Path Sensitive Static Analyser; Henry, Monniaux, Moy

Experiments on GNU programs and WCET benchmarks

Example

For each loop header, Pagai returns an invariant:

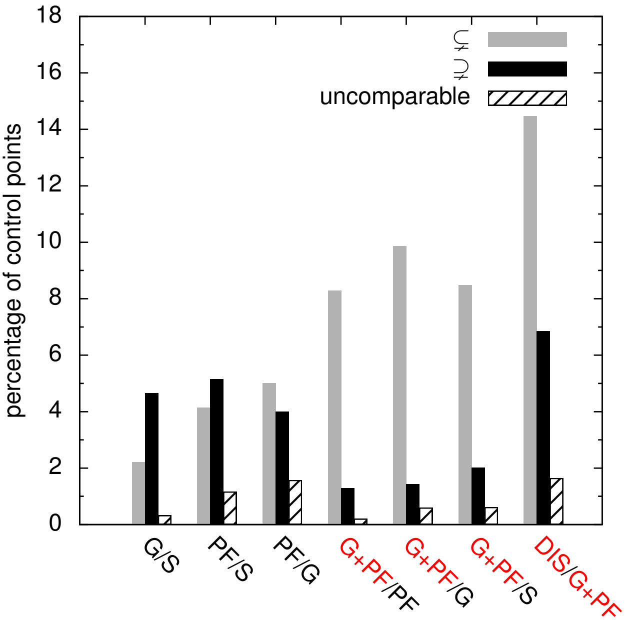
```
int main() {
        int x = 0;
        int y = 0;
        while (1) {
                /* invariant:
                102 + -1 * x + -1 * y >= 0
                y >= 0
                x + -1 * y >= 0
                */
                if (x <= 50) y++;
                else y--;
                if (y < 0) break;
                x++;
        }
}
```

Assert

```
int main() {
        int x = 0;
        int y = 0;
        while (1) {
                /* invariant:
                ...
                */
                if (x <= 50) y++;
                else y--;
                if (y < 0) break;
                x++;
        }
        /* assert OK */
        assert(x == 102);
}
```

Assume

```
void rate_limiter() {
  int x_old;
  int x;
  x_old = 0;
  while (1) {
    /* invariant:
      100000 + -1 * x >= 0
      100000 + x >= 0
    */
    x = input();
    assume (x >= -100000 && x <= 100000);
    if (x > x_old+10) x = x_old+10;
    if (x < x_old-10) x = x_old-10;
    x_old = x;
  }
/* UNREACHABLE */}
```


percentage of control points
18
16
14
12
10
8
6
4
2
0
⊊
⊋
uncomparable
G/S
PF/S
PF/G
G+PF/PF
G+PF/G
G+PF/S
DIS/G+PF

Time

	Size		Execution time (seconds)				
Name	kLOC	$\|P_R\|$	**S**	**G**	**PF**	**G+PF**	**DIS**
a2ps-4.14	55	2012	23	74	34	115	162
gawk-4.0.0	59	902	15	46	12	40	50
gnuchess-6.0.0	38	1222	50	220	81	312	351
gnugo-3.8	83	2801	77	159	92	766	1493
grep-2.9	35	820	41	85	22	65	122
gzip-1.4	27	494	22	268	91	303	230
lapack-3.3.1	954	16422	294	3740	3773	8159	10351
make-3.82	34	993	67	108	53	109	257
tar-1.26	73	1712	37	218	115	253	396

Table: Execution times