

ECON 6140 - Problem Set # 3

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Buckle up! This problem set as a lot of code and very few comments in it. I would have loved to make it neater, but I ran out of time and .

Problem #1

- (a) See `main.m`, `tauchen.m`, and `transition.m`. Moreover, see Tab b. Note that $E\{y_t\} = e^{\mu + \frac{\sigma^2}{2}}$ where μ and σ^2 are the expected moments of w_t . Hence, for $E\{y_t\} = 1$, we need

$$\bar{w} = -(1 - \rho) \frac{\sigma_\epsilon^2}{2(1 - \rho^2)}$$

- (b) See `main.m`, `tauchen.m`, `rouwenhorst.m`, and `transition.m`. The results are reported in Tab b. It seems that the Tauchen method does fairly well to approximate ρ and σ_ϵ . Note that increasing the number of states seems to have a ambiguous effectiveness. My guess is that with an even number of states, we are indirectly missing the point with the highest probability mass, i.e. μ . Hence, 9 states could potential do better than only 5 states or even 10 states.

	ρ	σ_ϵ
Model	.90	.24495
Tauchen (5 points)	.92665	.26813
Tauchen (10 points)	.89168	.27535

Table 1: Tauchen with 5 points and 10 points

- (c) See `main.m`, `tauchen.m`, `rouwenhorst.m`, and `transition.m`. The results are reported in Tab c. Clearly, the Rouwenhorst is yields the best approximation of ρ and σ_ϵ since it perfectly matches the parameters.

	ρ	σ_ϵ
Model	.90	.24495
Tauchen (5 points)	.92665	.26813
Rouwenhorst (5 points)	.90	.24495

Table 2: Tauchen with 5 points and Rouwenhorst 5 points

- (d) See `main.m`, `tauchen.m`, `rouwenhorst.m`, and `transition.m`. The results are reported in Tab d. Lo, and behold! Even with ρ close to 0, the Rouwenhorst perfectly matches ρ and σ_ϵ .

	ρ	σ_ϵ
Model	.98	.012505
Rouwenhorst (5 points)	.98	.012505

Table 3: Rouwenhorst 5 points and high ρ

Note that to match $var(w_t)$, we need to decrease σ_ϵ . In fact, we need

$$\sigma_\epsilon^2 = \frac{\sigma_{\epsilon,old}^2(1 - \rho^2)}{(1 - \rho_{old}^2)}$$

Problem #2

- (a) I modified Prof. Huckfedlt's code to get rid of the pesky globals and try to reduce everything to functions. See `main.m`, `policy_ip.m`, `euler_ip.m`, and `solve_ip.m`. Fig a and Fig a chose the decision rules for consumption and $a'(a, y)$ respectively.

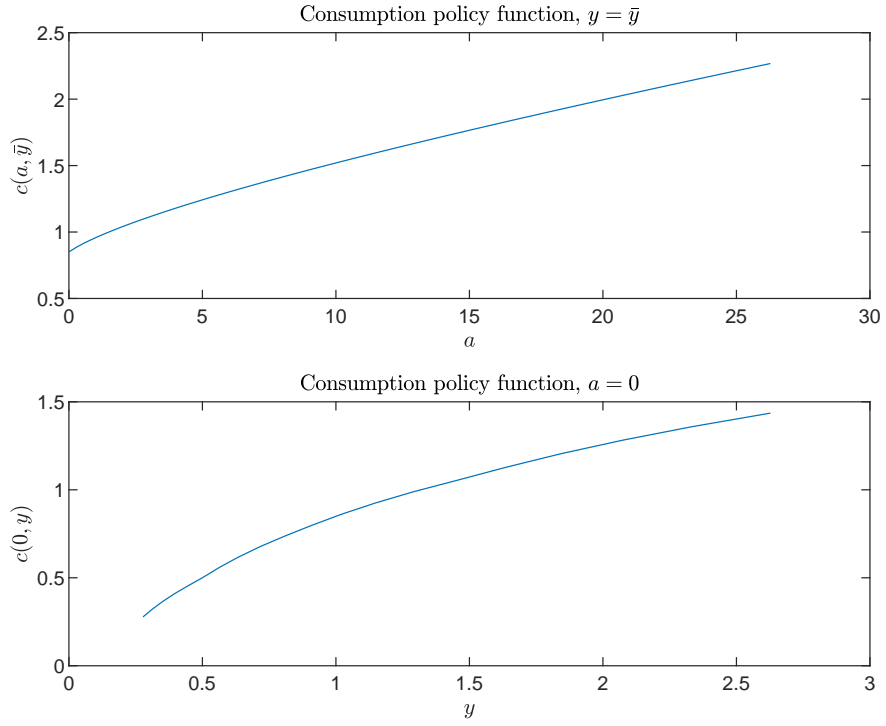


Figure 1: Consumption policy function

Note that for low y and $a = 0$, the borrowing constraints binds and we get the bent shape in our savings policy function.

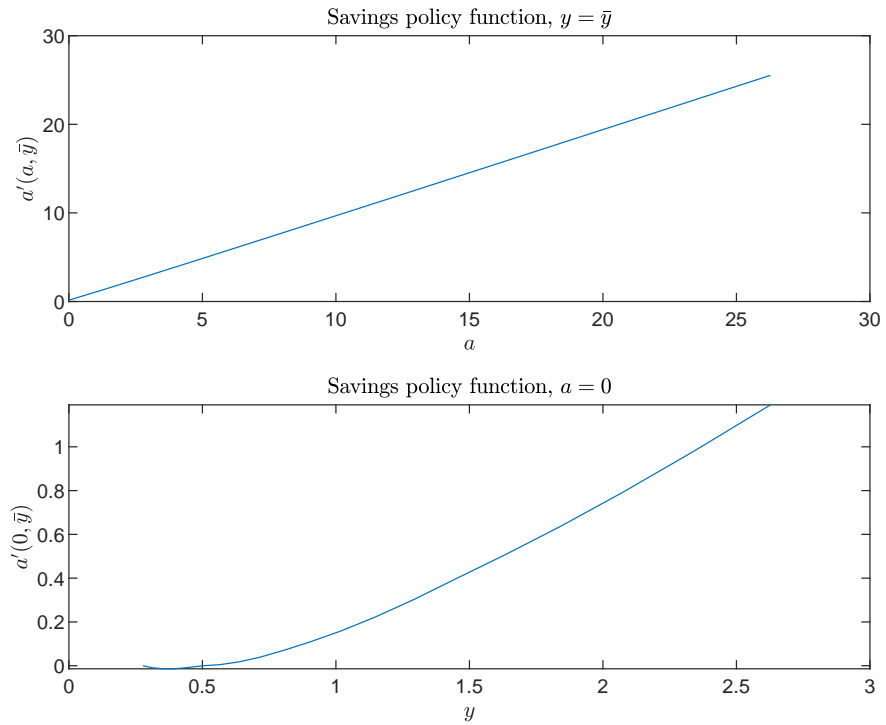


Figure 2: Savings policy function

- (b) See `markovchain.m`, and `markovprob.m` for the Markov chain simulation. The results for the unconditional standard deviation of c are reported in Tab b.

	σ_c
$\gamma = 1$.49118
$\gamma = 2$.44302
$\gamma = 5$.39500

Table 4: σ_c for different values of γ

- (c) Fig c plots the saving rates given $a = 0$ for different y .

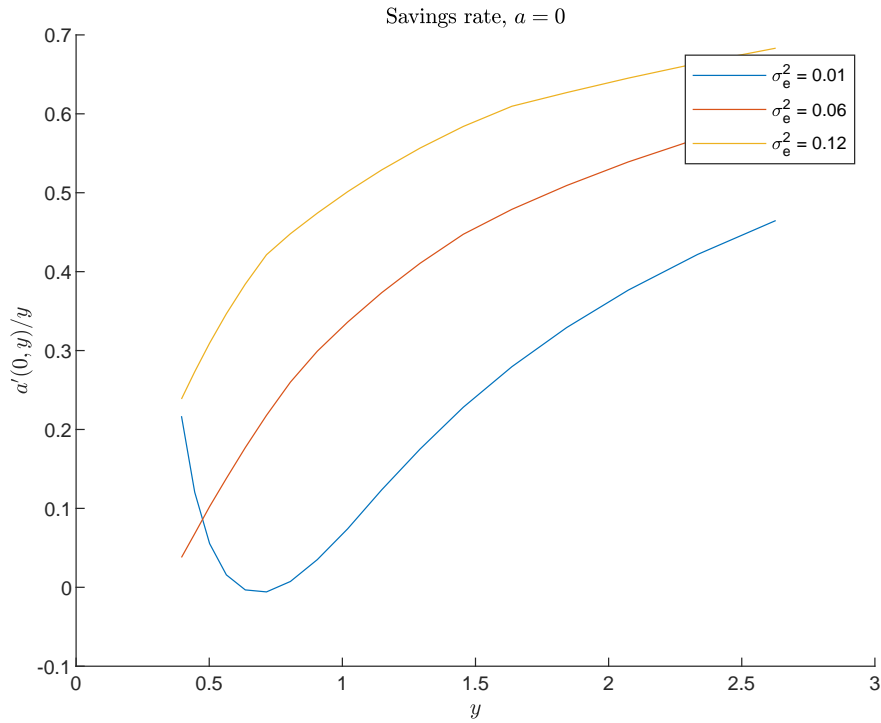


Figure 3: Savings rate

Note that as σ_ϵ increases, the savings rate also increases for all possible y . Hence, σ_ϵ increasing, implies higher volatility in income and therefore higher motive for precautionary savings. Please disregard the left part of the curve for $\sigma_\epsilon^2 = 0.01$. This is result is simply a consequence of how our grid was set up.

- (d) The results for the mean c with different borrowing constraints are reported in Tab d.

	\bar{c}
$a \geq 0$	1.0571
$a \geq -\frac{y_{min}}{r}$.92897

Table 5: \bar{c} for different borrowing constraints

It seems that having a looser borrowing constraint, implies lower average consumption. This is the results of having less motive for precautionary savings. Hence, by saving less, people get less consumption later on. While the value of utility might be higher due to the timing of consumption with the natural borrowing constraint, we get still get that on average consumption is lower than with no-borrowing.

- (e) The results for the insurance coefficient with different borrowing constraints are reported in Tab 6.

	ψ
$a \geq 0$.56445
$a \geq -\frac{y_{min}}{r}$.57244

Table 6: ψ for different borrowing constraints

Note that having a looser borrowing constraint, implies higher insurance coefficient. In fact, if ϵ_t and c_t are more correlated, then shocks to income will directly transfer to shocks to consumption. This in turns will decrease ψ , since the c_t are not protected against income volatility. Hence, natural borrowing constraint with its higher ψ implies that people are not a scared of shocks that could reduce their income since they are basically never restricted in their borrowing unlike $a \geq 0$.

Problem #3

- (a) Let $x_t = (1 + r_t)a_t + y_t$ or $x = Ra + y$. Then,

$$\begin{aligned}
c + a' &\leq Ra + y \\
c + \frac{x' - y'}{R} &\leq x \\
Rc + x' - y' &\leq Rx \\
x' &\leq R(x - c) + y'
\end{aligned}$$

and

$$\begin{aligned} a' &\geq 0 \\ \frac{x' - y'}{R} &\geq 0 \\ x' &\geq y' \\ x &\geq c \end{aligned}$$

Now, since our process is i.i.d., we have $\pi(y', y) = \pi(y')$. Hence, in this problem, y is not a state variable, but it does influence the expected value of x' since it is a function y' . Thus, our cash-on-hand problem can be summarized by

$$\begin{aligned} V(x) = \max_{c, x'} & \left\{ u(c) + \beta \sum_{y' \in Y} \pi(y') V(x') \right\} \\ \text{s. t. } & x' = R(x - c) + y' \\ & x \geq c \end{aligned}$$

Note that the Euler equation becomes

$$u'(c) \geq \beta R \mathbf{E}_t \{u'(c')\}$$

where $x' \leq R(x - c) + y'$ and it holds with equality when $x = c$.

- (b) Note that $\mathbf{E} \{y_t\} = e^{\mu + \frac{\sigma^2}{2}}$ where μ and σ^2 are the expected moments of w_t . Hence, for $\mathbf{E} \{y_t\} = 1$, we need

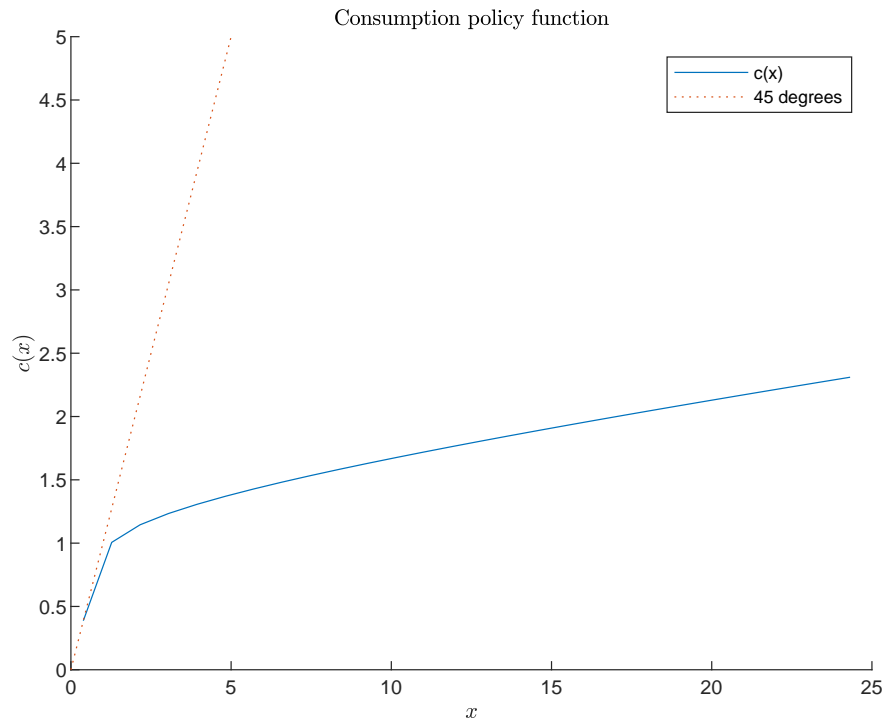
$$\bar{w} = -\frac{\sigma_\epsilon^2}{2}$$

Using `qnwnorm`, we get that this \bar{w} does imply $\mathbf{E} \{y_t\} = 1$. See `main.m` for the code.

- (c) See `main.m`, `policy_ca.m`, `euler_ca.m`, and `solve_ca.m` for the code. Note that I realized that there was a mistake in my code a bit too late to let the others know. I am truly sorry about that.

In this problem, we have only one state and that instead of using a transition matrix to compute the expected value, I decided to use the results from `qnwnorm` with 7 states from part (b) to compute the expectation in our Euler equation.

Fig c shows the policy function for $c(x)$.



Note that for low x , the borrowing constraint $x' \geq y$ is binding.

Code

main.m

```

1 clear; clc;
2 %% Problem 1
3 % declare parameters
4 param.ro=0.90;
5 param.se=sqrt(0.06);
6
7 % part a) – b)
8 param.k = 5;
9 [param, ro_ta5, se_ta5] = transition(param, "tauchen");
10 param.k = 10;
11 [param, ro_ta10, se_ta10] = transition(param, "tauchen");
12
13 tab_1b = table([param.ro; ro_ta5; ro_ta10],[param.se; se_ta5;
    se_ta10], 'VariableNames',{'rho','se'}, 'RowNames',{'Model','Five'; 'Ten'})
14
15 % part c)
16 param.k = 5;

```

```

17 [param, ro_rw, se_rw] = transition(param, "rouwenhorst");
18
19 tab_1c = table([param.ro; ro_ta5; ro_rw],[param.se; se_ta5; se_rw
    ], 'VariableNames',{ 'rho', 'se' }, 'RowNames',{ 'Model'; 'tauchen';
    'rouwenhorst' })
20
21 % part d)
22 old_var = param.se^2/(1-param.ro^2);
23 param.ro = 0.98;
24 param.se = old_var*(1-param.ro^2);
25 [param, ro_rw, se_rw] = transition(param, "rouwenhorst");
26
27 tab_1d = table([param.ro; ro_rw],[param.se; se_rw], 'VariableNames',{
    'rho', 'se' }, 'RowNames',{ 'Model'; 'rouwenhorst' })
28
29 %% Problem 2
30 % declare parameters
31 param.beta=0.95;
32 param.r=0.02;
33 param.ro=0.9;
34 param.se=sqrt(0.06);
35 param.gamma=2;
36 param.amin = 0;
37 param.n = 25;
38 param.k = 5;
39
40 % part a)
41 [param] = transition(param, "rouwenhorst");
42 [param,c,fspace,s,smin,smax] = policy_ip(param);
43
44 close all
45 sfine=gridmake(nodeunif(param.n*2,smin(1),smax(1)),param.ygrid);
46 xfine=funeval(c,fspace,sfine);
47
48 figure(1)
49 subplot(2,1,1)
50 sfine=gridmake(nodeunif(param.n*4,smin(1),smax(1)),0); %ygrid(
    floor(k/2)+2));
51 xfine=funeval(c,fspace,sfine);
52 plot(sfine(:,1),xfine)
53 xlabel({'$a$'}, 'Interpreter','latex')
54 ylabel({'$c(a,\bar{y})$'}, 'Interpreter','latex')
55 title({'Consumption policy function, $y=\bar{y}$'}, 'Interpreter','
    latex')
56 set(gca, 'FontSize',8);

```



```

57
58 subplot(2,1,2)
59 sfine=gridmake(0,nodeunif(param.k*4,smin(2),smax(2)));
60 xfine=funeval(c,fspace,sfine);
61 plot(exp(sfine(:,2)),xfine)
62 xlabel('$y$', 'Interpreter', 'latex')
63 ylabel('$c(0,y)$', 'Interpreter', 'latex')
64 title({'Consumption policy function, $a=0$'}, 'Interpreter', 'latex'
    )
65 set(gca, 'FontSize',8);
66 print -depsc fig1.eps
67
68 figure(2)
69 subplot(2,1,1)
70 sfine=gridmake(nodeunif(param.k*4,smin(1),smax(1)),0);
71 xfine=funeval(c,fspace,sfine);
72 plot(sfine(:,1),(1+param.r)*sfine(:,1)+exp(sfine(:,2))-xfine)
73 xlabel('$a$', 'Interpreter', 'latex')
74 ylabel('$a^{\backslash prime}(a,\backslash bar{y})$', 'Interpreter', 'latex')
75 title({'Savings policy function, $y=\backslash bar{y}$'}, 'Interpreter', '
    latex')
76 set(gca, 'FontSize',8);
77
78 subplot(2,1,2)
79 sfine=gridmake(0,nodeunif(param.k*4,smin(2),smax(2)));
80 xfine=funeval(c,fspace,sfine);
81 plot(exp(sfine(:,2)),(1+param.r)*sfine(:,1)+exp(sfine(:,2))-xfine)
82 xlabel('$y$', 'Interpreter', 'latex')
83 ylabel('$a^{\backslash prime}(0,\backslash bar{y})$', 'Interpreter', 'latex')
84 title({'Savings policy function, $a=0$'}, 'Interpreter', 'latex')
85 set(gca, 'FontSize',8);
86 print -depsc fig2.eps
87
88 % part b)
89 gamma = [1,2,5];
90 for i = 1:3
91     param.gamma = gamma(i);
92
93     [param] = transition(param, "rouwenhorst");
94     [param,c,fspace] = policy_ip(param);
95
96     con = markovchain(param,c,fspace, 10000); % Generate Markov
        chain
97     se_c(i) = std(con);
98 end

```

```

99
100 tab_2b = table(se_c , 'VariableNames' , { 'std_c' } , 'RowNames' , { 'Gamma
    = 1' ; 'Gamma = 2' ; 'Gamma = 5' })
101
102
103 % part c)
104 param.gamma = 2;
105 se = [0.01,0.06,0.12];
106
107 figure(3)
108 hold on
109 for i = 1:3
110     param.se = sqrt(se(i));
111
112     [param] = transition(param, "rouwenhorst");
113     [param,c,fspace] = policy_ip(param);
114
115     sfine=gridmake(0,nodeunif(param.k*4,smin(2),smax(2)));
116     xfine=funeval(c,fspace,sfine);
117     plot(exp(sfine(4:end,2)),1-xfine(4:end)./.exp(sfine(4:end,2)))
118 end
119 xlabel('y$', 'Interpreter', 'latex')
120 ylabel('$a^{\backslash\prime}(0,y)/y$', 'Interpreter', 'latex')
121 title({'Savings rate, $a=0$'}, 'Interpreter', 'latex')
122 legend('\sigma_e^2 = 0.01', '\sigma_e^2 = 0.06', '\sigma_e^2 = 0.12'
    )
123 set(gca, 'FontSize', 8);
124 hold off
125 print -depsc fig3.eps
126
127
128 % part d) – e)
129 % no-borrowing
130 param.gamma = 2;
131 param.se = sqrt(0.06);
132 param.amin = 0;
133
134 [param] = transition(param, "rouwenhorst");
135 [param,c,fspace] = policy_ip(param);
136
137 [con, s] = markovchain(param,c,fspace, 10000);
138 avg_c(1) = mean(con);
139
140 c_t = log(con(2:end))-log(con(1:end-1));
141 e_t = s(2:end,2)-param.ro*s(1:end-1,2);

```

```

142 sig = cov(c_t, e_t);
143 phi(1) = 1-sig(1,2)/sig(2,2);
144
145 % natural debt limit
146 param.amin = -min(exp(param.ygrid)+.01)/param.r;
147
148 [param] = transition(param, "rouwenhorst");
149 [param,c,fspace] = policy_ip(param);
150
151 [con, s] = markovchain(param,c,fspace, 10000);
152 avg_c(2) = mean(con);
153
154 c_t = log(con(2:end))-log(con(1:end-1));
155 e_t = s(2:end,2)-param.ro*s(1:end-1,2);
156 sig = cov(c_t, e_t);
157 phi(2) = 1-sig(1,2)/sig(2,2);
158
159 tab_2d = table(avg_c', phi', 'VariableNames', { 'mean_c', 'phi' }, '
      RowNames', { 'No borrowing'; 'Natural' })
160
161
162 %% Problem 3
163 % declare parameters
164 param.beta=0.95;
165 param.r=0.02;
166 param.ro=0;
167 param.se=sqrt(0.06);
168 param.gamma=2;
169 param.amin = 0;
170 param.n = 25;
171 param.k = 7;
172
173 % part b)
174 wbar = -param.se^2/2;
175 [x,w] = qnwnorm(7,wbar,param.se^2);
176 fprintf('E(y)=%f \n',exp(x)*w)
177
178 % part c)
179 % [param] = transition(param, "rouwenhorst");
180 param.ws = w';
181 param.ygrid = x;
182 [param,c,fspace,s,smin,smax] = policy_ca(param);
183
184 figure(5)
185 hold on

```

```

186 sfine=gridmake(nodeunif(param.k*4,smin(1),smax(1)));
187 xfine=funeval(c,fspace,sfine);
188 plot(sfine,xfine)
189 xlabel('$x$', 'Interpreter', 'latex')
190 ylabel('$c(x)$', 'Interpreter', 'latex')
191 title({'Consumption policy function'}, 'Interpreter', 'latex')
192 set(gca, 'FontSize', 8);
193 plot([0:5],[0:5], ':')
194 legend('c(x)', '45 degrees')
195 hold off
196 print -depsc fig4.eps

```

tauchen.m

```

1 function [state_grid, prob] = tauchen(rho, sig_eps, ns, dev)
2 %markovchain Discretize AR(1) process
3 % [prob, cum_prob, state_grid] = tauchen(ns, rho, sig_y, dev)
4 returns
5 % prob, transition matrix,
6 % cum_prob, cumulative distribution
7 % state_grid, value of states
8
9 sig_y = sig_eps/sqrt(1-rho^2); % Compute std of epsilon
10
11 state_grid = linspace(-dev*sig_y, dev*sig_y, ns)'; % Set state
12 grid
13 d = mean(diff(state_grid));
14 % Compute  $y^j_{t+1} - \rho y^i_t$ 
15 state_change = repmat(state_grid, 1, ns)' - rho*repmat(state_grid
16 , 1, ns);
17
18 prob = zeros(ns, ns); % Initialize transition matrix
19 % Compute transition matrix according to Tauchen (1986)
20 prob(:, 1) = cdf('norm', state_change(:, 1)+d/2, 0, sig_eps);
21 prob(:, end) = 1- cdf('norm', state_change(:, end) - d/2, 0, sig_eps);
22 prob(:, 2:end-1) = cdf('norm', state_change(:, 2:end-1)+d/2, 0, sig_eps)
23 -cdf('norm', state_change(:, 2:end-1)-d/2, 0, sig_eps);
24
25 state_grid = state_grid';
26 end

```

rouwenhorst.m

```

1 %rouwenhorst.m

```

```

2 %
3 %[zgrid , P] = rouwenhorst(rho , sigma_eps , n)
4 %
5 % rho is the 1st order autocorrelation
6 % sigma_eps is the standard deviation of the error term
7 % n is the number of points in the discrete approximation
8 %
9 function [zgrid , P] = rouwenhorst(rho , sigma_eps , n)
10
11 mu_eps = 0;
12
13 q = (rho+1)/2;
14 nu = ((n-1)/(1-rho^2))^(1/2) * sigma_eps;
15
16 P = [q 1-q; 1-q q];
17
18
19 for i=2:n-1
20     P = q*[P zeros(i,1); zeros(1,i+1)] + (1-q)*[zeros(i,1) P; zeros
        (1,i+1)] + ...
21     (1-q)*[zeros(1,i+1); P zeros(i,1)] + q*[zeros(1,i+1); zeros
        (i,1) P];
22     P(2:i,:) = P(2:i,:)/2;
23 end
24
25 zgrid = linspace(mu_eps/(1-rho)-nu, mu_eps/(1-rho)+nu, n);

```

transition.m

```

1 function [param, ro_tilde , se_tilde] = transition(param, method)
2 %UNTITLED2 Summary of this function goes here
3 % Detailed explanation goes here
4
5 var_w= param.se^2/(1-param.ro^2);
6 wbar = -var_w*(1-param.ro)/2;
7 % nodes for the distribution of income shocks
8
9 if method=="tauchen"
10     [e,w] = tauchen(param.ro , param.se , param.k , 3);
11 elseif method == "rouwenhorst"
12     [e,w] = rouwenhorst(param.ro , param.se , param.k); %
        Rouwenhorst method
13 end
14
15 param.yPP=w;

```

```

16 param.ygrid=e';
17
18 [~,D,V]=eig(w);
19 ws = V(:,1);
20 ws = ws/sum(ws);
21
22 param.ws = ws;
23 param.ygrid = param.ygrid + wbar/(1-param.ro);
24
25 ro_tilde = e*(w*e')/(e*e');
26 se_tilde = sqrt(e.^2*ws*(1-ro_tilde^2));
27 end

```

policy_ip.m

```

1 function [param,c,fspace,s,smin,smax] = policy_ip(param)
2 % Bounds for state space
3 ymin=min(param.ygrid);
4 ymax=max(param.ygrid);
5
6 amin = param.amin; % no borrowing
7 amax = 10*exp(ymax); % guess an upper bound on a,
   check later that do not exceed it
8
9 % Declare function space to approximate a'(a,y)
10 n=[param.n,param.k];
11
12 % Lower and higher bound for the state space (a,y)
13 smin=[amin,ymin];
14 smax=[amax,ymax];
15
16 scale=1/2;
17 fspace=fundef({ 'spli', nodeunif(n(1),(smin(1)-amin+.01).^scale,(
   smax(1)-amin+.01).^scale).(1/scale)+amin-.01,0,3},...
18   { 'spli', param.ygrid,0,1});
19
20 grid=funnode(fspace);
21 s=gridmake(grid); %collection of states (all a with y1... all a
   with y2... and so on)
22
23 c=funfitxy(fspace,s,param.r/(1+param.r)*s(:,1)+exp(s(:,2)));
   %guess that keep constant assets
24
25 tic
26 for it=1:101

```

```

27     cnew=c;
28     x = solve_ip(param,c, fspace , s , amin);
29     c=funfitxy( fspace , s , x);
30
31     fprintf( '%4i %6.2e\n' , [it , norm(c-cnew)] );
32     if norm(c-cnew)<1e-7, break , end
33 end
34 toc
35 end

```

solve_ip.m

```

1 function [x] = solve_ip(param,c, fspace , s , amin)
2 %UNTITLED3 Summary of this function goes here
3 % Detailed explanation goes here
4
5 ns=length(s);
6
7 a=.01*ones(ns,1);
8 b=(1+param.r)*s(:,1)+exp(s(:,2))-amin;
9 tol=1e-8; %tolerance level
10
11 fa=euler_ip(a,c, fspace , s , param);
12 fb=euler_ip(b,c, fspace , s , param);
13
14 x=zeros(ns,1);
15
16 % Start bisection
17 dx = 0.5*(b - a);
18
19 x = a + dx; % start at midpoint
20 sb=sign(fb);
21 dx = sb.*dx; % we increase or decrease x
    depending if f(b) is positive or negative
22
23 i=0;
24 while any(abs(dx)>tol)
25     i=i+1;
26     dx = 0.5*dx;
27     x = x - sign(euler_ip(x,c, fspace , s , param)).*dx;
28 end
29
30
31 x(fb>=0)=b(fb>=0);
32 end

```

euler_ip.m

```
1 function fval = euler_ip(x,c, fspace,s,param)
2
3 ns=size(s,1);
4 a=s(:,1);
5 y=exp(s(:,2));
6 aprime=(1+param.r)*a+y-x;
7 fval=x.^(-param.gamma);
8 as=ns/length(param.ygrid);
9
10 for i=1:length(param.ygrid)
11     cprime=funeval(c, fspace,[aprime,param.ygrid(i)*ones(ns,1)]);
12     for j=1:length(param.ygrid)
13         ypp(1+(j-1)*as:j*as,1)=param.yPP(j,i);
14     end
15     fval=fval-param.beta*(1+param.r)*ypp.*cprime.^(-param.gamma);
16 end
```

policy_ca.m

```
1 function [param,c, fspace,s,smin,smax] = policy_ca(param)
2 % Bounds for state space
3 ymin=min(param.ygrid);
4 ymax=max(param.ygrid);
5
6 xmin = exp(ymin); % no borrowing
7 xmax = 10*exp(ymax); % guess an upper bound on a,
   check later that do not exceed it
8
9 % Declare function space to approximate a'(a,y)
10 n=param.n;
11
12 % Lower and higher bound for the state space (a,y)
13 smin=xmin;
14 smax=xmax;
15
16 scale=1/2;
17 fspace=fundef({ 'spli', nodeunif(n(1),(.01).^scale,(xmax-xmin+.01)
   .^scale).^ (1/scale)+xmin-.01,0,3});
18
19 grid=funnode(fspace);
20 s=gridmake(grid); %collection of states (all a with y1... all a
   with y2... and so on)
21
```



```

22 c=funfitxy(fspace,s,s/2); %guess that keep constant
    assets
23
24 tic
25 for it=1:101
26     cnew=c;
27     x = solve_ca(param,c,fspace,s);
28     c=funfitxy(fspace,s,x);
29
30     fprintf('%4i %6.2e\n',[it,norm(c-cnew)]);
31     if norm(c-cnew)<1e-5, break, end
32 end
33 toc
34 end

```

solve_ca.m

```

1 function [x] = solve_ca(param,c,fspace,s)
2 %UNTITLED3 Summary of this function goes here
3 % Detailed explanation goes here
4
5 ns=length(s);
6
7 a=.01*ones(ns,1);
8 b=s;
9 tol=1e-8; %tolerance level
10
11 fa=euler_ca(a,c,fspace,s,param);
12 fb=euler_ca(b,c,fspace,s,param);
13
14 x=zeros(ns,1);
15
16 % Start bisection
17 dx = 0.5*(b - a);
18
19 x = a + dx; % start at midpoint
20 sb=sign(fb);
21 dx = sb.*dx; % we increase or decrease x
    depending if f(b) is positive or negative
22
23 i=0;
24 while any(abs(dx)>tol)
25     i=i+1;
26     dx = 0.5*dx;
27     x = x - sign(euler_ca(x,c,fspace,s,param)).*dx;

```

```

28     end
29
30
31 x(fb >= 0) = b(fb >= 0);
32 end

```

euler_ca.m

```

1 function fval = euler_ca(x, c, fspace, s, param)
2
3 fval = x.^(-param.gamma);
4 as = size(s, 1) / length(param.ygrid);
5
6 for i = 1:length(param.ygrid)
7     cprime = funeval(c, fspace, (1 + param.r) * (s - x) + exp(param.ygrid(i)));
8     ;
9     for j = 1:length(param.ygrid)
10        ypp(1 + (j - 1) * as : j * as, 1) = param.yPP(j, i);
11    end
12    ypp = param.ws(i);
13
14    fval = fval - param.beta * (1 + param.r) * ypp * cprime.^(-param.gamma);
15 end
16 end

```

markovchain.m

```

1 function [con, s] = markovchain(param, c, fspace, T)
2 %markovchain Generate Markov chain
3 % [chain] = markovchain(param, start)
4 s = ones(T, 2); % Initialize Markov Chain
5 con = ones(T, 1); % Initialize Markov Chain
6
7 s(1, :) = [0, param.ygrid(3)]; % Set starting value
8
9 cum_prob = cumsum(param.yPP, 2); % Compute cumulative distribution
10
11 % Generate Markov Chain using random numbers uniformly distributed
12 for t = 2:T
13     con(t - 1) = funeval(c, fspace, s(t - 1, :));
14     s(t, 1) = (1 + param.r) * s(t - 1, 1) + exp(s(t - 1, 2)) - con(t - 1);
15     s(t, 2) = param.ygrid(find(cum_prob(param.ygrid == s(t - 1, 2), :) >
16        rand(), 1));
17 end
18

```

18 end

markovprob.m

```
1 function [Gl, TM, CDF, sz] = markovprob(mue, p, s, N, m)
2 % markovprob - function
3 %% Arguments:
4 % mue = intercept of AR(1) process;
5 % p = slope coeff. of AR(1) process;
6 % s = std. dev. of residuals in AR(1) process;
7 % N = of grid points for the 'z' variable;
8 % m = Density of the grid for 'z' variable;
9 %% CODE:
10 sz = s / ((1-p^2)^(1/2)); % Std. Dev. of z.
11 zmin = -m * sz + mue/(1-p);
12 zmax = m * sz + mue/(1-p);
13 z = linspace(zmin, zmax, N); % Grid Points
14 %% Transition Matrix:
15 TM = zeros(N, N); % Transition Matrix
16 w = z(N) - z(N-1);
17 for j = 1:N;
18 TM(j, 1) = cdf('norm', (z(1)+w/2-mue-p*z(j))/s, 0, 1);
19 TM(j, N) = 1 - cdf('norm', (z(N)-w/2-mue-p*z(j))/s, 0, 1);
20 for k = 2:N-1;
21 TM(j, k) = cdf('norm', (z(k)+w/2-mue-p*z(j))/s, 0, 1) - cdf('norm', (z(k)
    -w/2-mue-p*z(j))/s, 0, 1);
22 end
23 end
24 %% Cumulative Distribution Function:
25 CDF = cumsum(TM, 2);
26 %% Invariant Distribution:
27 %% Grids:
28 Gl = exp(z');
29 fprintf('If we have a lognormal var. (log(z)) in AR(1) process, \n
    ');
30 fprintf('To make the interval finer at the lower end and coarser
    at the upper end.\n');
```