ECON 6140 - Problem Set # 2

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Government expenditure, corruption and output

(1)

$$\begin{aligned}
\max_{c_t,l_t,n_t,k_{t+1},c_t^g,x_t} &\sum_{t=0}^{\infty}\beta^t\left\{u(c_t,l_t)+v(c_t^g)\right\}\\
\text{s.t. } c_t+x_t+g_t &\leq zf(n_t,k_t)\\
k_{t+1} &\leq (1-\delta_k)k_t+x_{kt}\\
n_t+l_t &\leq 1\\
c_t^g &\leq \theta g_t\\
c_t,l_t,n_t,k_{t+1},c_t^g,x_t &\geq 0
\end{aligned}$$

(2)

$$\begin{aligned}
c_t &: u_c(c_t,l_t)-\lambda_t=0\\
c_t^g &: v_c(c_t^g)-\eta_t=0\\
n_t &: \lambda_t zf_n(n_t,k_t)-\psi_t=0\\
l_t &: u_l(c_t,l_t)-\psi_t=0\\
x_t &: \mu_t-\lambda_t=0\\
k_{t+1} &: \beta\lambda_{t+1}zf_n(n_{t+1},k_{t+1})+\beta\mu_{t+1}(1-\delta_k)-\mu_t=0\\
&: c_t+x_t+g_t\leq zf(n_t,k_t)\\
&: k_{t+1}\leq(1-\delta_k)k_t+x_{kt}\\
&: n_t+l_t\leq 1\\
&: c_t^g\leq\theta g_t\\
TVC &: \lim_{T\to\infty}\beta^T u_c(c_T,l_T)k_{T+1}=0\\
&+\ \text{complementary slackness conditions}
\end{aligned}$$

$$
\begin{aligned}
u_c(c_t, 1-n_t) &= \lambda_t \\
v_c(\theta g_t) &= \eta_t \\
\lambda_t z f_n(n_t, k_t) &= \psi_t \\
u_l(c_t, 1-n_t) &= \psi_t \\
\mu_t &= \lambda_t \\
\beta \lambda_{t+1} z f_n(n_{t+1}, k_{t+1}) + \beta \mu_{t+1}(1-\delta_k) &= \mu_t \\
c_t + k_{t+1} - (1-\delta_k)k_t + g_t &= z f(n_t, k_t) \\
\lim_{T\to\infty} \beta^T u_c(c_T, 1-n_T) k_{T+1} &= 0
\end{aligned}
$$

(3)

$$
\begin{aligned}
u_c(c, 1-n) &= \lambda \\
v_c(\theta g) &= \eta \\
\lambda z f_n(n,k) &= u_l(c, 1-n) \\
\beta[z f_n(n,k) + (1-\delta_k)] &= 1 \\
c + \delta_k k + g &= z f(n,k)
\end{aligned}
$$

$$
\begin{aligned}
\lim_{k\to 0} z f_n(n_t, k_t) &> \frac{1}{\beta} - (1-\delta_k) \\
\lim_{k\to \infty} z f_n(n_t, k_t) &< \frac{1}{\beta} - (1-\delta_k)
\end{aligned}
$$

$$
z f(n,k) > \delta_k k
$$

$$
z f_k(n,k) > \delta_k
$$

(4) (a)

$$
\beta \left[z f_n \left(1, \frac{k}{n}\right) + (1-\delta_k) \right] = 1
$$

(b)

(c)

(5) (a)

(b)

(c)

Skill-biased technical change

(1)

$$f_k(z,k_t,n_s,n_u) = \mu\lambda z(\lambda(\mu(k_t)^\rho + (1-\mu)(n_s)^\rho)^{\frac{\sigma}{\rho}} + (1-\lambda)n_u^\sigma)^{\frac{1}{\sigma}-1}(\mu(k_t)^\rho + (1-\mu)(n_s)^\rho)^{\frac{\sigma}{\rho}-1}k^{\rho-1}$$
$$f_{n_s}(z,k_t,n_s,n_u) = (1-\mu)\lambda z(\lambda(\mu(k_t)^\rho + (1-\mu)(n_s)^\rho)^{\frac{\sigma}{\rho}} + (1-\lambda)n_u^\sigma)^{\frac{1}{\sigma}-1}(\mu(k_t)^\rho + (1-\mu)(n_s)^\rho)^{\frac{\sigma}{\rho}-1}n_s^{\rho-1}$$

$$\frac{f_k}{f_{n_s}} = \frac{\mu}{1-\mu}\left(\frac{k}{n_s}\right)^{\rho-1}$$

$$\ln\left(\frac{f_k}{f_{n_s}}\right) = \ln\left(\frac{\mu}{1-\mu}\right) + (1-\rho)\ln\left(\frac{n_s}{k}\right)$$
$$\ln\left(\frac{n_s}{k}\right) = \frac{1}{1-\rho}\ln\left(\frac{f_k}{f_{n_s}}\right) - \frac{1}{1-\rho}\ln\left(\frac{\mu}{1-\mu}\right)$$

$$\epsilon = \frac{\partial \ln\left(\frac{n_s}{k}\right)}{\partial \ln\left(\frac{f_k}{f_{n_s}}\right)} = \frac{1}{1-\rho}$$

(2)

(3)

(4) (a)

(b)

(c)

(5) (a)

(b)

(c)

(6)