

ECON 7510 - BLP Replication

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Part 1

From the problem set description, we start by assuming that each consumer i has utility for product j as a function of price and product quality, δ_j of:

$$u_{ij} = \delta_j - \alpha_i p_j$$

The distribution of consumer price sensitivity, α_i , is exponential with parameter $\lambda = 4 \cdot 10^{-6}$. We further assume that $\delta_j = x_j \beta + \xi_j$ and that x_j is uncorrelated with x_i .

In this model, i choose the j that provides the highest utility. This implies that if $p_j > p_k$, we need $\delta_j > \delta_k$ otherwise no one would ever pick j . In the same vein, it is straightforward to see that the share of good j will depend only on its close neighbors in the price dimension. Hence, before deriving the result, we sort the data from lowest to highest price.

As mentionned above, to determine j share we only need to compare its u_{ij} to $j - 1$ and $j + 1$. In fact, i will choose j if and only if

$$u_{i,j} > u_{i,j-1} \text{ and } u_{i,j} > u_{i,j+1}$$

This boils down to the following condition for α_i

$$\frac{\delta_{j+1} - \delta_j}{p_{j+1} - p_j} < \alpha_i < \frac{\delta_j - \delta_{j-1}}{p_j - p_{j-1}}$$

and therefore the share of good j is given by

$$\begin{aligned} s_j &= \int_{\frac{\delta_{j+1} - \delta_j}{p_{j+1} - p_j}}^{\frac{\delta_j - \delta_{j-1}}{p_j - p_{j-1}}} f(x) dx \\ &= F\left(\frac{\delta_j - \delta_{j-1}}{p_j - p_{j-1}}\right) - F\left(\frac{\delta_{j+1} - \delta_j}{p_{j+1} - p_j}\right) \end{aligned}$$

where $F(x) = 1 - e^{-\lambda x}$.

Since we know s_j and p_j , we can recursively solve for δ_j , by noting that the following relationship holds

$$\ln\left(\sum_{k=0}^j s_k\right) = -\lambda\left(\frac{\delta_{j+1} - \delta_j}{p_{j+1} - p_j}\right)$$

Using our δ_j , we can compute our estimate for β with simple OLS since we assume that $\mathbb{E}\{x_j\xi_j\} = 0$. The results are stated in Table 1.

Table 1: Vertical model - Demand side only	
Demand side	
(Intercept)	7.5865×10^7 [3.9391×10^7 1.1234×10^8]
Weight	4.1722×10^4 [2.5642×10^4 5.7802×10^4]
Horsepower	3.1989×10^5 [1.2081×10^5 5.1897×10^5]
AC	1.1753×10^7 [-4.3171×10^6 2.7823×10^7]
FE	Yes
N	131
R ²	0.8175
[] 95% confidence interval	

Note that while the regression in Table 1 includes fixed effects for the firm, I simply don't report their values for the sake of conciseness.

Part 2

Note that if two goods, for example j and k , have the same price and strictly positive shares in this model we need that $\delta_k = \delta_j$ and hence $u_{ik} = u_{ij}$ for all consumer. If not, one good would have zero market share. This is a fairly extreme assumption.

Looking at Table 2, we can see that the model insinuates rather odd own and cross price elasticities.

Table 2: Vertical model - Price elasticities										
Car\Car	15	16	17	18	19	20	21	22	23	24
15	-0.0948	0.0003	0	0	0	0	0	0	0	0
16	0.0003	-0.0003	0.0000	0	0	0	0	0	0	0
17	0	0.0001	-0.0671	0.0673	0	0	0	0	0	0
18	0	0	0.0050	-0.0154	0.0104	0	0	0	0	0
19	0	0	0	0.0711	-0.5703	0.4997	0	0	0	0
20	0	0	0	0	0.1349	-0.1477	0.0126	0	0	0
21	0	0	0	0	0	0.0235	-0.0438	0.0202	0	0
22	0	0	0	0	0	0	1.2703	-1.8608	0.5881	0
23	0	0	0	0	0	0	0	0.0175	-0.0180	0.0004
24	0	0	0	0	0	0	0	0	0.0001	-0.0390

This weird pattern stems from the fact that cross price elasticities for any given good is going to be equal to zero apart from its direct neighbors. It is straightforward to show this fact by noting that $s_j = F\left(\frac{\delta_j - \delta_{j-1}}{p_j - p_{j-1}}\right) - F\left(\frac{\delta_{j+1} - \delta_j}{p_{j+1} - p_j}\right)$ is not a function of any price but p_j , p_{j-1} , and p_{j+1} .

Part 3

We now turn our attention to the supply side of our model. We are given the following equation for marginal cost $mc_j = x_j\gamma + \eta q_j + \omega_j$. Sadly we don't have the actual value for the marginal cost. We can circumvent this issue by relating marginal cost to price by assuming some pricing strategy for the firms. As a matter of fact, these pricing strategy will result in a markup over marginal cost given by the following formulas:

- Marginal cost pricing:

$$p_j = mc_j$$

- Single product firms:

$$p_j = mc_j + \Delta^{-1}s$$

where $\Delta_{jk} = \begin{cases} \frac{\partial s_j}{\partial p_k} & \text{if } j = k \\ 0 & \text{otherwise} \end{cases}$ and s is the vector of shares.

- Multiproduct firms:

$$p_j = mc_j + \Delta^{-1}s$$

where $\Delta_{jk} = \begin{cases} \frac{\partial s_j}{\partial p_k} & \text{if } k \in \text{firm}(j) \\ 0 & \text{otherwise} \end{cases}$ and s is the vector of shares.

- Perfect collusion:

$$p_j = mc_j + \Delta^{-1}s$$

where $\Delta_{jk} = \frac{\partial s_j}{\partial p_k}$ and s is the vector of shares.

From the previous question, it is easy to compute Δ and therefore mc_j .

With this in mind we can revert back to $mc_j = x_j\gamma + \eta q_j + \omega_j$ and estimate it like we would for $\delta_j = x_j\beta + \xi_j$. The only issue we are facing now is the potential endogeneity of q_j .

To remedy the situation, we need to find an instrument for q_j . There is plenty potential instruments, but after playing around for a while, I decide to follow Gandhi and Houde (2018) and use $\sum_k |x_j - x_k|$ and $\sum_k (x_j - x_k)^2$. To avoid any problem with collinearity and matrix inversion, I use the function `licols()` to reduce our set of instruments $Z = [x_j \sum_k (x_j - x_k)^2]$ to a full column rank matrix.

The result from our IV regression, using the different pricing strategy, are given in Table 3.

Table 3: Vertical model - supply side

	Marginal cost	Single product firms	Multiproduct firms	Collusion
(Intercept)	7.5865×10^7	7.5865×10^7	7.5865×10^7	7.5865×10^7
Weight	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$
Horsepower	4.1722×10^4	4.1722×10^4	4.1722×10^4	4.1722×10^4
	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$
	3.1989×10^5	3.1989×10^5	3.1989×10^5	3.1989×10^5
	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$
AC	1.1753×10^7	1.1753×10^7	1.1753×10^7	1.1753×10^7
	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$
FE	Yes	Yes	Yes	Yes
N	131	131	131	131
R ²	0.8175	0.8175	0.8175	0.8175
(Intercept)	7.5865×10^7	7.5865×10^7	7.5865×10^7	7.5865×10^7
Weight	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$
	4.1722×10^4	4.1722×10^4	4.1722×10^4	4.1722×10^4
	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$
Horsepower	3.1989×10^5	3.1989×10^5	3.1989×10^5	3.1989×10^5
	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$
AC	1.1753×10^7	1.1753×10^7	1.1753×10^7	1.1753×10^7
	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$
Quantity	7.5865×10^7	7.5865×10^7	7.5865×10^7	7.5865×10^7
	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$
FE	Yes	Yes	Yes	Yes
N	131	131	131	131
R ²	0.8175			

[] 95% confidence interval

Part 4

Part 5

Table 4: Vertical model - supply side

	Marginal cost	Multiple product firms
(Intercept)	7.5865×10^7	7.5865×10^7
	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$
Weight	4.1722×10^4	4.1722×10^4
	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$
Horsepower	3.1989×10^5	3.1989×10^5
	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$
AC	1.1753×10^7	1.1753×10^7
	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$
Price (α)	1.1753×10^7	1.1753×10^7
	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$
FE	Yes	Yes
N	131	131
R ²	0.8175	
(Intercept)		7.5865×10^7
		$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$
Weight		4.1722×10^4
		$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$
Horsepower		3.1989×10^5
		$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$
AC		1.1753×10^7
		$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$
Quantity		7.5865×10^7
		$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$
FE		Yes
N	131	131
R ²	0.8175	

[] 95% confidence interval

Table 5: Vertical model - Price elasticities

Car\Car	15	16	17	18	19	20	21	22	23	24
15	-0.0948	0.0003	0	0	0	0	0	0	0	0
16	0.0003	-0.0003	0.0000	0	0	0	0	0	0	0
17	0	0.0001	-0.0671	0.0673	0	0	0	0	0	0
18	0	0	0.0050	-0.0154	0.0104	0	0	0	0	0
19	0	0	0	0.0711	-0.5703	0.4997	0	0	0	0
20	0	0	0	0	0.1349	-0.1477	0.0126	0	0	0
21	0	0	0	0	0	0.0235	-0.0438	0.0202	0	0
22	0	0	0	0	0	0	1.2703	-1.8608	0.5881	0
23	0	0	0	0	0	0	0	0.0175	-0.0180	0.0004
24	0	0	0	0	0	0	0	0	0.0001	-0.0390

Part 6

Part 7

Part 8

Table 6: Vertical model - Demand side only

Demand side	
(Intercept)	7.5865×10^7
Weight	4.1722×10^4
Horsepower	3.1989×10^5
AC	1.1753×10^7
Price (α_1)	1.1753×10^7
Price (α_2)	1.1753×10^7
FE	Yes
N	131
R ²	0.8175

[] 95% confidence interval

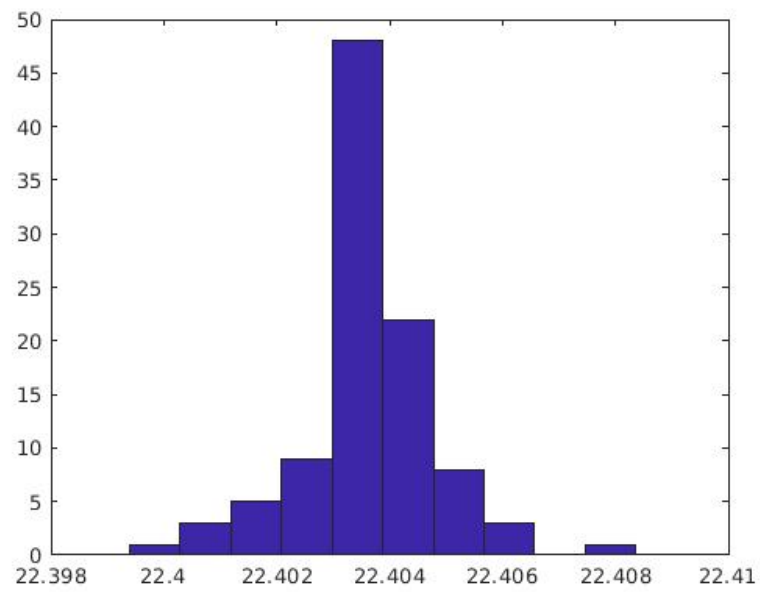


Figure 1:

Table 7: Vertical model - Price elasticities

Car\Car	15	16	17	18	19	20	21	22	23	24
15	-0.0948	0.0003	0	0	0	0	0	0	0	0
16	0.0003	-0.0003	0.0000	0	0	0	0	0	0	0
17	0	0.0001	-0.0671	0.0673	0	0	0	0	0	0
18	0	0	0.0050	-0.0154	0.0104	0	0	0	0	0
19	0	0	0	0.0711	-0.5703	0.4997	0	0	0	0
20	0	0	0	0	0.1349	-0.1477	0.0126	0	0	0
21	0	0	0	0	0	0.0235	-0.0438	0.0202	0	0
22	0	0	0	0	0	0	1.2703	-1.8608	0.5881	0
23	0	0	0	0	0	0	0	0.0175	-0.0180	0.0004
24	0	0	0	0	0	0	0	0	0.0001	-0.0390

Part 9

Code

Problem 2