ECON 7510 - BLP Replication

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Part 1

From the problem set description, we start by assuming that each consumer i has utility for product j as a function of price and product quality, δ_j of:

$$u_{ij} = \delta_j - \alpha_i p_j$$

The distribution of consumer price sensitivity, α_i, is exponential with parameter $\lambda = 4 \cdot 10^{-6}$. We further assume that $\delta_j = x_j\beta + \xi_j$ and that x_j is uncorrelated with ξ_j.

In this model, i choose the j that provides the highest utility. This implies that if $p_j > p_k$, we need $\delta_j > \delta_k$ otherwise no one would ever pick j. In the same vein, it is straightforward to see that the share of good j will depend only on its close neighbors in the price dimension. Hence, before deriving the result, we sort the data from lowest to highest price.

As mentionned above, to determine j share we only need to compare its u_{ij} to $j-1$ and $j+1$. In fact, i will choose j if and only if

$$u_{i,j} > u_{i,j-1} \text{ and } u_{i,j} > u_{i,j+1}$$

This boils down to the following condition for α_i

$$\frac{\delta_{j+1} - \delta_j}{p_{j+1} - p_j} < \alpha_i < \frac{\delta_j - \delta_{j-1}}{p_j - p_{j-1}}$$

and therefore the share of good j is given by

$$\begin{aligned} s_j &= \int_{\frac{\delta_{j+1}-\delta_j}{p_{j+1}-p_j}}^{\frac{\delta_j-\delta_{j-1}}{p_j-p_{j-1}}} f(x)dx \\ &= F\left(\frac{\delta_j - \delta_{j-1}}{p_j - p_{j-1}}\right) - F\left(\frac{\delta_{j+1} - \delta_j}{p_{j+1} - p_j}\right) \end{aligned}$$

where $F(x) = 1 - e^{-\lambda x}$.

Since we know s_j and p_j, we can recursively solve for δ_j, by noting that the following relationship holds

$$\ln\left(\sum_{k=0}^{j} s_k\right) = -\lambda\left(\frac{\delta_{j+1} - \delta_j}{p_{j+1} - p_j}\right)$$

Using our δ_j, we can compute our estimate for β with simple OLS since we assume that $\mathbb{E}\{x_j \xi_j\} = 0$. The results are stated in Table 1.

Table 1: Vertical model - Demand side only

	Demand side
(Intercept)	7.5865×10^7
	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$
Weight	4.1722×10^4
	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$
Horsepower	3.1989×10^5
	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$
AC	1.1753×10^7
	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$
FE	Yes
N	131
R^2	0.8175

[ ] 95% confidence interval

Note that while the regression in Table 1 includes fixed effects for the firm, I simply don't report their values for the sake of conciseness.

Part 2

Note that if two goods, for example j and k, have the same price and strictly positive shares in this model we need that $\delta_k = \delta_j$ and hence $u_{ik} = u_{ij}$ for all consumer. If not, one good would have zero market share. This is a fairly extreme assumption.

Looking at Table 2, we can see that the model insinuates rather odd own and cross price elasticities.

Table 2: Vertical model - Price elasticities

Car\Car	15	16	17	18	19	20	21	22	23	24
15	-0.0948	0.0003	0	0	0	0	0	0	0	0
16	0.0003	-0.0003	0.0000	0	0	0	0	0	0	0
17	0	0.0001	-0.0671	0.0673	0	0	0	0	0	0
18	0	0	0.0050	-0.0154	0.0104	0	0	0	0	0
19	0	0	0	0.0711	-0.5703	0.4997	0	0	0	0
20	0	0	0	0	0.1349	-0.1477	0.0126	0	0	0
21	0	0	0	0	0	0.0235	-0.0438	0.0202	0	0
22	0	0	0	0	0	0	1.2703	-1.8608	0.5881	0
23	0	0	0	0	0	0	0	0.0175	-0.0180	0.0004
24	0	0	0	0	0	0	0	0	0.0001	-0.0390

This weird pattern stems from the fact that cross price elasticities for any given good is going to be equal to zero apart from its direct neighbors. It is straightforward to show this fact by noting that $s_j = F\left(\frac{\delta_j - \delta_{j-1}}{p_j - p_{j-1}}\right) - F\left(\frac{\delta_{j+1} - \delta_j}{p_{j+1} - p_j}\right)$ is not a function of any price but p_j, p_{j-1}, and p_{j+1}.

Part 3

We know turn our attention to the supply side of our model. We are given the following equation for marginal cost $mc_j = x_j\gamma + \eta q_j + \omega_j$. Sadly we don't have the actual value for the marginal cost. We can circumvent this issue by relating marginal cost to price by assuming some pricing strategy for the firms. As a matter of fact, these pricing strategy will result in a markup over marginal cost given by the following formulas:

- Marginal cost pricing:
$$p_j = mc_j$$
- Single product firms:
$$p_j = mc_j + \Delta^{-1} s$$
where $\Delta_{jk} = \begin{cases} \frac{\partial s_j}{\partial p_k} & \text{if } j = k \\ 0 & \text{otherwise} \end{cases}$ and s is the vector of shares.
- Multiproduct firms:
$$p_j = mc_j + \Delta^{-1} s$$
where $\Delta_{jk} = \begin{cases} \frac{\partial s_j}{\partial p_k} & \text{if } k \in \text{firm}(j) \\ 0 & \text{otherwise} \end{cases}$ and s is the vector of shares.
- Perfect collusion:
$$p_j = mc_j + \Delta^{-1} s$$
where $\Delta_{jk} = \frac{\partial s_j}{\partial p_k}$ and s is the vector of shares.

From the previous question, it is easy to to compute Δ and therefore mc_j.

With this in mind we can revert back to $mc_j = x_j\gamma + \eta q_j + \omega_j$ and estimate it like we would for $\delta_j = x_j\beta + \xi_j$. The only issue we are facing now is the potential endogeneity of q_j.

To remedy the situation, we need to find an instrument for q_j. There is plenty potential instruments, but after playing around for a while, I decide to follow Gandhi and Houde (2018) and use $\sum_k |x_j - x_k|$ and $\sum_k (x_j - x_k)^2$. To avoid any problem with collinearity and matrix inversion, I use the function `licols()` to reduce our set of instruments $Z = [x_j, \sum_k |x_j - x_k|, \sum_k (x_j - x_k)^2]$ to a full column rank matrix.

The result from our IV regression, using the different pricing strategy, are given in Table ??.

Table 3: Vertical model - supply side

	Marginal cost	Single product firms	Multiproduct firms	Collusion
(Intercept)	7.5865×10^7	7.5865×10^7	7.5865×10^7	7.5865×10^7
	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$
Weight	4.1722×10^4	4.1722×10^4	4.1722×10^4	4.1722×10^4
	$[2.5642 \times 10^4\ 5.7802 \times 10^4]$	$[2.5642 \times 10^4\ 5.7802 \times 10^4]$	$[2.5642 \times 10^4\ 5.7802 \times 10^4]$	$[2.5642 \times 10^4\ 5.7802 \times 10^4]$
Horsepower	3.1989×10^5	3.1989×10^5	3.1989×10^5	3.1989×10^5
	$[1.2081 \times 10^5\ 5.1897 \times 10^5]$	$[1.2081 \times 10^5\ 5.1897 \times 10^5]$	$[1.2081 \times 10^5\ 5.1897 \times 10^5]$	$[1.2081 \times 10^5\ 5.1897 \times 10^5]$
AC	1.1753×10^7	1.1753×10^7	1.1753×10^7	1.1753×10^7
	$[-4.3171 \times 10^6\ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6\ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6\ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6\ 2.7823 \times 10^7]$
FE	Yes	Yes	Yes	Yes
N	131	131	131	131
R^2	0.8175	0.8175	0.8175	0.8175
(Intercept)	7.5865×10^7	7.5865×10^7	7.5865×10^7	7.5865×10^7
	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$
Weight	4.1722×10^4	4.1722×10^4	4.1722×10^4	4.1722×10^4
	$[2.5642 \times 10^4\ 5.7802 \times 10^4]$	$[2.5642 \times 10^4\ 5.7802 \times 10^4]$	$[2.5642 \times 10^4\ 5.7802 \times 10^4]$	$[2.5642 \times 10^4\ 5.7802 \times 10^4]$
Horsepower	3.1989×10^5	3.1989×10^5	3.1989×10^5	3.1989×10^5
	$[1.2081 \times 10^5\ 5.1897 \times 10^5]$	$[1.2081 \times 10^5\ 5.1897 \times 10^5]$	$[1.2081 \times 10^5\ 5.1897 \times 10^5]$	$[1.2081 \times 10^5\ 5.1897 \times 10^5]$
AC	1.1753×10^7	1.1753×10^7	1.1753×10^7	1.1753×10^7
	$[-4.3171 \times 10^6\ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6\ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6\ 2.7823 \times 10^7]$	$[-4.3171 \times 10^6\ 2.7823 \times 10^7]$
Quantity	7.5865×10^7	7.5865×10^7	7.5865×10^7	7.5865×10^7
	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$	$[3.9391 \times 10^7\ 1.1234 \times 10^8]$
FE	Yes	Yes	Yes	Yes
N	131	131	131	131
R^2	0.8175			

[ ] 95% confidence interval

Part 4

Part 5

We now assume that each consumer i has utility for product j as a function of price and product quality, δ_j of:

$$u_{ij} = \delta_j + \epsilon_{ij}$$

where $\delta_j = x_j\beta - \alpha p_j + \xi_j$, ξ_j is the unosbervable characteristics of good j and ϵ_{ij} follows a type I extreme value distribution.

As shown in class, we have that the share of good j is given by

$$s_j = \frac{e^{\delta_j}}{1 + \sum_k e^{\delta_k}}$$

or, equivalently,

$$\ln(s_j) - \ln(s_0) = x_j\beta - \alpha p_j + \xi_j$$

Armed with this regression, we can now estimate $[\beta, \alpha]$. Like with the supply side, we now have some endogeneity problem. To solve this, we use the same set of instruments, $Z = [x_j, \sum_k |x_j - x_k|, \sum_k (x_j - x_k)^2]$, and run an IV regression to get back both $[\beta, \alpha]$ and ξ. The result from our IV regression are reported in Table 4.

Moreover, Table 4 includes the estimation of demand and supply parameters assuming Nash-Bertrand equilibrium with multiproduct firms. Recall that with multiproduct firms, Nash-Bertrand equilibrium yields

$$p = mc + \Delta^{-1}s = \Delta^{-1}s + x\gamma + \eta q + \omega$$

where $\Delta_{jk} = \begin{cases} \frac{\partial s_j}{\partial p_k} & \text{if } k \in \text{firm}(j) \\ 0 & \text{otherwise} \end{cases}$ and s is the vector of shares. We can solve explicitly for Δ by noting that $\frac{\partial s_j}{\partial p_k} = \begin{cases} -\alpha s_j(1-s_j) & \text{if } j = k \\ \alpha s_j s_k & \text{otherwise} \end{cases}$. Let $\Delta^{-1}s = b(\alpha)$ since, unlike the vertical model, it depends on α in a non-linear fashion.

We can now define our moments equations:

$$\mathbb{E}\begin{Bmatrix} Z\xi_j \\ Z\omega_j \end{Bmatrix} = 0$$

or, equivalently

$$\mathbb{E}\begin{Bmatrix} Z(\delta_j - x_j\beta + \alpha p_j) \\ Z(p_j - b_j(\alpha) - x_j\gamma - \eta q_j) \end{Bmatrix} = 0$$

where Z is the set of instruments. Using these statcked moments conditions, we can use our usual GMM technique to estimate $\theta = [\alpha, \beta, \gamma, \eta]$. Since α enters non-linearly, to solve for it I use `fmincon()` to minize the objective function with respect to α and then solve for the rest of parameters using least squares techniques.

The only issue that I'm facing now is how to compute the standard errors of our estimate. Since, α enters non-linearly, I need to find the gradient of s_j with θ to be able to compute the GMM covariance matrix. Sadly, I ran out of time. Estimation results are reported in Table 4.

Table 4: Vertical model - supply side

	Marginal cost	**Multiple product firms**
(Intercept)	7.5865×10^7	7.5865×10^7
	$[3.9391 \times 10^7 \ 1.1234 \times 10^8]$	
Weight	4.1722×10^4	4.1722×10^4
	$[2.5642 \times 10^4 \ 5.7802 \times 10^4]$	
Horsepower	3.1989×10^5	3.1989×10^5
	$[1.2081 \times 10^5 \ 5.1897 \times 10^5]$	
AC	1.1753×10^7	1.1753×10^7
	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$	
Price (α)	1.1753×10^7	1.1753×10^7
	$[-4.3171 \times 10^6 \ 2.7823 \times 10^7]$	
FE	Yes	Yes
N	131	131
R^2	0.8175	
(Intercept)		7.5865×10^7
Weight		4.1722×10^4
Horsepower		3.1989×10^5
AC		1.1753×10^7
Quantity		7.5865×10^7
FE		Yes
N	131	131
R^2	0.8175	

[ ] 95% confidence interval

Again, we report the elasticities for a subset of the product in Table 5.

Table 5: Vertical model - Price elasticities

Car\Car	15	16	17	18	19	20	21	22	23	24
15	-0.0948	0.0003	0	0	0	0	0	0	0	0
16	0.0003	-0.0003	0.0000	0	0	0	0	0	0	0
17	0	0.0001	-0.0671	0.0673	0	0	0	0	0	0
18	0	0	0.0050	-0.0154	0.0104	0	0	0	0	0
19	0	0	0	0.0711	-0.5703	0.4997	0	0	0	0
20	0	0	0	0	0.1349	-0.1477	0.0126	0	0	0
21	0	0	0	0	0	0.0235	-0.0438	0.0202	0	0
22	0	0	0	0	0	0	1.2703	-1.8608	0.5881	0
23	0	0	0	0	0	0	0	0.0175	-0.0180	0.0004
24	0	0	0	0	0	0	0	0	0.0001	-0.0390

Looking at these elasticities, we have an improvment compared to the previous vertical model where most elasticities where equal to 0.

Part 6

Part 7

Part 8

Table 6: Vertical model - Demand side only

	Demand side
(Intercept)	7.5865×10^7
Weight	4.1722×10^4
Horsepower	3.1989×10^5
AC	1.1753×10^7
Price (α_1)	1.1753×10^7
Price (α_2)	1.1753×10^7
FE	Yes
N	131
R^2	0.8175

[ ] 95% confidence interval

Table 7: Vertical model - Price elasticities

Car\Price	15	16	17	18	19	20	21	22	23	24
15	-0.0948	0.0003	0	0	0	0	0	0	0	0
16	0.0003	-0.0003	0.0000	0	0	0	0	0	0	0
17	0	0.0001	-0.0671	0.0673	0	0	0	0	0	0
18	0	0	0.0050	-0.0154	0.0104	0	0	0	0	0
19	0	0	0	0.0711	-0.5703	0.4997	0	0	0	0
20	0	0	0	0	0.1349	-0.1477	0.0126	0	0	0
21	0	0	0	0	0	0.0235	-0.0438	0.0202	0	0
22	0	0	0	0	0	0	1.2703	-1.8608	0.5881	0
23	0	0	0	0	0	0	0	0.0175	-0.0180	0.0004
24	0	0	0	0	0	0	0	0	0.0001	-0.0390

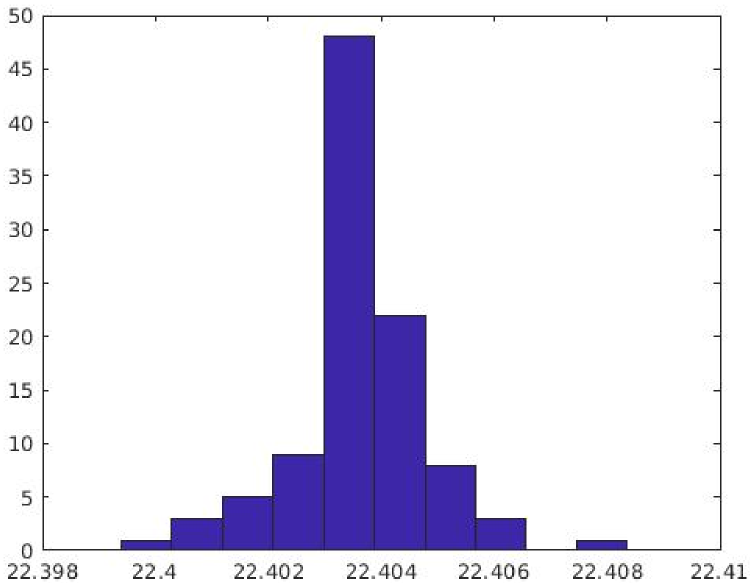

Figure 1:

Part 9

Code

Problem 2