Macroeconomic effects of household leverage regulations after the crisis[*]

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Abstract

This paper assesses the aggregate and distributional effects of policies that seek to reduce mortgage default by limiting a borrower's debt payment-to-income ratio. I document empirically that mortgage originations with a debt payment-to-income ratio above an institutional limit are associated with higher credit scores and lower leverage. I propose an overlapping-generations model with mortgage contract choice and equilibrium default risk consistent with the empirical findings. In the calibrated model, I show that a policy combining a constraint on the debt payment-to-income ratio with a costly option to relax that constraint is effective at lowering default and improving welfare, particularly for households in the middle of the wealth distribution who have low incomes.

1 Introduction

Loose underwriting standards are often identified as a contributing factor to the mortgage foreclosure crisis that precipitated the Great Recession, and policymakers have since sought to tighten them. In doing so, they confront a trade-off between lowering the risk of mortgage default on one hand and preserving opportunities for homeownership on the other. Policies aimed at mitigating default may curtail access to credit for some households, preventing some homeowners from accessing the equity stored in their homes or deterring some marginal agents from becoming owners at all. Although the aggregate effects of these policies are important, their distributional implications are relevant as well. This paper explores the heterogeneity in welfare changes that result from household leverage regulations.

One such regulation that has become more prominent since the crisis is a limit on the borrower's debt payment-to-income (DTI) ratio, which has been identified as a proxy for higher default risk.[1] I focus on two separate instances of DTI limits that policymakers have implemented in practice. The first policy is a requirement by Freddie Mac—one of the two large government-sponsored enterprises involved in the secondary mortgage market—that newly originated mortgages carry a DTI ratio of at most 45 percent in order to be eligible for sale to Freddie Mac. Since Freddie Mac's DTI limit has been in place since 2009, its effects on mortgage originations can be assessed empirically. The second policy is found in the Dodd-Frank Act, which combines a tighter DTI limit of 43 percent with a costly option to obtain a loan with a higher DTI ratio. A high-DTI loan under Dodd-Frank is costly in the sense that lenders incur greater legal liability if a borrower defaults on such a mortgage and that legal risk is passed on to borrowers in the form of higher interest rates. Since the vast majority of the U.S. residential mortgage market is exempt from the Dodd-Frank DTI limit until 2021, assessing its effects on household decisions requires a structural model.

My paper studies the aggregate and distributional effects of these two policies that limit a borrower's debt payment-to-income ratio. Using loan-level data from Freddie Mac, I document empirically the influence of its 45-percent DTI limit on mortgage originations. I propose an overlapping-generations model with mortgage contract choice and equilibrium default risk in order to study the distributional effects of DTI limits and evaluate the impending Dodd-Frank mortgage regulations relative to the existing Freddie Mac policy. My main finding is that the strict but flexible approach embodied by the Dodd-Frank DTI limit reduces default while improving aggregate welfare. Homeowners with low cash on hand benefit most from the Dodd-Frank policy. This is not necessarily an obvious result: since

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[1]E.g., the Consumer Financial Protection Bureau (2013): "All things being equal, consumers carrying loans with higher DTI ratios will be less able to absorb any such [negative] shocks and are more likely to default."

the Dodd-Frank regulations both tighten the DTI limit and impose higher costs on loans with DTI ratios exceeding that limit, one might expect such a policy to decrease welfare relative to the looser Freddie Mac DTI limit. However, my model suggests that, for this subset of households, the welfare benefits of the opt-out provision in Dodd-Frank outweigh those losses.

In my empirical analysis, I present two findings regarding the effect of Freddie Mac's DTI limit on mortgage originations. First, I illustrate that, after the limit was introduced, the share of mortgage loans with DTI ratios greater than the institutional limit has declined dramatically while, simultaneously, a growing fraction of borrowers have found themselves constrained by the limit. Second, I show that, following this policy change, high-DTI loans are associated with borrowers who are, on average, less leveraged and more creditworthy. This is a reversal from what is observed prior to the policy, when high-DTI loans were held by borrowers with above-average loan-to-value ratios and interest rates and below-average credit scores. The second finding is new to the literature, and it is consistent with a story in which policies that limit the DTI ratio are accompanied by a selection of more creditworthy borrowers into loans that exceed the limit.

I then develop an overlapping-generations model that confronts households with a choice between mortgage contracts and generates equilibrium default on mortgage debt. The Dodd-Frank Act's differential regulatory treatment of low- versus high-DTI loans motivates the inclusion of a mortgage contract choice in the model. This feature has the advantage of nesting different regulations on household leverage, making the model especially useful for policy evaluation. I calibrate the stationary distribution of the model under the Freddie Mac DTI limit to match moments from U.S. aggregate and cross-sectional data and the observed distribution of DTI ratios in the Freddie Mac loan-level data.

After validating the quantitative fit of the calibrated model, I use it to analyze and compare the consequences of the Freddie Mac and Dodd-Frank DTI limits. I present three sets of results. First, a DTI constraint reduces the aggregate default rate while lowering the homeownership rate as well. These declines are driven by borrower selection. In the absence of a DTI limit, low-income and low-net worth households obtain high-DTI loans in equilibrium, and these borrowers reduce their leverage most after the introduction of the Freddie Mac limit. Second, I show that costly option for a high-DTI loan under the Dodd-Frank regulations is primarily chosen by households in the middle of the wealth distribution with low current incomes. These agents value the option to borrow more while also being sufficiently creditworthy to obtain such mortgages in equilibrium. This is reflected in the fact that, relative to a setting without a DTI limit, the Dodd-Frank policy generates a stronger association between higher DTI ratios and lower interest rates and loan-to-value ratios. This

feature of the model is consistent with my empirical findings. Finally, I show that, relative to the looser Freddie Mac DTI limit that applies to all borrowers, the Dodd-Frank policy increases aggregate welfare and lowers default. Households disagree about the desirability of the Dodd-Frank regulation: roughly two-thirds of households benefit from it, and the distribution of welfare changes is closely correlated with homeownership. Welfare gains accrue to owners, particularly those who are liquidity constrained and value the option to get a high-DTI loan. The largest welfare losses are concentrated among marginal homeowners who find Dodd Frank's 43-percent limit too constraining and instead choose to rent.

The remainder of the paper proceeds as follows. I connect my paper to the relevant literature in Section 2 and discuss institutional details of the Freddie Mac DTI limit and the Dodd-Frank Act's mortgage regulations in Section 3. I present my empirical findings regarding the effects of the Freddie Mac DTI limit on mortgage originations in Section 4. I develop the theoretical model in Section 5, then discuss calibration and model fit in Section 6. My quantitative analysis of the Freddie Mac and Dodd-Frank regulations in the context of the calibrated model follows in Section 7. Section 8 concludes and discusses avenues for further research.

2 Related literature

On the theoretical side, my paper builds on the growing body of macroeconomic research that incorporates uninsurable idiosyncratic income risk, a life-cycle savings motive, illiquid housing wealth, and borrowing constraints in models of the U.S. housing and mortgage markets.[2] With its inclusion of long-term secured debt that carries the option to default, my paper also speaks to a related consumer finance literature that has used equilibrium models to study bankruptcy and foreclosure in stationary environments.[3] A distinguishing characteristic of my model is its inclusion of a borrower's mortgage choice problem, which is relatively understudied in the theoretical literature. Corbae and Quintin (2015) develop a model in which borrowers choose between low- or high-down payment fixed-rate mortgages subject to a debt payment-to-income constraint at origination and priced by risk-neutral financial intermediaries. For reasons of tractability, their model places strong restrictions

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[2]See, for instance, Beraja et al. (2018), Berger et al. (2017), Chatterjee and Eyigungor (2015), Chen, Michaux and Roussanov (2013), Corbae and Quintin (2015), Garriga and Hedlund (2017), Gorea and Midrigan (2018), Guren, Krishnamurty and McQuade (2018), Kaplan and Violante (2014), Kaplan, Mitman and Violante (2017), and Wong (2018). Guren et al. (2018) lay out the "new canonical model" of housing and consumption. Both Davis and Van Nieuwerburgh (2014) and Piazzesi and Schneider (2016) provide useful overviews of recent advances in macroeconomic research on housing and mortgage markets.

[3]See Campbell and Cocco (2015), Chatterjee et al. (2007), and Mitman (2016). Some of the papers mentioned in the previous footnote incorporate mortgage foreclosure into their analyses of the U.S. housing market.

on the choices available to agents, as well as the demographic structure of the economy.[4] I build on their environment by incorporating a more richly specified household problem that permits a more realistic study of household consumption and savings behavior. An earlier paper by Chambers, Garriga and Schlagenhauf (2009) focuses on the implications of mortgage choice in a general equilibrium setting by allowing owners to select between a fixed-rate mortgage and an alternative debt contract. The authors abstract from mortgage default, though, which is central to the objectives of the leverage regulations I study.

My paper also contributes to an ongoing discussion about the *ex ante* regulations of the residential mortgage market that were enacted in the aftermath of the financial crisis. On the empirical side, DeFusco, Johnson and Mondragon (2017) study the effect of the 43-percent DTI limit in the Dodd-Frank regulations on the jumbo loan market, where the constraint has already taken effect.[5] They find that the policy had a modest positive effect on prices but a much larger negative effect on quantities. Since such a result is inconsistent with existing estimates of mortgage elasticities, they posit that credit rationing is responsible for this outcome. Greenwald (2018) augments a saver-spender New Keynesian model with a DTI constraint and prepayable long-term mortgage debt in order to study the influence of the structure of mortgage finance on the transmission of macroeconomic shocks and evaluate the Dodd-Frank DTI limit as a tool for macroprudential policy. I complement this existing research on the Dodd-Frank mortgage regulations by using a nonlinear model that is suitable for analyzing mortgage default and household welfare.

3 Institutional background

My paper considers two separate regulations that placed constraints on a borrower's DTI ratio, Freddie Mac's 45-percent DTI limit and Dodd-Frank's 43-percent DTI limit with a costly opt-out provision. Their timing is important to the analysis that follows. The Freddie Mac limit was introduced in 2009, while the Dodd-Frank limit will be implemented in 2021 for the segment of the mortgage market that has already been shaped by Freddie Mac's underwriting standards. I therefore consider the latter to be representative of the current regulatory environment confronting the residential mortgage market and will use it as a baseline to evaluate the potential effects of the forthcoming Dodd-Frank rules.

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[4]For instance, only middle-aged agents can own houses, and mortgage refinancing and prepayment are ruled out by assumption.

[5]A jumbo loan is a mortgage that has an outstanding balance at origination that exceeds the conventional conforming loan limits set forward by the underwriting requirements of Fannie Mae and Freddie Mac.

3.1 Freddie Mac's underwriting standards

Freddie Mac is one of two large government-sponsored enterprises (GSEs) that purchase newly originated mortgage loans from lenders in the primary market and package them into mortgage-backed securities that are then sold to investors on the secondary market. Freddie Mac's stated purchase is to "provide liquidity, stability, and affordability to the U.S. housing market" by promoting the flow of capital to primary mortgage lenders.[6] The GSEs are only permitted to purchase mortgage loans—termed "conforming loans"—that meet certain guidelines. Although the GSEs do not lend to households directly, they exercise an outsize influence on the residential mortgage market by creating incentives for primary lenders to originate loans in accordance with their underwriting requirements. This influence has been particularly pronounced since the financial crisis. Since entering federal conservatorship in September 2008, the GSEs have jointly purchased or guaranteed around 75 percent of mortgage originations.[7]

In March 2009, Freddie Mac revised its underwriting standards to include a 45-percent maximum debt payment-to-income ratio for all manually underwritten mortgages.[8] Although their underwriting requirements had previously contained guidance regarding the borrower's debt payment-to-income ratio, this marked the first time an outright limit was placed on the DTI ratio.[9] Freddie Mac set out a few criteria that may be used to justify a higher DTI ratio: the borrower having sufficient liquid assets to constitute an ability to repay the mortgage regardless of income; a down payment on the purchase of a property of at least 25 percent; or a strong credit score (i.e., a FICO score of 740 or higher) in conjunction with the lender's assurance that "the borrower's credit reputation is excellent."[10]

3.2 The Dodd-Frank Wall Street and Consumer Protection Act

Signed into law in 2010, the Dodd-Frank Wall Street and Consumer Protection Act was intended to address some of the systemic risks that contributed the Great Recession,

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[6]See http://www.freddiemac.com/about/.

[7]This represents $100 billion per month in new mortgage lending. See http://www.fhfa.gov/AboutUs/Reports/ReportDocuments/20120221_StrategicPlanConservatorships_508.pdf.

[8]See http://www.freddiemac.com/singlefamily/guide/bulletins/pdf/bll112408.pdf. Manual underwriting is the process by which a lender assesses various components of the loan application in accordance with underwriting requirements and risk evaluation guidelines published in the Freddie Mac Seller/Servicer Guide. Alternatively, a lender can use Freddie Mac's proprietary automated underwriting system, Loan Product Advisor.

[9]Freddie Mac's selling guidelines have long required additional justification from a lender when originating a mortgage with a DTI ratio greater than 36 percent, but it will be seen in Section 4.2 that this requirement is little more than a formality and does not have any binding effect on loans purchased by Freddie Mac.

[10]See Section 5401.2 of the Freddie Mac Single-Family Seller/Servicer Guide, which can be accessed here: http://www.freddiemac.com/singlefamily/pdf/guide.pdf.

including the loose underwriting standards by some mortgage lenders in place prior to the foreclosure crisis. These were the focus of one particular provision of the Dodd-Frank Act, the ability-to-repay rule.[11] The ability-to-repay rule states:

> "...no creditor may make a residential mortgage loan unless the creditor makes a reasonable and good faith determination based on verified and documented information that, at the time the loan is consummated, the consumer has a reasonable ability to repay the loan."

The ability-to-repay rule itself does not explicitly ban certain loan features. If, however, a residential mortgage loan mets certain standards set forward by regulations, it will be presumed to be in accordance with the Dodd-Frank ability-to-repay rule. In other words, originating a mortgage satisfying those conditions is a sufficient way for the lender to comply with the rule; such a loan is, in the parlance of this legislation, called a qualified mortgage. The Dodd-Frank Act delegated authority to finalize these regulations to the newly created Consumer Financial Protection Bureau (CFPB). The CFPB announced the final rule in January 2013, and it officially came into effect in January 2014.[12]

The final rule applies to any consumer credit transaction secured by a dwelling.[13] First, it lays out the conditions that a loan must meet in order to be considered a qualified mortgage: (1) a ban on certain loan features deemed to be too risky, (2) a cap on points and fees due at origination, and (3) a maximum back-end DTI ratio of 43 percent.[14] Additionally, the final rule clarifies the nature of the legal protection given to a lender that originates a qualified mortgage. Qualified mortgages are granted safe harbor—that is, they are "conclusively presumed" to comply with the ability-to-repay rule, and a borrower who has difficulty repaying a qualified mortgage does not have legal standing to sue the lender. In contrast, if a borrower has trouble repaying a non-qualified mortgage and can successfully demonstrate in court that the mortgage violated the ability-to-repay rule, the lender would be liable for up to three years of interest payments and loan fees that the consumer has already paid, as well as legal fees incurred by the consumer. The final rule therefore creates a clear regulatory incentive for lenders to originate mortgages that satisfy the qualified mortgage criteria.[15]

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[11]The full text of the Dodd-Frank Act can be found at https://www.congress.gov/bill/111th-congress/house-bill/4173/text. The regulations that are the subject of this paper can be found in Sections 1411 and 1422 of Title XIV.

[12]The full text of the final rule can be found at https://www.federalregister.gov/documents/2013/01/30/2013-00736/ability-to-repay-and-qualified-mortgage-standards-under-the-truth-in-lending-act-regulation-z.

[13]This excludes open-ended credit lines, timeshare plans, reverse mortgages, and temporary loans.

[14]Prohibited loan features under the rule include negative amortization, interest-only payments, balloon payments, and loan terms exceeding 30 years.

[15]The CFPB writes, "Treating a qualified mortgage as a safe harbor provides greater legal certainty for creditors and secondary market participants than a rebuttable presumption of compliance. Increased legal

Notably, the final rule granted a transitional period during which mortgages that are eligible for sale to or guarantee by a GSE will still be considered a qualified mortgage if they satisfy the first two qualified mortgage requirements, thereby making these loans exempt from the 43-percent DTI limit. The GSE exemption will last until January 2021 or when the GSEs are removed from federal conservatorship, whichever occurs earliest. The residential mortgage market has therefore yet to see the full effect of the Dodd-Frank regulations.

Policymakers and relevant market participants have disagreed on the overall effect and desirability of the Dodd-Frank ability-to-repay rule. The Mortgage Bankers Association has argued that the potential legal liability associated with loans that do not satisfy the qualified mortgage criteria may lead lenders to charge their borrowers higher interest rates, even if they represent relatively low credit risk, or leave that part of the mortgage market altogether. In a similar vein, the American Bankers Association has advocated for the elimination of the 43-percent DTI standard altogether, calling it "arbitrary" and "inflexible."[16] The CFPB has countered by arguing that approximately 70 percent of mortgages originated between 1997 and 2003 would have satisfied the Dodd-Frank criteria, implying that the latter would have limited scope.[17]

For expositional simplicity, I will refer to qualified mortgages as low-DTI loans for the remainder of the paper because the debt payment-to-income ratio of the borrower at origination satisfies the 43-percent limit. Conversely, a high-DTI loan will be a mortgage that carries a DTI ratio greater than 43 percent.

4   Empirical evidence on the effect of the Freddie Mac DTI limit

The inclusion of an explicit 45-percent cap on the borrower's debt payment-to-income ratio in Freddie Mac's underwriting requirements in 2009 provides a setting in which to empirically assess the effects of a DTI limit on the allocation of credit in a large segment of the residential mortgage market. After establishing that the DTI limit had a significant effect on mortgage originations, I will draw on the empirical evidence in the calibration of the model and also use it validate the model's predictions about borrower behavior.

I use the Freddie Mac Single Family Loan-Level Dataset to document two facts regarding the DTI limit. Echoing results from Greenwald (2018) and DeFusco, Johnson and Mondragon (2017), the first is the emergence of bunching in the distribution of DTI ratios to the left

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certainty may benefit consumers if as a result creditors encouraged to make loans that satisfy the qualified mortgage criteria."

[16]See https://www.mba.org/issues/residential-issues/Dodd-Frank-improvements and https://www.aba.com/Advocacy/Documents/Mortgage-Reforms.PDF, respectively.

[17]See Moskins (2014).

of the 45-percent institutional limit. The second empirical finding—which is new to the literature—concerns a change in the composition of borrowers with high-DTI ratios that favored less leveraged and more creditworthy households after the DTI limit was introduced.

4.1 Freddie Mac Single Family Loan-Level Dataset

The Freddie Mac Single Family Loan-Level Dataset contains quarterly loan-level origination data on fully-amortizing 15-, 20-, and 30-year fixed-rate mortgages with full documentation that were purchased or guaranteed by Freddie Mac from 1999 to 2016. I limit my analysis to loans with 30-year terms that are collateralized by owner-occupied housing and have non-missing data on the DTI ratio, FICO score, interest rate, and loan-to-value (LTV) ratio.[18] This leaves approximately 18 million loans in my sample. I provide summary statistics in Section A.1 of the appendix.

4.2 Distribution of DTI ratios

My first empirical finding is that Freddie Mac's 45-percent DTI limit was binding and was associated with clear changes in the distribution of DTI ratios. Figure 1 plots the annual distribution of debt payment-to-income ratios at origination for the years 2005 through 2016. It is clear that the distribution of DTI ratios has changed markedly during this period, even as the distribution of loan-to-value ratios has remained relatively stable.[19] Prior to the introduction of the DTI limit in 2009, the DTI ratio distribution before the crisis does not display an indication of a binding constraint at any level. Afterwards, the share of originations with a DTI ratio greater than 45 percent has shrunk while that of loans with a DTI ratio just below the limit has increased. These observations indicate that, as underwriting requirement, the DTI limit has exercised greater influence on mortgage originations during the post-crisis years in a way that it previously had not.

Figure 2 provides another way to visualize this finding by plotting on an annual basis the share of mortgage originations that have a DTI ratio greater than 43 percent, between 43 and 45 percent, or above 45 percent. As of 2016, just under 20 percent of originations had a DTI ratio above 43 percent, the limit prescribed in the Dodd-Frank Act's mortgage regulations. Of these, slightly more than half have a DTI ratio between 43 and 45 percent, which is to say that they satisfy Freddie Mac's underwriting requirements but not the pending Dodd-Frank DTI limit. This suggests that, once the exemption of conforming loans from the

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[18]The FICO score is the borrower's credit score, named after the company—Fair, Isaac, and Company—that first introduced it. The loan-to-value ratio is the original mortgage loan amount divided by the mortgage property's appraised value.

[19]I plot the distribution of LTV ratios over this time in Section A.2 of the appendix.
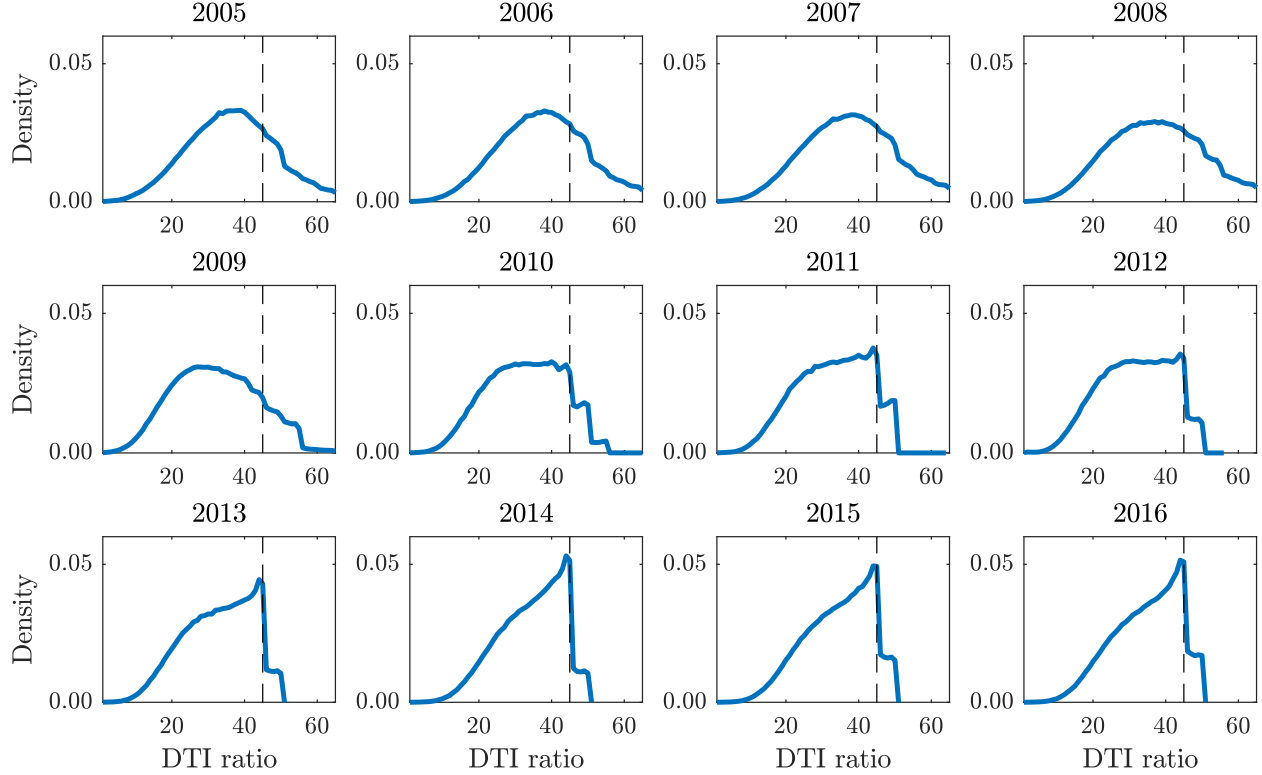
Figure 1

Distribution of DTI ratios at origination, 2005-2016. Mortgages are grouped into 1-percentage point bins. The black vertical line corresponds to the 45-percent DTI limit for manually underwritten loans.

latter expires, a nontrivial fraction of mortgages could be subject to increased legal liability, potentially altering the allocation of credit in meaningful ways.

4.3 Loan and borrower characteristics around the DTI limit

My second empirical finding, which this paper is the first the document, pertains to the divergence in the characteristics of high-DTI loans after the Freddie Mac DTI limit was introduced. The panels of Figure 3 plot the detrended interest rate, LTV ratio, and borrower's credit score before and after the the change in Freddie Mac's underwriting standards in the first quarter of 2009.[20] These three loan characteristics exhibit no visible differences on either side of the 45-percent DTI limit prior to the policy change. There are, furthermore, clear and nearly monotonic relationships between the DTI ratio and the three loan characteristics. The interest rate and LTV ratio are increasing in the DTI ratio on

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[20]I generate these figures by regressing the dependent variable in question on a vector of dummy variables that correspond to the quarter of loan origination, backing out the residual implied by the regression estimates, and separately computing the average residual over 1-percentage point DTI ratio bins for pre- and post-2009Q1 observations.
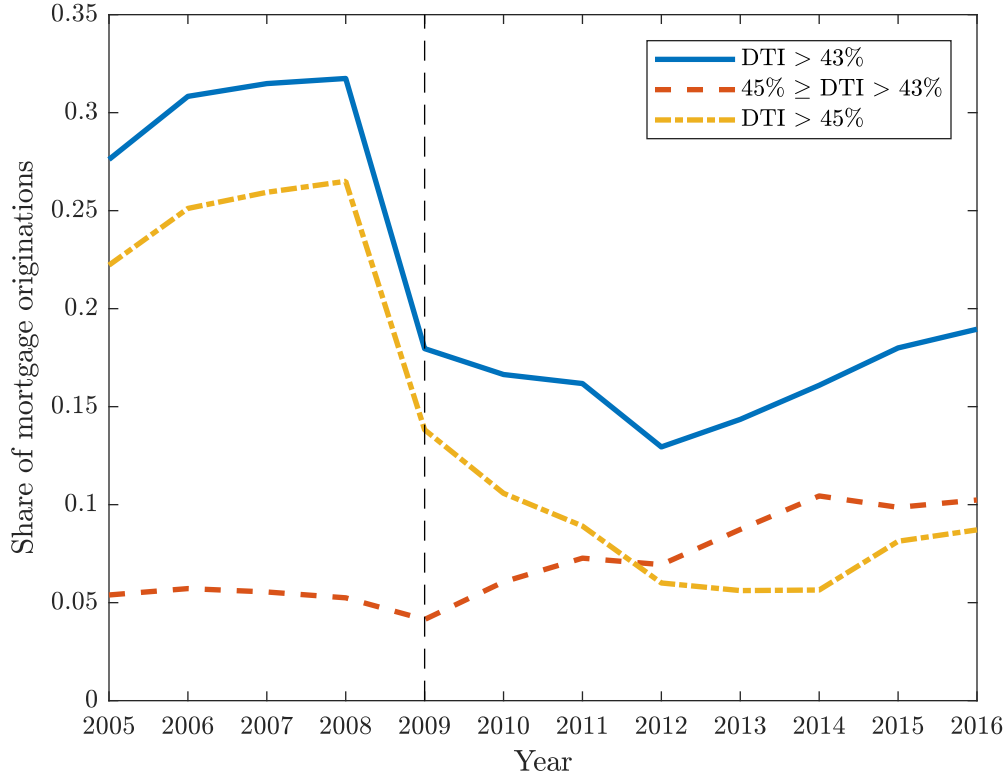
Figure 2

Shares of loans with a DTI ratio above 43 percent, between 43 and 45 percent, and above 45 percent, among mortgage originations purchased by Freddie Mac, 2005-2016. The dashed black line corresponds to the year in which the 43-percent DTI limit was introduced.

the loan, while the FICO score of the borrower is decreasing in the DTI ratio. During this period, loans with a DTI ratio above 45 percent carry above-average interest rates and LTV ratios and below-average credit scores.

After Freddie Mac added the DTI limit to their underwriting requirements, though, significant differences in these three loan characteristics emerge around the 45-percent DTI threshold. Loans with a DTI ratio just above the threshold have an interest rate that is almost 8 basis points lower than those at the threshold. Their average LTV ratio is also around 7 percentage points lower. The difference in credit scores mirrors that in interest rates: high-DTI loans are held by borrowers with a FICO score that is around 20 points higher than loans with DTI ratios exactly at the limit after the policy change. Notably, compared to high-DTI loans that were originated before the policy change, high-DTI loans originated afterwards are now associated with below-average interest rates, below-average LTV ratios, and above-average FICO scores. This stark reversal suggests a shift in borrower composition after the introduction of the DTI limit.

To control for variables that may affect loan and borrower characteristics, I use a
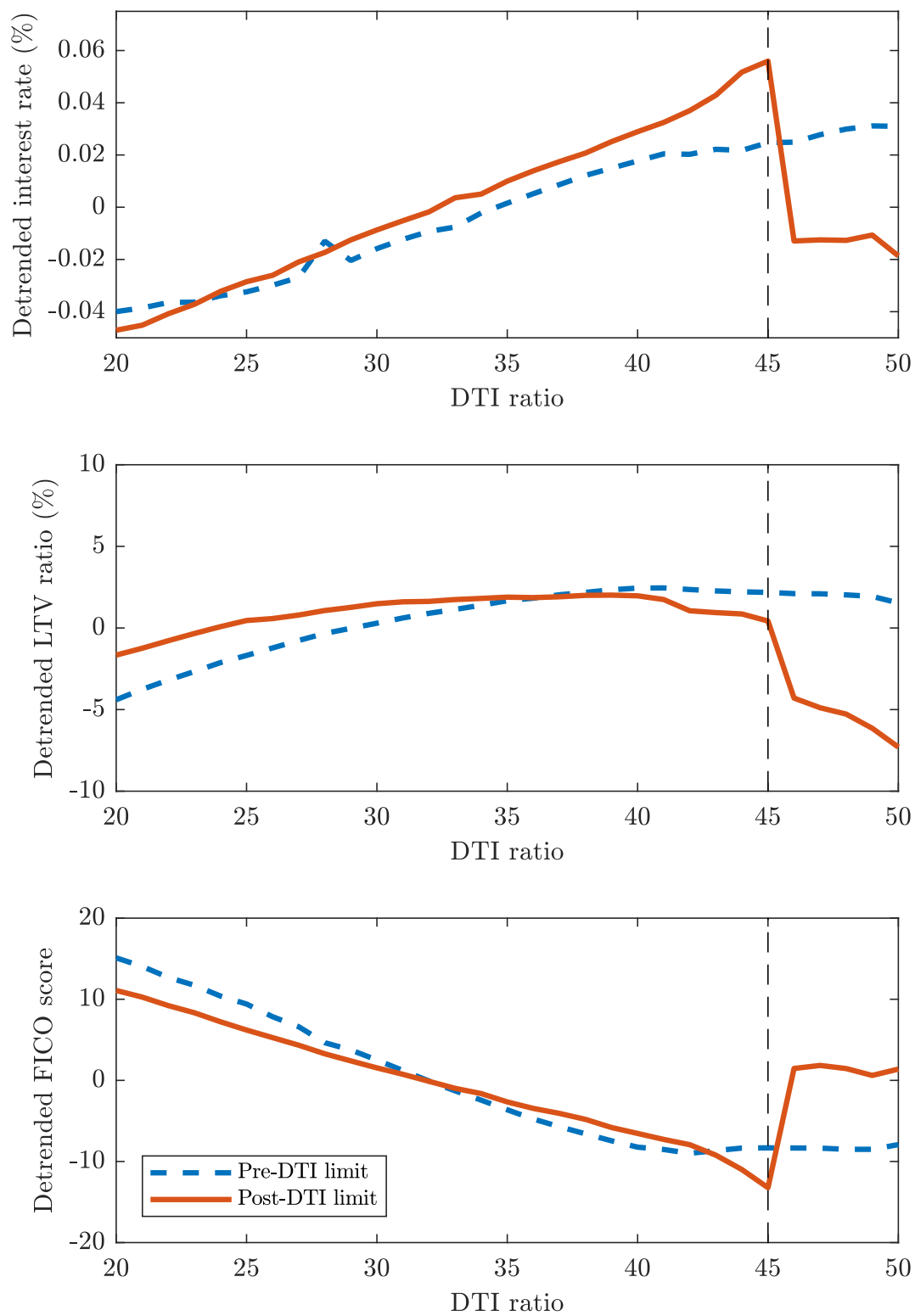
Figure 3

Loan and borrower characteristics around Freddie Mac's 45-percent DTI limit.

difference-in-differences design in the spirit of DeFusco, Johnson and Mondragon (2017) to estimate the change in the characteristics of a high-DTI loan, relative to a low-DTI loan, after the 45-percent DTI limit is added to Freddie Mac's underwriting requirements. The specification I use is

$$y_{it} = \alpha + \beta_1 \text{HighDTI}_i + \beta_2 \text{HighDTI}_i \times \text{Policy}_t + \gamma_t + X_i'\delta + \varepsilon_{it}, \tag{1}$$

where y_{it} is a particular loan characteristic of loan i originated in quarter t; α is the constant term; γ_t is a vector of quarter dummy variables; X_i is a vector of other loan, property, and borrower characteristics; and ε_{it} is an error term clustered at the state level.[21] HighDTI$_i$ is an indicator variable that takes a value of 1 if the DTI ratio of loan i exceeds 45 percent, and Policy$_t$ is an indicator variable that takes a value of 1 if the quarter is 2009Q1 or later.[22] The coefficient of interest is β_2, which is the differential change in the dependent variable for high-DTI loans relative to low-DTI loans following the introduction of the 45-percent DTI limit. When estimating the regression, I restrict my sample to loans with a DTI ratio in a symmetric window around 45-percent that were originated between 2006 and 2014.

	(1) Interest rate	(2) LTV ratio	(3) FICO score
DTI > 45	0.005***	0.007	-1.768***
	(0.000)	(0.900)	(0.000)
DTI > 45 × Policy	-0.033***	-4.786***	6.456***
	(0.000)	(0.000)	(0.000)
Observations	1,861,384	1,861,384	1,861,384

p-level in parentheses.

Robust standard errors are clustered at the state level.

* p < 0.10, ** p < 0.05, *** p < 0.01.

Table 1

Effect of the 45-percent DTI limit on high-DTI loans relative to low-DTI loans. The two rows correspond to estimated values for the coefficients β_1 and β_2 from the difference-in-differences specification in Equation (1) with the interest rate, LTV ratio, and FICO score serving as the dependent variable in turn.

Estimated values for the coefficients β_1 and β_2 in Equation (1) are presented in Table 1. The patterns I document regarding the interest rate, LTV ratio, and credit score in Figure 3 survive after controlling for other loan characteristics and are statistically significant at the 1

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[21]X_i includes dummy variables for the state in which the property is located, the purpose of the loan (i.e., if it is a refinance or purchase loan), and the type of property.

[22]It is not necessary to include Policy$_t$ as a regressor because it is absorbed by the time dummy variables.

percent level, though the magnitudes in the differences of these characteristics between low-
and high-DTI loans are somewhat smaller. Relative to low-DTI loans, the interest rate of
high-DTI loans declines by 3.3 basis points, the LTV ratio declines by 4.8 percentage points,
and the FICO score increases by 6.5 points after the Freddie Mac DTI limit is implemented.[23]

In sum, the 45-percent DTI limit added to Freddie Mac's underwriting requirements was
not only binding insofar as the debt payment-to-income ratio itself is concerned but also had
significant effects on other loan characteristics. In addition to simply establishing that this
policy did in fact have an effect on newly originated mortgage loans, the empirical evidence
laid out in this section serves two further purposes. First, I will use it to discipline the
parameterization of the model in order to ensure that the latter can generate the observed
bunching of loans at the DTI limit, which is a clear feature of the present state of the
conforming mortgage market. Second, I will verify if the model can generate the association
of higher DTI ratios with more creditworthy borrowers documented above.

5 An overlapping-generations model with mortgage contract choice and equilibrium default risk

The empirical evidence on the influence of Freddie Mac's DTI limit on mortgage
originations underscores the binding nature of the constraint on a significant fraction of
borrowers and a shift in the composition of borrowers with high-DTI ratios to encompass less
leveraged and more creditworthy households. Data alone, however, cannot reveal anything
about the welfare effects of such regulations and are insufficient for making predictions about
the impending Dodd-Frank DTI limit. To better understand these implications, I develop
a theoretical overlapping-generations model that features equilibrium default risk and a
discrete choice over mortgage contracts.

5.1 Model environment

The model features a constant population of overlapping generations of households.
Households are finitely lived and receive an age-specific income that is subject to uninsurable
idiosyncratic risk. They derive utility from nondurable consumption and housing services;
and can save in both a liquid asset and illiquid housing wealth. Housing services can be
obtained through a rental market or by purchasing a home, which yields a service flow every

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[23]As a robustness check, I use a more flexible specification that allows the effect of the DTI limit to vary
with the DTI ratio in order to demonstrate that these results are not being driven merely by the DTI ratio
being above the limit but, rather, a change precisely *at* the 45-percent cutoff. Details can be found in Section
A.3 of the appendix.

period. Owners can borrow using illiquid mortgage debt and have the option to default.

The model also features a continuum of competitive, risk-neutral, infinitely lived financial intermediaries that maximize expected discounted profit. Financial intermediaries store households' liquid savings at the risk-free rate and supply mortgage debt demanded by borrowers at an interest rate pinned down by the requirement that, in equilibrium, a lender breaks even on a loan-by-loan basis.

The key feature of the model is the inclusion of a choice between mortgage contracts. The choice of contract affects whether a borrower is constrained by a DTI limit and changes the financial intermediary's payoff in the event that a borrower defaults on their loan. The contract choice problem is flexible enough to nest different sets of leverage regulations, which are obtained in the model by appropriately setting parameter values and restricting household choices.

Preferences and endowments

A household maximizes expected discounted lifetime utility, defined as

$$\max \ \mathbb{E}\left\{\sum_{j=1}^{T}\left[\beta^{j-1}u\left(c_j, s_j\right)\right] + \beta^T \nu\left(W_T\right)\right\},$$

where j indexes the household's age and $u\left(\cdot\right)$ is the flow utility function satisfying standard Inada conditions. The flow utility function takes the form

$$u\left(c, s\right) = \frac{1}{1-\sigma}\left(c^\alpha s^{1-\alpha}\right)^{1-\sigma},$$

where c is nondurable consumption and s is housing services. Households have a bequest motive so that, in the terminal period of life T, they receive utility from end-of-life wealth W_T according to the function

$$\nu\left(W_T\right) = B\frac{W_T^{1-\sigma}}{1-\sigma},$$

where B controls the strength of the bequest motive. The bequest motive is needed to ensure that, consistent with the data, households reach the end of their lives with positive net worth.

A household supplies labor inelastically from age 1 until they retire at age T_R. The log income received by a household of age j is

$$\log y_j\left(z\right) = \begin{cases} \chi_j + z & \text{if } 1 \le j < T_R \\ \Phi\left(y_{T_R-1}\left(z\right)\right) & \text{if } T_R \le j \le T. \end{cases} \tag{2}$$

While working, log income is the sum of a deterministic component indexed by age χ_j and an idiosyncratic component z. The idiosyncratic component of income follows the first-order Markov process

$$z' = \rho z + \varepsilon', \ \varepsilon' \overset{i.i.d.}{\sim} \mathcal{N}\left(0, \sigma_\varepsilon^2\right).$$

Following Guvenen and Smith (2014), households in retirement receive a constant pension income that is a function of the income received in the last year of their working life, $y_{T_R-1}(z)$.

Asset technology

All households can save in a liquid asset a. The liquid asset takes the form of deposits held by financial intermediaries that earn a rate of return each period. Financial intermediaries have access to international capital markets where the risk-free rate, r, is determined by the net supply of safe financial assets. A zero-profit condition on deposits implies that the household will also earn the risk-free rate r on their deposits. A household's position in the liquid asset is subject to a no-borrowing constraint.

Households obtain housing services either through the rental market or the owner-occupied housing market. The supply of rented housing services is perfectly elastic so that a household can rent s units of housing services at a rate R per unit each period. I assume that $s \le h_{small}$, i.e., that a renter cannot obtain more housing services than those implied by the smallest possible house size to capture the fact that, in reality, rental properties lack many of the amenities that owner-occupied properties possess.[24] Adjusting the stock of rented housing services between time periods is costless.

The supply of owner-occupied housing is also perfectly elastic. A household can purchase housing stock h at a price normalized to 1, and, so long as the household remains an owner, the housing stock yields a one-to-one flow of housing services (i.e., $s = h$) every period. Adjustment of the housing stock incurs a transaction cost κ_h, and I assume that households may only own one house at any given time. As in Chatterjee and Eyigungor (2015), owner-occupied housing is subject to an i.i.d depreciation shock δ_h each period whereby

$$\delta_h = \begin{cases} \delta > 0 & \text{with probability } \zeta \\ 0 & \text{with probability } 1 - \zeta. \end{cases} \tag{3}$$

An owner hit by the depreciation shock thereby incurs a maintenance cost of $\delta_h h$. By definition, an owner is a household with $h > 0$, while a renter is a household with $h = 0$.

Owners can use their housing wealth as collateral for mortgage debt m, which is modeled

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[24]In the stationary equilibrium of the model, this constraint does not bind due to the selection of low-income, low-wealth households into renting.

after the fixed-rate, long-term mortgage contracts commonly used in the United States. Mortgage loans are supplied by infinitely lived risk-neutral financial intermediaries so that the interest rate on a newly originated loan is determined in equilibrium and is a function of the household's state. This interest rate remains fixed for the duration of the loan. The outstanding balance on a new loan is fully amortized over the remaining life of the borrower.[25] When an owner obtains a new loan, they incur a transaction cost of κ_m. A borrower may only hold one mortgage loan at a time.

The size of a new loan is subject to two constraints at origination. The first is a loan-to-value constraint,

$$m \leq \theta h,$$

which states that the face value of the loan m cannot exceed a fraction θ of the value of the home h. The second is a debt payment-to-income constraint,

$$\pi_{min,j}(m, r_m) \leq \lambda(q) y_j(z),$$

where $\pi_{min,j}(m, r_m)$ is the minimum mortgage payment defined by the standard amortization formula,

$$\pi_{min,j}(m, r_m) = \frac{(1 + r_m)^{T-(j-1)}}{(1 + r_m)^{T-(j-1)} - 1} r_m m. \tag{4}$$

This DTI constraint states that the minimum mortgage payment on a loan—which is a function of the borrower's age j, the loan size m, and the interest rate on the loan r_m—cannot exceed a fraction $\lambda(q)$ of the borrower's contemporaneous income. The tightness of the DTI constraint is the first instance in which the borrower's choice of mortgage contract appears and is a function of the contract type q in the following manner:

$$\lambda(q) = \begin{cases} \lambda & \text{if } q = L \\ \infty & \text{if } q = H. \end{cases} \tag{5}$$

Households who choose a low-DTI loan ($q = L$) are constrained to selecting a loan size that satisfies the institutional cap on the debt payment-to-income ratio, which is set by the parameter λ. Households who choose a high-DTI loan ($q = H$) can select a loan size that is not bound by this limit.

Conditional on the choice of contract, a mortgage originated to a household of age j specifies a face value m', interest rate r'_m, and maturity $T - j$ such that the household receives

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[25] A fully amortized fixed-rate loan is fully paid off by the end of its term and one in which the minimum loan payment due each period is constant.

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m' in the period of origination and promises to make a sequence of payments $\{\pi_j(m', r'_m)\}_{j+1}^T$ so that the balance on the loan equals zero after the household makes their final payment.[26] The law of motion on the balance of an outstanding loan is

$$m' = (1 + r_m) m - \pi_j(m, r_m),$$ (6)

where the mortgage payment is subject to the constraint

$$\pi_j(m, r_m) \geq \pi_{min,j}(m, r_m).$$

A mortgage loan is terminated prior to maturity if the borrower prepays it in its entirety, refinances an existing mortgage, or sells their house. In this case, the intermediary receives the outstanding balance on the loan plus interest,

$$(1 + r_m) m.$$

A borrower can also default on their mortgage, which sets their outstanding debt to zero at the cost of surrendering the underlying collateral to the intermediary, which is the value of the home net of depreciation.[27] The intermediary recovers the collateral less foreclosure costs,

$$(1 - \delta_h) h - \gamma(q),$$

where these foreclosure costs will depend on the type of contract upon which the household is defaulting in the following manner:

$$\gamma(q) = \begin{cases} \gamma_L & \text{if } q = L \\ \gamma_H & \text{if } q = H. \end{cases}$$ (7)

A critical restriction I place on the parameterization of the foreclosure costs is that, from the lender's perspective, default on a high-DTI loan is more costly than default on a low-DTI loan, i.e., $\gamma_H > \gamma_L$. This reflects the greater legal liability assigned by the Dodd-Frank legislation to mortgages with DTI ratios above the statutory limit.

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[26]I follow other papers in this literature, such as Kaplan, Mitman and Violante (2017) and Wong (2018), in amortizing mortgages over the remaining life of the household. This convention is consistent with the observed negative correlation between age and loan duration and ensures that I do not have to track loan maturity as an additional state variable in the model.

[27]This is the sale value of the house and what an owner would receive from liquidating the home.

5.2 Household's optimization problem

The household's optimization problem can be written in recursive form. The current state of an age-j household is summarized by the vector $\omega \equiv (a, h, \delta_h, m, q, r_m, z)$. An age-$j$ household has the value function

$$V_j(\omega) = \max \left\{ V_j^R(\omega), V_j^M(\omega), V_j^P(\omega), V_j^D(\omega) \right\}, \tag{8}$$

where the value functions inside the maximum operator correspond to the discrete choices available to a household in the current period. V_j^R is the value of renting, V_j^M is the value of owning and obtaining a new mortgage loan, V_j^P is the value of owning and making a payment on an existing loan, and V_j^D is the value of defaulting on outstanding debt.
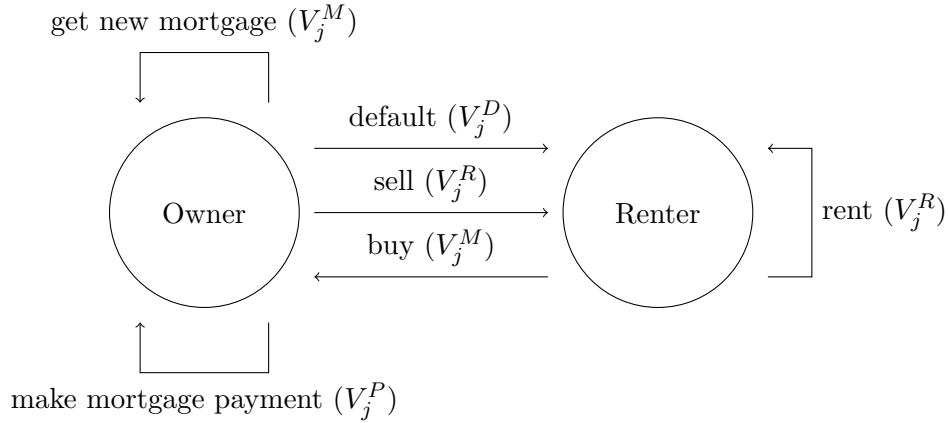
get new mortgage (V_j^M)

Figure 4

Discrete choices in the household's problem.

The timing of the model in each period is as follows. At the beginning of the period, a household in their working life receives their depreciation and income shocks; a retired household only receives a depreciation shock. A household makes a decision over the discrete choices available to them by solving the associated optimization problems and selecting the choice that yields the highest expected lifetime utility. The set of feasible discrete choices depends on whether the household is an existing renter or owner and is summarized in Figure 4. If the household is an existing renter, they either continue as a renter (V_j^R) or purchase a house (V_j^M). If the household is an existing owner, they can remain an owner either by making a payment on an existing mortgage loan (V_j^P) or by obtaining a new mortgage loan, which can occur if the owner refinances the loan or adjusts their housing stock (V_j^M). An existing owner can transition to renting by selling their house (V_j^R) or defaulting on outstanding debt, if any (V_j^D). Consumption occurs at the end of the period.

If a household chooses to rent, then they choose nondurable consumption, rented housing services, and liquid savings to solve

$$V_j^R(\omega) = \max_{c,s,a'} u(c,s) + \beta \mathbb{E}_{\delta_h',z'|z} \max \left\{ V_{j+1}^R(\omega'), V_{j+1}^M(\omega') \right\}$$

s.t.

$$c + Rs + a' \leq y_j(z) + (1+r)a + (1-\delta_h)h - (1+r_m)m - \mathbb{1}_{h\neq 0}\kappa_h \tag{9}$$

$$a' \geq 0$$

$$\omega' = (a', 0, \delta_h', 0, 0, 0, z').$$

The right-hand side of the flow budget constraint defines the household's cash on hand, which includes current income, liquid wealth, and, if the household is transitioning from owning to renting, the sale value of the house net of depreciation, repayment of any outstanding mortgage debt, and the housing adjustment cost. The continuation value reflects the fact that a household who rents today can continue renting tomorrow or become a homeowner.

If a household chooses to own and obtain a new mortgage, they choose nondurable consumption, liquid savings, housing wealth, mortgage debt, and type of mortgage contract to solve

$$V_j^M(\omega) = \max_{c,a',h',m',q'\in\{L,H\}} u(c,h') + \beta \mathbb{E}_{\delta_h',z'|z} V_{j+1}(\omega')$$

s.t.

$$c + a' + h' \leq y_j(z) + (1+r)a + (1-\delta_h)h - (1+r_m)m + m' - \mathbb{1}_{h'\neq h}\kappa_h - \kappa_m$$

$$m' \leq \theta h' \tag{10}$$

$$\pi_{min,j}\left(m', r_{m,j}'(\omega)\right) \leq \lambda(q')y_j(z)$$

$$a' \geq 0$$

$$\omega' = \left(a', h', \delta_h', m', q', r_{m,j}'(\omega), z'\right).$$

This optimization problem is central to the model. It both presents households with the broadest menu of choices and captures the interaction of borrowers with the mortgage market. As described in Section 5.1, the size of the new loan is subject to constraints on the LTV and DTI ratios, where the tightness of the DTI constraint $\lambda(q')$ depends on the type of mortgage contract chosen. Crucially, when a household chooses to obtain a new mortgage, the interest rate they receive on the new loan, $r_{m,j}'(\omega)$, is determined in equilibrium, is a function of the household's idiosyncratic state, and is carried into the future. Similarly, the borrower's choice of mortgage contract type q' is also carried forward into tomorrow's state. The flow budget constraint accommodates owners who sell their old house and purchase

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a new one, as well as owners who leave their housing stock unchanged but refinance their existing mortgage.

If a household chooses to own and continue with an existing mortgage loan, they choose nondurable consumption, liquid savings, and a mortgage payment to solve

$$V_j^P(\omega) = \max_{c,a',m'} u(c,h) + \beta \mathbb{E}_{\delta_h',z'|z} V_{j+1}(\omega')$$

$$\text{s.t.}$$

$$c + \delta_h h + a' \le y_j(z) + (1+r)a - (1+r_m)m + m'$$

$$m' \le (1+r_m)m - \pi_{min,j}(m, r_m)$$

$$a' \ge 0$$

$$\omega' = (a', h, \delta_h', m', q, r_m, z').$$

(11)

The inequality in the law of motion for mortgage debt gives borrowers the option to prepay their mortgage, if desired, without incurring any penalty. The household's state in the next period ω' accounts for the fact that the household continues with their predetermined housing stock, type of mortgage contract, and mortgage interest rate.

If an existing borrower exercises the option to default, they choose consumption, rented housing services, and liquid savings to solve

$$V_j^D(\omega) = \max_{c,s,a'} u(c,s) - \xi + \beta \mathbb{E}_{\delta_h',z'|z} \left[\varphi V_{j+1}^M(\omega') + (1-\varphi) V_{j+1}^R(\omega') \right]$$

$$\text{s.t.}$$

$$c + Rs + a' \le y_j(z) + (1+r)a$$

$$a' \ge 0$$

$$\omega' = (a', 0, \delta_h', 0, 0, 0, z').$$

(12)

A household that defaults does not make the mortgage payment or housing maintenance costs due in the current period and consequently loses their house, incurs a flow utility loss ξ, and is excluded from the owner-occupied housing and mortgage markets. The household begins the next period with zero outstanding mortgage debt and zero housing wealth and regains access to the owner-occupied housing and mortgage markets with exogenous probability φ. I assume that, regardless of the type of mortgage contract on which a borrower has defaulted, they solve the same optimization problem.

In the last period of life, the household must repay any outstanding mortgage debt and is prohibited from further borrowing. This places a restriction that $m' = 0$ on the problem in Equation (10), but the optimization problems that an age-T household solves are otherwise

unaltered. End-of-life wealth is defined as

$$W_T = (1 + r) a' + h'.$$

5.3 Financial intermediary's optimization problem

Mortgage contracts are issued by the financial intermediaries. Intermediaries discount the future at a rate $r + \phi$, where ϕ is a parameter that captures mortgage servicing costs.[28] I assume that financial intermediaries can perfectly observe a household's current state and decision rules. Competition ensures that each mortgage is priced so that the face value of the loan equals the expected present value of its future cash flows.

The present value of the existing mortgage held by an age-j household can be written as

$$\Pi_j (\omega) = \begin{cases} (1 + r_m) m & \text{if repay} \\ (1 - \delta_h) h - \gamma (q) & \text{if default} \\ (1 + r_m) m - m'_j (\omega) + \frac{1}{1+r+\phi} \mathbb{E}_{\delta'_h, z'|z} \Pi_{j+1} (\omega') & \text{otherwise,} \end{cases} \tag{13}$$

where $\omega' = \left(a'_j (\omega), h'_j (\omega), \delta'_h, m'_j (\omega), q'_j (\omega), r'_{m,j} (\omega), z' \right)$. This equation summarizes the three possible actions that a household with an existing loan could take. If the borrower repays the loan in full, then the intermediary receives the remaining balance on the loan plus interest. If the borrower defaults on the mortgage, then the intermediary recovers the depreciated value of the underlying collateral less foreclosure costs.[29] Recall from Equation (7) that the foreclosure costs are a function of the predetermined type of contract on which the household defaults such that

$$\gamma (q) = \begin{cases} \gamma_L & \text{if } q = L \\ \gamma_H & \text{if } q = H, \end{cases}$$

where $\gamma_L < \gamma_H$ by assumption. Finally, a borrower can continue with a loan by making a mortgage payment. The intermediary then receives the payment and the continuation value of the loan.

In the last period of life, a borrower either repays or defaults on all outstanding debt.

This requirement pins down the present value of a mortgage held by age-T household so that

$$\Pi_T (\omega) = (1 + r_m) m$$

in the case of repayment and

$$\Pi_T (\omega) = (1 - \delta_h) h - \gamma (q)$$

in the case of default. These terminal conditions make it possible to compute $\Pi_j (\omega)$ for $j \in \{1, 2, \ldots, T - 1\}$ using backward induction.

Free entry among financial intermediaries implies that, on a loan-by-loan basis, the face value of a newly originated loan equals the expected present value of future cash flows so that the lender earns zero profit:

$$m'_j (\omega) = \frac{1}{1 + r + \phi} \mathbb{E}_{\delta'_h, z'|z} \Pi_{j+1} (\omega') . \tag{14}$$

Thus, conditional on the household's current idiosyncratic state, the financial intermediary will offer an interest rate $r'_{m,j} (\omega)$ such that Equation (14) is satisfied.

5.4 Equilibrium definition

I solve for the stationary recursive equilibrium of the model. To establish notation, I define the state space W as Cartesian product $A \times H \times D \times M \times Q \times R_M \times Z$, and let the σ-algebra Σ_W be defined as $B_A \otimes B_H \otimes B_M \otimes B_{R_M} \otimes P (D) \otimes P (Z)$, where B_A, B_H, B_M, B_Q, and B_{R_M} are the Borel σ-algebras on A, H, M, Q, and R_M, respectively, and $P (D)$ and $P (Z)$ are the power sets of D and Z, respectively. Let $\Omega = \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \mathcal{Q} \times \mathcal{R_M}$ be the typical subset of Σ_W.

For a given parameterization of the model and a distribution of age-1 households μ_1, a stationary recursive equilibrium consists of

1. household value functions $\{V_j^R (\omega), V_j^M (\omega), V_j^P (\omega), V_j^D (\omega)\}$ and policy functions $\{c_j (\omega), s_j (\omega), a'_j (\omega), h'_j (\omega), m'_j (\omega), q'_j (\omega)\}$;

2. a mortgage interest rate schedule $r'_{m,j} (\omega)$; and

3. a stationary measure $\Lambda_j^* (\Omega)$;

such that,

1. given $r'_{m,j}(\omega)$, household value and policy functions solve the optimization problems in Equations (8), (9), (10), (11), and (12);

2. given the household's policy and value functions, $r'_{m,j}(\omega)$ is such that financial intermediaries' zero-profit condition in Equation (14) is satisfied; and

3. the invariant probability measure satisfies

$$\Lambda^*_{j+1}(\Omega) = \int_\Omega Q_j(\omega, \Omega) \left[\Lambda^*_{j+1}(d\omega) + \mu_1(d\omega) \right] \tag{15}$$

for all $\Omega \in \Sigma_W$ and where the transition function $Q_j(\omega, \Omega)$ is defined as

$$Q_j(\omega, \Omega) = \mathbb{1}_{a'_j(\omega) \in \mathcal{A}, \, h'_j(\omega) \in \mathcal{H}, \, m'_j(\omega) \in \mathcal{M}, \, q'_j(\omega) \in \mathcal{Q}, \, r'_{m,j}(\omega) \in \mathcal{R}_\mathcal{M}} \sum_{\delta'_h} \sum_{z'} \pi(\delta'_h) \pi(z'|z). \tag{16}$$

I solve the model numerically using backwards induction and provide a detailed outline of the solution algorithm in Section A.5 of the appendix.

6 Calibration

Before I can use the model to evaluate the distributional consequences of the Freddie Mac and Dodd-Frank policies and forecast the potential effects of the Dodd-Frank DTI limit given that the Freddie Mac regulation is already in place, I must ensure that the parameterized model is a reasonable environment for those quantitative exercises. To that end, the goal of the calibration is to have the stationary distribution of the model with the Freddie Mac policy in effect reproduce key features of aggregate, household-level, and loan-level data after the Great Recession, including the stylized facts from Section 4.2. The Freddie Mac policy is implemented in the model by setting the maximum DTI ratio on a low-DTI loan λ to 45 percent and restricting all borrowers to low-DTI loans (i.e., $q' = L$ for all households in all states). I calibrate a subset of parameters externally, and the remainder are calibrated internally to match moments in the data. I assess the fit of the model by comparing life-cycle profiles of household wealth accumulation and the distribution of loan characteristics in the data versus the model.

6.1 Externally calibrated parameters

Table 2 summarizes the externally calibrated parameters and their sources. These values are based either on direct empirical evidence or are taken from the existing literature.

	Description	Value	Source
μ_1	Distribution of age-1 households		SCF (2016)
$\{\chi_j\}_{j=1}^T$	Age-specific income		PSID (1999-2015)
λ	Maximum DTI ratio on low-DTI loan	0.45	Freddie Mac
φ	Exclusion from mortgage market	0.14	Equifax
r	Risk-free rate	0.017	FRED
ρ	Persistence of income shock	0.91	Floden and Lindé (2001)
σ	Risk aversion	2	Standard in literature
σ_ϵ	Standard deviation of income shock	0.21	Floden and Lindé (2001)
T	Number of model periods	59	U.S. life expectancy
T_R	Retirement age	44	Normal retirement age
θ	Maximum LTV ratio	0.85	Greenwald (2018)

Table 2

Externally calibrated parameters.

Initial distribution of households

I divide the measure of age-1 households μ_1 across idiosyncratic income states in accordance with the invariant distribution of z. Letting N_z be the number of income states after discretization, I stratify a sample of households with heads between the ages of 22 and 27 in the 2016 Survey of Consumer Finances (SCF) into N_z groups according to their labor income.[30] Within each income group, I compute the homeownership rate, the fraction of homeowners with mortgages, mean liquid assets, and mean home equity. I initialize the cross-sectional distribution of age-1 households in the model to match these means.

Preferences and endowments

One period in the model corresponds to one year. Households enter the model at age 22, retire at age 65, and die at age 80, implying $T = 59$ and $T_R = 44$. As is standard in the macroeconomics literature, the coefficient of relative risk aversion σ is set equal to 2.

Following Floden and Lindé (2001), I set the persistence of the idiosyncratic income process ρ to 0.91 and its standard deviation σ_ε to 0.21. As in Kaplan and Violante (2014), I use data from the 1999-2015 waves of the Panel Study of Income Dynamics to estimate the deterministic component of income $\{\chi_j\}_{j=1}^T$ by regressing log annual household labor income on a quartic polynomial in age for a sample of households whose heads are between ages 22 and 64. Pension income received in retirement is modeled after Guvenen and Smith (2014).[31]

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[30]I set $N_z = 5$ when solving the model numerically.

[31]Section A.4 in the appendix contains further details on the parameterization of the deterministic component of income and pension income.

Asset technology

I set the risk-free rate r equal to the 1-year Treasury constant maturity rate less the annual rate of CPI inflation and average the difference over the years 1971-2016. This results in a value of 1.27 percent. The probability that a household who has defaulted on their mortgage can regain access to the mortgage market in the next period, φ, is 0.143. This implies a mean duration of exclusion from borrowing of 7 years and corresponds to the length of time that a foreclosure flag remains on a consumer's credit report.[32] Since I calibrate the stationary distribution of the model with the assumption that the Freddie Mac policy is already in effect, I set the maximum DTI ratio on a low-DTI loan λ to 0.45. I set the maximum LTV ratio θ to 0.85 in line with Greenwald (2018).

6.2 Internally calibrated parameters

A set of 10 parameters in the model are selected jointly to minimize the weighted distance between a vector of moments from the stationary distribution and their empirical counterparts. I define the calibration loss function as

$$\sum_{i=1}^{10} \text{weight}_i \left(\frac{\text{model}_i - \text{data}_i}{1 + |\text{data}_i|} \right)^2. \tag{17}$$

The three moments that receive relatively more weight in the calibration loss function are the foreclosure rate (0.7 percent), the homeownership rate (62 percent), and the fraction of newly originated loans with DTI ratios between 43 and 45 percent (10 percent). The first two moments are relevant policy objectives, and the third is a measure of the bunching observed in the distribution of DTI ratios on loans purchased by Freddie Mac. I use the 2016 SCF to compute the cross-sectional moments regarding household wealth portfolios used in this procedure. I exclude households whose net worth is above the 95th percentile because the SCF is designed to over-sample relatively wealthy families for whom home equity is a less important form of savings.[33]

Table 3 provides a list of the 10 internally calibrated parameters plus the two foreclosure

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[32]See https://www.equifax.com/personal/understanding-credit/.

[33]The SCF over-samples families that are likely to be relatively wealthy in order to increase representation of the upper tail of the wealth distribution and to make possible analyses of less widely held asset classes— e.g., direct holdings of government bonds—that would otherwise require a prohibitively large sample size. These families correspond to the "list sample" since they are selected using specially edited individual tax returns provided by the Internal Revenue Service. In the 2004 SCF, the list sample accounts for only 15 percent of observations in the bottom 95 percent of the wealth distribution but 88 percent of observations in the top 5 percent. See https://www.federalreserve.gov/econresdata/scf/files/scf2001list.sampleredesign9.pdf for more information on the survey design of the SCF.

	Description	Value	Target	Data	Model
α	Nondurable consumption weight	0.898	Aggregate housing stock	2.73	2.13
B	Bequest weight	5.803	Share of retired owners with mortgage	0.37	0.33
β	Discount factor	0.905	Aggregate net worth	2.27	2.24
δ	Depreciation shock	0.481	Foreclosure rate (percent)	0.70	0.62
h_{small}	Smallest house size	9.284	Aggregate liquid wealth	0.64	0.65
κ_h	Housing transaction cost	0.287	Share of mortgages with DTI $\in (43, 45]$	0.10	0.09
κ_m	Mortgage transaction cost	0.093	Share of owners with mortgage	0.63	0.61
ϕ	Mortgage servicing cost	0.022	Mean mortgage interest rate (percent)	4.17	4.06
R	Housing rental rate	0.916	Homeownership rate	0.62	0.57
ξ	Utility loss from default	1.486	Mean LTV ratio	0.39	0.26
ζ	Depreciation shock probability	0.056	Mean depreciation rate (percent)	2.27	2.27
γ_L	Low-DTI loan default cost	0.287	Set to housing transaction cost κ_h		
γ_H	High-DTI loan default cost	2.138	CFPB cost-benefit analysis from *Federal Register*		

Table 3

Parameters chosen to match moments in the data. The values for aggregate housing stock, net worth, and liquid wealth are relative to mean household income in the stationary distribution.

costs, the moments targeted in the estimation, and the value of their model equivalents.[34] Although the parameters governing the foreclosure costs, γ_L and γ_H, are not directly involved in computing the calibration loss function, I place restrictions on their values that depend on the internally calibrated parameters and therefore include them in this table. I discuss the parameterization of the foreclosure costs later in this section.

Preferences and endowments

The weight on nondurable consumption α is calibrated to 0.898, and the discount factor β is calibrated to 0.905. Both are similar to values found in the literature. The parameter governing the strength of the bequest motive, B, is set to 5.803. Too small a value of B leads retired agents to obtain new mortgages near the end of life and leads to a large spike in the default rate at age T. The flow utility loss from default ξ is set to 1.486.

Asset technology

The smallest size house available for purchase, h_{small}, is set to 9.284, about 3.7 times larger than average household income in the model. Given the general rule of thumb that the price of a home should not exceed 2.5 to 3 times one's annual income, this is a plausible estimate. h_{small} is important in determining the fraction of borrowers who are near the 45-percent DTI limit. A large enough minimum house size is needed to push borrowers towards a loan size with a sufficiently large mortgage payment. If h_{small} is too large, though, marginal

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[34]Section A.4 details how the targeted moments were calculated.

homeowners choose to rent instead, resulting in a more creditworthy pool of borrowers who are more likely to choose loan sizes with low DTI ratios.

The housing and mortgage adjustment costs, κ_h and κ_m, are calibrated to 0.287 and 0.093, respectively. Taken together, these values imply that a household who finances the purchase of a new home with a mortgage loan will pay 4.1 percent of the value of the home in transaction costs and that a homeowner who refinances their mortgage must pay a loan origination fee equal to 3.7 percent of average household income.[35] The estimates for the two adjustment costs are broadly in line with previous values found in the literature.[36]

The depreciation shock δ is set to 0.481 and is essential in helping the model match the observed foreclosure rate of 0.7 percent.[37] Negative equity is a necessary—though not sufficient—condition for default in this class of models. Since the loan-to-value constraint prevents households from starting their tenure as owners with negative equity and the price of owner-occupied housing is constant in my model, a sufficiently large depreciation shock is needed to generate borrowers who are underwater on their mortgages. The probability of the depreciation shock ζ is set so that the expected value of the depreciation shock equals 2.27 percent, the depreciation rate of private residential investment estimated by the Bureau of Economic Analysis. Given the calibrated value of δ, this implies that ζ equals 0.056 so that, on average, a household experiences a positive depreciation shock once every 18 years.

The mortgage servicing cost ϕ is set to 0.022. Intuitively, this allows the model to match the average interest rate on the 30-year fixed rate mortgage of 4.17 percent.[38] The rental rate of housing R is set to 0.916, quite a bit higher than the user cost of housing implied by the risk-free rate, housing depreciation rate, and the cost of borrowing. This wedge generates additional incentive for homeownership in the model.

Lender's foreclosure costs

The parameterization of the lender's foreclosure costs, γ_L and γ_H, is critical to the quantitative results of the model. I restrict the cost of default on a low-DTI loan γ_L to equal to the internally calibrated value of the housing adjustment cost κ_h of 0.287. This restriction is motivated by the fact that, in reality, the lender is responsible for selling repossessed properties in the aftermath of foreclosure. It is thus reasonable to assume that,

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[35]This is assuming that the household buys a house of size h_{small}, which is indeed the most commonly chosen house size in the stationary distribution.

[36]Berger and Vavra (2015) estimate that the fraction of the value of durable goods lost to adjustment costs is 5.3 percent. Gorea and Midrigan (2018) arrive at a calibrated value for the fixed cost of obtaining a new home equity loan that is 2.3 percent of mean per-capita income in their model.

[37]See https://www.attomdata.com/news/heat-maps/2016-year-end-u-s-foreclosure-market-report/.

[38]The mean mortgage interest rate is the interest rate on 30-year fixed-rate mortgages less annual CPI inflation, averaged over 1971-2016. The data used for calculating this are from FRED.

if a household defaults on a mortgage, the intermediary will bear the costs of selling the underlying collateral.

Because the Dodd-Frank Act created a new legal liability relative to existing rules for lenders who originate high-DTI loans, I restrict the cost of default on a high-DTI loan γ_H to be greater than the cost of default on a low-DTI loan γ_L. As discussed in Section 3, if a borrower with a non-qualified mortgage brings a successful legal claim under the ability-to-repay rule, the lender is liable for up to three years of fees and finance charges, along with the borrower's legal expenses. This stands in contrast to the strong legal protection given to mortgage loans that satisfy the 43-percent DTI constraint.

To discipline the value of γ_H, I follow a cost-benefit analysis of the Dodd-Frank ability-to-repay rule that the Consumer Financial Protection Bureau (2013) included in its submission of the final rule to the *Federal Register*. Part of this analysis includes an estimate of the resource costs implied by the legal liability assigned to a high-DTI loan. To start, I define a typical mortgage in the calibrated model as having an initial balance of 6.16, an interest rate of 4.06 percent, a maturity of 30 years, and an origination fee of 0.093.[39] In line with the CFPB's analysis, I use the midpoint of the three-year window to calculate the interest payments and loan fees that would be owed by a lender. For the typical mortgage in the model, this amounts to 0.46 units of the nondurable consumption good. The CFPB estimates that lenders' and borrowers' legal expenses would sum to $34,500. This is equal to 66 percent of mean household income and translates to 1.68 units of nondurable consumption.[40] In total, the resource loss incurred by a lender due to default on a high-DTI loan, γ_H, is 2.14. This is around 35 percent of the initial loan size and more than 7 times the size of γ_L, the calibrated cost of foreclosure on a low-DTI loan to the lender.

In reality, not every borrower who is unable to repay a mortgage loan will bring a case against the responsible lender, as that decision will depend on, among other factors, whether the borrower lives in a judicial or non-judicial foreclosure state and their willingness and/or ability to obtain legal representation. Substantial evidence nonetheless suggests that mortgage lenders are indeed worried about these regulations.[41] Fuster, Lo and Willen (2017) estimate that, over 2008-2014, the price of intermediation in the mortgage market increased by around 30 basis points per year and that this trend appears to be driven by increased net costs of mortgage servicing and lenders growing more averse to liability risk. Kim et al.

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[39]I compute the initial loan balance by multiplying the average LTV ratio at origination in the stationary distribution, 0.66, by h_{small}. In the stationary distribution, the mean mortgage interest rate at origination is 4.06 percent. The origination fee is equal to κ_m, the calibrated mortgage transaction cost in the model.

[40]Mean household labor income in the 1998 SCF is $52,108 in 2013 CPI-U-RS adjusted dollars. Mean household labor income in the model is 2.53.

[41]The CFPB, for one, concedes that its estimate of litigation costs relies on "very conservative (likely unrealistic) assumptions."

(2018) document that, after the financial crisis, the GSEs and U.S. government required loan originators to repurchase mortgages collateralizing GSE securities if one or more of the "representations and warranties" made upon selling the loans to the GSEs was inaccurate. By the third quarter of 2015, these lender repurchases amounted to $76.1 billion. They also note that, under the auspices of the False Claims Act, the Department of Justice has litigated cases in which Federal Housing Administration loans were improperly originated and that cumulative settlements under this effort amounted to $6.6 billion. In light of this, the fact that Dodd-Frank creates the potential for increased legal claims against mortgage lenders, even if there is uncertainty about how many will claims will ultimately be brought, should be taken seriously.

6.3 Model fit

Overall, the model does a good job of matching the targeted moments. It is able to replicate the foreclosure rate and generates a homeownership rate that is close to what is seen in the data. The model also matches the share of newly originated mortgages that have a DTI ratio between 43 and 45 percent, thereby producing the bunching at the DTI limit documented in Section 4.2. The model understates the ratio of aggregate mortgage debt to aggregate housing stock relative to the data (0.26 versus 0.39, respectively) and has difficulty replicating the ratio of the aggregate housing stock to mean income (2.73 in the data versus 2.14 in the model). This may be a result of the fact that house prices in my model are fixed.

I evaluate the fit of the model by assessing its ability to account for non-targeted moments in the data. Figure 5 plots life-cycle profiles of wealth accumulation for households from the 2016 SCF and their model counterparts.[42] I focus on results from agents' working life (i.e., ages 22-64) since my model's approach to retirement is too parsimonious to adequately capture the sources of risk and financial decisions that confront aging households in reality.[43] As in the data, the model generates a gradual increase over the life cycle in net worth relative to income. It slightly overstates net worth of households in the years immediately preceding retirement, a feature that is driven by the over-accumulation of liquid assets in anticipation of lower expected income. This is likely driven by the fact that the model abstracts from other savings instruments that households use for smoothing consumption during retirement. The model captures well the steady accumulation of home equity over the life cycle, but households in the model at all ages hold less leveraged positions in their home than in the data. The model reproduces the increase, then gradual flattening out, of the homeownership

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[42]When computing these life-cycle profiles, I also exclude households in the top 5 percent of the net worth distribution in order to be consistent with the calibration strategy.

[43]Specifically, I am unable to produce the large accumulation of wealth, relative to labor income, observed in the data.
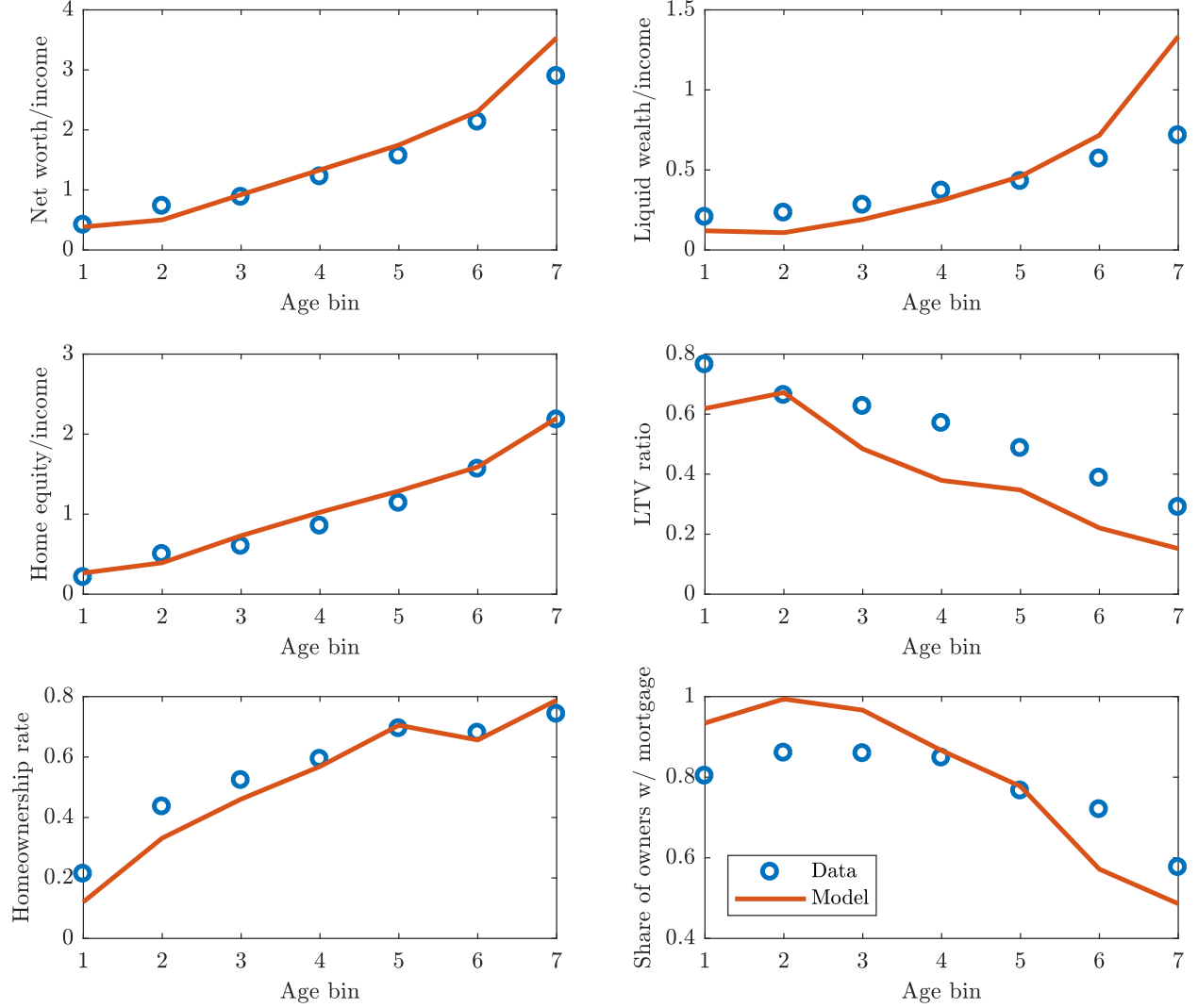
Figure 5

Life-cycle wealth accumulation in the data (blue circles) and the model (red line). Data moments are computed from the 2016 Survey of Consumer Finances. Model moments are computed assuming the Freddie Mac 45-percent DTI limit is in place. Age bins: 1 = 22-27, 2 = 28-33, 3 = 34-39, 4 = 40-45, 5 = 46-51, 6 = 52-57, 7 = 58-64.

rate as households age. It also able to replicate the fact that the share of owners with mortgage debt decreases over the life cycle.

Next, I compare mortgage originations in the stationary distribution of the model to those in the Freddie Mac Single Family Loan-Level Dataset. To make a consistent comparison, I compute the empirical moments from mortgage loans that were originated in 2016. As can be seen in Table 4, my model predicts that, on average, a newly originated mortgage loan has a LTV ratio of 66 percent, a DTI ratio of 18 percent, and an interest rate of 4.06 percent, compared to 77 percent, 35 percent, and 3.90 percent in the data, respectively.

	Data	Model
Mean LTV ratio at origination	76.6	66.3
Mean DTI ratio at origination	34.7	18.3
Mean mortgage interest rate at origination	3.90	4.06

Table 4

Mean loan characteristics at origination in the data and the model. Model moments are computed assuming the Freddie Mac 45-percent DTI limit is in place, and data moments are computed from mortgages originated in 2016 in order to ensure comparability with the model calibration.
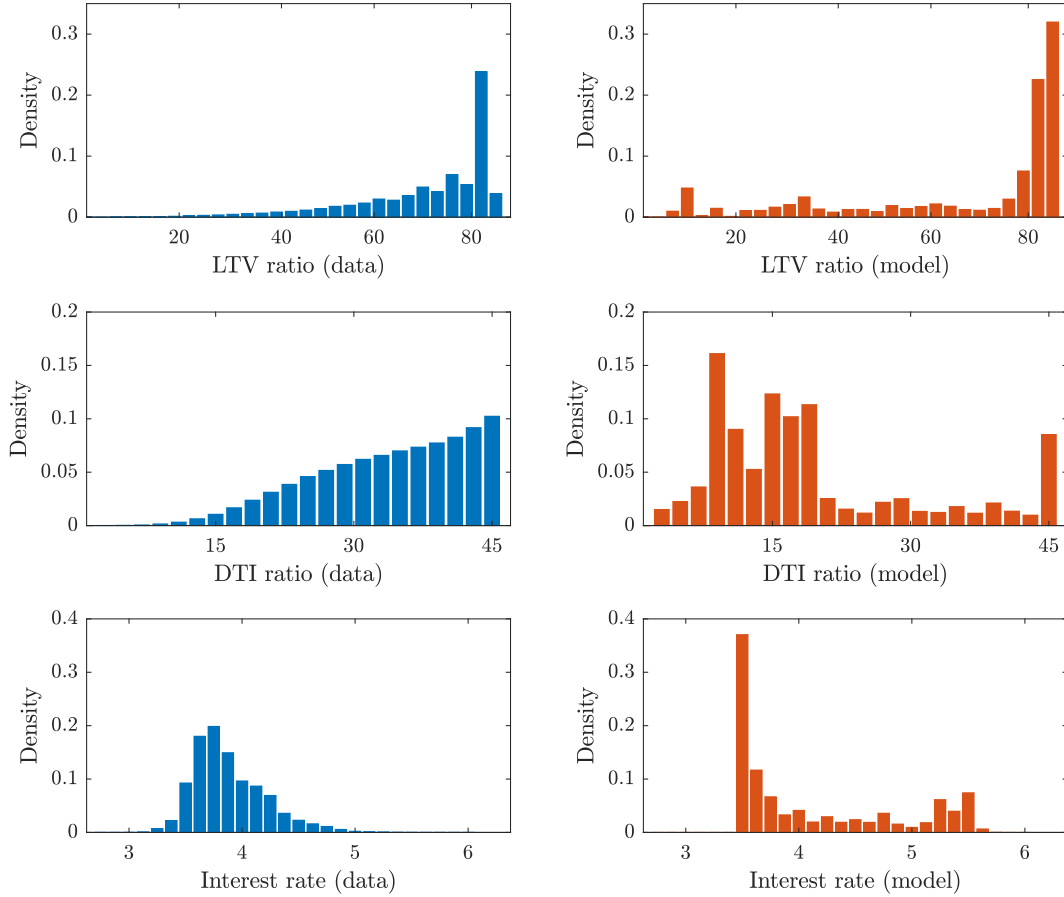

Figure 6

Distributions of LTV ratio, DTI ratio, and interest rate at origination in the data (blue bars) and the model with the Freddie Mac 45-percent DTI limit in place (red bars). The data distributions are computed from mortgages originated in 2016 in order to ensure comparability with the model calibration.

Figure 6 plots model-implied distributions of the LTV ratio, DTI ratio, and the interest rate at origination against their data counterparts. The model-implied distribution of LTV ratios at origination matches well the distribution in the data. By design, the distribution of DTI ratios in the model produces a mass of loans immediately to the left of the 45-percent

mark; however, it also generates a counterfactually large share of loans with very low DTI ratios. This is likely a consequence of making a maturity of the loan a function of household age. A borrower who obtain a mortgage very early in their life receives a mortgage whose maturity exceeds what it is seen in reality, where 30-year terms are the norm. *Ceterus paribus*, this mechanically implies a small minimum mortgage payment. Since these are not the borrowers that should be most directly affected by changes in the DTI constraint, though, it is less relevant for the policy analysis that follows. Finally, the model generates dispersion in the cost of borrowing, matching the fact that a plurality of borrowers face low mortgage interest rates while a smaller share of borrowers have a higher objective probability of default and consequently receive a higher interest rate.

7   Quantitative analysis of DTI limits

Having verified that the calibrated model is a good representation of the mortgage market as it currently stands while also capturing the life-cycle consumption and savings behavior of households, I use it to assess the aggregate and distributional consequences of two policies, Freddie Mac's 45-percent DTI limit and Dodd Frank's 43-percent DTI limit with a costly option to relax the limit. To illustrate the effects of these policies, I consider two policy changes. The first is a change from a setting characterized by the absence of a DTI limit— what I refer to as the no-DTI-limit baseline—to Freddie Mac's 45-percent DTI limit. The no-DTI-limit baseline is implemented by restricting borrowers to a high-DTI loan and setting the lender's foreclosure costs to its low value, γ_L, for all loans. This parameterization proxies a pre-crisis regulatory environment in which only the standard LTV constraint applies and mortgages with high DTI ratios are not penalized by greater legal costs. The second is a change from the Freddie Mac policy to the Dodd-Frank policy. I implement the Dodd-Frank regulation in the model by lowering the maximum DTI ratio on a low-DTI loan λ to 43 percent and allowing households to optimally choosen between a high- or low-DTI loan. These two sets of comparisons are designed to capture the observed progression of household leverage regulations and provide the appropriate initial policy against which to benchmark the pending Dodd-Frank regulations.[44] I summarize the mortgage contract parameters for each policy environment in Table 5.

I begin by quantifying the trade-off between mortgage default and homeownership that results from the two policies. I then study the patterns of borrower selection that can explain the aggregate decline in mortgage default. The Freddie Mac policy reduces leverage

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[44]I abstract from transition dynamics and assume that the economy unexpectedly jumps from one steady state to another.

	θ	λ	γ
No DTI limit			
High-DTI loan	0.85	∞	0.29
Freddie Mac			
Low-DTI loan	0.85	0.45	0.29
Dodd-Frank			
Low-DTI loan	0.85	0.43	0.29
High-DTI loan	0.85	∞	2.14

Table 5

Mortgage contract parameters under the no-DTI-limit baseline, the Freddie Mac policy, and the Dodd-Frank policy. θ is the maximum LTV ratio, λ is the maximum DTI ratio, and γ is the lender's foreclosure cost.

most among low-income and low-wealth households, and that in turn drives most of the observed decline in the aggregate default rate. Relative to the Freddie Mac DTI limit, the Dodd-Frank policy shapes household decisions in two opposing ways. On one hand, the more strict cap on the DTI ratio further reduces borrowing and foreclosure among the poorest households. On the other hand, wealthier households who have low incomes use the costly high-DTI contract to increase their leverage, thereby raising the default rate among this group. Because default is already extremely uncommon among wealthier households, though, the first channel dominates the second so that, overall, the aggregate foreclosure rate is lower. Finally, I show that the Dodd-Frank policy improves aggregate welfare relative to the Freddie Mac policy but produces heterogeneous changes in welfare across individual households: homeowners are better off, while renters are worse off.

7.1 Aggregate effects of DTI limits

In Table 6, I outline the consequences of the Freddie Mac and Dodd-Frank regulations for aggregate outcomes. I find that, in the model, a constraint on a borrower's debt payment-to-income ratio is effective at reducing mortgage foreclosure. Relative to the no-DTI-limit baseline, the Freddie Mac 45-percent DTI limit reduces the foreclosure rate by half from 1.20 percent to 0.62 percent. At the same time, the Freddie Mac policy lowers the homeownership rate from 65 to 57 percent. This is also a substantial decline.[45]

Relative to the Freddie Mac regulations, the Dodd-Frank policy further reduces the default and homeownership rates slightly. The lower foreclosure rate occurs in spite of the fact that, under the Dodd-Frank regulations, the share of high-DTI loans among newly

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[45]To place the decrease in the homeownership rate into perspective, the homeownership rate in the United States reached 69 percent during the peak of the housing boom before falling to 63 percent in 2016.

	No DTI limit	Freddie Mac	Dodd-Frank
Default rate (%)	1.20	0.62	0.52
Homeownership rate (%)	64.7	57.0	56.3
% owners with mortgage	67.8	61.2	62.6
LTV ratio (%)	71.8	66.3	65.7
DTI ratio (%)	51.9	18.3	49.8
Mortgage interest rate (%)	4.28	4.06	3.97
% mortgages with DTI > 43	30.0	8.53	20.6
Aggregate net worth	2.26	2.24	2.20
Aggregate liquid wealth	0.63	0.65	0.64
Aggregate home equity	1.62	1.60	1.56

Table 6

Aggregate effects of the Freddie Mac and Dodd-Frank DTI limits, compared to the no-DTI-limit baseline. All moments are computed from the stationary distribution of the model under the different leverage regulations. Aggregate net worth, liquid wealth, and home equity are reported relative to aggregate income.

originated mortgages increases substantially, with 20.6 percent of borrowers choosing the costly high-DTI loan option.

7.2 The importance of borrower selection in lowering default

Next, I study the changes in household behavior that explain the aggregate effects of the two DTI policies and underscore the role of borrower selection in driving those results. In Figure 7, I plot the default rate, the homeownership rate, the percent of households who obtain new loans, the LTV ratio at origination, the DTI ratio at origination, and the high-DTI loan share of newly originated loans conditional on household income across the three regulatory environments (i.e., no DTI limit, Freddie Mac, Dodd-Frank). I do the same in Figure 8, conditioning instead on household net worth.

I begin by comparing household decisions under the no-DTI-limit baseline versus Freddie Mac's 45-percent DTI limit and find that the large declines in the aggregate default and homeownership rates are driven by a reduction in leverage among households in the bottom half of the income and wealth distributions. As Figure 7 demonstrates, borrowers in the bottom 40 percent of the income distribution obtain almost all mortgages that are originated with a DTI ratio greater than 43 percent; the Freddie Mac policy affects these households most. These households respond by reducing borrowing along both the extensive and intensive margins: the share of households in the two lowest income quintiles who still obtain a new loan after the Freddie Mac policy is introduced falls by half, and, conditional on choosing to get a mortgage, poor borrowers choose lower loan-to-value ratios
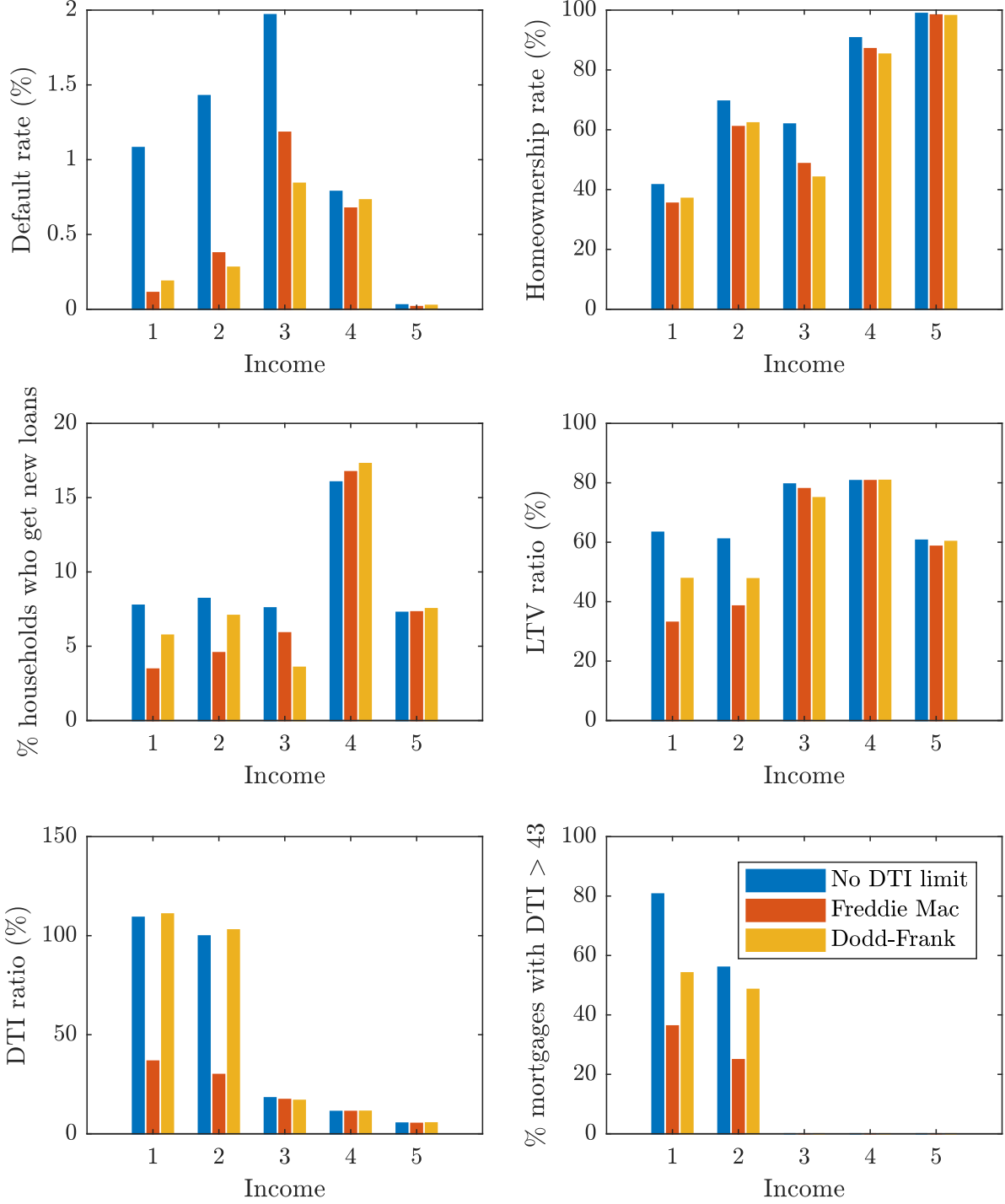
Figure 7

Default rate, homeownership rate, share of owners with mortgages, percent of households who obtain a new mortgage, LTV ratio at origination, DTI ratio at origination, and share of mortgage originations with a DTI ratio above 43 percent by income quintile. All moments are computed from the stationary distribution of the model under the different leverage regulations.
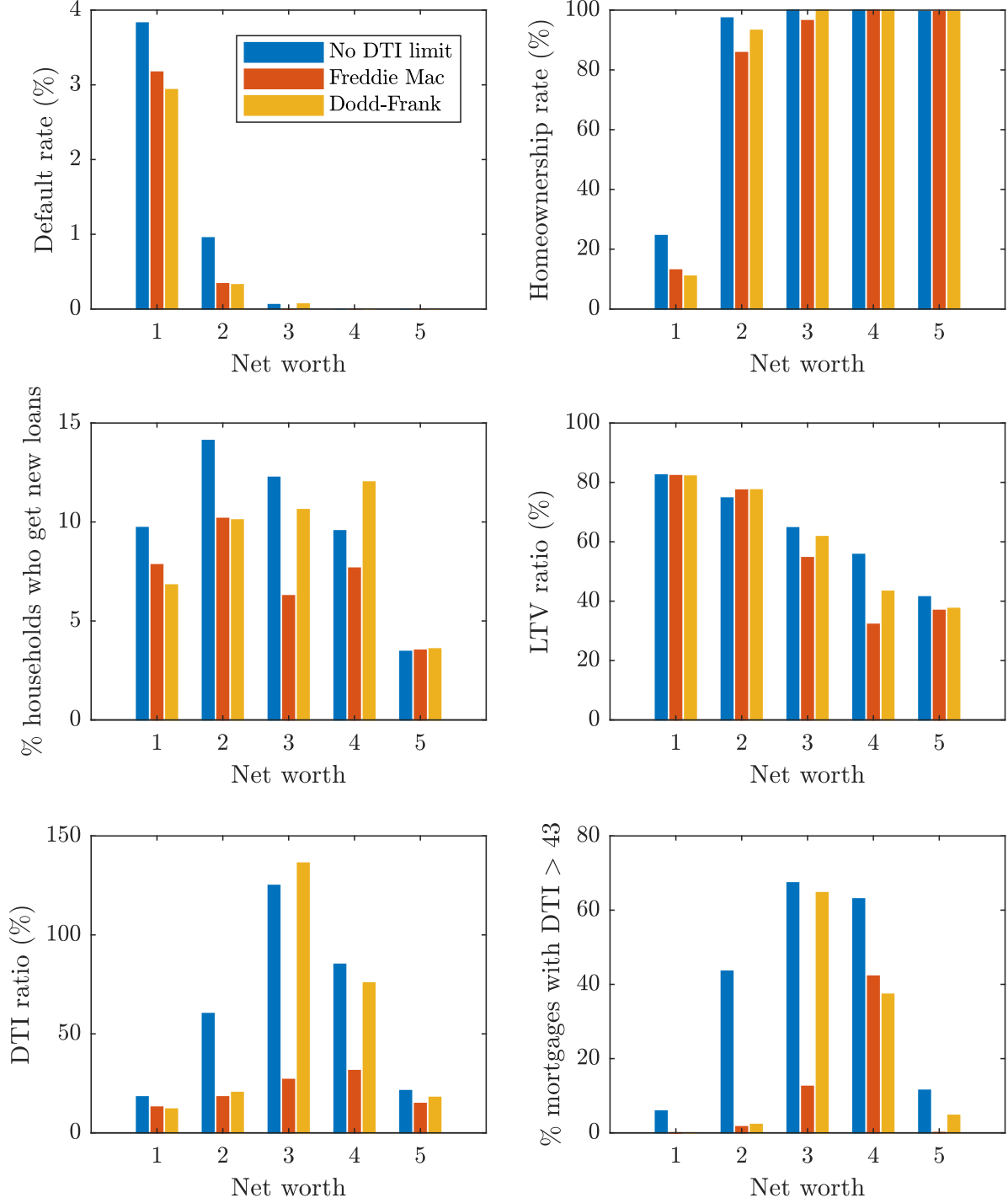
Figure 8

Default rate, homeownership rate, share of owners with mortgages, percent of households who obtain a new mortgage, LTV ratio at origination, DTI ratio at origination, and share of mortgage originations with a DTI ratio above 43 percent by net worth quintile. All moments are computed from the stationary distribution of the model under the different leverage regulations.

at origination. A similar result can be seen among borrowers who rank in the bottom 40 percent of the wealth distribution, as indicated by Figure 8.[46] The intuition for this result is straightforward. The introduction of a DTI limit will be most constraining for households with low contemporaneous income, high demand for mortgage debt, and a high minimum mortgage payment. In the presence of such a limit, many such borrowers will forgo a new loan altogether or, if they continue to choose to borrow, reduce the size of their loans. Since these households have a higher propensity for default, removing them from the borrower population reduces the overall default rate.

I next compare household decisions under the Freddie Mac policy to those under the Dodd-Frank policy, which lowers the maximum DTI ratio from 45 to 43 percent while providing the costly option to relax the limit. In Section 7.1, I found that, in the aggregate, the Dodd-Frank regulations slightly lowered the foreclosure rate while, at the same time, the fraction of borrowers with high-DTI loans more than doubled. Figures 7 and 8 clearly illustrate that it is households in the middle of the wealth distribution with low current incomes who exercise the costly high-DTI loan option after Dodd-Frank comes into effect. These are akin to the wealthy hand-to-mouth households discussed by Kaplan and Violante (2014). Intuitively, due to their lower cash on hand but high overall net worth, such borrowers have relatively high demand for mortgage debt for consumption smoothing purposes yet are also low credit risks from the financial intermediary's perspective. In equilibrium, then, these households are able to choose the costly high-DTI contract and many find it optimal to do so. The prevalence of high-DTI loans in the middle of the wealth distribution actually pushes up the default rate among this subset of borrowers. Because their probability of default is already near zero, however, this increase is outweighed by the large decline in foreclosures among the poorest households, many of whom, when faced with the more stringent DTI constraint on a low-DTI mortgage contract under Dodd-Frank, now choose to not obtain a new loan.

The selection of borrowers who are in the middle of the wealth distribution but have low cash on hand into high-DTI loans under the Dodd-Frank policy can also be seen in Figure 9. These graphs plot the model-implied mortgage interest rates and LTV ratios at origination as a function of the DTI ratio under both the Dodd-Frank policy and the no-DTI-limit baseline.[47] Although the model with the Dodd-Frank policy does not generate any visible discontinuity in either variable at the 43-percent DTI limit, the results are qualitatively consistent with my earlier empirical analysis of the differences in the characteristics of low-

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[46]Borrowers in the very middle of the wealth distribution also reduce their borrowing in response to the Freddie Mac policy; however, mortgage default is rare among these households.

[47]Since my model does not feature aggregate uncertainty, detrending is accomplished simply by subtracting from each variable their respective mean in the stationary distribution.

and high-DTI loans. The interest rate and LTV ratio on mortgages originated with higher DTI ratios are both less than those on mortgages with lower DTI ratios under both the no-DTI-limit setting and Dodd-Frank, but the difference is larger under the latter.
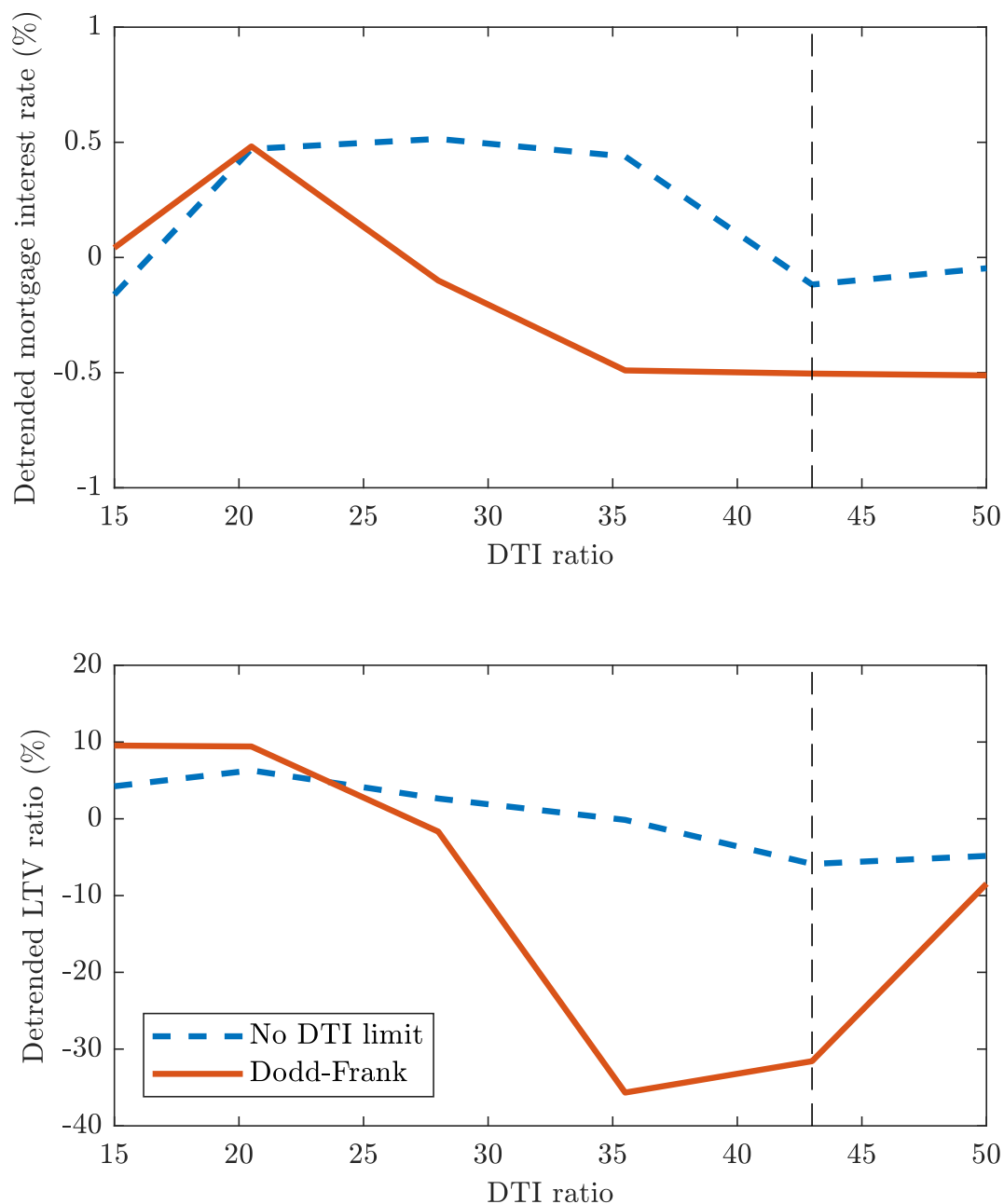

Figure 9

Model-implied interest rate and LTV ratio around the Dodd-Frank DTI limit versus the no-DTI limit. The dashed black line corresponds to the 43-percent limit.

7.3 The heterogeneous welfare consequences of DTI limits

Finally, I examine the consequences of the Freddie Mac and Dodd-Frank policies for household welfare and highlight the extent to which households are differentially affected by them. Let $V_j(\omega)$ and $\tilde{V}_j(\omega)$ be the value functions under the initial and new policies for an age-j household in state ω, respectively. Then the consumption-equivalent welfare change $\Delta C_j(\omega)$ is the percent by which the nondurable consumption of an age-j household in state ω would have to change to make them indifferent between the two policies:

$$\Delta C_j(\omega) = \left[\left(\frac{\tilde{V}_j(\omega)}{V_j(\omega)} \right)^{\frac{1}{\alpha(1-\sigma)}} - 1 \right] \times 100.$$

The average consumption-equivalent welfare change is thus

$$\Delta C = \int \Delta C_j(\omega) \, d\Lambda_j(\omega),$$

where $\Lambda_j(\omega)$ is the stationary distribution of households under the initial policy.

Although the uniformly applied DTI limit under the Freddie Mac policy results in a substantial decrease in the aggregate default rate and the average cost of borrowing, it lowers welfare. The average consumption-equivalent welfare loss across households is 0.92 percent of lifetime consumption, and households are unanimously opposed to the policy. Welfare losses are particularly high for borrowers who, in the absence of a DTI limit, would have chosen to obtain a loan with a DTI ratio greater than 45 percent in their respective states. The removal of the limit for these high-DTI borrowers would increase their average lifetime consumption by 2.55 percent, a substantial figure.[48]

If the economy were to transition immediately from the Freddie Mac policy to the Dodd-Frank regulations, which tighten the DTI constraint but provide households with a costly opportunity to circumvent it, households would be slightly better off: on average, their consumption-equivalent welfare increases by 0.20 percent. Table 7 indicates that households disagree about the desirability of the Dodd-Frank policy. 62 percent of households experience a welfare gain from the Dodd-Frank rules that is, on average, equivalent to 0.42 percent of their lifetime consumption, and the remaining 38 percent see their welfare fall by an average of 0.15 percent. Welfare gains and losses are concentrated among distinct groups in the

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[48]The large welfare loss for this group of households can be accounted for by the fact that a majority of them—55 percent—find it optimal to switch to renting after the Freddie Mac policy is introduced. Of the remaining 45 percent of high-DTI borrowers who continue to own under the Freddie Mac policy, 37 percent continue with an existing mortgage loan, thereby forgoing the opportunity to refinance their loans, and 8 percent continue to get a new mortgage after the policy change.

	Losers from Dodd-Frank	Winners from Dodd-Frank
% of households	38.3	61.3
Welfare change (%)	-0.15	0.42
Homeownership rate (%)	36.6	69.5
Net worth	0.70	3.13
Liquid wealth	0.14	0.94
Age	34	62

Table 7

Model-implied distribution of welfare changes under Dodd-Frank relative to Freddie Mac. Losers are households whose consumption-equivalent welfare change is strictly negative; winners are households whose consumption-equivalent welfare change is at least zero. Net worth and liquid wealth are reported relative to income.

population. Winners from the reform are older, have higher net worth, and more likely to be homeowners, while losers are younger, have lower net worth, and are also more likely to be renters.

	Baseline to Freddie Mac	Freddie Mac to Dodd-Frank
Own to own	87.8	94.1
Own to rent	12.2	5.9
Rent to own	0.9	6.0
Rent to rent	99.1	94.0

Table 8

Model-implied flows between homeownership and renting. The first column of numbers compares the no-DTI-limit baseline to the Freddie Mac 45-percent DTI limit, and the second column compares the Freddie Mac policy to the Dodd-Frank 43-percent DTI limi.

The distribution of welfare changes suggests a close link between welfare and homeownership. In spite of the illiquidity of home equity as an asset, the option to access mortgage debt is valued as a form of consumption insurance.[49] With this in mind, I split the households in the model into four groups based on whether they had chosen to own or rent under the initial policy and whether they stayed in that group or switched after the new policy is implemented. To provide context, I show the size of these flows between owning and renting in Table 8. Of households who had chosen to own in the no-DTI-limit setting, 12 percent of them rent under the Freddie Mac policy, while the flow from renting to owning is negligible. When the economy switches from the Freddie Mac policy to the Dodd-Frank policy, the

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[49]In an earlier version of their paper, Gorea and Midrigan (2018) demonstrate that households in their model have a smoother consumption than in an otherwise identical one-asset Bewley model for this reason, even though the illiquidity of home equity as an asset produces substantial welfare losses.

outflow of owners to renters is only half as large, and 6 percent of renters under the Freddie Mac policy now choose to own instead.
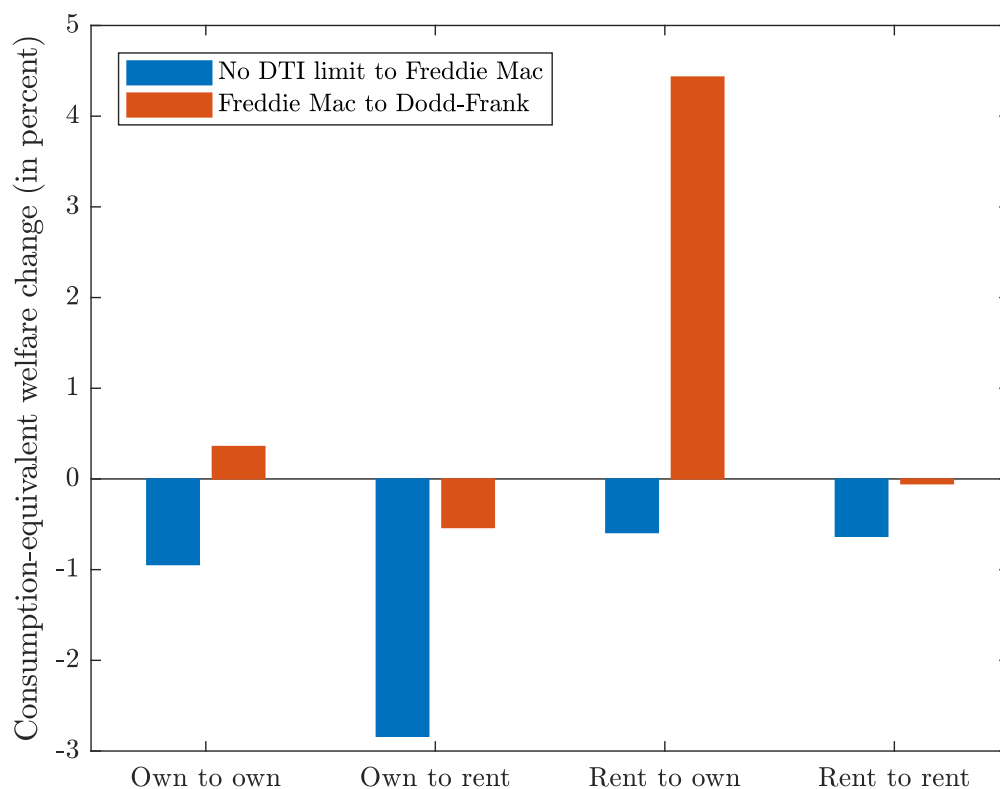

Figure 10

Welfare changes of own-to-own, own-to-rent, rent-to-own, and rent-to-rent households in the model. Blue bars depict the mean welfare change for each group when comparing the Freddie Mac policy to the no-DTI-limit baseline, and the orange bars depict the mean welfare change for each group when comparing the Dodd-Frank policy to the Freddie Mac policy.

I depict the mean consumption-equivalent welfare change across these four groups for the two policy changes in Figure 10. Relative to the no-DTI-limit baseline, all four groups experience an average welfare loss after the Freddie Mac policy is introduced.[50] The loss is greatest for households who, in their given state, had initially chosen to own but switch to renting after the policy change, and borrowers with DTI ratios above the Freddie Mac limit of 45 percent constitute the majority of this group. These are precisely the households who exhibit a higher objective probability of default, and their exit from the pool of borrowers after the introduction of a DTI limit drives the observed decline in the foreclosure rate.[51]

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[50]The welfare losses under the Dodd-Frank policy *relative to the baseline* are similar in magnitude.

[51]Own-to-rent switchers who had obtained a new mortgage loan in the baseline received a mortgage interest rate of 4.89 percent. This is 61 basis points greater than the mean mortgage interest rate at origination in the no-DTI-limit stationary distribution and 144 basis points greater than the mortgage interest rate implied by a mortgage interest rate with a default premium of zero.

By contrast, only 19 percent of households who own under both the no-DTI-limit baseline and the Freddie Mac policy had initially taken out high-DTI loans.

Relative to the Freddie Mac policy, welfare gains under the Dodd-Frank policy accrue to households who, after the new regulations are introduced, choose to own. Households who had chosen to rent under the Freddie Mac policy but now optimally own under Dodd-Frank experience an exceptionally large welfare gain of 4.45 percent. Of these rent-to-own households, a very small fraction—about 6 percent—are renters with a relatively large stock of liquid savings who need high-DTI loans in order to transition to homeownership. The remaining 94 percent consist of existing owners who, if they had not obtained a high-DTI loan, would have otherwise sold their homes. The option to sell is an alternative to default and exists as a way for households to convert their illiquid housing wealth to cash on hand. The fact that the high-DTI loan option under Dodd-Frank makes it optimal for this group of agents to keep their homes and do a cash-out refinance instead of selling suggests that they are liquidity constrained and value the option to obtain a less constrained loan. This finding accords with the pattern of borrower selection documented in Section 7.2, which demonstrated that the increase in high-DTI loans under Dodd-Frank is largely accounted for by households in the middle part of the wealth distribution who have low current income. Meanwhile, the group that experiences the largest welfare loss under the Dodd-Frank policy are those who had owned under Freddie Mac but now switch to renting. They correspond to poorer households for whom the tighter 43-percent DTI limit is binding and the option to obtain a high-DTI mortgage contract is either infeasible or suboptimal.

8   Conclusion

In this paper, I have evaluated the aggregate and distributional effects of post-crisis household leverage regulations that place limits on a borrower's debt payment-to-income ratio. I focused on two such policies, Freddie Mac's requirement that mortgages they purchase have a DTI ratio no greater than 45 percent, which has been in place since 2009, and the Dodd-Frank Act's 43-percent DTI limit with a costly option to relax that will come into effect for the majority of the U.S. mortgage market in 2021. Using a calibrated overlapping-generations model with mortgage contract choice and equilibrium default risk, I find that a limit on the DTI ratio is effective at lowering the aggregate default rate at the cost of reducing homeownership by improving the overall creditworthiness of the borrower population. I show that the strict but flexible approach embodied by the impending Dodd-Frank regulation is valuable to households from a welfare perspective compared to a policy that imposes a DTI constraint on all borrowers. This option is largely used by households in

the middle of the wealth distribution who have low incomes at the time of loan origination. These households value high-DTI loans for consumption smoothing purposes and have a sufficiently low probability of default for the financial intermediary to originate the more costly of the two mortgage contracts to them in equilibrium. Overall, my findings suggest that there is merit to incorporating greater flexibility in household leverage regulations.

Looking ahead, there are a number of ways in which the questions pursued in this paper could be explored further. In my theoretical environment, I have abstracted from movements in house prices. Incorporating them would permit a study of how effective the Dodd-Frank mortgage regulations would be at preventing foreclosures if there were to be a large drop in house prices like the one observed in the mid-2000s. I also abstract from changes in the risk-free rate, even though they are known to be a main driver of households' refinancing decisions and mortgage refinancing is an important mechanism by which monetary policy shocks are transmitted to the real economy. An environment in which the risk-free rate is determined endogenously would be more suitable for business cycle analysis. Finally, the model I have developed in this paper could be used to study the design of optimal limits on a borrower's debt payment-to-income ratio.

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A Appendix

A.1 Summary statistics from the Freddie Mac Single Family Loan-Level Dataset
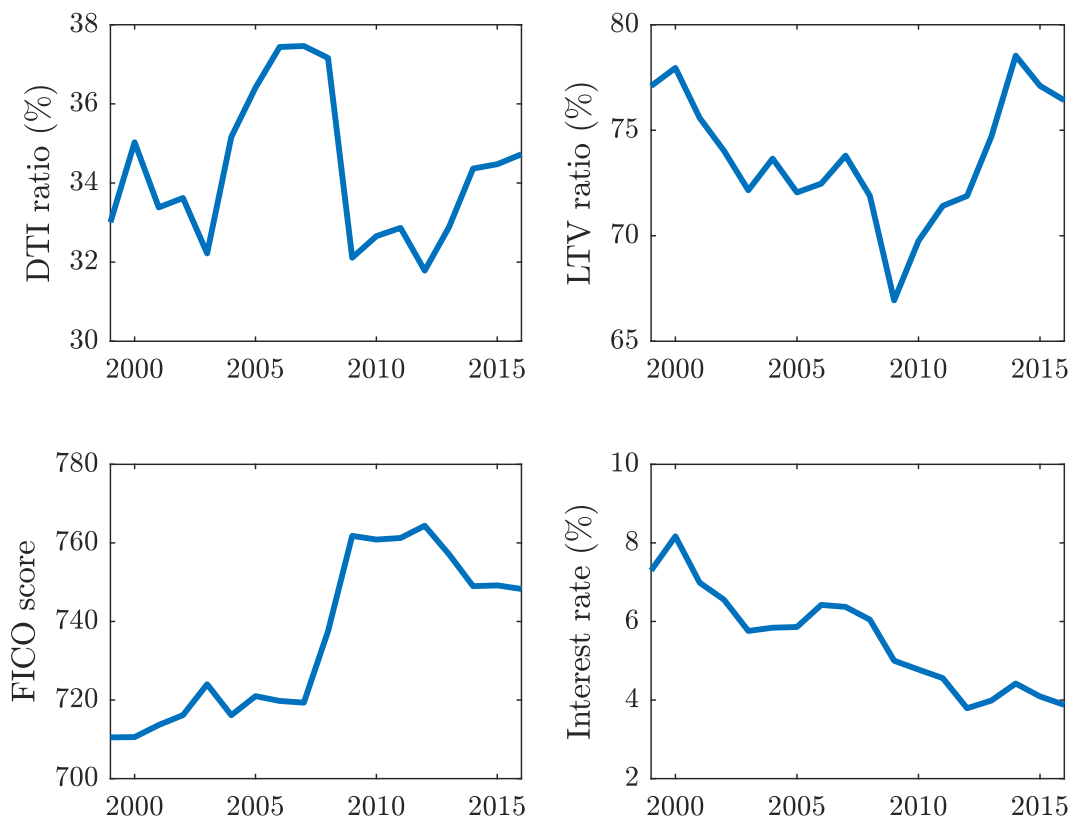

Figure 11

Average DTI ratio, LTV ratio, FICO score, and interest rate at origination, 1999-2016.

To provide some context for the empirical analysis, Figure 11 plots annual mean DTI ratio, LTV ratio, FICO score, and interest rate at origination for loans that were purchased or guaranteed by Freddie Mac between 1999 and 2016. These time series trace out a familiar tale: the DTI ratio rose and then fell with the housing boom and bust, the LTV ratio declined during the housing crisis before recovering, the average creditworthiness of a borrower has been much higher in the post-crisis era, and mortgage interest rates have followed a downward trend since the early 2000s.
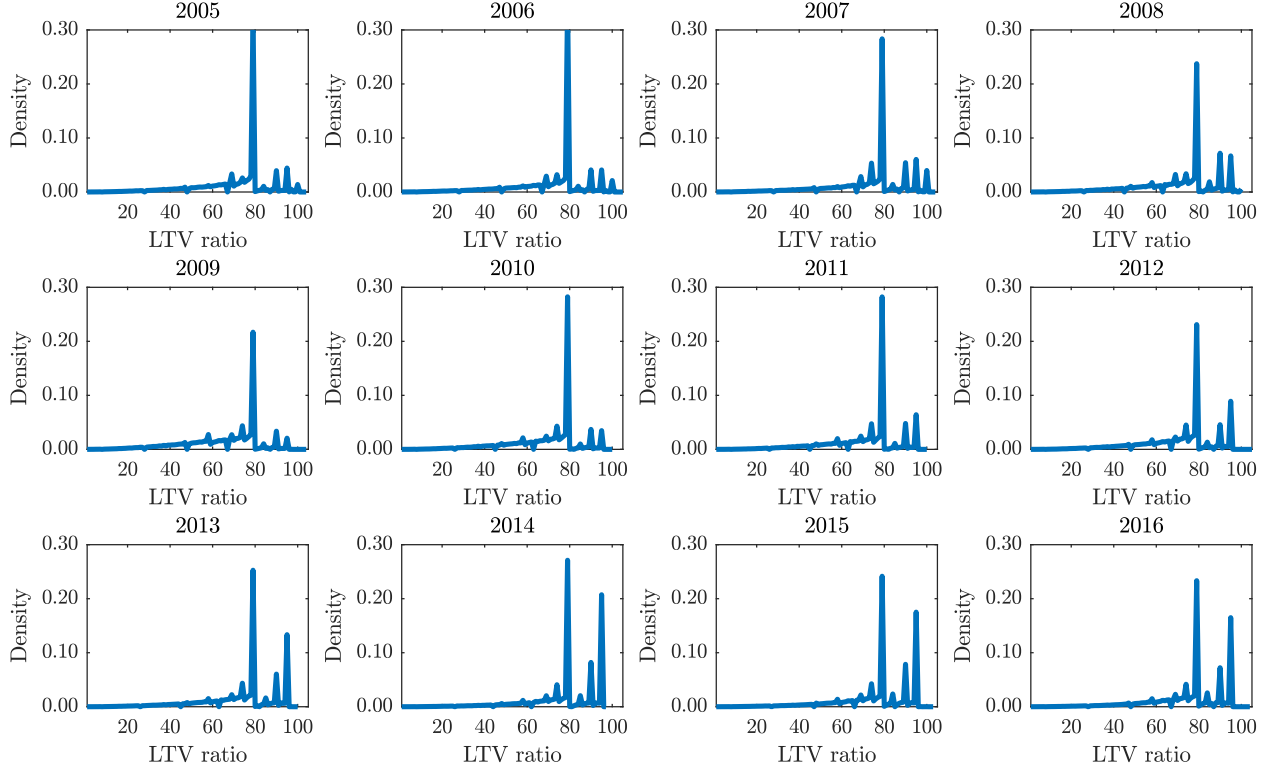
Figure 12

Distribution of loan-to-value (LTV) ratios, 2007-2014. Source: Freddie-Mac Single Family Loan-Level Dataset.

A.2 Distribution of loan-to-value ratios in the Freddie Mac Single Family Loan-Level Dataset

Figure 12 plots the distribution of LTV ratios at origination from 2007-2014 in the Freddie Mac data. Large spikes occur at GSE institutional limits. The overall shape of the distribution stays fairly constant over time in spite of significant macroeconomic upheaval during this period.

A.3 Effect of the Freddie Mac DTI limit on mortgage originations: a flexible difference-in-differences approach

To ensure that the results presented in Section 4.3 are not driven simply the fact that the DTI ratio on a particular loan is large but rather a change precisely at the 45-percent limit, I again follow DeFusco, Johnson and Mondragon (2017) in estimating a more flexible difference-in-differences specification that allows the effect of the policy to vary with the DTI ratio:

$$y_{it} = \alpha + \sum_{k=40}^{50} \left[\beta_1^k \mathbb{1}_{\{DTI_i=k\}} + \beta_2^k \mathbb{1}_{\{DTI_i=k\}} \times \text{Policy}_t \right] + \gamma_t + X_i'\delta + \varepsilon_{it}. \tag{18}$$

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$\mathbb{1}_{\{\text{DTI}_i=k\}}$ is an indicator variable that takes a value of 1 if the DTI ratio of loan i at origination is equal to k; all other regressors are as previously defined. The interpretation of β_2^k is now the differential change in the dependent variable for loans a DTI ratio of k after the 45-percent DTI limit is implemented. Like DeFusco, Johnson and Mondragon (2017), I make $k = 45$ the omitted DTI category so that the coefficients β_2^k on all other DTI categories estimate the effect of the DTI limit relative to loans exactly at the limit.

Figures 13, 14, and 15 plot point estimates for β_2^k with their respective 95-percent confidence intervals from the flexible difference-in-differences specification with the interest rate, LTV ratio, and FICO score in turn. These figures indicate that the effects documented in Section 4.3 are indeed the result of a change exactly at the 45-percent DTI ratio.
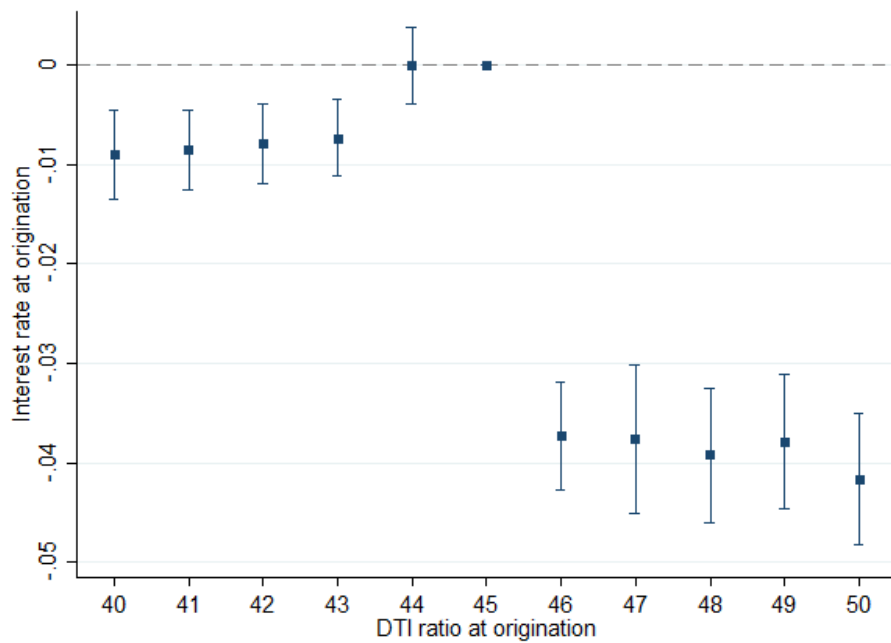

Figure 13

Change in interest rate for loans with DTI = k after the introduction of the 45-percent DTI limit.
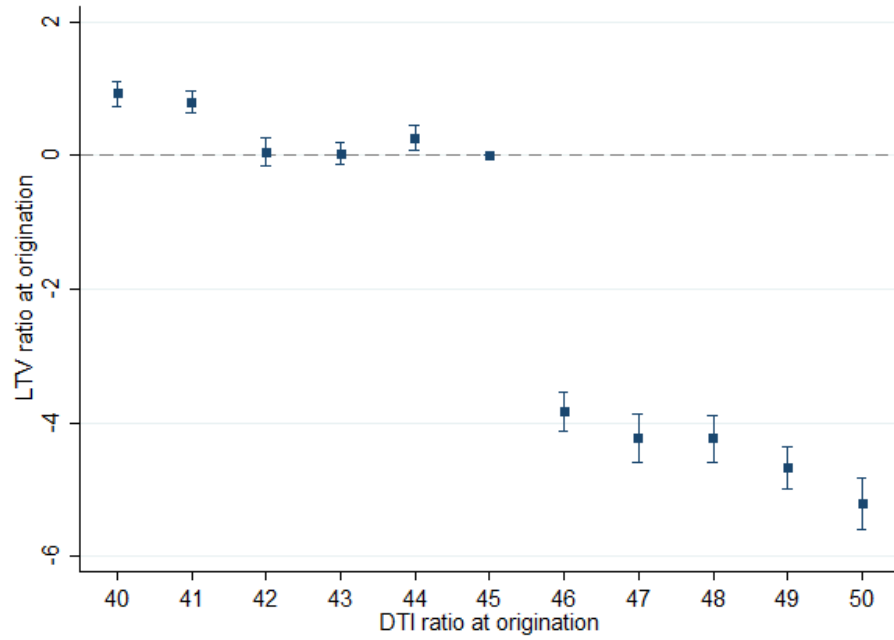
Figure 14

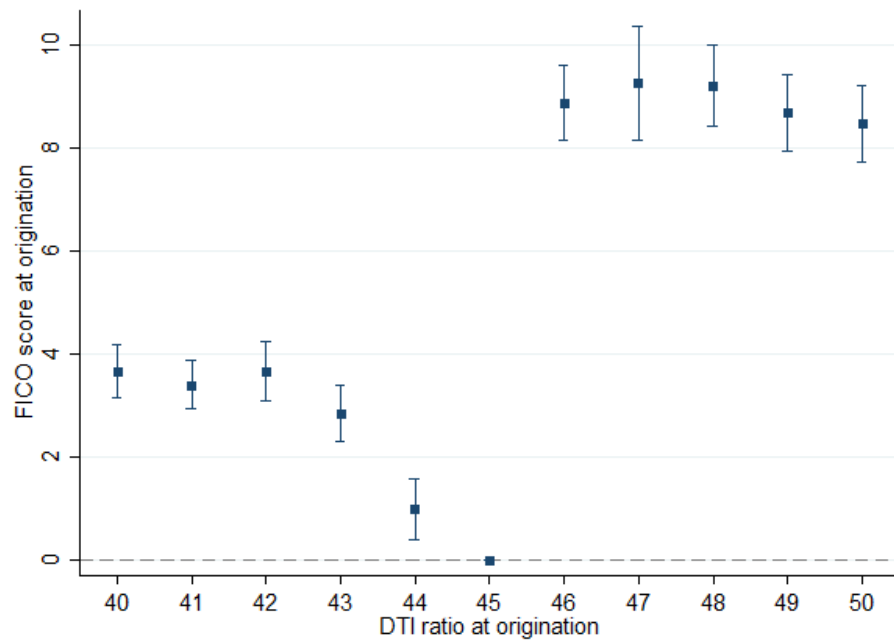Change in the LTV ratio for loans with DTI $= k$ after the introduction of the 45-percent DTI limit.

Figure 15

Change in the FICO score for loans with DTI $= k$ after the introduction of the 45-percent DTI limit.

A.4 Parameterization

Income process

To parameterize the deterministic component of income, I regress a quartic polynomial in age on log annual household income using a sample of households from the PSID (1999-2015) whose heads are between ages 22 and 64. The fitted values for log household income constitute the sequence $\{\chi_j\}_{j=1}^{T}$, and I normalize them so that $\chi_1 = 0$.

To parameterize pension income received in retirement, I follow the procedure from Guvenen and Smith (2014). I simulate for a given (ρ, σ_ϵ) pair the earnings of a panel of 10,000 households during their working years and regress average labor earnings on earnings in the last period of working life. I use the regression coefficients to predict average lifetime earnings for each possible realization of income in the last period of working life in the model, $\log y_{T_R-1} = \chi_{T_R-1} + z$. Letting $\log \hat{y}$ be the predicted average lifetime earnings, $\log \bar{y}$ be the economy-wide average annual labor earnings, and $\log \tilde{y} = \log \hat{y} / \log \bar{y}$, the function $\Phi(y_{T_R-1})$ is given by

$$
\Phi(y_{T_R-1}) = \begin{cases} 0.9 \log \hat{y} & \text{if } \log \tilde{y} \leq 0.3 \\ \log \bar{y} \left[0.27 + 0.32 \left(\log \tilde{y} - 0.3 \right) \right] & \text{if } 0.3 < \log \tilde{y} \leq 2 \\ \log \bar{y} \left[0.81 + 0.15 \left(\log \tilde{y} - 2 \right) \right] & \text{if } 2 < \log \tilde{y} \leq 4.1 \\ 1.13 \log \bar{y} & \text{if } \log \tilde{y} > 4.1. \end{cases}
$$

Cross-sectional moments

I use the 2016 Survey of Consumer Finances (SCF) to compute cross-sectional moments targeted in the calibration procedure. Net worth is defined as the sum of liquid assets and equity in the primary residence. Following Kaplan and Violante (2014), I define liquid assets as the sum of assets held in checking accounts, savings accounts, call accounts, directly held mutual accounts, directly held bonds, and directly held stocks. Equity in the primary residence is defined as the difference between the value of primary residential real estate and debt outstanding on the first mortgage secured by the primary residence. I use only the value of the primary residence and the first mortgage secured by it when calculating home equity because households in the model are only able to hold one property and mortgage at a time. Since relatively few households in the SCF report having more than one loan secured by their primary residence or owning a second home, however, the inclusion of these second mortgages (e.g., home equity loans or home equity lines of credit) is not quantitatively important for my results. I use labor income as my measure of household income. All means are computed with the sample weights included with the data, and I exclude households in

the top 5 percent of the net worth distribution (where net worth is defined as above).

A.5 Solution algorithm

I use the Rouwenhorst method described in Kopecky and Suen (2010) to discretize the idiosyncratic income process and generate the transition matrix for z. I create linearly spaced grids for housing wealth h, mortgage debt m, and the mortgage interest rate r_m. The grid for liquid assets a features more points clustered near the borrowing constraint. The grid for the depreciation shock δ_h consists of two points, 0 and δ, and the grid for q also consists of two points, L and H.

Choices of a', m', and r'_m are permitted to lie off the grid, and I use linear interpolation to evaluate the value and policy functions at off-grid points. I constrain the choice of h' to be on the grid for housing in order to capture the indivisibility of housing as an asset. I interpolate the policy and value functions arising from the household's problem over finer grids for the three continuous state variables. I use the finer policy and value functions when computing the stationary distribution Λ_j^* and the associated transition function Q_j.

1. Solve the problem of a household in the last period of life to obtain $V_T^R(\omega)$, $V_T^M(\omega)$, $V_T^P(\omega)$, and $V_T^D(\omega)$, along with all associated policy functions. By definition, $m'_T(\omega) = 0$. Also compute the present value of cash flows associated with a mortgage held by an age-T household. If the household repays their debt, then

$$\Pi_T(\omega) = (1 + r_m) m.$$

If the household defaults on their debt, then

$$\Pi_T(\omega) = (1 - \delta_h) h - \gamma(q).$$

2. Use backward induction to solve for value functions in Equations (9), (10), (11), and (12) for ages $j \in \{1, 2, \ldots, T - 1\}$.

   (a) *Solving the problem of a renter, $V_j^R(\omega)$:*

      (i) This option is available to all households.

      (ii) The assumption of Cobb-Douglas preferences over nondurable consumption and housing services implies

$$s = \frac{1 - \alpha}{\alpha R} c. \tag{19}$$

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Use this expression to substitute out s in the household's problem, and use the budget constraint to substitute out c from the flow utility function.

(iii) Solve for $V_j^R(\omega)$ and $a_j'^R(\omega)$ using Brent's method. Let c_{min} be the lowest possible value of nondurable consumption.[52] The requirement that $c \geq c_{min}$, along with the no-borrowing constraint on the liquid asset, characterizes the set of feasible solutions:

$$0 \leq a' \leq y_j(z) + (1+r)a + (1 - \delta_h)h - (1 + r_m)m - \mathbb{1}_{h \neq 0}\kappa_h - \frac{1}{\alpha}c_{min}.$$

(iv) Back $c_j^R(\omega)$ out from the flow budget constraint and use Equation (19) to find $s_j(\omega)$.

(v) By definition, $h_j'^R(\omega) = 0$ and $m_j'^R(\omega) = 0$. Since the household does not have any mortgage debt, $q_{m,j}'^R(\omega)$ and $r_{m,j}'^R(\omega)$ can be set to any arbitrary value.

(b) *Solving the problem of an owner who obtains a new mortgage, $V_j^M(\omega)$:*

(i) This option is available to all households.

(ii) For a given $r_{m,j}'(\omega)$, loop through all feasible pairs of (h', q').[53] For each feasible (h', q'), solve for $a_j'^M(\omega)$, $m_j'^M(\omega)$, and $V_j^M(\omega)$ using Nelder-Mead. We have $a_{min}' = 0$. Maximum feasible borrowing is determined by the LTV and DTI constraints. If $q' = L$, then

$$m_{max}' = \min\left\{\theta h', \lambda(q')\, y_j(z)\left(r_{m,j}'(\omega)\, \frac{\left(1 + r_{m,j}'(\omega)\right)^{T-j}}{\left(1 + r_{m,j}'(\omega)\right)^{T-j} - 1}\right)^{-1}\right\}.$$

where $\lambda(q')$ is defined in Equation (5). If $q' = H$, then

$$m_{max}' = \theta h'.$$

The non-negativity constraint on nondurable consumption implies that the set of feasible solutions is characterized by

$$0 \leq a' \leq y_j(z) + (1+r)a + (1 - \delta_h)h - (1 + r_m)m + m_{max}' - \mathbb{1}_{h' \neq h}\kappa_h - \kappa_m$$
$$- h' - c_{min}$$

---

[52]In the computation, c_{min} is set to 0.001.
[53]In this context, feasibility means that h' is in the household's budget set assuming the smallest possible down payment, liquid savings, and mortgage interest rate.

and

$$\max\{c_{min} + h' + a'_{min} + \mathbb{1}_{h' \neq h}\kappa_h + \kappa_m + (1 + r_m) m - y_j(z) - (1 + r) a -$$
$$(1 - \delta_h) h, 0\} \leq m' \leq m'_{max}.$$

Use the budget constraint to find $c^M_j(\omega)$. Select the (h', q') pair (and its associated choices of a', m', and V^M_j) that yields the highest value for the household.

(iii) For a given household to the solution's problem in the previous step, compute the financial intermediary's profit using Equation (14).

(iv) A bisection algorithm is used to find the mortgage interest rate offered to the household, $r'_{m,j}(\omega)$, that makes the financial intermediary break even on a newly originated mortgage. This algorithm exploits the fact that the financial intermediary's profit is increasing in $r'_{m,j}(\omega)$ and searches over the interval $[r_{m,min}, r_{m,max}]$.

(A) $r'_{m,T-1}(\omega) = r + \phi$. This follows from the fact that, if an age-T household repays their outstanding mortgage debt, then the zero profit condition is

$$\frac{1}{1 + r + \phi}\left(1 + r'_{m,T-1}(\omega)\right) m'_{T-1} = m'_{T-1},$$

implying $r'_{m,j}(\omega) = r + \phi$. Note that, if an age-$T$ household defaults, then, in equilibrium, an intermediary will not sell that household a mortgage contract when they are of age $T - 1$.

(B) If the financial intermediary's profit is negative when $r'_{m,j}(\omega) = r_{m,max}$, then the option to obtain a mortgage is not available to the household. Likewise, if the intermediary's profit is positive when $r'_{m,j}(\omega) = r_{m,min}$, then the household borrows at the rate $r + \phi$.

(c) *Solving the problem of an owner who makes a mortgage payment, $V^P_j(\omega)$:*

(i) This option is only available to existing homeowners ($h > 0$). Note that this problem is also solved by homeowners who do not have any debt. In this case, $m'^P_j(\omega) = 0$ and the owner only needs to solve for $a'^P_j(\omega)$.

(ii) Use the budget constraint to substitute c out of the flow utility function.

(iii) Solve for $V^P_j(\omega)$, $a'^P_j(\omega)$, and $m'^P_j(\omega)$ using Nelder-Mead. From the law of motion for mortgage debt, we have

$$m'_{max} = (1 + r_m) m - \pi_{min,j}(m, r_m),$$

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where $\pi_{min,j}(m, r_m)$ is defined by Equation (4). The requirement that $c \geq c_{min}$, combined with the fact that $a'_{min} = 0$, means that the feasible set of solutions is characterized by

$$0 \leq a' \leq y_j(z) + (1+r)a - (1+r_m)m + m'_{max} - \delta_h h - c_{min}$$

and

$$\max\{c_{min} + a'_{min} + \delta_h h - y_j(z) - (1+r)a, 0\} \leq m' \leq m'_{max}.$$

    (iv) Use the budget constraint to find $c_j^P(\omega)$. By definition, $h_j'^P(\omega) = h$, $q_j'^P(\omega) = q$, and $r_{m,j}'^P(\omega) = r_m$.

(d) *Solving the problem of a borrower who defaults, $V_j^D(\omega)$:*

    (i) This option is only available to existing borrowers ($h > 0$ and $m > 0$).

    (ii) Since a borrower who defaults in the current period must rent, Equation (19) can be used to substitute out s from the flow utility function and the budget constraint.

    (iii) Solve for $V_j^D(\omega)$ and $a_j'^R(\omega)$ using Brent's method. Non-negativity requirements on consumption and liquid savings characterize the set of feasible solutions:
$$0 \leq a' \leq y_j(z) + (1+r)a - \frac{1}{\alpha}c_{min}.$$

    (iv) Back $c_j^D(\omega)$ out from the flow budget constraint and use Equation (19) to find $s_j^D(\omega)$.

    (v) By definition, $h_j'^D(\omega) = 0$ and $m_j'^D(\omega) = 0$. Since the household does not have any mortgage debt, $q_{m,j}'^D(\omega)$ and $r_{m,j}'^D(\omega)$ can be set to any arbitrary value.

(e) Determine $V_j(\omega) = \max\{V_j^R(\omega), V_j^M(\omega), V_j^P(\omega), V_j^D(\omega)\}$.

(f) Compute $\Pi_j(\omega)$ using Equation (13).

3. Given the policy functions and the invariant distributions of z and δ_h, construct the $ns \times ns$ transition matrix Q_j for the distribution of agents over states according to Equation (16), where ns denotes the number of states.

4. Iterate on the transition matrix Q_j using the law of motion in Equation (15) until $\|\Lambda_{j+1} - \Lambda_j\| < \epsilon$ for some small ϵ.