ERGM and SAR

Julien Manuel Neves

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Model

Let G be the observed network and assume it is derived from the following distribution

$$P(G = g \mid \theta, \phi) = \frac{\exp\left\{\theta^T s(g) + \phi\xi\right\}}{c(\theta, \phi)}$$

where ξ is some vector of unobservables characteristics for the agents and $s(y)$ a vector of statistics for the network.

Let's assume that we are trying to estimate the following model

$$Y = \lambda GY + X\beta + \underbrace{\psi\xi + \nu}_{\epsilon}$$

where $\nu \sim N(0, \sigma_\nu^2)$.

The issue we face is that G is obviously endogenous and therefore we can't use the following moment conditions:

$$\mathbb{E}\left\{H'\epsilon(\lambda, \beta, \psi)\right\} = 0$$

where $H = \{X, GX, G^2X, \ldots\}$.

Estimation

To solve the endogeneity problem, first we need to fit an Exponential Random Graph Model (ERGM) using our G and MCMC. This should yields some, hopefully consistent, sample estimates $\hat{\theta}$ and $\hat{\xi}$.

Using these estimates, let's define the following ERGM

$$P(\tilde{G} = g \mid \theta, \phi) = \frac{\exp\left\{\hat{\theta}^T s(g)\right\}}{c(\theta, \phi)}$$

Let $\tilde{G}$ be some realization of the previous distribution. Since $\tilde{G}$ should technically get rid of ξ, do we expect that the following holds

$$\mathbb{E}\left\{\mathbb{E}\left\{\tilde{H}'\epsilon(\lambda, \beta, \psi) \mid \tilde{G}\right\}\right\} = 0$$

where $H = \left\{X, \tilde{G}X, \tilde{G}^2X, \ldots\right\}$? If so, can't we simply to use NLLS without having to do any method of simulated moments?

If instead, we are assuming that there is some function $\pi(\theta)$, that might not be known, such that

$$\mathbb{E}\left\{\mathbb{E}\left\{\tilde{H}'\epsilon(\lambda, \beta, \psi) \mid \tilde{G}\right\}\right\} = \pi(\theta)$$

Then, how do we generate $\epsilon(\lambda, \beta, \psi)$ without observing ξ? What kind of distributional assumption would we make for ϵ?