**Rappels physiques:**en cartésien : , en cylindrique :   
en sphérique :  **Gradient**

For a function in three-dimensional Cartesian coordinate variables, the gradient is the vector field:

More generally, for a function of *n* variables , also called a scalar field, the gradient is the vector field:

For a vector field written as a 1 × *n* row vector, also called a tensor field of order 1, the gradient or covariant derivative is the *n × n* Jacobian matrix:

For a tensor field of any order *k*, is a tensor field of order *k* + 1.

**Divergence**

In Cartesian coordinates, the divergence of a continuously differentiable vector field is the scalar-valued function:

The divergence of a tensor field of non-zero order *k* is written as , a contraction to a tensor field of order *k* − 1. Specifically, the divergence of a vector is a scalar. The divergence of a higher order tensor field may be found by decomposing the tensor field into a sum of outer products and using the identity,

where is the directional derivative in the direction of multiplied by its magnitude. Specifically, for the outer product of two vectors,

**Curl**

In Cartesian coordinates, for the curl is the vector field:

In Einstein notation, the vector field has curl given by:

where = ±1 or 0 is the Levi-Civita parity symbol.

**Laplacian**

In **Cartesian coordinates**, the Laplacian of a function is

For a tensor field, , the Laplacian is generally written as:

and is a tensor field of the same order.

In *Feynman subscript notation*,

where the notation means the subscripted gradient operates on only the factor .

**First derivative identities**

For scalar fields , and vector fields , , we have the following derivative identities.

Distributive properties

**Product rule**

We have the following generalizations of the product rule in single variable calculus.

In the second formula, the transposed gradient is an *n* × 1 column vector, is a 1 × *n* row vector, and their product is an *n × n* matrix: this may also be considered as the tensor product of two vectors, or of a covector and a vector*.*

**Quotient rule**

**Chain rule**

Let be a one-variable function from scalars to scalars, a parametrized curve, and a function from vectors to scalars. We have:

For a coordinate parametrization we have:

Here we take the trace of the product of two *n × n* matrices: the gradient of and the Jacobian of Φ.

**Dot product rule**

where denotes the Jacobian matrix of the vector field .

Alternatively, using Feynman subscript notation,

As a special case, when A = B,

The generalization of the dot product formula to Riemannian manifolds is a defining property of a Riemannian connection, which differentiates a vector field to give a vector-valued 1-form.

**Cross product rule**

**Second derivative identities**

The curl of the gradient of *any* continuously twice-differentiable scalar field is always the zero vector:

The divergence of the curl of *any* vector field is always zero:

Laplacian identities

Here 2 is the vector Laplacian operating on the vector field .

**Summary of important identities****Vector calculus identities**

(Binet-Cauchy Identity)

**Gradient**

**Divergence**

**Curl**

**Second derivatives**

et en physique (potentiel vecteur champ magnétique)

et en physique (potentiel électrique)

(scalar Laplacian)

(vector Laplacian)

(Green’s first identity)

(Green’s second identity)

(Green’s vector identity)

**Integration**

**Surface–volume integrals**  
Pour un ouvert borné régulier de , et un champ de vecteur , avec avec intégrale triple = dimension , double =

(Green-Ostrogradsky/Stokes divergence theorem)

(Green’s first identity) (par G.O. sur l’identité locale)

(Green’s second identity) (par G.O. sur l’identité locale)

(integration by parts) (par G.O. sur )  
Pour et pour , avec on a (integration par parties projetée)

**Curve–surface integrals**

In the following curve–surface integral theorems, *S* denotes a 2d open surface with a corresponding 1d boundary *C* = ∂*S* (a closed curve):

(Stokes’ theorem) (cas particulier de Green-Ostrogradski en 2d)

Integration around a closed curve in the clockwise sense is the negative of the same line integral in the counterclockwise sense.