Case 1: Calgary Desk Company

IEOR 240

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I. Introduction

This report recommends a production schedule for the month of September for Calgary Desk Company's (CALDESCO) entire line of desks. The sections of the report are Background Information, Formulation, Solution, Discussion, and an Appendix. The Background Information section provides an overview of CALDESCO's types of desks produced, materials used, resources available, and in-house quotas. The Formulation section describes decision variables, the objective function, constraints, and any assumptions that were made. The Solution section presents the optimal solution that maximizes profit and proves that it meets quota and resource constraints. The Discussion section presents an analysis of the solution and focuses on three parameters of interest. Finally, the Appendix includes the AMPL code and results.

II. Background Information

A. Desks/Orders

CALDESCO manufactures three different desk sizes in three different lines each for a total of 9 different desk types. The sizes are student (24 in. x 42 in.), standard (30 in. x 60 in.), and executive (42 in. x 72 in.), and the lines are economy, basic pine, and hand-crafted pine. The basic pine and hand-crafted pine are referred to as 'basic' and 'hand-crafted' respectively in this report.

B. Materials/Production

CALDESCO uses three different types of materials in their desks: aluminum, particle board, and pine sheets. The desk are comprised of parts that are produced on three different production lines based on materials. Production line 1 produces the aluminum drawers and base of the economy line. Production line 2 produces the particle board tops of the the economy line along with the pine sheet tops of the basic pine line. There are two lines of production line 3 to keep up with the production requirements of the drawers and base of the basic pine and hand-crafted pine desks. The amount of each material and time on each production line are resources with limited availability for the month, which are considered in this report.

C. Labor

The company requires craftsmen labor for production, finishing/assembly, and hand-crafting. There are two craftsmen required for each production line and each desk requires one craftsman for finishing/assembly. Additionally, the hand-crafted pine desks require a craftsman for hand-crafting. Given all of that information, the amount of worker-minutes required per desk, which translates into the labor resource, is found through Equation 1 below.

Equation 1: $$Worker\ Minutes\ per\ Desk\ =\ 2 * (Production\ Line\ Time)\ +\ Hand\ Crafting\ Time\ +\ Assembly/Finishing\ Time$$

D. Quotas

CALDESCO maintains its profit margins by imposing set quotas for production on each desk line and size. These maximum and minimum quotas are described further in the Formulation and Solution sections.

III. Formulation

Based on the information provided by the company, a linear program was formulated in order to determine a production schedule for CALDESCO's desks that maximizes their profits. This program was then solved using AMPL.

A. Assumptions

In the formulation of this linear program, a few assumptions were made where the provided information was ambiguous. First, it is assumed that the company does not require integers for their production schedule, so the problem is not restricted to integer programming. Meaning, the recommended schedule may include making a fraction of a desk. For example, it is recommended that CALDESCO produces 1657.5 basic-standard desks. Second, it is assumed that if the company produces more desks than the amount of September orders, then it will be able to sell all the additional desks produced. For example, the recommended schedule includes making 1069.24 hand-crafted-standard desks, but there are only 150 ordered for the month of September. The schedule assumes that the company will be able to sell the additional 919.24 desks to other customers.

B. Decision Variables:

The decision variables are the number of each type of desk CALDESCO needs to make in the month of September. These are given by X[j] where j goes from 1 to n, and n represents the 9 different types of desks. A summary of the decision variables with what they represent is given in the table below.

Table 1. Decision Variables

Desk Line	Desk Size	Decision Variable X[j] where j=1..n
Economy	Student	X1
	Standard	X2
	Executive	X3
Basic Pine	Student	X4
	Standard	X5
	Executive	X6
Hand-Crafted Pine	Student	X7
	Standard	X8
	Executive	X9

C. Objective Function

The objective is to maximize total profit for the month of September and therefore the objective function for the linear program is provided in Equation 2, where C[j] is the amount of profit to be made on each of the 9 types of desks. The profit amounts are obtained from the company in their problem statement and are shown in table 2.

Equation 2: $maximize \sum_{j=1}^{n} \quad C[j] * X[j]$

Table 2. Profits C[j]

Line	Size	j=1..n	Profit (Parameter C[j])
Economy	Student	1	20
	Standard	2	30
	Executive	3	40
Basic Pine	Student	4	50
	Standard	5	80
	Executive	6	125
Hand-Crafted Pine	Student	7	100
	Standard	8	250
	Executive	9	325

The objective function (Equation 2) is expanded out below by multiplying the profit for each desk type by the variables that represent the number of desks produced for each type.

maximize profit = 20*X[1] + 30*X[2] + 40*X[3] + 50*X[4] + 80*X[5] + 125*X[6] + 100*X[7] + 250*X[8] + 325*X[9]

D. Parameters and Constraints

In the formulation of the linear program, there were multiple parameters and constraints. In order to get a parameter that represents the resources required to produce each type of desk, first the worker-minutes for the labor resource needed be to calculated by Equation 1 given in the Labor section under Background Information. The worker-minutes were calculated from company given information about the time required for each production line along with assembly/finishing time and hand-crafting time for each type of desk. The calculated worker-minutes from the time information is given in the table below.

Table 3. Calculated Worker-Minutes needed to produce each type of desk (using Equation 1 on pg. 3)

Line	Size	Related Variable	Production Line 1 Time (min.)	Production Line 2 Time (min.)	Production Line 3 Time (min.)	Assembly/Finish Time (min.)	Hand-Crafting Time (min.)	Worker-Minutes needed (Labor) (Equation 1) (min.)
Economy	Student	X1	1.5	1		10		15
	Standard	X2	2	1		11		17
	Executive	X3	2.5	1		12		19
Basic Pine	Student	X4		1	3	15		23
	Standard	X5		1	4	18		28
	Executive	X6		1	5	20		32
Hand-Crafted Pine	Student	X7			3	20	50	76
	Standard	X8			4	25	60	93
	Executive	X9			5	30	70	110

In addition to the worker-minutes, there are other resources that need to be taken into consideration. As mentioned in the background information, the resources required to produce a desk are: labor, materials, and time on production lines. These required resources to produce each type of desk are taken into account in the linear program by the parameter A[i,j] where i goes from 1 to m (m represents the number of resources required which in this case is 7) and j goes from 1 to n as mentioned earlier. An overview of the required resources parameter is given in the table below.

Table 4. Required Resources Parameter A[i,j]

	Desk Line	Economy			Basic Pine			Hand-Crafted Pine		
	Desk Type	Student	Standard	Executive	Student	Standard	Executive	Student	Standard	Executive
	Related Variable	X1	X2	X3	X4	X5	X6	X7	X8	X9
Resources (Parameter A[i,j])	Worker-Minutes (Labor)	15	17	19	23	28	32	76	93	110
	Aluminum	14	24	30						

	(Sq Ft)									
	Particle Board (Sq Ft)	8	15	24						
	Pine Sheets (Sq Ft)				22	40	55	25	45	60
	Production Line 1 (min.)	1.5	2	2.5						
	Production Line 2 (min.)	1	1	1	1	1	1			
	Production Line 2 (min.)				3	4	5	2	4	5

The company has limited resources available for each month due to several factors like current workforce, storage space, suppliers, pricing, speed, and more. The resource availability for September was provided by the company and is represented by the parameter B[i] as seen below. Note that the worker-minutes availability was calculated by Equation 3 which uses the following formulation. The company currently has a workforce of 30 craftsmen that work for 160 hours per month, but only 80% of them are available at a given time during the month due to vacations, illnesses, etc. The equation results in a total of 230,400 worker-minutes per month.

Equation 3: $Avail\ Worker\ Minutes\ =\ 0.8 * (30\ Craftsmen/month) * (160\ hours/month) * (60\ minutes/month)$

Table 5. Resource Availability for September Parameter B[i]

Resource	i=1..m	Resource Availability for September (Parameter B[i])
Worker-Minutes (Labor)	1	230,400
Aluminum (Sq Ft)	2	65,000
Particle Board (Sq Ft)	3	60,000
Pine Sheets (Sq Ft)	4	175,000
Production Line 1 (minutes)	5	9,600
Production Line 2 (minutes)	6	9,600
Production Line 2 (minutes)	7	19,200

Since there is a limitation on the amount of resources, the resources used to make the desks need to be constrained. This is done in the formulation by constraint 1 which says that for each resource the amount of that resource available has to be greater than or equal to the amount of that resource used in production. In formulation for this was done Equation 4 below.

Equation 4: $\sum_{j=1}^{n} \quad A[i,j] * X[j] \leq B[i]$

If constraint 1 (Equation 4) was expanded out for all the 7 resources the following equations would be obtained.

1. Worker-Minutes (Labor): 15*X[1] + 17*X[2] + 19*X[3] + 23*X[4] + 28*X[5] + 32*X[6] + 76*X[7] + 93*X[8] + 110*X[9] <= 230400;
2. Aluminum (Sq Ft): 14*X[1] + 24*X[2] + 30*X[3] <= 65000;
3. Particle Board (Sq Ft): 8*X[1] + 15*X[2] + 24*X[3] <= 60000;

4. Pine Sheets (Sq Ft): 22*X[4] + 40*X[5] + 55*X[6] + 25*X[7] + 45*X[8] + 60*X[9] <= 175000;
5. Production Line 1 (minutes): 1.5*X[1] + 2*X[2] + 2.5*X[3] <= 9600;
6. Production Line 2 (minutes): X[1] + X[2] + X[3] + X[4] + X[5] + X[6] <= 9600;
7. Production Line 3 (minutes): 3*X[4] + 4*X[5] + 5*X[6] + 3*X[7] + 4*X[8] + 5*X[9] <= 19200;

Another important constraint is that CALDESCO needs to be able at least fulfill all of their given September orders. The given September orders were represented by parameter Ord[j], which is shown in the table below.

Table 6. September Orders Parameter Ord[j]

Line	Size	j=1..n	September Orders (Parameter Ord[j])
Economy	Student	1	20
	Standard	2	30
	Executive	3	40
Basic Pine	Student	4	50
	Standard	5	80
	Executive	6	125
Hand-Crafted Pine	Student	7	100
	Standard	8	250
	Executive	9	325

Then, to constrain based on orders, condition 2: $X[j] \geq Ord[j]$ is applied; which is the same as equation 5.

Equation 5: $-X[j] \leq -Ord[j]$.

This constraint expanded out gives:
1. Economy Student Orders: -X[1] <= -750
2. Economy Standard Orders: -X[2] <= -1500
3. Economy Executive Orders: -X[3] <= -100
4. Basic Pine Student Orders: -X[4] <= -400
5. Basic Pine Standard Orders: -X[5] <= -1500
6. Basic Pine Executive Orders: -X[6] <= -100
7. Hand-Crafted Pine Student Orders: -X[7] <= -25
8. Hand-Crafted Pine Standard Orders: -X[8] <= -150
9. Hand-Crafted Pine Executive Orders: -X[9] <= -50

In addition to constraining based on orders, also need to constrain based on the company's in-house quotas in order to maintain profit margins. The given quotas are taken into consideration in the linear program by the parameters min and max, which correspond to different desk lines and sizes as summarized in the table below.

Table 7. Quota Parameters

		Related Variables	Quota Min % (Parameter Min)	Quota Max % (Parameter Max)
Desk Line	Economy	X1 + X2 + X3	20	50
	Basic Pine	X4 + X5 + X6	40	60

	Hand-Crafted	**X7 + X8 + X9**	10	20
Desk Size	**Student**	**X1 + X4 + X7**	20	35
	Standard	**X2 + X5 + X8**	40	70
	Executive	**X3 + X6 + X9**	5	15

Expanding out constraint 3 that takes into consideration the minimum percentage quotas for the different desk lines you get the following equations:

1. Minimum Quota on Economy: -0.8*X[1] - 0.8*X[2] - 0.8*X[3] + 0.2*X[4] + 0.2*X[5] + 0.2*X[6] + 0.2*X[7] + 0.2*X[8] + 0.2*X[9] <= 0;
2. Minimum Quota on Basic Pine: 0.4*X[1] + 0.4*X[2] + 0.4*X[3] - 0.6*X[4] - 0.6*X[5] - 0.6*X[6] + 0.4*X[7] + 0.4*X[8] + 0.4*X[9] <= 0;
3. Minimum Quota on Hand-Crafted Pine: 0.1*X[1] + 0.1*X[2] + 0.1*X[3] + 0.1*X[4] + 0.1*X[5] + 0.1*X[6] - 0.9*X[7] - 0.9*X[8] - 0.9*X[9] <= 0;

Expanding out constraint 4 that takes into consideration the minimum percentage quotas for the different desk sizes you get the following equations:

1. Quota on Student: -0.8*X[1] + 0.2*X[2] + 0.2*X[3] - 0.8*X[4] + 0.2*X[5] + 0.2*X[6] - 0.8*X[7] + 0.2*X[8] + 0.2*X[9] <= 0
2. Minimum Quota on Standard: 0.4*X[1] - 0.6*X[2] + 0.4*X[3] + 0.4*X[4] - 0.6*X[5] + 0.4*X[6] + 0.4*X[7] - 0.6*X[8] + 0.4*X[9] <= 0
3. Minimum Quota on Executive: 0.05*X[1] + 0.05*X[2] - 0.95*X[3] + 0.05*X[4] + 0.05*X[5] - 0.95*X[6] + 0.05*X[7] + 0.05*X[8] - 0.95*X[9] <= 0

Expanding out constraint 5 that takes into consideration the maximum percentage quotas for the different desk lines you get the following equations:

1. Maximum Quota on Economy: 0.5*X[1] + 0.5*X[2] + 0.5*X[3] - 0.5*X[4] - 0.5*X[5] - 0.5*X[6] - 0.5*X[7] - 0.5*X[8] - 0.5*X[9] <= 0;
2. Maximum Quota on Basic Pine: -0.6*X[1] - 0.6*X[2] - 0.6*X[3] + 0.4*X[4] + 0.4*X[5] + 0.4*X[6] - 0.6*X[7] - 0.6*X[8] - 0.6*X[9] <= 0;
3. Maximum Quota on Hand-Crafted Pine: -0.2*X[1] - 0.2*X[2] - 0.2*X[3] - 0.2*X[4] - 0.2*X[5] - 0.2*X[6] + 0.8*X[7] + 0.8*X[8] + 0.8*X[9] <= 0;

Expanding out constraint 6 that takes into consideration the maximum percentage quotas for the different desk sizes you get the following equations:

1. Maximum Quota on Student: 0.65*X[1] - 0.35*X[2] - 0.35*X[3] + 0.65*X[4] - 0.35*X[5] - 0.35*X[6] + 0.65*X[7] - 0.35*X[8] - 0.35*X[9] <= 0;
2. Maximum Quota on Standard: -0.7*X[1] + 0.3*X[2] - 0.7*X[3] - 0.7*X[4] + 0.3*X[5] - 0.7*X[6] - 0.7*X[7] + 0.3*X[8] - 0.7*X[9] <= 0;
3. Maximum Quota on Executive: -0.15*X[1] - 0.15*X[2] + 0.85*X[3] - 0.15*X[4] - 0.15*X[5] + 0.85*X[6] - 0.15*X[7] - 0.15*X[8] + 0.85*X[9] <= 0;

Lastly, we must constrain the decision variables (X[j] = amount of desks of each type to produce) with a non-negativity constraint. This is because the desks are a physical thing and you can not have a negative amount of desks. Therefore, the program must have the following equation as a constraint.

Equation 6: $X[j] \geq 0$

IV. Solution

The recommended production schedule results in a maximum profit of 612,113 dollars and is displayed in Figure 1 and Table 8. Figure 1 shows the amount of each type of desk that should be produced for the September orders and the number of additional desks that should be produced to maximize total profit. Table 1 lists the number of each type of desk ordered alongside total production of that type of desk. Notice the total production is always greater than or equal to the amount of September orders for each desk type, thus the production schedule meets the demand. The production schedule recommends producing additional desks for certain size-line combinations under the assumption they will be sold and based on profit margin, production quotes, and materials needed. For example, the production schedule includes producing significantly more basic executive and hand-crafted standard desks than originally ordered because it will increase the total profit.

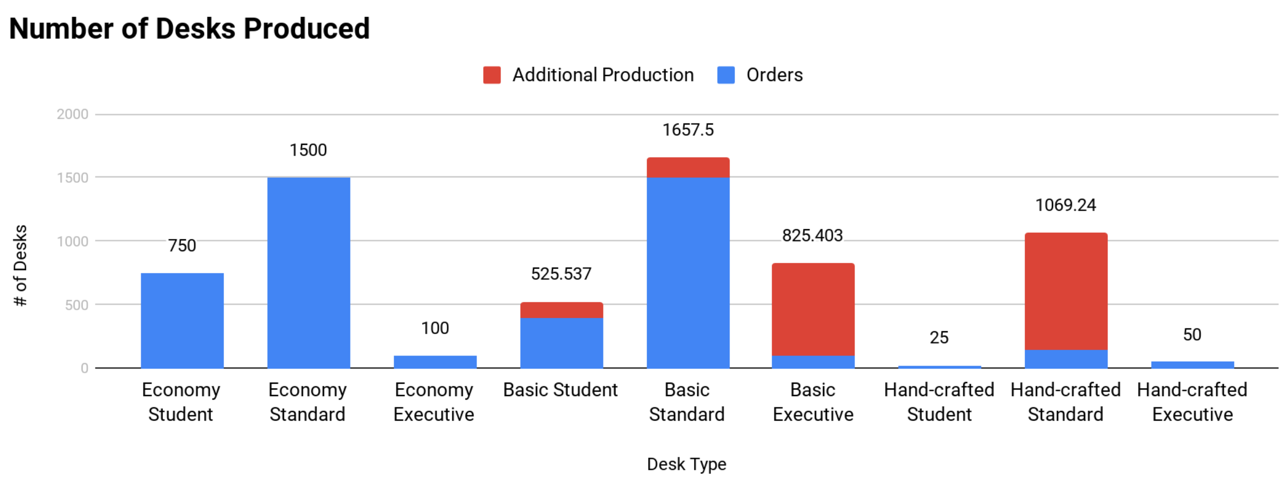

Figure 1

Table 8. Orders vs. Recommended Production

Number of Desks Produced			
Desk Line	**Desk Size**	**September Orders**	**Total Production**
Economy	**Student**	750	750
	Standard	1500	1500
	Executive	100	100
Basic	**Student**	400	525.537
	Standard	1500	1657.5
	Executive	100	825.403
Hand-crafted	**Student**	25	25
	Standard	150	1069.24
	Executive	50	50

Figure 2 and Table 9 show that the amount of each resource used in the recommended production schedule does not exceed the amount available. Figure 2 displays the percentage of each resource that is used in the production schedule, and Table 9 compares the amount used of each resource with the amount available.

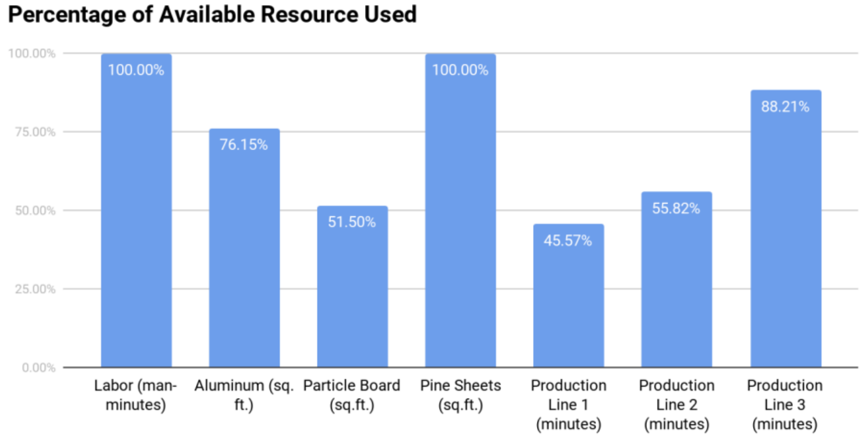

Figure 2

Table 9. Amount of Available Resources vs. Used Resources

Amount of Resources Used		
Resource	**Availability**	**Amount Used**
Labor (man-minutes)	230400	230400
Aluminum (sq.ft.)	65000	49500
Particle board (sq.ft.)	60000	30900
Pine sheets (sq.ft.)	175000	175000
Production Line 1 (minutes)	9600	4375
Production Line 2 (minutes)	9600	5358.4
Production Line 3 (minutes)	19200	16935.6

Figure 3 displays the percentage of each line of desk produced in the recommended production schedule. Table 10 lists CALDESCO's in-house line production quotas. For example, the economy line must make up at least 20 percent and at most 50 percent of the total desks produced. Figure 3 shows that the economy line makes up 36.1 percent of the total desks produced. Notice all of the line percentages fall within the min and max production quotas.

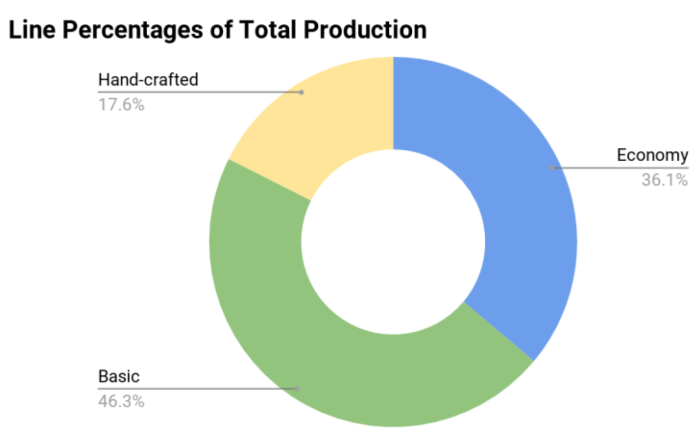

Figure 3

Table 10. In-house Line Production Quotas (for Desk Lines)

Line	**Min %**	**Max %**
Economy	20	50

Basic	40	60
Hand-crafted	10	20

Figure 4 displays the percentage of each size of desk produced in the recommended production schedule. Table 11 lists CALDESCO's in-house size production quotas. For example, the student size must make up at least 20 percent and at most 35 percent of the total desks produced. Figure 3 shows that the student size makes up 20 percent of the total desks produced. Notice all of the size percentages fall within the min and max production quotas.

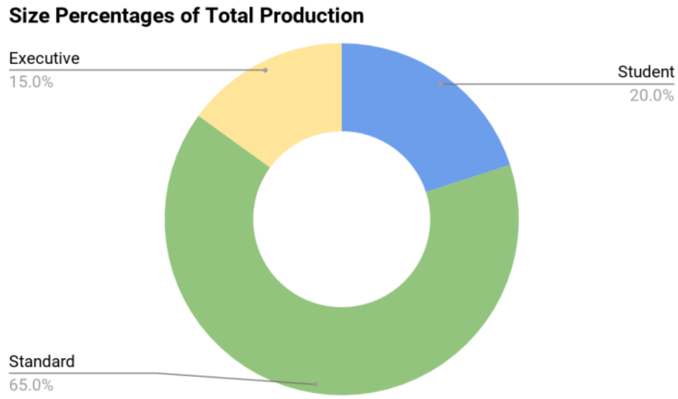

Figure 4

Table 11. In-house Size Production Quotas (for Desk Sizes)

Size	Min %	Max %
Student	20	35
Standard	40	70
Executive	5	15

The solution provided above maximizes CALDESCO's profits, given the provided information, but does not consider several real-life factors such as producing entire desks, working overtime, hiring new employees, increasing resources, and demand. The production schedule recommends producing fractions of desks for various types because it is not constrained to produce entire desks (integer programming). However, in real-life, a customer is not going to buy only the legs of a desk. If this constraint was considered, the production schedule would change. Also, the company only has 30 craftsmen that each work 40 hours per week. However, in real-life, many employees work overtime or the company hires additional employees. If the company considered overtime and hiring new employees, the production schedule may change and may produce a higher profit. Additionally, the company is constrained to a certain amount of each material resource. For example, the company has 65000 sq.ft. of aluminum available, but in real-life companies can purchase additional resources to increase production and profit. Lastly, the production schedule was created under the assumption the company could sell any production over the September orders. However, in real-life, companies must consider the demand of products, not just their profit margin on product. They have to balance production with demand as it is possible that if they over produce, they may not be able to sell all of additional the products. In this case, they would then have additional inventory costs associated with storing the additional products.

V. Discussion

This section is comprised of a sensitivity analysis of the solution outlined above. Table 12 displays an expanded version of Table 8, including the profit, reduced cost, profit lower bound, and profit upper bound. This is the first part of the sensitivity report produced in AMPL, and Table 13 contains the second part of the sensitivity report.

Table 12. Sensitivity Report Part 1

Line	Size	Related Variable	September Orders Ord[j]	Total Production	Unit Profit of Desk j C[j]	Reduced Cost r[j]	Unit Profit Lower Bound	Unit Profit Upper Bound
Economy	Student	X1	20	750	20	0	-1.00E+20	24.0623
	Standard	X2	30	1500	30	0	-1.00E+20	42.0494
	Executive	X3	40	100	40	1.77E-15	-1.00E+20	78.3373
Basic Pine	Student	X4	50	525.537	50	1.77E-15	45.3989	61.6671
	Standard	X5	80	1657.5	80	0	72.0293	83.0628
	Executive	X6	125	825.403	125	0	121.219	171.259
Hand-Crafted Pine	Student	X7	100	25	100	0	-1.00E+20	188.363
	Standard	X8	250	1069.24	250	-4.08E-14	246.892	270.172
	Executive	X9	325	50	325	0	-1.00E+20	328.766

Table 13. Sensitivity Report Part 2

Constraint		Shadow Price	Slack	Available Resource i B[i] (Con1)	Available Resource Lower Bound	Available Resource Upper Bound
Constraint 1 Resources	Labor	2.59735	0	230400	200443	239325
	Aluminum	0	15500	65000	49500	1.00E+20
	Particle Board	0	29100	60000	30900	1.00E+20
	Pine Sheets	0.234495	-2.91E-11	175000	169646	195940
	Line 1	0	5225	9600	4375	1.00E+20
	Line 2	0	4241.56	9600	5358.44	1.00E+20
	Line 3	0	2264.39	19200	16935.6	1.00E+20
Constraint 2 Orders	Economy Student	4.06234	0	0	0	0
	Economy Standard	12.0494	0	0	0	0
	Economy Executive	38.3373	0	0	0	0
	Basic Student	0	125.537	0	0	0
	Basic Standard	0	157.501	0	0	0
	Basic Executive	0	725.403	0	0	0
	Hand-crafted Student	88.3629	0	0	0	0
	Hand-crafted Standard	0	919.244	0	0	0
	Hand-crafted Executive	3.7655	0	0	0	0
Constraint 3	Economy	0	1049.46	0	-1049.46	1.00E+20

Min Quota on Lines	**Basic**	0	407.367	0	-407.367	1.00E+20
	Hand-crafted	0	493.976	0	-493.976	1.00E+20
Constraint 4 Min Quota on Sizes	**Student**	12.7924	-5.68E-14	0	-281.915	114.494
	Standard	0	1625.67	0	-1625.67	1.00E+20
	Executive	0	650.269	0	-650.269	1.00E+20
Constraint 5 Max Quota on Lines	**Economy**	0	901.343	0	-901.343	1.00E+20
	Basic	0	893.17	0	-893.17	1.00E+20
	Hand-crafted	0	156.293	0	-156.293	1.00E+20
Constraint 6 Max Quota on Sizes	**Student**	0	975.403	0	-975.403	1.00E+20
	Standard	0	325.134	0	-325.134	1.00E+20
	Executive	31.0932	-8.53E-14	0	-331.807	114.905

A. Sensitivity Analysis on Unit Profits (C[j])

In Table 12, the reduced cost is the amount the profit would have to change for the company to produce the desk. However, the company must produce certain amounts of every desk type because they must satisfy the September orders. Therefore, all of the reduced cost values are 0 or practically 0. In mathematical terms, all the reduced costs are 0 (or practically 0) by complementary slackness: (rj*xj* = 0 for all j), rj* = 0 for all j given that xj* are all positive.

In Table 12, the profit parameter is the amount of money CALDESCO makes on each type of desk. The profit lower bound and upper bound provide a range of profits for which this recommended schedule is still optimal. For example, management could increase the profit margin on the economy-student desk by 2 dollars, and the recommended schedule would remain the same. However, if management increased their profit margin C[1] on the same desk type above 24.0623, the recommended schedule would change.

Meanwhile, if we decrease C[1] from 20 to negative infinity, the optimal production level will not change either. The negative unit profit means that producing an additional unit of economy-student desk induces a loss. Given that the company has the obligation to produce no less than 750 economy-student desks under any circumstance, it must fulfill the order even if producing an additional unit of the economy-student desk induces a significant loss (i.e., C[1] approaches negative infinity). The same scenario holds for economy standard (X[2]), economy-executive (X[3]), hand-crafted student (X[7]) and hand-crafted executive (X[9]) given that the order obligation is a binding constraint for all the five decision variables above.

Variables X[4], X[5], X[6], X[8] are not bounded by the order constraints. Take basic-student desks (i.e., X[4]) as an example. From the sensitivity report, we can tell that if we increase the unit profit of the basic-student desks (C[4]) from 50 to 61.6671, the optimal production level will not change. If the increase of C[4] exceeds (61.6671 - 50 = 11.6671), the optimal production level will change.

Meanwhile, if we decrease C[4] from 50 to 45.3989, the optimal production level will not change either. If the decrease of C[4] exceeds (50 - 45.3989 = 4.6011), the optimal production level will change. The lower bound of allowable C[4] is no longer negative infinity in this case in that the order obligation is a non-binding constraint. If the unit profit of basic-student desks falls below a positive threshold, the production level of that desk will decrease because it becomes less profitable and there is no bottomline obligation. The same scenario holds for basic standard (X[5]), basic executive (X[6]) and hand-crafted standard (X[8]) given that the order obligation is a non-binding constraint for these decision variables.

B. Sensitivity Analysis on Resource Constraints (B[i])

The constraints (Con1) correspond to the maximum availability of 7 resources B[i] of different kinds and units: labor (man-minute), aluminum (square foot), particle board (square foot), pine sheets (square foot), production line 1 (minutes), production line 2 (minutes) and production line 3 (minutes).

Analysis on binding constraints (Con1[1] and Con2[4] correspond to labor B[1] and pine sheets B[4]):
From the sensitivity report, labor and pine sheets are binding constraints because their respective slacks (si*) are zero and the shadow prices (yi*) are positive. In non-technical terms, the labor and pine sheets have are the only two resources that are entirely used up in the recommended schedule. Therefore, if the company wanted to produce more of any size of the basic line, the entire production schedule would need to change.

Take the labor constraint (Con1[1]) as an example. The shadow price with respect to labor is 2.59735, implying that 2.59735 is the maximum price that the company should be willing to pay for each additional unit (man-minute) of labor. The optimal production levels will change if the available labor (man-minute) decreases, but the shadow price (y1* = 2.59735) remains the same as long as the available labor does not fall below 200,443 man-minutes. If the available labor falls by 400 man-minutes and is now 230,000 man-minutes, the maximized profit is reduced by 400*2.59735 = 1,038.94 dollars. If the available labor falls below 200,443 man-minutes, the maximized profit is expected to decrease more than (230,400-200,443)*2.59735, but we no longer know the exact drop in profit because the shadow price begins to change.

If the available labor increases by 600 man-minutes and is now 231,000 man-minutes, the maximized profit is increased by 600*2.59735 = 1,540.41 dollars. If the available labor increases above 239,325 man-minutes, the maximized profit is expected to increase no less than (239,325 - 230,400) *2.59735, but we no longer know the exact increase in profit because the shadow price begins to change. As labor becomes universally abundant (the right-hand-side of Con1[1] gets arbitrarily large), the labor constraint becomes non-binding, leading the shadow price to be reduced to zero.

Looking at the labor constraint from another perspective, if the company were to hire 1 additional person (increase the workforce from 30 to 31 workers), the labor availability would increase by 7680 man-minutes (from 230400 to 238080 man-minutes). After re-configuring for the new amount of labor while keeping everything else the same, the new profit will boost from $612113 to $632061, therefore, as the the labor increases by 1 craftsmen, the profit will increase by 3%. In the original problem, labor is constrained by 30 craftsmen working 160 hours a week, 80% of the time. In the real world, businesses can hire more workers if needed to match demand. In addition, they can offer incentives or overtime pay in order to increase the percentage of time the workers are available to work, which would also increase the labor availability (man-minutes).

Analysis on non-binding constraints (aluminum, particle board, and production lines 1-3):
From the sensitivity report, aluminum, particle board, production line 1, production line 2, and production line 3 are non-binding constraints because their respective slacks (si*) are positive and the shadow prices (yi*) are zero. In non-technical terms, aluminum (square foot), particle board (square foot), production line 1 (minute), production line 2 (minute) and production line 3 (minute) are not entirely used up in the recommended production schedule.

Take the aluminum constraint (Con1[2]) as an example. The shadow price with respect to aluminum is zero, implying that the company is not willing to pay any extra money for an additional unit (square foot) of aluminum. This makes sense from the business standpoint because 15,500 square feet of aluminum is not being used, so a wise decision maker will not pay more for extra aluminum. The optimal production levels will not change if the available aluminum (square foot) increases up to infinity. Again, the reason is that some of the aluminum is already redundant so that any additional amount of aluminum will not lead to an increase in desk production. The optimal production levels will not change either if the available aluminum (in square foot) decreases by fewer than 15,500 square feet. 15,500 is the slack of the aluminum constraint, and thus represents the leftover aluminum. If we reduce the available aluminum by more than 15,500 square feet, aluminum becomes a binding constraint. The original production levels (i.e., optimal solution to the primal problem) are no longer feasible, requiring the recommended production schedule to change. Similar scenarios hold for particle board (i.e., Con1[3]), production line 1 (Con1[5]), production line 2 (Con1[6]) and production line 3 (Con1[7]).

VI. Appendix

A. AMPL Results

```
ampl: include Case1.run;
maximize prof:
 20*X[1] + 30*X[2] + 40*X[3] + 50*X[4] + 80*X[5] + 125*X[6] + 100*X[7] + 250*X[8] + 325*X[9];
subject to Con1[1]:
 15*X[1] + 17*X[2] + 19*X[3] + 23*X[4] + 28*X[5] + 32*X[6] + 76*X[7] + 93*X[8] + 110*X[9] <= 230400;
subject to Con1[2]:
        14*X[1] + 24*X[2] + 30*X[3] <= 65000;
subject to Con1[3]:
        8*X[1] + 15*X[2] + 24*X[3] <= 60000;
subject to Con1[4]:
        22*X[4] + 40*X[5] + 55*X[6] + 25*X[7] + 45*X[8] + 60*X[9] <= 175000;
subject to Con1[5]:
        1.5*X[1] + 2*X[2] + 2.5*X[3] <= 9600;
subject to Con1[6]:
        X[1] + X[2] + X[3] + X[4] + X[5] + X[6] <= 9600;
subject to Con1[7]:
        3*X[4] + 4*X[5] + 5*X[6] + 3*X[7] + 4*X[8] + 5*X[9] <= 19200;
subject to Con2[1]:
        -X[1] <= -750;
subject to Con2[2]:
        -X[2] <= -1500;
subject to Con2[3]:
        -X[3] <= -100;
subject to Con2[4]:
        -X[4] <= -400;
subject to Con2[5]:
        -X[5] <= -1500;
subject to Con2[6]:
        -X[6] <= -100;
subject to Con2[7]:
        -X[7] <= -25;
subject to Con2[8]:
        -X[8] <= -150;
subject to Con2[9]:
        -X[9] <= -50;
subject to Con3[1]:
 -0.8*X[1] - 0.8*X[2] - 0.8*X[3] + 0.2*X[4] + 0.2*X[5] + 0.2*X[6] + 0.2*X[7]+0.2*X[8]+0.2*X[9] <= 0;
subject to Con3[2]:
 0.4*X[1] + 0.4*X[2] + 0.4*X[3] - 0.6*X[4] - 0.6*X[5] - 0.6*X[6] + 0.4*X[7] + 0.4*X[8] + 0.4*X[9] <= 0;
subject to Con3[3]:
 0.1*X[1] + 0.1*X[2] + 0.1*X[3] + 0.1*X[4] + 0.1*X[5] + 0.1*X[6] - 0.9*X[7] - 0.9*X[8] - 0.9*X[9] <= 0;
subject to Con4[1]:
 -0.8*X[1] + 0.2*X[2] + 0.2*X[3] - 0.8*X[4] + 0.2*X[5] + 0.2*X[6] - 0.8*X[7] + 0.2*X[8] + 0.2*X[9] <= 0;
subject to Con4[2]:
 0.4*X[1] - 0.6*X[2] + 0.4*X[3] + 0.4*X[4] - 0.6*X[5] + 0.4*X[6] + 0.4*X[7] - 0.6*X[8] + 0.4*X[9] <= 0;
subject to Con4[3]:
 0.05*X[1] + 0.05*X[2] - 0.95*X[3] + 0.05*X[4] + 0.05*X[5]-0.95*X[6]+0.05*X[7]+0.05*X[8]-0.95*X[9] <= 0;
subject to Con5[1]:
```

```
 0.5*X[1] + 0.5*X[2] + 0.5*X[3] - 0.5*X[4] - 0.5*X[5] - 0.5*X[6] - 0.5*X[7] - 0.5*X[8] - 0.5*X[9] <= 0;
subject to Con5[2]:
 -0.6*X[1] - 0.6*X[2] - 0.6*X[3] + 0.4*X[4] + 0.4*X[5] + 0.4*X[6] - 0.6*X[7] - 0.6*X[8] - 0.6*X[9] <= 0;
subject to Con5[3]:
 -0.2*X[1] - 0.2*X[2] - 0.2*X[3] - 0.2*X[4] - 0.2*X[5] - 0.2*X[6] + 0.8*X[7] + 0.8*X[8] + 0.8*X[9] <= 0;
 subject to Con6[1]:
0.65*X[1] - 0.35*X[2] - 0.35*X[3] + 0.65*X[4] - 0.35*X[5]-0.35*X[6]+0.65*X[7]-0.35*X[8]-0.35*X[9] <= 0;
subject to Con6[2]:
 -0.7*X[1] + 0.3*X[2] - 0.7*X[3] - 0.7*X[4] + 0.3*X[5] - 0.7*X[6] -0.7*X[7] + 0.3*X[8] - 0.7*X[9] <= 0;
subject to Con6[3]:
-0.15*X[1] - 0.15*X[2] + 0.85*X[3] - 0.15*X[4] - 0.15*X[5]+0.85*X[6]-0.15*X[7]-0.15*X[8]+0.85*X[9] <= 0;

CPLEX 12.8.0.0: sensitivity
CPLEX 12.8.0.0: optimal solution; objective 612113.3832
12 dual simplex iterations (2 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
# $1 = _objname
:      $1      _obj  _varname _var    _var.rc        _var.down      _var.up
 :=
1   prof   612113   'X[1]' 750     0         -1e+20          24.0623
2      .        .    'X[2]'   1500         0         -1e+20          42.0494
3      .        .    'X[3]'        100     0         -1e+20          78.3373
4      .        .    'X[4]'        525.5371.77636e-15     45.398961.6671
5      .        .    'X[5]'   1657.5       -1.77636e-15     72.0293   83.0628
6      .        .    'X[6]'        825.4030              121.219171.259
7      .        .    'X[7]'        25      0         -1e+20          188.363
8      .        .    'X[8]'   1069.24      -4.08562e-14     246.892   270.172
9      .        .    'X[9]'        50      0         -1e+20          328.766
;
: _var.current    :=
1      20
2      30
3      40
4      50
5      80
6      125
7      100
8    250
9      325
;

# $4 = _con.current
:      _conname       _con    _con.slack      $4    _con.down        _con.up
 :=
1      'Con1[1]'      2.59735         0        230400   200443     239325
2      'Con1[2]'      0        15500           65000    49500      1e+20
3      'Con1[3]'      0        29100           60000   30900      1e+20
4      'Con1[4]'      0.234495        -2.91038e-11   175000   169646          195940
5      'Con1[5]'      0        5225            9600    4375      1e+20
6      'Con1[6]'      0        4241.56         9600    5358.44    1e+20
7      'Con1[7]'      0        2264.39         19200   16935.6    1e+20
```

```
8	'Con2[1]'	4.06234	0	0	0	0
9	'Con2[2]'	12.0494	0	0	0	0
10	'Con2[3]'	38.3373	0	0	0	0
11	'Con2[4]'	0	125.537	0	0	0
12	'Con2[5]'	0	157.501	0	0	0
13	'Con2[6]'	0	725.403	0	0	0
14	'Con2[7]'	88.3629	0	0	0	0
15	'Con2[8]'	0	919.244	0	0	0
16	'Con2[9]'	3.7655	0	0	0	0
17	'Con3[1]'	0	1049.46	0	-1049.46	1e+20
18	'Con3[2]'	0	407.367	0	-407.367	1e+20
19	'Con3[3]'	0	493.976	0	-493.976	1e+20
20	'Con4[1]'	12.7924	-5.68434e-14	0	-281.915	114.494
21	'Con4[2]'	0	1625.67	0	-1625.67	1e+20
22	'Con4[3]'	0	650.269	0	-650.269	1e+20
23	'Con5[1]'	0	901.343	0	-901.343	1e+20
24	'Con5[2]'	0	893.17	0	-893.17	1e+20
25	'Con5[3]'	0	156.293	0	-156.293	1e+20
26	'Con6[1]'	0	975.403	0	-975.403	1e+20
27	'Con6[2]'	0	325.134	0	-325.134	1e+20
28	'Con6[3]'	31.0932	-8.52651e-14	0	-331.807	114.905
;
```

B. AMPL Data File

```
# j: Types of desk
	# 1 = economy student
	# 2 = economy standard
	# 3 = economy executive
	# 4 = basic student
	# 5 = basic standard
	# 6 = basic executive
	# 7 = hand-crafted student
	# 8 = hand-crafted standard
	# 9 = hand-crafted executive
# i: Resources
	# 1 = labor in worker minutes
	# 2 = aluminum in square foot
	# 3 = particle board in square foot
	# 4 = pine sheets in square foot
	# 5 = production line 1 in minute
	# 6 = production line 2 in minute
	# 7 = production line 3 in minute
# q: Desk class for min/max quotas
	# 1 = economy
	# 2 = basic pine
	# 3 = hand crafted
	# 4 = student
	# 5 = standard
	# 6 = executive
param n := 9; # Number of types of desks
param m := 7; # Number of resources
# C: Objective coefficients of the primal problem
	# Cj: Unit profit earned by producing one desk type j
```

```
param   C :=
        1   20
        2   30
        3   40
        4   50
        5   80
        6   125
        7   100
        8   250
        9   325;
# A: Coefficient matrix
        # Aij: Amount of resource i needed to produce one desk of type j
param   A:  1   2   3   4   5   6   7   8   9:= # desk types=j=1..9, resources=i=1..7
        1   15  17  19  23  28  32  76  93  110
        2   14  24  30  0   0   0   0   0   0
        3   8   15  24  0   0   0   0   0   0
        4   0   0   0   22  40  55  25  45  60
        5   1.5 2   2.5 0   0   0   0   0   0
        6   1   1   1   1   1   1   0   0   0
        7   0   0   0   3   4   5   3   4   5;
# B: Right-hand-side constraints of resources
        # Bi: The maximum amount that resource i is available
param   B :=
        1   230400
        2   65000
        3   60000
        4   175000
        5   9600
        6   9600
        7   19200;
 # Ord: Orders that give the minimum number of each desk to be produced
        # Ord_j: Orders = Minimum number of desk type j to be produced
param   Ord :=
        1   750
        2   1500
        3   100
        4   400
        5   1500
        6   100
        7   25
        8   150
        9   50;
# Min: Minimum quota
        # Min_q: minimum quota of a certain q class of desks
param   Min :=
        1   20
        2   40
        3   10
        4   20
        5   40
        6   5;

# Max: Maximum quota
        # Max_q: Maximum quota of a certain q class of desks
```

```
param   Max :=
        1   50
        2   60
        3   20
        4   35
        5   70
        6   15;
```

C. AMPL Mod File

```
# PARAMETERS
param n; # Number of types of desks
param m; # Number of resources

set J := {1..n}; # j: Types of desk
set I := {1..m}; # i: Resources
set Q := {1..sqrt(n)}; # q: Desk class for min/max quotas

# j: Types of desk
        # 1 = economy student
        # 2 = economy standard
        # 3 = economy executive
        # 4 = basic student
        # 5 = basic standard
        # 6 = basic executive
        # 7 = hand-crafted student
        # 8 = hand-crafted standard
        # 9 = hand-crafted executive
# i: Resources
        # 1 = labor in worker minutes
        # 2 = aluminum in square foot
        # 3 = particle board in square foot
        # 4 = pine sheets in square foot
        # 5 = production line 1 in minute
        # 6 = production line 2 in minute
        # 7 = production line 3 in minute
# q: Desk class for min/max quotas
        # 1 = economy
        # 2 = basic pine
        # 3 = hand crafted
        # 4 = student
        # 5 = standard
        # 6 = executive

param C {J} >= 0; # Cj: Unit profit earned by producing one desk type j
param A {I,J} >= 0; # Aij: Amount of resource i needed to produce one desk of type j
param B {I} >= 0; # Bi: The maximum amount that resource i is available
param Ord {J} >= 0; # Ord_j: Orders = Minimum number of desk type j to be produced
param Min {J} >= 0; # Min_q: minimum quota of a certain q class of desks
param Max {J} >= 0; # Max_q: Maximum quota of a certain q class of desks

# DECISION VARIABLES
var X {J} >= 0; # Xj = number of each j type of desk to produce
# OBJECTIVE FUNCTION
```

```
maximize prof: sum {j in J} C[j] * X[j]; # prof: total production profit
# CONSTRAINTS
# Constraint 1: Can not use more resources than available
s.t. Con1 {i in I}:
	sum {j in J} A[i,j] * X[j] <= B[i];
# Constraint 2: Must at least produce enough desks to fulfill orders
s.t. Con2 {j in J}:
	-X[j] <= -Ord[j];
# Constraint 3: Must satisfy minimum percentage quotas for the different desk lines
s.t. Con3 {q in Q}:
	-(X[3*q-2] + X[3*q-1] + X[3*q]) <= -(Min[q]/100 * sum {j in J} X[j]);
# Contraint 4: Must satisfy minimum percentage quotas for the different desk sizes
s.t. Con4 {q in Q}:
	-(X[q] + X[3+q] + X[6+q]) <= -(Min[3+q]/100 * sum {j in J} X[j]);
# Contraint 5: Must satisfy maximum percentage quotas for the different desk lines
s.t. Con5 {q in Q}:
	X[3*q-2] + X[3*q-1] + X[3*q] <= Max[q]/100 * sum {j in J} X[j];
# Constraint 6: Must satisfy maximum percentage quotas for the different desk sizes
s.t. Con6 {q in Q}:
	X[q] + X[3+q] + X[6+q] <= Max[3+q]/100 * sum {j in J} X[j];
```

C. AMPL Run File

```
reset;
model Case1.mod;
data Case1.dat;
expand prof, Con1, Con2, Con3, Con4, Con5, Con6;

option solver cplex;
option presolve 1;
option cplex_options 'sensitivity';

solve;

display _objname, _obj, _varname, _var, _var.rc, _var.down, _var.up, _var.current;
display _conname, _con, _con.slack, _con.current, _con.down, _con.up;
display X, prof, _objname, _obj, _varname, _var, _var.rc, _var.down, _var.up, _var.current, _conname, _con,
_con.slack, _con.current, _con.down, _con.up >> Case1upd.txt;
```