Case 2: Giant Motor Company

IEOR 240

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I. Introduction

This report gives the Giant Motor Company (GMC) recommendations on retooling specific production plants and lays out a production plan for each of its plants for the upcoming year. The sections of the report are Background Information, Formulation, Solution, Discussion, and an Appendix. The Background Information section provides an overview of GMC's problems and considerations. The Formulation section describes decision variables, the objective function, constraints, and any assumptions that were made. The Solution section presents the optimal solution that maximizes profit, while meeting constraints. The Discussion section presents an analysis of the solution and gives the company recommendations. Finally, the Appendix includes the AMPL code and results.

II. Background Information

A. Retooling/ Production

The Giant Motor Company produces three lines of cars: Lyras (inexpensive subcompact), Libras (sporty compact), and Hydras (luxury). The company currently produces each of these cars on its own production line but is considering retooling its Lyra and/or Libra plants. The company needs to figure out if retooling these plants is a worthy investment. They also need plan production for each plant in order to meet the forecasted demand for the coming year.

On one hand, retooling these plants will increase the fixed costs of those plants. Fixed costs are annual costs independent of the number of cars produced and include property taxes, insurance, loan payments and more. If the plant is retooled, the new fixed cost includes the fixed cost of the original plant and the renovation costs. On the other hand, retooling the plants will increase production capacity, marginal profit contributions, and flexibility. The increased flexibility will allow GMC to produce more than one line of cars at the new retooled plants.

The fixed costs, production capacity, profit margins, and flexibility associated with each type of plant are all considered in the formulation section of this report in order to come up with recommendations on retooling/expansion and production planning.

B. Demand/ Demand Diversion

GMC has been able to come up with reliable forecast demands for each of their car lines. They know that if they strictly look at production capacity versus forecasted demand, they can not meet demand. However, they have figured from the past that some of this is offset by demand diversion.

Demand diversion is where if there are not enough cars of one type, the salesperson can convince the customer to purchase a higher up car. GMC has been able to determine what percentage of unmet demand of one type of GMC car can materialize into demand of a better GMC car, while the remaining unfulfilled demand goes to its competitors.

GMC provided their forecasted demands as well as their demand diversion matrix, both of which are used as considerations in formulation.

III. Formulation

In order to help GMC solve their production planning and capacity expansion problem, a mixed integer linear program (MILP) was designed to optimize GMC's profit while remaining constrained to its parameters and business model.

A. Assumptions

It was assumed that the plants could not produce a fraction of a car, thus all amounts of cars produced (x[i,j]) are integers. Also, it was assumed that the direct unsatisfied demand (y[i]) and the number of customers diverted to more expensive car types and the competitor could be continuous values. For example, the plan includes convincing 7142.5 unsatisfied Lyra customers to buy a Libra instead. While in the real-world this would not be possible, these assumptions were made because the company uses a mixed integer linear program (MILP) to determine their production and retooling plans, thus some variables can be continuous while others are discrete.

B. Decision Variables

GMC's main pending decisions are: 1) whether or not to retool their Lyra and/or Libra Plants 2) production plans for each of their plants. In addition, to facilitate these decisions, GMC also needs to determine how much unsatisfied demand the can redirect and the total fixed costs. The MILP takes each of these decisions into account by the following decision variables.

In order to determine whether or not to retool the Lyra and/or Libra Plants, binary decision variables b1 and b2 were used respectively. The variable would equal to 1 (true) if the plant should be retooled and 0 (false) if it isn't a worthy investment and should not be retooled. A summary of this is in Table 1.

Table 1: Decision Variables- Binary- Retooling Decision (b)

Plant	Binary Decision Variables	Plant Should be Retooled	Should Not be Retooled
Lyra Plant	b1	1	0
Libra Plant	b2	1	0

The next set of decision variables were to determine the number of cars each plant should produce and this was represented by the matrix x[i,j]. These variables were constrained to be non-negative integer variables because cars are real items and you can not produce a fraction of car or a negative car. The car type was denoted by the rows (i) and the plant they were produced by was represented the columns (j) of the x[i,j] decision variables matrix as shown below.

Table 2: Decision Variables- Integer (number of cars) - Production Planning (x[i,j])

x [i,j]	Plant [j]	Original Lyra	Original Libra	Hydra	New Lyra	New Libra
Car Type [i]	**i \ j**	**1**	**2**	**3**	**4**	**5**
Lyra	**1**	x[1,1]	x[1,2]	x[1,3]	x[1,4]	x[1,5]
Libra	**2**	x[2,1]	x[2,2]	x[2,3]	x[2,4]	x[2,5]
Hydra	**3**	x[3,1]	x[3,2]	x[3,3]	x[3,4]	x[3,5]

The third set of variables are related to demand diversion and are shown in the table below. The decision variables, y[i], denote the amount of unsatisfied demand for each car type (i). Percentages of each unmet demand can be diverted into demand for another car type or to the competition, and this is discussed further in the Solution section. These variables are non-negative as demand for cars can not be negative. Also, these variables, as stated in the assumptions section of the report, are considered to be continuous not integers because the company wants to use an MILP instead of a ILP. However, in the real-world these variables are actually integers since it is impossible to convince half a customer to buy a different car type.

Table 3: Unmet Demand Variables (y[i])

Car Type [i]	i	Unmet Demand y[i]
Lyra	1	y1
Libra	2	y2
Hydra	3	y3

Lastly, a non-negative variable representing the total fixed cost (tfc) is needed for the MILP. This variable takes into account the fixed costs associated with each of the plant types and is dependent on whether or not the two plants are re-tooled.

C. Objective Function

The objective is to maximize total profit for GMC and therefore Equation 1 represents the objective function for the linear program.

Equation 1 $$\textit{maximize profit}: \sum_{j=1}^{P} \sum_{i=1}^{C} (pm[i,j] * x[i,j]) - tfc$$

The equation takes into consideration the profit margins (pm[i,j]) on each car (x[i,j]) and the total fixed costs (tfc) associated with the production plants. The profit margins are obtained from GMC in their problem statement and are shown in Table 4. Note that the index j with set P represent the five choices of production plants and index i with set C represent the three car types. In the data file, both the profit margin and the fixed cost are in units of $1,000. Hence, there is no need to convert unit in the objective function.

Table 4: Profit Margins Parameter (pm[i,j]) (in $ Thousands) of each type of car produced at each plant

pm [i,j]	Plant [j]	Original Lyra	Original Libra	Hydra	New Lyra	New Libra
Car Type [i]	**i \ j**	**1**	**2**	**3**	**4**	**5**
Lyra	**1**	2			2.5	2.3
Libra	**2**		3		3	3.5
Hydra	**3**			5		4.8

Note that in the profit margin parameter (pm[i,j]), the empty cells mean that the associated car type (i) can not be produced by the corresponding plant (j).

The objective function (Equation 2) is expanded out with the profit margin parameter plugged in results in the following.

*maximize profit = 2*x[1,1] + 2.5*x[1,4] + 2.3*x[1,5] + 3*x[2,2] + 3*x[2,4] + 3.5*x[2,5] + 5*x[3,3] + 4.8*x[3,5] - tfc*

D. Parameters and Constraints

In the formulation of the linear program, there were multiple parameters and constraints to account for the all of the background information and GMC's business model.

An important note for many of the constraints that are dependent on retooling or not, is that this dependency is represented in the constraints by either 1-b1 and b1 (for Lyra plants) or 1-b2 and b2 (for Libra plants). The parameter value associated with the original plant is multiplied by 1-b so that if the plant is retooled (b=1) then this goes to zero and is not counted. On the other hand, if the plant is not retooled (b=0) then 1-b equals one and the original gets counted. The parameter value associated with the new retooled plant is multiplied by b so that if the plant is retooled (b=1), it gets counted, and if it is not retooled (b=0), it gets ignored. By having this manipulation in the constraints, we ensure that only one of the plants, original or new retooled, is counted in the constraints.

-- Total fixed cost constraint:

The first of the constraints is the total fixed cost (tfc) definition, which is dependent on the retooling options and is based off of the fixed cost parameter (fc) that the company provided. The constraint along with its associated parameter are shown below.

*tfc = fc[1] * (1-b1) + fc[2] * (1-b2) + fc[3] + fc[4] * b1 + fc[5] * b2;*

Table 7: Fixed Cost Parameter fc[j]

Plant [j]	Original Lyra	Original Libra	Hydra	New Lyra	New Libra
j	1	2	3	4	5
Fixed Cost ($ Thousands) Parameter (fc[j])	2,000,000	2,000,000	2,600,000	3,400,000	3,700,000

Notice that this is one of the cases of the retooling dependency. In this case, 1-b1 and b1 paired with fc[1] and fc[4] respectively mean that if Lyra (original plant fc[1]) was used then the New Lyra plant (fc[4]) would not be and vice versa. This is the same for 1-b2 and b2 paired with fc[2] and fc[5] corresponding to the original and new Libra plants. This manipulation ensures that only one of each type of plant is counted in the total fixed cost.

-- Production capacity constraints:

The next set of constraints are for production capacity. Since each plant has a production cap, which is represented by the parameter cp[j], and can only produce certain types of cars, we have to take this into consideration. A summary of this is in Table 8. As with the fixed cost constraint, we have a retooling dependency because we can only have one type each plant either original or new retooled. In order to take this into account the same 1-b and b manipulations were added to the production constraints, which are shown below.

1. Original Lyra Plant Capacity: *x[1,1] <= cp[1] * (1 - b1)*
2. Original Libra Plant Capacity: *x[2,2] <= cp[2] * (1 - b2)*
3. Hydra Plant Capacity: *x[3,3] <= cp[3]*
4. New Lyra Plant Capacity: *x[1,4] + x[2,4] <= cp[4] * b1*
5. New Libra Plant Capacity: *x[1,5] + x[2,5] + x[3,5] <= cp[5] * b2;*

Table 8: Production Capability and Capacity Parameter cp[j]

Plant [j]	Original Lyra	Original Libra	Hydra	New Lyra	New Libra
j	1	2	3	4	5
Plant Capability to Produce Car Type (i)	Lyra	Libra	Hydra	Lyra , Libra	Lyra , Libra, Hydra
Car Type (i) Each Plant (j) Can Produce	x[1,1]	x[2,2]	x[3,3]	x[1,4] + x[2,4]	x[1,5] + x[2,5] + x[3,5]
Production Capacity (# of Cars) (cp[j])	1,000,000	800,000	900,000	1,600,000	1,800,000

-- Demand Diversion Constraints:

The diverted demand occurs when a GMC dealer convinces unsatisfied customers to purchase a more expensive model, because there were not enough cars of they type they originally wanted. It is important to note that the dealer can only convince a certain percentage of customers to upgrade, thus some of the unmet demand is lost to competitors. Table 9 lists the demand (dm[i]) for each car type and the demand diversion (dv[i, j]). The demand diversion is the percentage of unsatisfied customers that originally wanted to buy car type i and were instead convinced to buy car type j due to the short supply. For example, 30 percent of customers who originally wanted to buy the Lyra but cannot because of short supply can be convinced to to purchase the Libra instead.

Table 9: Demand Diversion by Car Type

Car Type Customer Originally Wanted	Demand (dm[i])	Demand Diversion (dv[i, j])		
		Lyra	Libra	Hydra
Lyra	1,400,000		30 %	5 %
Libra	1,100,000	0		10 %
Hydra	800,000	0	0	

There are three demand diversion constraints listed below, one for each type of car. Each constraint requires y[i] to be less than or equal to the amount of unsatisfied customers. The amount of unsatisfied customers is calculated by subtracting the total amount of each car type produced from the demand for that car type. Note the the sign in the constraints in less than or equal to instead of strictly equal to, because the dealer does not need to release his or her full potential of persuasion. In other words, not all unsatisfied demand for car i must be harnessed by GMC dealers.

1. Excess demand for Lyra to be harnessed: *y[1] <= dm[1] - (x[1,1] + x[1,4] + x[1,5])*
2. Excess demand for Libra to be harnessed: *y[2] <= dm[2]-(x[2,2] + x[2,4]+ x[2,5])+dv[1,2]* y[1]*
3. Excess demand for Hydra to be harnessed: *y[3] <= dm[3]-(x[3,3]+x[3,5]) + dv[1,3] * y[1]+dv[2,3] * y[2]*

Note all non negativity constraints are discussed in the Decision Variables subsection of the Formulation section.

IV. Solution

The recommended retooling and production plan include retooling only the Lyra plant and producing 1,257,150 Lyras, 1,142,850 Libras, and 807,143 Hydras. This plan will result in a maximum profit of 2.60714 billion dollars. Table 10 lists the demand and production by car type, identifying excess demand and excess production. Figure 1 displays the demand and production of each car type. Notice the plan recommends producing more Libras and Hydras than demanded and less Lyras than demanded in hopes that some customers can be diverted to purchasing more expensive models.

Table 10: Demand and Production by Car Type

Car Type	Demand	Production	Excess Production	Excess Demand
Lyra	1,400,000	1,257,150		142,850
Libra	1,100,000	1,142,850	42,850	
Hydra	800,000	807,143	7,143	

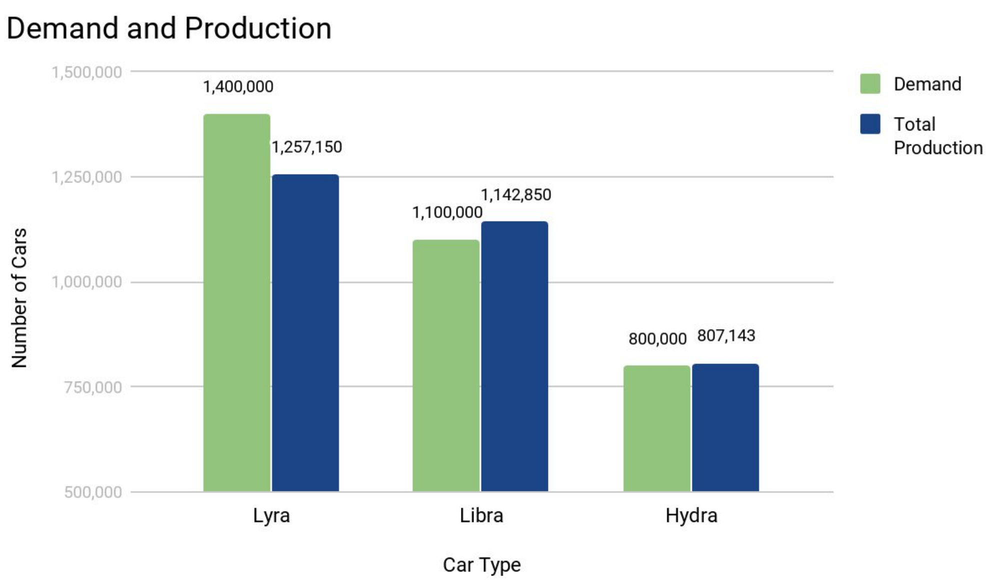

Figure 1

The MILP determined it is in GMC's best interest to retool the Lyra plant but not the Libra plant, because that investment would not be profitable. Thus, the company will have the new Lyra plant, Hydra plant, and original Libra plant to produce cars. Table 11 breaks down the production at each plant by car type, lists the total number of cars produced at each plant, and lists the plant capacities. Notice that the new Lyra plant and original libra plant are at maximum capacity, but the Hydra plant is 92,857 cars under capacity. Figure 2 displays the production of each car by plant and Figure 3 compares the recommended production to capacity of each plant.

Table 11: Amount Produced of Each Car Type by Plant

Car Type	New Lyra Plant	Hydra Plant	Original Libra Plant
Lyra	1,257,150	0	0

Libra	342,850	0	800,000
Hydra	0	807,143	0
Total Cars Produced	1,600,000	807,143	800,000
Car Production Capacity	1,600,000	900,000	800,000

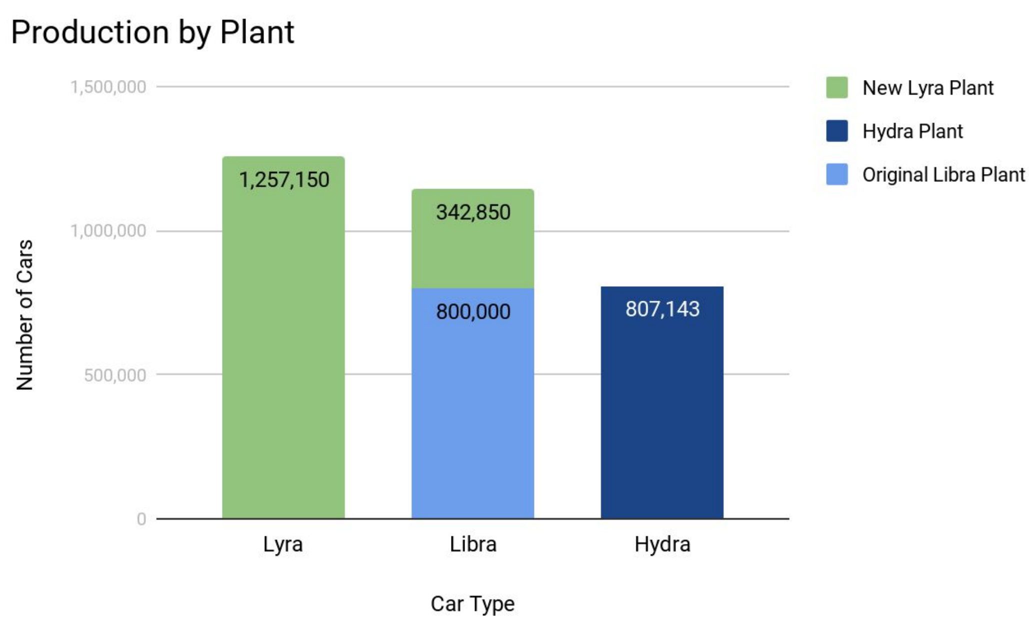

Figure 2

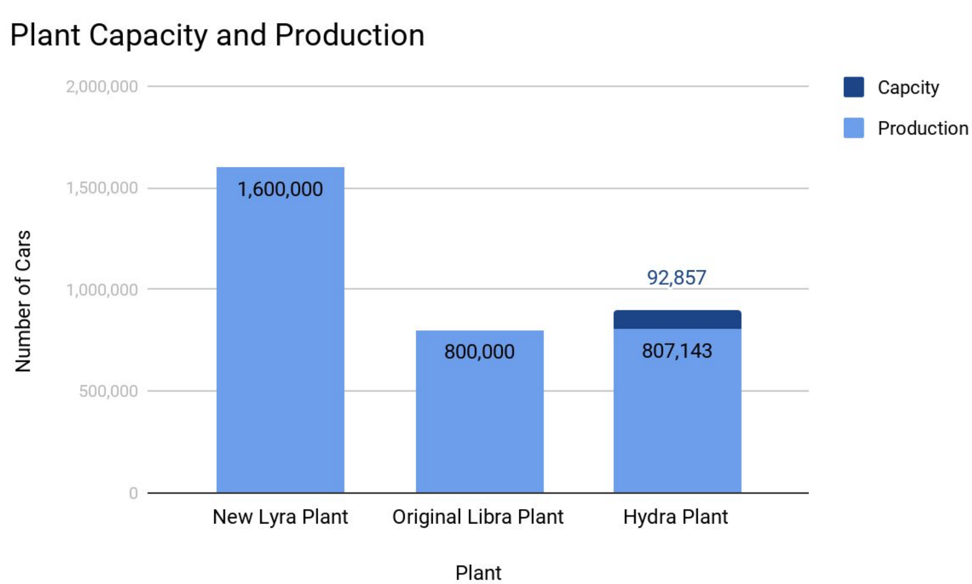

Figure 3

As mentioned earlier, the recommended plan includes producing 142,850 Lyras under demand, 42,850 Libras over demand, and 7,143 Hydras over demand. GMC projects that 30% or 42,855 unsatisfied customers who were hoping to buy the Lyra can be convinced to buy the Libra instead and 5% or 7,142.5 unsatisfied customers can be convinced to by buy the Hydra instead. GMC projects that 10% or 0.5 unsatisfied customers who were hoping to buy the Libra can be convinced to buy the Hydra instead. Table 12 lists the excess demand diversion by car type. Notice in Table 12 that the remaining 65% or 92,852.5 unsatisfied Lyra customers and 90% or 4.5 unsatisfied Libra customers who were not convinced to buy a higher model will be diverted to the competition. Figure 4 displays the percentages of unsatisfied Lyra and Libra customers who will be diverted to buying a higher model or diverted to the competition.

Table 12: Excess Demand Diversion by Car Type

Car Type Demanded	**Unsatisfied Customers**	**Car Type Diverted**			**Diverted to Competition**
		Lyra	Libra	Hydra	
Lyra	142,850		42,855	7,142.5	92,852.5
Libra	5	0		0.5	4.5
Hydra	0	0	0		0

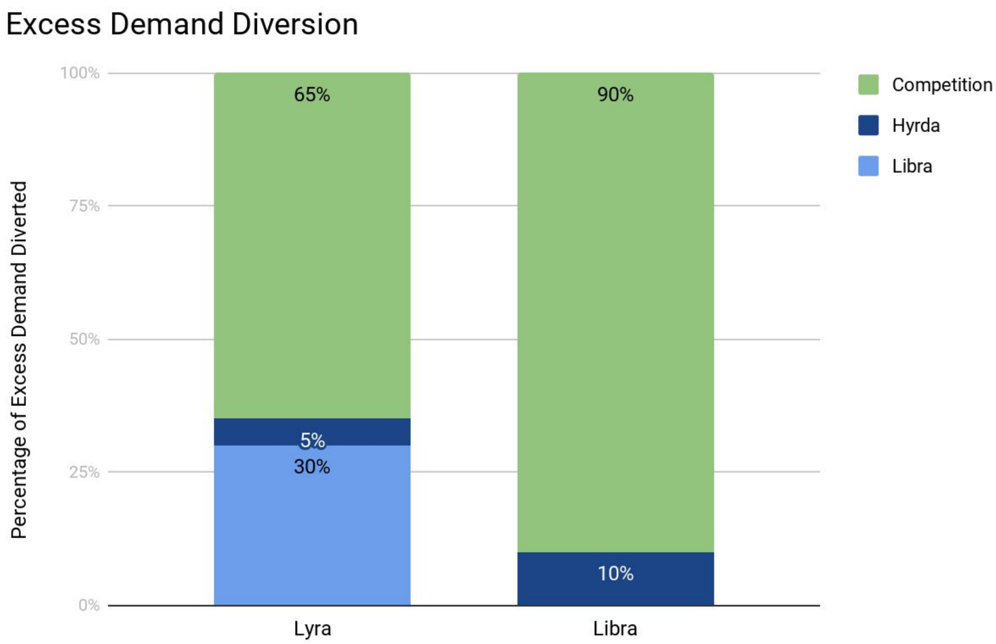

Figure 4

Note this report uses the term ‘revenue’ to describe the amount of money the company makes before considering the fixed costs. This report uses the term ‘profit’ to describe the amount of money the company makes after considering fixed costs.

Table 13 lists the amount of revenue each car is predicted to generate, based on sales predictions and profit margins, with this retooling and production plan. The company is expected to generate a total of 10.60714 billion dollars in revenue. Figure 5 shows the breakdown of revenue in percentages by car type. The Hydra will account for approximately 38%, the Libra will account for approximately 32.3%, and the Lyra will account for approximately 29.6%. Table 13 breaks down the fixed cost by plant and lists the total fixed cost to be 8 billion dollars. The New Lyra plant was the largest percentage of the total fixed cost at 3.4 billion because of the retooling. Nonetheless, GMC will make 2.60714 billion in profit, which is the total fixed cost subtracted from the total revenue. Table 13 also breaks down the revenue by plant. Notice that every plant's revenue surpasses the fixed cost, even the new Lyra plant which had the highest fixed cost due to retooling.

Table 13: Revenue by Car Type

Car Type	Sales Prediction (# of Cars)	Profit Margin Per Car ($ Thousands)	Revenue ($ Thousands)
Lyra	1,257,150	2.5	3,142,875
Libra	1,142,850	3	3,428,550
Hydra	807,143	5	4,035,715
		Total Revenue ($)	**10,607,140**

Table 13: Fixed Cost by Plant

Plant	Car Type Produced	Revenue ($ Thousands)	Fixed Cost ($ Thousands)
New Lyra Plant	Lyra and Libra	4,171,425	3,400,000
Original Libra Plant	Libra	2,400,000	2,000,000
Hydra Plant	Hydra	4,035,715	2,600,000
		Total Fixed Cost ($)	**8,000,000**

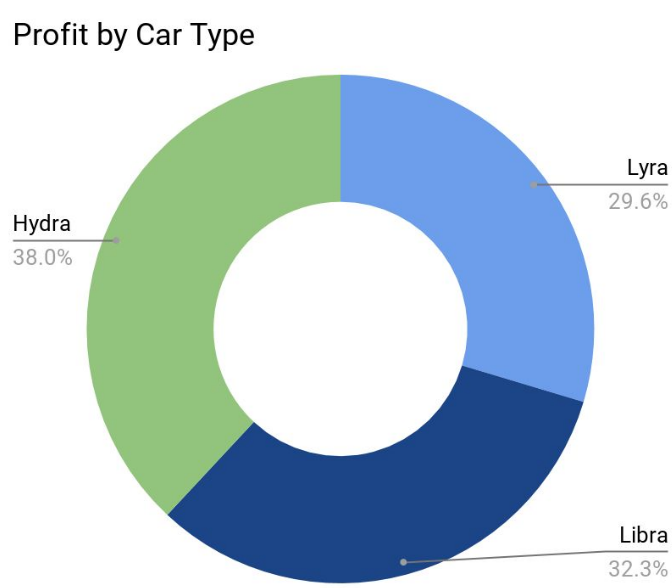

Figure 5

V. Discussion

Some of GMC's biggest decisions for the upcoming year were whether or not to retool the Lyra and Libra plants. This report recommends retooling the Lyra Plant but not the Libra plant, based on the profit the new plants can generate compared to the original plants. The new Lyra plant has a capacity of 1,600 cars and is projected to generate $4.171425 billion dollars in revenue and $771.425 million in profit after considering the fixed cost. If the original Lyra plant was at maximum capacity of 1,000 Lyra cars, it would only generate $2 billion in revenue and $0 in profit due to the fixed cost of $ 2 billion. Thus, the new Lyra plant is predicted to make $771.425 million more in profit than the original Lyra plant. Note these revenue and profit calculations for each plant are for operating at maximum capacity, but the original Lyra plant's maximum capacity is 800,000 cars less than the new Lyra plant. Since the MILP determined it was not profitable to retool the Libra plant, it did not calculate the amount of each car the Lyra plant would have produced. Therefore, it is not possible to project how profitable this plant would have been compared to the original, but by the MILP's decision to not retool, it can be assumed that the new plant would not have made as much money as the original plant. This result is not surprising due to the high fixed cost of the new Libra plant and the lower marginal costs of producing Lyras and Hydras compared to the new Lyra plant and Hydra plant respectively.

When considering this recommended retooling and production plan, the most concerning aspect from a business perspective is the amount of customers that GMC will be losing to the competition due to insufficient supply. With 92,852.5 unsatisfied Lyra customers diverted to the competition, GMC is missing out on $232.13125 million dollars in Lyra revenue. With 4.5 unsatisfied Libra customers diverted to the competition, GMC is missing out on $13,500 dollars in Libra revenue. Two options for GMC to decrease the amount of customers it is losing to the competition are to retool the Hydra plant to produce both Hydras and Lyras and/or invest in more training for dealers. In this retooling and production plan, the Hydra plant can only produce Hydras and is 92,857 cars under capacity. The demand for Hydras is not enough to get the Hydra plant to maximum capacity, but GMC could retool the Hydra plant to produce both Hydras and Lyras. By producing 92,852.5 more Lyras at the retooled Hydra plant, the company would not be losing so many customers to the competition and the Hydra plant would be closer to its capacity (assuming the capacity of the new Hydra plant was the same as the original Hydra plant). Note this would impact the amount of unsatisfied Lyra customers that are diverted to purchasing a higher model. Thus, this is a balancing game, because GMC does want to divert some customers to buying the higher model to increase profit. The other option is for GMC to invest in more training for the dealers, such that they are more convincing when persuading unsatisfied customers to buy the higher model. This strategy would also increased the amount of customers retained and increase revenue when the customers buy the higher priced car types.

When considering the proposed solution, notice it is not possible for a fraction of a customer to be diverted to the competition. For example, the company cannot lose exactly 92,852.5 and 4.5 unsatisfied Lyra and Libra customers to the competition respectively. These fractions are due to solving this problem with an mixed integer linear program (MILP), which means some parts of the solution such as the number of cars produced are whole numbers while other parts of the solution are allowed to be fractions like the customers diverted. If the problem was restricted to diverting entire customers (not fractions), the solution would change and would be more realistic.

VI. Appendix

<u>A. AMPL Results and Conclusions</u>

```
: _objname    _obj    _varname    _var    :=
1   profit   2607140   b1           1
```

```
2     .     .    b2           0
3     .     .    'x[1,1]'     0
4     .     .    'x[1,2]'     0
5     .     .    'x[1,3]'     0
6     .     .    'x[1,4]'   1257150
7     .     .    'x[1,5]'     0
8     .     .    'x[2,1]'     0
9     .     .    'x[2,2]'    8e+05
10    .     .    'x[2,3]'     0
11    .     .    'x[2,4]'   342850
12    .     .    'x[2,5]'     0
13    .     .    'x[3,1]'     0
14    .     .    'x[3,2]'     0
15    .     .    'x[3,3]'   807143
16    .     .    'x[3,4]'     0
17    .     .    'x[3,5]'     0
18    .     .    'y[1]'     142850
19    .     .    'y[2]'       5
20    .     .    'y[3]'       0
21    .     .    tfc         8e+06
;
```

```
# Conclusions:
# 1. GMC needs to retool the Lyra plant, but not the Libra plant.
# 2. The retooled Lyra plant produces 1,257,150 Lyras and 342,850 Libras (full capacity).
  # Unsatisfied demand for Lyra: 142,850
  # Diversion to Libra: 142,850 * 30% = 42,855
  # Diversion to Hydra: 142,850 * 5% = 7,142.5
  # Lyra demand equilibrium: 1,400,000 = 1,257,150 + 142,850
# 3. The original Libra plant produces 800,000 Libras (full capacity).
  # Unsatisfied demand for Libra: 5
  # Diversion to Hydra: 5 * 10% = 0.5
  # Libra demand equilibrium: 1,100,000 + 42,855 = 800,000 + 342,850 + 5
# 4. The original Hydra plant produces 807,143 Hydras (all Hydra demand satisfied).
  # Hydra demand equilibrium: 800,000 + 7,142.5 + 0.5 = 807,143
# 5. Total fixed cost is 8,000,000 thousand dollars = 8 billion dollars.
# 6. Total profit is 2,607,140 thousand dollars = 2.60714 billion dollars.
```

B. AMPL Data File

```
param   pm:  1   2   3   4   5 :=
        1    2   0   0   2.5 2.3
        2    0   3   0   3   3.5
        3    0   0   5   0   4.8;

param   fc :=
        1    2000000
```

```
        2   2000000
        3   2600000
        4   3400000
        5   3700000;

param   cp :=
        1   1000000
        2   800000
        3   900000
        4   1600000
        5   1800000;

param   dm :=
        1   1400000
        2   1100000
        3   800000;

param   dv:  1   2   3 :=
        1   0   0.3 0.05
        2   0   0   0.1
        3   0   0   0;

# pm: Profit Margin
  # rows (cars): Lyra, Libra, and Hydra
  # columns (plants): (orig) Lyra, (orig) Libra, Hydra, new Lyra, and new Libra
# fc: Fixed Cost (in $1,000, by plant)
# cp: Production Capacity (by plant)
# dm: Demand (by car)
# dv: Diversion Table (cheaper cars to more luxurious ones)
```

C. AMPL Mod File

```
set P := {1..5}; # Five choices of plants
set C := {1..3}; # Three lines of cars

param pm{i in C, j in P}; # Profit margin
param fc{j in P}; # Fixed cost
param cp{j in P}; # Production capacity
param dm{i in C}; # Demand
param dv{i in C, k in C}; # Diversion table

# Decision Variables
var b1 binary; # b1 = 1 if retooled and b1 = 0 if not
var b2 binary; # b2 = 1 if retooled and b2 = 0 if not
# b1 (binary) denotes whether the Lyra plant should be retooled.
```

```
# b2 (binary) denotes whether the Libra plant should be retooled.
var x{i in C, j in P} >= 0 integer;
# x[i,j] is the number of car i to be produced by plant j.
var y{i in C} >= 0;
# y[i] is the unsatisfied demand for car i that GMC plans to harness.
var tfc >= 0 ; # tfc denotes total fixed cost.

# Objective Function
maximize profit:
sum {i in C, j in P} (pm[i,j] * x[i,j]) - tfc;
# Important Note: In our data file, both the profit margin and the fixed cost are in $1,000.
# Hence, there is no need to convert unit.

# Constraints
# I. Fixed Cost
s.t. t_f_c: tfc = fc[1] * (1-b1) + fc[2] * (1-b2) + fc[3] + fc[4] * b1 + fc[5] * b2;
# Total fixed cost depends on the retooling plan.

# II. Plants
s.t. p1: x[1,1] <= cp[1] * (1 - b1);            # Production constraint for (Orig) Lyra
s.t. p2: x[2,2] <= cp[2] * (1 - b2);            # Production constraint for (Orig) Libra
s.t. p3: x[3,3] <= cp[3];                       # Production constraint for (Orig) Hydra
s.t. p4: x[1,4] + x[2,4] <= cp[4] * b1;          # Production constraint for New Lyra
s.t. p5: x[1,5] + x[2,5] + x[3,5] <= cp[5] * b2; # Production constraint for New Libra

# III. Direct unsatisfied demand
s.t. d1: y[1] <= dm[1] - (x[1,1] + x[1,4] + x[1,5]);                # Excess demand for Lyra to be harnessed.
s.t. d2: y[2] <= dm[2] - (x[2,2] + x[2,4] + x[2,5]) + dv[1,2] * y[1];  # Excess demand for Libra to be harnessed.
s.t. d3: y[3] <= dm[3] - (x[3,3] + x[3,5]) + dv[1,3] * y[1] + dv[2,3] * y[2]; # Excess demand for Hydra to be
harnessed.
# Note: y[i] does not need to be equal to the right hand side, "<=" suffices.
# Reason: the salesperson does not need to release his or her full potential of persuasion.
```

D. AMPL Run File

This file is used to run AMPL and solve the MILP.

```
reset;
option solver cplex;
model case2.mod;
data case2.dat;
solve;
display _varname, _var;
display  _objname,  _obj,  _varname,  _var > case2.txt;
```