Technical University of Denmark

42380 Supply Chain Analytics

Group 25

Inventory routing

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1 Demand analysis

1.1 Demand Distribution

As the forecast is given in units of products sold, we start by multiplying the forecast with 100, as we know that 100g of salmon is used per product. As we would like to have the data in kg, we then divide it with 1000. The data is now ready for use.

The mean and standard deviation for salmon at each DC on each of the days is found by creating a pivot table in excel. The mean is found by summing the total demand for each day at each DC. The standard deviation is found by taking the square of the RMSE, and summing it for each day at each DC. Afterwards the square root of this sum is taken and it is divided by 10 to get the data in kg.

Calculations can be seen in the Excel file in the appendix. The mean is displayed in table 1 & the standard deviation in table 2 below:

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC1	75,9	75,9	75,9	75,9	75,9	85,7	85,7	75,9
DC2	62,9	94,8	94,8	94,8	94,8	68,65	36,75	62,9
DC3	67,2	67,2	67,2	67,2	67,2	67,15	67,15	67,2
DC4	102,3	156	156	156	156	119,05	65,35	102,3
DC5	107,4	130,1	130,1	130,1	130,1	65,15	42,45	107,4

Table 1: Mean

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC1	11,87	11,87	11,87	11,87	11,87	14,69	14,69	11,87
DC2	10,91	13,89	13,89	13,89	13,89	14,14	11,22	10,91
DC3	11,27	11,27	11,27	11,27	11,27	12,86	12,86	11,27
DC4	13,19	16,03	16,03	16,03	16,03	15,05	11,98	13,19
DC5	13,04	14,00	14,00	14,00	14,00	11,08	9,84	13,04

Table 2: Standard Deviation

1.2 Pros/Cons of having only Salmon

Having salmon in each of the three products can have both pros and cons:

Pros of only using salmon:

- **Better Focus & Efficiency:** There is only one good to take care of, and therefore all the focus and attention can be on this good, hereby securing the best quality and timeliness.
- **Lower Marketing Cost**: Content, adds etc. only has to be done for one single good, hereby lowering marketing costs.

- **Optimizes Supply Chain Management**: Multiple goods, may lead to multiple suppliers, hereby increasing the risk of something going wrong, delays etc. Even if all the goods come from the same supplier, this enhances the risk as well that something might go wrong, as there may suddenly be a shortage on one of the fish's for example.
- **Usability**: If there is a much higher demand for sandwich on one day then anticipated, the salmon intended for the wraps and salads may be used. This also makes sure that the goods intended for wraps and salads can be used for other products, if the demand for those options are low on a given day.

Cons of only using salmon:

- **More Pressure & market research**: More pressure on choosing the right good. This will demand an extended market research, which might be expensive.
- **Fewer Opportunities**: Having only one product, will lead to fewer opportunities, as customers that need several products will probably turn to an other supplier that can deliver all of them.
- **Less spread of Risk**: Having only one good and one supplier may increase the risk of not being able to supply the company's customers. This could for example be the case, if there was a sudden shortage of salmon. Having only one fish, would result in the fresh-food producer not being able to produce their products. If the supplier was to have multiple fish's, they would still be able to deliver these, hereby still enabling the fresh-food producer to product products. Having multiple goods may therefore spread the risk.

2 Solving the problem

In this section a mathematical model for optimizing the inventory routing between the supplier & the DC's will be implemented. One part of the model consists of finding the optimal partitioning and sequencing of the DC's, and the other part of finding the optimal inventory for each DC on each day. The model takes into account the different DC's storage capacity, the routing cost, and the delivery trucks capacity.

2.1 Building and solving an inventory routing model

The aim of the inventory routing model is to minimize the total transportation and inventory costs. It does this by combining the Vehicle Routing Problem with the Inventory Problem.

Definition of Parameters & Variables

The first step is defining the different Parameters & Variables of the model

Parameters:
N Set of nodes, one of them is the depot, the rest customers
N^- Set of customers
K Number of vehicles at the depot
d_i Quantity to be delivered to customer i (demand)
c_{ij} Transportation cost for going directly from node i to node j
I Set of customers (previously N^-)
r_t Supply available at depot at the start of period t
C_i Inventory holding capacity of customer $i \in I$
D_k Capacity of vehicle k
h_i Holding cost per unit of inventory at node $i \in N$ per period
I_{i0} Inventory level at node $i \in N$ at the beginning of the planning horizon
SI_i Inventory level at node $i \in N$ at the beginning of the planning horizon

Variables:
x_{ijk} Binary, if vehicle k goes directly from node i to node j
y_{ik} Binary, if vehicle k visits customer i
q_{ikt} Quantity delivered to customer i by vehicle k at start of period t
z_{ikt} Load of truck k when arriving at node i in period t
I_{it} Inventory level at node $i \in N$ at the end of period $t \in T$

Modeling the inventory routing model

The second step is defining an objective function which minimizes the total costs, as well as some constraints to make sure the results obtained from the model are feasible.

Objective function

The following costs are minimized:

- The holding cost at each inventory in time period t
- The total cost of shipping goods with vehicle k to DC i in time period t

Constraints

1. The start inventory is defined
2. Flow balance constraints, inflow has to be equal to outflow at the depot in each period t
3. Flow balance constraints, inflow has to be equal to outflow at each DC i in each period t
4. Inventory capacity constraints, the capacity for each DC i in each period t can not be exceeded
5. Inflow has to be equal to the outflow
6. Subtours are eliminated by assigning a load variable to each truck, such that when a truck visits DC i the load of truck k is lower than it was before the visit.
7. Linking the VRP to the inventory part, the quantity delivered to each DC in period t, can not exceed the total capacity
8. Linking the VRP to the inventory part, the quantity delivered to each DC in period t, can not exceed the demand

Model

The finished model is displayed below:

$$\text{minimize} \sum_{i \in N} \sum_{t \in T} (h_i \cdot I_{it}) + \sum_{t \in T} \sum_{k=1} \sum_{i \in N} \sum_{j \in N} c_{ij} \cdot x_{ijkt}$$

subject to

$$I_{i,0} = SI_i \quad i \in I$$

$$I_{1,t-1} + r_t = I_{1,t} + \sum_{k \in K} \sum_{i \in I} q_{ikt} \quad t \in T$$

$$I_{i,t-1} + \sum_{k \in K} q_{ikt} = I_{it} + d_{it} \quad i \in I, t \in T$$

$$I_{i,t-1+} \sum_{k \in K} q_{ikt} \leq C_i i \in I, t \in T$$

$$\sum_{j \in N} x_{jikt} = \sum_{j \in N} x_{ijkt} = y_{ikt} i \in N, k = 1, \ldots, 1K, t \in T$$

$$z_{ikt} - d_{it} \geq z_{jkt} - (1 - x_{ijkt}) \cdot \sum_{i \in N} \cdot d_{it} \quad i \in I, j \in I, k \in K, t \in T$$

$$q_{ikt} \leq C_i y_{ikt} \quad i \in I, k \in K, t \in T$$

$$\sum_{i \in I} q_{ikt} \leq \mathrm{D}_k y_{1kt} \quad k \in K, t \in T$$

$$x_{ijkt} \in \{0, 1\} \quad \forall i \in N \quad \forall j \in N \quad \forall k \in K \quad \forall t \in NT$$

$$y_{ikt} \geq 0 \quad \forall i \in N \quad \forall k \in K \quad \forall t \in T$$

$$q_{ikt} \geq 0 \quad \forall i \in N \quad \forall k \in K \quad \forall t \in T$$

$$I_{it} \geq 0 \quad \forall i \in N \quad \forall t \in T$$

The model is now solved using Julia.

2.1.1 Results

The full overview over the routes used and how much is delivered to the different DC's in the different time periods can be seen below:

Period	Truck	Route	Kg. Product
1	1	1-4	92
		4-5	235
		5-3	292
		3-2	281
		2-1	0
	2	1-6	527
		6-1	0
3	1	1-4	202
		4-5	468
		5-6	230
		6-1	0
6	1	1-2	247
		2-3	165
		3-5	287
		5-4	201
		4-1	0

Table 3: Final Results

This results in an optimal cost of 4481.57kr. Based on this it can be concluded that there is no need for the company having 3 trucks available at all time, as they maximum use a total of 2 trucks in the different time periods.

2.2 Building a simulation

The producer also has a fill rate target, defined as the percentage of that day's demand that could be fulfilled from stock at the DC. The fill rate target is that 99 out of 100 times, the DC's satisfy more than 95% of customer demand. Using Julia, we simulate the situation 10000 times and each time, the actual demands are generated using the given demand distribution. In listing 1, the code snippet of the simulation is shown.

The results obtained from the simulation are shown in table 4. Most of the DC's satisfy the target rate on some days, however the DC's struggles with insufficient stock on day 2, 5 and 8, meaning the fill rate target is not met. Relating this to the results obtained in 3, this makes good sense as day 2, 5 and 8 are the days just before next delivery.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC1	100%	100%	100%	99%	55%	100%	99%	54%
DC2	100%	100%	100%	100%	56%	99%	97%	54%
DC3	100%	70%	100%	99%	56%	100%	99%	55%
DC4	100%	65%	100%	100%	64%	100%	99%	56%
DC1	100%	100%	100%	100%	100%	100%	99%	55%

Table 4: Probability of meeting 95% of the demand for each DC.

Listing 1: Simulation of fill rates

```
Random.seed!();
iterations = 10000
filled= zeros((N, T));
sales = zeros((N, T));
for r = 1:iterations
    actual_demand = demand_mean + randn(N,T).*std;
    new_inv = value.(I)
    for t=1:T #T
        for i =1:N #N
            delivered = 0;
            for k = 1:K
                delivered = delivered + JuMP.value.(q[i,k,t]);
            end
            # update inventory and calculate fill rate:
            if t==1
                new_inv[i,t] = init_inventory[i] + delivered - actual_demand[i,t]
                sales[i,t] = min(new_inv[i,t] + delivered, actual_demand[i,t])
            else
                new_inv[i,t] = new_inv[i,t-1] + delivered - actual_demand[i,t]
                sales[i,t] = min(new_inv[i,t-1]+ delivered, actual_demand[i,t])
            end
            if value(sales[i,t]) >= actual_demand[i,t] * 0.95
                filled[i,t] += 1;
            end
        end
    end
end
```

2.3 Introducing safety stock

From previous simulation exercise it was shown that the fill target rate was not met, and the inventory routing model is not robust to uncertainty in demand. To adapt for uncertainty in the model, we introduce a safety stock. We update the mean demands in table 1 by adding a safety stock calculated by the standard deviation multiplied by a safety factor.

We then update the safety factor in our simulations until the fill rate target is met. From our analysis, it is estimated that the model requires a **safety factor of 1.93** in order to meet the target, resulting in an increase in **total cost to 5774.**

3 Capacity analysis

3.1 Capacity utilization of original solution

In this part we are going to analyze the capacity utilization of the solution found in the previous part. This is to identify a possible bottleneck and is done by calculating the percentage of utilization of storage capacity for every distribution center on each day, and the percentage of truck capacity utilized on the start of every trip. Table 5 shows the utility of each DC at the beginning of each day, meaning that the demand has not been delivered to the customers, and table 6 shows the utilization of each truck at the beginning of each day.

DC/Day	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC1	75,60%	60,48%	45,36%	30,24%	15,12%	49,26%	32,19%	15,12%
DC2	90,59%	77,70%	58,28%	38,85%	19,43%	34,49%	20,42%	12,89%
DC3	63,00%	32,45%	91,64%	61,09%	30,55%	91,59%	61,07%	30,55%
DC4	53,15%	32,10%	97,08%	64,98%	32,88%	58,99%	34,50%	21,05%
DC5	82,48%	68,01%	81,58%	64,04%	46,51%	28,98%	20,20%	14,47%

Table 5: Storage utilization of original solution

Truck/Day	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
Truck 1	0%	0%	100%	0%	0%	0%	0%	0%
Truck 2	100%	0%	0%	0%	0%	0%	0%	0%
Truck 3	58.48%	0%	0%	0%	0%	100%	0%	0%

Table 6: Truck utility

Since there are never more than 2 out of 3 trucks in use at the same time, it can be concluded that the storage capacity is the bottleneck.

3.2 Capacity analysis

In this section we are going to consider three capacity scenarios a) truck capacity is increased by a factor of 10, b) storage capacity is increased by a factor of 10 and c) both storage and truck capacity are increased by a factor of 10.

3.2.1 Scenario a: inflating truck capacity

When the truck capacity is increased by a factor of 10 in the original model, the total cost is reduced to 4288.97. In order to meet the fill rate target in this scenario it requires a safety factor of 1 resulting in a total cost of 5352.72. The amount of safety stock needed each day is summarized in table 7.

DC/Day	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC1	69,94%	52,45%	34,97%	17,48%	74,96%	57,48%	37,48%	17,48%
DC2	81,94%	66,82%	44,55%	22,27%	64,19%	41,92%	24,95%	15,13%
DC3	71,34%	35,67%	71,34%	35,67%	72,04%	36,37%	72,04%	35,67%
DC4	59,16%	35,40%	70,79%	35,40%	62,99%	27,59%	39,67%	23,76%
DC5	35,65%	19,42%	91,81%	72,39%	52,97%	33,55%	23,28%	16,23%

Table 7: Storage utilization of scenario a

3.2.2 Scenario b: inflating storage capacity

When the storage capacity is increased by a factor of 10 in the original model, the total cost is reduced to 3311.37. In order to meet the fill rate target in this scenario it requires a safety factor of 1.39 resulting in a total cost of 3601.32. Since we identified the storage capacity as the bottleneck earlier on, it was expected that this scenario would lead to a greater decrease in the total cost compared to an increase in the truck capacity. The amount of safety stock needed each day is summarized in table 8.

DC/Day	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC1	3,68%	1,84%	11,59%	9,75%	7,91%	6,07%	3,95%	1,84%
DC2	15,43%	13,83%	11,50%	9,16%	6,82%	4,48%	2,67%	1,60%
DC3	30,33%	26,56%	22,80%	19,03%	15,26%	11,50%	7,63%	3,77%
DC4	5,69%	21,72%	18,05%	14,39%	10,72%	7,05%	4,17%	2,48%
DC5	3,71%	2,02%	9,58%	7,57%	5,55%	3,53%	2,45%	1,69%

Table 8: Storage utilization of scenario b

3.2.3 Scenario c: inflating both truck and storage capacity

When combining the two scenarios, the total cost is reduced to 1883,07. In order to meet the fill rate target in this scenario it requires a safety factor of 0,567 resulting in a total cost of 1908.50. The amount of safety stock needed each day is summarized in table 9.

DC/Day	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
DC1	13,62%	11,98%	10,33%	8,68%	7,04%	5,39%	3,52%	1,65%
DC2	13,70%	12,29%	10,18%	8,08%	5,97%	3,87%	2,30%	1,42%
DC3	26,84%	23,49%	20,15%	16,80%	13,46%	10,11%	6,73%	3,35%
DC4	22,21%	19,96%	16,56%	13,16%	9,77%	6,37%	3,74%	2,26%
DC5	12,15%	10,60%	8,74%	6,88%	5,02%	3,16%	2,19%	1,55%

Table 9: Storage utilization of scenario c

With regards to bottleneck, it is obviously still the storage capacity that is the bottleneck in scenario a. In scenario b, the truck capacity is the bottleneck, and in scenario c there is no bottleneck since we are able to deliver all demand in the first time period, meaning that increasing the capacities further would not yield a lower total cost.

3.3

Looking in the case where both storage capacity and truck capacity are inflated, we obtain a total cost of 1908.5 and an optimal route of 1 truck visiting Depot-6-2-3-5-4-Depot in the first period. Having high transportation costs and low holding costs while level of demand remains the same, it is clear that the model seeks to minimize any transportation and prefer to hold stock in any given period.

In this case, inflating both capacities increases flexibility by far and allows for aggregating forecasts. By aggregating demand streams and stocking up in the DC's, the flexibility in random fluctuations in demand increases, meaning by pooling demand streams the amount of safety stock required to meet at given fill rate level can be reduced. This is called the risk-pooling effect.

A more technical explanation of this, is that the amount of safety stock required is proportional to the standard deviation of demand, as given in the model. The standard deviation of demand at an aggregate level of the DC's is smaller than the sum of the standard deviations as derived in the slides from week 7. However, it is important to take in to account that all other parameters are kept constant in this experiment, meaning the flexibility and advantage is extreme. In real world settings, increasing truck- and storage capacities also increases shipping- and holding costs which might offset the pooling effect.

4 Flexibility analysis

In the current situation, the DC's place their orders 1 day in advance and the planning horizon of the truck routes are made for the following 8 days. Based on the information given in the project description, it is assumed that this process is repeated every. Now the situation has changed and the DC's now place their orders 2 days in advance, meaning the planned route have a 2 day buffer from where it can be changed, which is again repeated on a daily basis. In other words, the lead time would be extended with one day, but the day of the order delivery would remain the same. Causing no negative effect to the customers. This is visualized in figure 1.

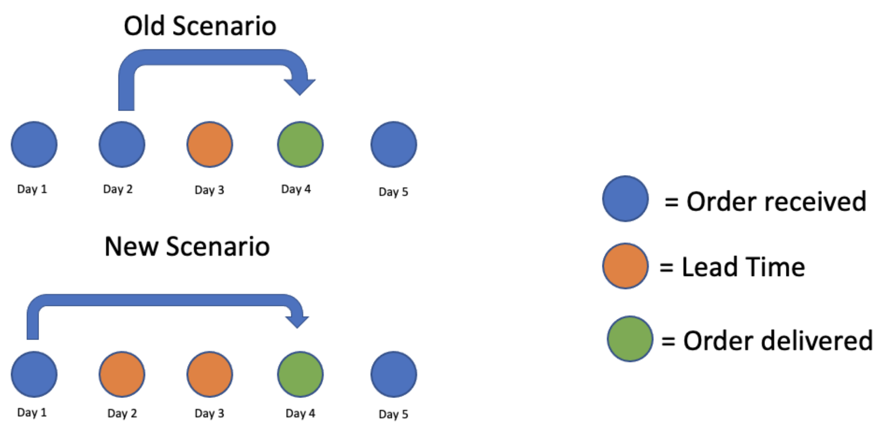

Figure 1: Lead Time

A longer lead time adds some flexibility in the planning of the routes, as we now have a longer buffer period and are able to 1) change the routes and adjust for uncertainty and sudden events in the demand forecast and 2) utilize the storage capacity in all DC's in a better way.

From the storage utilization in original solution in table 5, we see that there is still some unused storage capacity for each day. Allowing for allocation of salmons on the DC's before delivery day might open up for utilizing the third truck, that the current solution does not use.

4.1 Visual Comparison of the two scenarios:

When looking at the old scenario as displayed in figure 2, it can be seen that on day 4, the day of the order delivery, the demand is known for the corresponding day, as well as for day 5.

When looking at the new scenario, the demand is on the other hand known both for the corresponding day, day 5, as well as day 6. This means the demand is known for an extra day, giving the company the possibility of shipping all orders for those 3 days, on the same day, hereby saving money.

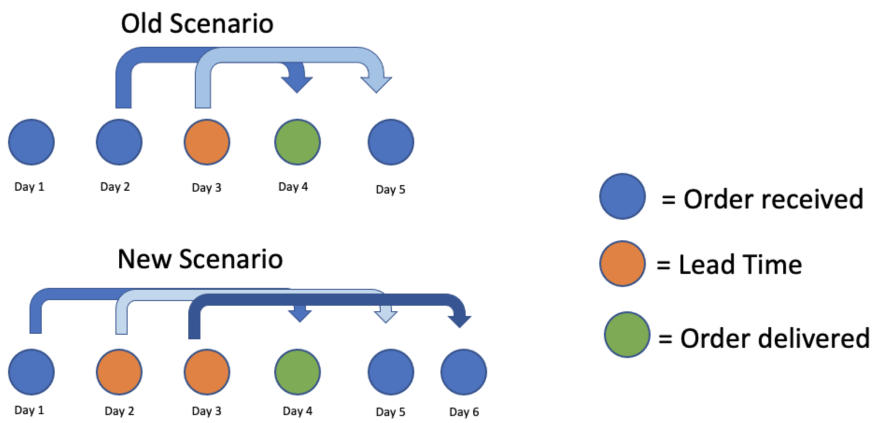

Figure 2: Lead Time

In terms of the model from section 2.1, this means a relaxation of constraint 3, adding the inventory from the previous day to the constraint as well. This expands the original solution space, hereby providing a lower bound to the original model.

5 Contribution Table

Name	**Student Number**	**Contribution**
Frederik Bjørnstrup	S184301	Part 1 & 3
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Julie Olin	s184317	Part 2 & 3
Peter Vilhelmsen	s184292	Part 2 & 4

Table 10

Overall everyone has contributed evenly to the project and helped in every Part.