On the Excitation and Propagation of Zonal Winds in an Atmosphere with Newtonian Cooling

Robert E. Dickinson

Massachusetts Institute of Technology, Cambridge

(Manuscript received 24 July 1967, in revised form 23 October 1967)

ABSTRACT

A system of equations, including Newtonian cooling and valid away from the equator, is derived for atmospheric zonal winds and the associated zonal temperature and meridional circulation fields. A time scaling is introduced to distinguish three regimes of motion. For short time scales a variation of the usual nondissipative forced symmetric vortex problem is recovered. For intermediate time scales, a "diffusive model" is defined, while for long time scales a "steady state" model is defined. Two examples of the solution of the diffusive equation are given. From the first example we infer that our model is capable of explaining many of the observed features of the downward progression of the stratospheric biennial oscillation. Momentum is carried downward by Coriolis torques resulting from propagating meridional cells. The solutions of our second example describe transient zonal winds which resemble those observed to propagate downward from the stratopause in Meteorological Rocket Network data time-height sections. In general, dissipation relaxes the constraint of conservation of potential vorticity so that wave-like motions are possible when Newtonian cooling is present. The "steady state" model equation is solved by the method of Green's functions to obtain the forced temperature. For this model, there is a direct response to heating and only momentum fluxes force meridional circulations to give subsidence heating. It is found that the domain of influence of the temperature response to momentum fluxes lies below the source point. Our model suggests that observed long-period departures from radiative equilibrium in the atmosphere below the mesopause can be explained as due either to heating by horizontal eddy heat transports or to subsidence heating forced by eddy momentum fluxes.

1. Introduction

One of the basic problems in the theory of atmospheric motions is the manner in which a symmetric vortex adjusts to hydrostatic and geostrophic balance. We shall analyze here the dynamics of such balanced zonal motions, when Newtonian cooling is assumed. The variation on the symmetric vortex problem considered here was essentially initiated by Leovy (1964), who constructed models of the great circumpolar jets of the winter and summer stratosphere-mesosphere. Leovy assumed a realistic heat input of annual periodicity and derived resulting motions, taking as dissipation Newtonian cooling as well as eddy thermal diffusion, eddy momentum diffusion and linear drag. He found that the maximum amplitude of the predicted zonal winds depended primarily on the Newtonian cooling. Values agreeing with observation were obtained with a temperature cooling coefficient of magnitude 10^{-6} $\sec^{-1}$. On the other hand, the predicted meridional circulations and vertical decrease of winds with height in the mesosphere depended primarily on assumed linear drag or eddy viscosity.

More recently, Lindzen (1966) studied the dynamics of a geostrophic, symmetric vortex at the equator damped by radiative-photochemical relaxation. Lindzen's (1966) theory, while rather unsuccessful in modeling the biennial oscillation, is of great intrinsic interest in that it provides an example of how diabatic damping can shift the phase of the zonal temperature field relative to the zonal wind field, so that the geostrophic zonal winds can propagate as a wave. In the absence of such dissipation, zonal winds can change in time only as a consequence of external forcing. In this paper we assume a simplified form of the dissipative mechanism used by Lindzen and assume in the manner of Kuo (1956) that eddy heat transports and eddy momentum transports are to be specified by meteorological observations. (Leovy assumed such transports could be parameterized as dissipation.)

Various analytical models for zonal vortices are analyzed here to provide some insight into the dynamic explanation of questions raised by several interesting observational phenomena in the stratosphere and mesosphere. That is: a) Why do the spring and fall change-overs (Miers, 1963) and mid-winter warming easterlies (Morris and Miers, 1964) of the circumpolar vortices of the upper stratosphere usually start at one height and then move downward and possibly upward (at rates of 1–5 km day^{-1})? b) Why does the biennial oscillation (Reed and Rogers, 1962) progress downward through the tropical lower stratosphere (at the rate of 1 km month^{-1})? c) What is the cause of anomalously warm temperature in the winter mesosphere in high latitudes? d) Why is there found in the winter lower stratosphere a poleward increase of temperature from the subtropics to mid-latitudes with an accompanying high pressure ridge up to the stratopause, so that the stratospheric summer easterlies intrude into the winter hemisphere? This anomaly is associated with a semi-annual wind oscillation over the equator, giving easterlies at the solstices and westerlies at the equinoxes, as described by Reed (1966). What causes the transient strengthening and poleward motions of this ridge at the stratopause during the so called "winter storm period" (Webb, 1964)?

A common aspect of all of the above mentioned features is that there occurs a large deviation of the zonal temperature from what is thought to be radiative-photochemical equilibrium. If this is the case, such zonal temperature perturbations must be created and maintained by phenomena connected with atmospheric motions. We shall only discuss here possible explanations involving warming by adiabatic heating, either by eddy heat transports (Newell, 1963) or by mean meridional circulations (Leovy, 1964). Leovy's models, driven by solar heating alone, suggested that the warm winter polar mesosphere could be explained by adiabatic warming due to subsidence in a meridional circulation forced by the heating contrast from the summer to winter hemisphere, but this result depended on rather artificial terms in his zonal momentum equation.

In the next section we introduce a system of equations for atmospheric zonal motions which is reduced to a single partial differential equation for geopotential height. We distinguish three possible scalings of the time variable. For time scales small compared to the diabatic relaxation time scale, we obtain as a first approximation an adiabatic model equation which is analogous to the equations studied in earlier zonal vortex theories. For intermediate time scales, we obtain a system where the thermodynamic equation describes a balance between diabatic cooling by relaxation or adiabatic cooling by symmetric vertical motion and the external heating. Diffusion-like propagation of perturbations occurs as a result of induced meridional circulations which change the zonal wind by Coriolis torques and hence, consistent with the thermal wind equation, change the zonal temperature. The "diffusion regime" model can thus describe the vertical propagation of zonal wind systems excited by either some impulsive or some kind of periodic addition of heat or momentum. This model appears able to provide a partial answer to questions a) and b) raised above. Expressions for propagation speeds which agree with those observed for these phenomena are derived. For sufficiently long time scales only those vertical motions driven by Reynolds' stresses are found to give significant adiabatic heating. Such heating, together with that by eddy heat transport, can maintain deviations of the zonal temperatures from radiative-photochemical equilibrium. Partial explanations of questions c) and d) appear to be provided by the solutions of this steady-state model.

2. Mathematical formulation

We shall assume an unbounded Cartesian geometry with t= time, y= poleward coordinate, and $z=\log(p_0/p)$, used as vertical coordinate, where p is pressure and p_0 some reference pressure. Horizontal distances are assumed to be nondimensionalized with respect to a, the earth's radius, so that as a northward coordinate we use $y=\varphi$ where φ is latitude in radians. Time is nondimensionalized with respect to $(2\Omega)^{-1}$ where Ω is the earth's rotation frequency. (The Coriolis parameter f is then equal to $\sin\varphi$.) The variables u, v denote nondimensional horizontal velocities in the longitudinal and latitudinal directions; $w=dz/dt$; $\theta=RT/(2\Omega a)^2$, where T is temperature; $h=(2\Omega a)^{-2}$ times the geopotential is the nondimensional geopotential height; and $S=[(\partial\bar{\bar{\theta}}/\partial z)+\kappa\bar{\bar{\theta}})$ is nondimensional static stability. Single bars denote zonal averages and double bars hemispheric averages; primes denote eddies. (When there can be no confusion, we shall omit the single bars on zonally averaged quantities.) We consider atmospheric motions below the mesopause and of low enough frequency that the thermal wind equation is valid. We scale the equations for a zonal strip at least 5° from the equator and omit second-order terms. Using subscripts to denote differentiation with respect to an independent variable, we then find to lowest order the following linearized nondimensional system of equations for zonal perturbation motions:

$$\bar{u}_t - f\bar{v} = -\tau, \tag{1}$$

$$\bar{v}_y + \bar{w}_z - \bar{w} = 0, \tag{2}$$

$$\bar{\theta}_t + \alpha(\bar{\theta} - \bar{\theta}_e) + \bar{w}S = q, \tag{3}$$

$$-f\bar{u}_z = \bar{h}_{yz} = \bar{\theta}_y. \tag{4}$$

The above are the equations of zonal momentum and continuity, the thermodynamic equation with Newtonian cooling, and the thermal wind equation, respectively. The term $\bar{\theta}_e$ is some specified radiative-photochemical equilibrium temperature. The terms τ and $q+\alpha\bar{\theta}_e$ describe the rate of momentum and heat addition, respectively, by eddies or externally specified sources; τ is the Reynolds' stress $\overline{(u'v')}_y$; and $q=\kappa Q/(2\Omega a)^2$, where $\kappa=R/C_p$, R is the gas constant, C_p the specific heat, and Q the eddy plus external rate of heat addition per unit mass.

The parameter α is the Newtonian cooling coefficient, which may be considered the result of radiative-photochemical damping. Strictly speaking, an equation for perturbations in the ozone concentration should then also be included (cf. Lindzen and Goody, 1965), but to a rough first approximation we may take α to be approximated by an infrared cooling-to-space coefficient below

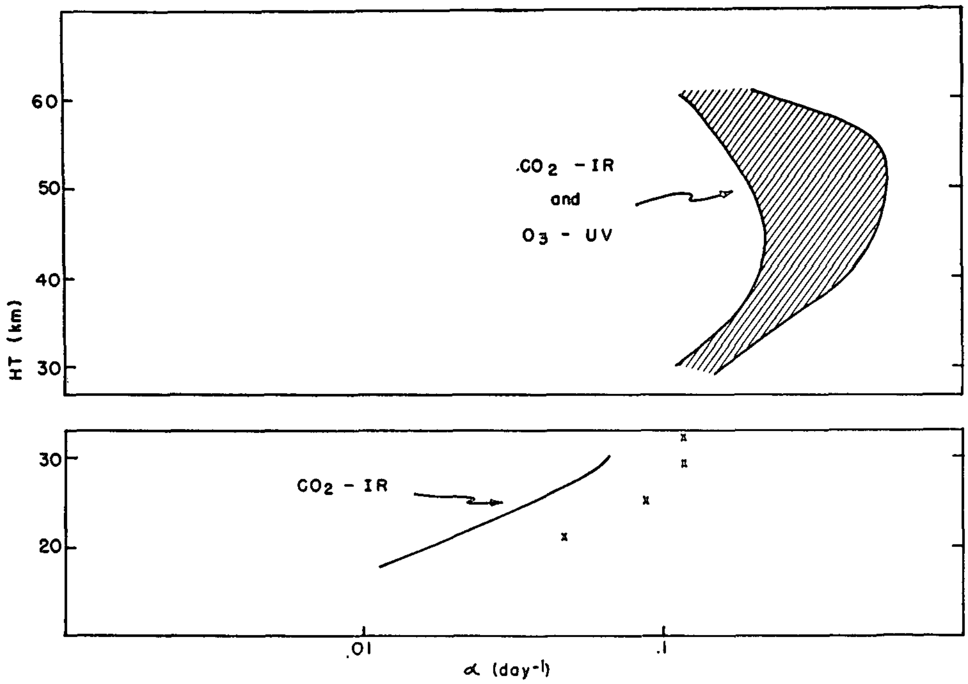

FIG. 1. Newtonian coefficient α(day^{-1}), determined from infrared cooling by CO_2 below 30 km and by CO_2 cooling and ultraviolet heating by perturbation ozone above 30 km. The crosses indicate values of α obtained by Hering et al. (1967). The hatched area gives the range of α in the upper atmosphere consistent with the range of ozone observations.

10 mb (Rodgers and Walshaw, 1966) and assume that above 10 mb the ozone perturbations satisfy the classical photochemical theory and are in photochemical equilibrium with temperature perturbations. We then have an uncoupled thermodynamic equation, with enhanced Newtonian cooling [cf. Leovy, 1964, Eq. (20); Lindzen and Goody, 1965, Eq. (38)]. A rough sketch of α so obtained is shown in Fig. 1. We have also given in the figure the values of α obtained by Hering *et al.* (1967) from observations. The effective α for the lower stratosphere probably lies between our theoretical curve and the values obtained by Hering *et al.* The indicated variation of α in the upper stratosphere is based on the range of ozone observations (Leovy[1]). In the following development we shall omit $\bar{\theta}_e$ by taking $\bar{\theta}$ to be the deviation from the equilibrium temperature. We accordingly modify the definitions of $\bar{h}$ and $\bar{u}$.

It is convenient to use (2) to introduce a meridional stream function ψ so that

$$\left.\begin{aligned}\bar{v}&=e^z\psi_z\\ \bar{w}&=-e^z\psi_y\end{aligned}\right\}. \tag{5}$$

Also note that (4) integrates to

$$\left.\begin{aligned}f\bar{u}&=-\bar{h}_y\\ \bar{\theta}&=\bar{h}_z\end{aligned}\right\}. \tag{6}$$

[1] Leovy, C., 1967: Energetics of the middle atmosphere. Paper prepared for presentation at Survey Symposium on Measurements in the Upper Atmosphere (ICMUA, IAMAP), XIV General Assembly of IUGG.

We shall denote the net external source of zonal potential vorticity by X/f, where X is defined as

$$X(y,z,t)=f\tau_y+f^2e^z[e^{-z}(q/S)]_z. \tag{7}$$

Eqs. (5) and (6) are substituted into (1) and (3) to obtain equations with h and ψ as dependent variables. These two equations are combined to give

$$f^2e^z\left[\frac{e^{-z}}{S}(h_{tz}+\alpha h_z)\right]_z+h_{yyt}=X, \tag{8}$$

which may be used to determine h, while (1) then integrates to

$$\psi=\frac{1}{f^2}\int_\infty^z e^{-z}(f\tau-h_{yt})dz. \tag{9}$$

We now discuss possible further simplifications that may be used when the motion is assumed to be characterized by specific time scales. Let us define new independent variables $\hat{t}$, $\hat{y}$ and $\hat{z}$, each taken to be of order unity, by

$$\hat{t}=t/\epsilon,$$
$$\hat{y}=y/L,$$
$$\hat{z}=z/D,$$

where ϵ, L and D are constants describing the time, latitudinal and vertical scales of motion, respectively. Also, let $\hat{S}=(D^2/f_0{}^2L^2)S$ be a new static stability parameter, which is chosen to measure the amplitude of h_{yyt} relative to the other terms on the left hand side of (8). We take f_0 to be a constant value of $\sin\varphi$. We then write (8) as

$$\left(\frac{f}{f_0}\right)^2 e^{\hat{z}D}\left[\frac{e^{-\hat{z}D}}{\hat{S}}(h_{\hat{t}\hat{z}}+\alpha\epsilon h_{\hat{z}})\right]_{\hat{z}}+h_{\hat{y}\hat{y}\hat{t}}=\epsilon L^2 X(\hat{y},\hat{z},\hat{t}). \tag{10}$$

For the purpose of scaling, let us assign typical constant values to the parameters S and α. Then for given forcing X and distance scales L and D, the solutions to (10) will depend on the two parameters $\hat{S}$ and $\delta=\alpha\epsilon$. The parameter δ measures the ratio of the motion time scale to the time scale for diabatic damping. We distinguish three regimes of motion, characterized by different values the parameters δ, $\hat{S}$ may assume. These are denoted:

a) the adiabatic regime, where

$$\delta \ll 1; \tag{11}$$

b) the "diffusive" regime, where

$$\delta \gg 1, \quad \delta/\hat{S}=O(1); \tag{12}$$

c) the "steady" regime, where

$$\delta \gg 1, \quad \delta/\hat{S}\gg 1. \tag{13}$$

To solve (10) for the adiabatic regime, we assume (11) and seek an asymptotic power series solution in δ,

$$h=h_0+\delta h_1+\delta^2 h_2+\cdots, \tag{14}$$

while to solve (10) for the diffusive and steady regimes we seek asymptotic power series solutions in δ^{-1},

$$h=h_0+\delta^{-1}h_1+\delta^{-2}h_2+\cdots. \tag{15}$$

First let us assume (14) and equate powers of δ. Then reverting back to the earlier variables, Eq. (10) to lowest order becomes

$$f^2 e^z\left(\frac{e^{-z}}{S}h_{tz}\right)_z+h_{tyy}=X(y,z,t), \tag{16}$$

which is essentially the symmetric vortex model introduced by Eliassen (1951), Kuo (1956), and many later authors.

We shall not discuss this model further except to note that no steady zonal wind and temperature is possible for nonvanishing $X(y,z,t)$. This results from the fact that there is no dissipation in this model to balance the zonal kinetic and potential energy generated by sources. The following sections are devoted to the analysis and discussion of the two regimes of motion dominated by Newtonian cooling, which we have called the diffusive and steady-state regimes.

3. The diffusive motion regime

Assume the scaling (12) and, following (15), expand solutions to (8) in an asymptotic power series in δ^{-1}. Then the solution, to lowest order in δ^{-1}, must satisfy the model equation,

$$f^2 e^z\left[e^{-z}\left(\frac{\alpha}{S}\right)h_z\right]_z+h_{yyt}=X(y,z,t). \tag{17}$$

This is a parabolic partial differential equation which has "diffusion wave" solutions. By "diffusion wave" we mean that the energy of solutions decays in time but that solutions have wave-like propagation of phases. To see this, let us examine the solutions to some elementary problems that may be posed for (17). For this purpose, assume (α/S) is a constant independent of z, and that f is a constant.

We may then put (17) into a more convenient form by defining a new dependent variable h^* by

$$h^*=e^{-z/2}h. \tag{18}$$

Then in terms of h^*, (17) is written

$$f^2\left(\frac{\alpha}{S}\right)(h^*_{zz}-\tfrac{1}{4}h^*)+h^*_{yyt}=e^{-z/2}X(y,z,t). \tag{19}$$

Let us examine a simple example that illustrates the manner in which time-periodic disturbances described by (17) will propagate downward from some source region. For this purpose assume there is a lid at $z=z_0$ where we apply the condition, $h^*(y,z_0,t)=\mathrm{Re}\,e^{i\nu t}\cos ly$ (we use Re to denote real part of). Then assuming a solution of the form, $h^*=\mathrm{Re}\,\tilde{h}(z)e^{i\nu t}\cos ly$, to introduce a new variable $\tilde{h}$ and neglecting X, (19) becomes

$$\tilde{h}_{zz}-2i\gamma^2\tilde{h}=0, \tag{20}$$

with the boundary condition $\tilde{h}(z_0)=1$, and where we define γ as

$$\gamma=\left[\frac{l^2}{f^2}\left(\frac{S\nu}{2\alpha}\right)-\tfrac{1}{8}i\right]^{\frac{1}{2}}. \tag{21}$$

For simplicity assume $|\gamma|\simeq(l/f)(S\nu/2\alpha)^{\frac{1}{2}}\gg\frac{1}{4}$. Then (19) has the solution

$$h^*=e^{-\gamma(z_0-z)}\cos[\gamma(z_0-z)-\nu t]\cos ly. \tag{22}$$

In general, solutions to (17) below some source region will be of a form similar to (22).

For example, if we take as appropriate to the biennial oscillation $l=4$ (corresponding to a disturbance half-wavelength of 45° of latitude), $f\simeq\frac{1}{5}$ (corresponding to latitude of 12N), $\nu=(2\pi/750)$ days, $\alpha=1/25$ day^{-1}, and $S\simeq0.02$, we find $\gamma\simeq0.92-0.07i$ with a corresponding downward propagation phase speed, $c=\nu/\gamma$, of approximately one-fourth a scale height per month. This agrees roughly with the speed of the downward progression of the biennial oscillation (Reed and Rogers, 1962). With the values assumed above we have $\alpha\nu^{-1}=\delta\simeq5$, which may be used as a large scaling parameter. As characteristic distance scales we have $D\sim\gamma^{-1}\simeq1$, and $L\sim l^{-1}=\frac{1}{4}$,

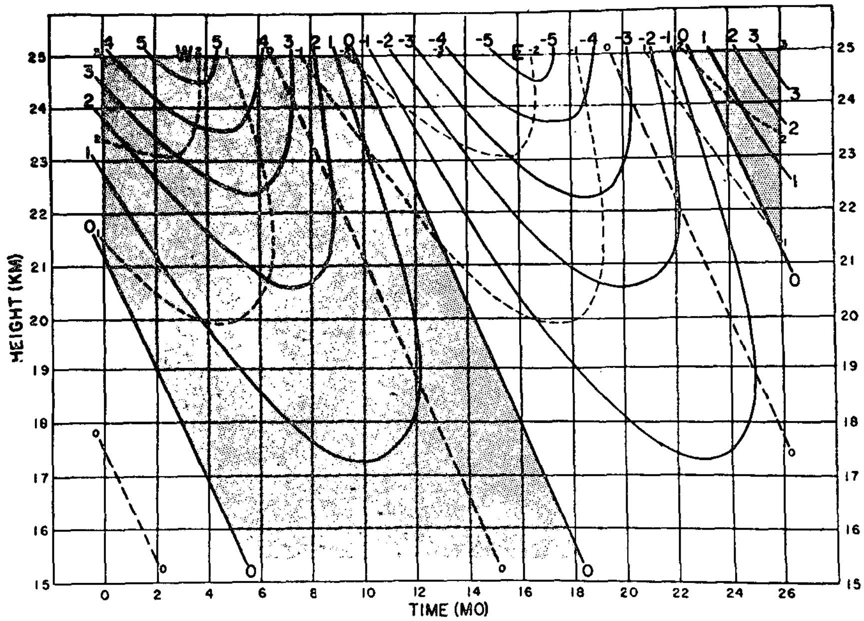

FIG. 2. Theoretical downward diffusion of the biennial temperature wave, taken from Staley (1963). Solid lines are winds, dashed lines are temperature fluctuations.

so that the parameter $\hat{S}$ is roughly 10. We thus verify that, for the assumed parameters, the diffusive scaling given by (12) applies to the biennial oscillation.

If a biennial source of momentum is imposed at some level, say in the vicinity of 10 mb, as described by Wallace and Newell (1966), then the present model will propagate the momentum downward through the lower stratosphere. When momentum is added right at the equator, the present model predicts a resulting meridional circulation that becomes infinite at the equator, reflecting the fact that Eq. (9) is not a valid first approximation within a few degrees of the equator. In this respect our theory may be viewed as complementary to that of Lindzen (1966), whose lowest order solution is valid for a latitudinal belt of a few degrees width centered at the equator. For a sketch of the solution (22) as applied to the biennial oscillation see Fig. 2, taken from Staley (1963), who derived an equation equivalent to (20) by assuming the downward progression of the biennial oscillation might be regarded simply as eddy heat diffusion driven by an oscillating heat source.

Taking (22) to describe the perturbation height field, we find using (6) that the temperature field has the same latitudinal phase as the height, but the $-z$ or t temperature phase leads by $\pi/4$. Also from (6), the zonal winds have the same z and t phase as the height but are shifted a quarter wavelength in latitudinal phase. From (9) and (5) the v's are found to have the same latitudinal phase as the u's but to lead the u's in $-z$ or t by a quarter wavelength or period, while the w's have the same latitudinal phase as the temperatures and are exactly out of phase in z or t with the temperature. Keeping in mind these phase relations, we show schematically in Fig. 3 how the biennial oscillation might propagate downward by the Coriolis torque mechanism. The left half of the figure indicates a half wavelength in latitude and a wavelength in z of (22). The pattern is then reflected about the center, assumed

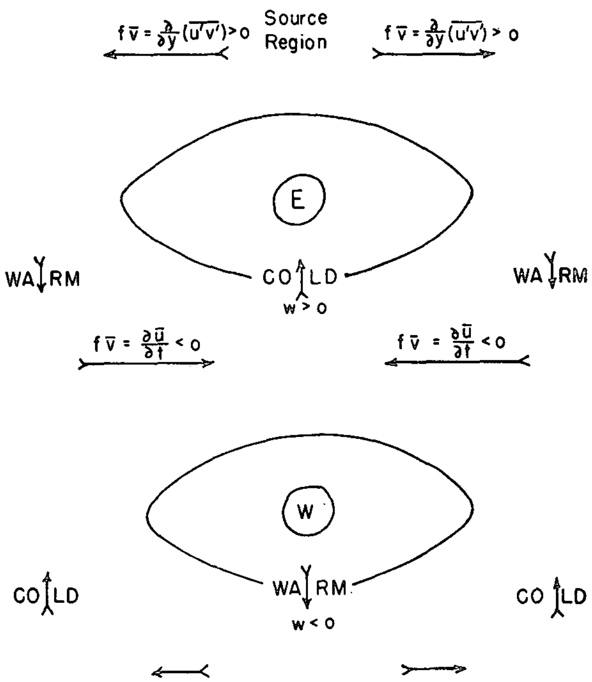

FIG. 3. Schematic depiction of the kinematics of the theoretical downward progression of a biennial wave centered at the equator.

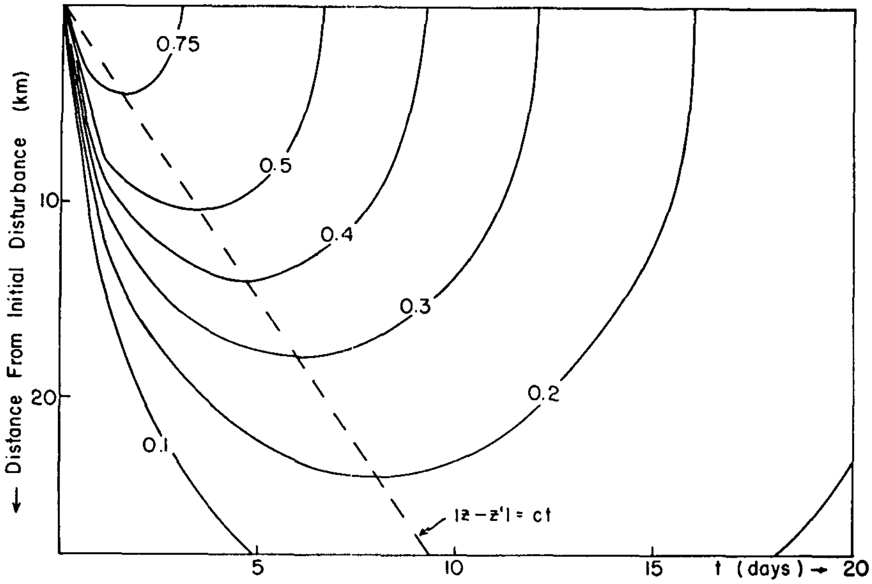

FIG. 4. Time-height section of isolines of $\exp[-\phi(z,t)]$, indicating possible downward progression of zonal wind disturbances from the stratopause into the lower stratosphere.

to be the equator, and the zonal winds are drawn with maxima at the equator, rather than above the maximum v's. These downward propagating cells appear similar to those described by Reed (1964) for which the temperature perturbations were taken to lead the winds by a quarter wavelength in $-z$ or t. As indicated at the top of Fig. 3, our solution requires a source (indicated to be eddy stresses) to excite the diffusive wave motion. The wave phases propagate away from the source region.

As a second simple example of a solution to (19), which illustrates the propagation of impulsively generated disturbances, again assume that $X(y,z,t)=0$ and that initial conditions are that at $t=0$, h^* is given at some level, $z=z'$, as $h^*=\cos ly$, but $h^*=0$ for $z\neq z'$. Let $\hat{h}$ be the Laplace transform of $h^*/(\cos ly)$ so that h^* is given by

$$h^*=\cos ly\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}e^{\sigma t}\hat{h}(y,z,\sigma)d\sigma, \tag{23}$$

where $\hat{h}$ is found from (19) to satisfy

$$\hat{h}_{zz}-\tfrac{1}{4}\hat{h}-c^{-1}\sigma\hat{h}=0, \tag{24}$$

with the boundary condition that $\hat{h}=1$ at $z=z'$, and where we define

$$c=\alpha f^2/(l^2S). \tag{25}$$

Eq. (24) is solved by

$$\hat{h}=\exp\left[-\left(\frac{c}{4}+\sigma\right)^{\frac{1}{2}}\frac{|z-z'|}{c^{\frac{1}{2}}}\right]. \tag{26}$$

Evaluation of (23) then gives

$$h^*=A(z,t)\cos ly e^{-\phi(z,t)}, \tag{27}$$

where we define

$$\left.\begin{aligned}\phi(z,t)&=\tfrac{1}{4}c[t+(z-z')^2/c^2t]\\ A(z,t)&=\frac{c}{[4\pi(z-z')^2t^3]^{\frac{1}{2}}}\end{aligned}\right\}. \tag{28}$$

The coefficient $A(z,t)$ depends on the assumed initial condition, but solutions may be expected to be proportional to $e^{-\phi(z,t)}$ for a wide variety of initial conditions, so the exponent of the solution given by (28) will be discussed further. For fixed z the function $\phi(z,t)$, considered as a function of t only, has a minimum for $|z-z'|=ct$, so that c may be considered the speed of propagation of the disturbance in a time-height section. As typical values of the parameters we might use $f\simeq\frac{1}{2}$, $l\simeq3$, $S\simeq0.03$, giving $c\simeq\alpha$ from (25), which indicates that the downward and upward propagation of the disturbance will be roughly proportional to the coefficient of Newtonian cooling. In dimensional units we have

$$c=\mathrm{O}(1)\alpha H, \tag{29}$$

where H is atmospheric scale height and O(1) denotes order of magnitude one. When a time independent source $X(y,z)$ is suddenly turned on, the ensuing steady forced motion, described by our steady-state model, will propagate downward with the speed c.

In Fig. 4 we have plotted contours of constant $\exp[-\phi(z,t)]$, assuming $c=\alpha H$ and taking $\alpha=0.4$ day^{-1}, $H=7\frac{1}{2}$ km. This plot bears an intriguing resemblance to the time-height sections described by Miers (1963) and

Morris and Miers (1964). However, to verify the relationship between our zonal wind theory and such observed downward propagation of winds, it would be necessary to obtain hemispheric data coverage for the upper stratosphere. Since such data is not presently available, it is not possible to distinguish between planetary waves and zonal winds.

In summary then, we infer from this example that if zonal heat or momentum is added suddenly at one height, the zonal wind and associated temperature perturbation and meridional circulation system propagates downward and upward with a speed proportional to the Newtonian cooling constant.

4. The steady-state regime

Assuming the scaling (13), we obtain as an approximation to (8),

$$\left[f^2\left(\frac{\alpha}{S}\right)e^{-z}h_z\right]_z = e^{-z}X(y,z,t). \tag{30}$$

It is sufficient for our purpose here to assume α and S are exponential in z and hence taking a and s to be constants we have

$$\left.\begin{aligned}\alpha &= \alpha_0 e^{az}\\ S &= S_0 e^{sz}\end{aligned}\right\}. \tag{31}$$

In general, (31) will only give a local fit to the actual plots of α and S, but a more realistic description of these parameters would not qualitatively change our conclusions.

Let $r=(1+s-a)$. We introduce a new dependent variable h^* by

$$h(y,z,t) = h^*(y,z,t)e^{\frac{1}{2}rz}, \tag{32}$$

which puts (30) into the normal form

$$h^*_{zz} - \tfrac{1}{4}r^2 h^* = \frac{S}{\alpha f^2}e^{-\frac{1}{2}rz}X(y,z,t). \tag{33}$$

We note that Green's function equation,

$$G_{zz} - (\tfrac{1}{4})r^2 G = -\delta(z-z'),$$

has the solution

$$G = (1/r)\exp(-r/2|z-z'|).$$

Using this fact, we solve (33). Substitution of the solution of (33) into (32) then gives

$$h = -\int_{-\infty}^{\infty}\left\{\frac{S(z')}{rf^2\alpha(z')}X(y,z',t) \times \exp\left[\frac{r}{2}(z-z') - \frac{r}{2}|z-z'|\right]\right\}dz'. \tag{34}$$

The zonal wind and temperature are then obtained from (6). For convenience in comparison with observations, we shall give the perturbation temperature in dimensional units (°K). After substituting (7) into (34) and further manipulations, we find a dimensional temperature perturbation given by the two terms,

$$T = T_H + T_M, \tag{35}$$

where (when the theory is derived is spherical coordinates) we obtain the definitions.

$$T_H = \frac{1}{\alpha}\left[\frac{Q}{C_P} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial\varphi}(\overline{v'T'}\cos\varphi)\right], \tag{36}$$

$$T_M = -(R\sin\varphi\cos^2\varphi)^{-1} \times \int_z^{\infty}\left[\left(\frac{2\Omega S}{\alpha}\right)e^{-r(z'-z)}\frac{\partial^2}{\partial\varphi^2}(\overline{u'v'}\cos^2\varphi)\right]dz'. \tag{37}$$

In (36) and (37) all parameters except S are dimensional. Eqs. (36) and (37) give the departure of T from some average value due to eddy and solar heating and due to eddy momentum forced meridional circulations, respectively. An important feature of (37) is that T_M depends only on momentum transports that occur above a point of observation. For more general α and S, source points will, strictly speaking, influence points above, but the resulting influence function for $z'<z$ will decay rapidly compared to a scale height, and the same conclusions are approximately valid.

One finds typically that in the middle of jets the curvature of the eddy angular momentum transport is negatively correlated with transport itself. As illustration of this fact we plot in Fig. 5 the average $(\overline{u'v'})$ and $\cos^{-2}\varphi\,\partial^2/\partial\varphi^2(\overline{u'v'}\cos^2\varphi)$ for 10 mb during the month of January 1965, from data[2] analyzed by Richards (personal communication). Assuming these transports are independent of height and taking as "typical values" $r\simeq 1$, $S\simeq 0.02$, we obtain from (37) the subsidence heating indicated in Fig. 6 which balances αT. We see that the meridional circulation forced by eddy momentum stresses acts to heat middle latitudes and cool the subtropical latitudes and hence to give a winter poleward increase of temperature from the subtropics to middle latitudes, as observed in the lower stratosphere. Divergence of eddy heat fluxes cancels some of the middle latitude subsidence heating. A Coriolis parameter occurs in the denominator of (37), so tropical momentum transports will be relatively more effective in forcing zonal temperature perturbations. It seems likely that the maintenance and transient poleward motions of the winter subtropical ridge in the upper stratosphere is the result of eddy momentum forced meridional circulations.

[2] Computed from the ESSA IQSY stratospheric data tapes.

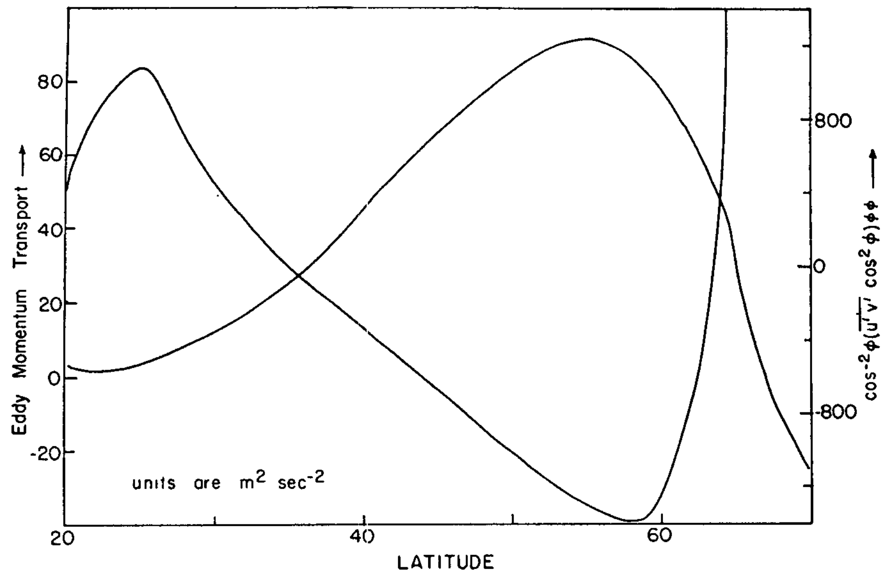

FIG. 5. Momentum transport $\overline{u'v'}$ (positive in middle latitudes) and curvature $\cos^{-2}\varphi(\overline{u'v'}\cos^2\varphi)_{\varphi\varphi}$ (negative in middle latitudes) adopted from 10-mb data of Richards for January 1965.

5. Discussion

When heat or momentum is suddenly added to an unbounded stratified atmosphere in constant rotation, it is well known that, according to linear theory, the resulting motion consists of internal gravity waves and a geostrophically balanced potential vorticity motion. After a day or so the gravity waves "radiate to infinity" and there remains the invariant potential vorticity motion. When heat or momentum is added slowly, the ensuing motion given by linear theory is determined locally by the net addition of potential vorticity. Only if the constraint of conservation of potential vorticity by the motion is relaxed can low frequency disturbances travel like waves from one region to another. As one well known mechanism for such propagation, there are Rossby waves, which may exist when parcels of fluid

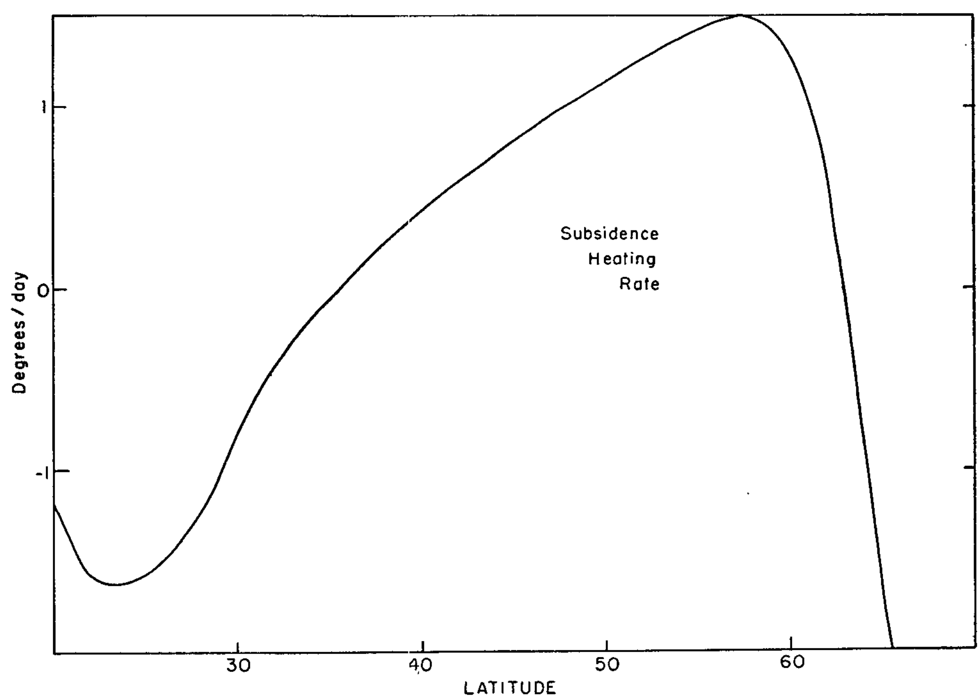

FIG. 6. Subsidence heating estimated for the middle stratosphere from momentum transport data of previous figure.

can gain potential vorticity of motion by losing vorticity of the earth's rotation or vice versa.

The constraint of potential vorticity conservation is also relaxed when dissipation is included in the dynamics, and disturbances may be expected in some fashion to propagate. The energy of such wavelike motion will, in the absence of external sources, decay monotonically with increase of time as does the energy of solutions to a diffusion equation. The investigation of such diffusive wave propagation as a part of the theory of atmospheric motions is in its initial stages. We have studied in this paper some simple propagation models for atmospheric zonal winds when a Newtonian cooling is present. There also exists a large class of dynamical problems with the major dissipation resulting from a linear drag term in the equations of motion. There is a rational basis for such theory, for example, in the middle thermosphere where "ion drag" is dominant (cf. Geisler, 1966). More complicated problems involving wind propagation as a result of viscosity or heat conduction can readily be formulated. Applications would include description of various phenomena in the thermosphere.

The biennial oscillation is currently one of the most well documented examples of a zonal wind propagation phenomenon. Lindzen (1966) first attempted to provide a dynamical explanation of the downward progression of these zonal winds. In his theory, the downward progression is a consequence of vertical motions of biennial periodicity, which generate momentum by advecting downward the shear of a zero-frequency mean wind. Propagation occurs because the temperature field, in thermal wind balance with the zonal wind, is forced by dissipation to be approximately in phase with downward vertical motion. Given mean shears of the observed sign, Lindzen's theory would give upward progression of phases towards a source region around 30 km, which is opposite to that which is observed. Wallace (1967) has more recently alternatively conjectured that the zonal winds are advected downward by a mean vertical motion opposite to that expected from radiative balance studies. Both the above mentioned mechanisms are of primary relevance to the momentum balance within a degree or less of the equator, where Coriolis torques are vanishingly small. The biennial periodicity in the stratospheric zonal winds, however, extends over nearly half the earth. It may easily be shown by scaling arguments that Coriolis torques are always much larger, at distances a few degrees or greater from the equator, than any kind of momentum advection by vertical meridional motions. Hence, any theory which relies only on vertical motions to transport momentum down can at best satisfactorily describe the downward progression of but a small fraction of the total momentum involved.

Our theory suggests that the most important mechanism for carrying momentum downward in the biennial oscillation is the Coriolis torques associated with downward propagating meridional cells. The example given is too idealized to be useful for accurate quantitative description of the observed downward propagation. However, the model can easily be generalized for numerical studies to include variables f, α, S, and momentum sources $\bar{v}\bar{u}_y$, $\bar{w}\bar{u}_z$ in the zonal equation of motion, and meridional heat transport by $\bar{v}$ in the thermodynamic equation. A numerical model can be solved for all wavelengths of disturbance, and for observed forcing by momentum convergences or heating. Wallace and Holton (1968) have recently developed such a numerical model for study of the biennial oscillation and have carried out a number of integrations. For the source distributions they assumed, the Coriolis torque effect was too small to carry down the biennial oscillation. Since our theory shows downward propagation by Coriolis torques is very sensitive to the scales of the source, further numerical work seems desirable.

The two most uncertain parameters in the vertical wavenumber γ, Eq. (21), of our simple example, are the effective Coriolis parameter f and effective latitudinal wavenumber l. Note that for $|\gamma| \gg \frac{1}{4}$, the vertical propagation phase speed ν/γ is proportional to f/l. If the biennial oscillation were concentrated more near the equator, l should increase and f decrease, so that the phase speed would decrease. Year-to-year differences in the latitudinal structure of the forcing, such as by eddy momentum convergences might thus be used to explain variations in the downward progression speed from one cycle to the next, and especially the fact that westerly winds move downward with roughly twice the speed of easterly winds (Wallace, 1967). The fact that the amplitude of the biennial wave above 50 mb changes little (the so-called neutral region) can be reproduced with our model, provided eddy momentum sources are, over an extended region, assumed at these levels as indicated by Wallace and Newell (1966). (Our example assumed momentum added in an infinitesimal layer.)

When $(l^2/f^2)(S\nu/\alpha)$ decreases to $\frac{1}{4}$ or less, it may be seen from (20) and (21) that the exponential decay dominates over the wavelike behavior of solutions. This suggests a partial explanation for the confinement of the disturbance to tropical latitudes since $l/f > 10$ seems necessary to give propagating solutions. Planetary scale disturbances feeling a mid-latitude Coriolis parameter will not propagate with a biennial periodicity. The steady regime scaling (13) is appropriate to such disturbances. Also, when the oscillation frequency ν increases to the magnitude of the cooling coefficient α, disturbances are dominated by exponential decay in z, rather than wavelike propagation. Thus, for example, it is not possible to have near 100 mb in the lower stratosphere a downward phase progression of a biennial oscillation. In summary, a filtering of high frequency disturbances, as emphasized by Lindzen (1966), occurs because the scaling condition of (12) that $\delta \gg 1$ breaks down for such disturbances, while disturbances with too small a latitudinal wavenumber or too large a Coriolis parameter do not satisfy $\delta/\hat{S}=O(1)$, and so are filtered from downward wave propagation.

The biennial temperature wave given by our example with $|\gamma|\gg\frac{1}{4}$ [cf. (21)] leads the zonal wind by three months. The $-\frac{1}{8}i$ in the definition of γ, or the effect of local temperature changes, which was omitted by the scaling (12), both reduce the lead to less than three months. Reed (1964) has found that above 20 mb the temperature appears to lead the winds by five months with a lead decreasing to one month as the system progresses downward to 80 mb. Using the "exact" equation (8) but still taking $|\gamma|\gg\frac{1}{4}$, we find the phase lag of the wind to be $[\pi/4-\tan^{-1}(\nu/2\alpha)]$, so the decrease of the phase lag to one month can be explained as due to the decrease of α to less than ν in the lower stratosphere, as is seen to occur (cf. Fig. 1). At levels around 10 mb, as indicated by the theory of Lindzen and Goody (1965), Newtonian cooling is not a good approximation. The ozone photochemical relaxation-time scale is of the magnitude of the biennial oscillation time scale, so that coupling of the ozone wave with the temperature wave becomes important. When this coupling is included, a phase difference greater than three months between temperature and winds is found, but it is not yet clear whether this coupling can reproduce the phase differences described by Reed.

While the theory appears to supply a reasonable explanation of the downward progression of the biennial oscillation once external forcing is supplied, it contains no explanation for the basic drive, which is presently believed to be biennial fluxes of eddy momentum (cf. Wallace and Newell, 1966).

As a second possible application, we hypothesized that the downward progression of winds from the stratopause as observed in time-height sections prepared from Meteorological Rocket Network data is a manifestation of behavior described by our diffusive model. We inferred the result that the downward speed of propagation would be roughly proportional to the Newtonian cooling coefficient. The propagation speed also depends on the square of the latitudinal wave number, with disturbances of larger wavelength traveling more swiftly. A specification of actual sources and initial conditions would be necessary in order to apply quantitatively our model to explanation of the details of these events.

In Section 4, we formally examined the consequences of our steady-state scaling. The resulting motion is characterized by a balance between the destruction of temperature perturbations by diabatic relaxation and the generation of temperature perturbations by either eddy heating and perturbation radiational heating or by meridional subsidence forced by eddy momentum transports. Note from Eq. (37) that the forced temperature perturbation is proportional to the Newtonian cooling relaxation time, which decreases by more than an order of magnitude from the troposphere to the upper stratosphere. One would hence expect in the troposphere that jets would be much narrower in latitude than they are in the stratosphere. Such is observed synoptically, but this difference is smeared out when space and time mean zonal winds are defined.

In the steady-state model, either eddy or solar heating at a given latitude and height can, to lowest order, only influence the temperature at that latitude and height. The adiabatic heating by meridional circulations forced by these sources is a higher order correction to this result. Consequently, during the winter when the direct solar heating decreases monotonically poleward, the steady-state model predicts that, in the absence of eddies, zonal westerly winds will monotonically increase with height; in order to have one jet stacked upon another one (e.g., the stratospheric winter jet above the tropospheric jet), it is necessary that there exist at some levels either a) forced subsidence due to eddy momentum transports with upward motion equatorward and downward motion poleward of the jet, or b) convergence of eddy heat transports poleward and divergence equatorward of the jet.

In the troposphere and lower stratosphere, zonal winds with annual periodicity change rapidly enough relative to the diabatic damping time that they should probably be discussed using the general model, Eq. (8). It is not possible to define the relative importance of the above two mechanisms for damping the tropospheric jet in the lower stratosphere until further theoretical and observational calculations are pursued. Both the above mechanisms appear to result in heating and cooling rates of the same order of magnitude, so both tentatively need be included in such an analysis. Likewise, it is not possible to ascertain the relative importance of a) or b) for the maintenance of the warm winter mesospheric temperatures, but the theory suggests at least one of these mechanisms must occur.

The adiabatic, diffusive and equilibrium models may be nicely described by the constraints on energy transformations they imply. From (8) and omitting the external heat forcing q from the discussion, we have

$$\frac{\partial}{\partial t}(K_z+P_z)=\langle K_E\cdot K_z\rangle+G_z, \tag{38}$$

where

$$K_z=\tfrac{1}{2}[(f^{-1}h_y)^2],\quad P_z=\tfrac{1}{2}[S^{-1}h_z^2],$$
$$\langle K_E\cdot K_z\rangle=[f^{-1}h_y\tau],\quad G_z=-\tfrac{1}{2}[\alpha S^{-1}h_z^2],$$

and where the notation [ ] is a mass integral over the area of integration which is assumed to be unbounded, i.e.,

$$[(\ )]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-z}(\ )dydz.$$

Zonal kinetic energy may be generated from eddy kinetic energy by means of the Reynolds' stress term $[f^{-1}h_y\tau]$. This generation of zonal kinetic energy balances the increase of K_z plus conversion to P_z. This conversion, given by $[vh_y]$, occurs as a result of meridional circulations and in our geostrophic models, at a rate sufficient to insure the thermal wind balance is always satisfied. The conversion to P_z must balance the increase of P_z plus the destruction of P_z by diabatic damping given by G_z. Note that G_z as defined is negative definite.

For the short-time scale, adiabatic model, we neglect G_z, so that $\partial/\partial t(K_z+P_z)=\langle K_E\cdot K_z\rangle$. For the medium and long-time scale diffusive and equilibrium models all the energy transformed from K_z to P_z in order to maintain thermal wind balance is annihilated through $-G_z$ with $\partial P_z/\partial t$ being negligible. For the equilibrium model, $\partial K_z/\partial t$ and $\partial P_z/\partial t$ are both assumed negligible, so we have $\langle K_E\cdot K_z\rangle+G_z=0$, while in the diffusive model, $\partial K_z/\partial t$ is important. The balance between oscillations of K_E and radiative dissipation of P_E gives the diffusive wave propagation phenomena.

Given that δ, the ratio of the time scale of motion phenomena to the Newtonian time scale, is large, the distinction between the diffusive and steady state regimes is whether $\delta/\hat{S}\sim(f^2L^2\alpha\epsilon)/(D^2S)$ is of order one or large. Thus, for small enough Newtonian cooling, small enough Coriolis parameter, and small enough ratio of horizontal to vertical distance scales, diffusive wave motions will occur for a given time scale of motion. When $f^2L^2/D^2\sim 1$, which is appropriate to the planetary scale features of the great stratospheric summer and winter jets, then the diffusive regime scaling given by (12) can no longer exist, and there is a direct transition from the adiabatic to the steady regime.

We note that the assumption of a constant Coriolis parameter and the neglect of the sphericity of the earth are not necessary for separating out the latitudinal dependence of solutions, and that we can easily develop the examples of Section 3 with height dependent Newtonian cooling for an equatorial strip or spherical earth geometry. Since the appropriate latitudinal eigenfunctions do not resemble plausible source distributions, no further simple insight is gained by such an exercise. However, such an eigenfunction expansion can be used for obtaining accurate numerical solutions to analytic models. The theory of zonal wind systems on a spherical earth, but neglecting nonlinear terms as we have done, is equivalent to "atmospheric tidal theory" for symmetric motions with time scales long compared to a day. Lindzen and McKenzie (1967) have shown how in general one may separate out the latitudinal dependence of the atmospheric tidal equations with height dependent Newtonian cooling. The theory of three-dimensional small amplitude perturbations for a planar atmosphere in constant rotation is only trivially different from the two-dimensional situation treated here.

Acknowledgments. The author is grateful for discussion of the cooling-to-space approximation with Dr. Clive Rodgers, for typing of the manuscript by Mrs. Cynthia Webster, and for improvements suggested by Jr. Jack Geisler. The author was supported in this work by a National Science Foundation Grant, (GA-400).

REFERENCES

Eliassen, A. N., 1951: Slow thermally or frictionally controlled meridional circulations in a circular vortex. *Astrophys. Norv.*, **5**, 19–60.

Geisler, J., 1966: Atmospheric winds in the middle latitude F-region. *J. Atmos. Terr. Phys.*, **28**, 703–720.

Hering, W., C. Touart and T. Borden, 1967: Ozone heating and radiative equilibrium in the lower stratosphere. *J. Atmos. Sci.*, **24**, 402–413.

Kuo, H. L., 1956: Forced and free meridional circulations in the atmosphere. *J. Meteor.*, **13**, 561–568.

Leovy, C., 1964: Simple models of thermally driven mesospheric circulation. *J. Atmos. Sci.*, **21**, 327–341.

Lindzen, R., 1966: Radiative and photochemical processes in mesospheric dynamics: Part II, Vertical propagation of long period disturbances at the equator. *J. Atmos. Sci.*, **23**, 334–343.

——, and R. Goody, 1965: Radiative and photochemical processes in mesospheric dynamics: Part I, Models for radiative and photochemical processes. *J. Atmos. Sci.*, **22**, 341–348.

——, and D. McKenzie, 1967: Tidal theory with Newtonian cooling. *Pure Appl. Geophys.*, **66**, 90–96.

Miers, B. T., 1963: Zonal wind reversal between 30 and 80 km over the southwestern United States. *J. Atmos. Sci.*, **20**, 87–93.

Morris, J. E., and B. T. Miers, 1964: Circulation disturbances between 25 and 70 kilometers associated with the sudden warming of 1963. *J. Geophys. Res.*, **69**, 201–214.

Newell, R. E., 1963: Preliminary study of quasi-horizontal eddy fluxes from meteorological rocket data. *J. Atmos. Sci.*, **20**, 213–225.

Reed, R. J., 1964: A tentative model of the 26-month oscillation in tropical latitudes. *Quart. J. Roy. Meteor. Soc.*, **90**, 441–466.

——, 1966: Zonal wind behavior in the equatorial stratosphere and lower mesosphere. *J. Geophys. Res.*, **71**, 4223–4233.

——, and D. Rogers, 1962: The circulation of the tropical stratosphere in the years 1954–1960. *J. Atmos. Sci.*, **19**, 127–135.

Rodgers, C. D., and C. D. Walshaw, 1966: The computation of infrared cooling rate in planetary atmospheres. *Quart. J. Roy. Meteor. Soc.*, **92**, 67–91.

Staley, D. O., 1963: A partial theory of the 26-month oscillation of the zonal wind in the equatorial stratosphere. *J. Atmos. Sci.*, **20**, 506–515.

Wallace, J. M., 1967: On the role of mean meridional motions in the biennial oscillation. *Quart. J. Roy. Meteor. Soc.*, **93**, 176–185.

——, and J. Holton, 1968: A diagnostic numerical model of the quasi-biennial oscillation. *J. Atmos. Sci.* **25**, 280–292.

——, and R. E. Newell, 1966: Eddy fluxes and the biennia stratospheric oscillation. *Quart. J. Roy. Meteor. Soc.*, **92**, 481–489.

Webb, W. L., 1964: Stratospheric solar response. *J. Atmos. Sci.*, **21**, 582–591.