Method of Parameterization for Infrared Cooling between Altitudes of 30 and 70 Kilometers

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A simple parameterization of infrared cooling between altitudes of 30 and 70 km consists of the sum of the exact cooling for a reference temperature profile and a Newtonian cooling approximation for departures from this profile. The parameterization is developed by using the 1962 standard atmosphere for the reference temperature. The exact line-by-line calculation of cooling gives total infrared cooling rates that in the vicinity of the stratopause are somewhat greater ($\sim$1 deg day^{-1}) than the values obtained by previous investigators, apparently because the present study includes 'second hot bands.'

Radiative sources and sinks that depend on temperature are usually found for 'standard atmosphere' or climatological mean temperature configurations. For typical temperature profiles of the terrestrial upper stratosphere and lower mesosphere, infrared emission by 15-μ bands of CO_2 provides the largest source of radiative cooling, but cooling by the 9.6-μ bands of O_3 is also significant in the vicinity of the stratopause. The global mean balance between absorption of ultraviolet solar radiation by O_3 and the temperature-dependent infrared cooling determines the mean temperature profile in the 30- to 70-km region. The maximum solar heating around 50 km reflects the upward decrease of the O_3 mixing ratio in conjunction with a mean optical depth for the absorbed solar radiation of essential unity between that level and space.

Plass [1956a, b] and *Murgatroyd and Goody* [1958] first attempted to determine infrared cooling rates in the region under consideration. Effects of nonlocal thermodynamic equilibrium (NLTE) were first included in the latter study. Many more recent studies have been aimed at improving this calculation with more current physical parameters or exploring in greater detail certain related questions, e.g., the radiative photochemical equilibrium model of *Leovy* [1964] and the model for relaxation of deviations from radiative-photochemical equilibrium [*Lindzen and Goody*, 1965]. More recently, *Kuhn and London* [1969] and *Drayson* [1967]

have carried out improved versions of the infrared cooling calculation.

The determination of infrared cooling for a given temperature profile has generally required considerable time on the fastest computers available. However, hydrodynamic calculations with radiative sources and sinks require the evaluation of infrared cooling at many different points in the horizontal as well as at frequent intervals in time. Consequently, approximate methods must be developed that are computationally much faster than those generally used for the most exact calculations.

The objective of the present paper is to suggest a very fast procedure for approximate evaluation of infrared cooling rates in the upper stratosphere and mesosphere. The principle of this approach is the 'exact' evaluation of cooling rates for some reference temperature profile, together with a Newtonian cooling correction for departures of the actual temperature from the reference temperature.

To provide an example of this approach, we have developed the suggested parameterization for a 1962 standard atmosphere temperature profile. The exact evaluation of cooling for this profile and the Newtonian cooling parameterization for departures from this profile are based on the methodology and computer program described by *Dickinson* [1972]. Small changes needed to adapt the program to terrestrial conditions are described in the next section.

The cooling calculation does not assume local thermodynamic equilibrium, and so in principle it can be performed up to altitudes where in-

frared cooling by CO_2 is no longer important. However, the Newtonian parameterization developed here does not appear to give a useful approximation to perturbation cooling above 70 km. The relatively cold layers between 70 and 95 km absorb an amount of thermal radiation from lower layers comparable to the thermal radiation that they emit to space. Furthermore, CO_2 'hot bands' can make a significant contribution to the net infrared cooling in these layers, and it is not known how to calculate correctly the NLTE contribution from these bands. Our treatment of overlap of CO_2 rotational-vibrational lines below 30 km is also probably too crude to be satisfactory, and our calculation for O_3 infrared transfer does not properly describe the region in the lower stratosphere where O_3 absorbs a significant amount of infrared radiation emitted from the ground. Thus in this paper we present only numerical results for the cooling parameterization between 30 and 70 km.

RADIATIVE COOLING CALCULATION

The details of the radiative emission and NLTE radiative transfer calculation for CO_2 are given by *Dickinson* [1972] and need only be summarized here. At pressures low enough that overlap of lines can be neglected, we calculate essentially the contribution to cooling from each individual vibrational-rotational Voigt line and then sum these contributions to get the cooling by a CO_2 vibrational band. The approximate procedure of *Goody and Belton* [1967] is used for line overlap. A harmonic vibrator-rotator approximation determines the partitioning of a band intensity into individual line intensities. For reasons of computational expediency [*Dickinson*, 1972], we have developed approximate expressions for Voigt line escape functions and have partitioned the lines in a band into groups, each group being represented by an equivalent line.

The most significant changes between previous Venusian and present terrestrial calculation in the physics of the radiative transfer are the change from 100% CO_2 to the terrestrial particle mixing ratio of 3.3×10^{-4} and the use of rotational line widths from *Yamamoto et al.* [1969] for N_2 broadening rather than self pressure broadening. Because of the smaller range in pressure for the present calculation, we have used layers with width in log pressure of 0.125 (~ 1 km) rather than the previous 0.25. The collective error in individual band cooling rates between 30 and 70 km is estimated to be a few percent and certainly less than 10%.

To calculate the NLTE population of the first excited level of the various CO_2 isotopes, we use a standard temperature-pressure collisional relaxation time of 1.04×10^{-5} sec [*Merrill and Amme*, 1969], together with the variation with temperature suggested by *Houghton* [1969], and assume that coupling to all levels except the ground state can be neglected. Table 1 shows the calculated source functions relative to LTE source functions for the significant isotopes of CO_2 and the 1962 standard atmosphere. Table 2 compares our calculated source function with that of *Drayson* [1967] for a constant 10^{-5} sec value of the collisional relaxation time used in both calculations. The agreement between the two calculations is quite good.

The population of higher excited levels is calculated assuming a collisional equilibrium defined by the vibrational temperature of the first excited level. This assumption is quite poor for the terrestrial problem, and so the calculated hot band cooling rates at levels of significant NLTE are probably far from correct. Its use is dictated by convenience and inadequate knowledge of the actual energy transfer rates between excited levels of the 15-μ system of CO_2. The cooling by the 9.6-μ band of O_3 has also been evaluated by summing the contributions of individual vibrational-rotational lines. A symmetric top approximation (cf. discussion

TABLE 1. Ratio of Source Function to LTE Source Function for 1962 Standard Atmosphere Temperature

Altitude, km	$^{12}C^{16}O_2$	$^{13}C^{16}O_2$	$^{12}C^{16}O^{18}O$	$^{12}C^{16}O^{17}O$	$^{13}C^{16}O^{18}O$
30	1.0000	0.9999	0.9998	0.9996	0.999
35	1.0000	0.9998	0.9996	0.999	0.998
40	1.0000	0.9997	0.9992	0.998	0.996
45	0.9999	0.9995	0.998	0.995	0.992
50	0.9999	0.999	0.995	0.990	0.98
55	0.9998	0.997	0.990	0.98	0.97
60	0.9997	0.991	0.98	0.96	0.95
65	0.9994	0.98	0.96	0.94	0.90
70	0.9985	0.97	0.99	0.97	0.86
75	0.996	1.06	1.21	1.23	0.96
80	1.024	1.41	1.87	2.03	1.58
85	0.80	1.35	1.97	2.20	1.70
90	0.56	1.30	1.98	2.25	1.74
95	0.28	0.90	1.38	1.55	1.19
100	0.16	0.63	0.98	1.09	0.79

TABLE 2. Ratio of Source Function to LTE Source Function for Vibrational-Collisional Relaxation Rate of 10^{-5} sec^{-1} Compared to Values (in Parentheses) Obtained by *Drayson* [1967]

Altitude, km	$^{12}C^{16}O_2$	$^{13}C^{16}O_2$	$^{12}C^{16}O^{18}O$	$^{12}C^{16}O^{17}O$
70	1.00 (1.00)	0.99 (0.99)	0.99 (0.99)	0.98 (0.98)
80	1.01 (1.01)	1.27 (1.24)	1.60 (1.64)	1.71 (1.74)
90	0.70 (0.72)	1.27 (1.18)	1.91 (1.89)	2.15 (2.10)
100	0.21 (0.20)	0.64 (0.59)	0.98 (0.95)	1.09 (1.05)

by *Herzberg* [1945]) has been used to distribute the band intensity among the individual lines. *Clough and Kneizys* [1965] carried out a more exact calculation of 9.6-μ line positions and strengths. Comparison of our calculated line strengths with their tabulated values indicates that the differences between our approximate and the more exact strengths are sufficiently small that the resulting errors in cooling rates will be small compared to errors from uncertainties in the ozone concentrations. There are $O(10^3)$ individual lines that must be included in determining the cooling by this band.

Following earlier studies, we assume LTE for the 9.6-μ O_3 emission, since the resulting cooling is negligible at levels where NLTE would be significant. For the ozone concentration, we have used a simple analytic profile consisting of a particle mixing ratio of 1.06×10^{-5} below 37 km and exponential decay with $\frac{1}{2}$ the atmospheric scale height above this level. This profile lies within the observational range of ozone observations between 40 and 60 km, the region where infrared cooling by ozone is significant.

The 1962 standard atmosphere temperature cooling rates obtained for O_3 alone and the sum of O_3 and CO_2 are shown in Figure 1. The calculated cooling by CO_2 is significantly greater than the values obtained by other authors, including *Drayson* [1967] and *Kuhn and London* [1969]; the greatest discrepancy, near 55 km, is nearly 2 deg day^{-1}. It is possible to compare the calculated cooling rates in greater detail with the band-by-band results tabulated by *Drayson* [1967]; this comparison is made in Tables 3 and 4. The agreement between the two calculations in cooling rates by each band is reasonably good, and the differences are generally within the range expected owing to uncertainties in the calculation. The summed cooling rates apparently differ, however, largely because of

Drayson's omission of downward transitions from CO_2 energy levels higher than $v_2 = 2$. The contribution from various bands is summarized in Figure 2. It is seen that the second hot bands neglected by Drayson contribute >1 deg day^{-1} cooling between 45 and 60 km. The kinks in the individual band cooling profiles in Figure 2 are artifacts of the broken-line temperature profile of the model atmosphere and disappear with a smoother temperature profile. Figure 1 was somewhat smoothed by hand to remove such features due to the temperature derivative discontinuities.

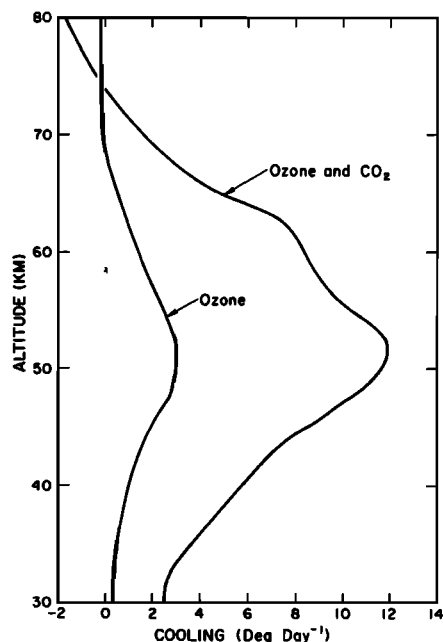The significant contribution made by hot and isotopic bands to CO_2 cooling in the upper

Fig. 1. Infrared radiative cooling rate in deg day^{-1} for 1962 standard atmosphere temperature profile and due to O_3 9.6 μ alone or to the sum of O_3 and all the 15-μ CO_2 bands.

TABLE 3. Band-by-Band Calculated Cooling Rates ($^\circ$K day^{-1}) for Fundamental Bands of Different Isotopes in Comparison with Values (in Parentheses) Obtained by *Drayson* [1967]

Altitude, km	$^{12}C^{16}O_2$	$^{13}C^{16}O_2$	$^{12}C^{16}O^{18}O$	$^{12}C^{16}O^{17}O$	$^{13}C^{16}O^{18}O$
44.0	2.88 (2.85)	0.28 (0.31)	0.29 (0.30)	0.15 (0.15)	0.02
49.5	3.30 (3.23)	0.39 (0.39)	0.53 (0.47)	0.22 (0.21)	0.02
53.0	3.02 (3.08)	0.50 (0.45)	0.61 (0.55)	0.23 (0.21)	0.02
56.0	2.40 (2.38)	0.49 (0.44)	0.57 (0.52)	0.21 (0.20)	0.02
60.0	1.87 (1.89)	0.58 (0.50)	0.53 (0.48)	0.17 (0.16)	0.01
64.0	1.36 (1.36)	0.50 (0.45)	0.36 (0.34)	0.11 (0.11)	0.01
68.0	0.85 (0.86)	0.26 (0.26)	0.12 (0.12)	0.04 (0.04)	0.00
71.0	0.65 (0.81)	0.09 (0.11)	-0.01 (-0.02)	0.00 (0.00)	0.00

stratosphere and mesosphere again illustrates, though not so dramatically, the point made by *Dickinson* [1972] that quite weak lines can contribute significantly to the cooling relative to much stronger lines when the strong lines have Voigt profiles, since photons can escape to space much more readily in the weak lines. For example, a photon emitted near the stratopause in the $^{12}C^{16}O_2$ fundamental band has, on the average, a 6×10^{-3} probability of escape to space, whereas a photon released in the $^{12}C^{16}O^{18}O$ band has a 500 times greater probability of escape. Thus the molecules of $^{12}C^{16}O^{18}O$ give nearly 20% as much cooling as $^{12}C^{16}O_2$, in agreement with the values given in Table 3, even though the relative mixing ratio of the former to the latter is only 4×10^{-3}.

Previous CO_2 cooling calculations other than that of *Drayson* [1967] were not presented in terms of contributions by individual bands, and so we cannot explain the somewhat smaller 15-μ cooling rates previously obtained. It is evident, however, that if 15-μ lines of strength 3 to 4 orders of magnitude less than the strongest lines

were not adequately represented, significantly less cooling would result.

PARAMETERIZATION OF COOLING FOR PERTURBATION FROM THE MODEL TEMPERATURE

The most important aspect of radiative cooling from the viewpoint of dynamic calculations is its variation with variation of temperature. The damping of temperature fluctuations associated with motion systems damps the motions as well and thus can be an important factor in determining the variation in time and space of winds in the upper atmosphere. Perturbations in radiative cooling, as well as the cooling itself, at each level can be decomposed into a 'cool-to-space' term and a term representing the exchange of radiation at that level with all other levels. The cool-to-space term depends only on the temperature at that level, whereas the layer exchange term also depends on the temperatures at all other levels.

If possible, it is desirable to parameterize the perturbation cooling at a given level in terms

TABLE 4. Band-by-Band Calculated Cooling Rates ($^\circ$K day^{-1}) for Hot Bands of $^{12}C^{16}O_2$ Isotopes in Comparison with Values (in Parentheses) Obtained by *Drayson* [1967]

Altitude, km	(02°0) $\leftarrow$ (01'0)	(02°0) $\leftarrow$ (01'0)	(100) $\leftarrow$ (01'0)	ν_2=3 $\leftarrow$ ν_2=2
44.0	0.79 (0.85)	0.30 (0.34)	0.32 (0.33)	0.79
49.5	1.06 (1.07)	0.44 (0.45)	0.46 (0.46)	1.58
53.0	1.26 (1.19)	0.55 (0.53)	0.57 (0.55)	1.73
56.0	1.12 (1.11)	0.51 (0.51)	0.52 (0.53)	1.40
60.0	1.32 (1.34)	0.59 (0.62)	0.61 (0.62)	1.02
64.0	1.22 (1.33)	0.51 (0.60)	0.54 (0.56)	0.43
68.0	0.80 (0.88)	0.29 (0.37)	0.32 (0.29)	-0.05
71.0	0.50 (0.55)	0.14 (0.21)	0.16 (0.12)	-0.21

of the perturbation temperature at that level alone. The most obvious approach to this parameterization would be to use only the cool-to-space term. The Planck function dependence of this term can be analytically differentiated with respect to temperature, and the derivative evaluated for the reference temperature profile can be used as Newtonian cooling coefficient. Multiplication of this coefficient by the perturbation temperature then would give the perturbation cooling. There are, however, several shortcomings to this approach. First, the cooling contribution from exchange between layers, although generally less than the cool-to-space contribution below 70 km, is still significant and its contribution will also vary with temperature. Second, the strong dependence of the strengths of hot bands on temperature is not easily included, since the consequent variation of photon emission rate is partially compensated by variation in the probability of photon escape within that band. The degree of this compensation depends on the optical depth of an emitting layer.

Thus we resorted to an entirely numerical approach for obtaining a Newtonian cooling coefficient $a_0(z)$ for small departures from the reference temperature profile $T_0(z)$. That is, if $Q(T)$ is the infrared cooling rate for a temperature profile $T(z)$, then

$$a_0(z) = \tfrac{1}{2}\,\delta^{-1}[Q(T_0 + \delta) - Q(T_0 - \delta)] \quad (1)$$

where δ is a small temperature perturbation (we used $\delta = 0.1°K$). The cooling coefficient $a_0(z)$ obtained for perturbations from the 1962 standard atmosphere is shown in Figure 3. To correct for variation of the cooling coefficient with variations of temperature in order to get a cooling coefficient $a(z)$ valid over a wider range of temperatures, we use

$$a(z) = a_0(z)\{1 + b[T(z) - T_0(z)]\} \quad (2)$$

where b is a constant, chosen in some sense to minimize the resulting error. On the basis of a simple fit to the second term in a Taylor's series expansion of the 15-μ Planck function, a tentative choice for b is

$$b = b_0 = \frac{0.0033}{T_0(z) - 135} \quad (3)$$
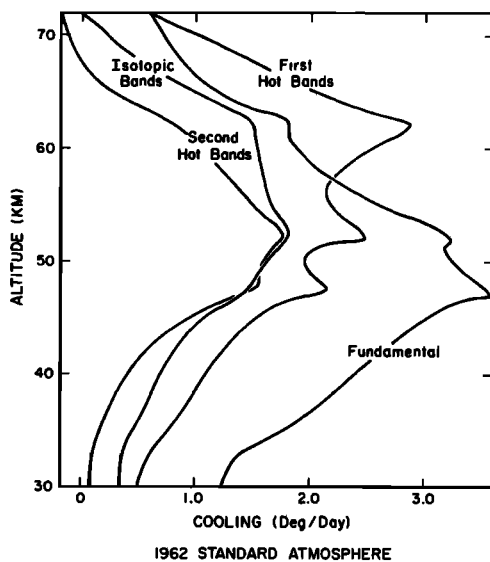
which is used for $p > 0.2$ mb. For $p < 0.2$ mb,

Fig. 2. Same as Figure 1 but showing the cooling given by different types of CO_2 bands, i.e., for $^{12}C^{16}O_2$, the fundamental ($\nu_2 = 1 \leftarrow \nu_2 = 0$), all the first hot bands, and all the fundamental bands of the other isotopes (isotopic bands). Also included in the total cooling, but not shown in this figure, is a small contribution by the hot bands of other isotopes.
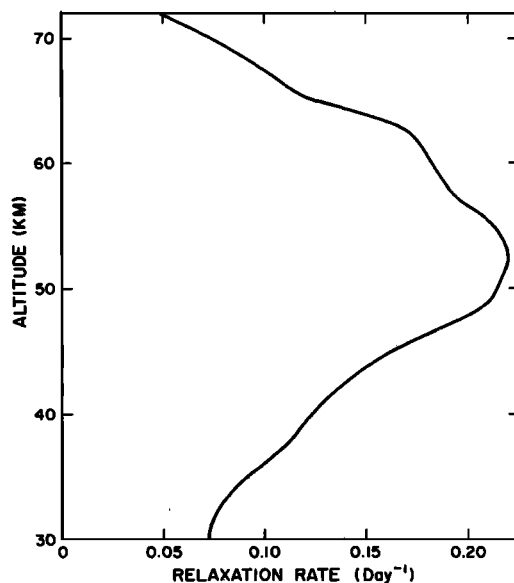

Fig. 3. The Newtonian cooling coefficient which, when multiplied by a small departure from the 1962 standard atmosphere temperatures at a given pressure level, gives the deviation from the cooling of Fig. 1 referred to the same pressure level.

a stronger variation with temperature is suggested empirically; for example,

$$b = b_0 + 0.04 \times [1 - 5p(\text{mb})] \quad (4)$$

These expressions for b undoubtedly could be improved with more elaborate empirical fitting. They are given here to suggest a form that more correct expressions might assume.

As an indication of the magnitude of error that might be associated with use of the Newtonian cooling approximation described above, we have used it to calculate the cooling for a drastically altered temperature profile, namely a 230°K isothermal atmosphere, and have compared this result in Figure 4 with the cooling obtained using the complete radiative transfer calculation. The agreement is quite good below 60 km and within a factor of 2 up to 70 km. Actual departures from the model temperature profile, and thus errors in the cooling rates, would generally be considerably less. Also, the exact cooling in Figure 4 above 60 km is somewhat doubtful, because the relatively large contribution from hot bands would be expected to be less with a more proper NLTE calculation for the hot bands.

TABLE 5. Total Infrared Cooling Rates and Newtonian Cooling Coefficients Derived from 1962 Standard Atmosphere at Various Pressure Levels

log (1 mb/p)	Cooling, deg day^{-1}	$a_0(z)$, day^{-1}
-3.0	2.0	0.06
-2.1	2.8	0.08
-0.8	6.7	0.135
0.0	11.4	0.212
0.5	12.1	0.220
1.0	9.5	0.200
1.8	7.6	0.172
2.2	4.2	0.125
3.125	0.7	0.062
4.5	-1.7	0.016

Intermediate values are adequately represented by linear interpolation.

DISCUSSION AND CONCLUSIONS

A simple parameterization is suggested for infrared cooling at levels between 30 and 70 km in the terrestrial atmosphere. The cooling is given by the sum of the exact cooling for a reference temperature profile and a correction term defined as the product of a Newtonian cooling coefficient and the departure of the actual temperature from the reference temperature. The reference profile cooling and the cooling coefficient for small temperature departures depend only on pressure level. Strictly speaking, the parameterization applies only to temperature perturbations whose amplitudes are independent of altitude but should give a reasonable approximation for perturbations whose vertical scale is comparable to or greater than one scale height.

A NLTE line-by-line procedure has been developed for relatively accurate calculation of infrared cooling above 30 km. This calculation is used in conjunction with a 1962 standard atmosphere temperature profile to illustrate the cooling rate parameterization. The standard atmosphere NLTE source functions and the cooling rates for individual bands are in reasonable agreement with *Drayson* [1967], but greater total 15-μ CO_2 cooling is obtained in the present calculation because of the inclusion of the second hot bands of $^{12}C^{16}O_2$ and, to a lesser extent, the hot bands of other isotopes. The hot bands, in general, will make a relatively larger contribution to the cooling coefficient than to the cooling rates because of the strong variation of these band strengths with temperature.
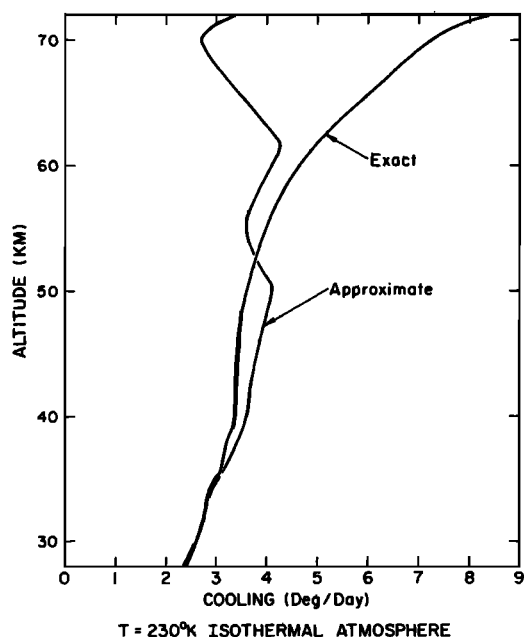

T = 230°K ISOTHERMAL ATMOSPHERE

Fig. 4. The cooling for a 230°K isothermal atmosphere according to the parameterization of this paper compared with that given by the complete radiative transfer calculation.

The calculated standard atmosphere cooling rates and cooling coefficients versus log pressure appear in Table 5. A sufficient number of levels is included so that the cooling rates and cooling coefficients at intermediate levels can be obtained within the accuracy of the calculation by using linear interpolation.

The parameterization of this paper is used to calculate cooling rates from a given temperature profile at a given level by the following steps: (1) determine the log of pressure at that level; (2) generate the temperature of the 1962 standard atmosphere at the same pressure; (3) calculate the difference between the given and standard atmosphere temperatures at this pressure level; (4) interpolate the reference cooling rate and Newtonian cooling coefficients of Table 5 versus log pressure to the same level; (5) if necessary, correct the cooling coefficients for large departures of the temperature from the reference temperature; and (6) calculate the total infrared cooling at the given pressure level as the sum of the reference cooling rate and the product of the temperature difference with the cooling coefficient. These steps are readily incorporated into a computer program that requires sufficiently little computer time to give infrared cooling values; i.e., it can be used in conjunction with three-dimensional time-dependent dynamic models.

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