



Agenda

- ↗ Lecture: Navigation I: Path Planning
- ↗ Demo Time: Pset 3b Follower
- ↗ Upcoming:
 - ↗ **ROOM CHANGE:** March 15 in Pierce 213 (Brooks room)
 - ↗ We will do EKF lab, very important!
 - ↗ Also please complete online lab safety
 - ↗ Pset 4 starts week after spring break (yay spring break)
- ↗ References:
 - ↗ "Intro to AI Robotics", chapter 9 and 10, Robin Murphy, 2000.
 - ↗ "Intro to Autonomous Mobile Robots", chapter 5.5 , 6.1-2, Seigwart et al, 2004
 - ↗ "Robot Motion Planning", Lecture Notes, Choset and others (CMU 16-735)

What Does it Mean to be Autonomous?



The diagram illustrates the autonomy loop:

```

    graph TD
        PERCEPTION[PERCEPTION] --> COGNITION[COGNITION]
        COGNITION --> ACTION[ACTION]
        ACTION --> PHYSICS[PHYSICS OF THE WORLD]
        PHYSICS --> PERCEPTION
    
```

The components are:

- PERCEPTION**: Represented by an orange box.
- COGNITION**: Represented by a yellow box.
- ACTION**: Represented by a blue box.
- PHYSICS OF THE WORLD**: Represented by a green box.

Arrows indicate the flow of information: PERCEPTION feeds into COGNITION, COGNITION feeds into ACTION, ACTION feeds into PHYSICS OF THE WORLD, and PHYSICS OF THE WORLD feeds back into PERCEPTION.

Today: Robots Navigating the World



Scenarios

- Hospital Helper (e.g. Diligent, Tugs)
- Office security or mail-delivery (e.g. Cobal, Savioke)
- Tour Guide robot in a museum (Minerva)
- Autonomous Car with GPS and Nav system

Biological analogies:
Humans, bees and ants, migrating birds, herds

Today: Robots Navigating the World

Second Part of CS189: High-level reasoning

From finite state machines to complex representation and memory

↗ **Path Planning:** *How to I get to my Goal?*

↗ **Localization:** *Where am I?*

↗ **Mapping:** *Where have I been?*

↗ **Exploration:** *Where haven't I been?*

What is Path Planning?

↗ **Simple Question:** *How do I get to my Goal?*

↗ **Not a simple answer!**

↗ Can you see your goal?

Do you have a map?

Are obstacles unknown or dynamic?

↗ Does it matter *how fast* you get there?

Does it matter *how smooth* the path is ?

↗ How much compute power do you have?

How precise is your motion control?

↗ **Path Planning is best thought of as a Collection of Algorithms**

↗ You have to match the method to the “*ecological niche*”

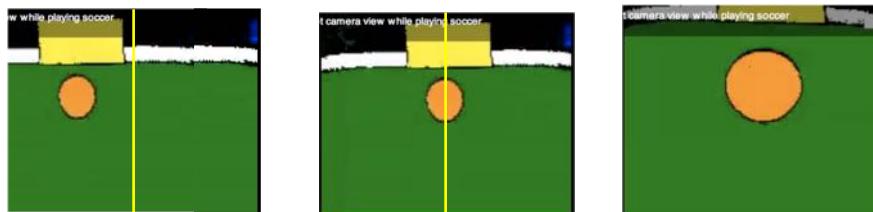
↗ 3 Things: Environment, Success metrics, Robot capability.

Types of Path Planning Approaches

- Reminder of the Basics
 - Visual homing (Purely local sensing and feedback control)
 - Inverse Kinematics (Turn-move-turn to get from A to B)
- Bug-based Path Planning (mostly-local without a map)
 - Robots can see the Goal (direction and distance)
 - But there are unknown obstacles in the way (No map)
- Metric (A*) Path Planning (global with a map)
 - Assumes that you have a *map (distance or graph)* and you know where you and the goal are located in it.
 - Path is represented as a series of waypoints (*directions*)

Basics: Visual Homing

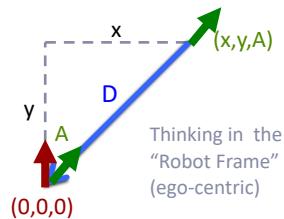
- Purely Reactive Navigation
 - Measure Visual (x,y) Position of Goal
 - Move to bring goal to Visual Center
 - Proportional Control (if you see the goal), Random walk (if you don't)



Basics: Inverse Kinematics

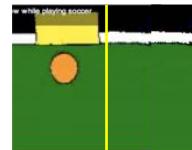
↗ Getting from Here to Point B

- ↗ Popular Option: Turn-Move-Turn [Lecture: Autonomy 1]
- ↗ No obstacles (like in visual homing example)



Path Planned is: Turn A then Move D

Turn A = $\text{atan2}(x/y) = W \times \text{duration}$
 Move D = $\sqrt{x^2 + y^2} = L \times \text{duration}$
 (Turn again, to end in new orientation)



*Example:
Line up
Ball & Goal*

Bug-based Path Planning



What if the Robot has obstacles in the way?

- ↗ Always have Goal direction and/or distance (Global)
 But No Map: Only local knowledge of environment (Local)
- ↗ *Example Scenario:*
 - ↗ Outdoor robot knows GPS location of goal, but building in the way.
 - ↗ Indoor robot see goal location, but furniture in the way.

"Bug" Algorithms depend on simple but provable behaviors!

- ↗ Don't need to build a map
- ↗ Simple Computation: Visual Homing + Wall-following + Odometry

- ↗ Very intuitive class of algorithms – but surprisingly powerful

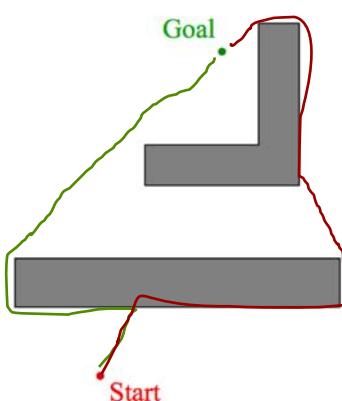
Basic Idea: Bug 0



- ↗ Robot
 - ↗ Known direction to goal
 - ↗ Wall-following
- ↗ Bug 0 Algorithm
 - ↗ Head towards goal
 - ↗ If obstructed, follow obstacle wall until you can head towards goal again.
- ↗ Continue

Adapted from Choset 16-735

Basic Idea: Bug 0

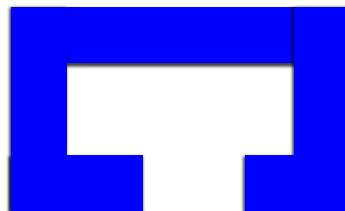


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What map will foil Bug o?

Goal
*



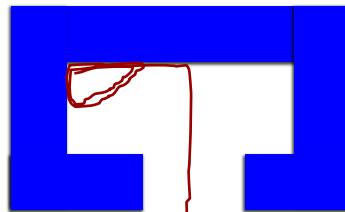
*
Start

- ↗ Robot
 - ↗ Known direction to goal
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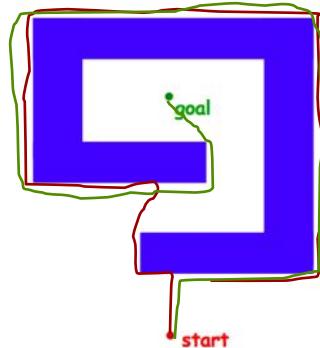


Robot

- ↗ Known direction to goal
 - ↗ Wall-following
- ### Bug 0 Algorithm
- ↗ Head towards goal
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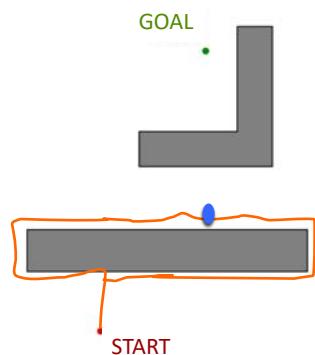


Robot

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- ↗ Head towards goal
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Adapted from Choset 16-735

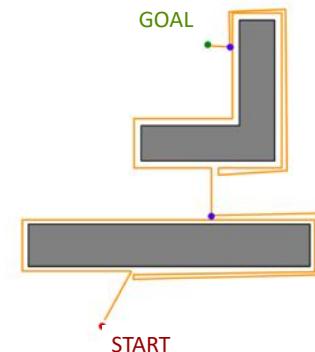
A Better Bug: Bug 1



- ↗ Robot
 - ↗ Known direction to goal
 - ↗ Wall-following
 - ↗ Measure distance to goal
 - ↗ Odometry with encoders
- ↗ Bug 1 Algorithm
 - ↗ Head towards goal
 - ↗ If obstructed,
circumnavigate the
obstacle and remember
the point P on the
perimeter that is closest to
the goal
 - ↗ Return to that closest
point and continue.

Adapted from Choset 16-735

A Better Bug: Bug 1



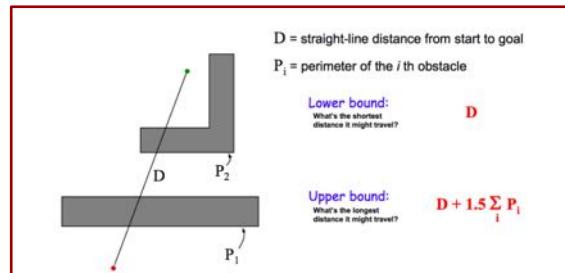
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Adapted from Choset 16-735

What map will foil Bug 1?

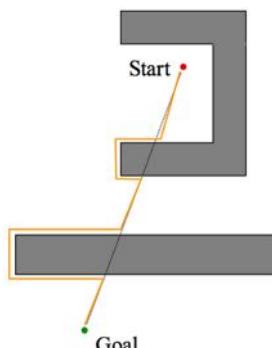
↗ **None!**

- ↗ *Any reasonable world* (finite number of obstacles with finite perimeter)
- ↗ Analysis: It is possible to bound worst and best case trajectories
- ↗ Discussion: What do you think are the pros and cons of this approach?



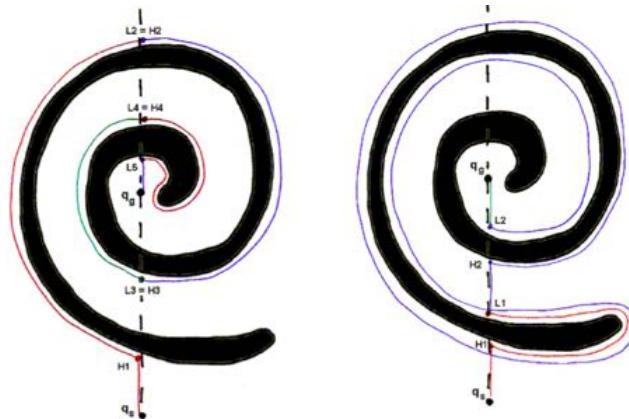
Adapted from Choset 16-735

An Alternative: Bug 2



- ↗ **Robot**
 - ↗ Known direction to goal
 - ↗ Wall-following
 - ↗ Measure distance to goal
 - ↗ Odometry with encoders
or orientation to goal
- ↗ **M-line**
 - ↗ Line from the start to goal
- ↗ **Bug 2 Algorithm**
 - ↗ Head toward goal **on the m-line**
 - ↗ If an obstacle is in the way, follow it until **you encounter the m-line again and you are closer to the goal**.
 - ↗ Leave the obstacle and continue toward the goal

Some Fun Examples: Bug2



16-735, Howie Choset with slides from G.D. Hager and Z. Dodds

Adapted from Choset 16-735

Many Types of Bug Algorithms!

- Recent Variant: **i-Bug** (intensity-Bug, Lavalle etc al)
 - Proved that you can exit an obstacle at the first point “closer” to the goal (don’t need to keep track of m-line)
- Attractive for many reasons
 - Simplicity of implementation and robot assumptions, ability to deal with unknown and dynamic environments, and *the analogy to ant behavior*.

Open question: Do ants (bugs) use the bug algorithms?

Many Types of Bug Algorithms!

Collective Strategy for Obstacle Navigation
during Cooperative Transport by Ants



Helen F. McCreery, Zachary A. Dix, Michael D. Breed, Radhika Nagpal
University of Colorado and Harvard University
Journal of Experimental Biology, Nov 2016
[Overview Video](#)

Types of Path Planning Approaches

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 - ↗ Visual homing (Purely local sensing and feedback control)
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- ↗ **Bug-based Path Planning (mostly-local without a map)**
 - ↗ Robots can see the Goal (direction and distance)
 - ↗ But there are unknown obstacles in the way (No map)
- ↗ **Metric (A*) Path Planning (global with a map)**
 - ↗ Assumes that you have a *map (distance or graph)* and you know where you and the goal are located in it.
 - ↗ Path is represented as a series of waypoints

Metric/Global Path Planning

↗ What if the Robot has Full Knowledge

- ↗ A map of the environment and robot + goal's locations
- ↗ Goal: Find a "optimal" path (typically distance but other possibilities)
- ↗ We will focus on robots, but it's a general problem (*think Google maps*)

↗ Two Components

↗ Map Representation ("graph"):

- ↗ Feature based maps (office numbers, landmarks)
- ↗ Grid based maps (cartesian, quadtrees)
- ↗ Polygonal maps (geometric decompositions)

↗ Path Finding Algorithms:

- ↗ Shortest-Path Graph Algorithms (Breadth-First-Search, A* Algorithm)

Map Representation: Feature based

↗ Also known as a Topological or Landmark-based Map

↗ Features your robot can recognize:

- ↗ Includes both natural landmarks (corner, doorway, hallways) and artificial ones (office door numbers; or robot-friendly tags)
- ↗ Gateways are landmarks that represent decisions (e.g. intersection)
- ↗ Distinguishable places are unique landmarks

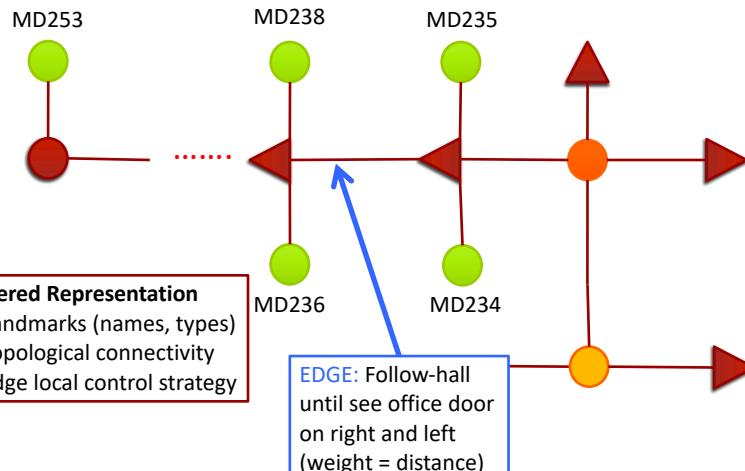
↗ World is a graph that connects landmarks

- ↗ Edges represent **actual motion**: how to get from landmark A to landmark B
Usually visual/reactive navigation is possible along an edge
- ↗ Edges can also keep **extra attributes**: distance, time it takes, etc.

↗ Google Maps are topological maps for humans (e.g. turn at intersection)

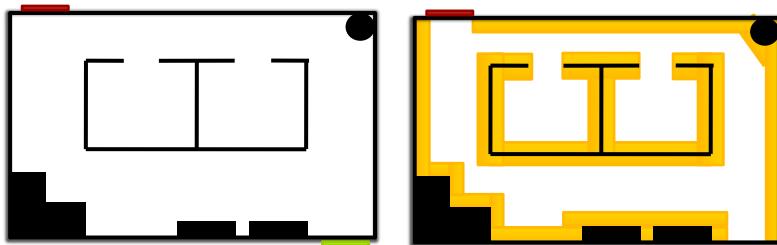
- ↗ Caveat: *Much less easy to construct topological maps for robots!*

Example: Maxwell-Dworkin



Map Representation: Grid based

- ↗ Ignore any notion of Features
- ↗ Instead, Convert the map into a grid-graph
 - ↗ Step 1: Grow the boundaries (by robot size)
 - ↗ Step 2: Overlay a grid



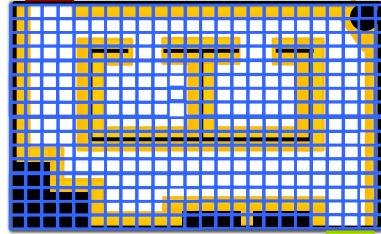
Adapted from Murphy 2000

Map Representation: Grid based

↗ Basic: An Occupancy Matrix

↗ Problem:

- ↗ How do you choose the “resolution” of the grid?
- ↗ Too small – computationally expensive, jagged paths
- ↗ Too big – might miss paths



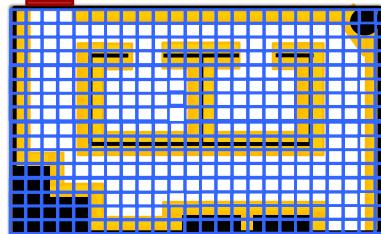
Note: Occupancy Grids will be more useful later, when the robot is responsible for making the map!

Map Representation: Grid based

↗ Basic: An Occupancy Matrix

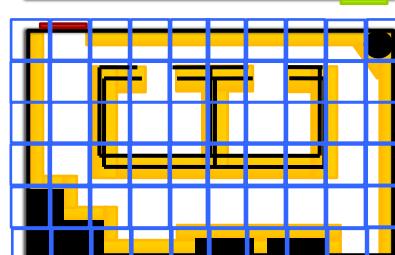
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↗ Quadtree

- ↗ Create a grid recursively!
- ↗ Start with very coarse grid;
- ↗ Then for each grid section, if there is an obstacles, refine.
- ↗ Outcome: Captures large open spaces as a single big grid point

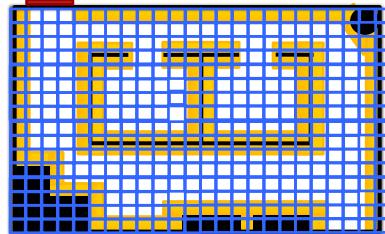


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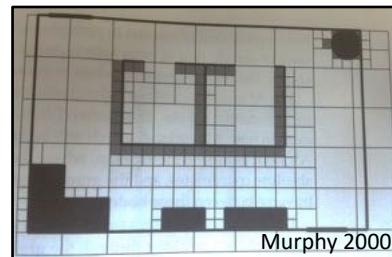
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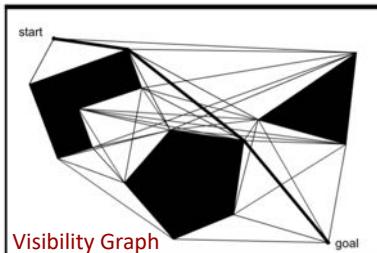
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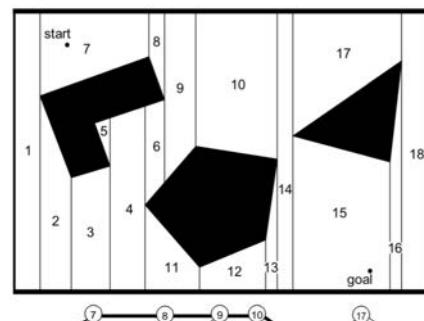
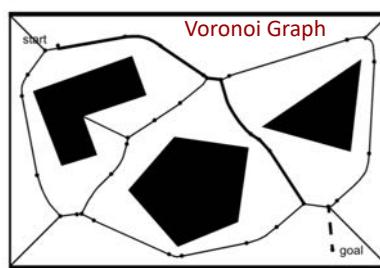


Murphy 2000

More Map Representations



From “Introduction to Autonomous Mobile Robots”,
Chapter 5 and 6, Seigwart and Nourbakhsh, 2004.



Metric/Global Path Planning

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Path Finding Algorithms

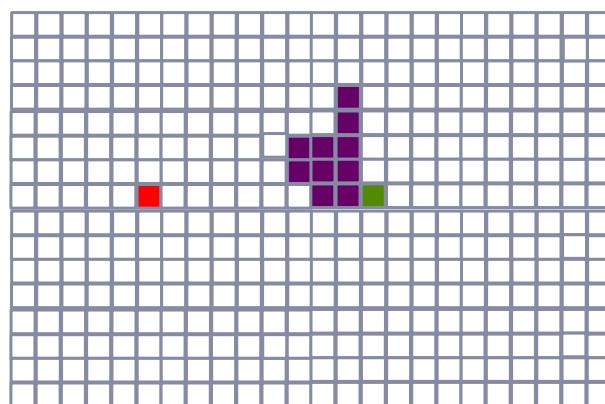
↗ All Map Representations are a weighted "graph"

- ↗ Nice part is that you only need to do this once (amortize computation)

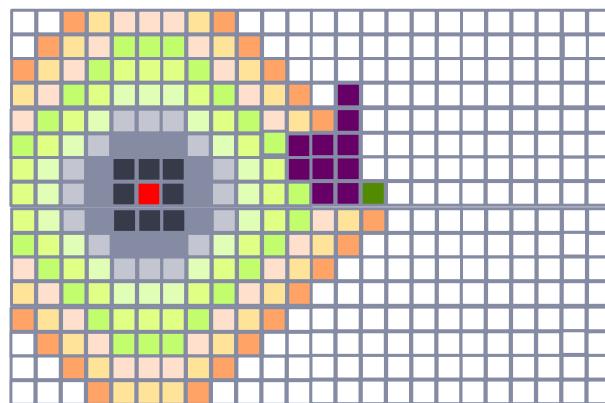
↗ Algorithm: Compute shortest paths in the graph

- ↗ Path is represented by a series of waypoints
- ↗ **Single Path Search Algorithms:** Find shortest path A to B
 - ↗ Breadth-First-Search (simple graphs); Dijkstra's (weighted)
 - ↗ A* search for large graphs (BFS + Heuristic)
- ↗ **Gradient Path Algorithms:** Find *all paths* towards B
 - ↗ E.g. Fixed Basestation: BFS, Dijkstra's, Wavefront algorithms, etc

Breadth-First Search

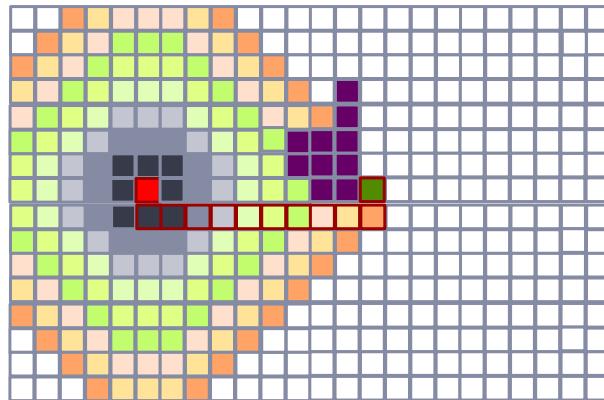


Breadth-First Search



Breadth-First Search

Side Note: Here bug 2 ("m-line") would have worked well too!
If few obstacles, then bug is good enough



A* Algorithm

A* Algorithm

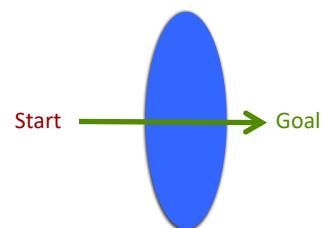
Similar to BFS but choose next node to expand based on two things

1. Distance from start (like BFS)
2. Expected distance from goal (H)

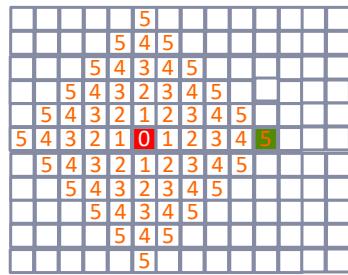
H is the *heuristic*. The theory shows that so long as the heuristic is “optimistic” then A* returns the best path.

Key point:
Average behavior can be awesome!

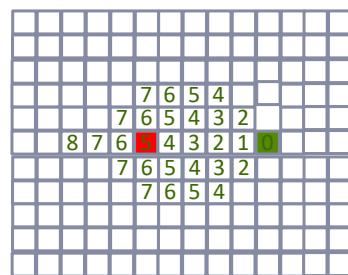
For maps,
 $H = \text{straight-line distance}$ is a good heuristic



How A* works



How BFS would
Explore the space

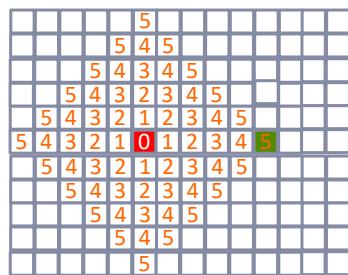


Manhattan distance to Green
(easy to compute directly)
(no obstacles considered)

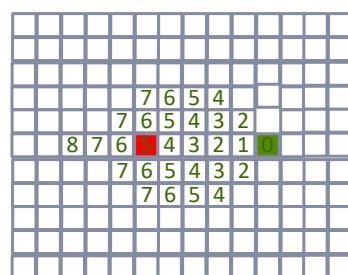


A* criteria = BFS+Manhattan

How A* works



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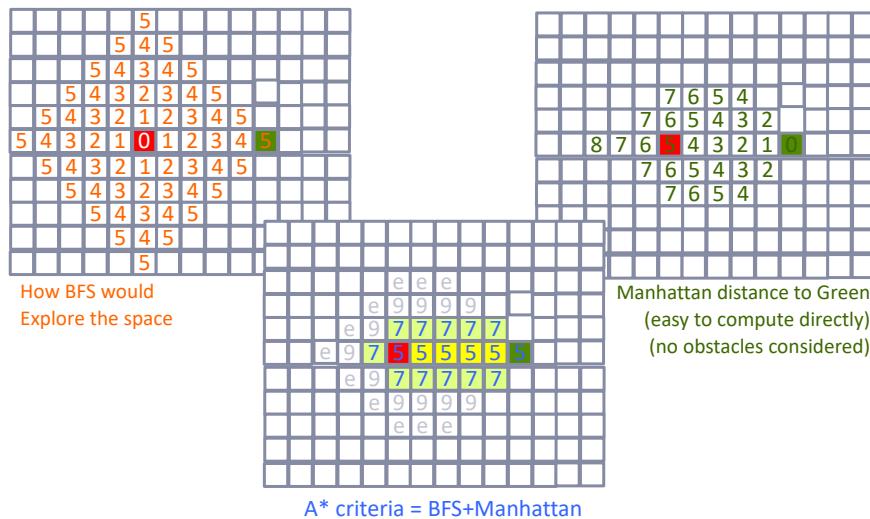


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A* Algorithm

A* Algorithm

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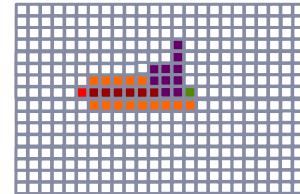
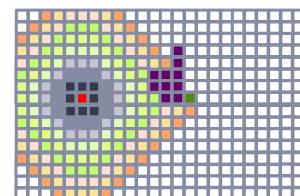
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A* is a "general" graph search (AI, game tree, orbitz, etc); see Murphy 2000 chapter 10 for more details

Case Studies and AAAI Competitions



Given a “map” of the environment with some landmarks.

Given initial position (not pose) and final goal

Unknown obstacles might be introduced

AAAI 1992 and 1994 Mobile robot competitions [Murphy 2000]

DARPA Urban Challenge (2007)



Final Thoughts

- ↗ Robot systems must combine many ideas
 - ↗ Interleave bug like navigation with serious path planning
 - ↗ High-level maps and low-level primitives
 - ↗ e.g. collision avoidance, feature recognition, etc
 - ↗ Ecological niche matters!
 - ↗ E.g. Robot soccer is very different from a mail-delivery robot.

- ↗ Cool New Methods
 - ↗ RRT: Rapidly exploring Random Trees
 - ↗ Combining with Probabilistic localization

RRT solves hard problems



**High-dimensional Spaces
Complex movement constraints**

(Parallel parking is hard, but now imagine parking a car with three hitched trailers!)

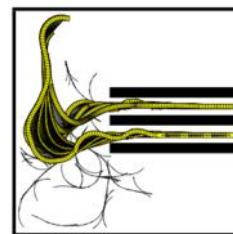
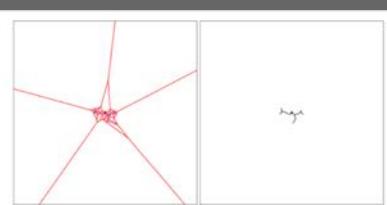


Figure 13: A nonholonomic planning problem that involves a car pulling three trailers. There are seven degrees of freedom.

RRT

↗ Sample

- ↗ Pick some random points
- ↗ Based on voronoi areas
 - ↗ Bias towards open spaces)
 - ↗ Bias towards goal, if one exists



↗ Extend

- ↗ Connect the new point to your old path by seeing how close your robot can get to that point
- ↗ *extend using actual (complex) dynamics model of the robot*

