1.

a) for E.Coli, 1 unit of OD600 = 0.39 gDW/L

$$\left(\frac{0.39\ gDW}{L \cdot unit}\right)(0.1\ unit)\left(\frac{1\ L}{1000\ mL}\right) = 3.9\times10^{-5}\ \frac{gDW}{mL}$$

b) $\dot{m}_i = r_{x,i}\,\bar{u}_i - (\mu + \theta_{m,i})\,m_i$

$\dot{m}^* = r_x^*\,\bar{u}^* - (\mu + \theta_m^*)\,m^*$

at SS, $\dot{m}^* = 0$ $\left(\frac{dm^*}{dt} = 0\right)$

$0 = r_x^*\,\bar{u}^* - (\mu + \theta_m^*)\,m^*$

$(\mu + \theta_m^*)\,m^* = r_x^*\,\bar{u}^*$ $\quad \mu = 0$ since $\dot{V} = 0$

$m^* = \frac{r_x^*\bar{u}^*}{\theta_m^*}$ $\quad m^* = \bar{r}_x(G,\theta)\,\bar{u}(I,K)$

$G^*(=)\,nM$

$r_x^* = k_E^{*x}\,(G_j^* : R_x)_o$ $\quad$ rate of transcription is first order in open complex conc.

$m^* = \frac{\frac{nM}{s}}{1/s} = nM$

from 23Jan20 notes:

$r_x^* = k_{E,x}\,R_{x,T}\left(\frac{G_j^*}{\tau_x^* K_x^* + (\tau_x^* + 1)G^*}\right)$ $\quad r_x^* = \frac{1}{s} \cdot nM\left(\frac{nM}{s\,nM + s\,nM}\right)$

$m^* = \frac{k_{E,x}^*\,R_{x,T}\,G^* \cdot \bar{u}^*}{(\tau_x^* K_x^* + (\tau_x^* + 1)G^*)\,\theta_m^*}$

$(G^* : R_x)_o (=)\ nM \cdot \frac{nM}{s \cdot nM + s \cdot nM}$

$$\boxed{\bar{r}_x(G,\theta) = \frac{k_{E,x}^*\,R_{x,T}\,G^*}{(\tau_x^* K_x^* + (\tau_x^* + 1)G^*)\,\theta_m^*}}$$

$m^* = \frac{s^{-1}\,nM\ nM}{()\,nM + ()\,nM \cdot \frac{1}{s}}$

c) > estimate τ with McClure fig 2:

use A2 promoter curve, assume $R_x = 0.05\,nM$

$\tau_x^* = 27.40$ sec

> given: $G^* = 2$

> calculate θ_m^*:

halflife = 5 min

$1 = 2e^{-\theta t}$

$\frac{1}{2} = e^{-5\theta}$

$\ln\frac{1}{2} = -5\theta$

$\theta = \frac{\ln 0.5}{5} = 0.139\ s^{-1}$

> elongation rate constant $k_{E,x}^*$:

$\langle k_{E,x}^* \rangle = \frac{e_x}{L}$ $\quad L_j = 1000$ nt (given)

$e_x = 25\ \frac{nt}{s}$ (Chen 2015)

$k_{E,x}^* = \langle k_{E,x}^* \rangle\left(\frac{L}{L_j}\right) = \frac{25}{L} \cdot \frac{L}{L_j} = 0.025\ s^{-1}$

> saturation constant for lacZ

$K_x^* = 0.003$ mM

> $G_j = 2$ copies

$(2\ copies)\left(\frac{1}{4.7\times10^{-10}\,nL}\right)\left(\frac{mol}{6.02\times10^{23}}\right)\left(\frac{10^6\ nmol}{1\ mol}\right)\left(\frac{10^6\ nL}{1\ L}\right) = 0.0050\ nM$

assume IPTG induced PTAC promoter (Moon Supplemental IV)

$> u(I,\theta) = \left(\frac{K_1}{1 + K_1 + K_2 f_T}\right)$

Moon Table S2 $\begin{cases} K_1 = 53 \\ K_2 = 1950 \end{cases}$ $\quad K_D = 0.003$ mM, $n = 1.6$

$f_T = 1 - \frac{L^n}{K_D^n + L^n}$

$$\boxed{m^* = \frac{(0.025\ s^{-1})(0.05\ nM)(0.0050\ nM)}{[27.40 \cdot 3\,nM + (27.4+1)(0.0050\ nM)]\ 0.139\ s^{-1}} \cdot \bar{u}^*}$$

→

2)

a) let $\tilde\alpha_x, \tilde\alpha_z$ (basal production rate of X, Z) $\frac{conc}{time}$

$\tilde\beta_x$: signal induction $time^{-1}$

$\tilde\delta_x, \tilde\delta_z$: degradation rates

$$\frac{d\tilde X}{d\tilde t} = \frac{\tilde\alpha_x + \tilde\beta_x S}{1+S+\left(\frac{\tilde Z}{\tilde Z_x}\right)^{n_{zx}}} - \tilde\delta_x \tilde X$$

$$\frac{d\tilde Z}{d\tilde t} = \frac{\tilde\alpha_z}{1+\left(\frac{\tilde X}{\tilde X_z}\right)^{n_{xz}}} - \tilde\delta_z \tilde Z$$

$[\;] = time \cdot \frac{1}{time} = [\;]$

b) $\delta_x = \frac{\tilde\delta_x}{\tilde\delta_x} = 1$ $\delta_z = \frac{\tilde\delta_z}{\tilde\delta_x}$ $\boxed{t = \tilde t\, \tilde\delta_x}$ incorrectly given as $t = \tilde t \delta_x$ in paper

$\Rightarrow \tilde t = \frac{t}{\tilde\delta_x}$

$\alpha_x = \frac{\tilde\alpha_x}{\tilde\alpha_z}$ $\boxed{\beta_x = \frac{\tilde\beta_x}{\tilde\alpha_z}}$

$Z_x = \frac{\tilde Z_x \tilde\delta_x}{\tilde\alpha_z}$ (=) $\frac{conc \cdot \frac{1}{time}}{\frac{conc}{time}}$ $\Rightarrow \tilde Z_x = \frac{Z_x \tilde\alpha_z}{\tilde\delta_x}$ ... $X_z = \frac{\tilde X_z \tilde\delta_x}{\tilde\alpha_z}$ $\Rightarrow \tilde X_z = \frac{X_z \tilde\alpha_z}{\tilde\delta_x}$

$X = \frac{\tilde X \tilde\delta_x}{\tilde\alpha_z}$, $Z = \frac{\tilde Z \tilde\delta_x}{\tilde\alpha_z}$ $\Rightarrow \tilde Z = \frac{Z\tilde\alpha_z}{\tilde\delta_x}$

$\Downarrow$

$\tilde X = \frac{X\tilde\alpha_z}{\tilde\delta_x}$ $\tilde Z = \frac{Z\tilde\alpha_z}{\tilde\delta_x}$

$$\frac{d\left(\frac{X\tilde\alpha_z}{\tilde\delta_x}\right)}{d\left(\frac{t}{\tilde\delta_x}\right)} = \frac{\alpha_x\tilde\alpha_z + \beta_x\tilde\alpha_z S}{1+S+\left(\frac{\frac{Z\tilde\alpha_z}{\tilde\delta_x}}{\frac{Z_x\tilde\alpha_z}{\tilde\delta_x}}\right)^{n_{zx}}} - \cancel{\tilde\delta_x}\frac{X\tilde\alpha_z}{\cancel{\tilde\delta_x}}$$

$$\frac{\frac{\tilde\alpha_z}{\cancel{\tilde\delta_x}}}{\cancel{\tilde\delta_x}}\frac{dx}{dt} = \frac{\cancel{\tilde\alpha_z}(\alpha_x+\beta_x S)}{1+S+\left(\frac{z}{z_x}\right)^{n_{zx}}} - x\cancel{\tilde\alpha_z}$$

$$\boxed{\frac{dx}{dt} = \frac{(\alpha_x+\beta_x S)}{1+S+\left(\frac{z}{z_x}\right)^{n_{zx}}} - X}$$

$$\frac{d\left(\frac{Z\tilde\alpha_z}{\tilde\delta_x}\right)}{d\left(\frac{t}{\tilde\delta_x}\right)} = \frac{\tilde\alpha_z}{1+\left(\frac{\frac{X\tilde\alpha_z}{\tilde\delta_x}}{\frac{X_z\tilde\alpha_z}{\tilde\delta_x}}\right)^{n_{xz}}} - \delta_z\cancel{\tilde\delta_x}\frac{Z\tilde\alpha_z}{\cancel{\tilde\delta_x}}$$

$$\cancel{\tilde\alpha_z}\frac{dz}{dt} = \frac{\cancel{\tilde\alpha_z}}{1+\left(\frac{x}{x_z}\right)^{n_{xz}}} - \delta_z z\cancel{\tilde\alpha_z}$$

$$\boxed{\frac{dz}{dt} = \frac{1}{1+\left(\frac{x}{x_z}\right)^{n_{xz}}} - \delta_z z}$$

$$\frac{d\left(\frac{x\tilde\alpha_z}{\cancel{\tilde\delta_x}}\right)}{d\left(\frac{t}{\cancel{\tilde\delta_x}}\right)} = \frac{\alpha_x\cancel{\tilde\alpha_z} + \beta_x\cancel{\tilde\alpha_z}S}{1+S+\left(\frac{\frac{z\tilde\alpha_z}{\tilde\delta_x}}{\frac{z_x\tilde\alpha_z}{\tilde\delta_x}}\right)^{n_{zx}}} - \cancel{\tilde\delta_x}\frac{x\cancel{\tilde\alpha_z}}{\cancel{\tilde\delta_x}}$$

$$\frac{dx}{dt} = \frac{\alpha_x+\beta_x S}{1+S+\left(\frac{z}{z_x}\right)^{n_{zx}}} - x$$

$$\frac{d\left(\frac{z\tilde\alpha_z}{\cancel{\tilde\delta_x}}\right)}{d\left(\frac{t}{\cancel{\tilde\delta_x}}\right)} = \frac{\cancel{\tilde\alpha_z}}{1+\left(\frac{\frac{x\tilde\alpha_z}{\tilde\delta_x}}{\frac{x_z\tilde\alpha_z}{\tilde\delta_x}}\right)^{n_{xz}}} - \cancel{\tilde\delta_z}\frac{z\cancel{\tilde\alpha_z}}{\cancel{\tilde\delta_x}}\,\delta_z\cancel{\tilde\delta_x}$$

$$\frac{dz}{dt} = \frac{1}{1+\left(\frac{x}{x_z}\right)^{n_{xz}}} - \delta_z z$$

→

c) @ SS: $\frac{dx}{dt} = 0$, $\frac{dz}{dt} = 0$

$$\boxed{\frac{dx}{dt} = \frac{\alpha_x + \beta_x S}{1 + S + \left(\frac{z}{z_x}\right)^{n_{zx}}} - x} = 0$$

$$\boxed{\frac{dz}{dt} = \frac{1}{1 + \left(\frac{x}{x_z}\right)^{n_{xz}}} - \delta_z z} = 0$$

$$x = \frac{\alpha_x + \beta_x S}{1 + S + \left(\frac{z}{z_x}\right)^{n_{zx}}}$$

$$z = \frac{1}{\delta_z\left(1 + \left(\frac{x}{x_z}\right)^{n_{xz}}\right)}$$

$$\alpha_x + \beta_x S = x\left(1 + S + \left(\frac{z}{z_x}\right)^{n_{zx}}\right)$$

$$\beta_x S = x\left(1 + S + \left(\frac{z}{z_x}\right)^{n_{zx}}\right) - \alpha_x$$

$$\beta_x S - xS = x\left(1 + \left(\frac{z}{z_x}\right)^{n_{zx}}\right) - \alpha_x$$

$$S = \frac{x + x\left(\frac{z}{z_x}\right)^{n_{zx}} - \alpha_x}{\beta_x - x}$$

It appears that the black solid lines of Figure 1B are qualitatively reproducible

d) I'm unsure about n_{zx} and n_{xz} and n_{yz}: they're not specified in Fig S.1.

c) $S = 0.1$

$$X_{ss} = \frac{0.008 + 0.001}{2} = 0.0045$$

$$Y_{ss} = \frac{0.95 + 0.20}{2} = 0.575$$

$$Z_{ss} = \frac{0.12 + 0.01}{2} = 0.065$$