## Project Summary: Multigraded Homological Algebra and Geometry

**Overview.** This proposal develops tools in multigraded commutative algebra to generalize and extend several classical results in algebraic geometry. The first project expands our understanding of ways that syzygies encode geometry by generalizing a classical conjecture of Green's concerning canonical curves. The second project is devoted to developing new tools in multigraded commutative algebra to gain a better understanding of the geometry of multigraded Hilbert schemes.

**Intellectual Merit.** The PI's first project expands our understanding of connections between minimal graded free resolutions and the geometry of algebraic varieties. More specifically, this project generalizes Green's conjecture concerning the syzygies of canonical curves to the canonical rings of stacky curves. The PI proposes developing algebraic and geometric tools to study properties of the canonical rings of stacky curves. Further, in order to develop such a conjecture this project will carry out a computational exploration of the minimal graded free resolution of simple stacky curves. As an approach to proving such a conjecture the project also extends classical results characterizing subvarieties of minimal degree in projective space to other toric varieties. This project includes a number proposed applications including to a Bertini-like theorem for weighted projective space and minimal graded free resolutions of subvarieties of toric varieties.

A second project proposed by the PI deepens our understanding of the geometry of multigraded Hilbert schemes. Despite their increasing usefulness throughout algebraic geometry and commutative algebra, the geometry of multigraded Hilbert schemes remains quite mysterious. In particular, unlike their classical counterparts, very little is known about when multigraded Hilbert schemes are non-empty, connected, or smooth. This project seeks to characterize when mutligraded Hilbert schemes are non-empty, connected, or smooth by developing new tools in multigraded commutative algebra. In particular, the project looks to generalize a number of classical results and tools in commutative algebra – like Macaualy's theorem, Gotzmann's theorem, and lex ideals – to the multigraded setting. This project contains significant connections to computation and combinatorics.

**Broader Impacts.** As an LGBTQ woman, the PI has worked hard to promote diversity, inclusivity, and justice in the mathematical community. This proposal will further the PI's work in this direction by her continued involvement as a mentor and research collaborator to one undergraduate and two graduate students (a majority of whom identify with generally underrepresented groups) she is working with. The PI also plans to organize an undergraduate research program exploring questions related to the second project in this proposal for 1-3 students during Summer 2024.

The PI has served as board member for *Spectra, the Association for LBGTQ+ Mathematicians* since Fall 2020, and is currently serving as its inaugural president. Since 2020 the PI has organized an annual conference for transgender and non-binary mathematicians. The PI plans to continue organizing this conference, with the first in-person iteration occurring in 2024. Further, the PI plans to organize an online mentoring program aimed at LGBTQ+ undergraduate math students applying for graduate school.

The PI has organized a number of conferences including the *Western Algebraic Geometry Symposium (WAGS)* (2021), *Gender Equity in the Mathematical Study of Combinatorics* (2021), the *Graduate Workshop in Commutative Algebra for Women and Mathematicians of Minority Genders* (2019), a workshop bringing together algebraic geometers and number theorists *Geometry & Arithmetic of Surfaces* (2019), and a five-day conference dedicated to developing open-source computer software for algebraic geometry and commutative algebra *M2@UW* (2018). The PI plans to organize a follow-up to *Graduate Workshop in Commutative Algebra for Women and Mathematicians of Minority Genders* tentatively planned for Summer 2023, as well as a conference highlighting recent advances in multigraded algebra and geometry.

future tense

maybe ok, maybe awkward?

is "when" correct? should this be like, "what happens when" or "how to process" or some formal version of that phrase?

Should you embellish on these connections?

proper grammar rules, it should be: "with whom she is working." (but this is considered formal and less common now, I suppose)

recommendation: put "inaugural president" earlier in this sentence, and "board member" maybe later in the sentence. Reason being: you don't want someone who is skimming this paragraph to miss the bigger achievement. If that makes sense in this context.

comma here? Not sure though, it's ambiguous which title the description belongs to. Next one too — unless this is a common format that I'm just not familiar with.