*sitename*www.ams.org**2023 – 10 – 23

AMS Standard Cover Sheet

Last Name: *Bruce* Middle Name: ______

First Name: *Juliette*

Complete mailing address:

990 Duncan St. #G207

San Francisco, CA 94131

Home Phone ______

e-mail address

juliette_bruce1@brown.edu

Current Institutional Affiliation:

Postdoctoral Research Associate, 08/2022 - Present

Brown University

Department of Mathematics

Skype Name ______

Work Phone ______

Cell Phone ______

Highest Degree held or expected *PhD in Mathematics*

Granting Institution *University of Wisconsin - Madison, Department of Mathematics* Date *08/2020*

Ph.D. Advisor: *Daniel Erman*

Thesis Title *Asymptotic Syzygies in Algebraic Geometry*

Primary Interest (MSC# only) *13* Secondary Interests *14*

Give a very brief synopsis of your current research interests in the box below (e.g. finite group actions on four-manifolds).

My research interests lie in commutative algebra, algebraic geometry, and arithmetic geometry. In particular, I am interested in using homological methods to study the geometry of zero loci of systems of polynomials (i.e. algebraic varieties). I am also interested in studying the arithmetic properties of varieties over finite fields. Further, I am passionate about promoting inclusivity, diversity, and justice in the mathematics community.
https://juliettebruce.github.io

Most recent position held, if any, post Ph.D.

University or Company *Brown University*

Position Title *Postdoctoral Research Associate* Dates *08/2022 - Present*

Eligible for positions which requires U.S. citizenship or U.S. permanent residency: ☒ Yes ☐ No

If unsuccessful for this position, would you like to be considered for a temporary position?

☐ Yes ☒ No If yes, please check the appropriate boxes.

☐ Postdoctoral Position ☐ 2+ Year Position ☐ 1 Year Position

List the names and affiliations of individuals who will provide letters of recommendation if asked.

1. *Daniel Erman, University of Wisconsin, Madison, erman@wisc.edu*
2. *Christine Berkesch, University of Minnesota, cberkesc@umn.edu*
3. *Melody Chan, Brown University, melody_chan@brown.edu*
4. *Shirin Malekpour, University of Wisconsin, Madison, shirin.malekpour@wisc.edu (teaching)*
5. *Gregory Smith, Queen's University, gregory.george.smith@gmail.com*
6. *David Eisenbud, University of California, Berkeley, de@berkeley.edu*

BROWN

Juliette Bruce
Postdoctoral Researcher
Department of Mathematics
juliette_bruce1@brown.edu

October 30, 2023

Dear Committee Members,

I am writing to apply for a tenure-track assistant professor position in the Department of Mathematics at the University of Southern California. Currently, I am a postdoctoral researcher in the Mathematics Department at Brown University, a position I have held since August 2022. I received my Ph.D. in Mathematics from the University of Wisconsin-Madison under the guidance of Professor Daniel Erman in 2020. From 2020-2022 I was an NSF Postdoctoral Fellow in the Mathematics Department at the University of California, Berkeley. Additionally, I was a postdoctoral fellow at the Mathematical Sciences Research Institute for the 2020-2021 academic year.

My research interests lie in the intersection of algebraic geometry and commutative algebra with connections to combinatorics and number theory. I am interested in using homological, combinatorial, and computational methods to study the geometry of algebraic varieties. Currently, my research program has two broad directions.

(i) I have sought to deepen and expand our understanding of the ways homological algebra can be used to study the geometry of toric varieties. This seeks to generalize a very classical story using homological algebra to understand subvarieties of projective space.

(ii) I have been studying the geometry and topology of various moduli spaces, e.g., the moduli space of (principally polarized) abelian varieties of a fixed dimension, via combinatorially and homological methods. This has led to novel applications to the cohomology of certain arithmetic groups.

Further, I am passionate about promoting inclusivity, diversity, and justice in the mathematics community. This passion extends throughout my teaching where I am dedicated to creating an interactive and supportive classroom environment that helps students thrive.

My research output includes 15 papers, with publications in journals such as *Algebra & Number Theory*, *Geometry & Topology*, and *Experimental Mathematics*, as well as, multiple published software packages. Below are a few of the non-research highlights of my file.

- I was awarded a *NSF Postdoctoral Research Fellowship*, a *NSF Graduate Research Fellowship*, and I have secured over $100,000 in conference grants including 4 NSF conference grants.
- I have organized 12+ conferences, workshops, and special sessions, including multiple events aimed at supporting and promoting mathematicians from generally underrepresented groups, especially women and LGBTQ+ mathematicians.
- I was awarded the highest departmental and campus-wide teaching awards at the University of Wisconsin - Madison, the Capstone Teaching Award (2019) and the Teaching Assistant Award for Exceptional Service (2018), awarded to 1 and 3 students each year respectively.

With my application, I include the standard AMS cover sheet, a curriculum vitae, a research statement, a teaching statement, and a diversity statement. I will have six letters of recommendation, five research letters: Christine Berkesch (cberkesc@umn.edu), Melody Chan (melody_chan@brown.edu), David Eisenbud (de@berkeley.edu), Daniel Erman (erman@hawaii.edu), and Gregory G. Smith (ggsmith@mast.queensu.ca), and one teaching letter from Shirin Malekpour (shirin.malekpour@wisc.edu).

Please do not hesitate to contact me with any questions, or if there is anything else I can provide, and thank you in advance for your consideration.

Sincerely,

Juliette E. Bruce

Juliette Bruce
Postdoctoral Research Associate

Department of Mathematics
Brown University 151 Thayer Street Providence, RI 02912
Phone: (401) 863-2708 Fax: (401) 863-9013 Web: https://www.brown.edu/math/

Juliette Bruce's Research Statement

My research interests lie in algebraic geometry, commutative algebra, and arithmetic geometry. In particular, I am interested in using homological and combinatorial methods to study the geometry of zero loci of systems of polynomials (i.e. algebraic varieties). I am also interested in studying the arithmetic properties of varieties over finite fields. Further, I am passionate about promoting inclusivity, diversity, and justice in the mathematics community. Broadly speaking my current research follows these ideas in two directions.

- **Homological Algebra on Toric Varieties:** A classical story in algebraic geometry is that homological methods and tools like minimal free resolutions and Castelnuovo–Mumford regularity capture the geometry of subvarieties of projective space in nuanced ways. My work has sought to generalize this story by developing ways homological algebra can be used to study the geometry of toric varieties (i.e., "nice' compactifications of the torus $(\mathbb{C}^{\times})^n$).
- **Cohomology of Moduli Spaces and Arithmetic Groups:** Despite its importance in algebraic geometry and number theory much remains unknown about the topology of $\mathcal{A}_g$, the moduli space of abelian varieties of dimension g. I have been working to study a canonical "part" of the cohomology of $\mathcal{A}_g$, called the top-weight cohomology. This turns out to be closely connected to the study of cohomology of various arithmetic groups like $\mathrm{GL}_g(\mathbb{Z})$ and $\mathrm{Sp}_{2g}(\mathbb{Z})$, as well as the study of automorphic forms.

1. Homological Algebra on Toric Varieties

Given a graded module M over a graded ring R, a helpful tool for understanding the structure of M is its minimal graded free resolution. In essence, a minimal graded free resolution is a way of approximating M by a sequence of free R-modules. More formally, a *graded free resolution* of a module M is an exact sequence

$$\cdots \to F_k \xrightarrow{d_k} F_{k-1} \xrightarrow{d_{k-1}} \cdots \xrightarrow{d_1} F_0 \xrightarrow{\epsilon} M \to 0$$

where each F_i is a graded free R-module, and hence can be written as $\bigoplus_j R(-j)^{\beta_{i,j}}$. The module $R(-j)$ is the ring R with a twisted grading, so that $R(-j)_d$ is equal to R_{d-j} where R_{d-j} is the graded piece of degree $d-j$. The $\beta_{i,j}$'s are the *Betti numbers* of M, and they count the number of i-syzygies of M of degree j. We will use syzygy and Betti number interchangeably throughout.

Given a projective variety X embedded in $\mathbb{P}^r$, we associate to X the ring $S_X = S/I_X$, where $S = \mathbb{C}[x_0, \ldots, x_r]$ and I_X is the ideal of homogenous polynomials vanishing on X. As S_X is naturally a graded S-module we may consider its minimal graded free resolution, which is often closely related to both the extrinsic and intrinsic geometry of X. An example of this phenomenon is Green's Conjecture, which relates the Clifford index of a curve with the vanishing of certain $\beta_{i,j}$ for its canonical embedding [Voi02, Voi05, AFP+19]. See also [Eis05, Conjecture 9.6] and [Sch86, BE91, FP05, Far06, AF11, FK16, FK17].

Much of my work can be viewed as understanding how minimal graded free resolutions capture the geometry when the role of $\mathbb{P}^r$ is replaced by another variety Y. In particular, I have focused on the case when Y is a toric variety, i.e., a compactification of the torus $(\mathbb{C}^{\times})^r$ where the action of the torus extends to the boundary. Examples of toric varieties include projective space, products of projective spaces, and Hirzebruch surfaces. Work of Cox shows there is a correspondence between (toric) subvarieties of a fixed toric variety and quotients of a polynomial ring similar to the story discussed above for $\mathbb{P}^r$ [Cox95]. As such recent years have seen substantial work looking to use homological algebra and to better understand the geometry of toric varieties [ABLS20, BES20, BE22, BE23a, BB21, CEVV09, EES15, GVT15, MS04, MS05].

1.1 Asymptotic Syzygies Broadly speaking, asymptotic syzygies is the study of the graded Betti numbers (i.e. the syzygies) of a projective variety as the positivity of the embedding grows. In many ways, this perspective dates back to classical work on the defining equations of curves of high degree and projective normality [Mum66, Mum70]. However, the modern viewpoint arose from the pioneering work of Green [Gre84a, Gre84b] and later Ein and Lazarsfeld [EL12].

To give a flavor of the results of asymptotic syzygies we will focus on the question: In what degrees do non-zero syzygies occur? Going forward we will let $X \subset \mathbb{P}^{r_d}$ be a smooth projective variety embedded by a very ample line bundle L_d. Following [EY18] we set,

$$\rho_q(X, L_d) \quad := \quad \frac{\#\left\{p \in \mathbb{N} \mid \mid \beta_{p,p+q}(X, L_d) \neq 0\right\}}{r_d},$$

which is the percentage of degrees in which non-zero syzygies appear [Eis05, Theorem 1.1]. The asymptotic perspective asks how $\rho_q(X; L_d)$ behaves along the sequence of line bundles $(L_d)_{d\in\mathbb{N}}$.

With this notation in hand, we may phrase Green's work on the vanishing of syzygies for curves of high degree as computing the asymptotic percentage of non-zero quadratic syzygies.

Theorem 1.1. [Gre84a] *Let $X \subset \mathbb{P}^r$ be a smooth projective curve. If $(L_d)_{d\in\mathbb{N}}$ is a sequence of very ample line bundles on X such that $\deg L_d = d$ then*

$$\lim_{d\to\infty} \rho_2(X; L_d) = 0.$$

Put differently, asymptotically the syzygies of curves are as simple as possible, occurring in the lowest possible degree. This inspired substantial work, with the intuition being that syzygies become simpler as the positivity of the embedding increases [OP01, EL93, LPP11, Par00, PP03, PP04].

In a groundbreaking paper, Ein and Lazarsfeld showed that for higher dimensional varieties this intuition is often misleading. Contrary to the case of curves, they show that for higher dimensional varieties, asymptotically syzygies appear in every possible degree.

Theorem 1.2. [EL12, Theorem C] *Let $X \subset \mathbb{P}^r$ be a smooth projective variety, $\dim X \geq 2$, and fix an index $1 \leq q \leq \dim X$. If $(L_d)_{d\in\mathbb{N}}$ is a sequence of very ample line bundles such that $L_{d+1} - L_d$ is constant and ample then*

$$\lim_{d\to\infty} \rho_q(X; L_d) = 1.$$

My work has focused on the behavior of asymptotic syzygies when the condition that $L_{d+1} - L_d$ is constant and ample is weakened to assuming $L_{d+1} - L_d$ is semi-ample. Recall a line bundle L is *semi-ample* if $|kL|$ is base point free for $k \gg 0$. The prototypical example of a semi-ample line bundle is $\mathcal{O}(1, 0)$ on $\mathbb{P}^n \times \mathbb{P}^m$. My exploration of asymptotic syzygies in the setting of semi-ample growth thus began by proving the following nonvanishing result for $\mathbb{P}^n \times \mathbb{P}^m$ embedded by $\mathcal{O}(d_1, d_2)$.

Theorem 1.3. [Bru19, Corollary B] *Let $X = \mathbb{P}^n \times \mathbb{P}^m$ and fix an index $1 \leq q \leq n+m$. There exist constants $C_{i,j}$ and $D_{i,j}$ such that*

$$\rho_q(X; \mathcal{O}(d_1, d_2)) \geq 1 - \sum_{\substack{i+j=q \\ i\leq n,\, j\leq m}} \left(\frac{C_{i,j}}{d_1^i d_2^j} + \frac{D_{i,j}}{d_1^{n-i} d_2^{m-j}}\right) - O\begin{pmatrix} \text{lower ord.} \\ \text{terms} \end{pmatrix}.$$

Notice if both $d_1 \to \infty$ and $d_2 \to \infty$ then $\rho_q(\mathbb{P}^n \times \mathbb{P}^m; \mathcal{O}(d_1, d_2)) \to 1$, recovering the results of Ein and Lazarsfeld for $\mathbb{P}^n \times \mathbb{P}^m$. However, if d_1 is fixed and $d_2 \to \infty$ (i.e. semi-ample growth) my results bound the asymptotic percentage of non-zero syzygies away from zero. This together with work of Lemmens [Lem18] has led me to conjecture that, unlike in previously studied cases, in the semi-ample setting $\rho_q(\mathbb{P}^n \times \mathbb{P}^m; \mathcal{O}(d_1, d_2))$ does not approach 1. Proving this would require a vanishing result for asymptotic syzygies, which is open even in the ample case [EL12, Conjectures 7.1, 7.5].

The proof of Theorem 1.3 is based upon generalizing the monomial methods of Ein, Erman, and Lazarsfeld. Such a generalization is complicated by the difference between the Cox ring and homogenous coordinate ring of $\mathbb{P}^n \times \mathbb{P}^m$. A central theme in this work is to exploit the fact that a key regular sequence I use has a number of non-trivial symmetries.

This work suggests that the theory of asymptotic syzygies in the setting of semi-ample growth is rich and substantially different from the other previously studied cases. Going forward I plan to use this work as a jumping-off point for the following question.

Question 1.4. *Let $X \subset \mathbb{P}^{r_d}$ be a smooth projective variety and fix an index $1 \leq q \leq \dim X$. Let $(L_d)_{d\in\mathbb{N}}$ be a sequence of very ample line bundles such that $L_{d+1} - L_d$ is constant and semi-ample, can one compute $\lim_{d\to\infty} \rho_q(X; L_d)$?*

A natural next case in which to consider Question 1.4 is that of Hirzebruch surfaces. I addressed a different, but related question for a narrow class of Hirzebruch surfaces in [Bru22].

1.2 Syzygies via Highly Distributed Computing It is quite difficult to compute examples of syzygies. For example, until recently the syzygies of the projective plane embedded by the d-uple Veronese embedding were only known for $d \leq 5$. My co-authors and I exploited recent advances in numerical linear algebra and high-throughput high-performance computing to generate a number of new examples of Veronese syzygies. A follow-up project used similar computational approaches to compute the syzygies of $\mathbb{P}^1 \times \mathbb{P}^1$ in over 200 new examples. This data provided support for several existing conjectures and led to a number of new conjectures [BEGY20, BEGY21, BCE+22].

1.3 Multigraded Castelnuovo–Mumford Regularity Introduced by Mumford, the Castelnuovo–Mumford Regularity of a projective variety $X \subset \mathbb{P}^r$ is a measure of the complexity of X given in terms of the vanishing of certain cohomology groups of X. Roughly speaking one should think about Castelnuovo–Mumford regularity as being a numerical measure of geometric complexity. Mumford was interested in such a measure as it plays a key role in constructing Hilbert and Quot schemes. In particular, being d-regular implies that $\mathcal{F}(d)$ is globally generated. However, Eisenbud and Goto showed that regularity is also closely connected to interesting homological properties.

Theorem 1.5. [EG84] *Let $\mathcal{F}$ be a coherent sheaf on $\mathbb{P}^r$ and $M = \bigoplus_{e\in\mathbb{Z}} H^0(\mathbb{P}^r, \mathcal{F}(e))$ the corresponding section ring. The following are equivalent:*

(1) M is d-regular;
(2) $\beta_{p,q}(M) = 0$ for all $p \geq 0$ and $q > d + i$;
(3) $M_{\geq d}$ has a linear resolution.

Maclagan and Smith introduced multigraded Castelnuovo–Mumford regularity, where $\mathbb{P}^r$ can be replaced by any toric variety. Similarly to the definition in the classical setting multigraded Castelnuovo–Mumford regularity is defined in terms of the vanishing of certain cohomology groups, however, the multigraded Castelnuovo–Mumford regularity of a subvariety or module is not a single number, but instead an infinite subset of $\mathbb{Z}^r$.

As an example, let us consider the case of products of projective spaces. Fixing a dimension vector $\mathbf{n} = (n_1, n_2, \dots, n_r) \in \mathbb{N}^r$ we let $\mathbb{P}^{\mathbf{n}} := \mathbb{P}^{n_1} \times \mathbb{P}^{n_2} \times \cdots \times \mathbb{P}^{n_r}$ and $S = \mathbb{K}[x_{i,j} \mid 1 \leq i \leq r, 0 \leq j \leq n_i]$ be the Cox ring of $\mathbb{P}^{\mathbf{n}}$ with the $\mathrm{Pic}(X) \cong \mathbb{Z}^r$-grading given by $\deg x_{i,j} = \mathbf{e}_i \in \mathbb{Z}^r$, where $\mathbf{e}_i$ is the i-th standard basis vector in $\mathbb{Z}^r$. Fixing some notation given $\mathbf{d} \in \mathbb{Z}^r$ and $i \in \mathbb{Z}_{\geq 0}$ we let:

$$L_i(\mathbf{d}) := \bigcup_{\mathbf{v}\in\mathbb{N}, |\mathbf{v}|=i} (\mathbf{d} - \mathbf{v}) + \mathbb{N}^r.$$

Note when $r = 2$ the region $L_i(\mathbf{d})$ looks like a staircase with $(i + 1)$-corners. Roughly speaking we define regularity by requiring the i-th cohomology of certain twists of $\mathcal{F}$ to vanish on L_i.

Definition 1.6. [MS04, Definition 6.1] *A coherent sheaf $\mathcal{F}$ on $\mathbb{P}^{\mathbf{n}}$ is* $\mathbf{d}$*-regular if and only if*

$$H^i\left(\mathbb{P}^{\mathbf{n}}, \mathcal{F}(\mathbf{e})\right) = 0 \qquad \text{for all } \mathbf{e} \in L_i(\mathbf{d}).$$

The multigraded Castelnuovo–Mumford regularity of $\mathcal{F}$ is then the set:

$$\operatorname{reg}(\mathcal{F}) := \left\{\mathbf{d} \in \mathbb{Z}^r \;\middle|\; \mathcal{F} \text{ is } \mathbf{d}\text{-regular}\right\} \subset \mathbb{Z}^r.$$

The obvious approaches to generalize Theorem 1.5 to a product of projective spaces turn out not to work. For example, the multigraded Betti numbers do not determine multigraded Castelnuovo–Mumford regularity [BCHS21, Example 5.1] Despite this we show that part (3) of Theorem 1.5 can be generalized. To do so we introduce the following generalization of linear resolutions.

Definition 1.7. *A complex $F_\bullet$ of $\mathbb{Z}^r$-graded free S-modules is* $\mathbf{d}$*-quasilinear if and only if F_0 is generated in degree $\mathbf{d}$ and each twist of F_i is contained in $L_{i-1}(\mathbf{d}-\mathbf{1})$.*

Theorem 1.8. [BCHS21, Theorem A] *Let M be a (saturated) finitely generated $\mathbb{Z}^r$-graded S-module:*

$$M \text{ is } \mathbf{d}\text{-regular} \iff M_{\geq \mathbf{d}} \text{ has a } \mathbf{d}\text{-quasilinear resolution.}$$

The proof of Theorem 1.8 is based in part on a spectral sequence argument that relates the Betti numbers of $M_{\geq \mathbf{d}}$ to the Fourier–Mukai transform of $\widetilde{M}$ with Beilinson's resolution of the diagonal as the kernel. Recent breakthroughs [HHL23, BE23b] understanding resolutions of the diagonal on arbitrary toric varieties mean that there is hope one may be able to generalize the above argument to arbitrary toric varieties. With this in mind, I am interested in pursuing the following question

Question 1.9. *How can Theorem 1.8 be generalized to arbitrary smooth projective toric varieties? in particular, what is the correct definition of quasilinear resolutions?*

1.3.1 Multigraded Regularity of Powers of Ideals Building on the work of many people [BEL91, Cha97], Cutkosky, Herzog, Trung [CHT99] and independently Kodiyalam [Kod00] showed the Castelnuovo–Mumford regularity for powers of ideals on a projective space $\mathbb{P}^r$ has surprisingly predictable asymptotic behavior. In particular, given an ideal $I \subset \mathbb{K}[x_0, \ldots, x_r]$, there exist constants $d, e \in \mathbb{Z}$ such that $\operatorname{reg}(I^t) = dt + e$ for $t \gg 0$.

Building upon our work discussed above, my collaborators and I generalized this result to arbitrary toric varieties. In particular, Definition 1.6 can be extended to all toric varieties by letting S be Cox ring of the toric variety X, replacing $\mathbb{Z}^r$ with the Picard group of X, and replacing $\mathbb{N}^r$ with the nef cone of X. My collaborators and I show that the multigraded regularity of powers of ideals is bounded and translates in a predictable way. In particular, the regularity of I^t essentially translates within $\operatorname{Nef} X$ in fixed directions at a linear rate.

Theorem 1.10. [BCHS22, Theorem 4.1] *There exists a degree $\mathbf{a} \in \operatorname{Pic} X$, depending only on I, such that for each integer $t > 0$ and each pair of degrees $\mathbf{q}_1, \mathbf{q}_2 \in \operatorname{Pic} X$ satisfying $\mathbf{q}_1 \geq \deg f_i \geq \mathbf{q}_2$ for all generators f_i of I, we have*

$$t\mathbf{q}_1 + \mathbf{a} + \operatorname{reg} S \subseteq \operatorname{reg}(I^t) \subseteq t\mathbf{q}_2 + \operatorname{Nef} X.$$

A key aspect of the proof of this theorem is showing that the multigraded regularity of an ideal is finitely generated, in the sense that there exist vectors $\mathbf{v}, \mathbf{w} \in \mathbb{Z}^r$ such that $\mathbf{v} + \operatorname{Nef} X \subset \operatorname{reg}(I) \subset \mathbf{w} + \operatorname{Nef} X$. Perhaps somewhat surprisingly, my co-authors and I showed that this can fail for arbitrary modules [BCHS22]. This naturally raises the question of whether one can characterize when multigraded regularity is finitely generated.

Question 1.11. *Let X be a smooth projective toric variety. Can one characterize when $\operatorname{reg}(M)$ is finitely generated for a module M over the Cox ring of X?*

An first case of this question that I think would make a lovely first research project for a student is to attempt to answer Question 1.11 when M is the Cox ring of a torus fixed-point. In this special case, the question reduces to a delicate combinatorial question about vector partition functions.

2. Cohomology of Moduli Spaces and Arithmetic Groups

Some of the most classical objects in algebraic geometry are moduli spaces, i.e., spaces that parameterize a given collection of geometric objects. The quintessential example of a moduli space is $\mathcal{M}_g$, the moduli space of (smooth) genus g curves, also known as the moduli space of compact Riemann surfaces of genus g. Despite their classical nature, much remains unknown about the geometry of many moduli spaces. For example, the rational cohomology of $\mathcal{M}_g$ is only known for $g \leq 4$. However, classical results suggest that $\mathcal{M}_g$ should have a lot of cohomology because its Euler characteristic grows super exponentially. Recent groundbreaking work of Chan, Galatius, and Payne has shed the first direct light on this phenomenon by constructing new non-trivial cohomology classes, and showing that the dimension of certain cohomology groups of $\mathcal{M}_g$ grow at least exponentially.

Theorem 2.1. [CGP21, Theorem 1.1] *For $g \geq 2$ the dimension of $H^{4g-6}(\mathcal{M}_g;\mathbb{Q})$ grows at least exponentially. In particular $\dim H^{4g-6}(\mathcal{M}_g;\mathbb{Q}) > \beta^g$ for any real number $\beta < \beta_0$ where $\beta_0 \approx 1.3247\ldots$ is the real solution of $t^3 - t - 1 = 0$.*

Much of my recent work has sought to build up the groundwork laid by Chan, Galatius, and Payne to study the rational cohomology of other moduli spaces. Of particular, interest to me has been the moduli space of abelian varieties and various generalizations. This work has deep connections to the cohomology of various arithmetic groups like $\mathrm{Sp}_{2g}(\mathbb{Z})$ and $\mathrm{GL}_g(\mathbb{Z})$.

2.1 Cohomology of $\mathcal{A}_g$ The moduli space of (principally polarized) abelian varieties of dimension g, is a smooth variety $\mathcal{A}_g$ (truthfully a smooth Delinge–Mumford stack) whose points are in one to one correspondence with isomorphism classes of principally polarized abelian varieties of dimension g. Concretely, we may view it as the quotient $[\mathbb{H}_g/\mathrm{Sp}_{2g}(\mathbb{Z})]$ where H_g is the Siegel upper half-space. Notice this means that $\mathcal{A}_g$ is a rational classifying space for the integral symplectic group $\mathrm{Sp}_{2g}(\mathbb{Z})$.

Similar to the moduli space of curves $\mathcal{A}_g$ has long been studied, but much remains unknown about its geometry. For example, the (singular) cohomology is $\mathcal{A}_g$ is only fully known for $g \leq 3$, with $g = 0, 1$ being relatively easy, $g = 2$ which is a classical result of Igusa, and $g = 3$ due to work of Hain. In fact the cohomology of $\mathcal{A}_g$ is so mysterious until recently work by myself and co-authors it was unknown whether $H^{2i+1}(\mathcal{A}_g;\mathbb{Q}) \neq 0$ for some g and i. This was a question posed by Gruskestkky that my recent work answered.

Building upon the work of Chan, Galatius, and Payne, my co-authors and I developed new methods for understanding a certain canonical quotient of the cohomology of $\mathcal{A}_g$. In particular, our results construct non-trivial cohomology classes in $H^k(\mathcal{A}_g;\mathbb{Q})$ in a number of new cases.

Theorem 2.2. [BBC+22, Theorem A] *The rational cohomology $H^k(\mathcal{A}_g;\mathbb{Q}) \neq 0$ for:*

$$(g,k) = (5,15), (5,20), (6,30), (7,28), (7,33), (7,37), \textit{ and } (7,42).$$

For broader context, since $\mathcal{A}_g$ is a rational classifying space for $\mathrm{Sp}_{2g}(\mathbb{Z})$ there is natural isomorphism $H^*(\mathcal{A}_g;\mathbb{Q}) \cong H^*(\mathrm{Sp}_{2g}(\mathbb{Z});\mathbb{Q})$. In particular, the above results provide new non-vanishing results for $H^*(\mathrm{Sp}_{2g}(\mathbb{Z});\mathbb{Q})$. However, my work takes advantage of the fact that since $\mathcal{A}_g$ is a smooth and separated Deligne Mumford stack with a coarse moduli space which is an algebraic variety, permitting Deligne's mixed Hodge theory to be applied to study the rational cohomology of these groups. In particular, the rational cohomology of a complex algebraic variety X of dimension d admits a weight filtration with graded pieces $\mathrm{Gr}^W_j H^k(X;\mathbb{Q})$. As $\mathrm{Gr}^W_j H^k(X;\mathbb{Q})$ vanishes whenever $j > 2d$, $\mathrm{Gr}^W_{2d} H^k(X;\mathbb{Q})$ is referred to as the *top-weight* part of $H^k(X;\mathbb{Q})$.) In this way we deduce Theorem 2.2 above as a corollary to computing the top-weight cohomology of $\mathcal{A}_g$ for all $g \leq 7$.

2.2 Cohomology of $\mathcal{A}_g(m)$ The moduli space $\mathcal{A}_g$ actually a special instance of the moduli space of (principally polarized) abelian varieties of dimension g with level m-structure. Denoted by $\mathcal{A}_g(m)$, we may view it as the quotient $[\mathbb{H}_g/\mathrm{Sp}_{2g}(\mathbb{Z})](m)$ where $\mathrm{Sp}_{2g}(\mathbb{Z})(m)$ is the principal congruence subgroup $\ker(\mathrm{Sp}_{2g}(\mathbb{Z}) \to \mathrm{Sp}_{2g}(\mathbb{Z}/m\mathbb{Z})$. Note that when $m = 1$, we have that $\mathcal{A}_g(m)$ is isomorphic to $\mathcal{A}_g$. From this perspective, one may hope to generalize Theorem 2.2 and underlying methods my co-authors and I developed in [BBC+22] to studying the rational cohomology of $\mathcal{A}_g(m)$ and $\mathrm{Sp}_{2g}(\mathbb{Z})(m)$. In ongoing work, Melody Chan and I are developing such generalizations.

Goal Theorem 2.3. *Let $d = \binom{g+1}{2}$ be the dimension of $\mathcal{A}_g(m)$. For any integers $m \geq 1$ and $g \geq 0$ there exists a cellular complex $LA_g(m)^{\mathrm{trop}}$ such that for all $i \geq 0$ there is a natural isomorphism*

$$\tilde{H}_{i-1}\left(LA_g(m)^{\mathrm{trop}}; \mathbb{Q}\right) \cong \mathrm{Gr}^W_{2d} H^i\left(\mathcal{A}_g(m); \mathbb{Q}\right),$$

The methods behind Goal Theorem 2.3 show new connections between the cohomology of $\mathcal{A}_g(m)$ and the cohomology of $\mathrm{GL}_g(\mathbb{Z})(m)$. The cohomology of $\mathrm{Sp}(2g, \mathbb{Z})(m)$ – and hence $\mathcal{A}_g(m)$ – and $\mathrm{GL}_g(\mathbb{Z})(m)$ are closely connected to automorphic forms. Thus it is natural to wonder whether our methods for computing the top-weight cohomology of $\mathcal{A}_g(m)$ shed new light on automorphic forms. In particular, since the top-weight cohomology of $\mathcal{A}_g(m)$ comes from understanding the boundary of a locally symmetric space, one may hope it is related to Siegel–Eisenstein series. In an ongoing conversation with, Melody Chan, and Peter Sarnak we hope to address this question.

Question 2.4. *What is the relationship between the top-weight cohomology of $\mathcal{A}_g(m)$ and Siegel Eisenstein series?*

2.3 Matroid Complexes and Cohomology of $\mathcal{A}_g^{\mathbf{mat}}$ A key step in the proof of Theorem 2.2 is constructing a chain complex $P_\bullet^{(g)}$ whose homology is precisely the top-weight cohomology of $\mathcal{A}_g$. A major hurdle to pushing our results on the cohomology of $\mathcal{A}_g$, further, is that this chain complex very quickly becomes extremely large and complicated. However, with my co-authors, I identified a subcomplex $R_\bullet^{(g)} \subset P_\bullet^{(g)}$, called the regular matroid complex, which has rich combinatorics. In particular, $R_k^{(g)}$ is spanned by isomorphism classes of regular matroids on k elements of rank $\leq g$. I am working to study this complex from a number of perspectives. As an example, the following goal theorem is a result that I am working on with three graduate students.

Goal Theorem 2.5. *Compute the homology of the matroid complex $R_\bullet^{(g)}$ for all $g \geq 14$.*

Currently by combining theoretical results and large-scale computations to compute the cohomology for all $g \leq 9$. Computing the homology of the regular matroid complex is interesting, not only because it provides a new approach for studying the combinatorics of matroids, but also because it is closely related to the cohomology of partial compactification of $\mathcal{A}_g$ called the matroidal (partial) compactification $\mathcal{A}_g^{\mathrm{mat}}$. In ongoing work with Madeline Brandy and Daniel Corey, I am looking to show that one can compute the top-weight cohomology of $\mathcal{A}_g^{\mathrm{mat}}$ from the regular matroid complex.

Goal Theorem 2.6. *Compute the top-weigh cohomology of $\mathcal{A}_g^{\mathrm{mat}}$ for all $g \leq 10$.*

Work of Willwacher [Wil15] and Kpntsevich [Kon93, Kon94] on graph complexes suggests that one may hope for $R_\bullet^{(g)}$ to have rich algebraic structure beyond just that of chain complex.

Question 2.7. *Does the complex $R_\bullet^{(g)}$ carry a natural Lie bracket, endowing it with the structure of a differentially graded Lie algebra?*

Constructing such a Lie bracket likely relies on developing a new understanding of the ways one can combine two matroids. Ongoing work with the graduate students mentioned above is studying this problem in the special cases of graphic and co-graphic matroids. The existence of similar Lie structure was crucial to achieving the exponential bounds in Theorem 2.1.

References

[ABLS20] Ayah Almousa, Juliette Bruce, Michael Loper, and Mahrud Sayrafi, *The virtual resolutions package for Macaulay2*, J. Softw. Algebra Geom. **10** (2020), no. 1, 51–60.

[AFP+19] Marian Aprodu, Gavrill Farkas, Ştefan Papadima, Claudiu Raicu, and Jerzy Weyman, *Koszul modules and Green's conjecture*, Inventiones mathematicae (2019).

[AF11] Marian Aprodu and Gavril Farkas, *Green's conjecture for curves on arbitrary $K3$ surfaces*, Compos. Math. **147** (2011), no. 3, 839–851.

[BE91] Dave Bayer and David Eisenbud, *Graph curves*, Adv. Math. **86** (1991), no. 1, 1–40. With an appendix by Sung Won Park.

[BEL91] Aaron Bertram, Lawrence Ein, and Robert Lazarsfeld, *Vanishing theorems, a theorem of Severi, and the equations defining projective varieties*, J. Amer. Math. Soc. **4** (1991), no. 3, 587–602.

[BES20] Christine Berkesch, Daniel Erman, and Gregory G. Smith, *Virtual resolutions for a product of projective spaces*, Algebr. Geom. **7** (2020), no. 4, 460–481, DOI 10.14231/ag-2020-013. MR4156411

[BBC+22] Madeline Brandt, Juliette Bruce, Melody Chan, Margarida Melo, Gwyneth Moreland, and Corey Wolfe, *On the top-weight rational cohomology of $\mathcal{A}_g$*, Geometry & Topology (2022). to appear.

[BE22] Michael K. Brown and Daniel Erman, *Tate resolutions on toric varietie* (2022). Pre-print: arxiv:2108.03345.

[BE23a] ———, *Linear syzygies of curves in weighted projective space* (2023). Pre-print: arxiv:2301.0915.

[BE23b] ———, *A short proof of the Hanlon-Hicks-Lazarev Theorem* (2023). Pre-print: arxiv:2303.14319.

[BEGY20] Juliette Bruce, Daniel Erman, Steve Goldstein, and Jay Yang, *Conjectures and computations about Veronese syzygies*, Exp. Math. **29** (2020), no. 4, 398–413.

[BEGY21] ———, *The Schur-Veronese package in Macaulay2*, J. Softw. Algebra Geom. **11** (2021), no. 1, 83–87.

[Bru19] Juliette Bruce, *Asymptotic syzygies in the setting of semi-ample growth* (2019). Pre-print: arxiv:1904.04944.

[Bru22] ———, *The quantitative behavior of asymptotic syzygies for Hirzebruch surfaces*, J. Commut. Algebra **14** (2022), no. 1, 19–26.

[BCE+22] Juliette Bruce, Daniel Corey, Daniel Erman, Steve Goldstein, Robert P. Laudone, and Jay Yang, *Syzygies of $\mathbb{P}^1 \times \mathbb{P}^1$: data and conjectures*, J. Algebra **593** (2022), 589–621.

[BCHS21] Juliette Bruce, Lauren Cranton Heller, and Mahrud Sayrafi, *Characterizing Multigraded Regularity on Products of Projective Spaces* (2021). Pre-print: arxiv:2110.10705.

[BCHS22] ———, *Bounds on Multigraded Regularity* (2022). Pre-print: arxiv:2208.11115.

[BB21] Weronika Buczyńska and Jarosław Buczyński, *Apolarity, border rank, and multigraded Hilbert scheme*, Duke Math. J. **170** (2021), no. 16, 3659–3702.

[CEVV09] Dustin A. Cartwright, Daniel Erman, Mauricio Velasco, and Bianca Viray, *Hilbert schemes of 8 points*, Algebra Number Theory **3** (2009), no. 7, 763–795.

[CGP21] Melody Chan, Søren Galatius, and Sam Payne, *Tropical curves, graph complexes, and top weight cohomology of $\mathcal{M}_g$*, J. Amer. Math. Soc. **34** (2021), no. 2, 565–594.

[Cha97] Karen A. Chandler, *Regularity of the powers of an ideal*, Comm. Algebra **25** (1997), no. 12, 3773–3776.

[Cox95] David A. Cox, *The homogeneous coordinate ring of a toric variety*, J. Algebraic Geom. **4** (1995), no. 1, 17–50. MR1299003

[CHT99] S. Dale Cutkosky, Jürgen Herzog, and Ngô Viêt Trung, *Asymptotic behaviour of the Castelnuovo-Mumford regularity*, Compositio Mathematica **118** (1999), no. 3, 243–261.

[EL93] Lawrence Ein and Robert Lazarsfeld, *Syzygies and Koszul cohomology of smooth projective varieties of arbitrary dimension*, Invent. Math. **111** (1993), no. 1, 51–67.

[EL12] ———, *Asymptotic syzygies of algebraic varieties*, Invent. Math. **190** (2012), no. 3, 603–646.

[EES15] David Eisenbud, Daniel Erman, and Frank-Olaf Schreyer, *Tate resolutions for products of projective spaces*, Acta Math. Vietnam. **40** (2015), no. 1, 5–36, DOI 10.1007/s40306-015-0126-z. MR3331930

[EG84] David Eisenbud and Shiro Goto, *Linear free resolutions and minimal multiplicity*, J. Algebra **88** (1984), no. 1, 89–133.

[Eis05] David Eisenbud, *The geometry of syzygies*, Graduate Texts in Mathematics, vol. 229, Springer-Verlag, New York, 2005. A second course in commutative algebra and algebraic geometry.

[EY18] Daniel Erman and Jay Yang, *Random flag complexes and asymptotic syzygies*, Algebra Number Theory **12** (2018), no. 9, 2151–2166.

[GVT15] Elena Guardo and Adam Van Tuyl, *Arithmetically Cohen-Macaulay sets of points in* $\mathbb{P}^1 \times \mathbb{P}^1$, SpringerBriefs in Mathematics, Springer, Cham, 2015. MR3443335

[FP05] Gavril Farkas and Mihnea Popa, *Effective divisors on $\overline{\mathcal{M}}_g$, curves on $K3$ surfaces, and the slope conjecture*, J. Algebraic Geom. **14** (2005), no. 2, 241–267.

[Far06] Gavril Farkas, *Syzygies of curves and the effective cone of $\overline{\mathcal{M}}_g$*, Duke Math. J. **135** (2006), no. 1, 53–98.

[FK16] Gavril Farkas and Michael Kemeny, *The generic Green-Lazarsfeld secant conjecture*, Invent. Math. **203** (2016), no. 1, 265–301.

[FK17] ———, *The Prym-Green conjecture for torsion line bundles of high order*, Duke Math. J. **166** (2017), no. 6, 1103–1124.

[GLL15] Ofer Gabber, Qing Liu, and Dino Lorenzini, *Hypersurfaces in projective schemes and a moving lemma*, Duke Math. J. **164** (2015), no. 7, 1187–1270.

[Gre84a] Mark L. Green, *Koszul cohomology and the geometry of projective varieties*, J. Differential Geom. **19** (1984), no. 1, 125–171.

[Gre84b] ———, *Koszul cohomology and the geometry of projective varieties. II*, J. Differential Geom. **20** (1984), no. 1, 279–289.

[HHL23] Andrew Hanlon, Jeff Hicks, and Oleg Lazarev, *Resolutions of toric subvarieties by line bundles and applications* (2023). Pre-print: arxiv:2303.03763.

[Kod00] Vijay Kodiyalam, *Asymptotic behaviour of Castelnuovo-Mumford regularity*, Proc. Amer. Math. Soc. **128** (2000), no. 2, 407–411.

[Kon93] Maxim Kontsevich, *Formal (non)commutative symplectic geometry*, The Gelfand Mathematical Seminars, 1990–1992, Birkhäuser Boston, Boston, MA, 1993, pp. 173–187.

[Kon94] ———, *Feynman diagrams and low-dimensional topology*, First European Congress of Mathematics, Vol. II (Paris, 1992), Progr. Math., vol. 120, Birkhäuser, Basel, 1994, pp. 97–121.

[LPP11] Robert Lazarsfeld, Giuseppe Pareschi, and Mihnea Popa, *Local positivity, multiplier ideals, and syzygies of abelian varieties*, Algebra Number Theory **5** (2011), no. 2, 185–196.

[Lem18] Alexander Lemmens, *On the n-th row of the graded Betti table of an n-dimensional toric variety*, J. Algebraic Combin. **47** (2018), no. 4, 561–584.

[M2] Daniel R. Grayson and Michael E. Stillman, *Macaulay 2, a software system for research in algebraic geometry*. Available at `http://www.math.uiuc.edu/Macaulay2/`.

[MS05] Diane Maclagan and Gregory G. Smith, *Uniform bounds on multigraded regularity*, J. Algebraic Geom. **14** (2005), no. 1, 137–164.

[MS04] ———, *Multigraded Castelnuovo-Mumford regularity*, J. Reine Angew. Math. **571** (2004), 179–212.

[Mum70] David Mumford, *Varieties defined by quadratic equations*, Questions on Algebraic Varieties (C.I.M.E., III Ciclo, Varenna, 1969), Edizioni Cremonese, Rome, 1970, pp. 29–100.

[Mum66] D. Mumford, *On the equations defining abelian varieties. I*, Invent. Math. **1** (1966), 287–354.

[OP01] Giorgio Ottaviani and Raffaella Paoletti, *Syzygies of Veronese embeddings*, Compositio Math. **125** (2001), no. 1, 31–37.

[Par00] Giuseppe Pareschi, *Syzygies of abelian varieties*, J. Amer. Math. Soc. **13** (2000), no. 3, 651–664.

[PP03] Giuseppe Pareschi and Mihnea Popa, *Regularity on abelian varieties. I*, J. Amer. Math. Soc. **16** (2003), no. 2, 285–302.

[PP04] ———, *Regularity on abelian varieties. II. Basic results on linear series and defining equations*, J. Algebraic Geom. **13** (2004), no. 1, 167–193.

[Sch86] Frank-Olaf Schreyer, *Syzygies of canonical curves and special linear series*, Math. Ann. **275** (1986), no. 1, 105–137.

[Voi02] Claire Voisin, *Green's generic syzygy conjecture for curves of even genus lying on a $K3$ surface*, J. Eur. Math. Soc. (JEMS) **4** (2002), no. 4, 363–404.

[Voi05] ———, *Green's canonical syzygy conjecture for generic curves of odd genus*, Compos. Math. **141** (2005), no. 5, 1163–1190.

[Wil15] Thomas Willwacher, *M. Kontsevich's graph complex and the Grothendieck-Teichmüller Lie algebra*, Invent. Math. **200** (2015), no. 3, 671–760, DOI 10.1007/s00222-014-0528-x. MR3348138

Juliette Bruce's Teaching Statement

I. Introduction. My goal as an educator is to be an active guide for students, providing them with environments where they feel supported and encouraged to let their own mathematical and quantitative curiosities guide how they engage and learn. By taking this approach, I hope to engage with students as the complete people that they are, asking them to bring all of their experiences, backgrounds, identities, and knowledge into the learning environment. I want students to experience mathematics in a humanistic way, seeing how mathematics and quantitative thinking are integral aspects of their lives. As one of my former students noted, "Juliette obviously wants us to succeed not only in math but in life." Recognizing that learning mathematics is not necessarily confined to the classroom I have sought out new and non-traditional teaching opportunities. My teaching has been recognized through both awards and student evaluations:

- In 2018, I was one of three graduate students at the University of Wisconsin-Madison recognized campus-wide with the Teaching Assistant Award for Exceptional Service.
- I received two TA awards from the math department at the University of Wisconsin-Madison, the TA Service Award (2018) and the Capstone Teaching Award (2019), the latter of which is awarded to just one teaching assistant each year, for an exceptional record of teaching excellence and service.
- My student evaluations are very positive; for instance, for one course 100% of students agreed that I was an effective teacher.

I have sought to develop and refine my skills as an educator, both by viewing each teaching assignment as my own opportunity for growth and learning and by actively seeking out learning from other educators and education experts. In particular, I have implemented evidence-based techniques to support and engage students from diverse backgrounds

II. Teaching Experiences. As a graduate student at the University of Wisconsin - Madison, I served as a teaching assistant and course coordinator for Calculus I for multiple semesters, the instructor of record for Math for Early Education Majors, and the instructor of record for a Calculus I course providing students from generally under-represented groups additional support during their first college math course. Additionally, for several semesters, I held a non-traditional teaching assistantship for my role as the organizer of the Madison Math Circle outreach program. My passion for promoting an interest in and excitement for math – especially for people from generally underrepresented groups – led me to take on teaching and outreach roles through the *Girls Math Night Out* program and the *Wisconsin Directed Reading Program*.

My postdoctoral positions at Brown University and the University of California, Berkeley did not allow me to have formal teaching responsibilities, however, I have actively sought out non-traditional teaching opportunities and mentoring opportunities. For example, in 2020, in response to the COVID-19 pandemic, I helped Ravi Vakil and others organize *Algebraic Geometry in the Time of COVID*, a massive open-access virtual algebraic geometry course, which drew over 1500 participants from around the world. Inspired by this experience, in 2021, I organized an online open access course, *Virtual Directed Reading in Geometry & Algebra*, aimed at undergraduates. During this time I continued to seek to grow as an educator. For example, while at the University of California, Berkeley I actively participated in a reading/working group exploring antiracist and anti-oppressive pedagogy in the mathematics classroom. Further, I personally sought to engage with ways to humanize mathematics and support underrepresented students by exploring the works of Pamela E. Harris, Aris Winger, Rochelle Gutiérrez, Luis Leyva, and Francis Su.

III. Teaching Philosophy and Strategies for Classroom Success. As an instructor, I view my role is to be an active guide. I encourage my students to explore, engage with, and question the course material for themselves. I try to structure much of the course around guided group work that gives students opportunities to develop and discuss their understanding and confusion with their fellow students. In addition to encouraging students to take an active role in learning, this format also helps students to learn to vocalize their thought processes and ideas.

Active learning presents challenges to me and my students, most notably, the challenge of managing student mistakes. In many ways, the most significant moments during the learning process are not necessarily the moments of success, but the moments of failure. It is at this moment that students can recognize errors and gaps in their understanding of a subject and begin trying to correct them. It is also the moment that as an instructor I can understand what my students are finding difficult and nudge the conversation in such a way as to overcome these hurdles.

Making mistakes is hard, and most students, like most people, would prefer not to make mistakes. With this in mind, I think it is crucial to promote an inclusive environment where all students feel comfortable and safe participating. This environment encourages students to be open about what confuses them and where they are making mistakes. Creating an inclusive classroom environment requires active attention and work to maintain. However, in my experience, this work is well worth it.

My approach to creating an inclusive classroom environment has been influenced by the semester-long course *Inclusive Practices in the College Classroom*, which I took through the *Delta Program for Integrating Research, Teaching and Learning.* For example, one activity I implemented successfully asked students to brainstorm attributes from classes they found productive and attributes from classes they found less productive. After collecting a list of such attributes, we use this as a jumping-off point for forming community standards that we wish to shape our classroom environment. Examples of such community standards that my classes have often adopted include: "Respect everyone" and "Address the problem, not the person when discussing mistakes". I have found this helps the students buy into the belief that the classroom is an inclusive space where it is safe to make mistakes.

However, beyond simply creating an inclusive learning environment I have also found it important to create a space where students feel comfortable bringing their whole selves, including all of their experiences, backgrounds, challenges, identities, struggles, and knowledge. For example, I recognize that all students, like all people, will have days when negative experiences outside the classroom affect their ability to engage in the classroom. This is even more true for students who face racism, sexism, homo/transphobia, and other systems of oppression. On such a day when students enter the classroom, I look to try to meet the students where they are. For example, sometimes this means I will walk the student to the campus mental health or cultural center, or sometimes it means I create new problems specifically to help keep the student's mind off of whatever is troubling them. I try to make sure my students know I am there to provide them with whatever resources they need to succeed both in the classroom and in their life beyond. However, this human-centered approach also leads to many beautiful moments. For example, by allowing students to bring all of themselves to class they experience mathematics in a humanistic way, seeing how mathematics and quantitative thinking are an integral aspect of their life. I have found this often increases students' motivation, as well as opens themselves up to making mistakes, growing, and learning.

IV. Sample Student Feedback. The effectiveness of my teaching is highlighted in student comments:

- "I've always struggle with math and I've had a lot of teachers that didn't believe in me so because of this I've always dreaded math courses. But Juliette always showed she cared, was constantly encouraging, believed in our class, and taught the material really clearly. From her constant availability to help and great instructing, her class became one of my favorites and I am more successful in a math course than I've ever been before."
- "She went around and tried helping each student... She cared about each student's success in the class and tried her best to make everyone understand the material."
- "Juliette obviously wants us to succeed not only in math but in life. She is always making sure we know our resources especially when it comes to health. She also always wishes us a good day/weekend and that is awesome."

V. Conclusion. As a graduate student and postdoctoral scholar, I have found teaching to be extremely rewarding. I developed a passion for supporting and engaging students from diverse backgrounds. Going forward, I am excited for new opportunities to grow and learn as a teacher, continue to promote inclusivity, diversity, and justice in my teaching, and create human-centered learning environments for my students.

Juliette Bruce's Diversity Statement

I. Introduction. I believe strongly in the importance of inclusivity, diversity, and justice, and I am passionate about promoting these values within the mathematical community. I have worked hard to create a learning community that was as open and inclusive to as many people as possible. For example, I have worked to make mathematics more inclusive of people from underrepresented groups; by founding events like *Trans Math Day* and leading *Spectra: the Association for LGBTQ+ Mathematicians*. Further, to promote the success of mathematicians from underrepresented groups I organized numerous national and international workshops and conferences. Going forward, I am excited to continue working hard to promote these values through my research, teaching, and service.

II. Organizational Service. I have organized 10+ national/international conferences including *M2@UW* (45 participants), *Geometry and Arithmetic of Surfaces* (40 participants), *Graduate Workshop in Commutative Algebra for Women & Mathematicians of Minority Genders* (35 participants), *CAZoom* (70 participants), *Western Algebraic Geometry Symposium* (100 participants), *GEMS of Combinatorics* (40 participants), Spec($\overline{\mathbb{Q}}$) (50 participants), *BATMOBILE* (30 participants), *GEMS of Combinatorics II* (30 participants), and *GEMS of Commutative Algebra* (40 participants). Additionally, I organized three special sessions at AMS Sectional Meetings and the Joint Math Meetings.

Multiple of these conferences were specifically aimed a supporting mathematicians from underrepresented groups. For example, *Graduate Workshop in Commutative Algebra for Women & Mathematicians of Minority Genders* focused on forming communities of women and non-binary researchers, and Spec($\overline{\mathbb{Q}}$) highlighted the research of LGBTQ+ mathematicians in algebra, geometry, and number theory. Further, the "GEMS" workshops sought to build a diverse community of mathematicians to address gender equity in the mathematical community from new perspectives. Going forward I am interested in expanding these "GEMS" workshops to other areas of mathematics and creating cross-field discussions that broaden the standard notion of gender equity in mathematics.

When organizing these conferences I paid particular attention to making them as inclusive of women and non-binary researchers as possible. For example, I designed the registration form to be thoughtful of the concerns of transgender researchers and highlighted the locations of single occupancy and ADA-compliant restrooms. The importance of such efforts was highlighted by the following comment I received from a participant, "I just wanted to thank you for making this workshop inclusive for people with all gender identifications. ... I have always felt out of place when I participated in conferences/workshops for women when they do not specify that non-binary people are welcome ... I really appreciate those questions you put in the registration form. It means a lot to me."

III. National & International Advocacy. As a postdoc, I looked to deepen the impact of my work by attempting to promote underrepresented groups in mathematics beyond just campus. For example, I have worked with the Executive Committee of the *Association for Women in Mathematics* to consider ways they could expand their support of women and non-binary mathematicians. In Winter 2023 I joined MSRI's *Committee on Women in Mathematics*. Since Fall 2020 I have organized *Trans Math Day*, an annual virtual conference promoting the work of transgender and non-binary mathematicians. This conference regularly has 50 participants. Highlighting the importance of such conferences one participant said, "I've been really considering leaving mathematics. [Trans Math Day] reminded me why I'm here and why I want to stay. If a conference like this had been around for me five years ago, my life would have been a lot better." Further, I have been a board member for *Spectra: The Association for LGBTQ+ Mathematicians* since 2020, including as the inaugural president in 2022. In this role, I have overseen the growth and formalization of the organization, including the creation and adoption of bylaws, the creation of an invited lecture at the Joint Mathematics Meetings, and a $20,000+ fundraising campaign. Spectra has 500 members.

Going forward I am excited to continue my work supporting LGBTQ+ students, and would love to continue building organizations to do so. In particular, given the amazing successes of programs like MSRI-UP and the EDGE Program, I would love to organize a summer REU program specifically aimed at supporting and promoting LGBTQ+ mathematicians. Further, I am in the early stages of planning a mentorship program to help guide LGBTQ+ undergraduates through the process of applying to graduate programs in mathematics and helping young LGBTQ+ graduate students establish themselves. The plan would be to break participants into groups with each group having LGBTQ+ mathematicians at various career stages, thus allowing participants to exchange advice, find support, and build mentoring networks.

IIV A More Inclusive Learning Community. During the Fall of 2016, in response to a growing climate of hate, bias, and discrimination on campus, I led the creation of the Mathematics Department's *Committee on Inclusivity and Diversity.* As a member of this committee, I drafted what would become the department's commitment to inclusivity and non-discrimination. I also created syllabi statements that let students know about these department policies, and that inform them of campus resources. Everyone within the department is encouraged to use these statements.

While a graduate student I co-founded oSTEM@UW as a resource for LGBTQ+ students in STEM, which eventually grew to over fifty members. As one member said, "It made me very happy to see other friendly LGBTQ+ faces around... Thanks so much for organizing this stuff – it's really helpful". From 2017-2020 I led the campus social organization for LGBTQ+ graduate students, which had over 350 members. In this role, I have co-organized a weekly coffee social hour intended to give LGBTQ+ graduate students a place to relax, make friends, and discuss the challenges of being LGBTQ+ at the UW - Madison.

IV. Mentoring. Inspired by the mentoring that helped me navigate the challenges of being a woman in mathematics, I have worked hard to mentor people from underrepresented groups. While a graduate student, I led reading courses with three undergraduates. One of these students, an undergraduate woman, worked with me for over a year, during which time I helped her apply for summer research projects. Working with *Girls' Math Night Out* I lead two girls in high school through a project exploring cryptography. During 2018-2019, I mentored 6 first-year graduate students (all women, non-binary students, or students of color).

As a postdoc, I began research projects with three graduate students (a majority of whom identify with a generally underrepresented group). These projects have resulted in two pre-prints, with additional projects still ongoing. Throughout the Spring and Summer of 2022, I did a reading course with a first-year graduate woman on algebraic geometry. Additionally, I advised two summer research projects for undergraduate students. The first of these projects ran virtually during Summer 2021 when 6 undergraduates worked on a question related to the moduli space of Abelian varieties. In Summer 2022 I advised an undergraduate student on a research project related to my work on syzygies. This work is ongoing and will hopefully result in a paper. This student is now in graduate school for math and was awarded an NSF Graduate Fellowship.

V. Virtual Mathematics In response to the COVID-19 pandemic and the shift of many mathematical activities to virtual formats, I worked to find ways for these online activities to reach those often at the periphery. During the Summer and Fall of 2020, I helped with Ravi Vakil's *Algebraic Geometry in the Time of Covid* project. This massive online open-access course in algebraic geometry brought together $\sim 1,500$ participants from around the world. In Spring 2021, I organized an 8-week virtual reading course for undergraduates in algebraic geometry and commutative algebra.

IV. Expanding the Learning Community. The Madison Math Circle (MMC) is an outreach program sponsored by the UW - Madison Math Department. Its goal is to kindle excitement and appreciation of math in middle and high school students. In Fall 2014, I began volunteering with the MMC. At the time, the circle's main programming was a weekly on-campus lecture given by a member of the math department. After a year of volunteering, I stepped into the role of organizer. During my three years as an organizer, I worked to build stronger connections between the MMC, local schools and other outreach organizations focused on underrepresented groups. These ties helped the weekly attendance more than double, and grow substantially more diverse. I also led the creation of a new outreach arm of the MMC, which visits high schools around the state of Wisconsin to better serve students from underrepresented groups. This program has dramatically expanded the reach of the circle, and during my final year as an organizer, the MMC reached over 300 students.

VII. Conclusion. I have worked hard to develop programs, policies, and practices that promoted diversity, inclusion, and justice within the mathematical community. Going forward, I will continue promoting these values through my research, teaching, and service.