Given a graded module M over a graded ring R, in essence, a minimal graded free resolution is a way of approximating M by a sequence of free R-modules. More formally, a *graded free resolution* of a module M is an exact sequence

$$0 \longleftarrow M \xleftarrow{\epsilon} F_0 \xleftarrow{d_1} \cdots \qquad\qquad \cdots \xleftarrow{d_{k-1}} F_{k-1} \xleftarrow{d_k} F_k \longleftarrow \cdots$$

where each F_p is a graded free R-module, and hence can be written as $\bigoplus_q R(-p)^{\beta_{p,q}}$. The module $R(-q)$ is the ring R with a twisted grading, so that $R(-q)_d$ is equal to R_{d-q} where R_{d-q} is the graded piece of degree $d-q$. The $\beta_{p,q}$'s are the *Betti numbers* of M, and they count the number of p-syzygies of M of degree q. We will use syzygy and Betti number interchangeably throughout.

Given a projective variety X embedded in $\mathbb{P}^n$, we associate to X the ring $S_X = S/I_X$, where $S = \mathbb{C}[x_0, \ldots, x_n]$ and I_X is the ideal of homogeneous polynomials vanishing on X. As S_X is a graded S-module we may consider its minimal graded free resolution, which is often closely related to both the extrinsic and intrinsic geometry of X. Much of my work has focused on studying the asymptotic properties of syzygies of projective varieties. Broadly, asymptotic syzygies is the study of the graded Betti numbers of a projective variety as the positivity of the embedding grows.

To give a flavor of the results of asymptotic syzygies we will focus on the question: in what degrees do non-zero syzygies occur? Going forward we will let $X \subset \mathbb{P}^{n_d}$ be a smooth projective variety embedded by a very ample line bundle L_d, and we set,

$$\rho_q(X, L_d) \;\; := \;\; \tfrac{1}{n_d} \cdot \#\left\{p \in \mathbb{N} | \;\middle|\; \beta_{p,p+q}(X, L_d) \neq 0\right\}$$

which is the percentage of degrees in which non-zero syzygies appear. The asymptotic perspective asks how $\rho_q(X; L_d)$ behaves along the sequence of line bundles $(L_d)_{d\in\mathbb{N}}$. With this notation in hand, we may phrase Green's work on the vanishing of syzygies for curves of high degree as computing the asymptotic percentage of non-zero quadratic syzygies.

Theorem A. *Let X be a smooth projective curve. If $(L_d)_{d\in\mathbb{N}}$ is a sequence of very ample line bundles on X such that* $\deg L_d = d$ *then $\rho_2(X; L_d) \to 0$ as $d \to \infty$.*

Put differently, asymptotically the syzygies of curves are as simple as possible, occurring in the lowest possible degree. This inspired substantial work, with the intuition being that syzygies become simpler as the positivity of the embedding increases. Ein and Lazarsfeld showed that for higher dimensional varieties this intuition is often misleading. Contrary to the case of curves, they show that for higher dimensional varieties, asymptotically syzygies appear in every possible degree.

Theorem B. *Let X be a smooth projective variety,* $\dim X \geq 2$. *Fix $1 \leq q \leq \dim X$. If (L_d) is a sequence of very ample line bundles such that $L_{d+1} - L_d$ is constant and ample then $\rho_q(X; L_d) \to 1$ as $d \to \infty$.*

My work has focused on the behavior of asymptotic syzygies when the condition that $L_{d+1} - L_d$ is constant and ample is weakened to assuming $L_{d+1} - L_d$ is semi-ample. A line bundle L is *semi-ample* if $|kL|$ is base point free for $k \gg 0$. The prototypical example of a semi-ample line bundle is $\mathcal{O}(1,0)$ on $\mathbb{P}^n \times \mathbb{P}^m$. My exploration of asymptotic syzygies in the setting of semi-ample growth thus began by proving the following nonvanishing result for $\mathbb{P}^n \times \mathbb{P}^m$ embedded by $\mathcal{O}(d_1, d_2)$.

Theorem C. *Let $X = \mathbb{P}^n \times \mathbb{P}^m$ and fix $1 \leq q \leq n+m$. There exist constants $C_{i,j}$, $D_{i,j}$ such that*

$$\rho_q\left(X; \mathcal{O}\left(d_1, d_2\right)\right) \geq 1 - \sum_{\substack{i+j=q \\ i \leq n,\, j \leq m}} \left(\frac{C_{i,j}}{d_1^i d_2^j} + \frac{D_{i,j}}{d_1^{n-i} d_2^{m-j}}\right) - O\begin{pmatrix} \textit{lower ord.} \\ \textit{terms} \end{pmatrix}.$$

Notice if both $d_1 \to \infty$ and $d_2 \to \infty$ then $\rho_q\left(\mathbb{P}^n \times \mathbb{P}^m; \mathcal{O}(d_1, d_2)\right) \to 1$, recovering the results of Ein and Lazarsfeld for $\mathbb{P}^n \times \mathbb{P}^m$. However, if d_1 is fixed and $d_2 \to \infty$ (i.e. semi-ample growth) my results bound the asymptotic percentage of non-zero syzygies away from zero. This together with work of Lemmens has led me to conjecture that, unlike in previously studied cases, in the semi-ample setting $\rho_q\left(\mathbb{P}^n \times \mathbb{P}^m; \mathcal{O}(d_1, d_2)\right)$ does not approach 1. Proving this would require a vanishing result for asymptotic syzygies, which is open even in the ample case

Relevance of Visit. Being a postdoctoral fellow at MSRI would be extremely beneficially towards my career goals as it would provide me with a stimulating mathematical environment, and the opportunity to engage and work with a number of leading researchers in commutative algebra. My research interest align extremely well with topics the of semester long program. For example, one ongoing project I am working on concerns better understanding the homological implications of certain generalizations of Castelnuovo–Mumford regularity to toric varieties, and in a separate project I am beginning to explore ways Green's conjecture may be generalized to describe the syzygies of canonical stacky curves. Both of these projects touch upon many of the topics included in the program description. As such begin a postdoc at MSRI would likely prove valuable toward progressing both of these projects, as well as a fantastic opprotunity to being new collaborations with other participants. Moreover, given the the ways my research interest align with the program I feel I would be able to find numerous ways to myself contribute to the program.

Overall, being a postdoctoral fellow for this semester program in commutative algebra presents an amazing opportunity to connect and build relationships within the research area that has long felt like my research community/home. And such connections would significantly advance my goals of being a math professor at a research university.

Building Mathematical Community. As an LGBTQ+ woman, I have worked hard to promote diversity, inclusivity, and justice in the mathematical community by mentoring students, supporting women and LGBTQ+ people in mathematics, and organizing conferences and outreach programs.

As a postdoc, I have put significant effort into mentoring, advising, and working with students, especially those from underrepresented groups. During Summer 2021 in order to help fill gaps caused by the COVID-19 pandemic I organized a virtual summer undergraduate research program for 6 undergraduates from around the world. In Summer 2022 I advised an undergraduate student on a research project related to my work on asymptotic syzygies. This student is now applying to graduate schools in math. I began research projects with multiple graduate students, in which I played a substantial mentoring and guiding role. In the Spring and Summer of 2022, I did a reading course with a first-year graduate woman who is now working in commutative algebra.

Since Fall 2020 I have organized an annual virtual conference promoting the work of transgender and non-binary mathematicians. Highlighting the importance of such conferences one participant said, "I've been really considering leaving mathematics. [Trans Math Day 2020] reminded me why I'm here and why I want to stay. ... If a conference like this had been around for me five years ago, my life would have been a lot better." Trans Math Day regularly has 50 participants.

For over the last two years I have served as a board member for *Spectra: The Association for LGBTQ+ Mathematicians*. As a board member I have overseen the growth and formalization of the organization, including the creation and adoption of bylaws, the creation of an invited lecture at the Joint Mathematics Meetings, and a fundraising campaign that has raised over $20,000 to support LGBTQ+ students and mathematicians. I am currently the inaugural president of Spectra, and as of right now we have over 500 people on our mailing/membership lists.

In response to the COVID-19 pandemic and the shift of many mathematical activities to virtual formats, I worked to find ways for these online activities to reach those often at the periphery. During Summer and Fall 2020, I helped with Ravi Vakil's *Algebraic Geometry in the Time of Covid* project. This massive online open-access course in algebraic geometry brought together $\sim 2,000$ participants from around the world. In Spring 2021, I organized an 8-week virtual reading course for undergraduates in algebraic geometry and commutative algebra.

I have organized over 15 conferences, special sessions, and workshops including: *M2@UW* (45 participants), *Graduate Workshop in Commutative Algebra for Women & Mathematicians of Minority Genders* (35 participants), *CAZoom* (70 participants), *Western Algebraic Geometry Symposium* (100 participants)x, and Spec($\overline{\mathbb{Q}}$) (50 participants). When organizing these conferences I have given paid special attention to promoting women and other underrepresented groups in mathematics. For example, *Graduate Workshop in Commutative Algebra for Women & Mathematicians of Other Minority Genders* and *GEMS in Combinatorics* focused on forming communities of women and non-binary researchers in commutative algebra and combinatorics respectively. Further, Spec($\overline{\mathbb{Q}}$) highlighted the research of LGBTQ+ mathematicians in algebra, geometry, and number theory.