Project Summary: Multigraded Homological Algebra and Geometry

Overview. This proposal develops new connections between multigraded algebra and geometry. The first project expands our understanding of the relationship between minimal graded free resolutions and geometry by generalizing Green's conjecture on the syzygies of canonical curves. The second project is devoted to developing new tools in multigraded commutative algebra to gain a better understanding of the geometry of multigraded Hilbert schemes.

Intellectual Merit. The PI's first project expands our understanding of the connections between minimal graded free resolutions and the geometry of algebraic varieties. More specifically, this project generalizes Green's conjecture concerning the syzygies of canonical curves to the canonical rings of stacky curves. Towards this goal, the PI proposes developing algebraic and geometric tools to study the properties of the canonical rings of stacky curves. This includes a substantial program to extend classical results characterizing subvarieties of projective space of minimal degree (and their minimal graded free resolutions) to other toric varieties. This project includes several applications like a large-scale computational exploration of syzygies and homological algebra on weighted projective spaces and other toric varieties.

A second project proposed by the PI deepens our understanding of the geometry of multigraded Hilbert schemes. Despite their increasing usefulness throughout algebraic geometry and commutative algebra, the geometry of multigraded Hilbert schemes remains quite mysterious. In particular, unlike their classical counterparts, very little is known about when multigraded Hilbert schemes are non-empty, connected, or smooth. This project develops new tools in multigraded commutative algebra to approach answering such geometric questions. In particular, the project looks to generalize a number of classical results and tools in commutative algebra – like Macaulay's theorem, Gotzmann's theorem, and lex ideals – to the multigraded setting. This project contains significant connections to computation and combinatorics.

Broader Impacts. As an LGBTQ+ woman, the PI has worked hard to promote diversity, inclusivity, and justice in the mathematical community. This proposal will further the PI's work in these directions as she continues to mentor students, to work to support women and LGBTQ+ people in mathematics, and organize conferences. The PI plans to continue mentoring one undergraduate and two graduate students (a majority of whom identify with generally underrepresented groups) she is working with on research projects. In Summer 2024 the PI will organize an undergraduate research program exploring questions in this proposal for students from underrepresented groups.

The PI is the inaugural president of *Spectra, the Association for LBGTQ+ Mathematicians*, and has served as a board member for several years. Since 2020 the PI has organized an annual conference for transgender and non-binary mathematicians to continue organizing this conference, with the first in-person iteration scheduled for 2024. In Fall 2023 PI will organize a mentoring program supporting LGBTQ+ undergraduate students applying to graduate school.

The PI has organized many conferences including Spec($\overline{\mathbb{Q}}$) a conference for LGBTQ+ mathematicians in algebra, geometry, and number theory (Summer 2022), *Western Algebraic Geometry Symposium* (2021), *Gender Equity in the Mathematical Study of Combinatorics* (2021), the *Graduate Workshop in Commutative Algebra for Women and Mathematicians of Minority Genders* (2019), *Geometry & Arithmetic of Surfaces* (2019), and a five-day conference dedicated to developing open-source computer software for algebraic geometry and commutative algebra *M2@UW* (2018). The PI plans to organize a follow-up to *Graduate Workshop in Commutative Algebra for Women and Mathematicians of Minority Genders* tentatively planned for Summer 2023, as well as a conference highlighting recent advances in multigraded algebra and geometry.