



Juliette Bruce  
Postdoctoral Researcher  
Department of Mathematics  
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November 13, 2023

Dear Committee Members,

I am writing to apply for the tenure-stream assistant professor position in the Department of Mathematics of The Pennsylvania State University. Currently, I am a postdoctoral researcher in the Mathematics Department at Brown University, a position I have held since August 2022. I received my Ph.D. in Mathematics from the University of Wisconsin-Madison under the guidance of Professor Daniel Erman in 2020. From 2020-2022 I was an NSF Postdoctoral Fellow in the Mathematics Department at the University of California, Berkeley. Additionally, I was a postdoctoral fellow at the Mathematical Sciences Research Institute – now the Simons Laufer Mathematical Sciences Institute – for the 2020-2021 academic year.

My research interests lie in the intersection of algebraic geometry and commutative algebra with connections to combinatorics, number theory, and topology. I am interested in using homological, combinatorial, and computational methods to study the geometry of algebraic varieties. Currently, my research program has two broad directions.

- (i) I have sought to deepen and expand our understanding of the ways homological algebra can be used to study the geometry of toric varieties. This seeks to generalize a very classical story using homological algebra to understand subvarieties of projective space.
- (ii) I have been studying the geometry and topology of various moduli spaces, e.g., the moduli space of (principally polarized) abelian varieties of a fixed dimension, via combinatorially and homological methods. This has led to novel applications to the cohomology of certain arithmetic groups.

Further, I am passionate about promoting inclusivity, diversity, and justice in the mathematics community. This passion extends throughout my teaching where I am dedicated to creating an interactive and supportive classroom environment that helps students thrive.

My research output includes 15 papers, with publications in journals such as *Algebra & Number Theory*, *Geometry & Topology*, and *Experimental Mathematics*, as well as, multiple published software packages. Below are a few of the non-research highlights of my file.

- I was awarded an *NSF Postdoctoral Research Fellowship*, an *NSF Graduate Research Fellowship*, and I have secured over \$100,000 in conference grants, including 4 NSF conference grants.
- I have organized 12+ conferences, workshops, and special sessions, including multiple events aimed at supporting and promoting mathematicians from generally underrepresented groups, especially women and LGBTQ+ mathematicians.
- I was awarded the highest departmental and campus-wide teaching awards at the University of Wisconsin-Madison, the Capstone Teaching Award (2019) and the Teaching Assistant Award for Exceptional Service (2018), awarded to 1 and 3 students each year respectively.



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With my application, I include the AMS cover sheet, a curriculum vitae, a research statement, a teaching statement, a publication list, and a diversity statement. I will have six letters of recommendation. Five research letters: Christine Berkesch (cberkesch@umn.edu), Melody Chan (melody\_chan@brown.edu), David Eisenbud (de@berkeley.edu), Daniel Erman (erman@hawaii.edu), and Gregory G. Smith (ggsmith@mast.queensu.ca), and one teaching letter from Shirin Malekpour (shirin.malekpour@wisc.edu).

Please do not hesitate to contact me with any questions, or if there is anything else I can provide, and thank you in advance for your consideration.

Sincerely,

*Juliette E. Bruce*

Juliette Bruce  
Postdoctoral Research Associate

# Juliette Bruce

October 23, 2023

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## Employment

- |  |                                  |
|--|----------------------------------|
| • <b>Brown University</b><br><i>Postdoctoral Research Associate</i>                    | Providence, RI<br>2022 – Present |
| • <b>University of California, Berkeley</b><br><i>NSF Postdoctoral Research Fellow</i> | Berkeley, CA<br>2020– 2022       |
| • <b>Mathematical Sciences Research Institute</b><br><i>Postdoctoral Fellow</i>        | Berkeley, CA<br>2020– 2021       |

## Education

- |   |                                   |
|---|-----------------------------------|
| • <b>University of Wisconsin</b><br><i>Ph.D. Mathematics</i><br>– Advisor: Daniel Erman | Madison, WI<br>2014 – August 2020 |
| • <b>University of Wisconsin</b><br><i>M.A. Mathematics</i>                             | Madison, WI<br>2014 – 2016        |
| • <b>University of Michigan</b><br><i>B.S. in Mathematics &amp; Political Science</i>   | Ann Arbor, MI<br>2010 – 2014      |

## Research Interests

Algebraic Geometry, Commutative Algebra, Arithmetic Geometry. Specifically, homological and combinatorial methods in algebraic geometry and commutative algebra.

## Publications

15. M. Brandt, J. Bruce, M. Chan, M. Melo, G. Moreland, C. Wolfe. On the Top-weight Cohomology of  $\mathcal{A}_g$ . *Geometry & Topology*, To appear. E-Print: [arXiv:2012.02892](https://arxiv.org/abs/2012.02892)
14. J. Bruce, D. Corey, D. Erman, S. Goldstein, R. Laudone, and J. Yang. Syzygies of  $\mathbb{P}^1 \times \mathbb{P}^1$ : Data and Conjectures. *Journal of Algebra*, **593** (2022) no. 1, 589-621. E-Print: [arXiv:2104.14598](https://arxiv.org/abs/2104.14598)
13. J. Bruce, D. Erman, S. Goldstein, and J. Yang. The Schur-Veronese package in Macaulay2. *Journal of Software for Algebra and Geometry*, **11** (2021), 83-87 E-print: [arXiv:1905.12661](https://arxiv.org/abs/1905.12661).
12. J. Bruce. The Quantitative Behavior of Asymptotic Syzygies for Hirzebruch Surfaces. *Journal of Commutative Algebra*, To appear. E-Print: [arXiv:1906.07333](https://arxiv.org/abs/1906.07333).

11. A. Almousa, J. Bruce, M. Loper, and M. Sayrafi. The Virtual Resolutions Package for Macaulay2. *Journal of Software for Algebra and Geometry*, **10** (2020), 50-60. E-print: [arXiv:1905.07022](https://arxiv.org/abs/1905.07022).
10. J. Bruce and D. Erman. A probabilistic approach to systems of parameters and Noether normalization. *Algebra and Number Theory*, **13** (2019), no. 9, 2081–2102. E-print: [arXiv:1604.01704](https://arxiv.org/abs/1604.01704).
9. J. Bruce and W. Li. Effective bounds on the dimensions of Jacobians covering abelian varieties. *Proc. Amer. Math. Soc.*, **148** (2020), no. 2, 535-551. E-print: [arXiv:1804.11015](https://arxiv.org/abs/1804.11015).
8. J. Bruce, D. Erman, S. Goldstein, and J. Yang. Conjectures and computations about Veronese syzygies. *Experimental Mathematics*, **29** (2020), 398-413. E-print: [arXiv:1711.03513](https://arxiv.org/abs/1711.03513).
7. M. Brandt, J. Bruce, T. Brysiewicz, R. Krone, and E. Robeva. The degree of  $SO(n)$ . *Combinatorial Algebraic Geometry*, 207-224, Fields Inst. Commun. **80** (2017). E-print: [arXiv:1701.03200](https://arxiv.org/abs/1701.03200).
6. J. Bruce, M. Logue, and R. Walker. Monomial valuations, cusp singularities, and continued fractions. *Journal of Commutative Algebra*, **7** (2015) no. 4, 495-522. E-print: [arXiv:1311.6493](https://arxiv.org/abs/1311.6493).
5. J. Bruce, P. Kao, E. Nash, B. Perez, and P. Vermeire. Betti tables of reducible algebraic curves. *Proc. Amer. Math. Soc.* **142** (2014) 4039-4051. E-print: [arXiv:1210.3064](https://arxiv.org/abs/1210.3064).

## Pre-Prints

4. M. Brandt, J. Bruce, D. Corey. The virtual Euler characteristic for binary matroids. *Submitted*. E-Print: [arXiv:2301.10108](https://arxiv.org/abs/2301.10108)
3. J. Bruce, L. Cranton Heller, M. Sayrafi. Bounds on Multigraded Regularity. *Submitted*. E-Print: [arXiv:2208.11115](https://arxiv.org/abs/2208.11115)
2. J. Bruce, L. Cranton Heller, M. Sayrafi. Characterizing Multigraded Regularity on Products of Projective Space. *Submitted*. E-Print: [arXiv:2110.10705](https://arxiv.org/abs/2110.10705)
1. J. Bruce. Asymptotic Syzygies in the Setting of Semi-Ample Growth. *Submitted*. E-Print: [arXiv:1904.04944](https://arxiv.org/abs/1904.04944)

## Software

4. SchurVeronese, (with D. Erman, S. Goldstein, and J. Yang). Submitted for distribution with future releases of Macaulay2, a compute algebra system focused on computations in algebraic geometry and commutative algebra.
3. VirtualResolutions, (with A. Almousa, M. Loper, and M. Sayrafi). Distributed with version 1.14 of Macaulay2 (2019).

2. FrobeniusThresholds, (with D. Hernández, K. Schwede, D. Smolkin, P. Teixeira, and E. Witt). Distributed with version 1.14 of Macaulay2 (2019).
1. TestIdeals, (with E. Bela, A. Boix, D. Ellingson, D. Hernández, Z. Kadyrsizova, M. Katzman, S. Malec, M. Mastroeni, M. Mostafazadehfard, M. Robinson, K. Schwede, D. Smolkin, P. Teixeira, and E. Witt). Distributed with version 1.14 of Macaulay2 (2019).

## Non-Research Articles

2. J. Bruce. A Word from... Juliette Bruce, Inaugural President of Spectra. *Notices of the AMS*, **69** (2022) no. 6, 898–899.
1. A. Bonato, J. Bruce, and R. Buckmire. Spaces for All: The Rise of LGBTQ+ Mathematics Conferences. *Notices of the AMS*, **68** (2021) no. 6, 998–1003. <https://doi.org/10.1090/noti2288>

## Multimedia

1. [SyzygyData.com](https://syzygydata.com), (with D. Erman, S. Goldstein, and J. Yang). An online public database on large-scale syzygy computations.

## Grants

- **Conference Grant DMS 2332592 – \$20,000** August 2023  
*National Science Foundation*
- **Conference Grant – \$23,000** July 2022  
*Fields Institute / National Science Foundation*
- **Postdoctoral Research Fellowship – \$150,000** 2020 – 2022  
*National Science Foundation*
- **Conference Grant DMS-1908799 – \$15,000** March 2019  
*National Science Foundation*
- **Graduate Research Fellowship** 2015 – 2018  
*National Science Foundation*
- **Conference Grant DMS-1812462 – \$15,000** February 2018  
*National Science Foundation*
- **Professional Development Grant – \$1000** December 2016  
*Graduate School – University of Wisconsin*

## Awards & Honors

- **US Junior Oberwolfach Fellow** April 2022  
*Awarded to outstanding junior scientists from US to participate in activities at Oberwolfach.*
- **Capstone Teaching Award** October 2019  
*Awarded to one student in the math department for an exceptional record of teaching excellence.*
- **Excellence in Mathematical Research Award** October 2019  
*Recognizes significant and substantial contributions to research as part of their thesis.*
- **Elizabeth Hirschfelder Prize** October 2018  
*Awarded to an outstanding female student.*
- **Mathematics TA Service Award** April 2018  
*Dept. of Mathematics - University of Wisconsin*
- **Teaching Assistant Award for Exceptional Service** February 2018  
*Campus-wide award recognizing TA's who perform exceptional service*
- **Outstanding Achievement in Mathematics** May 2014  
*Dept. of Mathematics – University of Michigan*
- **Phi Beta Kappa** April 2014  
*University of Michigan*
- **Chancellor's Opportunity Award** April 2014  
*University of Wisconsin*

## Seminar and Colloquium Talks

- San Francisco State University - Algebra, Geometry, & Combinatorics Seminar September 2023
- University of Vermont - Mathematics Colloquium May 2023
- Princeton University - Algebraic Geometry Seminar April 2023
- Dartmouth College - Algebra & Number Theory Seminar April 2023
- Bowdoin College - Mathematics Colloquium February 2023
- University of Texas - Austin - Geometry Seminar December 2022
- Harvard/MIT - Algebraic Geometry Seminar November 2022
- University of Kentucky - Algebra Seminar November 2022
- San Francisco State University - Mathematistas Raising Voices Colloquium April 2022
- COGENT Seminar - Held Virtually March 2022
- Simon Fraser University - Number Theory & Algebraic Geometry Seminar March 2022
- Reed College - Mathematics Colloquium April 2021
- San Francisco State University - Algebra, Geometry, & Combinatorics Seminar April 2021
- Algebra, Geometry, and Combinatorics Online - Held Virtually February 2021
- MPI MiS - Non-linear Algebra Seminar Online February 2021
- Syracuse University - Algebra Seminar November 2020
- Arizona State University - AWM Seminar October 2020
- Arizona State University - Number Theory Seminar October 2020
- Louisiana State University - Algebra & Number Theory Seminar October 2020
- University of Georgia - Algebraic Geometry Seminar October 2020
- Stanford University - Algebraic Geometry Seminar October 2020

• MSRI - The Fellowship of the Ring	September 2020
• University of Washington - Algebra & Algebraic Geometry Seminar	May 2020 <sup>†</sup>
• University of Nebraska - Commutative Algebra Seminar	April 2020 <sup>†</sup>
• University of Michigan - Commutative Algebra Seminar	December 2019
• University of Notre Dame - Algebraic Geometry Seminar	November 2019
• DePaul University - Algebra, Combinatorics, and Number Theory Seminar	October 2019
• Lawrence University - Mathematics Colloquium	October 2019
• University of Utah - Algebraic Geometry Seminar	September 2019
• Stanford University - Algebraic Geometry Seminar	May 2019
• University of Kentucky - Algebra Seminar	April 2019
• University of Minnesota - Commutative Algebra Seminar	April 2019
• Rice University- Algebraic Geometry and Number Theory Seminar	September 2018
• DePaul University - Algebra, Combinatorics, and Number Theory Seminar	March 2018
• University of Michigan - Commutative Algebra Seminar	December 2017

## Conference Talks

• Joint Math Meetings (x2) - San Francisco, CA	January 2023
• Workshop for Graduate Students: Syzygies & Regularity - UIC	April 2023
• Spring AMS Southeastern Sectional - Georgia Tech University	March 2023
• Algebraic Geometry Northeast Series (AGNES) - UMass Amherst	November 2022
• Fall AMS Western Sectional - University of Utah	October 2022
• Fall AMS Southeastern Sectional - University of Tennessee, Chattanooga	October 2022
• BATMOBILE - Brown University	May 2022
• CA+ - Iowa State University	April 2022
• Joint Math Meetings (x2) - Seattle, WA	January 2022 <sup>†</sup>
• CMS Winter Meetings - Held Virtually	December 2021
• Queer Research Showcase, QAtCanSTEM - Held Virtually	November 2021
• Fall AMS Central Sectional - Held Virtually	October 2021
• SIAM Conference on Applied Algebraic Geometry 2021	August 2021
• Queer and Trans in Combinatorics	June 2021
• Math Summer Program for Inclusive Excellence	June 2021
• Spring AMS Western Sectional - Held Virtually	May 2021
• Spring AMS Central Sectional - Held Virtually	April 2021
• Bay Area Discrete (BAD) Math Day - Held Virtually	April 2021
• Spring AMS Southeastern Sectional - Held Virtually	March 2021
• LGBTQ+Math - Fields Institute	November 2020
• Written Geometry: Commutative Algebra - CIRM	October 2020 <sup>†</sup>
• Early Commutative Algebra Researchers (eCARS) - Held Virtually	June 2020
• Foundations of Computational Mathematics - Simon Fraser University	June 2020 <sup>†</sup>

<sup>†</sup>Canceled due to the COVID-19 pandemic.

- Western Algebraic Geometry Online (WAGON) - Held Virtually April 2020
- CA+ - Iowa State University April 2020<sup>+</sup>
- Joint Math Meetings - Denver, CO January 2020
- Fall AMS Central Sectional - University of Wisconsin September 2019
- SIAM Conference on Applied Algebraic Geometry 2019 July 2019
- KUMUNUjr - University of Nebraska March 2019
- Spring AMS Southeastern Sectional - Auburn University March 2019
- Joint Math Meetings - Baltimore, MD January 2019
- Fall AMS Central Sectional - University of Michigan October 2018
- Structures on Free Resolutions - Texas Tech University October 2017
- Midwest Algebraic Geometry Graduate Conference - University of Illinois, Chicago April 2015

## Poster Talks

- AWM Poster Session - Joint Math Meetings January 2020
- Summer School on Randomness and Learning in NLA - Max Plank Institute, Leipzig July 2019
- 2019 AWM Research Symposium - Rice University April 2019
- AWM Poster Session - Joint Math Meetings January 2018
- AGNES Poster Session - Brown University September 2018
- Lectures on Arithmetic Geometry - Rice University February 2017
- Introductory Workshop: Combinatorial Algebraic Geometry - Fields Institute August 2016
- Commutative Algebra and Its Interactions with Algebraic Geometry July 2016
- Midwest Commutative Algebra and Algebraic Geometry Conference May 2016

## Conference & Seminar Organizing

- **GEMS in Commutative Algebra** University of Minnesota  
*November 10-13, 2023*
- **GEMS in Combinatorics** AIM  
*March 27-31, 2023*
- **Trans Math Day** Held Virtually  
*December 3, 2022*
- **BATMOBILE** Amherst College  
*September 30, 2022*
- **$\text{Spec}(\overline{\mathbb{Q}})$**  Fields Institute  
*July 6-8, 2022*
- **LGBTQ+ Math Day** Fields Institute  
*November 18, 2021*
- **GEMS in Combinatorics** Held Virtually  
*September 1-2, 2021*



- **Trans Math Day** Held Virtually  
*June 14-15, 2021*
- **Western Algebraic Geometry Symposium (WAGS)** Held Virtually  
*April 23 - April 24, 2021*
- **Trans Math Day** Held Virtually  
*December 5, 2020*
- **CAZoom** Held Virtually  
*April 25 - April 26, 2020*
- **GWCAWMMG** University of Minnesota  
*April 12 - April 14, 2019*
- **Geometry & Arithmetic of Surfaces** University of Wisconsin  
*February 9 - February 10, 2019*
- **M2@UW** University of Wisconsin  
*April 14-17, 2018*
- **Math Careers Beyond Academia** University of Wisconsin  
*April 14, 2017*

## Seminar & Session Organizing

- **Special Session on Combinatorial Algebraic Geometry** AMS Sectional  
*October 1 - October 2, 2022*
- **Commutative Algebra & Algebraic Geometry Seminar** UC Berkeley  
*September 2021 - May 2022*
- **Special Session on Commutative Algebra** AMS Sectional  
*May 1 - May 2, 2021*
- **Experimental Talks in Algebraic Geometry** Held Virtually  
*May - July, 2020*
- **Special Session on Combinatorial Algebraic Geometry** AMS Sectional  
*September 14 - September 15, 2019*

## Outreach Activities

- **Michigan Research Experience for Graduate Students** University of Michigan  
*Project Leader*  
  - Lead a diverse group of 4 early-stage graduate students on a project in algebraic geometry and combinatorics.
  - I expect this project to eventually result in a publication and software package.
- **Virtual Directed Readings in Geometry** Held Virtually  
*Organizer*  
  - Created a virtual, open access, directed reading program for undergraduate students.
  - Approximately 30 students signed up for the 5-week reading group.

- **Virtual Directed Readings in Geometry** Held Virtually  
*Organizer* *February 2021 – April 2021*

  - Created a virtual, open access, directed reading program for undergraduate students.
  - Approximately 30 students signed up for the 5-week reading group.
- **Algebraic Geometry in the Time of COVID** Held Virtually  
*Shepard* *June 2020 – October 2020*

  - A virtual, open access introductory course in algebraic geometry.
  - Approximately 1600 registered participants.
- **Undergrad Directed Reading Program** University of Wisconsin  
*Mentor* *January 2018 – May 2019*

  - Lead two semester long reading projects on commutative algebra and algebraic geometry.
  - Lead an undergraduate women on a two semester reading project, and provided guidance on applying for REU's and graduate school.
- **Graduate Peer Mentoring** University of Wisconsin  
*Mentor* *September 2018 – December 2018*

  - Mentored 5 first year graduate students from minority genders, organizing monthly dinners where the mentees could discuss issues they were facing.
- **Girls Math Night Out** University of Wisconsin  
*Mentor* *September 2018 – December 2018*

  - Lead 2 women from local high schools on a semesters long project about cryptography.
- **Madison Math Circle** University of Wisconsin  
*Lead Organizer* *January 2016 – December 2018*

  - Lead the creation of a new outreach program, which directly visits high schools around the state of Wisconsin to better serve students from underrepresented groups.
  - Expanded the total number of students reached per year from 25 to >250.
- **Madison Math Circle** University of Wisconsin  
*Student Volunteer* *January 2015 – December 2018*
- **Out in STEM (oSTEM) @ UW-Madison** University of Wisconsin  
*co-Founder* *July 2017 – Math 2018*

  - Founded, at the time. the only campus resource specifically for LGBTQ+ individuals in STEM, and grew the organization to over 50 members.
  - Secured a travel grant to help 11 members (undergraduate and graduate students) attend the national oSTEM conference.
- **Madison Mega Math Meet** University of Wisconsin  
*Graded* *May 2015*
- **Bonding Undergraduate and Graduate Students** University of Wisconsin  
*Mentor for Undergraduate* *September 2014 – December 2014*
- **Michigan Math Circle** University of Michigan  
*Organizer* *January 2013 – June 2014*

## Teaching Experience

- **Math 221: Calculus and Analytic Geometry I** University of Wisconsin  
*Teaching Assistant* *Fall 2014/2018/2019*
  - Selected as a TA coordinator in 2018 and 2019, and was responsible for overseeing all other TA's and mentoring first year TA's.
  - Sections ranged from 25-35 students on average, with typically teaching two sections at a time.
  - Average score 4.9/5.0
- **Math 228: Wisconsin Emerging Scholars** University of Wisconsin  
*Instructor* *Fall 2018*
  - Course providing students from underrepresented groups additional support.
  - Course had approximately 10-15 students.
  - Average score: 5.0/5.0, 100% amongst all TA's
- **Math 132: Wisconsin Emerging Scholars** University of Wisconsin  
*Instructor* *Spring 2015*
  - Course had approximately 15-25 students.
  - Responsible for all aspects of the course.
- **Inquiry Based Learning Courses** University of Michigan  
*Course Assistant* *2012-2014*
  - Assisted with advanced undergraduate courses on topology, analysis, and probability.
  - Courses had approximately 25-35 students.
  - Facilitated inquiry based learning in the classroom, and responsible for office hours, grading, and review sessions.

## Outreach Panels

- **Holding AMS Meetings in Localities with Discriminatory Practices** Joint Math Meetings  
*Panelist* *January 2022*
- **AWM 101: What You Need to Know about Women in Math** Joint Math Meetings  
*Panelist* *January 2022*
- **Equity, Diversity, and Inclusion in Mathematics** CAIMS  
*Panelist* *June 2021*
- **Diversity and Inclusion Panel** Womxn in Math at Berkeley  
*Panelist* *April 2021*
- **How to Stay Productive as a Researcher** Lunch in the Time of COVID  
*Panelist* *June 2020*
- **Mathematics Research Online: Hosting Virtual Events** Held Virtually  
*Panelist* *May 2020*
- **Supporting Transgender and Non-binary Students** MAA Panel at the JMM  
*Organizer & Moderator* *January 2020*
- **Out in Math: Professional Issues Facing LGBTQ Mathematicians** MAA Panel at the JMM  
*Panelist* *January 2018*

## Service

- **Spectra: The Association for LGBTQ+ Mathematicians**  
*President, Immediate Past President* *September 2020 – Present*
- **MSRI: Committee on Women in Mathematics**  
*Committee Member* *March 2023 – Present*
- **Communications in Algebra**  
*Communications Editor* Editorial Board  
*September 2020 – September 2022*
- **Spectra: The Association for LGBTQ+ Mathematicians**  
*Board Member* *September 2020 – January 2022*
- **AMS Graduate Student Blog**  
*Writer & Editor* American Mathematical Society  
*September 2015 – September 2018*
- **Committee on Inclusivity and Diversity**  
*Member* UW Dept. of Mathematics  
*November 2016 – August 2017*
  - Created policies seeking to make the department a more welcoming, inclusive, and comfortable place. This included drafting the department’s statement on inclusivity, and creating similar statements for syllabi to be used throughout the department.
- **Committee on TA Pay and Performance**  
*Member* UW Dept. of Mathematics  
*September 2015 – August 2017*
  - Developed and implemented a new system to evaluate TA performance, with the goal of creating a more transparent, useful, and non-biased system.
- **Instructor Excellence Program**  
*Teaching Mentor* UW Dept. of Mathematics  
*September 2015 – May 2016*
- **Society of Undergraduate Math Students**  
*President and Founder* University of Michigan  
*December 2012 – June 2014*

## Referee Work

- Algebra & Number Theory, Communications in Algebra, Journal of Algebra, Journal of Commutative Algebra, Journal of Pure and Applied Algebra, Mathematica Scandinavia, Research in the Mathematical Sciences, Rocky Mountain Journal of Mathematics

## Memberships

- Society of Industrial and Applied Mathematics January 2017 – Present
- Association for Women in Mathematics January 2016 – Present
- American Mathematical Society September 2014 – Present

## Juliette Bruce’s Research Statement

My research interests lie in algebraic geometry, commutative algebra, and arithmetic geometry. In particular, I am interested in using homological and combinatorial methods to study the geometry of zero loci of systems of polynomials (i.e. algebraic varieties). I am also interested in studying the arithmetic properties of varieties over finite fields. Further, I am passionate about promoting inclusivity, diversity, and justice in the mathematics community. Broadly, my current research follows these ideas in two directions.

- **Homological Algebra on Toric Varieties:** A classical story in algebraic geometry is that homological methods and tools like minimal free resolutions and Castelnuovo–Mumford regularity capture the geometry of subvarieties of projective space in nuanced ways. My work has sought to generalize this story by developing ways homological algebra can be used to study the geometry of toric varieties (i.e., “nice” compactifications of the torus  $(\mathbb{C}^\times)^n$ ).
- **Cohomology of Moduli Spaces and Arithmetic Groups:** Despite its importance in algebraic geometry and number theory much remains unknown about the topology of  $\mathcal{A}_g$ , the moduli space of abelian varieties of dimension  $g$ . I have been working to study a canonical “part” of the cohomology of  $\mathcal{A}_g$ , called the top-weight cohomology. This turns out to be closely connected to the study of cohomology of various arithmetic groups like  $\mathrm{GL}_g(\mathbb{Z})$  and  $\mathrm{Sp}_{2g}(\mathbb{Z})$ , as well as the study of automorphic forms.

### 1. Homological Algebra on Toric Varieties

Given a graded module  $M$  over a graded ring  $R$ , a helpful tool for understanding the structure of  $M$  is its minimal graded free resolution. In essence, a minimal graded free resolution is a way of approximating  $M$  by a sequence of free  $R$ -modules. More formally, a *graded free resolution* of a module  $M$  is an exact sequence

$$\cdots \rightarrow F_k \xrightarrow{d_k} F_{k-1} \xrightarrow{d_{k-1}} \cdots \xrightarrow{d_1} F_0 \xrightarrow{\epsilon} M \rightarrow 0$$

where each  $F_i$  is a graded free  $R$ -module, and hence can be written as  $\bigoplus_j R(-j)^{\beta_{i,j}}$ . The module  $R(-j)$  is the ring  $R$  with a twisted grading, so that  $R(-j)_d$  is equal to  $R_{d-j}$  where  $R_{d-j}$  is the graded piece of degree  $d-j$ . The  $\beta_{i,j}$ ’s are the *Betti numbers* of  $M$ , and they count the number of  $i$ -syzygies of  $M$  of degree  $j$ . We will use syzygy and Betti number interchangeably throughout.

Given a projective variety  $X$  embedded in  $\mathbb{P}^r$ , we associate to  $X$  the ring  $S_X = S/I_X$ , where  $S = \mathbb{C}[x_0, \dots, x_r]$  and  $I_X$  is the ideal of homogenous polynomials vanishing on  $X$ . As  $S_X$  is naturally a graded  $S$ -module we may consider its minimal graded free resolution, which is often closely related to both the extrinsic and intrinsic geometry of  $X$ . An example of this phenomenon is Green’s Conjecture, which relates the Clifford index of a curve with the vanishing of certain  $\beta_{i,j}$  for its canonical embedding [Voi02, Voi05, AFP<sup>+</sup>19]. See also [Eis05, Conjecture 9.6] and [Sch86, BE91, FP05, Far06, AF11, FK16, FK17].

Much of my work can be viewed as understanding how minimal graded free resolutions capture the geometry when the role of  $\mathbb{P}^r$  is replaced by another variety  $Y$ . In particular, I have focused on the case when  $Y$  is a toric variety, i.e., a compactification of the torus  $(\mathbb{C}^\times)^r$  where the action of the torus extends to the boundary. Examples of toric varieties include projective space, products of projective spaces, and Hirzebruch surfaces. Work of Cox shows there is a correspondence between (toric) subvarieties of a fixed toric variety and quotients of a polynomial ring similar to the story discussed above for  $\mathbb{P}^r$  [Cox95]. As such recent years have seen substantial work looking to use homological algebra and to better understand the geometry of toric varieties [?almousaBruce19, BES20, BE22, BE23a, BB21, CEVV09, EES15, MS04, MS05].

**1.1 Asymptotic Syzygies** Broadly speaking, asymptotic syzygies is the study of the graded Betti numbers (i.e. the syzygies) of a projective variety as the positivity of the embedding grows. In many ways, this perspective dates back to classical work on the defining equations of curves of high degree and projective normality [Mum66, Mum70]. However, the modern viewpoint arose from the pioneering work of Green [Gre84a, Gre84b] and later Ein and Lazarsfeld [EL12].

To give a flavor of the results of asymptotic syzygies we will focus on the question: In what degrees do non-zero syzygies occur? Going forward we will let  $X \subset \mathbb{P}^{r,d}$  be a smooth projective variety embedded by a

very ample line bundle  $L_d$ . Following [EY18] we set,

$$\rho_q(X, L_d) := \frac{\#\{p \in \mathbb{N} \mid \beta_{p,p+q}(X, L_d) \neq 0\}}{r_d},$$

which is the percentage of degrees in which non-zero syzygies appear [Eis05, Theorem 1.1]. The asymptotic perspective asks how  $\rho_q(X; L_d)$  behaves along the sequence of line bundles  $(L_d)_{d \in \mathbb{N}}$ .

With this notation in hand, we may phrase Green's work on the vanishing of syzygies for curves of high degree as computing the asymptotic percentage of non-zero quadratic syzygies.

**Theorem 1.1.** [Gre84a] *Let  $X \subset \mathbb{P}^r$  be a smooth projective curve. If  $(L_d)_{d \in \mathbb{N}}$  is a sequence of very ample line bundles on  $X$  such that  $\deg L_d = d$  then  $\lim_{d \rightarrow \infty} \rho_2(X; L_d) = 0$ .*

Put differently, asymptotically the syzygies of curves are as simple as possible, occurring in the lowest possible degree. This inspired substantial work, with the intuition being that syzygies become simpler as the positivity of the embedding increases [OP01, EL93, LPP11, Par00, PP03, PP04].

In a groundbreaking paper, Ein and Lazarsfeld showed that for higher dimensional varieties this intuition is often misleading. Contrary to the case of curves, they show that for higher dimensional varieties, asymptotically syzygies appear in every possible degree.

**Theorem 1.2.** [EL12, Theorem C] *Let  $X \subset \mathbb{P}^r$  be a smooth projective variety,  $\dim X \geq 2$ , and fix an index  $1 \leq q \leq \dim X$ . If  $(L_d)_{d \in \mathbb{N}}$  is a sequence of very ample line bundles such that  $L_{d+1} - L_d$  is constant and ample then  $\lim_{d \rightarrow \infty} \rho_q(X; L_d) = 1$ .*

My work has focused on the behavior of asymptotic syzygies when the condition that  $L_{d+1} - L_d$  is constant and ample is weakened to assuming  $L_{d+1} - L_d$  is semi-ample. Recall a line bundle  $L$  is *semi-ample* if  $|kL|$  is base point free for  $k \gg 0$ . The prototypical example of a semi-ample line bundle is  $\mathcal{O}(1, 0)$  on  $\mathbb{P}^n \times \mathbb{P}^m$ . My exploration of asymptotic syzygies in the setting of semi-ample growth thus began by proving the following nonvanishing result for  $\mathbb{P}^n \times \mathbb{P}^m$  embedded by  $\mathcal{O}(d_1, d_2)$ .

**Theorem 1.3.** [Bru19, Corollary B] *Let  $X = \mathbb{P}^n \times \mathbb{P}^m$  and fix an index  $1 \leq q \leq n + m$ . There exist constants  $C_{i,j}$  and  $D_{i,j}$  such that*

$$\rho_q(X; \mathcal{O}(d_1, d_2)) \geq 1 - \sum_{\substack{i+j=q \\ i \leq n, j \leq m}} \left( \frac{C_{i,j}}{d_1^i d_2^j} + \frac{D_{i,j}}{d_1^{n-i} d_2^{m-j}} \right) - O\left(\frac{\text{lower ord.}}{\text{terms}}\right).$$

Notice if both  $d_1 \rightarrow \infty$  and  $d_2 \rightarrow \infty$  then  $\rho_q(\mathbb{P}^n \times \mathbb{P}^m; \mathcal{O}(d_1, d_2)) \rightarrow 1$ , recovering the results of Ein and Lazarsfeld for  $\mathbb{P}^n \times \mathbb{P}^m$ . However, if  $d_1$  is fixed and  $d_2 \rightarrow \infty$  (i.e. semi-ample growth) my results bound the asymptotic percentage of non-zero syzygies away from zero. This together with work of Lemmens [Lem18] has led me to conjecture that, unlike in previously studied cases, in the semi-ample setting  $\rho_q(\mathbb{P}^n \times \mathbb{P}^m; \mathcal{O}(d_1, d_2))$  does not approach 1. Proving this would require a vanishing result for asymptotic syzygies, which is open even in the ample case [EL12, Conjectures 7.1, 7.5].

The proof of Theorem 1.3 is based upon generalizing the monomial methods of Ein, Erman, and Lazarsfeld. Such a generalization is complicated by the difference between the Cox ring and homogenous coordinate ring of  $\mathbb{P}^n \times \mathbb{P}^m$ . A central theme in this work is to exploit the fact that a key regular sequence I use has a number of non-trivial symmetries.

This work suggests that the theory of asymptotic syzygies in the setting of semi-ample growth is rich and substantially different from the other previously studied cases. Going forward I plan to use this work as a jumping-off point for the following question.

**Question 1.4.** *Let  $X \subset \mathbb{P}^{r_d}$  be a smooth projective variety and fix an index  $1 \leq q \leq \dim X$ . Let  $(L_d)_{d \in \mathbb{N}}$  be a sequence of very ample line bundles such that  $L_{d+1} - L_d$  is constant and semi-ample, can one compute  $\lim_{d \rightarrow \infty} \rho_q(X; L_d)$ ?*

A natural next case in which to consider Question 1.4 is that of Hirzebruch surfaces. I addressed a different, but related question for a narrow class of Hirzebruch surfaces in [Bru22].

**1.2 Multigraded Castelnuovo–Mumford Regularity** Introduced by Mumford, the Castelnuovo–Mumford Regularity of a projective variety  $X \subset \mathbb{P}^r$  is a measure of the complexity of  $X$  given in terms of the vanishing of certain cohomology groups of  $X$ . Roughly speaking one should think about Castelnuovo–Mumford regularity as being a numerical measure of geometric complexity. Mumford was interested in such a measure as it plays a key role in constructing Hilbert and Quot schemes. In particular, being  $d$ -regular implies that  $\mathcal{F}(d)$  is globally generated. However, Eisenbud and Goto showed that regularity is also closely connected to interesting homological properties.

**Theorem 1.5.** [EG84] *Let  $\mathcal{F}$  be a coherent sheaf on  $\mathbb{P}^r$  and  $M = \bigoplus_{e \in \mathbb{Z}} H^0(\mathbb{P}^r, \mathcal{F}(e))$  the corresponding section ring. The following are equivalent: (1)  $M$  is  $d$ -regular; (2)  $\beta_{p,q}(M) = 0$  for all  $p \geq 0$  and  $q > d + i$ ; (3)  $M_{\geq d}$  has a linear resolution.*

MacLagan and Smith introduced multigraded Castelnuovo–Mumford regularity, where  $\mathbb{P}^r$  can be replaced by any toric variety. Similarly to the definition in the classical setting multigraded Castelnuovo–Mumford regularity is defined in terms of the vanishing of cohomology groups, however, the multigraded Castelnuovo–Mumford regularity of a subvariety or module is not a single number, but an infinite subset of  $\mathbb{Z}^r$ .

As an example, let us consider the case of products of projective spaces. Fixing a dimension vector  $\mathbf{n} = (n_1, n_2, \dots, n_r) \in \mathbb{N}^r$  we let  $\mathbb{P}^{\mathbf{n}} := \mathbb{P}^{n_1} \times \mathbb{P}^{n_2} \times \dots \times \mathbb{P}^{n_r}$  and  $S = \mathbb{K}[x_{i,j} \mid 1 \leq i \leq r, 0 \leq j \leq n_i]$  be the Cox ring of  $\mathbb{P}^{\mathbf{n}}$  with the  $\text{Pic}(X) \cong \mathbb{Z}^r$ -grading given by  $\deg x_{i,j} = \mathbf{e}_i \in \mathbb{Z}^r$ , where  $\mathbf{e}_i$  is the  $i$ -th standard basis vector in  $\mathbb{Z}^r$ . Fixing some notation given  $\mathbf{d} \in \mathbb{Z}^r$  and  $i \in \mathbb{Z}_{\geq 0}$  we let  $L_i(\mathbf{d}) := \bigcup_{\mathbf{v} \in \mathbb{N}_i, |\mathbf{v}|=i} (\mathbf{d} - \mathbf{v}) + \mathbb{N}^r$ . Note when  $r = 2$  the region  $L_i(\mathbf{d})$  looks like a staircase with  $(i + 1)$ -corners. Roughly speaking we define regularity by requiring the  $i$ -th cohomology of certain twists of  $\mathcal{F}$  to vanish on  $L_i$ .

**Definition 1.6.** [MS04, Definition 6.1] *A coherent sheaf  $\mathcal{F}$  on  $\mathbb{P}^{\mathbf{n}}$  is  $\mathbf{d}$ -regular if and only if  $H^i(\mathbb{P}^{\mathbf{n}}, \mathcal{F}(\mathbf{e})) = 0$  for all  $\mathbf{e} \in L_i(\mathbf{d})$ . The multigraded Castelnuovo–Mumford regularity of  $\mathcal{F}$  is then the set:*

$$\text{reg}(\mathcal{F}) := \{\mathbf{d} \in \mathbb{Z}^r \mid \mathcal{F} \text{ is } \mathbf{d}\text{-regular}\} \subset \mathbb{Z}^r.$$

The obvious approaches to generalize Theorem 1.5 to a product of projective spaces turn out not to work. For example, the multigraded Betti numbers do not determine multigraded Castelnuovo–Mumford regularity [BCHS21, Example 5.1] Despite this we show that part (3) of Theorem 1.5 can be generalized. To do so we introduce the following generalization of linear resolutions.

**Definition 1.7.** *A complex  $F_{\bullet}$  of  $\mathbb{Z}^r$ -graded free  $S$ -modules is  $\mathbf{d}$ -quasilinear if and only if  $F_0$  is generated in degree  $\mathbf{d}$  and each twist of  $F_i$  is contained in  $L_{i-1}(\mathbf{d} - \mathbf{1})$ .*

**Theorem 1.8.** [BCHS21, Theorem A] *Let  $M$  be a (saturated) finitely generated  $\mathbb{Z}^r$ -graded  $S$ -module:*

$$M \text{ is } \mathbf{d}\text{-regular} \iff M_{\geq \mathbf{d}} \text{ has a } \mathbf{d}\text{-quasilinear resolution.}$$

The proof of Theorem 1.8 is based in part on a spectral sequence argument that relates the Betti numbers of  $M_{\geq \mathbf{d}}$  to the Fourier–Mukai transform of  $\widetilde{M}$  with Beilinson’s resolution of the diagonal as the kernel. Recent breakthroughs [HHL23, BE23b] understanding resolutions of the diagonal on arbitrary toric varieties mean that there is hope one may be able to generalize the above argument to arbitrary toric varieties. With this in mind, I am interested in pursuing the following question

**Question 1.9.** *How can Theorem 1.8 be generalized to arbitrary smooth projective toric varieties? in particular, what is the correct definition of quasilinear resolutions?*

**1.2.1 Multigraded Regularity of Powers of Ideals** Building on the work of many people [BEL91, Cha97], Cutkosky, Herzog, Trung [CHT99] and independently Kodiyalam [Kod00] showed the Castelnuovo–Mumford regularity for powers of ideals on a projective space  $\mathbb{P}^r$  has surprisingly predictable asymptotic behavior. In particular, given an ideal  $I \subset \mathbb{K}[x_0, \dots, x_r]$ , there exist constants  $d, e \in \mathbb{Z}$  such that  $\text{reg}(I^t) = dt + e$  for  $t \gg 0$ . Building upon our work discussed above, my collaborators and I generalized this result to arbitrary toric varieties. In particular, Definition 1.6 can be extended to all toric varieties by letting  $S$  be Cox ring of the toric variety  $X$ , replacing  $\mathbb{Z}^r$  with the Picard group of  $X$ , and replacing  $\mathbb{N}^r$  with the nef cone of  $X$ . My collaborators and I show that the multigraded regularity of powers of ideals is bounded and translates in a predictable way. In particular, the regularity of  $I^t$  essentially translates within  $\text{Nef } X$  in fixed directions at a linear rate.

**Theorem 1.10.** [BCHS22, Theorem 4.1] *There exists a degree  $\mathbf{a} \in \text{Pic } X$ , depending only on  $I$ , such that for each integer  $t > 0$  and each pair of degrees  $\mathbf{q}_1, \mathbf{q}_2 \in \text{Pic } X$  satisfying  $\mathbf{q}_1 \geq \deg f_i \geq \mathbf{q}_2$  for all generators  $f_i$  of  $I$ , we have*

$$t\mathbf{q}_1 + \mathbf{a} + \text{reg } S \subseteq \text{reg}(I^t) \subseteq t\mathbf{q}_2 + \text{Nef } X.$$

A key aspect of the proof of this theorem is showing that the multigraded regularity of an ideal is finitely generated, in the sense that there exist vectors  $\mathbf{v}, \mathbf{w} \in \mathbb{Z}^r$  such that  $\mathbf{v} + \text{Nef } X \subset \text{reg}(I) \subset \mathbf{w} + \text{Nef } X$ . Perhaps somewhat surprisingly, my co-authors and I showed that this can fail for arbitrary modules [BCHS22]. This naturally raises the following question, which I hope to work on.

**Question 1.11.** *Let  $X$  be a smooth projective toric variety. Can one characterize when  $\text{reg}(M)$  is finitely generated for a module  $M$  over the Cox ring of  $X$ ?*

An first case of this question that I think would make a lovely first research project for a student is to attempt to answer Question 1.11 when  $M$  is the Cox ring of a torus fixed-point. In this special case, the question reduces to a delicate combinatorial question about vector partition functions.

## 2. Cohomology of Moduli Spaces and Arithmetic Groups

Some of the most classical objects in algebraic geometry are moduli spaces, i.e., spaces that parameterize a given collection of geometric objects. The quintessential example of a moduli space is  $\mathcal{M}_g$ , the moduli space of (smooth) genus  $g$  curves, also known as the moduli space of compact Riemann surfaces of genus  $g$ . Despite their classical nature, much remains unknown about the geometry of many moduli spaces. For example, the rational cohomology of  $\mathcal{M}_g$  is only known for  $g \leq 4$ . However, classical results suggest that  $\mathcal{M}_g$  should have a lot of cohomology because its Euler characteristic grows super exponentially. Recent groundbreaking work of Chan, Galatius, and Payne has shed the first direct light on this phenomenon by constructing new non-trivial cohomology classes, and showing that the dimension of certain cohomology groups of  $\mathcal{M}_g$  grow at least exponentially.

**Theorem 2.1.** [CGP21, Theorem 1.1] *For  $g \geq 2$  the dimension of  $H^{4g-6}(\mathcal{M}_g; \mathbb{Q})$  grows at least exponentially. In particular  $\dim H^{4g-6}(\mathcal{M}_g; \mathbb{Q}) > \beta^g$  for any real number  $\beta < \beta_0$  where  $\beta_0 \approx 1.3247 \dots$  is the real solution of  $t^3 - t - 1 = 0$ .*

Much of my recent work has sought to build up the groundwork laid by Chan, Galatius, and Payne to study the rational cohomology of other moduli spaces. Of particular interest to me has been the moduli space of abelian varieties and various generalizations. This work has deep connections to the cohomology of various arithmetic groups like  $\text{Sp}_{2g}(\mathbb{Z})$  and  $\text{GL}_g(\mathbb{Z})$ .

**2.1 Cohomology of  $\mathcal{A}_g$**  The moduli space of (principally polarized) abelian varieties of dimension  $g$ , is a smooth variety  $\mathcal{A}_g$  (truthfully a smooth Deligne–Mumford stack) whose points are in one to one correspondence with isomorphism classes of principally polarized abelian varieties of dimension  $g$ . Concretely, we may view it as the quotient  $[\mathbb{H}_g / \text{Sp}_{2g}(\mathbb{Z})]$  where  $\mathbb{H}_g$  is the Siegel upper half-space. Notice this means that  $\mathcal{A}_g$  is a rational classifying space for the integral symplectic group  $\text{Sp}_{2g}(\mathbb{Z})$ .

Similar to the moduli space of curves  $\mathcal{A}_g$  has long been studied, but much remains unknown about its geometry. For example, the (singular) cohomology of  $\mathcal{A}_g$  is only fully known for  $g \leq 3$ , with  $g = 0, 1$  being relatively easy,  $g = 2$  which is a classical result of Igusa [Igu62], and  $g = 3$  due to work of Hain [Hai02]. In fact the cohomology of  $\mathcal{A}_g$  is so mysterious until recently work by myself and co-authors it was unknown whether  $H^{2i+1}(\mathcal{A}_g; \mathbb{Q}) \neq 0$  for some  $g$  and  $i$ . This was a question posed by Grushevsky. that my recent work answered [Gru09]. Building upon the work of Chan, Galatius, and Payne, my co-authors and I developed new methods for understanding a certain canonical quotient of the cohomology of  $\mathcal{A}_g$ . In particular, our results construct non-trivial cohomology classes in  $H^k(\mathcal{A}_g; \mathbb{Q})$  in a number of new cases.

**Theorem 2.2.** [BBC<sup>+</sup>22, Theorem A] *The rational cohomology  $H^k(\mathcal{A}_g; \mathbb{Q}) \neq 0$  for:*

$$(g, k) = (5, 15), (5, 20), (6, 30), (7, 28), (7, 33), (7, 37), \text{ and } (7, 42).$$



For broader context, since  $\mathcal{A}_g$  is a rational classifying space for  $\mathrm{Sp}_{2g}(\mathbb{Z})$  there is an isomorphism  $H^*(\mathcal{A}_g; \mathbb{Q}) \cong H^*(\mathrm{Sp}_{2g}(\mathbb{Z}); \mathbb{Q})$ . In particular, the above results provide new non-vanishing results for  $H^*(\mathrm{Sp}_{2g}(\mathbb{Z}); \mathbb{Q})$ . However, my work takes advantage of the fact that  $\mathcal{A}_g$  is a smooth and separated Deligne Mumford stack with a coarse moduli space which is an algebraic variety, permitting Deligne's mixed Hodge theory to be applied to study the rational cohomology of these groups. The rational cohomology of a complex algebraic variety  $X$  of dimension  $d$  admits a weight filtration with graded pieces  $\mathrm{Gr}_j^W H^k(X; \mathbb{Q})$ . As  $\mathrm{Gr}_j^W H^k(X; \mathbb{Q})$  vanishes whenever  $j > 2d$ ,  $\mathrm{Gr}_{2d}^W H^k(X; \mathbb{Q})$  is referred to as the *top-weight* part of  $H^k(X; \mathbb{Q})$ . In this way we deduce Theorem 2.2 above by computing the top-weight cohomology of  $\mathcal{A}_g$  for all  $g \leq 7$ .

**2.2 Cohomology of  $\mathcal{A}_g(m)$**  The moduli space  $\mathcal{A}_g$  actually a special instance of the moduli space of (principally polarized) abelian varieties of dimension  $g$  with level  $m$ -structure. Denoted by  $\mathcal{A}_g(m)$ , we may view it as the quotient  $[\mathbb{H}_g/\mathrm{Sp}_{2g}(\mathbb{Z})](m)$  where  $\mathrm{Sp}_{2g}(\mathbb{Z})(m)$  is the principal congruence subgroup  $\ker(\mathrm{Sp}_{2g}(\mathbb{Z}) \rightarrow \mathrm{Sp}_{2g}(\mathbb{Z}/m\mathbb{Z}))$ . Note that when  $m = 1$ , we have that  $\mathcal{A}_g(m)$  is isomorphic to  $\mathcal{A}_g$ . From this perspective, one may hope to generalize Theorem 2.2 and underlying methods my co-authors and I developed in [BBC<sup>+</sup>22] to studying the rational cohomology of  $\mathcal{A}_g(m)$  and  $\mathrm{Sp}_{2g}(\mathbb{Z})(m)$ . In ongoing work, Melody Chan and I are developing such generalizations.

**Goal Theorem 2.3.** *Let  $d = \binom{g+1}{2}$  be the dimension of  $\mathcal{A}_g(m)$ . For any integers  $m \geq 1$  and  $g \geq 0$  there exists a cellular complex  $LA_g(m)^{\mathrm{trop}}$  such that for all  $i \geq 0$  there is a natural isomorphism*

$$\tilde{H}_{i-1}(LA_g(m)^{\mathrm{trop}}; \mathbb{Q}) \cong \mathrm{Gr}_{2d}^W H^i(\mathcal{A}_g(m); \mathbb{Q}),$$

The methods behind Goal Theorem 2.3 show new connections between the cohomology of  $\mathcal{A}_g(m)$  and the cohomology of  $\mathrm{GL}_g(\mathbb{Z})(m)$ . It is natural to wonder whether our methods for computing the top-weight cohomology of  $\mathcal{A}_g(m)$  shed new light on automorphic forms. Since the top-weight cohomology of  $\mathcal{A}_g(m)$  comes from understanding the boundary of a locally symmetric space, one may hope it is related to Siegel–Eisenstein series. Ongoing work with Melody Chan, Peter Sarnak and I hope to address this.

**Question 2.4.** *Is there a relationship between the top-weight cohomology of  $\mathcal{A}_g(m)$  and Siegel–Eisenstein series?*

**2.3 Matroid Complexes and Cohomology of  $\mathcal{A}_g^{\mathrm{mat}}$**  A key step in the proof of Theorem 2.2 is constructing a chain complex  $P_\bullet^{(g)}$  whose homology is precisely the top-weight cohomology of  $\mathcal{A}_g$ . A major hurdle to pushing our results on the cohomology of  $\mathcal{A}_g$ , further, is that this chain complex very quickly becomes extremely large and complicated. However, with my co-authors, I identified a subcomplex  $R_\bullet^{(g)} \subset P_\bullet^{(g)}$ , called the regular matroid complex, which has rich combinatorics. In particular,  $R_k^{(g)}$  is spanned by isomorphism classes of regular matroids on  $k$  elements of rank  $\leq g$ . I am working to study this complex from a number of perspectives. As an example, the following goal theorem is a result that I am working on with three graduate students.

**Goal Theorem 2.5.** *Compute the homology of the matroid complex  $R_\bullet^{(g)}$  for all  $g \geq 14$ .*

Currently by combining theoretical results and large-scale computations to compute the cohomology for all  $g \leq 9$ . Computing the homology of the regular matroid complex is interesting, not only because it provides a new approach for studying the combinatorics of matroids, but also because it is closely related to the cohomology of partial compactification of  $\mathcal{A}_g$  called the matroidal (partial) compactification  $\mathcal{A}_g^{\mathrm{mat}}$ . In ongoing work with Madeline Brandy and Daniel Corey, I am looking to show that one can compute the top-weight cohomology of  $\mathcal{A}_g^{\mathrm{mat}}$  from the regular matroid complex.

Work of Willwacher [Wil15] and Kontsevich [Kon93, Kon94] on graph complexes suggests that one may hope for  $R_\bullet^{(g)}$  to have rich algebraic structure beyond just that of chain complex.

**Question 2.6.** *Does the complex  $R_\bullet^{(g)}$  carry a natural Lie bracket, endowing it with the structure of a differentially graded Lie algebra?*

Constructing such a Lie bracket likely relies on developing a new understanding of the ways one can combine two matroids. Ongoing work with the graduate students mentioned above is studying this problem in the special cases of graphic and co-graphic matroids. The existence of similar Lie structure was crucial to achieving the exponential bounds in Theorem 2.1.

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## Juliette Bruce's Teaching Statement

**I. Introduction.** My goal as an educator is to be an active guide for students, providing them with environments where they feel supported and encouraged to let their own mathematical and quantitative curiosities guide how they engage and learn. By taking this approach, I hope to engage with students as the complete people that they are, asking them to bring all of their experiences, backgrounds, identities, and knowledge into the learning environment. I want students to experience mathematics in a humanistic way, seeing how mathematics and quantitative thinking are integral aspects of their lives. As one of my former students noted, “Juliette obviously wants us to succeed not only in math but in life.” Recognizing that learning mathematics is not necessarily confined to the classroom I have sought out new and non-traditional teaching opportunities. My teaching has been recognized through both awards and student evaluations:

- In 2018, I was one of three graduate students at the University of Wisconsin-Madison recognized campus-wide with the Teaching Assistant Award for Exceptional Service.
- I received two TA awards from the math department at the University of Wisconsin-Madison, the TA Service Award (2018) and the Capstone Teaching Award (2019), the latter of which is awarded to just one teaching assistant each year, for an exceptional record of teaching excellence and service.
- My student evaluations are very positive; for instance, for one course 100% of students agreed that I was an effective teacher.

I have sought to develop and refine my skills as an educator, both by viewing each teaching assignment as my own opportunity for growth and learning and by actively seeking out learning from other educators and education experts. In particular, I have implemented evidence-based techniques to support and engage students from diverse backgrounds

**II. Teaching Experiences.** As a graduate student at the University of Wisconsin - Madison, I served as a teaching assistant and course coordinator for Calculus I for multiple semesters, the instructor of record for Math for Early Education Majors, and the instructor of record for a Calculus I course providing students from generally under-represented groups additional support during their first college math course. Additionally, for several semesters, I held a non-traditional teaching assistantship for my role as the organizer of the Madison Math Circle outreach program. My passion for promoting an interest in and excitement for math – especially for people from generally underrepresented groups – led me to take on teaching and outreach roles through the *Girls Math Night Out* program and the *Wisconsin Directed Reading Program*.

My postdoctoral positions at Brown University and the University of California, Berkeley did not allow me to have formal teaching responsibilities, however, I have actively sought out non-traditional teaching opportunities and mentoring opportunities. For example, in 2020, in response to the COVID-19 pandemic, I helped Ravi Vakil and others organize *Algebraic Geometry in the Time of COVID*, a massive open-access virtual algebraic geometry course, which drew over 1500 participants from around the world. Inspired by this experience, in 2021, I organized an online open access course, *Virtual Directed Reading in Geometry & Algebra*, aimed at undergraduates. During this time I continued to seek to grow as an educator. For example, while at the University of California, Berkeley I actively participated in a reading/working group exploring antiracist and anti-oppressive pedagogy in the mathematics classroom. Further, I personally sought to engage with ways to humanize mathematics and support underrepresented students by exploring the works of Pamela E. Harris, Aris Winger, Rochelle Gutiérrez, Luis Leyva, and Francis Su.

**III. Teaching Philosophy and Strategies for Classroom Success.** As an instructor, I view my role is to be an active guide. I encourage my students to explore, engage with, and question the course material for themselves. I try to structure much of the course around guided group work that gives students opportunities to develop and discuss their understanding and confusion with their fellow students. In addition to encouraging students to take an active role in learning, this format also helps students to learn to vocalize their thought processes and ideas.

Active learning presents challenges to me and my students, most notably, the challenge of managing student mistakes. In many ways, the most significant moments during the learning process are not necessarily the moments of success, but the moments of failure. It is at this moment that students can recognize errors and gaps in their understanding of a subject and begin trying to correct them. It is also the moment that as an instructor I can understand what my students are finding difficult and nudge the conversation in such a way as to overcome these hurdles.

Making mistakes is hard, and most students, like most people, would prefer not to make mistakes. With this in mind, I think it is crucial to promote an inclusive environment where all students feel comfortable and safe participating. This environment encourages students to be open about what confuses them and where they are making mistakes. Creating an inclusive classroom environment requires active attention and work to maintain. However, in my experience, this work is well worth it.

My approach to creating an inclusive classroom environment has been influenced by the semester-long course *Inclusive Practices in the College Classroom*, which I took through the *Delta Program for Integrating Research, Teaching and Learning*. For example, one activity I implemented successfully asked students to brainstorm attributes from classes they found productive and attributes from classes they found less productive. After collecting a list of such attributes, we use this as a jumping-off point for forming community standards that we wish to shape our classroom environment. Examples of such community standards that my classes have often adopted include: “Respect everyone” and “Address the problem, not the person when discussing mistakes”. I have found this helps the students buy into the belief that the classroom is an inclusive space where it is safe to make mistakes.

However, beyond simply creating an inclusive learning environment I have also found it important to create a space where students feel comfortable bringing their whole selves, including all of their experiences, backgrounds, challenges, identities, struggles, and knowledge. For example, I recognize that all students, like all people, will have days when negative experiences outside the classroom affect their ability to engage in the classroom. This is even more true for students who face racism, sexism, homo/transphobia, and other systems of oppression. On such a day when students enter the classroom, I look to try to meet the students where they are. For example, sometimes this means I will walk the student to the campus mental health or cultural center, or sometimes it means I create new problems specifically to help keep the student’s mind off of whatever is troubling them. I try to make sure my students know I am there to provide them with whatever resources they need to succeed both in the classroom and in their life beyond. However, this human-centered approach also leads to many beautiful moments. For example, by allowing students to bring all of themselves to class they experience mathematics in a humanistic way, seeing how mathematics and quantitative thinking are an integral aspect of their life. I have found this often increases students’ motivation, as well as opens themselves up to making mistakes, growing, and learning.

**IV. Sample Student Feedback.** The effectiveness of my teaching is highlighted in student comments:

- “I’ve always struggle with math and I’ve had a lot of teachers that didn’t believe in me so because of this I’ve always dreaded math courses. But Juliette always showed she cared, was constantly encouraging, believed in our class, and taught the material really clearly. From her constant availability to help and great instructing, her class became one of my favorites and I am more successful in a math course than I’ve ever been before.”
- “She went around and tried helping each student... She cared about each student’s success in the class and tried her best to make everyone understand the material.”
- “Juliette obviously wants us to succeed not only in math but in life. She is always making sure we know our resources especially when it comes to health. She also always wishes us a good day/weekend and that is awesome.”

**V. Conclusion.** As a graduate student and postdoctoral scholar, I have found teaching to be extremely rewarding. I developed a passion for supporting and engaging students from diverse backgrounds. Going forward, I am excited for new opportunities to grow and learn as a teacher, continue to promote inclusivity, diversity, and justice in my teaching, and create human-centered learning environments for my students.

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## Publications

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14. J. Bruce, D. Corey, D. Erman, S. Goldstein, R. Laudone, and J. Yang. Syzygies of  $\mathbb{P}^1 \times \mathbb{P}^1$ : Data and Conjectures. *Journal of Algebra*, **593** (2022) no. 1, 589-621. E-Print: [arXiv:2104.14598](https://arxiv.org/abs/2104.14598)
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## Software

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3. VirtualResolutions, (with A. Almousa, M. Loper, and M. Sayrafi). Distributed with version 1.14 of Macaulay2 (2019).
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## Non-Research Articles

2. J. Bruce. A Word from... Juliette Bruce, Inaugural President of Spectra. *Notices of the AMS*, **69** (2022) no. 6, 898–899.
1. A. Bonato, J. Bruce, and R. Buckmire. Spaces for All: The Rise of LGBTQ+ Mathematics Conferences. *Notices of the AMS*, **68** (2021) no. 6, 998–1003. <https://doi.org/10.1090/noti2288>

## Multimedia

1. [SyzygyData.com](https://syzygydata.com), (with D. Erman, S. Goldstein, and J. Yang). An online public database on large-scale syzygy computations.

## Juliette Bruce's Diversity Statement

**I. Introduction.** I believe strongly in the importance of inclusivity, diversity, and justice, and I am passionate about promoting these values within the mathematical community. Going forward, I am excited to continue working hard to promote these values through my research, teaching, and service.

**II. Organizational Service.** I have organized 10+ national/international conferences including *M2@UW* (45 participants), *Geometry and Arithmetic of Surfaces* (40 participants), *Graduate Workshop in Commutative Algebra for Women & Mathematicians of Minority Genders (GWCAWMMG)* (35 participants), *CAZoom* (70 participants), *Western Algebraic Geometry Symposium* (100 participants), *GEMS of Combinatorics* (40 participants), *Spec( $\mathbb{Q}$ )* (50 participants), *BATMOBILE* (30 participants), *GEMS of Combinatorics II* (30 participants), and *GEMS of Commutative Algebra* (40 participants). Additionally, I organized three special sessions at AMS Sectional Meetings and the Joint Math Meetings.

Multiple of these conferences were aimed at supporting mathematicians from underrepresented groups. For example, *GWCAWMMG* focused on forming communities of women and non-binary researchers, and *Spec( $\mathbb{Q}$ )* highlighted the research of LGBTQ+ mathematicians in algebra, geometry, and number theory. Further, the “GEMS” workshops sought to build a diverse community of mathematicians to address gender equity in the mathematics from new perspectives. I am interested in expanding these “GEMS” workshops to other areas of mathematics to create cross-field discussions broadening the notion of gender equity in mathematics.

**III. National & International Advocacy.** As a postdoc, I looked to deepen the impact of my work by attempting to promote underrepresented groups in mathematics beyond just campus. For example, I have worked with the Executive Committee of the *Association for Women in Mathematics* to consider ways they could expand their support of women and non-binary mathematicians. In Winter 2023 I joined MSRI's *Committee on Women in Mathematics*. Since Fall 2020 I have organized *Trans Math Day*, an annual virtual conference promoting the work of transgender and non-binary mathematicians. This conference regularly has 50 participants. Highlighting the importance of such conferences one participant said, “I’ve been really considering leaving mathematics. [Trans Math Day] reminded me why I’m here and why I want to stay. If a conference like this had been around for me five years ago, my life would have been a lot better.” Further, I have been a board member for *Spectra: The Association for LGBTQ+ Mathematicians* since 2020, including as the inaugural president in 2022. In this role, I have overseen the growth and formalization of the organization, including the creation and adoption of bylaws, the creation of an invited lecture at the Joint Mathematics Meetings, and a \$20,000+ fundraising campaign. *Spectra* has 500 members.

Going forward I am excited to continue my work supporting LGBTQ+ students, and would love to continue building organizations to do so. In particular, given the amazing successes of programs like MSRI-UP and the EDGE Program, I would love to organize a summer REU program specifically aimed at supporting and promoting LGBTQ+ mathematicians. Further, I am in the early stages of planning a mentorship program to help guide LGBTQ+ undergraduates through the process of applying to graduate programs in mathematics and helping young LGBTQ+ graduate students establish themselves.

**IV. Mentoring.** Inspired by the mentoring that helped me navigate the challenges of being a woman in mathematics, I have worked hard to mentor people from underrepresented groups. While a graduate student, I led reading courses with three undergraduates. One of these students, an undergraduate woman, worked with me for over a year, during which time I helped her apply for summer research projects. Working with *Girls' Math Night Out* I lead two girls in high school through a project exploring cryptography. During 2018-2019, I mentored 6 first-year graduate students (all women, non-binary students, or students of color).

As a postdoc, I began research projects with three graduate students (a majority of whom identify with a generally underrepresented group). These projects have resulted in two pre-prints, with additional projects still ongoing. During 2022, I did a reading course with a first-year graduate woman on algebraic geometry. Additionally, I advised two summer research projects for undergraduate students. The first of these projects ran virtually during Summer 2021 with 6 undergraduates. In Summer 2022 I advised an undergraduate student on a research project related to my work on syzygies. This work is ongoing and will hopefully result in a paper. This student is now in graduate school for math and was awarded an NSF Graduate Fellowship.