Discussion Syllabus

Course: Math 221 - Calculus I
Course Homepage: See Canvas.

Instructor: Juliette Bruce
Email: `juliette.bruce@math.wisc.edu`
Office: Van Vleck 418
Office Hours: M 10:00am-11:00am, T 1:00pm-2:00pm, by appointment

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Welcoming Message

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treated with dignity and respect and that discrimination and harassment will not be tolerated. We further commit ourselves to making the department a supportive, inclusive, and safe environment for all students, faculty, staff, and visitors, regardless of race, religion, national origin, sexual orientation, gender identity, disability, age, pregnancy, or any other aspect of identity. For more information, see `https://math.wisc.edu/climate`.

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Academic Integrity

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Wisconsin Emerging Scholars (WES) Syllabus

Course: Math 228 - WES Calculus Supplement
Course Homepage: See Canvas.
Times & Locations: MWF 11:00am - 11:50am in 38 Ag Hall

Instructor: Juliette Bruce
Email: juliette.bruce@math.wisc.edu
Office: Van Vleck 418
Office Hours: M 12:00pm-1:00pm, Th 10:00am-11:00am, by appointment

Course Designations: None
Instructional Mode: Blended
Credits: 2

Course Description

This course covers differential and integral calculus, plane analytic geometry, applications; transcendental functions, etc. The Math 221-222 sequence is the first two semesters of the standard three-semester calculus sequence, completed with 234, which is normally required for all higher level math courses and should be taken by those preparing for major study in mathematics, the physical sciences, computer sciences, or engineering. It is also recommended for students in the social and life sciences who may want a more substantial introduction to calculus than is offered in the Math 211-213 sequence.

Prerequisites

The prerequisites for this course as listed on the course guide are: (I) Advanced math competency-algebra and suitable placement scores, or Math 112 and (II) Advanced math competency-trigonometry and suitable placement scores, or Math 113; or Math 114. These prerequisites are enforced by the department, and I have no control over this. If you feel you are in the wrong course please see the "Placement" below.

Learning Objectives

By the end of Math 228 you should be able to:

- Apply differential calculus to quantify rates of change, and in particular to model physical and biological phenomena.
- Analyze the behavior of functions of one variable, including their asymptotic behavior, local behavior and existence of extrema.
- Apply integral calculus to model the cumulative effects of continuous processes.
- Articulate mathematical knowledge and understanding of differential and integral calculus in a written context.

How Credit Hours are met by the Course:

The two credit hours are met by two 50-minute meetings and a minimum of four hours of out of class student work per week for 15 weeks.

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Grading

Since you are in this section that means you are actually enrolled in two courses Math 221 and Math 228. As such you will be given two grades at the end of the semester:

- Your grade for the five credits of Math 221 will be determined as laid out in the Math 221 syllabus. This breakdown will be the same for all discussions in Prof. Meyer's lecture. If you have any questions regarding your grade for Math 221 please consult the course syllabus, or email me.
- Your grade for the two credits of Math 228 will be based on four components:
  1. Homework 60%: Throughout the semester I will occasionally give short homework assignments, which will be collected and graded. These homework assignments are not meant to be tedious and, or to substantially to your workload. They are intended to be a way to help hold you accountable to the material, and help me know how students are understanding the material.
  2. Team Quizzes 25%: At various points throughout the semester we will have short team quizzes. These quizzes will always be announced at least one class period in advance.
  3. Office Hours 15%: Throughout the semester I expect you attend a office hours at least twice. One of your visits to office hours must occur before October 10, 2018 – the date of the first exam. If you have conflicts with my scheduled office hours, or do not believe you will be able to attend two office hours for some reason, please contact me as soon as possible.
  4. Attendance: Attendance in discussion is mandatory, and will count towards your final grade in Math 228. You are allowed four absences, barring emergency, before your grade is affected. For each absence after your fourth your final grade will be lowered by half of a letter grade. For example, if at the end of the semester your grade based on the above components (worksheets/homework, team quizzes, and office hours) is an A/B and you miss five discussions then your final grade will be a B; miss six discussions and your grade would be a B/C, etc. *If you know you have to miss a discussion, especially a number of discussions, please talk to me as soon as possible.*

Grading Scale

The grading scale for this course will be no less lenient than the standard university-wide grading scale:

- A - 90%
- A/B - 88%
- B - 80%
- B/C - 78%
- C - 70%
- D - 60%
- F - less than 60%

As the instructor I reserve the right to modify this grading scale, and "curve" the course, at the end of the semester. However, if I do I it will only work in your favor.

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Worksheet #17 - A Famous Optimization Problem About Slides

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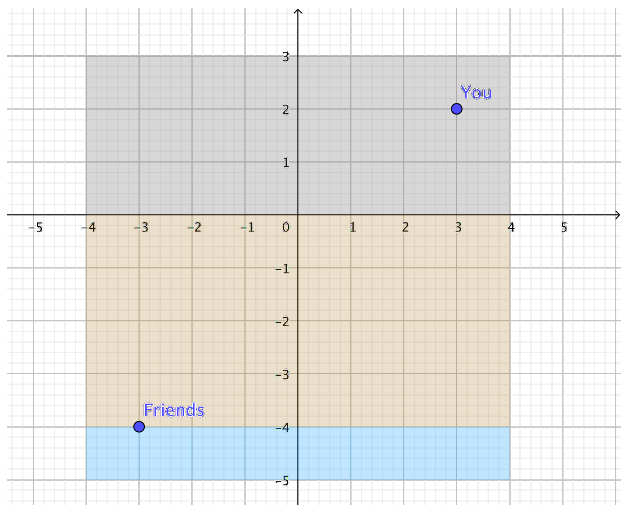
Running Through the Beach

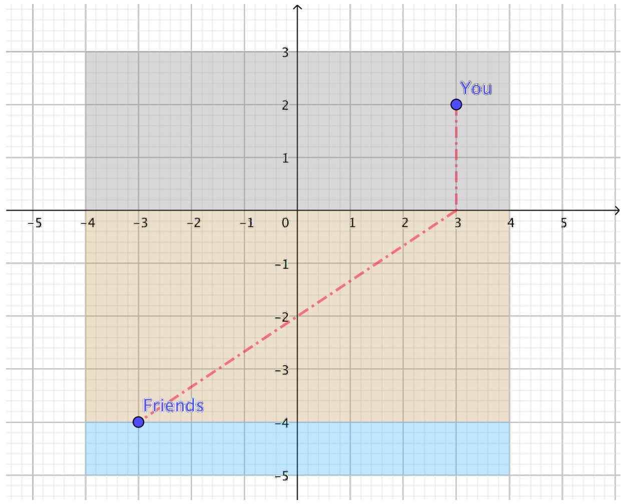
1. On vacation you arrive at the beach, and cannot wait to join your friends in the water and cool off. In order to join your friends in the lake you need to run across the parking lot and then across the beach as shown in the diagram below. (All units are in meters.)



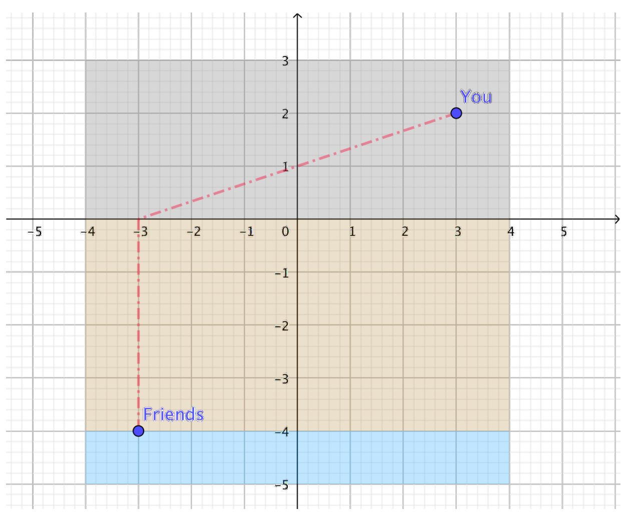
(a) How far are your friends from you when you are standing in the parking lot.

(b) Suppose that in the parking lot you can run at a (constant) speed of 2 m/s, but on the beach the sand slows you down, and you are only able to run at a (constant) speed of .75 m/s. How long would it take you reach your friends if you take the shortest possible path?

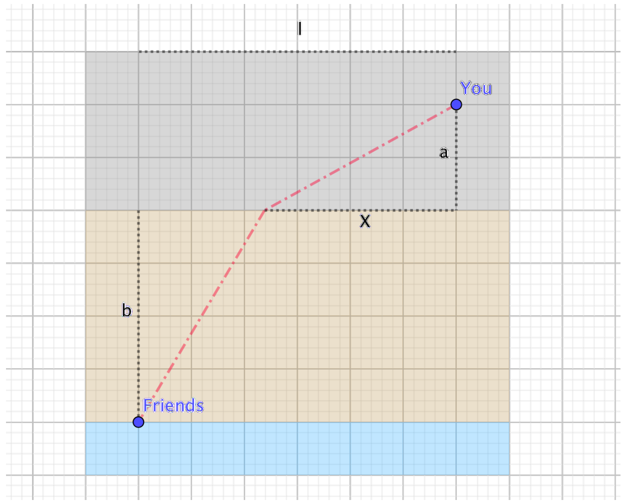
(c) How long would it take you to reach your friends if you take the path shown in red?



(d) How long would it take you to reach your friends if you take the path shown in red?

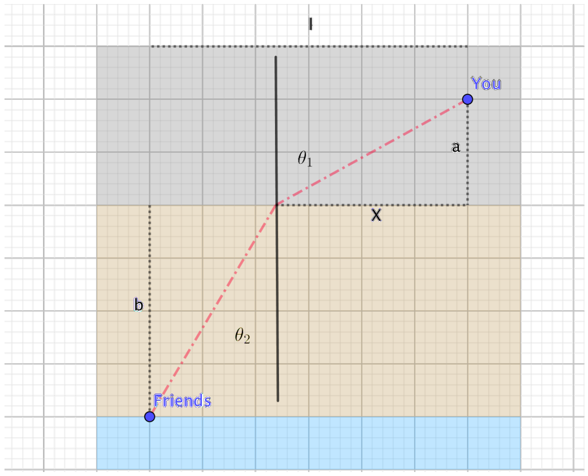


(e) We now wish to find the distance for a "general" path, and how long it will take us to traverse this path and reach our friends. Write down expressions for the distance of the path, and for the how long it will take you to run the path in terms of a, b, x, and ℓ.



(f) Since a, b, and ℓ are all constants we may view the expressions writen above as functions of x. Thinking of the time it takes you to reach your friends as a function of x compute its derivative.

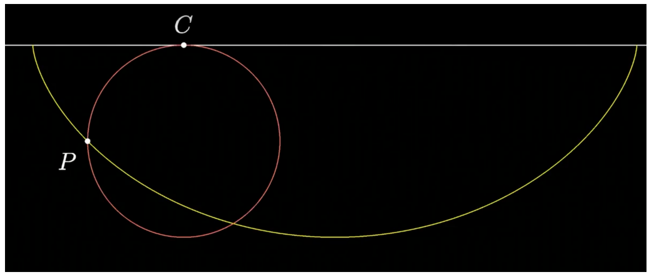
(g) In the diagram below solid black line is perpendicular to the boundary between the parking lot and the beach. Compute $\sin(\theta_1)$ and $\sin(\theta_2)$ as in the diagram below in terms of a, b, x and ℓ:



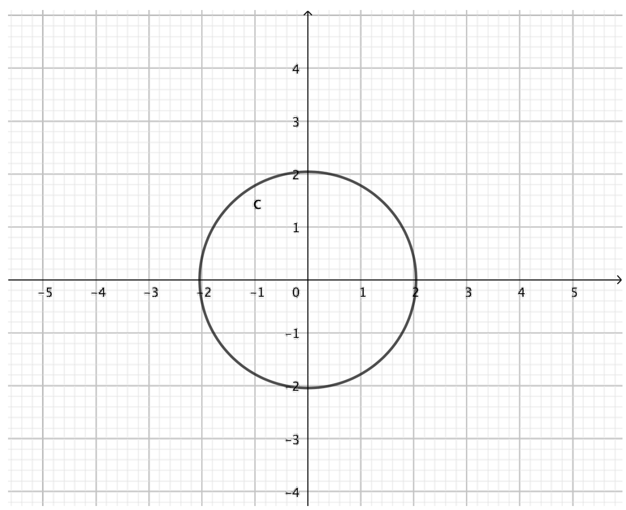
(h) Combining parts (f) and (g) deduce a condition on $\sin(\theta_1)$ and $\sin(\theta_2)$ minimizing the total amount of time it takes you to reach your friend.

A Hard Geometry Problem

1. Imagine a circle (pink) of radius D rolling along the ceiling with a point marked P on the circle. As the circle rolls the path of point P traces out a curve (yellow) as shown in the picture below. The goal of this exercise is to begin exploring the path.

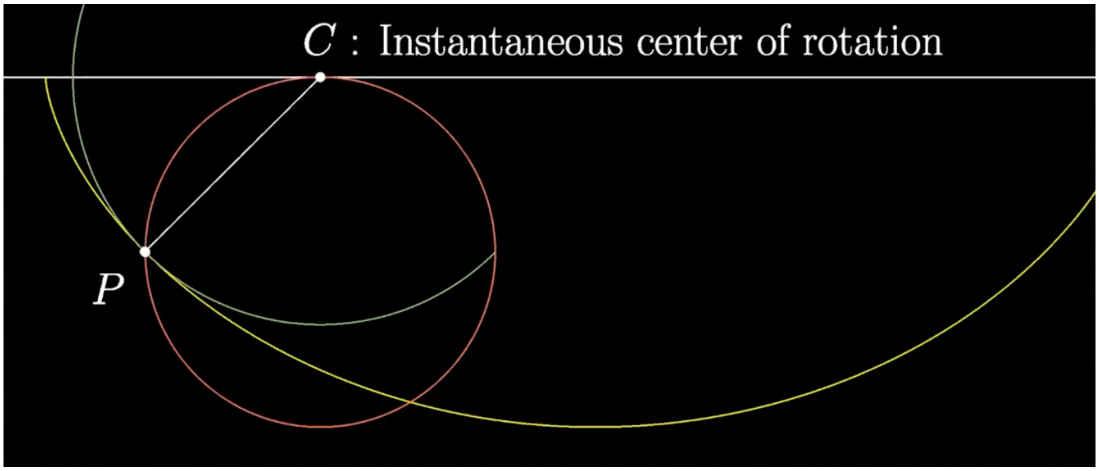


(a) Consider the circle given by the equation $x^2 + y^2 = 4$. Compute the equation of the line going through the point $(0, 0)$ and the point $(\sqrt{2}, \sqrt{2})$. Compute the equation of the line tangent to the circle at the point $(\sqrt{2}, \sqrt{2})$. Draw both on the diagram below. Do you see a relationship?

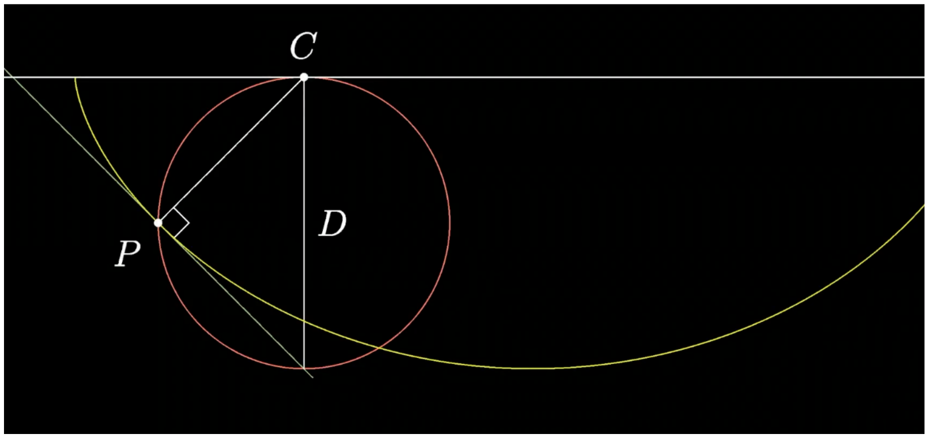


(b) More generally can you guess a relationship between the tangent line to a circle at a point A and a line of radius passing through point A?

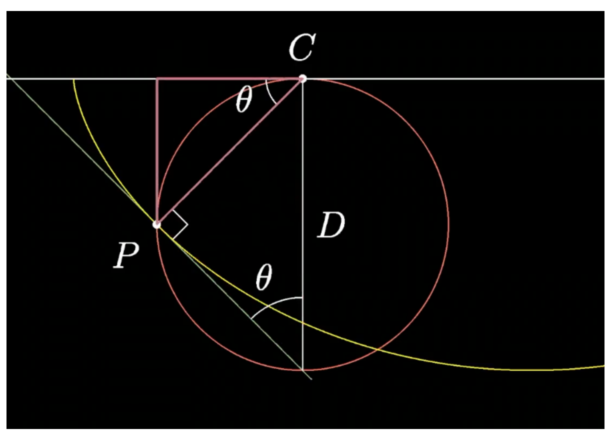
(c) Consider the line segment between the point C and point P. Drawing a circle (green) centered at C which has CP as a radius gives the picture below. What can we saw about the line tangent to the yellow curve at P and the line segment CP?



(d) Using our answer to the previous part we know we can draw a right triangle inside the pink circle. Using this triangle find the length of CP in terms of D and θ



(e) Finally using similar triangles show find the distance P is from the ceiling (i.e. the horizontal line).



Worksheet #2 - Function Composition & Trigonometric Functions

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Warm-Up

1. Find the equation of the line passing through the points $(11, 23)$ and $(6, 5)$.

2. Find the equation of the line passing through the points $(2, 3)$ and $(-13, 17)$.

3. Find the equation of the line passing through the points $\left(1, \frac{1}{2}\right)$ and $\left(\frac{3}{1}, 1\right)$.

4. Find the equation of the line passing through the points $\left(\frac{3}{5}, \frac{2}{3}\right)$ and $(7, 12)$.

5. Factor $x^3 + 2x^2 + x$.

6. Factor $3y^2 - 8y + 5$.

7. Factor $x^3 + 3x^2 - 4$.

8. Factor $x^2 + \frac{x}{4} - \frac{3}{8}$.

9. Factor $t^4 + 10t^3 + 35t^2 + 50t + 24$.

10. Find the equation of the line parallel to the line $y = \frac{1}{8}x + 11$ passing through the point $(4, 5)$.

11. Find the equation of the line perpendicular to the line give by $y = 2x + 3$ passing through the point $(1, 2)$.

Function Composition & Inverse Functions

If f and g are functions, then the composition of f and g, which we denote $(f \circ g)(x)$, is defined to be $(f \circ g)(x) = f(g(x))$. Further if we have the that $f(g(x)) = x$ for all x in the domain of g and $g(f(y)) = y$ for all y in the domain of f, we say that g is the "inverse function" of f. If f has an inverse function we generally call it $f^{-1}(x)$.

1. In general $(f \circ g)$ is not equal to $(g \circ f)$. Find two functions f and g such that $(f \circ g) \neq (g \circ f)$.

2. If $f(x) = \dfrac{1}{x+1}$, find $\underbrace{f(f(\cdots\cdots(f(}_{\text{2018 times}}x))\cdots)$ and $\underbrace{f^{-1}(f^{-1}(\cdots\cdots(f^{-1}(}_{\text{2018 times}}x))\cdots)$.

3. Find a function which is equal to its own inverse function.

4. Find a function, which *does not* have an inverse.

0.1 Graphing Inverse Functions

1. Let $f(x)$ be the function described by the table below. Completely the table describing $f^{-1}(x)$.

x	3	10	25	100
$f(x)$	2	4	9	25

x				
$f^{-1}(x)$				

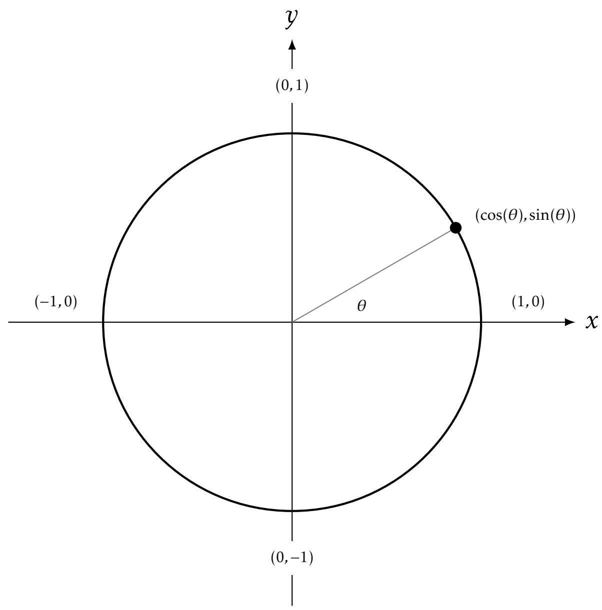
2. If $h(t) = 3t - 1$ find $h^{-1}(t)$. Graph both $h(t)$ and $h^{-1}(t)$.

3. Suppose $f(x)$ is an invertible function. Can you describe how the graph $y = g(x)$ and $x = g^{-1}(y)$ relate?

4. Show that if f is a function which has an inverse, then $f(a) = f(b)$ implies $a = b$. Using this find a way to test whether a function $f(x)$ has an inverse by looking at the graph of $y = f(x)$.

Trigonometric Functions

One way of defining the trigonometric functions $\sin(\theta)$ and $\cos(\theta)$ is via the unit circle. In particular, if we draw the circle of radius one centered at the origin (i.e. the circle given by the equation $x^2 + y^2 = 1$) then line making an angle θ with the positive x-axis intersects this circle at the point $(\cos(\theta), \sin(\theta))$.



1. Using the unit circle definition of sin and cos explain why the following identities are true:

(a) $\sin(\theta + \pi) = -\sin(\theta)$

(b) $\cos(\theta + \pi) = -\cos(\theta)$

(c) $\sin\left(\theta \pm \frac{\pi}{2}\right) = \pm\cos(\theta)$

(d) $\cos\left(\theta \pm \frac{\pi}{2}\right) = \pm\sin(\theta)$.

2. Explain why the function $f(\theta) = \sin(\theta)$, defined for all real numbers θ, *doesn't* have an inverse function, but the function $g(\theta) = \sin(\theta)$, defined only for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, *does* have an inverse function.

3. How would you compute the inverse of $\sin(\theta)$ for any value of θ?

4. Using the previous two exercises we found a way to compute the inverse of $\sin(\theta)$ for any value of θ. Can you do the same for $\cos(\theta)$? How about $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$?

5. Let $f(\theta) = \sin(\theta)$. Write down a function $m(a)$ describing the slope of the secant line between the points $(0, f(0))$ and $(a, f(a))$. (Hint: your answer may be a function of θ and a.)