Juliette Bruce's Research Statement

The goal of this project would be to explore monomial ideals in multigraded polynomial rings with certain extremal properties. For example, in the standard $\mathbb{Z}$-graded setting, classical results of Macaulay and Bigatti, Hulett, and Pardue show that out of all ideals with a fixed Hilbert function Lex monomial ideals have the largest possible Hilbert functions and graded Betti numbers. For more general gradings no analog is fully known, although there has been recent work of Maclagan and Smith exploring the case of two-dimensional polynomial rings. My suggestion for a project would be to explore how these results might be generalized to other gradings. Progress in this direction might also have interesting implications for the geometry of multigraded Hilbert schemes.