A projective variety $X \subset \mathbb{P}^n$, is defined by a homogenous ideal I_X in the standard $\mathbb{Z}$-graded polynomial ring $\mathbb{C}[x_0, \ldots, x_n]$. The minimal graded free resolution of I_X often captures not only interesting algebraic information about the defining equations of X, but also infromation about the extrinsic and intrinsic geometry of X. When $\mathbb{P}^n$ is replaced by another toric variety X the ways minimal multigraded free resolutions over the Cox ring of X capture algebraic and geometric information about subvarieties of X remains quite mysterious. Much of my recent work has focused on developing tools in multigraded homological algebra to better understand the geometry of toric varieties. As a taste of this I will focus on my recent work studying the homological properties of multigraded Castelnuovo–Mumford regularity.

Castelnuovo–Mumford regularity is a measure of geometric complexity that has proven extremely useful in studying subvarieties of projective space. As introduced originally introduced by Mumford the regularity of a coherent sheaf $\mathcal{F}$ on $\mathbb{P}^n$ is given in terms of certain cohomological vanishing conditions. Roughly, one can think of the regularity of $\mathcal{F}$ as being an effective bound for Serre vanishing. Mumford was interested such a measure as it plays a key role in constructing Hilbert schemes. However, as the following result of Eisenbud and Goto shows it is also closely connected to minimal graded free resolutions.

Theorem A (Eisenbud-Goto)**.** *Let $\mathcal{F}$ be a coherent sheaf on $\mathbb{P}^n$ and $M = \bigoplus_{e \in \mathbb{Z}} H^0(\mathbb{P}^n, \mathcal{F}(e))$ the corresponding section ring. The following are equivalent: (1) M is d-regular; (2) $\operatorname{Tor}_p^S(M, \mathbb{C})_q = 0$ for all $p \geq 0$ and $q > d + i$; (3) $M_{\geq d}$ has a linear minimal graded free resolution.*

Maclagan and Smith introduced a generalization of Castelnuovo–Mumford regularity to coherent sheaves on other toric varieties. Following Mumford they define the *multigraded Castelnuovo–Mumford regularity* of a coherent sheaf $\mathcal{F}$ on a toric variety X in terms of certain cohomological vanishing. For example, if $X = \mathbb{P}^{n_1} \times \mathbb{P}^{n_2} \times \cdots \times \mathbb{P}^{n_r}$ then $\mathcal{F}$ is $\mathbf{d}$-regular if and only if

$$H^i\left(\mathbb{P}^{\mathbf{n}}, \mathcal{F}(\mathbf{e})\right) = 0 \qquad \text{for all } \mathbf{e} \in L_i(\mathbf{d}) = \bigcup_{\mathbf{v} \in \mathbb{N}, |\mathbf{v}| = i} (\mathbf{d} - \mathbf{v}) + \mathbb{N}^r.$$

My collaborators and I have worked to understand how multigraded Castelnuovo–Mumford regularity can be characterized in terms of minimal multigraded free resolutions. Containing focus on the the case when X is a product of projective spaces, let $X = \mathbb{P}^{n_1} \times \mathbb{P}^{n_2} \times \cdots \times \mathbb{P}^{n_r}$ and $S = \mathbb{C}[x_{i,j} \mid 1 \leq i \leq r, 0 \leq j \leq n_i]$ be the corresponding Cox ring with the $\mathbb{Z}^r$-grading given by $\deg x_{i,j} = \mathbf{e}_i \in \mathbb{Z}^r$, where $\mathbf{e}_i$ is the i-th standard basis vector in $\mathbb{Z}^r$.

We were able to find examples showing that generalizing Theorem A to mulitgraded regularity is subtle. For example, the multigraded Betti numbers of a module do not determine its multigraded Castelnuovo–Mumford regularity, and M being $\mathbf{d}$-regular does not imply that the truncation $M_{\geq \mathbf{d}}$ does not have a linear free resolution. However, we show that multigraded regularity can be characterized by the truncation $M_{\geq \mathbf{d}}$ having a certain type of free resolution, which generalizes the notion of a linear resolutions. We say that a complex $F_\bullet$ of $\mathbb{Z}^r$-graded free S-modules is $\mathbf{d}$-quasilinear if and only if F_0 is generated in degree $\mathbf{d}$ and each twist of F_i is contained in $L_{i-1}(\mathbf{d} - \mathbf{1})$.

Theorem B. *Let M be a finitely generated $\mathbb{Z}^r$-graded S-module with $H^0_B(M) = 0$:*

$$M \text{ is } \mathbf{d}\text{-regular} \iff M_{\geq \mathbf{d}} \text{ has a } \mathbf{d}\text{-quasilinear resolution.}$$

The proof of Theorem B is based in part on a Čech–Koszul spectral sequence that relates the multigraded Betti numbers of $M_{\geq \mathbf{d}}$ to the Fourier–Mukai transform of $\widetilde{M}$.

This work together with additional work I have done studying the asymptotic behavior of multigraded regularity for powers of ideals, as well was work of Berkesch, Brown, Chardin, Erman, and Smith, suggest that further developing tools in multigraded homolgocal algebra will both highlight new algebraic phenomena as well as prove fruitful for understanding some of the rich geometry of toric varieties. . Currently, my collaborators and I are exploring other ways minimal multigraded free resolutions on toric varieties capture geometric and algebraic of subvarieties of toric varieties including via things like Bayer-Stillman type results for multigraded regularity and generalizations of Green's conjecture for canonical stacky curves.

Relevance of Visit. Being a postdoctoral fellow at MSRI would be extremely beneficially towards my career goals as it would provide me with a stimulating mathematical environment, and the opportunity to engage and work with a number of leading researchers in commutative algebra. My research interest align very well with the topics of the program, and multiple projects I am working on directly touch upon many of the topics included in the program description For example, one project I am working on concerns better understanding multigraded Castelnuovo–Mumford regularity, and in a separate project I am exploring ways Green's conjecture can be generalized to canonical stacky curves. Being a postdoc at MSRI would likely prove valuable toward progressing both of these projects, as well as a fantastic opportunity to being new collaborations with other participants. Further, given the the ways my research interest align with the program I feel I would be able to find numerous ways to contribute to the program, for example, I would love to help organize a seminar and social actives for myself and the other postdocs.

I am specifically applying for a postdoctoral position, since my current position would make it extremely difficult for me to participate in the semester otherwise (i.e., as a research member or occasional visitor). Overall, being a postdoctoral fellow for this semester program in commutative algebra presents an amazing opportunity to connect and build relationships within the research area that has long felt like my research community and home. And such connections would significantly advance my goals of being a math professor at a research university.

Building Mathematical Community. I have worked hard to promote diversity, inclusivity, and justice in the mathematical community by mentoring students, supporting women and LGBTQ+ people in mathematics, and organizing conferences and outreach programs.

I have put significant effort into mentoring and working with students, especially those from underrepresented groups. During 2021 I organized a virtual summer research program for 6 undergraduates. In 2022 I advised an undergraduate student on a research project in commutative algebra. This student is now applying to graduate schools in math. I began research projects with multiple graduate students, in which I played a substantial mentoring role. In 2022, I led a reading course with a first-year graduate woman in commutative algebra.

Since 2020 I have organized an annual conference promoting the work of transgender and non-binary mathematicians, which regularly has over 50 participants. Highlighting the importance of such conferences one participant said, "I've been really considering leaving mathematics. [Trans Math Day 2020] reminded me why I'm here and why I want to stay. ... If a conference like this had been around for me five years ago, my life would have been a lot better." For the last two years I have been a board member for *Spectra: The Association for LGBTQ+ Mathematicians*, and currently I am the inaugural president. As a board member I led the creation and adoption of bylaws, the creation of an invited lecture at the Joint Mathematics Meetings, and a fundraising campaign that has raised over $20,000 to support LGBTQ+ students and mathematicians.

In response to the COVID-19 pandemic, I worked to find ways for these online activities to reach those often at the periphery. During 2020, I helped with Ravi Vakil's *Algebraic Geometry in the Time of Covid* project; an online open-access course in algebraic geometry binging together $\sim 2,000$ participants. In 2021, I organized a virtual undergraduate reading course in commutative algebra.

I have organized over 15 conferences, special sessions, and workshops including: *M2@UW* (45 participants), *CAZoom* (70 participants), *Western Algebraic Geometry Symposium* (100 participants), and Spec($\overline{\mathbb{Q}}$) (50 participants). When organizing these conferences I have given paid special attention to promoting women and other underrepresented groups in mathematics. For example, *Graduate Workshop in Commutative Algebra for Women & Mathematicians of Other Minority Genders* and *GEMS in Combinatorics* focused on forming communities of women and non-binary researchers in commutative algebra and combinatorics respectively. Further, Spec($\overline{\mathbb{Q}}$) highlighted the research of LGBTQ+ mathematicians in algebra, geometry, and number theory.