Castelnuovo–Mumford regularity is a measure of geometric complexity that has proven extremely useful in studying subvarieties of projective space. As introduced originally introduced by Mumford the regularity of a coherent sheaf $\mathcal{F}$ on $\mathbb{P}^n$ is given in terms of certain cohomological vanishing conditions. Roughly, one can think of the regularity of $\mathcal{F}$ as being an effective bound for Serre vanishing. Mumford was interested such a measure as it plays a key role in constructing Hilbert schemes. However, the connection between coherent sheaves on projective space and graded modules over the standard $\mathbb{Z}$-graded polynomial ring $S = \mathbb{C}[x_0, \ldots, x_n]$ has meant that Castelnuovo–Mumford regularity has proven a fruitful area of active research in commutative algebra.

In recent years an active area of research has been in developing the connections between toric geometry and multigraded commutative/homological algebra. This has included work by Maclagan and Smith generalizing Castelnuovo–Mumford regularity to coherent sheaves on other toric varieties. Following Mumford their definition of *multigraded Castelnuovo–Mumford regularity* in terms of certain cohomological vanishing. Much of my recent work has focused on exploring the algebraic properties of multigraded regularity.

In order to give a taste of some of my recent work in these directions. Let me focus on the following theorem of Eisenbud and Goto, which states that the Castelnuovo–Mumford regularity of a module over the standard graded polynomial ring S can be characterized solely in terms of homological properties of minimal graded free resolutions.

Theorem A (Eisenbud-Goto)**.** *Let $\mathcal{F}$ be a coherent sheaf on $\mathbb{P}^n$ and $M = \bigoplus_{e\in\mathbb{Z}} H^0(\mathbb{P}^n, \mathcal{F}(e))$ the corresponding section ring. The following are equivalent:*

(1) M is d-regular;

(2) $\operatorname{Tor}_p^S(M, \mathbb{C})_q = 0$ for all $p \geq 0$ and $q > d + i$;

(3) $M_{\geq d}$ has a linear resolution.

My collaborators and I have worked to understand how such a result can be extended to characterize multigraded Castelnuovo–Mumford regularity in terms of minimal resolutions. To give a taste of our results let us focus on the the case when X is a product of projective spaces. In particular, flet $X = \mathbb{P}^{n_1} \times \mathbb{P}^{n_2} \times \cdots \times \mathbb{P}^{n_r}$ and $S = \mathbb{C}[x_{i,j} \mid 1 \leq i \leq r, 0 \leq j \leq n_i]$ be the corresponding Cox ring with the $\operatorname{Pic}(X) \cong \mathbb{Z}^r$-grading given by $\deg x_{i,j} = \mathbf{e}_i \in \mathbb{Z}^r$, where $\mathbf{e}_i$ is the i-th standard basis vector in $\mathbb{Z}^r$.

Definition 0.1. *A coherent sheaf $\mathcal{F}$ on $\mathbb{P}^{\mathbf{n}}$ is $\mathbf{d}$-regular if and only if*

$$H^i\left(\mathbb{P}^{\mathbf{n}}, \mathcal{F}(\mathbf{e})\right) = 0 \qquad \textit{for all } \mathbf{e} \in L_i(\mathbf{d}) = \bigcup_{\mathbf{v}\in\mathbb{N}, |\mathbf{v}|=i} (\mathbf{d} - \mathbf{v}) + \mathbb{N}^r.$$

The multigraded Castelnuovo–Mumford regularity of $\mathcal{F}$ is then the set:

$$\operatorname{reg}(\mathcal{F}) \coloneqq \{\mathbf{d} \in \mathbb{Z}^r \;\mid\; \mathcal{F} \textit{ is } \mathbf{d}\textit{-regular}\} \subset \mathbb{Z}^r.$$

With my collaborators, I was able to find examples showing that generalizing Theorem A to mulitgraded regularity is quite subtle. For example, we showed that the multigraded Betti numbers of a module do not determine multigraded Castelnuovo–Mumford regularity, and there exists a finitely generated module M that is $\mathbf{d}$-regular, but where the truncation $M_{\geq \mathbf{d}}$ does not have a linear free resolution. this we show that part (3) of Theorem A can be generalized. To do so we introduce the following generalization of linear resolutions.

Definition 0.2. *A complex $F_\bullet$ of $\mathbb{Z}^r$-graded free S-modules is* $\mathbf{d}$*-quasilinear if and only if F_0 is generated in degree* $\mathbf{d}$ *and each twist of F_i is contained in $L_{i-1}(\mathbf{d}-\mathbf{1})$.*

Theorem B. *Let M be a finitely generated $\mathbb{Z}^r$-graded S-module with $H^0_B(M) = 0$:*

$$M \text{ is } \mathbf{d}\text{-regular} \iff M_{\geq \mathbf{d}} \text{ has a } \mathbf{d}\text{-quasilinear resolution.}$$

The proof of Theorem **??** is based in part on a Čech–Koszul spectral sequence that relates the Betti numbers of $M_{\geq \mathbf{d}}$ to the Fourier–Mukai transform of $\widetilde{M}$ with Beilinson's resolution of the diagonal as the kernel. Precisely, if M is $\mathbf{d}$-regular and $H^0_B(M) = 0$ we prove the that

$$\dim_{\mathbb{C}} \operatorname{Tor}^S_j(M_{\geq \mathbf{d}}, \mathbb{C})_{\mathbf{a}} = h^{|\mathbf{a}|-j}\big(\mathbb{P}^{\mathbf{n}}, \widetilde{M} \otimes \mathcal{O}^{\mathbf{a}}_{\mathbb{P}^{\mathbf{n}}}(\mathbf{a})\big) \quad \text{for } |\mathbf{a}| \geq j \geq 0,$$

where the $\mathcal{O}^{\mathbf{a}}_{\mathbb{P}^{\mathbf{n}}}$ are cotangent sheaves on $\mathbb{P}^{\mathbf{n}}$. The result then follows from showing that M being $\mathbf{d}$-regular is equivalent to certain vanishings of the right-hand side above.

Relevance of Visit. Being a postdoctoral fellow at MSRI would be extremely beneficially towards my career goals as it would provide me with a stimulating mathematical environment, and the opportunity to engage and work with a number of leading researchers in commutative algebra. My research interest align very well with the topics of the program, and multiple projects I am working on directly touch upon many of the topics included in the program description For example, one project I am working on concerns better understanding multigraded Castelnuovo–Mumford regularity, and in a separate project I am exploring ways Green's conjecture can be generalized to canonical stacky curves. Being a postdoc at MSRI would likely prove valuable toward progressing both of these projects, as well as a fantastic opportunity to being new collaborations with other participants. Further, given the the ways my research interest align with the program I feel I would be able to find numerous ways to contribute to the program, for example, I would love to help organize a seminar and social actives for myself and the other postdocs.

I am specifically applying for a postdoctoral position, since my current position would make it extremely difficult for me to participate in the semester otherwise (i.e., as a research member or occasional visitor). Overall, being a postdoctoral fellow for this semester program in commutative algebra presents an amazing opportunity to connect and build relationships within the research area that has long felt like my research community and home. And such connections would significantly advance my goals of being a math professor at a research university.

Building Mathematical Community. I have worked hard to promote diversity, inclusivity, and justice in the mathematical community by mentoring students, supporting women and LGBTQ+ people in mathematics, and organizing conferences and outreach programs.

I have put significant effort into mentoring and working with students, especially those from underrepresented groups. During Summer 2021 I organized a virtual summer undergraduate research program for 6 undergraduates. In 2022 I advised an undergraduate student on a research project in commutative algebra. This student is now applying to graduate schools in math. I began research projects with multiple graduate students, in which I played a substantial mentoring role. In 2022, I led a reading course with a first-year graduate woman in commutative algebra.

Since 2020 I have organized an annual conference promoting the work of transgender and non-binary mathematicians, which regularly has over 50 participants. Highlighting the importance of such conferences one participant said, "I've been really considering leaving mathematics. [Trans Math Day 2020] reminded me why I'm here and why I want to stay. ... If a conference like this had been around for me five years ago, my life would have been a lot better." For the last two years I have been a board member for *Spectra: The Association for LGBTQ+ Mathematicians*, and currently I am the inaugural president. As a board member I led the creation and adoption of bylaws, the creation of an invited lecture at the Joint Mathematics Meetings, and a fundraising campaign that has raised over $20,000 to support LGBTQ+ students and mathematicians.

In response to the COVID-19 pandemic, I worked to find ways for these online activities to reach those often at the periphery. During Summer and Fall 2020, I helped with Ravi Vakil's *Algebraic Geometry in the Time of Covid* project. This massive online open-access course in algebraic geometry brought together $\sim 2,000$ participants from around the world. In Spring 2021, I organized an 8-week virtual reading course for undergraduates in algebraic geometry and commutative algebra.

I have organized over 15 conferences, special sessions, and workshops including: *M2@UW* (45 participants), *Graduate Workshop in Commutative Algebra for Women & Mathematicians of Minority Genders* (35 participants), *CAZoom* (70 participants), *Western Algebraic Geometry Symposium* (100 participants)x, and Spec($\overline{\mathbb{Q}}$) (50 participants). When organizing these conferences I have given paid special attention to promoting women and other underrepresented groups in mathematics.

For example, *Graduate Workshop in Commutative Algebra for Women & Mathematicians of Other Minority Genders* and *GEMS in Combinatorics* focused on forming communities of women and non-binary researchers in commutative algebra and combinatorics respectively. Further, $\mathrm{Spec}(\overline{\mathbb{Q}})$ highlighted the research of LGBTQ+ mathematicians in algebra, geometry, and number theory.