Juliette Bruce's Statement of Purpose

Overview. My research focuses on questions in commutative algebra, algebraic geometry, and arithmetic geometry, and during my stay at MSRI I am proposing to work on three projects in these areas. Two projects involve studying the geometry of algebraic varieties via homological commutative algebra. A third project lies in the intersection of algebraic geometry and arithmetic geometry and seeks to extend classical results in algebraic geometry to finite fields.

Intellectual Merit. My first project seeks to expand our understanding of syzygies of algebraic varieties from both the asymptotic and computational perspectives. In recent years substantial work has been done studying the syzygies of algebraic varieties as the positivity of the embedding grows. This project builds upon my thesis and seeks to extend these results by weakening the positivity conditions previously considered. In particular, the I will explore the asymptotic syzygies of varieties in the setting of semi-ample growth. Further, by utilizing new advances in high-performance computing and numerical linear algebra this project will generate new data regarding the syzygies of Hirzebruch surfaces. This data will be publicly disseminated via the online syzygy database *syzygydata.com*, which I maintain.

A second project I propose working on while visiting MSRI will deepen our understanding of curves in toric 3-folds, for example, curves in $\mathbb{P}^1 \times \mathbb{P}^2$, by generalizing existing results about the liaison theory of curves in $\mathbb{P}^3$. This project will make use of recent developments in the homological properties of varieties embedded in spaces other than $\mathbb{P}^n$. This project has the potential to yield applications to the study of toric varieties and toric vector bundles, as well as potential applications to unirationality of $\mathcal{M}_g$ or other moduli spaces.

The third project proposes builds upon recent work generalizing classical Bertini Theorems in algebraic geometry to the setting of finite fields. More specifically, this project hopes to prove more general and uniform Bertini Theorems over finite fields. Such results will shed light on both the geometry and arithmetic of varieties of finite fields, and potentially prove useful in studying things like rational points on varieties.

Relevance of Visit. Visiting MSRI would be extremely beneficially towards my career goals as it would provide me with a stimulating mathematical environment, and the opportunity to work with David Eisenbud. Working with David Eisenbud would provide the opportunity to learn from one of the foremost experts in algebraic geometry and commutative algebra. Eisenbud has done foundational work concerning the syzygies of varieties, liaison theory, and the geometry of curves. Working with him would likely prove valuable towards the first two projects I plan on working on while visiting MSRI. In short, working with Eisenbud would significantly advance my goals of being a math professor at a research university.

Additionally, MSRI is located near both the University of California, Berkeley and Stanford University, both of which have a large number of faculty in algebraic geometry and arithmetic geometry (Martin Olsson, Melanie Matchett Wood, Ravi Vakil, etc.). This will provide a rich mathematical environment. In particular, both Melanie Matchett Wood and Ravi Vakil are experts on Bertini theorems over finite fields, and being near them would likely prove valuable toward my third project.