Juliette Bruce's Project Summary

Overview. The PI proposes to study questions in commutative algebra, algebraic geometry, and arithmetic geometry. Two projects involve studying the geometry of algebraic varieties via homological commutative algebra. A third project lies in the intersection of algebraic geometry and arithmetic geometry and seeks to extend classical results in algebraic geometry to finite fields.

Intellectual Merit. The PI's first project seeks to expand our understanding of syzygies of algebraic varieties from both the asymptotic and computational perspectives. In recent years substantial work has been done studying the syzygies of algebraic varieties as the positivity of the embedding grows. This project builds upon the PI's thesis and seeks to extend these results by weakening the positivity conditions previously considered. In particular, the PI will explore the asymptotic syzygies of varieties in the setting of semi-ample growth. Further, by utilizing new advances in high-performance computing and numerical linear algebra this project will generate new data regarding the syzygies of Hirzebruch surfaces. This data will be publicly disseminated via the online syzygy database *syzygydata.com* maintained by the PI.

A second project proposed to by the PI will deepen our understanding of curves in toric 3-folds, for example, curves in $\mathbb{P}^1 \times \mathbb{P}^2$, by generalizing existing results about the liaison theory of curves in $\mathbb{P}^3$. This project will make use of recent developments in the homological properties of varieties embedded in spaces other than $\mathbb{P}^n$. This project has the potential to yield applications to the study of toric varieties and toric vector bundles, as well as potential applications to unirationality of $\mathcal{M}_g$ or other moduli spaces (similar to [BS15, Theorem 4.5]).

The third project the PI proposes builds upon recent work generalizing classical Bertini Theorems in algebraic geometry to the setting of finite fields. More specifically, this project hopes to prove more general and uniform Bertini Theorems over finite fields. Such results will shed light on both the geometry and arithmetic of varieties of finite fields, and potentially prove useful in studying things like rational points on varieties.

Broader Impacts. As an LGBTQ woman, the PI has worked hard to promote diversity, inclusivity, and justice in the mathematical community. This proposal will further the PI's work in this direction by her continued involvement as a mentor to multiple undergraduate women via the Association for Women in Mathematics's mentor network. The PI also plans to mentor undergraduates via the Berkeley Directed Reading Program and the MSRI-Up program.

At the 2018 Joint Mathematica Meeting, the PI served on a panel organized by *Spectra*, the association of LGBTQ+ mathematicians, titled *Professional Issues Facing LGBTQ Mathematicians*. At the 2020 Joint Meetings, the PI is organizing a *Spectra* panel on transgender inclusion in mathematics. Going forward the PI plans to continue being involved in *Spectra*, potentially stepping into more leadership and organizing roles. The PI has also founded and organized numerous groups for LGBTQ+ students at the University of Wisconsin. Since 2017 she has organized the campus social group for LGBTQ+ graduate students, which has over 350 members. While at Berkeley she plans to be involved in similar organizing efforts.

The PI has organized a number of conferences including the *Graduate Workshop in Commutative Algebra for Women and Mathematicians of Minority Genders* (2019), a workshop bringing together algebraic geometers and number theorists *Geometry & Arithmetic of Surfaces* (2019), and a five day conference dedicated to developing open-source computer software for algebraic geometry and commutative algebra *M2@UW* (2018). The PI plans to organize a follow-up to *Graduate Workshop in Commutative Algebra for Women and Mathematicians of Minority Genders* tentatively planned for Spring 2021, as well as a conference for LGBTQ+ mathematicians in algebraic geometry and commutative algebra.