BROWN

Juliette Bruce
Postdoctoral Researcher
Department of Mathematics
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November 9, 2023

Dear Committee Members,

I am writing to apply for the tenure-track position in number theory and related areas in the Department of Mathematics and Statistics at McGill University. Currently, I am a postdoctoral researcher in the Mathematics Department at Brown University, a position I have held since August 2022. I received my Ph.D. in Mathematics from the University of Wisconsin-Madison under the guidance of Professor Daniel Erman in 2020. From 2020-2022 I was an NSF Postdoctoral Fellow in the Mathematics Department at the University of California, Berkeley. Additionally, I was a postdoctoral fellow at the Mathematical Sciences Research Institute – now the Simons Laufer Mathematical Sciences Institute – for the 2020-2021 academic year.

My research interests lie in the intersection of algebraic geometry and commutative algebra with connections to combinatorics and number theory. I am interested in using homological, combinatorial, and computational methods to study the geometry of algebraic varieties. Currently, my research program has two broad directions.

(i) I have sought to deepen and expand our understanding of the ways homological algebra can be used to study the geometry of toric varieties. This seeks to generalize a very classical story using homological algebra to understand subvarieties of projective space.

(ii) I have been studying the geometry and topology of various moduli spaces, e.g., the moduli space of (principally polarized) abelian varieties of a fixed dimension, via combinatorially and homological methods. This has led to novel applications to the cohomology of certain arithmetic groups.

Further, I am passionate about promoting inclusivity, diversity, and justice in the mathematics community. This passion extends throughout my teaching where I am dedicated to creating an interactive and supportive classroom environment that helps students thrive.

My research output includes 15 papers, with publications in journals such as *Algebra & Number Theory*, *Geometry & Topology*, and *Experimental Mathematics*, as well as, multiple published software packages. Below are a few of the non-research highlights of my file.

- I was awarded an *NSF Postdoctoral Research Fellowship*, an *NSF Graduate Research Fellowship*, and I have secured over $100,000 in conference grants, including 4 NSF conference grants.
- I have organized 12+ conferences, workshops, and special sessions, including multiple events aimed at supporting and promoting mathematicians from generally underrepresented groups, especially women and LGBTQ+ mathematicians.
- I was awarded the highest departmental and campus-wide teaching awards at the University of Wisconsin-Madison, the Capstone Teaching Award (2019) and the Teaching Assistant Award for Exceptional Service (2018), awarded to 1 and 3 students each year respectively.

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With my application, I include a curriculum vitae, a research statement, and a teaching statement with sample evaluations. I will have six letters of recommendation, five research letters: Christine Berkesch (cberkesc@umn.edu), Melody Chan (melody_chan@brown.edu), David Eisenbud (de@berkeley.edu), Daniel Erman (erman@hawaii.edu), and Gregory G. Smith (ggsmith@mast.queensu.ca), and one teaching letter from Shirin Malekpour (shirin.malekpour@wisc.edu).

Please do not hesitate to contact me with any questions, or if there is anything else I can provide, and thank you in advance for your consideration.

Sincerely,

Juliette E. Bruce

Juliette Bruce
Postdoctoral Research Associate

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Juliette Bruce

October 23, 2023

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Employment

- **Brown University** — Providence, RI
  Postdoctoral Research Associate — *2022 – Present*
- **University of California, Berkeley** — Berkeley, CA
  NSF Postdoctoral Research Fellow — *2020– 2022*
- **Mathematical Sciences Research Institute** — Berkeley, CA
  Postdoctoral Fellow — *2020– 2021*

Education

- **University of Wisconsin** — Madison, WI
  Ph.D. Mathematics — *2014 – August 2020*
  – Advisor: Daniel Erman
- **University of Wisconsin** — Madison, WI
  M.A. Mathematics — *2014 – 2016*
- **University of Michigan** — Ann Arbor, MI
  B.S. in Mathematics & Political Science — *2010 – 2014*

Research Interests

Algebraic Geometry, Commutative Algebra, Arithmetic Geometry. Specifically, homological and combinatorial methods in algebraic geometry and commutative algebra.

Publications

15. M. Brandt, J. Bruce, M. Chan, M. Melo, G. Moreland, C. Wolfe. On the Top-weight Cohomology of $\mathcal{A}_g$. *Geometry & Topology*, To appear. E-Print: arXiv:2012.02892

14. J. Bruce, D. Corey, D. Erman, S. Goldstein, R. Laudone, and J. Yang. Syzygies of $\mathbb{P}^1 \times \mathbb{P}^1$: Data and Conjectures. *Journal of Algebra*, **593** (2022) no. 1, 589-621. E-Print: arXiv:2104.14598

13. J. Bruce, D. Erman, S. Goldstein, and J. Yang. The Schur-Veronese package in Macaulay2. *Journal of Software for Algebra and Geometry*, **11** (2021), 83-87 E-print: arXiv:1905.12661.

12. J. Bruce. The Quantitative Behavior of Asymptotic Syzygies for Hirzebruch Surfaces. *Journal of Commutative Algebra,* To appear. E-Print: arXiv:1906.07333.

11. A. Almousa, J. Bruce, M. Loper, and M. Sayrafi. The Virtual Resolutions Package for Macaulay2. *Journal of Software for Algebra and Geometry*, **10** (2020), 50-60. E-print: arXiv:1905.07022.

10. J. Bruce and D. Erman. A probabilistic approach to systems of parameters and Noether normalization. *Algebra and Number Theory*, **13** (2019), no. 9, 2081–2102. E-print: arXiv:1604.01704.

9. J. Bruce and W. Li. Effective bounds on the dimensions of Jacobians covering abelian varieties. *Proc. Amer. Math. Soc.*, **148** (2020), no. 2, 535-551. E-print: arXiv:1804.11015.

8. J. Bruce, D. Erman, S. Goldstein, and J. Yang. Conjectures and computations about Veronese syzygies. *Experimental Mathematics*, **29** (2020), 398-413. E-print: arXiv:1711.03513.

7. M. Brandt, J. Bruce, T. Brysiewicz, R. Krone, and E. Robeva. The degree of $SO(n)$. *Combinatorial Algebraic Geometry*, 207-224, Fields Inst. Commun. **80** (2017). E-print: arXiv:1701.03200.

6. J. Bruce, M. Logue, and R. Walker. Monomial valuations, cusp singularities, and continued fractions. *Journal of Commutative Algebra*, **7** (2015) no. 4, 495-522. E-print: arXiv:1311.6493.

5. J. Bruce, P. Kao, E. Nash, B. Perez, and P. Vermeire. Betti tables of reducible algebraic curves. *Proc. Amer. Math. Soc.* **142** (2014) 4039-4051. E-print: arXiv:1210.3064.

Pre-Prints

4. M. Brandt, J. Bruce, D. Corey. The virtual Euler characteristic for binary matroids. *Submitted.* E-Print: arXiv:2301.10108

3. J. Bruce, L. Cranton Heller, M. Sayrafi. Bounds on Multigraded Regularity. *Submitted.* E-Print: arXiv:2208.11115

2. J. Bruce, L. Cranton Heller, M. Sayrafi. Characterizing Multigraded Regularity on Products of Projective Space. *Submitted.* E-Print: arXiv:2110.10705

1. J. Bruce. Asymptotic Syzygies in the Setting of Semi-Ample Growth. *Submitted.* E-Print: arXiv:1904.04944

Software

4. SchurVeronese, (with D. Erman, S. Goldstein, and J. Yang). Submitted for distribution with future releases of Macaulay2, a compute algebra system focused on computations in algebraic geometry and commutative algebra.

3. VirtualResolutions, (with A. Almousa, M. Loper, and M. Sayrafi). Distributed with version 1.14 of Macaulay2 (2019).

2. FrobeniusThresholds, (with D. Hernández, K. Schwede, D. Smolkin, P. Teixeira, and E. Witt). Distributed with version 1.14 of Macaulay2 (2019).

1. TestIdeals, (with E. Bela, A. Boix, D. Ellingson, D. Hernández, Z. Kadyrsizova, M. Katzman, S. Malec, M. Mastroeni, M. Mostafazadehfard, M. Robinson, K. Schwede, D. Smolkin, P. Teixeira, and E. Witt). Distributed with version 1.14 of Macaulay2 (2019).

Non-Research Articles

2. J. Bruce. A Word from... Juliette Bruce, Inaugural President of Spectra. *Notices of the AMS*, **69** (2022) no. 6, 898–899.

1. A. Bonato, J. Bruce, and R. Buckmire. Spaces for All: The Rise of LGBTQ+ Mathematics Conferences. *Notices of the AMS*, **68** (2021) no. 6, 998-1003. https://doi.org/10.1090/noti2288

Multimedia

1. SyzygyData.com, (with D. Erman, S. Goldstein, and J. Yang). An online public database on large-scale syzygy computations.

Grants

- **Conference Grant DMS 2332592 – $20,000** August 2023
  National Science Foundation
- **Conference Grant – $23,000** July 2022
  Fields Institute / National Science Foundation
- **Postdoctoral Research Fellowship – $150,000** 2020 – 2022
  National Science Foundation
- **Conference Grant DMS-1908799 – $15,000** March 2019
  National Science Foundation
- **Graduate Research Fellowship** 2015 – 2018
  National Science Foundation
- **Conference Grant DMS-1812462 – $15,000** February 2018
  National Science Foundation
- **Professional Development Grant – $1000** December 2016
  Graduate School – University of Wisconsin

Awards & Honors

- **US Junior Oberwolfach Fellow** April 2022
  Awarded to outstanding junior scientists from US to participate in activities at Oberwolfach.
- **Capstone Teaching Award** October 2019
  Awarded to one student in the math department for an exceptional record of teaching excellence.
- **Excellence in Mathematical Research Award** October 2019
  Recognizes significant and substantial contributions to research as part of their thesis.
- **Elizabeth Hirschfelder Prize** October 2018
  Awarded to an outstanding female student.
- **Mathematics TA Service Award** April 2018
  Dept. of Mathematics - University of Wisconsin
- **Teaching Assistant Award for Exceptional Service** February 2018
  Campus-wide award recognizing TA's who perform exceptional service
- **Outstanding Achievement in Mathematics** May 2014
  Dept. of Mathematics – University of Michigan
- **Phi Beta Kappa** April 2014
  University of Michigan
- **Chancellor's Opportunity Award** April 2014
  University of Wisconsin

Seminar and Colloquium Talks

- San Francisco State University - Algebra, Geometry, & Combinatorics Seminar September 2023
- University of Vermont - Mathematics Colloquium May 2023
- Princeton University - Algebraic Geometry Seminar April 2023
- Dartmouth College - Algebra & Number Theory Seminar April 2023
- Bowdoin College - Mathematics Colloquium February 2023
- University of Texas - Austin - Geometry Seminar December 2022
- Harvard/MIT - Algebraic Geometry Seminar November 2022
- University of Kentucky - Algebra Seminar November 2022
- San Francisco State University - Mathematistas Raising Voices Colloquium April 2022
- COGENT Seminar - Held Virtually March 2022
- Simon Fraser University - Number Theory & Algebraic Geometry Seminar March 2022
- Reed College - Mathematics Colloquium April 2021
- San Francisco State University - Algebra, Geometry, & Combinatorics Seminar April 2021
- Algebra, Geometry, and Combinatorics Online - Held Virtually February 2021
- MPI MiS - Non-linear Algebra Seminar Online February 2021
- Syracuse University - Algebra Seminar November 2020
- Arizona State University - AWM Seminar October 2020
- Arizona State University - Number Theory Seminar October 2020
- Louisiana State University - Algebra & Number Theory Seminar October 2020
- University of Georgia - Algebraic Geometry Seminar October 2020
- Stanford University - Algebraic Geometry Seminar October 2020

- MSRI - The Fellowship of the Ring September 2020
- University of Washington - Algebra & Algebraic Geometry Seminar May 2020†
- University of Nebraska - Commutative Algebra Seminar April 2020†
- University of Michigan - Commutative Algebra Seminar December 2019
- University of Notre Dame - Algebraic Geometry Seminar November 2019
- DePaul University - Algebra, Combinatorics, and Number Theory Seminar October 2019
- Lawerence University - Mathematics Colloquium October 2019
- University of Utah - Algebraic Geometry Seminar September 2019
- Stanford University - Algebraic Geometry Seminar May 2019
- University of Kentucky - Algebra Seminar April 2019
- University of Minnesota - Commutative Algebra Seminar April 2019
- Rice University- Algebraic Geometry and Number Theory Seminar September 2018
- DePaul University - Algebra, Combinatorics, and Number Theory Seminar March 2018
- University of Michigan - Commutative Algebra Seminar December 2017

Conference Talks

- Joint Math Meetings (x2) - San Francisco, CA January 2023
- Workshop for Graduate Students: Syzygies & Regularity - UIC April 2023
- Spring AMS Southeastern Sectional - Georgia Tech University March 2023
- Algebraic Geometry Northeast Series (AGNES) - UMass Amherst November 2022
- Fall AMS Western Sectional - University of Utah October 2022
- Fall AMS Southeastern Sectional - University of Tennessee, Chattanooga October 2022
- BATMOBILE - Brown University May 2022
- CA+ - Iowa State University April 2022
- Joint Math Meetings (x2) - Seattle, WA January 2022†
- CMS Winter Meetings - Held Virtually December 2021
- Queer Research Showcase, QAtCanSTEM - Held Virtually November 2021
- Fall AMS Central Sectional - Held Virtually October 2021
- SIAM Conference on Applied Algebraic Geometry 2021 August 2021
- Queer and Trans in Combinatorics June 2021
- Math Summer Program for Inclusive Excellence June 2021
- Spring AMS Western Sectional - Held Virtually May 2021
- Spring AMS Central Sectional - Held Virtually April 2021
- Bay Area Discrete (BAD) Math Day - Held Virtually April 2021
- Spring AMS Southeastern Sectional - Held Virtually March 2021
- LGBTQ+Math - Fields Institute November 2020
- Written Geometry: Commutative Algebra - CIRM October 2020†
- Early Commutative Algebra Researchers (eCARS) - Held Virtually June 2020
- Foundations of Computational Mathematics - Simon Fraser University June 2020†

†Canceled due to the COVID-19 pandemic.

- Western Algebraic Geometry Online (WAGON) - Held Virtually — April 2020
- CA+ - Iowa State University — April 2020†
- Joint Math Meetings - Denver, CO — January 2020
- Fall AMS Central Sectional - University of Wisconsin — September 2019
- SIAM Conference on Applied Algebraic Geometry 2019 — July 2019
- KUMUNUjr - University of Nebraska — March 2019
- Spring AMS Southeastern Sectional - Auburn University — March 2019
- Joint Math Meetings - Baltimore, MD — January 2019
- Fall AMS Central Sectional - University of Michigan — October 2018
- Structures on Free Resolutions - Texas Tech University — October 2017
- Midwest Algebraic Geometry Graduate Conference - University of Illinois, Chicago — April 2015

Poster Talks

- AWM Poster Session - Joint Math Meetings — January 2020
- Summer School on Randomness and Learning in NLA - Max Plank Institute, Leipzig — July 2019
- 2019 AWM Research Symposium - Rice University — April 2019
- AWM Poster Session - Joint Math Meetings — January 2018
- AGNES Poster Session - Brown University — September 2018
- Lectures on Arithmetic Geometry - Rice University — February 2017
- Introductory Workshop: Combinatorial Algebraic Geometry - Fields Institute — August 2016
- Commutative Algebra and Its Interactions with Algebraic Geometry — July 2016
- Midwest Commutative Algebra and Algebraic Geometry Conference — May 2016

Conference & Seminar Organizing

- **GEMS in Commutative Algebra** — University of Minnesota
  November 10-13, 2023
- **GEMS in Combinatorics** — AIM
  March 27-31, 2023
- **Trans Math Day** — Held Virtually
  December 3, 2022
- **BATMOBILE** — Amherst College
  September 30, 2022
- **Spec**$(\overline{\mathbb{Q}})$ — Fields Institute
  July 6-8, 2022
- **LGBTQ+ Math Day** — Fields Institute
  November 18, 2021
- **GEMS in Combinatorics** — Held Virtually
  September 1-2, 2021

- **Trans Math Day** Held Virtually
  June 14-15, 2021
- **Western Algebraic Geometry Symposium (WAGS)** Held Virtually
  April 23 - April 24, 2021
- **Trans Math Day** Held Virtually
  December 5, 2020
- **CAZoom** Held Virtually
  April 25 - April 26, 2020
- **GWCAWMMG** University of Minnesota
  April 12 - April 14, 2019
- **Geometry & Arithmetic of Surfaces** University of Wisconsin
  February 9 - February 10, 2019
- **M2@UW** University of Wisconsin
  April 14-17, 2018
- **Math Careers Beyond Academia** University of Wisconsin
  April 14, 2017

Seminar & Session Organizing

- **Special Session on Combinatorial Algebraic Geometry** AMS Sectional
  October 1 - October 2, 2022
- **Commutative Algebra & Algebraic Geometry Seminar** UC Berkeley
  September 2021 - May 2022
- **Special Session on Commutative Algebra** AMS Sectional
  May 1 - May 2, 2021
- **Experimental Talks in Algebraic Geometry** Held Virtually
  May - July, 2020
- **Special Session on Combinatorial Algebraic Geometry** AMS Sectional
  September 14 - September 15, 2019

Outreach Activities

- **Michigan Research Experience for Graduate Students** University of Michigan
  Project Leader *July – August 2023*
  - Lead a diverse group of 4 early-stage graduate students on a project in algebraic geometry and combinatorics.
  - I expect this project to eventually result in a publication and software package.
- **Virtual Directed Readings in Geometry** Held Virtually
  Organizer *February 2021 – April 2021*
  - Created a virtual, open access, directed reading program for undergraduate students.
  - Approximately 30 students signed up for the 5-week reading group.

- **Virtual Directed Readings in Geometry** Held Virtually
  Organizer *February 2021 – April 2021*
  - Created a virtual, open access, directed reading program for undergraduate students.
  - Approximately 30 students signed up for the 5-week reading group.
- **Algebraic Geometry in the Time of COVID** Held Virtually
  Shepard *June 2020 – October 2020*
  - A virtual, open access introductory course in algebraic geometry.
  - Approximately 1600 registered participants.
- **Undergrad Directed Reading Program** University of Wisconsin
  Mentor *January 2018 – May 2019*
  - Lead two semester long reading projects on commutative algebra and algebraic geometry.
  - Lead an undergraduate women on a two semester reading project, and provided guidance on applying for REU's and graduate school.
- **Graduate Peer Mentoring** University of Wisconsin
  Mentor *September 2018 – December 2018*
  - Mentored 5 first year graduate students from minority genders, organizing monthly dinners where the mentees could discuss issues they were facing.
- **Girls Math Night Out** University of Wisconsin
  Mentor *September 2018 – December 2018*
  - Lead 2 women from local high schools on a semesters long project about cryptography.
- **Madison Math Circle** University of Wisconsin
  Lead Organizer *January 2016 – December 2018*
  - Lead the creation of a new outreach program, which directly visits high schools around the state of Wisconsin to better serve students from underrepresented groups.
  - Expanded the total number of students reached per year from 25 to >250.
- **Madison Math Circle** University of Wisconsin
  Student Volunteer *January 2015 – December 2018*
- **Out in STEM (oSTEM) @ UW-Madison** University of Wisconsin
  co-Founder *July 2017 – Math 2018*
  - Founded, at the time. the only campus resource specifically for LGBTQ+ individuals in STEM, and grew the organization to over 50 members.
  - Secured a travel grant to help 11 members (undergraduate and graduate students) attend the national oSTEM conference.
- **Madison Mega Math Meet** University of Wisconsin
  Graded *May 2015*
- **Bonding Undergraduate and Graduate Students** University of Wisconsin
  Mentor for Undergraduate *September 2014 – December 2014*
- **Michigan Math Circle** University of Michigan
  Organizer *January 2013 – June 2014*

Teaching Experience

- **Math 221: Calculus and Analytic Geometry I** — University of Wisconsin
  Teaching Assistant — *Fall 2014/2018/2019*
  - Selected as a TA coordinator in 2018 and 2019, and was responsible for overseeing all other TA's and mentoring first year TA's.
  - Sections ranged from 25-35 students on average, with typically teaching two sections at a time.
  - Average score 4.9/5.0
- **Math 228: Wisconsin Emerging Scholars** — University of Wisconsin
  Instructor — *Fall 2018*
  - Course providing students from underrepresented groups additional support.
  - Course had approximately 10-15 students.
  - Average score: 5.0/5.0, 100% amongst all TA's
- **Math 132: Wisconsin Emerging Scholars** — University of Wisconsin
  Instructor — *Spring 2015*
  - Course had approximately 15-25 students.
  - Responsible for all aspects of the course.
- **Inquiry Based Learning Courses** — University of Michigan
  Course Assistant — *2012-2014*
  - Assisted with advanced undergraduate courses on topology, analysis, and probability.
  - Courses had approximately 25-35 students.
  - Facilitated inquiry based learning in the classroom, and responsible for office hours, grading, and review sessions.

Outreach Panels

- **Holding AMS Meetings in Localities with Discriminatory Practices** — Joint Math Meetings
  Panelist — *January 2022*
- **AWM 101: What You Need to Know about Women in Math** — Joint Math Meetings
  Panelist — *January 2022*
- **Equity, Diversity, and Inclusion in Mathematics** — CAIMS
  Panelist — *June 2021*
- **Diversity and Inclusion Panel** — Womxn in Math at Berkeley
  Panelist — *April 2021*
- **How to Stay Productive as a Researcher** — Lunch in the Time of COVID
  Panelist — *June 2020*
- **Mathematics Research Online: Hosting Virtual Events** — Held Virtually
  Panelist — *May 2020*
- **Supporting Transgender and Non-binary Students** — MAA Panel at the JMM
  Organizer & Moderator — *January 2020*
- **Out in Math: Professional Issues Facing LGBTQ Mathematicians** — MAA Panel at the JMM
  Panelist — *January 2018*

Service

- **Spectra: The Association for LGBTQ+ Mathematicians**
  President, Immediate Past President — *September 2020 – Present*
- **MSRI: Committee on Women in Mathematics**
  Committee Member — *March 2023 – Present*
- **Communications in Algebra** — Editorial Board
  Communications Editor — *September 2020 – September 2022*
- **Spectra: The Association for LGBTQ+ Mathematicians**
  Board Member — *September 2020 – January 2022*
- **AMS Graduate Student Blog** — American Mathematical Society
  Writer & Editor — *September 2015 – September 2018*
- **Committee on Inclusivity and Diversity** — UW Dept. of Mathematics
  Member — *November 2016 – August 2017*
  - Created policies seeking to make the department a more welcoming, inclusive, and comfortable place. This included drafting the department's statement on inclusivity, and creating similar statements for syllabi to be used throughout the department.
- **Committee on TA Pay and Performance** — UW Dept. of Mathematics
  Member — *September 2015 – August 2017*
  - Developed and implemented a new system to evaluate TA performance, with the goal of creating a more transparent, useful, and non-biased system.
- **Instructor Excellence Program** — UW Dept. of Mathematics
  Teaching Mentor — *September 2015 – May 2016*
- **Society of Undergraduate Math Students** — University of Michigan
  President and Founder — *December 2012 – June 2014*

Referee Work

- Algebra & Number Theory, Communications in Algebra, Journal of Algebra, Journal of Commutative Algebra, Journal of Pure and Applied Algebra, Mathematica Scandinavia, Research in the Mathematical Sciences, Rocky Mountain Journal of Mathematics

Memberships

- Society of Industrial and Applied Mathematics — January 2017 – Present
- Association for Women in Mathematics — January 2016 – Present
- American Mathematical Society — September 2014 – Present

Juliette Bruce's Research Statement

My research interests lie in algebraic geometry, commutative algebra, and arithmetic geometry. In particular, I am interested in using homological and combinatorial methods to study the geometry of zero loci of systems of polynomials (i.e. algebraic varieties). I am also interested in studying the arithmetic properties of varieties over finite fields. Further, I am passionate about promoting inclusivity, diversity, and justice in the mathematics community. Broadly speaking my current research follows these ideas in two directions.

- **Homological Algebra on Toric Varieties:** A classical story in algebraic geometry is that homological methods and tools like minimal free resolutions and Castelnuovo–Mumford regularity capture the geometry of subvarieties of projective space in nuanced ways. My work has sought to generalize this story by developing ways homological algebra can be used to study the geometry of toric varieties (i.e., "nice' compactifications of the torus $(\mathbb{C}^{\times})^n$).
- **Cohomology of Moduli Spaces and Arithmetic Groups:** Despite its importance in algebraic geometry and number theory much remains unknown about the topology of $\mathcal{A}_g$, the moduli space of abelian varieties of dimension g. I have been working to study a canonical "part" of the cohomology of $\mathcal{A}_g$, called the top-weight cohomology. This turns out to be closely connected to the study of cohomology of various arithmetic groups like $\mathrm{GL}_g(\mathbb{Z})$ and $\mathrm{Sp}_{2g}(\mathbb{Z})$, as well as the study of automorphic forms.

1. Homological Algebra on Toric Varieties

Given a graded module M over a graded ring R, a helpful tool for understanding the structure of M is its minimal graded free resolution. In essence, a minimal graded free resolution is a way of approximating M by a sequence of free R-modules. More formally, a *graded free resolution* of a module M is an exact sequence

$$\cdots \to F_k \xrightarrow{d_k} F_{k-1} \xrightarrow{d_{k-1}} \cdots \xrightarrow{d_1} F_0 \xrightarrow{\epsilon} M \to 0$$

where each F_i is a graded free R-module, and hence can be written as $\bigoplus_j R(-j)^{\beta_{i,j}}$. The module $R(-j)$ is the ring R with a twisted grading, so that $R(-j)_d$ is equal to R_{d-j} where R_{d-j} is the graded piece of degree $d-j$. The $\beta_{i,j}$'s are the *Betti numbers* of M, and they count the number of i-syzygies of M of degree j. We will use syzygy and Betti number interchangeably throughout.

Given a projective variety X embedded in $\mathbb{P}^r$, we associate to X the ring $S_X = S/I_X$, where $S = \mathbb{C}[x_0, \ldots, x_r]$ and I_X is the ideal of homogenous polynomials vanishing on X. As S_X is naturally a graded S-module we may consider its minimal graded free resolution, which is often closely related to both the extrinsic and intrinsic geometry of X. An example of this phenomenon is Green's Conjecture, which relates the Clifford index of a curve with the vanishing of certain $\beta_{i,j}$ for its canonical embedding [Voi02, Voi05, AFP+19]. See also [Eis05, Conjecture 9.6] and [Sch86, BE91, FP05, Far06, AF11, FK16, FK17].

Much of my work can be viewed as understanding how minimal graded free resolutions capture the geometry when the role of $\mathbb{P}^r$ is replaced by another variety Y. In particular, I have focused on the case when Y is a toric variety, i.e., a compactification of the torus $(\mathbb{C}^{\times})^r$ where the action of the torus extends to the boundary. Examples of toric varieties include projective space, products of projective spaces, and Hirzebruch surfaces. Work of Cox shows there is a correspondence between (toric) subvarieties of a fixed toric variety and quotients of a polynomial ring similar to the story discussed above for $\mathbb{P}^r$ [Cox95]. As such recent years have seen substantial work looking to use homological algebra and to better understand the geometry of toric varieties [ABLS20, BES20, BE22, BE23a, BB21, CEVV09, EES15, GVT15, MS04, MS05].

1.1 Asymptotic Syzygies Broadly speaking, asymptotic syzygies is the study of the graded Betti numbers (i.e. the syzygies) of a projective variety as the positivity of the embedding grows. In many ways, this perspective dates back to classical work on the defining equations of curves of high degree and projective normality [Mum66, Mum70]. However, the modern viewpoint arose from the pioneering work of Green [Gre84a, Gre84b] and later Ein and Lazarsfeld [EL12].

To give a flavor of the results of asymptotic syzygies we will focus on the question: In what degrees do non-zero syzygies occur? Going forward we will let $X \subset \mathbb{P}^{r_d}$ be a smooth projective variety embedded by a very ample line bundle L_d. Following [EY18] we set,

$$\rho_q\left(X, L_d\right) \;\; := \;\; \frac{\#\left\{p \in \mathbb{N} \mid \mid \beta_{p,p+q}\left(X, L_d\right) \neq 0\right\}}{r_d},$$

which is the percentage of degrees in which non-zero syzygies appear [Eis05, Theorem 1.1]. The asymptotic perspective asks how $\rho_q(X; L_d)$ behaves along the sequence of line bundles $(L_d)_{d\in\mathbb{N}}$.

With this notation in hand, we may phrase Green's work on the vanishing of syzygies for curves of high degree as computing the asymptotic percentage of non-zero quadratic syzygies.

Theorem 1.1. [Gre84a] *Let $X \subset \mathbb{P}^r$ be a smooth projective curve. If $(L_d)_{d\in\mathbb{N}}$ is a sequence of very ample line bundles on X such that $\deg L_d = d$ then*

$$\lim_{d\to\infty} \rho_2\left(X; L_d\right) = 0.$$

Put differently, asymptotically the syzygies of curves are as simple as possible, occurring in the lowest possible degree. This inspired substantial work, with the intuition being that syzygies become simpler as the positivity of the embedding increases [OP01, EL93, LPP11, Par00, PP03, PP04].

In a groundbreaking paper, Ein and Lazarsfeld showed that for higher dimensional varieties this intuition is often misleading. Contrary to the case of curves, they show that for higher dimensional varieties, asymptotically syzygies appear in every possible degree.

Theorem 1.2. [EL12, Theorem C] *Let $X \subset \mathbb{P}^r$ be a smooth projective variety, $\dim X \geq 2$, and fix an index $1 \leq q \leq \dim X$. If $(L_d)_{d\in\mathbb{N}}$ is a sequence of very ample line bundles such that $L_{d+1} - L_d$ is constant and ample then*

$$\lim_{d\to\infty} \rho_q\left(X; L_d\right) = 1.$$

My work has focused on the behavior of asymptotic syzygies when the condition that $L_{d+1} - L_d$ is constant and ample is weakened to assuming $L_{d+1} - L_d$ is semi-ample. Recall a line bundle L is *semi-ample* if $|kL|$ is base point free for $k \gg 0$. The prototypical example of a semi-ample line bundle is $\mathcal{O}(1,0)$ on $\mathbb{P}^n \times \mathbb{P}^m$. My exploration of asymptotic syzygies in the setting of semi-ample growth thus began by proving the following nonvanishing result for $\mathbb{P}^n \times \mathbb{P}^m$ embedded by $\mathcal{O}(d_1, d_2)$.

Theorem 1.3. [Bru19, Corollary B] *Let $X = \mathbb{P}^n \times \mathbb{P}^m$ and fix an index $1 \leq q \leq n+m$. There exist constants $C_{i,j}$ and $D_{i,j}$ such that*

$$\rho_q\left(X; \mathcal{O}\left(d_1, d_2\right)\right) \geq 1 - \sum_{\substack{i+j=q \\ i\leq n,\, j\leq m}} \left(\frac{C_{i,j}}{d_1^i d_2^j} + \frac{D_{i,j}}{d_1^{n-i} d_2^{m-j}}\right) - O\begin{pmatrix}\textit{lower ord.} \\ \textit{terms}\end{pmatrix}.$$

Notice if both $d_1 \to \infty$ and $d_2 \to \infty$ then $\rho_q\left(\mathbb{P}^n \times \mathbb{P}^m; \mathcal{O}(d_1, d_2)\right) \to 1$, recovering the results of Ein and Lazarsfeld for $\mathbb{P}^n \times \mathbb{P}^m$. However, if d_1 is fixed and $d_2 \to \infty$ (i.e. semi-ample growth) my results bound the asymptotic percentage of non-zero syzygies away from zero. This together with work of Lemmens [Lem18] has led me to conjecture that, unlike in previously studied cases, in the semi-ample setting $\rho_q\left(\mathbb{P}^n \times \mathbb{P}^m; \mathcal{O}(d_1, d_2)\right)$ does not approach 1. Proving this would require a vanishing result for asymptotic syzygies, which is open even in the ample case [EL12, Conjectures 7.1, 7.5].

The proof of Theorem 1.3 is based upon generalizing the monomial methods of Ein, Erman, and Lazarsfeld. Such a generalization is complicated by the difference between the Cox ring and homogenous coordinate ring of $\mathbb{P}^n \times \mathbb{P}^m$. A central theme in this work is to exploit the fact that a key regular sequence I use has a number of non-trivial symmetries.

This work suggests that the theory of asymptotic syzygies in the setting of semi-ample growth is rich and substantially different from the other previously studied cases. Going forward I plan to use this work as a jumping-off point for the following question.

Question 1.4. *Let $X \subset \mathbb{P}^{r_d}$ be a smooth projective variety and fix an index $1 \leq q \leq \dim X$. Let $(L_d)_{d \in \mathbb{N}}$ be a sequence of very ample line bundles such that $L_{d+1} - L_d$ is constant and semi-ample, can one compute $\lim_{d \to \infty} \rho_q(X; L_d)$?*

A natural next case in which to consider Question 1.4 is that of Hirzebruch surfaces. I addressed a different, but related question for a narrow class of Hirzebruch surfaces in [Bru22].

1.2 Syzygies via Highly Distributed Computing It is quite difficult to compute examples of syzygies. For example, until recently the syzygies of the projective plane embedded by the d-uple Veronese embedding were only known for $d \leq 5$. My co-authors and I exploited recent advances in numerical linear algebra and high-throughput high-performance computing to generate a number of new examples of Veronese syzygies. A follow-up project used similar computational approaches to compute the syzygies of $\mathbb{P}^1 \times \mathbb{P}^1$ in over 200 new examples. This data provided support for several existing conjectures and led to a number of new conjectures [BEGY20, BEGY21, BCE+22].

1.3 Multigraded Castelnuovo–Mumford Regularity Introduced by Mumford, the Castelnuovo–Mumford Regularity of a projective variety $X \subset \mathbb{P}^r$ is a measure of the complexity of X given in terms of the vanishing of certain cohomology groups of X. Roughly speaking one should think about Castelnuovo–Mumford regularity as being a numerical measure of geometric complexity. Mumford was interested in such a measure as it plays a key role in constructing Hilbert and Quot schemes. In particular, being d-regular implies that $\mathcal{F}(d)$ is globally generated. However, Eisenbud and Goto showed that regularity is also closely connected to interesting homological properties.

Theorem 1.5. [EG84] *Let $\mathcal{F}$ be a coherent sheaf on $\mathbb{P}^r$ and $M = \bigoplus_{e \in \mathbb{Z}} H^0(\mathbb{P}^r, \mathcal{F}(e))$ the corresponding section ring. The following are equivalent:*

(1) M is d-regular;
(2) $\beta_{p,q}(M) = 0$ for all $p \geq 0$ and $q > d + i$;
(3) $M_{\geq d}$ has a linear resolution.

Maclagan and Smith introduced multigraded Castelnuovo–Mumford regularity, where $\mathbb{P}^r$ can be replaced by any toric variety. Similarly to the definition in the classical setting multigraded Castelnuovo–Mumford regularity is defined in terms of the vanishing of certain cohomology groups, however, the multigraded Castelnuovo–Mumford regularity of a subvariety or module is not a single number, but instead an infinite subset of $\mathbb{Z}^r$.

As an example, let us consider the case of products of projective spaces. Fixing a dimension vector $\mathbf{n} = (n_1, n_2, \ldots, n_r) \in \mathbb{N}^r$ we let $\mathbb{P}^{\mathbf{n}} := \mathbb{P}^{n_1} \times \mathbb{P}^{n_2} \times \cdots \times \mathbb{P}^{n_r}$ and $S = \mathbb{K}[x_{i,j} \mid 1 \leq i \leq r, 0 \leq j \leq n_i]$ be the Cox ring of $\mathbb{P}^{\mathbf{n}}$ with the $\operatorname{Pic}(X) \cong \mathbb{Z}^r$-grading given by $\deg x_{i,j} = \mathbf{e}_i \in \mathbb{Z}^r$, where $\mathbf{e}_i$ is the i-th standard basis vector in $\mathbb{Z}^r$. Fixing some notation given $\mathbf{d} \in \mathbb{Z}^r$ and $i \in \mathbb{Z}_{\geq 0}$ we let:

$$L_i(\mathbf{d}) := \bigcup_{\mathbf{v} \in \mathbb{N}, |\mathbf{v}| = i} (\mathbf{d} - \mathbf{v}) + \mathbb{N}^r.$$

Note when $r = 2$ the region $L_i(\mathbf{d})$ looks like a staircase with $(i+1)$-corners. Roughly speaking we define regularity by requiring the i-th cohomology of certain twists of $\mathcal{F}$ to vanish on L_i.

Definition 1.6. [MS04, Definition 6.1] *A coherent sheaf $\mathcal{F}$ on $\mathbb{P}^{\mathbf{n}}$ is* $\mathbf{d}$*-regular if and only if*

$$H^i\left(\mathbb{P}^{\mathbf{n}}, \mathcal{F}(\mathbf{e})\right) = 0 \qquad \text{for all } \mathbf{e} \in L_i(\mathbf{d}).$$

The multigraded Castelnuovo–Mumford regularity of $\mathcal{F}$ is then the set:

$$\operatorname{reg}(\mathcal{F}) \coloneqq \left\{\mathbf{d} \in \mathbb{Z}^r \;\middle|\; \mathcal{F} \text{ is } \mathbf{d}\text{-regular}\right\} \subset \mathbb{Z}^r.$$

The obvious approaches to generalize Theorem 1.5 to a product of projective spaces turn out not to work. For example, the multigraded Betti numbers do not determine multigraded Castelnuovo–Mumford regularity [BCHS21, Example 5.1] Despite this we show that part (3) of Theorem 1.5 can be generalized. To do so we introduce the following generalization of linear resolutions.

Definition 1.7. *A complex $F_\bullet$ of $\mathbb{Z}^r$-graded free S-modules is* $\mathbf{d}$*-quasilinear if and only if F_0 is generated in degree $\mathbf{d}$ and each twist of F_i is contained in $L_{i-1}(\mathbf{d}-\mathbf{1})$.*

Theorem 1.8. [BCHS21, Theorem A] *Let M be a (saturated) finitely generated $\mathbb{Z}^r$-graded S-module:*

$$M \text{ is } \mathbf{d}\text{-regular} \iff M_{\geq \mathbf{d}} \text{ has a } \mathbf{d}\text{-quasilinear resolution.}$$

The proof of Theorem 1.8 is based in part on a spectral sequence argument that relates the Betti numbers of $M_{\geq \mathbf{d}}$ to the Fourier–Mukai transform of $\widetilde{M}$ with Beilinson's resolution of the diagonal as the kernel. Recent breakthroughs [HHL23, BE23b] understanding resolutions of the diagonal on arbitrary toric varieties mean that there is hope one may be able to generalize the above argument to arbitrary toric varieties. With this in mind, I am interested in pursuing the following question

Question 1.9. *How can Theorem 1.8 be generalized to arbitrary smooth projective toric varieties? in particular, what is the correct definition of quasilinear resolutions?*

1.3.1 Multigraded Regularity of Powers of Ideals Building on the work of many people [BEL91, Cha97], Cutkosky, Herzog, Trung [CHT99] and independently Kodiyalam [Kod00] showed the Castelnuovo–Mumford regularity for powers of ideals on a projective space $\mathbb{P}^r$ has surprisingly predictable asymptotic behavior. In particular, given an ideal $I \subset \mathbb{K}[x_0, \ldots, x_r]$, there exist constants $d, e \in \mathbb{Z}$ such that $\operatorname{reg}(I^t) = dt + e$ for $t \gg 0$.

Building upon our work discussed above, my collaborators and I generalized this result to arbitrary toric varieties. In particular, Definition 1.6 can be extended to all toric varieties by letting S be Cox ring of the toric variety X, replacing $\mathbb{Z}^r$ with the Picard group of X, and replacing $\mathbb{N}^r$ with the nef cone of X. My collaborators and I show that the multigraded regularity of powers of ideals is bounded and translates in a predictable way. In particular, the regularity of I^t essentially translates within $\operatorname{Nef} X$ in fixed directions at a linear rate.

Theorem 1.10. [BCHS22, Theorem 4.1] *There exists a degree $\mathbf{a} \in \operatorname{Pic} X$, depending only on I, such that for each integer $t > 0$ and each pair of degrees $\mathbf{q}_1, \mathbf{q}_2 \in \operatorname{Pic} X$ satisfying $\mathbf{q}_1 \geq \deg f_i \geq \mathbf{q}_2$ for all generators f_i of I, we have*

$$t\mathbf{q}_1 + \mathbf{a} + \operatorname{reg} S \subseteq \operatorname{reg}(I^t) \subseteq t\mathbf{q}_2 + \operatorname{Nef} X.$$

A key aspect of the proof of this theorem is showing that the multigraded regularity of an ideal is finitely generated, in the sense that there exist vectors $\mathbf{v}, \mathbf{w} \in \mathbb{Z}^r$ such that $\mathbf{v} + \operatorname{Nef} X \subset \operatorname{reg}(I) \subset \mathbf{w} + \operatorname{Nef} X$. Perhaps somewhat surprisingly, my co-authors and I showed that this can fail for arbitrary modules [BCHS22]. This naturally raises the question of whether one can characterize when multigraded regularity is finitely generated.

Question 1.11. *Let X be a smooth projective toric variety. Can one characterize when $\operatorname{reg}(M)$ is finitely generated for a module M over the Cox ring of X?*

An first case of this question that I think would make a lovely first research project for a student is to attempt to answer Question 1.11 when M is the Cox ring of a torus fixed-point. In this special case, the question reduces to a delicate combinatorial question about vector partition functions.

2. Cohomology of Moduli Spaces and Arithmetic Groups

Some of the most classical objects in algebraic geometry are moduli spaces, i.e., spaces that parameterize a given collection of geometric objects. The quintessential example of a moduli space is $\mathcal{M}_g$, the moduli space of (smooth) genus g curves, also known as the moduli space of compact Riemann surfaces of genus g. Despite their classical nature, much remains unknown about the geometry of many moduli spaces. For example, the rational cohomology of $\mathcal{M}_g$ is only known for $g \le 4$. However, classical results suggest that $\mathcal{M}_g$ should have a lot of cohomology because its Euler characteristic grows super exponentially. Recent groundbreaking work of Chan, Galatius, and Payne has shed the first direct light on this phenomenon by constructing new non-trivial cohomology classes, and showing that the dimension of certain cohomology groups of $\mathcal{M}_g$ grow at least exponentially.

Theorem 2.1. [CGP21, Theorem 1.1] *For $g \ge 2$ the dimension of $H^{4g-6}(\mathcal{M}_g;\mathbb{Q})$ grows at least exponentially. In particular $\dim H^{4g-6}(\mathcal{M}_g;\mathbb{Q}) > \beta^g$ for any real number $\beta < \beta_0$ where $\beta_0 \approx 1.3247\ldots$ is the real solution of $t^3 - t - 1 = 0$.*

Much of my recent work has sought to build up the groundwork laid by Chan, Galatius, and Payne to study the rational cohomology of other moduli spaces. Of particular, interest to me has been the moduli space of abelian varieties and various generalizations. This work has deep connections to the cohomology of various arithmetic groups like $\mathrm{Sp}_{2g}(\mathbb{Z})$ and $\mathrm{GL}_g(\mathbb{Z})$.

2.1 Cohomology of $\mathcal{A}_g$ The moduli space of (principally polarized) abelian varieties of dimension g, is a smooth variety $\mathcal{A}_g$ (truthfully a smooth Delinge–Mumford stack) whose points are in one to one correspondence with isomorphism classes of principally polarized abelian varieties of dimension g. Concretely, we may view it as the quotient $[\mathbb{H}_g/\mathrm{Sp}_{2g}(\mathbb{Z})]$ where H_g is the Siegel upper half-space. Notice this means that $\mathcal{A}_g$ is a rational classifying space for the integral symplectic group $\mathrm{Sp}_{2g}(\mathbb{Z})$.

Similar to the moduli space of curves $\mathcal{A}_g$ has long been studied, but much remains unknown about its geometry. For example, the (singular) cohomology is $\mathcal{A}_g$ is only fully known for $g \le 3$, with $g = 0, 1$ being relatively easy, $g = 2$ which is a classical result of Igusa [Igu62], and $g = 3$ due to work of Hain [Hai02]. In fact the cohomology of $\mathcal{A}_g$ is so mysterious until recently work by myself and co-authors it was unknown whether $H^{2i+1}(\mathcal{A}_g;\mathbb{Q}) \neq 0$ for some g and i. This was a question posed by Grushevsky. that my recent work answered [Gru09].

Building upon the work of Chan, Galatius, and Payne, my co-authors and I developed new methods for understanding a certain canonical quotient of the cohomology of $\mathcal{A}_g$. In particular, our results construct non-trivial cohomology classes in $H^k(\mathcal{A}_g;\mathbb{Q})$ in a number of new cases.

Theorem 2.2. [BBC+22, Theorem A] *The rational cohomology $H^k(\mathcal{A}_g;\mathbb{Q}) \neq 0$ for:*

$$(g,k) = (5,15), (5,20), (6,30), (7,28), (7,33), (7,37), \text{ and } (7,42).$$

For broader context, since $\mathcal{A}_g$ is a rational classifying space for $\mathrm{Sp}_{2g}(\mathbb{Z})$ there is natural isomorphism $H^*(\mathcal{A}_g;\mathbb{Q}) \cong H^*(\mathrm{Sp}_{2g}(\mathbb{Z});\mathbb{Q})$. In particular, the above results provide new non-vanishing results for $H^*(\mathrm{Sp}_{2g}(\mathbb{Z});\mathbb{Q})$. However, my work takes advantage of the fact that since $\mathcal{A}_g$ is a smooth and separated Deligne Mumford stack with a coarse moduli space which is an algebraic variety, permitting Deligne's mixed Hodge theory to be applied to study the rational cohomology of these groups. In particular, the rational cohomology of a complex algebraic variety X of dimension d admits a weight filtration with graded pieces $\mathrm{Gr}_j^W H^k(X;\mathbb{Q})$. As $\mathrm{Gr}_j^W H^k(X;\mathbb{Q})$ vanishes whenever $j > 2d$, $\mathrm{Gr}_{2d}^W H^k(X;\mathbb{Q})$ is referred to as the *top-weight* part of $H^k(X;\mathbb{Q})$.) In this way we deduce Theorem 2.2 above as a corollary to computing the top-weight cohomology of $\mathcal{A}_g$ for all $g \le 7$.

2.2 Cohomology of $\mathcal{A}_g(m)$ The moduli space $\mathcal{A}_g$ actually a special instance of the moduli space of (principally polarized) abelian varieties of dimension g with level m-structure. Denoted by $\mathcal{A}_g(m)$, we may view it as the quotient $[\mathbb{H}_g/\mathrm{Sp}_{2g}(\mathbb{Z})](m)$ where $\mathrm{Sp}_{2g}(\mathbb{Z})(m)$ is the principal congruence subgroup $\ker(\mathrm{Sp}_{2g}(\mathbb{Z}) \to \mathrm{Sp}_{2g}(\mathbb{Z}/m\mathbb{Z})$. Note that when $m = 1$, we have that $\mathcal{A}_g(m)$ is isomorphic to $\mathcal{A}_g$. From this perspective, one may hope to generalize Theorem 2.2 and underlying methods my co-authors and I developed in [BBC+22] to studying the rational cohomology of $\mathcal{A}_g(m)$ and $\mathrm{Sp}_{2g}(\mathbb{Z})(m)$. In ongoing work, Melody Chan and I are developing such generalizations.

Goal Theorem 2.3. *Let $d = \binom{g+1}{2}$ be the dimension of $\mathcal{A}_g(m)$. For any integers $m \geq 1$ and $g \geq 0$ there exists a cellular complex $LA_g(m)^{\mathrm{trop}}$ such that for all $i \geq 0$ there is a natural isomorphism*

$$\tilde{H}_{i-1}\left(LA_g(m)^{\mathrm{trop}}; \mathbb{Q}\right) \cong \mathrm{Gr}^W_{2d} H^i\left(\mathcal{A}_g(m); \mathbb{Q}\right),$$

The methods behind Goal Theorem 2.3 show new connections between the cohomology of $\mathcal{A}_g(m)$ and the cohomology of $\mathrm{GL}_g(\mathbb{Z})(m)$. The cohomology of $\mathrm{Sp}(2g, \mathbb{Z})(m)$ – and hence $\mathcal{A}_g(m)$ – and $\mathrm{GL}_g(\mathbb{Z})(m)$ are closely connected to automorphic forms. Thus it is natural to wonder whether our methods for computing the top-weight cohomology of $\mathcal{A}_g(m)$ shed new light on automorphic forms. In particular, since the top-weight cohomology of $\mathcal{A}_g(m)$ comes from understanding the boundary of a locally symmetric space, one may hope it is related to Siegel–Eisenstein series. In an ongoing conversation with, Melody Chan, and Peter Sarnak we hope to address this question.

Question 2.4. *What is the relationship between the top-weight cohomology of $\mathcal{A}_g(m)$ and Siegel Eisenstein series?*

2.3 Matroid Complexes and Cohomology of $\mathcal{A}_g^{\mathbf{mat}}$ A key step in the proof of Theorem 2.2 is constructing a chain complex $P_\bullet^{(g)}$ whose homology is precisely the top-weight cohomology of $\mathcal{A}_g$. A major hurdle to pushing our results on the cohomology of $\mathcal{A}_g$, further, is that this chain complex very quickly becomes extremely large and complicated. However, with my co-authors, I identified a subcomplex $R_\bullet^{(g)} \subset P_\bullet^{(g)}$, called the regular matroid complex, which has rich combinatorics. In particular, $R_k^{(g)}$ is spanned by isomorphism classes of regular matroids on k elements of rank $\leq g$. I am working to study this complex from a number of perspectives. As an example, the following goal theorem is a result that I am working on with three graduate students.

Goal Theorem 2.5. *Compute the homology of the matroid complex $R_\bullet^{(g)}$ for all $g \geq 14$.*

Currently by combining theoretical results and large-scale computations to compute the cohomology for all $g \leq 9$. Computing the homology of the regular matroid complex is interesting, not only because it provides a new approach for studying the combinatorics of matroids, but also because it is closely related to the cohomology of partial compactification of $\mathcal{A}_g$ called the matroidal (partial) compactification $\mathcal{A}_g^{\mathrm{mat}}$. In ongoing work with Madeline Brandy and Daniel Corey, I am looking to show that one can compute the top-weight cohomology of $\mathcal{A}_g^{\mathrm{mat}}$ from the regular matroid complex.

Goal Theorem 2.6. *Compute the top-weigh cohomology of $\mathcal{A}_g^{\mathrm{mat}}$ for all $g \leq 10$.*

Work of Willwacher [Wil15] and Kpntsevich [Kon93, Kon94] on graph complexes suggests that one may hope for $R_\bullet^{(g)}$ to have rich algebraic structure beyond just that of chain complex.

Question 2.7. *Does the complex $R_\bullet^{(g)}$ carry a natural Lie bracket, endowing it with the structure of a differentially graded Lie algebra?*

Constructing such a Lie bracket likely relies on developing a new understanding of the ways one can combine two matroids. Ongoing work with the graduate students mentioned above is studying this problem in the special cases of graphic and co-graphic matroids. The existence of similar Lie structure was crucial to achieving the exponential bounds in Theorem 2.1.

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Juliette Bruce's Teaching Statement

I. Introduction. My goal as an educator is to be an active guide for students, providing them with environments where they feel supported and encouraged to let their own mathematical and quantitative curiosities guide how they engage and learn. By taking this approach, I hope to engage with students as the complete people that they are, asking them to bring all of their experiences, backgrounds, identities, and knowledge into the learning environment. I want students to experience mathematics in a humanistic way, seeing how mathematics and quantitative thinking are integral aspects of their lives. As one of my former students noted, "Juliette obviously wants us to succeed not only in math but in life." Recognizing that learning mathematics is not necessarily confined to the classroom I have sought out new and non-traditional teaching opportunities. My teaching has been recognized through both awards and student evaluations:

- In 2018, I was one of three graduate students at the University of Wisconsin-Madison recognized campus-wide with the Teaching Assistant Award for Exceptional Service.
- I received two TA awards from the math department at the University of Wisconsin-Madison, the TA Service Award (2018) and the Capstone Teaching Award (2019), the latter of which is awarded to just one teaching assistant each year, for an exceptional record of teaching excellence and service.
- My student evaluations are very positive; for instance, for one course 100% of students agreed that I was an effective teacher.

I have sought to develop and refine my skills as an educator, both by viewing each teaching assignment as my own opportunity for growth and learning and by actively seeking out learning from other educators and education experts. In particular, I have implemented evidence-based techniques to support and engage students from diverse backgrounds

II. Teaching Experiences. As a graduate student at the University of Wisconsin - Madison, I served as a teaching assistant and course coordinator for Calculus I for multiple semesters, the instructor of record for Math for Early Education Majors, and the instructor of record for a Calculus I course providing students from generally under-represented groups additional support during their first college math course. Additionally, for several semesters, I held a non-traditional teaching assistantship for my role as the organizer of the Madison Math Circle outreach program. My passion for promoting an interest in and excitement for math – especially for people from generally underrepresented groups – led me to take on teaching and outreach roles through the *Girls Math Night Out* program and the *Wisconsin Directed Reading Program*.

My postdoctoral positions at Brown University and the University of California, Berkeley did not allow me to have formal teaching responsibilities, however, I have actively sought out non-traditional teaching opportunities and mentoring opportunities. For example, in 2020, in response to the COVID-19 pandemic, I helped Ravi Vakil and others organize *Algebraic Geometry in the Time of COVID*, a massive open-access virtual algebraic geometry course, which drew over 1500 participants from around the world. Inspired by this experience, in 2021, I organized an online open access course, *Virtual Directed Reading in Geometry & Algebra*, aimed at undergraduates. During this time I continued to seek to grow as an educator. For example, while at the University of California, Berkeley I actively participated in a reading/working group exploring antiracist and anti-oppressive pedagogy in the mathematics classroom. Further, I personally sought to engage with ways to humanize mathematics and support underrepresented students by exploring the works of Pamela E. Harris, Aris Winger, Rochelle Gutiérrez, Luis Leyva, and Francis Su.

III. Teaching Philosophy and Strategies for Classroom Success. As an instructor, I view my role is to be an active guide. I encourage my students to explore, engage with, and question the course material for themselves. I try to structure much of the course around guided group work that gives students opportunities to develop and discuss their understanding and confusion with their fellow students. In addition to encouraging students to take an active role in learning, this format also helps students to learn to vocalize their thought processes and ideas.

Active learning presents challenges to me and my students, most notably, the challenge of managing student mistakes. In many ways, the most significant moments during the learning process are not necessarily the moments of success, but the moments of failure. It is at this moment that students can recognize errors and gaps in their understanding of a subject and begin trying to correct them. It is also the moment that as an instructor I can understand what my students are finding difficult and nudge the conversation in such a way as to overcome these hurdles.

Making mistakes is hard, and most students, like most people, would prefer not to make mistakes. With this in mind, I think it is crucial to promote an inclusive environment where all students feel comfortable and safe participating. This environment encourages students to be open about what confuses them and where they are making mistakes. Creating an inclusive classroom environment requires active attention and work to maintain. However, in my experience, this work is well worth it.

My approach to creating an inclusive classroom environment has been influenced by the semester-long course *Inclusive Practices in the College Classroom*, which I took through the *Delta Program for Integrating Research, Teaching and Learning.* For example, one activity I implemented successfully asked students to brainstorm attributes from classes they found productive and attributes from classes they found less productive. After collecting a list of such attributes, we use this as a jumping-off point for forming community standards that we wish to shape our classroom environment. Examples of such community standards that my classes have often adopted include: "Respect everyone" and "Address the problem, not the person when discussing mistakes". I have found this helps the students buy into the belief that the classroom is an inclusive space where it is safe to make mistakes.

However, beyond simply creating an inclusive learning environment I have also found it important to create a space where students feel comfortable bringing their whole selves, including all of their experiences, backgrounds, challenges, identities, struggles, and knowledge. For example, I recognize that all students, like all people, will have days when negative experiences outside the classroom affect their ability to engage in the classroom. This is even more true for students who face racism, sexism, homo/transphobia, and other systems of oppression. On such a day when students enter the classroom, I look to try to meet the students where they are. For example, sometimes this means I will walk the student to the campus mental health or cultural center, or sometimes it means I create new problems specifically to help keep the student's mind off of whatever is troubling them. I try to make sure my students know I am there to provide them with whatever resources they need to succeed both in the classroom and in their life beyond. However, this human-centered approach also leads to many beautiful moments. For example, by allowing students to bring all of themselves to class they experience mathematics in a humanistic way, seeing how mathematics and quantitative thinking are an integral aspect of their life. I have found this often increases students' motivation, as well as opens themselves up to making mistakes, growing, and learning.

IV. Sample Student Feedback. The effectiveness of my teaching is highlighted in student comments:

- "I've always struggle with math and I've had a lot of teachers that didn't believe in me so because of this I've always dreaded math courses. But Juliette always showed she cared, was constantly encouraging, believed in our class, and taught the material really clearly. From her constant availability to help and great instructing, her class became one of my favorites and I am more successful in a math course than I've ever been before."
- "She went around and tried helping each student... She cared about each student's success in the class and tried her best to make everyone understand the material."
- "Juliette obviously wants us to succeed not only in math but in life. She is always making sure we know our resources especially when it comes to health. She also always wishes us a good day/weekend and that is awesome."

V. Conclusion. As a graduate student and postdoctoral scholar, I have found teaching to be extremely rewarding. I developed a passion for supporting and engaging students from diverse backgrounds. Going forward, I am excited for new opportunities to grow and learn as a teacher, continue to promote inclusivity, diversity, and justice in my teaching, and create human-centered learning environments for my students.

Summary of Student Evaluations

During her time as a graduate student at the University of Wisconsin - Madison Juliette Bruce has held appointments as a teaching assistant for eleven semesters. In her first year Juliette was a teaching assistant for Math 221: Calculus I and the instructor for Math 132, a math course for education majors. In these roles her student evaluations were generally positive with particularly strong student comments. Since then she has held a non-standard teaching assistantship with the Madison Math Circle outreach program for six semesters. More recently Juliette has twice served as a teaching assistant and coordinator for Math 221: Calculus I, and once been the instructor for the accompanying Math 228: Wisconsin Emerging Scholars course. (Math 228 is a course taken in addition to Calculus I to provide students from generally underrepresented groups additional support and community.) In all of these courses her student evaluations were near perfect, and she received a number of glowing student comments. An overview of the student evaluations she has received is below:

Semester	Course #	Course Title	Rating	Overall	Course Percentile
Fall 2019	Math 221	Calculus & Analytic Geometry I	Superior	4.91	100%
Fall 2018	Math 228	Wisconsin Emerging Scholars	Superior	5.00	100%
Fall 2018	Math 221	Calculus & Analytic Geometry I	Superior	5.00	100%
Fall 2018	n/a	Madison Math Circle	n/a	n/a	n/a
Spring 2018	n/a	Madison Math Circle	n/a	n/a	n/a
Fall 2017	n/a	Madison Math Circle	n/a	n/a	n/a
Spring 2017	n/a	Madison Math Circle	n/a	n/a	n/a
Fall 2016	n/a	Madison Math Circle	n/a	n/a	n/a
Spring 2016	n/a	Madison Math Circle	n/a	n/a	n/a
Spring 2015	Math 132	Problem Solving in Algebra, Statistics, & Probability	Satisfactory	3.96	n/a
Fall 2014	Math 221	Calculus & Analytic Geometry I	Satisfactory Plus	4.74	n/a

These ratings were determinate by the Mathematics Department's Committee on Teaching Assistant Performance and Pay, which consists of faculty, academic staff and teaching assistants. The committee bases its rating (Unsatisfactory, Needs Improvement, Satisfactory Minus, Satisfactory, Satisfactory Plus, Superior) on numerical scores and student comments. The overall score is the mean of 14 questions on a scale of 1-5. The course percentile is compared to instructors teaching the same course in recent years, and was only reported beginning in 2018.

Note the (relative) small number of students in a Math 132 class, and the small number of sections make the numerical evaluation scores extremely noisy and unstable. For example, when Juliette taught Math 132 only six out twenty of students completed the student evaluations. As such student comments are extremely important when evaluating someone teaching Math 132. In Juliette's case her comments are quite positive, and indicate she was effective in creating a classroom atmosphere in which students felt comfortable participating. This is generally in line with her evaluations for Math 221, which were exceptional.

Because of the non-standard nature of the Madison Math Circle teaching assistantship student evaluations are not done. Comments from parents and participants are collected, however, and some have been included in the excepted student comments section.

Excerpted Student Comments

Fall 2014 – Math 221 (TA for 2 Discussion Sections)

- In response to the question, "How could the TA improve his/her teaching?"
  - "Our TA is perfect. She is great at teaching math problems and concepts."
  - "She arrives on time and when we don't have questions she has a plan for the whole hour, she has been very effective and helpful in her teaching."
  - "I believe that my TA is doing a great job as is, and does not need to change anything."
- In response to the question, "What do you like most about the TA's teaching?"
  - "She gets the whole class to participate."
  - "She is very clear and helpful, and she really cares about our learning."
  - "Our TA is very encouraging and optimistic. She treats everyone fairly. She is great at solving math problems of varying difficulties."
  - "Very friendly and willing to help; tries to create a comfortable atmosphere open to questions and discussion."
  - "She is always there for us and is willing to help us in any shape, way, or form. Holds review sessions, most TA's don't."
  - "She is willing to find extra time to help out."
  - "She takes a very confusing lecture and makes it understandable. If it weren't for discussion my grade would be far lower."
  - "I like that my TA is always willing to help, and honestly wants us all to succeed, which is reflected in her teaching methods."
- In response to the question, "Any further comments?"
  - "TA should be the professor."
  - "Very good TA!"
  - "I think Juliette Bruce is a very good TA."
  - "By far one of the best TA's."
  - "She is essentially the best math teacher I've ever had."
- Misc. comments
  - "The TA in charge of discussion appears to really care, ..."
  - "If it weren't for your discussion I would still be lost ..."
  - "Thank you for all that you have done. I have thoroughly enjoyed your discussion, you made math actually enjoyable at 8:50 in the morning. I also found your discussion very helpful compared to lecture. I appreciate it!"

Spring 2015 – Math 132 (Instructor for 1 Discussion Section)

- In response to the question, "What do you like most about the TA's teaching?"
  - "Constantly open to changing teaching style based on what we need."

  - "Willingness to talk through confusion and answer questions."
  - "Wanted us to succeed – tried to make concepts connect to our lives."
  - "She is very clear spoken and is always willing to go above and beyond to meet or clarify mistakes or confusion."
- Misc. comments
  - "I really appreciate your willingness to help and desire for us succeed – especially when it comes to showing the material differently, or trying a different method to the course concepts."

Spring 2016/2017, Fall 2016/2017 – Madison Math Circle (TA Student Organizer)

- Each semester we collect comments from parents, since I have been an organizer these have included:
  - "You guys are doing great stuff! Keep up the wonderful work!"
  - "Great job. This is a fantastic service that you provide. And all your presenters are so enthusiastic. Thank you for sharing your love of mathematics with us all."
  - "My daughter loves the Math Circle and we're very grateful for this opportunity. It has been an amazing experience for her!"
  - "Thank you for holding this course. My children like to be challenged in math and this is a great opportunity for them to experience math outside the box. They especially like when there is hands on group activities."
  - "[Student] struggles with Math, not her strong suit, so I very much appreciate this resource!"

Fall 2018 – Math 221/228 (TA/Instructor for 1 Discussion Section)

- In response to the question, "What do you like most about the instructor's teaching?"
  - "Juliette made sure that I knew all the material and was comfortable with it. When I did have confusion she was really understanding and broke down problems step by step through asking me questions and having me solve through the work in ways I understood. She is always so positive, supportive, and enthusiastic in class and made me excited to learn every day. Additionally, she provided challenging material broken down in simpler ways so we were able to expand our problem solving skills of complex problems in interesting attainable ways."
  - "She explains thoroughly step by step, which makes learning the material much easier to understand. Anytime I have any questions or am confused in any way, she makes the time to explain it and I'd able to explain it in more than one way so that I can understand. She?s also super supportive and helpful and creates and really great environment in which I look forward to coming to Discussion because I know it?ll be productive and helpful for me."
  - "She always has a simpler way to explain stuff - Respectful - She brings interesting topics to class - challenges us effectively"
  - "I really appreciate how she explains things very clearly and goes through problems step-by-step. I also feel that she does an amazing job of preparing us for what is going to be on exams and explains exactly how we should write our answers to be the most clear possible in our understanding. She really knows what she is talking about, and is able to effectively communicate that understanding to someone who may be confused on the material."
  - "She is very effective and takes time to make sure that everyone understands the materials that are taught in lecture. It is very obvious that she cares about how we do and that we are comfortable asking questions."

    - “Juliette will clearly work through all the problems for us. She has a good sense of what material we understand and what we don’t understand. Additionally, she is very supportive and encouraging. Juliette will effectively use time and I never feel like coming to class is ever a waste.”

- In response to the question, ‘How could the instructor improve his/her teaching?”
    - “Hmmm, tough question. Can a non massless particle reach the speed of light? Not really. The same goes here, I don?t think there’s a better teaching atyle than this”
    - “I think she already does a fantastic job teaching this course and do not have any recommendations for improvements at this time.”
    - “Juliette is honestly the best math instructor I’ve ever had so I can’t think of many ways she could improve her teaching.”

- Misc. comments
    - “I’ve always struggle with math and I’ve had a lot of teachers that didn’t believe in me so because of this I’ve always dreaded math courses. But Juliette always showed she cared, was constantly encouraging, believed in our class, and taught the material really clearly. From her constant availability to help and great instructing, her class became one of my favorites and I am more successful in a math course than I’ve ever been before.”
    - “I would highly recommend this instructor to anyone. She is incredibly good at explaining challenging concepts and is also a very kind and caring individual.”
    - “Juliette is an amazing TA and she has really made me interested in calculus.”
    - “She’s one of if not the best math teacher I?ve had and I strongly recommend anyone to take a class with her.”
    - “I went to the review today and I just wanted to let you know that everyone around me kept commenting about how good of a teacher you are. The people behind me said many times that they wished that you were our professor instead of [the professor]. I just wanted to let know that everyone was saying nice things about you and I am really so blessed to have had you for a TA :) ”
    - “I am sending this email to let you know that I appreciate all the support you have given us this semester. It has been a true pleasure being in your class this semester. And even though, I did not expect to learn much this semester, you proved me wrong so thank you. ”

Fall 2019 – Math 221 (TA for 2 Discussion Sections)

- In response to the question, ‘How could the instructor improve his/her teaching?”
    - “She is the best TA I have had and I have learned so much from this discussion.”
    - “I really appreciate how available Juliette is and how she genuinely cares about my understanding and success in this class. I have a slower processing speed than other students and therefore require testing accommodations. Juliette was very understanding about it and worked with me to make sure we had a plan in place so I can feel confident with this class. She is also willing to work with me one on one if there is a specific problem I am not understanding or concept I want to talk through. Whenever she is explaining problems she walks through step by step which is very clear and helpful. Overall she is the best TA I have had here and I love everything about how she teaches”
    - “She really seemed to care about each of her students and was extremely good at explaining problems and answering everyones issues. Overall, I felt she was very fair and is my favorite TA

this semester. She goes over everything very effectively and does a very good job about clearly explaining and breaking down a complex subject."

- "She went around and tried helping each student instead of giving broad answers to the whole class. She cared about each student's success in the class and tried her best to make everyone understand the material."
- "Juliette Bruce has been, by far, my favorite TA because of her ability to relay the and teach the material. She is especially good at clarifying difficult material from lecture. For example, when we covered the delta epsilon process in the actual definition of a limit the concept was difficult for most students to wrap there head around but Juliette was able to clarify this topic enough where I ended up teaching my fellow students in a way they understood it by using the way she taught my discussion group. This is a small example but I by far have retained the most information from my math discussion compared to all of my other discussion sections and I believe this is due to our TA."
- "Juliette is an incredibly personable teacher. The charisma, humor, and brightness she brings into the classroom both brightens students day, and in NO WAY detracts from the con tent we are learning. She is able to fully describe the process of all of the problems that we work on, as well as offer ing her own tips and tricks to help further our understanding. Tuesday's are my busiest day, and I often find my self skipping some of my Tuesday classes, but Juliette's class I will never willing miss. As difficult as I find this course, Juliette goes above and beyond in the reinforcement of ideas went over in lecture, and her ability to clarify content is very helpful."
- "Juliette obviously wants us to succeed not only in math but in life. She is always making sure we know our resources especially when it comes to health. She also always wishes us a good day/weekend and that is awesome."

Department of Mathematics
COLLEGE OF LETTERS & SCIENCE
UNIVERSITY OF WISCONSIN–MADISON

DATE: January 10, 2020

TO: Bruce,Juliette

FROM: TA Evaluation Committee

SUBJECT: Fall 2019 MATH 221 Evaluation

You will find your complete TA evaluation results at aefis.wisc.edu. You will need to log in with your net ID and password. For your information:

The percentage of your students who agreed or strongly agreed with the statement “The instructor was an effective teacher” (question 13) is **100%**. This places you at the **100th** percentile of all TAs who taught a similar course this semester and at the **100th** percentile of all TAs teaching a mathematics course this semester.

CTAPP has read your teaching evaluations and instructor feedback. CTAPP has deemed your teaching to be **superior**. Congratulations!

Here are some selected student quotes CTAPP would like to highlight:

Juliette is an incredibly personable teacher. The charisma, humor, and brightness she brings into the classroom both brightens students day, and inNO WAY detracts from the con tent we are learning. She is able to fully describe the process of all of the problems that we work on, as well as offering her own tips and tricks to help further our understanding. Tuesday’s are my busiest day, and I often find my self skipping some of my Tuesday classes, but Juliette’s class I will never willing miss. As difficult as I find this course, Juliette goes above and beyond in the reinforcement of ideaswent over in lecture, and her ability to clarify content is very helpful.

I really appreciate how available Juliette is and how she genuinely cares about my understanding and success in this class. I have a slowerprocessing speed than other students and therefore require testing accommodations. Juliette was very understanding about it and worked with meto make sure we had a plan in place so I can feel confident with this class. She is also willing to work with me one on one if there is a specificproblem I am not understanding or concept I want to talk through. Whenever she is explaining problems she walks through step by step which is veryclear and helpful. Overall she is the best TA I have had here and I love everything about how she teaches.

Here is feedback from your teaching observer/mentor:

n/a

CTAPP has the following feedback for you:

You are doing an amazing job!!!

We recommend that you read your students' comments carefully to find in what areas you are doing well and where you should improve. Read the enclosed Teaching Evaluation Feedback handout for guidance on interpreting your results. If you have questions about your evaluations or this ranking, please speak to one of the committee members.

TA EVALUATION COMMITTEE

Daniel Erman	Justin Sukiennik	Jaeun Park
Xianghong Gong	Michel Alexis	Kyriakos Sergiou
Fabian Waleffe	Geoff Bentsen	Rajula Srivastava
Bobby Grizzard	Solly Parenti	Polly Yu
Oh Hoon Kwon		

Department of Mathematics
COLLEGE OF LETTERS & SCIENCE
UNIVERSITY OF WISCONSIN–MADISON

DATE: December 27, 2018

TO: Bruce,Juliette

FROM: TA Evaluation Committee

SUBJECT: Fall 2018 MATH 221 Evaluation

You will find your complete TA evaluation results at aefis.wisc.edu. You will need to log in with your net ID and password. For your information:

The percentage of your students who agreed or strongly agreed with the statement “The instructor was an effective teacher” (question 13) is **100%**. This places you at the **100th** percentile of all TAs who taught a similar course this semester and at the **100th** percentile of all TAs teaching a mathematics course this semester.

CTAPP has read your teaching evaluations and instructor feedback. CTAPP has deemed your teaching to be **superior**. Congratulations!

Here are some selected student quotes CTAPP would like to highlight:

"Juliette made sure that I knew all the material and was comfortable with it. When I did have confusion she was really understanding and broke down problems step by step through asking me questions and having me solve through the work in ways I understood. She is always so positive, supportive, and enthusiastic in class and made me excited to learn every day. Additionally, she provided challenging material broken down in simpler ways so we were able to expand our problem solving skills of complex problems in interesting attainable ways.", "I've always struggle with math and I've had a lot of teachers that didn't believe in me so because of this I've always dreaded math courses. But Juliette always showed she cared, was constantly encouraging, believed in our class, and taught the material really clearly. From her constant availability to help and great instructing, her class became one of my favorites and I am more successful in a math course than I've ever been before."

Here is feedback from your teaching observer/mentor:

Juliette really cares, puts in an enormous effort, to help her students before the first exam. She is the lead TA for one of my lectures.Her work sheets, which are shared with the other TAs and are used in all discussions, are excellent and comprehensive. This makes the discussions of all other TAs much better. I really appreciate her work.

CTAPP has the following feedback for you:

Amazing job!

We recommend that you read your students' comments carefully to find in what areas you are doing well and where you should improve. Read the enclosed Teaching Evaluation Feedback handout for guidance on interpreting your results. If you have questions about your evaluations or this ranking, please speak to one of the committee members.

TA EVALUATION COMMITTEE

Daniel Erman	Bobby Grizzard	Jaeun Park
Sergey Bolotin	Kyle Martinez	Soumya Sankar
Xianghong Gong	Justin Sukiennik	Kyriakos Sergiou
Chanwoo Kim	Eva Elduque	Vladimir Sotirov
Mariya Soskova	Christian Geske	Polly Yu
Soledad Benguria	Vefa Goksel	