Project Summary: Tropicalizations of Shimura Varieties

Overview.
This proposal develops new connections between homological algebra, algebraic geometry, and combinatorics by developing a theory of tropicalizations of Shimura varieties. Broadly there are four projects: (i) constructing an abstract theory of tropical Shimura varieties via toroidal compactifications, (ii) giving a combinatorial interpretation of the tropical Shimura varieties analogous to the connection between metric graphs and the tropical moduli space of curves, (iii) using the tropical moduli spaces to study the cohomology of Shimura varieties and arithmetic groups, and (iv) constructing homological structures on the cohomology of Shimura varieties.

Intellectual Merit.
The PI's proposal expands our understanding of the connections between homological algebra, algebraic geometry, and combinatorics. More specifically, this proposal generalizes recent breakthroughs studying the moduli space of curves combinatorial to the setting of Shimura varieties. Towards this goal, PI proposes developing a method to topicalize Shimura varieties that is geometrically and combinatorially meaningful. Using these tropicalizations the PI then seeks to further our understanding of the topology of Shimura varieties by studying and computing their weight-zero compactly supported cohomology. Finally, the PI looks to construct rich algebraic structures on the cohomology of Shimura varieties. These projects contain significant connections to computation and combinatorics. Further, they seek to strengthen connections between algebraic geometry and topology by using methods from algebraic geometry to study arithmetic groups in new ways.

While the concrete projects in this proposal focus on Shimura varieties, many can be viewed as strong test cases for deepening our understanding of tropical geometry in general. For example, the PI seeks to show that the tropicalization of a Shimura variety "realizes" its weight-zero compactly supported cohomology by constructing an explicit proper map from the Shimura variety to its tropicalization such that the induced map factors through the weight-zero piece. It is easily imaginable that such results may hold more generally in tropical geometry and that the methods developed in the proposal will be a strong step in proving such a result.

Broader Impacts.
As an LGBTQ+ woman, the PI has worked hard to promote diversity, inclusivity, and justice in the mathematical community. This proposal will further the PI's work in these directions as she continues to mentor students, work to support women and LGBTQ+ people in mathematics, and organize conferences. The PI plans to continue mentoring one undergraduate and one graduate student (a majority of whom identify with underrepresented groups). In Summer 2026 the PI will organize an undergraduate research program exploring questions in this proposal for students from underrepresented groups.

The PI served as the inaugural president of *Spectra, the Association for LBGTQ+ Mathematicians*, and has served as a board member for several years. Since 2020 the PI has organized an annual conference for transgender and non-binary mathematicians and she plans to continue organizing this conference, with the first in-person iteration scheduled for 2024. In Fall 2025 PI will organize a mentoring program supporting LGBTQ+ undergraduate students applying to graduate school. The PI serves as a member of SLMath's *Committee on Women in Math.*

The PI has organized many conferences including *Algebraic Geometry Northeastern Series* (2024), *Gender Equity in the Mathematical Study (GEMS) of Commutative Algebra* (2023), Spec($\overline{\mathbb{Q}}$) a conference for LGBTQ+ mathematicians (2022), *Western Algebraic Geometry Symposium* (2021), *Gender Equity in the Mathematical Study of Combinatorics* (2021/2023), the *Graduate Workshop in Commutative Algebra for Women and Mathematicians of Minority Genders* (2019), *Geometry & Arithmetic of Surfaces* (2019), and a conference developing software for algebra and geometry *M2@UW* (2018). The PI plans to organize *GEMS in Commutative Algebra* again in Fall 2025.