Openning

Q: Do any of you have the same knot?

Q: What does it mean to have the same knot?

Introduction

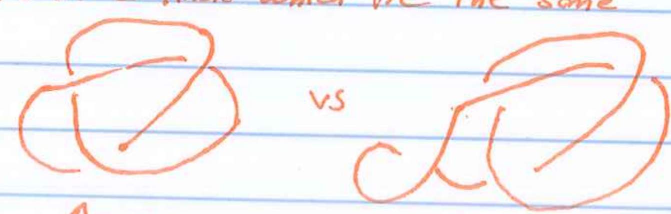
Def: A knot is a piece of string with the ends tied together

↳ This isn't the formal definition, which is surprisingly complicated but it is good enough for us, and (hopefully) agrees with our intution.

Problem: Some times things look different, but are actually the same knot...

Q: When should two knots be thought of as the same knot.

Do examples of 2 knots which are the same



↑ Trifoil ↑ Trifoil with twist

Def: Two knots are the same (or equivalent) if we can make one into the other without breaking the rope.

~~Reference~~

Q: How can we tell if 2 knots are the same? (Hard)

(Haken '60's showed deceibablity. Hass, Lagarias, Pippenger '99 showed in NP by showing ~ finding etremal ray of cone. Kuperburg 2011 GRH ⇒ is co-NP) All for detecting the unknot.

Q: How can we tell when something is the unknot?

~~This~~ → This is much much harder than it sounds....

→ Show unknot.

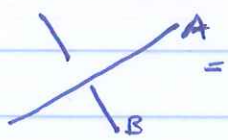
Before we can opprach this we need a better way to talk about knots.

Knot Diagrams:

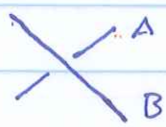
• A knot diagmam is a way to repesent a knot on paper....

Algorthim: Lie your knot flat on the table, so no strands are ~~crossing~~ lieing comeletely on top of eaach other and draw what you see.

This is called a crossing →



= means strand A goes over strand B

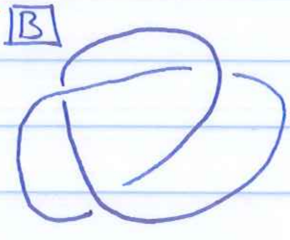


= means strand A goes under strand B

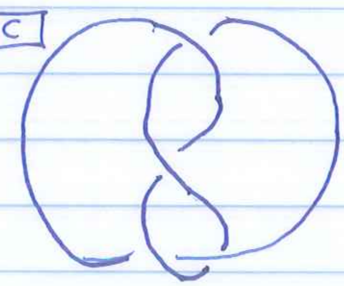
Ex:

A

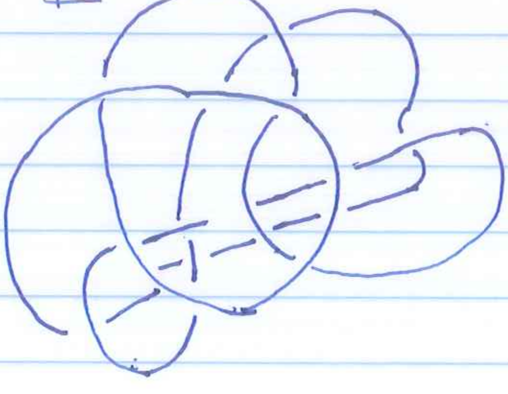
B



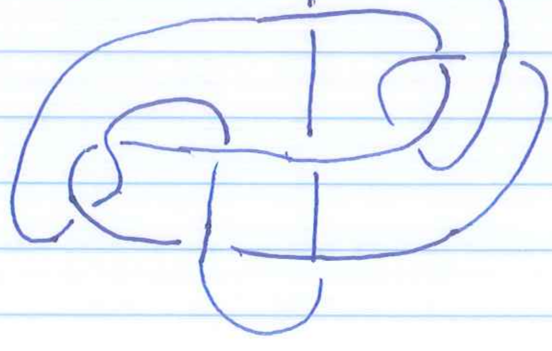
C



D



E



Put up the Guordian Knot.....

Q: Which of A - E / Gourdian Knot are the same as the unknot?

Answer: A, D, E, Gourdian Knot

Q: What about B & G? Are the unknots in disguise?

↳ They are certainly less complicated then D, E, and so even complicated looking things can be the unknot.

↳ We need away to be sure they are not the unknot.

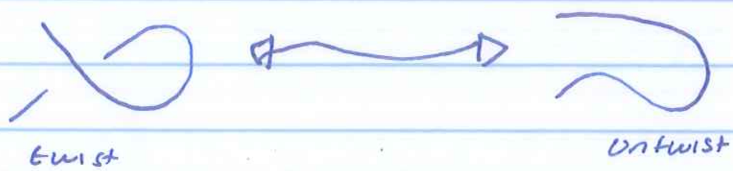
Q: How could we do this?

Gameplan: Find some property that the unknot has, and show these two knots do not have it..... (such a property is called an invariant).

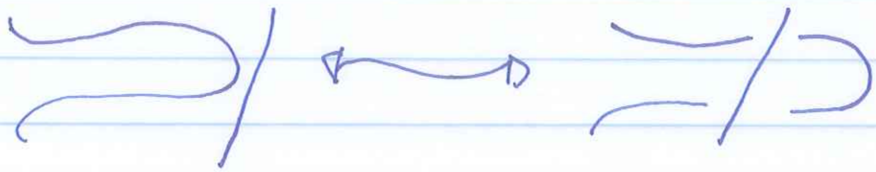
Reidemeister Moves:

Q: How does changing our knot change the knot diagram?

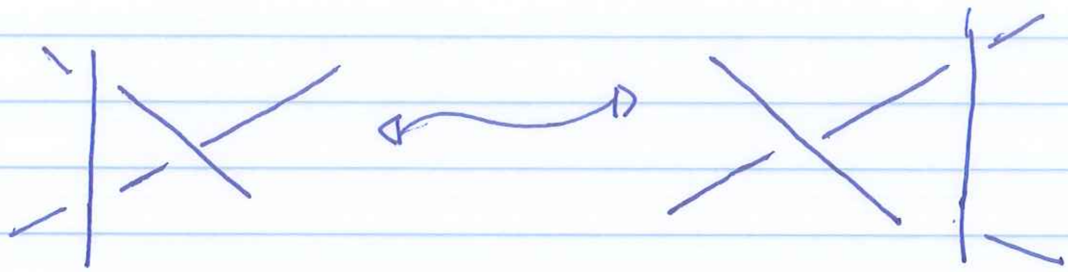
Theorem: Two knots are the same if and only if there knot digrams are related by a series of Reidemeister moves.

RI 

RII



RIII



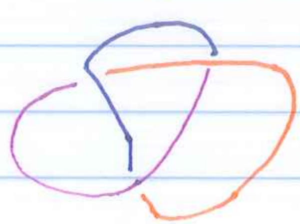
↳ Note this theorem is not all that helpful as it does not tell us how many, or what type of moves we need to do....

· There are many different ways to diagram a knot, and so if we wish to talk about properties of knots via the knot diagrams we need to be sure that our property does not depend on the diagram we choose. This is where reidemeister moves come in......

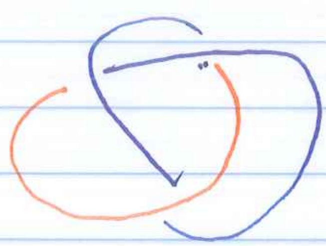
· Tricolorability :

· Def: A knot diagram is tricolorable if each arc can be colored one of three colors such that:

1) At least 2 colors are used
2) At a crossing all three colors are used or only one color is used.



↑ IS a tricoloring



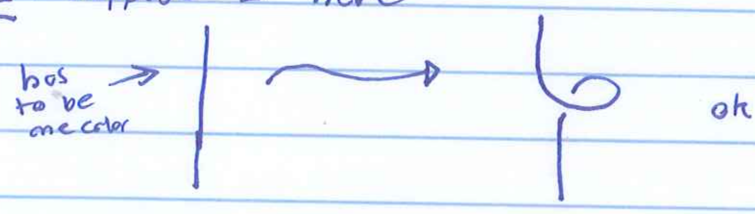
↑ not a tricoloring

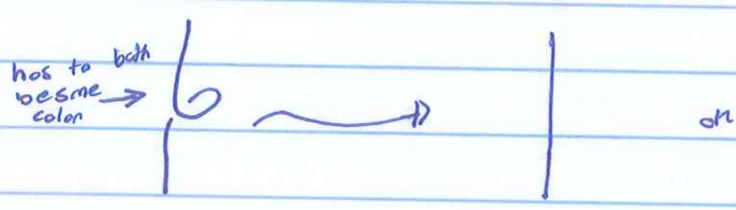
Theorem: If a Knot diagram is tricolorable then ~~changing~~ the diagram resulting from any Reidmeister move is also tri-colorablity.

~~Proof: we need to check each reidemeister m~~

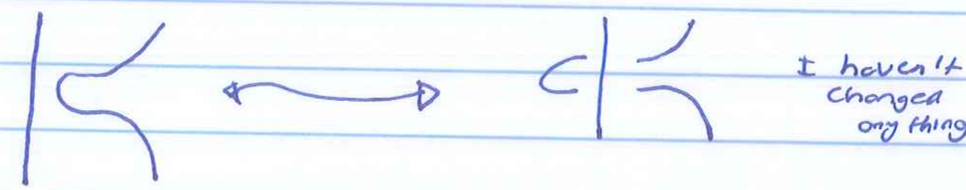
Proof: Suppose I have a tricolored Knot diagram, we need to check ~~nothing~~ that after making a Reideimester move it is still tricolorable

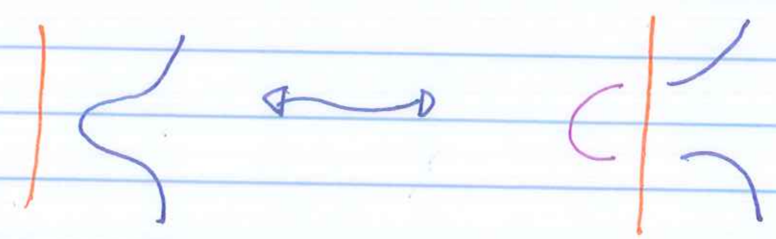
I: Suppose I have





II:





- This means if a knot has one knot diagram, which is tricolorable then all its diagrams are tricolorable.

So it makes sense to talk about the tricolorability of a knot.

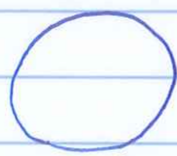
<u>Def</u>: A knot is tricolorable if its diagrams are tricolorable.

• Remember ~~this means~~ 2 Knots are the same if there diagrams are related by a sequence of moves...

<u>Cor</u>: If Knot A has a tricolorable diagram and Knot B has a diagram, which is not tricolorable then A is not the same as B.

<u>Cor</u>: The triefoil is not the unknot

<u>Proof</u>:



unknot has no 3-coloring

∎

<u>Q</u>: What about the Figure 8-Knot?