Relay Assignment in Multiple Source-Destination Cooperative Networks With Limited Feedback

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***Abstract*—We consider in this paper relay assignment for cooperative systems with mutiple source-destination pairs. The objective here is to assign relays to the source-destination pairs in such a way that all pairs achieve the maximum diversity. In networks with mutiple source-destination pairs, it is normally difficult for destinations to acquire the channel state information (CSI) of the entire network without feedback. To this end, we design a practical limited feedback strategy in conjunction with two relay assignment schemes, i.e., fullset selection and subset selection, which are based on maximizing the minimum end-to-end (E2E) signal to noise ratio (SNR) among all pairs. In this strategy, each destination acquires its SNR, quantizes it, and feeds it back to the relays. The relays then construct the E2E SNR table and select the relay assignment permutation from all possible relay assignment permutations or only a subset of these permutations. We analyze the performance of these schemes over independent Rayleigh fading channels in terms of the worst E2E SNR. We derive closed-form expressions for the E2E bit error rate (BER) and investigate the asymptotic performance at high SNR. We show that relay assignment with quantized CSI can achieve the same first-order diversity as that of the full CSI case, but there is a second-order diversity loss. We also demonstrate that increasing the quantization levels yields performance that is close to that of having full knowledge of the CSI.**

***Index Terms*—Cooperative diversity, decode-and-forward, diversity order, limited feedback, relay assignment.**

I. Introduction

COOPERATIVE communications has proven to be an effective way to combat wireless fading by allowing devices to share their antennas to achieve spatial diversity [1]–[3]. An active research area in cooperative communications is *selection diversity*, which aims at utilizing the system/network resources in a more efficient way [4]–[10]. Specifically, in the presence of multiple relays, only one or a subset of the relays are selected to cooperate, while maintaining full diversity.

Relay selection based on the exact end-to-end (E2E) signal-to-noise ratio (SNR) is studied in [4] and it is shown that this scheme achieves full diversity. A relay selection scheme based on the max-min criterion for both amplify-and-forward (AF) and decode-and-forward (DF) is proposed in [5] and [6]. The diversity-multiplexing trade-off is shown to be the same as that of the space-time coding scheme proposed in [12]. According to whether relay selection is performed before or after actual data transmission, this selection scheme can be classified into two main relay selection methods: proactive and reactive opportunistic relaying. In proactive opportunistic relaying, relay selection is based solely on the quality of the subchannels, which takes place before the source actually transmits its signal. Specifically, the relays are ordered according to their respective weakest subchannels, i.e., bottlenecks, and the one exhibiting the best bottleneck is chosen. In reactive opportunistic relaying, on the other hand, relay selection is performed after the source transmission over the first hop. That is, the selected relay is the one that has successfully decoded the source's message and whose relay-destination subchannel is the strongest. Both proactive and reactive opportunistic relaying are extensively studied in [7]–[10].

Most of the above works consider selecting the best relay, according to a certain criterion, to serve a pair of nodes in a network. Relay assignment for multiple pairs in relay networks is considered in [13]–[18]. A relay assignment scheme based on the location of the relays is proposed in [13], but it was demonstrated that the performance can be poor. In [14], the authors propose an assignment scheme, which is based on maximizing the minimum capacity among all pairs. The authors focus on reducing the complexity by developing a polynomial time algorithm, which has a linear complexity for each iteration. In [15] and [16], we consider two relay assignment schemes, fullset and subset selection, which are based on maximizing the minimum E2E SNR among all pairs. Compared to fullset selection, subset selection significantly reduces the search complexity while achieving the same diversity order. These two schemes can be viewed as an extension of the opportunistic proactive relaying scheme proposed in [5] and [6] to the case of multiple pairs. Fullset selection is also investigated in [17] and [18] with an effort to reduce the search complexity and investigate the performance analytically.

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Most of the existing works on relay assignment [14]–[18] assume that there is a central controller in the network that knows the channel state information (CSI) of all the links. However, for a network with multiple source-destination pairs, from a practical point of view, none of the nodes can acquire the CSI of the entire network without feedback. Therefore, it is crucial to design a practical limited feedback strategy in conjunction with relay assignment. This is addressed in this paper. In particular, we examine the fullset and subset relay assignment schemes with quantized CSI and analyze their performance in terms of the E2E bit error rate (BER). We also carry out the asymptotic analysis (i.e., at high SNR) to show the form of the optimal thresholds used in the quantization process and the achievable diversity.

Some related work in the field of limited feedback can be found in [20]–[24]. The performance of multiuser diversity with limited feedback is studied in [20]. Beamforming with quantized feedback for one transmitting pair is investigated in [21] and it is generalized to interference networks with multiple transmitting pairs in [22]. More references can be found in [23]. In [24], the authors consider several partner selection schemes with limited feedback. However, the schemes in [24] are based on quantized average CSI and no theoretical insight about the impact of limited feedback has been provided. Therefore, to the best of our knowledge, few works have been done to study the impact of using quantized instantaneous CSI on the performance of relay assignment in multiple source–destination cooperative networks. This motivates our work.

In the following, we summarize the contributions of the paper.

1) We present a limited feedback quantization strategy and the corresponding relay assignment schemes. In particular, the relay assignment is performed based on quantized CSI instead of full CSI, which is a practical scenario.
2) For both subset and fullset selection, we derive the exact E2E BER expressions in terms of the worst E2E SNR among all pairs.
3) We examine the asymptotic performance at high SNR in terms of the worst E2E SNR among all pairs. The optimal threshold function is identified and confirmed by simulation results.
4) We adopt a generalized diversity measure defined in [22] to determine the achievable diversity gain. It is shown that the presented relay assignment schemes can achieve diversity $(n, -(n-1))$ with quantized CSI and $(n, 0)$ with full CSI, where n is the number of relays. So even with only quantized CSI, our relay assignment schemes achieve the first-order full diversity. However, we show that there is a second-order diversity loss.

The remainder of the paper is organized as follows. The system model is presented in Section II. In Section III, the proposed limited feedback quantization strategy and the corresponding relay assignment schemes are presented. We analyze the E2E BER performance for both subset and fullset selection in Section IV, and examine its asymptotic performance at high SNR in Section V. We present several numerical examples in Section VI, and conclude the paper in Section VII.

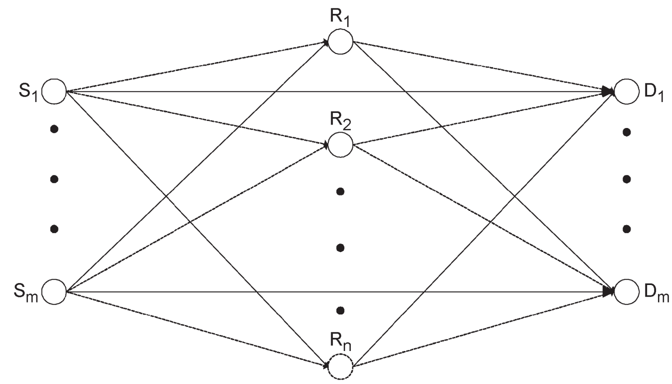

Fig. 1. A cooperative network with m communication pairs and n relays.

II. SYSTEM MODEL

We consider the system model shown in Fig. 1, in which the network consists of m pairs and n relays where $n \geq m$.[1] Each of the nodes is equipped with a single antenna and operates in a half-duplex mode. In the first time slot, the source of each pair transmits its signal, i.e., m nodes transmit simultaneously in the first time slot using frequency division multiple access (FDMA) [12]. In the second time slot, the selected relays transmit. Note that only one relay is assigned to each pair, and this assignment is done before actual transmission takes place. As such, each relay will have to decode only the signal coming from the pair it is assigned to. We assume there is no direct path between the sources and the destinations.

Let $h_{S_iR_j}$ and $h_{R_jD_i}$ (for $i = 1, \ldots, m, j = 1, \ldots, n$) denote the fading coefficient between the ith source-jth relay and jth relay-ith destination, respectively. Let $y_{S_iR_j}$ denote the received signal at the relay from the ith source, which is expressed as $y_{S_iR_j} = \sqrt{\rho} h_{S_iR_j} x_{S_i} + n_{S_iR_j}$, where x_{S_i} is the signal transmitted from the ith source and $n_{S_iR_j}$ is an additive white complex Gaussian noise (AWGN) sample corresponding to the ith source-jth relay link, with zero mean and unit variance and $\rho = E_b/N_0$ is the per-bit SNR. The relay forwards the detected signal to the destination. The signal received from the selected relay at the ith destination is expressed as $y_{R_jD_i} = \sqrt{\rho} h_{R_jD_i} \hat{x}_{S_iR_j} + n_{R_jD_i}$, where $\hat{x}_{S_iR_j}$ is a hard decision made based on $y_{S_iR_j} h^*_{S_iR_j}$.

The channels are assumed to experience independent, slow and frequency-nonselective Rayleigh fading. Let $\gamma_{S_iR_j}$ and $\gamma_{R_jD_i}$ denote the instantaneous SNRs for the links $S_i \rightarrow R_j$ and $R_j \rightarrow D_i$, respectively. For DF relaying, the E2E instantaneous SNR is well approximated as $\gamma_{ij} = \min(\gamma_{S_iR_j}, \gamma_{R_jD_i})$ [11], and its probability density function (pdf) is expressed as

$$f_\gamma(\gamma) = \frac{1}{\overline{\gamma}} e^{-\frac{1}{\overline{\gamma}}\gamma}, \tag{1}$$

where $\overline{\gamma} = \overline{\gamma}_{SR}\overline{\gamma}_{RD}/(\overline{\gamma}_{SR} + \overline{\gamma}_{RD})$ and $\overline{\gamma}_{SR} = \rho E[|h_{SR}|^2]$ and $\overline{\gamma}_{RD} = \rho E[|h_{RD}|^2]$ are the average SNRs for the links $S_i \rightarrow R_j$ and $R_j \rightarrow D_i$, respectively. We drop the index here

[1] In our paper, we assume that a single relay is assigned to a single pair at any given time, suggesting that the number of relays should be at least as many as the number of pairs. This is a realistic assumption because any node in the network can serve as a relay.

since the average SNRs are assumed to be the same for the source to relay links and relay to destination links, respectively. This corresponds to a network with clustered sources, clustered relays and clustered destinations. Thus the E2E instantaneous SNR in this paper refers to $\gamma_{ij} = \min(\gamma_{S_i R_j}, \gamma_{R_j D_i})$.

III. Relay Assignment With Limited Feedback

In this section, we elaborate on the limited feedback quantization strategy and the corresponding relay assignment schemes. To illustrate our relay assignment scheme in conjunction with quantized CSI, as an example, consider the case in which $m = 2$ and $n = 3$. Consequently, there are six possible relay assignment permutations, which are illustrated in the table below.

		pair 1	pair 2
subset 1		R_1	R_2
		R_2	R_3
		R_3	R_1
subset 2		R_1	R_3
		R_2	R_1
		R_3	R_2

In the table, the first entry of each row indicates the relay assigned to the first pair, the second is the relay assigned to the second pair. As we can see, there is correlation between certain rows of the table. For instance, rows one and four are correlated since in both cases, R_1 is assigned to the first pair. To eliminate the correlation, the six relay assignment permutations can be divided into two subsets as shown in the table. The objective is to divide the entire set of permutations into subsets such that no two or more permutations within a subset have the same relay assigned to the same pair. As a consequence, the rows are mutually independent in each subset. More details of the steps to construct subsets can be found in [16].

Let γ_{ij} denote the E2E instantaneous SNR of pair i when the j_{th} relay helps it. Then γ_{ij}^q represents the corresponding quantized E2E SNR. As such, the corresponding E2E SNR with limited feedback is shown in the table below.

		pair 1	pair 2
subset 1		γ_{11}^q	γ_{22}^q
		γ_{12}^q	γ_{23}^q
		γ_{13}^q	γ_{21}^q
subset 2		γ_{11}^q	γ_{23}^q
		γ_{12}^q	γ_{21}^q
		γ_{13}^q	γ_{22}^q

Note that for relay assignment with full CSI, the corresponding entries in the table are the exact value of the instantaneous E2E SNR, i.e., γ_{ij}. Let $\gamma_{k,\min}$ denote the worst E2E SNR of the kth assignment choice. Thus, the index of the selected assignment choice for fullset selection is obtained as

$$k^* = \arg\max_k \left\{ \gamma_{k,\min},\ k = 1, 2, \ldots, \frac{n!}{(n-m)!} \right\}. \tag{2}$$

We remark that the above selection criterion is for both full CSI and quantized CSI depending on the form of the E2E SNR table. While for subset selection, the best choice is selected within a subset with only n permutations instead of all permutations. We should emphasize that the relay assignment choice may not be unique, especially for the case of quantized CSI. In such a case, the target performance measure of our relay assignment schemes, i.e., the worst E2E SNR among all pairs, cannot be improved no matter which permutation is selected. So when there are more than two available choices, we just select the one with the smallest index k.

In the following, we outline the steps for the limited feedback quantization strategy and the corresponding relay assignment schemes.

1) At the end of the training period, each destination will have acquired the CSI for all source-relay links, as well as the links between all the relays and its own receiving channels.[2] Possible ways for the destination to obtain the CSI are illustrated in [17] and [22].[3]
2) The instantaneous SNR range $[0, \infty]$ is divided by $N-1$ thresholds, γ_{thl} $(l = 1, 2, \ldots, N-1)$, into N quantization levels. Then the destination uses $\log_2 N$ bits to feedback the quantization level. Therefore, each destination sends $n \log_2 N$ feedback bits for each channel coherence time. The feedback channels between each destination to the relays are assumed to be error free.
3) Upon receiving the feedbacks from all the destinations, the relays construct the E2E SNR table for fullset selection or subset selection. Then the relays calculate $\gamma_{k,\min}$ and determine the relay assignment choice according to the assignment criterion in (2). Since the feedback channels are assumed to be error free, the relays will have the same E2E SNR table and make the same relay assignment decision independently. So there is no need for the relays to communicate with each other.[4]

IV. The End-to-End Bit Error Rate

A. Preliminaries

As shown in the previous section, the worst E2E SNR, $\gamma_{k^*,\min}$, of the selected assignment choice is critical in the process of relay assignment. Therefore, in this section, we derive the exact E2E BER performance with N quantization levels in terms of $\gamma_{k^*,\min}$ for both subset selection and fullset selection. We consider a general modulation scheme for which the conditional error probability takes the form of $d \cdot \operatorname{erfc}\sqrt{a\gamma}$

[2]Note that each destination can only acquire the CSI of the source-relay links and its own receiving channel, and each relay can only obtain the CSI of all the sources to itself by training. Therefore, it is impossible for the destinations to perform relay assignment without CSI feedback. Furthermore, it is not sufficient to ask the destinations to feedback the relay-destination CSI to the relays.

[3]The destination can acquire the CSI of the links between the relays and itself via training. The relays can acquire the CSI of the source-relay link by training. The relays can amplify and forward the received training signals from the sources to the destination, so that the destination can estimate the product of the source-relay and relay-destination links. Since the CSI of the relay-destination links is known by the destination, the CSI of the source-relay links can be estimated.

[4]We can design the feedback strategy in such a way that one node collects all the quantized CSI and makes the decision. However, there will be additional overheads since this node needs to notify the relays to help which pairs. While for the proposed scheme, the relays make decisions by themselves in a distributed manner and without a need for additional overheads.

[25], where γ is the instantaneous SNR, and (d, a) are constants depending on the modulation scheme (e.g., for binary phase shift keying (BPSK) $d = 1$ and $a = 2$). Therefore, tailoring the BER expressions for M-ary phase shift keying (M-PSK) modulation and M-ary quadrature amplitude modulation (M-QAM) is straightforward. For example, in Appendix A, we adapt the obtained expression to M-QAM.

Let γ_{thl} $(l = 1, 2, \ldots, N-1)$ denote the thresholds separating different quantization levels and the value of γ_{thl} increases with l. Thus $\gamma_{k^*,\min}$ can be either less than γ_{th1}, greater than γ_{thN-1} or belong to the interval $[\gamma_{thl}, \gamma_{th(l+1)}]$. To simplify the presentation of the derivation in this part, we assume that $\gamma_{th0} = 0$ and $\gamma_{thN} = \infty$. According to the value of $\gamma_{k^*,\min}$ of the selected assignment permutation, we divide the calculation of P_e to N separate parts as

$$P_e = \sum_{l=0}^{N-1} P_{el}, \tag{3}$$

where P_{el} represents the BER that $\gamma_{k^*,\min}$ belongs to the interval $[\gamma_{thl}, \gamma_{th(l+1)}]$, which is given by

$$\begin{aligned} P_{el} = & P_r\left(\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)}\right) \\ & \cdot P\left(\varepsilon \,\middle|\, \gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)}\right), \end{aligned} \tag{4}$$

where $P_r(\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)})$ represents the probability that $\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)}$ and $P(\varepsilon|\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)})$ is the BER conditioned on $\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)}$.

B. Subset Selection

Lemma 1: For a network with m source-destination pairs and n relays, using $N-1$ quantization thresholds and subset selection results in

$$\begin{aligned} P_e = & \left(1 - e^{-\frac{\gamma_{th1}}{\overline{\gamma}_{\min}}}\right)^n d \cdot \operatorname{erfc}\left(a, \overline{\gamma}_{\min}, 0, \gamma_{th1}\right) \\ & + \sum_{l=1}^{N-1} \sum_{j=1}^{n} \frac{n!}{j!(n-j)!} \left(e^{-\frac{\gamma_{thl}}{\overline{\gamma}_{\min}}} - e^{-\frac{\gamma_{th(l+1)}}{\overline{\gamma}_{\min}}}\right)^j \\ & \cdot \left(1 - e^{-\frac{\gamma_{thl}}{\overline{\gamma}_{\min}}}\right)^{n-j} d \cdot \operatorname{erfc}\left(a, \overline{\gamma}_{\min}, \gamma_{thl}, \gamma_{th(l+1)}\right), \end{aligned} \tag{5}$$

where [26]

$$\begin{aligned} I\left(a, b, \gamma_{thl}, \gamma_{th(l+1)}\right) = & \int_{\gamma_{thl}}^{\gamma_{th(l+1)}} \operatorname{erfc}\sqrt{a\gamma} \frac{1}{b} e^{-\frac{1}{b}\gamma} d\gamma \\ = & e^{-\frac{1}{b}\gamma_{thl}} \operatorname{erfc}\sqrt{a\gamma_{thl}} \\ & - \sqrt{\frac{ab}{1+ab}} \operatorname{erfc}\sqrt{\gamma_{thl}\left(a + \frac{1}{b}\right)} \\ & - e^{-\frac{1}{b}\gamma_{th(l+1)}} \operatorname{erfc}\sqrt{a\gamma_{th(l+1)}} \\ & + \sqrt{\frac{ab}{1+ab}} \operatorname{erfc}\sqrt{\gamma_{th(l+1)}\left(a + \frac{1}{b}\right)}, \end{aligned}$$

and $\overline{\gamma}_{\min} = (1/m)(\overline{\gamma}_{SR}\overline{\gamma}_{RD}/(\overline{\gamma}_{SR} + \overline{\gamma}_{RD}))$.

Proof: See Appendix A. ■

C. Fullset Selection

Lemma 2: For a network with two source-destination pairs and n relays, using $N-1$ quantization thresholds and fullset selection results in

$$\begin{aligned} P_e = & \sum_{l=0}^{N-1} \left[1 - P_r\left(\gamma_{k^*,\min} > \gamma_{th(l+1)}\right) - P_r(\gamma_{k^*,\min} < \gamma_{thl})\right] \\ & \cdot d \cdot \operatorname{erfc}\left(a, \overline{\gamma}, \gamma_{thl}, \gamma_{th(l+1)}\right) \end{aligned}$$

where the probability that $\gamma_{k^*,\min}$ is greater than a particular threshold, $\gamma_{th(l+1)}$, is derived as

$$\begin{aligned} & P_r\left(\gamma_{k^*,\min} > \gamma_{th(l+1)}\right) \\ & = \frac{n!}{(n-2)!} e^{-\frac{2\gamma_{th(l+1)}}{\overline{\gamma}}} \left(1 - e^{-\frac{\gamma_{th(l+1)}}{\overline{\gamma}}}\right)^{2n-2} \\ & \quad + \sum_{t=3}^{n} \left(\frac{2n!}{t!(2n-t)!} - 2\frac{n!}{t!\,(n-t)!}\right) e^{-\frac{t\gamma_{th(l+1)}}{\overline{\gamma}}} \\ & \quad \cdot \left(1 - e^{-\frac{\gamma_{th(l+1)}}{\overline{\gamma}}}\right)^{2n-t} \\ & \quad + \sum_{t=n+1}^{2n} \frac{2n!}{t!\,(2n-t)!} e^{-\frac{t\gamma_{th(l+1)}}{\overline{\gamma}}} \\ & \quad \cdot \left(1 - e^{-\frac{\gamma_{th(l+1)}}{\overline{\gamma}}}\right)^{2n-t}, \end{aligned} \tag{6}$$

and the probability that $\gamma_{k^*,\min}$ is less than a particular threshold, γ_{thl}, is derived as

$$\begin{aligned} & P_r(\gamma_{k^*,\min} < \gamma_{thl}) \\ & = \left(1 - e^{-\frac{\gamma_{thl}}{\overline{\gamma}}}\right)^{2n} + 2ne^{-\frac{\gamma_{thl}}{\overline{\gamma}}} \left(1 - e^{-\frac{\gamma_{thl}}{\overline{\gamma}}}\right)^{2n-1} \\ & \quad + \left(\frac{2n!}{2\,(2n-2)!} - \frac{n!}{(n-2)!}\right) \cdot e^{-\frac{2\gamma_{thl}}{\overline{\gamma}}} \left(1 - e^{-\frac{\gamma_{thl}}{\overline{\gamma}}}\right)^{2n-2} \\ & \quad + \sum_{t=3}^{n} \frac{2n!}{t!\,(n-t)!} e^{-\frac{t\gamma_{thl}}{\overline{\gamma}}} \left(1 - e^{-\frac{\gamma_{thl}}{\overline{\gamma}}}\right)^{2n-t}. \end{aligned} \tag{7}$$

Proof: See Appendix B. ■

So far, we have obtained closed-form expressions for the E2E BER for both subset and fullset selection. As expected, these expressions are functions of the average E2E SNR and thresholds. Since the E2E BER expressions for both subset and fullset selection are not invertible, the exact optimal thresholds can only be obtained by numerically minimizing the E2E BER with exhaustive grid search [19], and this is the approach followed in this paper. That is, we have used our mathematical derivation to obtain the exact optimal thresholds numerically.

V. Asymptotic E2E BER Performance

Since the final expressions derived above are exact and the optimal thresholds are obtained numerically, they do not give much insight about the diversity order achieved and the form of the optimal thresholds. So in this section, we analyze the behavior of the E2E BER at high SNR while using the optimal thresholds for relay assignment with limited feedback. Since the BER performance of one threshold is an upper bound on the BER performance of multiple thresholds, we focus on the asymptotic analysis for one threshold, which is denoted as γ_{th}.

A. Asymptotic E2E BER

Lemma 3: The E2E BER for subset selection can be upper bounded as

$$P_e^{asy} \leq \frac{\gamma_{th}^{n-1}}{4\rho^n} + \frac{e^{-\gamma_{th}}}{4\rho}. \tag{8}$$

Proof: See Appendix C. ∎

B. Asymptotic Optimal Threshold

To find the asymptotic optimal threshold, γ_{opt}, that minimizes (8), we differentiate (8) with respect to γ_{th}, which yields

$$\frac{\partial P_e^{asy}}{\partial \gamma_{th}} = \frac{(n-1)\gamma_{th}^{n-2}}{4\rho^n} - \frac{e^{-\gamma_{th}}}{4\rho} = 0. \tag{9}$$

It is not tractable to solve the optimal threshold γ_{opt} for any n directly from this equation. While for $n = 2$, we have

$$\frac{1}{4\rho^2} - \frac{e^{-\gamma_{th}}}{4\rho} = 0. \tag{10}$$

By solving (10), we have

$$\gamma_{opt} = \log \rho.$$

Note that $\gamma_{opt} = \log \rho = (2-1)\log \rho$, where 2 is the number of relays. Therefore, based on the insight from the asymptotic optimal threshold derived above for $n = 2$, we extrapolate that the optimal threshold function could be in the form of

$$\gamma_{opt} = (n-1)\log c\rho. \tag{11}$$

where c is a constant, which is independent of ρ. To validate our observation, we plug $\gamma_{th} = (n-1)\log c\rho$ into (9), which yields

$$\frac{\partial P_e^{asy}}{\partial \gamma_{th}} = \frac{(n-1)^{n-1}(\log c\rho)^{n-2}}{4\rho^n} - \frac{1}{4c^{n-1}\rho^n}. \tag{12}$$

For sufficiently large ρ, it is obvious that

$$\frac{\partial P_e^{asy}}{\partial \gamma_{th}} \Big|_{\gamma_{th}=(n-1)\log c\rho} > 0. \tag{13}$$

Then we plug $\gamma_{th} = (n-1)\log(c\rho/\log c\rho)$ into (9), then we have

$$\frac{\partial P_e^{asy}}{\partial \gamma_{th}} = \frac{(n-1)^{n-1}\left(\log c\rho - \log(\log c\rho)\right)^{n-2}}{4\rho^n} - \frac{(\log c\rho)^{n-1}}{4c^{n-1}\rho^n}. \tag{14}$$

For sufficiently large ρ, we have

$$\frac{\partial P_e^{asy}}{\partial \gamma_{th}} \Big|_{\gamma_{th}=(n-1)\log\left(\frac{c\rho}{\log c\rho}\right)} < 0. \tag{15}$$

Since $\partial P_e^{asy}/\partial \gamma_{th}$ is a monotonically increasing function with γ_{th}, we have

$$(n-1)\log\left(\frac{c\rho}{\log c\rho}\right) < \gamma_{opt} < (n-1)\log c\rho. \tag{16}$$

Then we conclude that $\gamma_{opt} = (n-1)\log c\rho - o(\log c\rho)$. This validates our observation. This result is also confirmed by simulations in Section VI.

C. Achievable Diversity

In this part, we analyze the diversity order achieved by relay assignment with quantized CSI based on the asymptotic E2E BER and the optimal threshold function derived in subsections A and B above. For comparison purposes, we also include the diversity analysis of relay assignment with full CSI.

We adopt the following generalized measure of diversity defined in [22],

$$d = (d_1, d_2),$$

where d_1 and d_2 are the first-order diversity and second-order diversity, respectively. They are given as

$$d_1 = -\lim_{\rho\to\infty} \frac{\log(BER)}{\log(\rho)}, \tag{17}$$

and

$$d_2 = -\lim_{\rho\to\infty} \frac{\log(BER) + d_1\log(\rho)}{\log\log(\rho)}. \tag{18}$$

As we can see from the definition, the generalized diversity measure not only encapsulates the conventional one as the first-order diversity but also incorporates the second-order diversity which captures the $\log \rho^{d_1}$ term in the error rate expression and its effect on the performance.

Proposition 1: Subset selection with quantized CSI can achieve diversity order of $(n, -(n-1))$.

Proof: From (8), we have

$$P_e^{asy} = P_{e1}^{asy} + P_{e2}^{asy}, \tag{19}$$

where $P_{e1}^{asy} = \gamma_{th}^{n-1}/4\rho^n$ and $P_{e2}^{asy} = e^{-\gamma_{th}}/4\rho$. Plugging $\gamma_{opt} = (n-1)\log c\rho$ into P_{e1}^{asy} and P_{e2}^{asy}, respectively, we have

$$P_{e1}^{asy} = \frac{((n-1)\log c\rho)^{n-1}}{4\rho^n}, \tag{20}$$

and

$$P_{e2}^{asy} = \frac{1}{4c^{n-1}\rho^n}. \tag{21}$$

Plugging (20) into (17) and (18) and after some simple algebraic manipulations, we can obtain the diversity achieved by P_{e1}^{asy}, which is $(n, -(n-1))$. Similarly, plugging (21) into (17) and (18), we obtain the diversity for P_{e2}^{asy}, which is $(n, 0)$. Since the performance of P_e^{asy} in (19) is dominated by the term with the lower diversity order, we conclude that subset selection with quantized CSI and one threshold can achieve a diversity order of $(n, -(n-1))$. Since the BER performance of one threshold is an upper bound on the BER performance of multiple thresholds, we can also conclude that the subset selection with quantized CSI can achieve diversity order of $(n, -(n-1))$ which is stated in Proposition 1. Therefore, the diversity order of subset selection with quantized CSI is at least $(n, -(n-1))$ for any number of quantization levels. ■

Till now, we have analyzed the asymptotic performance of subset selection. Since the BER performance of subset selection is an upper bound on that of fullset selection, we conclude that fullset selection can also achieve the same asymptotic performance.

Proposition 2: Subset selection with full CSI can achieve diversity order $(n, 0)$.

Proof: The E2E BER in terms of the worst E2E SNR for subset selection with full CSI can be expressed as

$$P_e = \int_0^\infty \frac{1}{2}\mathrm{erfc}\sqrt{\rho h} f_{h_{k^*,\min}}(h)\, dh, \tag{22}$$

where $f_{h_{k^*,\min}}(h)$ is the pdf of the worst E2E channel gain corresponding to the selected assignment choice, which can be expressed as [7]

$$f_{h_{k^*,\min}}(h) = ne^{-2mh}(1-e^{-2mh})^{n-1}. \tag{23}$$

Then plugging (23) into (22), we have

$$\begin{aligned} P_e &= \int_0^\infty \frac{1}{2}\mathrm{erfc}\sqrt{\rho h}\left[ne^{-2mh}(1-e^{-2mh})^{n-1}\right] dh \\ &\le \int_0^\infty \frac{1}{4}\left[ne^{-(\rho+2m)h}(1-e^{-2mh})^{n-1}\right] dh & (24) \\ &= \frac{n!}{8m}\left(\prod_{i=1}^{n}\left(\frac{\rho}{2m}+i+1\right)\right)^{-1} & (25) \\ &\le \frac{n!(2m)^n}{8m}\frac{1}{\rho^n}, & (26) \end{aligned}$$

where (24) follows from $\mathrm{erf}c\sqrt{x} \le (1/2)e^{-x}$ and (25) follows from Equation B.1 in [28]. Plugging (26) into (17) and (18), we obtain $d_1 = n$ and $d_2 = 0$. As such, subset selection with full CSI can achieve diversity order of $(n, 0)$. Since the BER performance of subset selection is an upper bound on that of fullset selection, we conclude that fullset selection can also achieve diversity $(n, 0)$. ■

Therefore, relay assignment with limited feedback suffers from a second-order diversity loss compared to relay assignment with full CSI. This fact is also confirmed by simulations in the next section.

VI. Simulation Results

We present in this section numerical examples that aim at validating the E2E BER expressions derived for fullset and subset selection. The performance of relay assignment with quantized CSI and full CSI are also compared. Throughout the simulations, we assume all channel variances are set to 0.25 and all nodes use BPSK.[5] The curves in this part are generated by using the exact optimal thresholds, which are obtained by minimizing the E2E BER expressions.

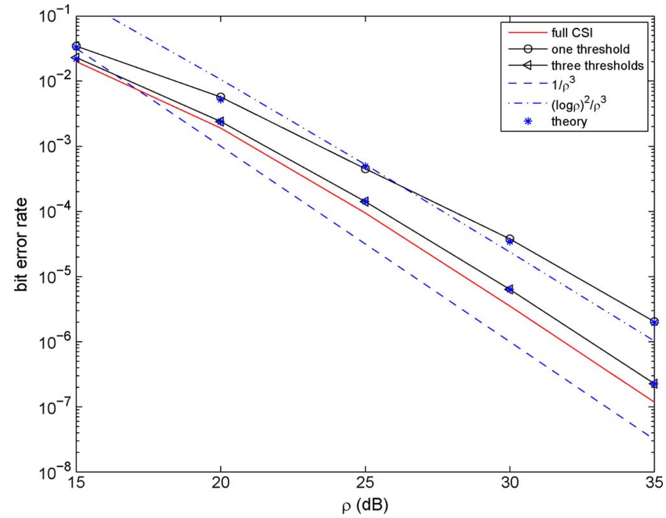

Fig. 2. Bit error rate performance (theory and simulation) of subset selection with different number of thresholds.

In Fig. 2, we show the performance results for a network with $m = 2$ and $n = 3$ for subset selection. We also include the curves obtained by using $1/\rho^3$ and $(\log\rho)^2/\rho^3$. They are used as references since these two curves achieve diversity $(3, 0)$ and $(3, -2)$, respectively. By comparing the slopes of the curves, subset selection with full CSI and limited feedback achieves diversity gains (3, 0) and $(3, -2)$, respectively, which is expected. This confirms our analysis of diversity. In addition, we can see the degradation in SNR due to using only quantized CSI, which is about 3 dB at BER 10^{-4} for one threshold. This degradation diminishes as the number of thresholds increases from 1 to 3. Finally, we can also see the perfect match between theory and simulations, which validates the derived BER expression for subset selection.

The BER performance for fullset selection with $m = 2$ and $n = 3$ is shown in Fig. 3, with similar observations. We observe fullset selection with limited feedback also achieves diversity gain $(3, -2)$. As expected, the performance improves as the number of thresholds increases. In addition, the simulation results match the theoretical results, which validates the derived BER expression for fullset selection.

In Fig. 4, we present the theoretical BER performance for the following cases: $m = 2$ and $n = 2, 4, 10$. We consider both fullset selection and subset selection. As shown in the figure, fullset selection achieves the same diversity as subset selection. We can also see the degradation in SNR due to the subset selection scheme, which is about 2 dB at BER 10^{-5} for $n = 4$.

In Fig. 5, we demonstrate the optimal threshold values and their corresponding asymptotic values as a function of ρ for

[5]We remark that these assumptions are used merely to demonstrate the efficacy of the proposed schemes. That is, the proposed schemes and the conclusions do not depend on the modulation scheme adopted.

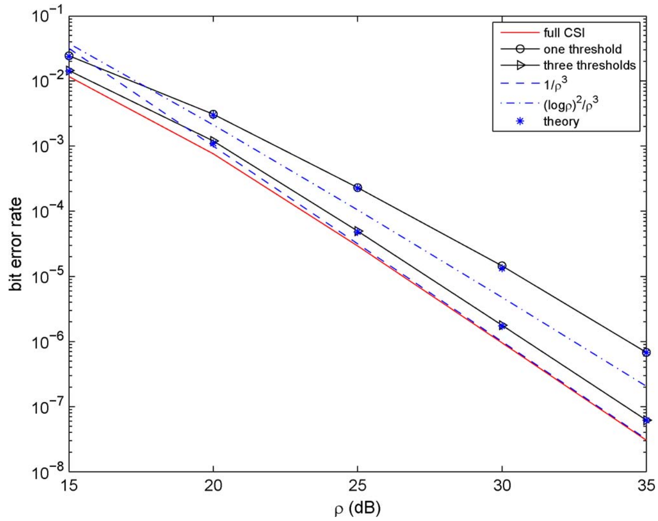

Fig. 3. Bit error rate performance (theory and simulation) of fullset selection with different number of thresholds.

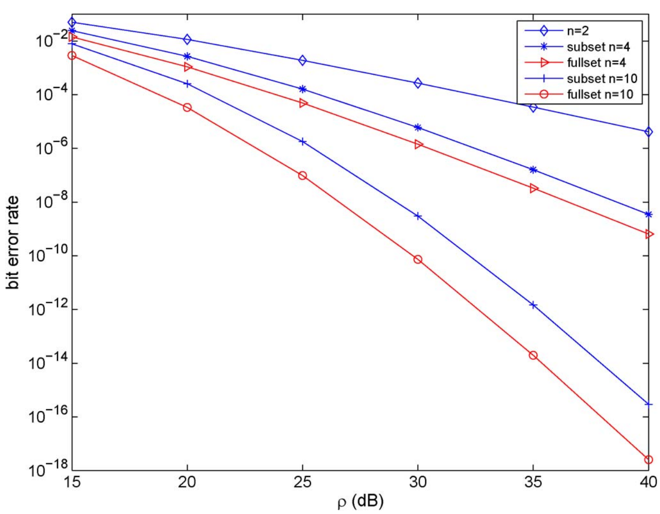

Fig. 4. Theoretical bit error rate performance comparison between the fullset selection and subset selection.

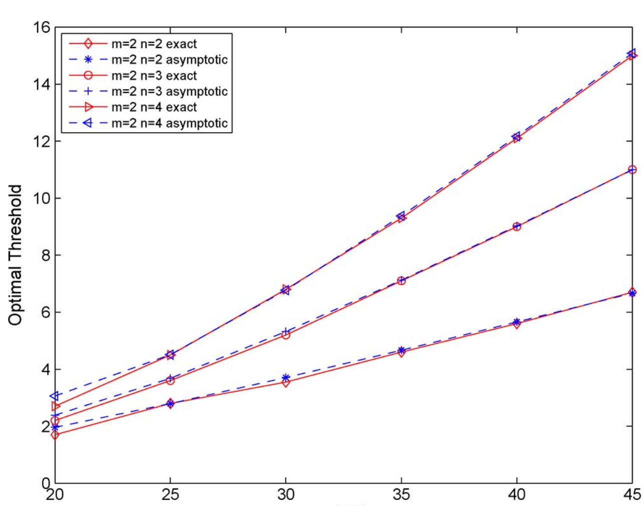

Fig. 5. The optimal threshold values (exact and asymptotic) as a function of ρ for subset selection.

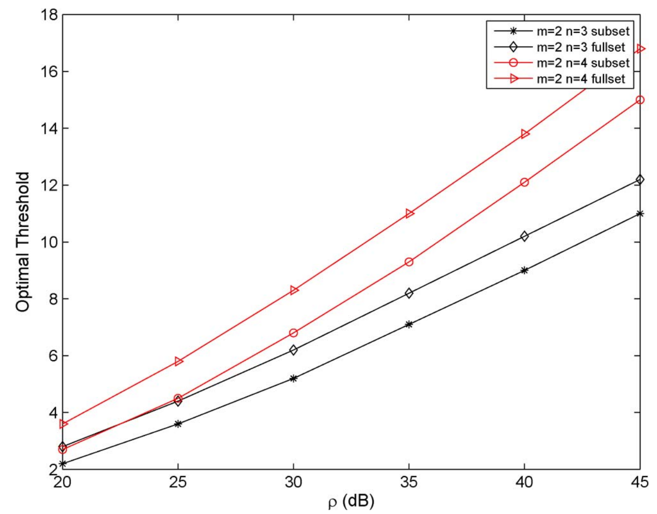

Fig. 6. Comparision of the optimal threshold values as a function of ρ for subset selection and fullset selection.

the following cases: $m = 2$ and $n = 2, 3, 4$. For all cases, we consider the case of one threshold. For the asymptotic optimal thresholds, we use the function $(n-1)\log(c\rho/\log c\rho)$ by setting $c = 0.22$, $c = 0.058$, and $c = 0.0336$ for $n = 2,\ 3,$ and 4, respectively. It shows that the asymptotic values perfectly match the exact ones, which validates our observation for the form of the optimal threshold in Section V. We also notice that the optimal thresholds increase as ρ and n increase.

In Fig. 6, we compare the values of the optimal thresholds for fullset selection and subset selection with $m = 2$, and $n = 3, 4$. We consider the case when there is one threshold. As can be seen in the figure, the curves for fullset selection and subset selection are parallel. It indicates that they have the same asymptotic optimal threshold function. In addition, the threshold value of fullset selection is greater than that of subset selection for the same n.

VII. Conclusion

In this paper, we presented a limited feedback quantization strategy and the corresponding relay assignment schemes for relay networks comprising multiple source-destination pairs. We examined two assignment schemes, fullset selection, which is based on searching over all possible assignment permutations, and subset selection, which is based on searching over only a subset on the possible permutations. We have derived BER expressions for both selection schemes based on the worst E2E SNR. By studying the asymptotic performance at high SNR, we found that the asymptotic optimal threshold function is in the form of $(n-1)\log c\rho$. We also compared the performance of relay assignment with limited feedback and full CSI in terms of the achievable diversity order and resulting E2E BER. For the diversity analysis, we adopted a generalized measure of diversity and showed that they can achieve diversity gains $(n, -(n-1))$ and $(n, 0)$, respectively. So there is a second-order diversity loss if only quantized CSI is available. As for the E2E BER, we observed that little loss in performance was experienced as compared to that of full CSI.

APPENDIX

A. Proof of Lemma 1

For subset selection, $\gamma_{k,\min}$ varies identically and independently and its pdf for a network with m pairs and n relays can be expressed as [7]

$$f_{\gamma_{\min}}(\gamma_{\min}) = \frac{1}{\overline{\gamma}_{\min}} e^{-\frac{\gamma_{\min}}{\overline{\gamma}_{\min}}}, \tag{27}$$

where $\overline{\gamma}_{\min} = (1/m)(\overline{\gamma}_{SR}\overline{\gamma}_{RD}/(\overline{\gamma}_{SR}+\overline{\gamma}_{RD}))$. Since the instantaneous SNR range $[0,\infty]$ is divided by $N-1$ thresholds, γ_{thl} $(l=1,2,\ldots,N-1)$, there are N quantization levels. If the worst E2E SNR of the selected choice, $\gamma_{k^*,\min}$, belongs to the quantization level $[\gamma_{thl},\gamma_{th(l+1)}]$, it means that at least one of $\gamma_{k,\min}$ belongs to this quantization level and no $\gamma_{k,\min}$ are greater than $\gamma_{th(l+1)}$. For the probability term in (4) when $l=0$, i.e., $\gamma_{k^*,\min}$ is less than γ_{th1}, it means that all $\gamma_{k,\min}$ are less than γ_{th1}. Then we have

$$\begin{aligned} P_r(&\gamma_{k^*,\min} < \gamma_{th1}) \\ &= (P_r(\gamma_{\min} < \gamma_{th1}))^n \\ &= \left(\int_0^{\gamma_{th1}} \frac{1}{\overline{\gamma}_{\min}} e^{-\frac{\gamma_{\min}}{\overline{\gamma}_{\min}}} d\gamma_{\min}\right)^n \\ &= \left(1-e^{-\frac{\gamma_{th1}}{\overline{\gamma}_{\min}}}\right)^n. \end{aligned} \tag{28}$$

When $l \geq 1$, the probability terms in (4) can be expressed as

$$\begin{aligned} P_r&\left(\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)}\right) \\ &= \sum_{j=1}^{n} \frac{n!}{j!(n-j)!} \left(P_r(\gamma_{thl} < \gamma_{\min} < \gamma_{th(l+1)})\right)^j \\ &\quad \cdot (P_r(\gamma_{\min} < \gamma_{thl}))^{n-j} \\ &= \sum_{j=1}^{n} \frac{n!}{j!(n-j)!} \left(\int_{\gamma_{thl}}^{\gamma_{th(l+1)}} \frac{1}{\overline{\gamma}_{\min}} e^{-\frac{\gamma_{\min}}{\overline{\gamma}_{\min}}} d\gamma_{\min}\right)^j \\ &\quad \cdot \left(\int_0^{\gamma_{thl}} \frac{1}{\overline{\gamma}_{\min}} e^{-\frac{\gamma_{\min}}{\overline{\gamma}_{\min}}} d\gamma_{\min}\right)^{n-j} \\ &= \sum_{j=1}^{n} \frac{n!}{j!(n-j)!} \left(e^{-\frac{\gamma_{thl}}{\overline{\gamma}_{\min}}} - e^{-\frac{\gamma_{th(l+1)}}{\overline{\gamma}_{\min}}}\right)^j \\ &\quad \cdot \left(1-e^{-\frac{\gamma_{thl}}{\overline{\gamma}_{\min}}}\right)^{n-j}. \end{aligned} \tag{29}$$

The BER conditioned on $\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)}$ can be expressed as

$$\begin{aligned} P&\left(\varepsilon|\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)}\right) \\ &= \int_0^{\infty} d \cdot \operatorname{erfc}\sqrt{a\gamma_{\min}} f_{\gamma_{\min}|\gamma_{thl}<\gamma_{\min}<\gamma_{th(l+1)}}(\gamma_{\min}) d\gamma_{\min}, \end{aligned} \tag{30}$$

where $f_{\gamma_{\min}|\gamma_{thl}<\gamma_{\min}<\gamma_{th(l+1)}}(\gamma_{\min})$ is the conditional pdf of $\gamma_{\min}$ conditioned on $\gamma_{thl} < \gamma_{\min} < \gamma_{th(l+1)}$, which is derived as

$$f_{\gamma_{\min}|\gamma_{thl}<\gamma_{\min}<\gamma_{th(l+1)}}(\gamma_{\min}) = \frac{\frac{1}{\overline{\gamma}_{\min}} e^{-\frac{1}{\overline{\gamma}_{\min}}\gamma_{\min}}}{e^{-\frac{1}{\overline{\gamma}_{\min}}\gamma_{thl}} - e^{-\frac{1}{\overline{\gamma}_{\min}}\gamma_{th(l+1)}}}. \tag{31}$$

Plugging (31) into (30) and carrying out the integration, we obtain

$$P\left(\varepsilon|\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)}\right) = \frac{d \cdot \operatorname{erfc}\left(a,\overline{\gamma}_{\min},\gamma_{thl},\gamma_{th(l+1)}\right)}{e^{-\frac{1}{\overline{\gamma}_{\min}}\gamma_{thl}} - e^{-\frac{1}{\overline{\gamma}_{\min}}\gamma_{th(l+1)}}}. \tag{32}$$

Plugging (28), (29), and (32) into (4) and then into (3) yields (5), which completes the proof.

Note that the obtained results can be adapted to other modulation schemes. For instance, for M-QAM, the exact BER, conditioned on the instantaneous SNR, γ, is given by [27]

$$\begin{aligned} P_e^{M\text{-QAM}}(\gamma) &= \frac{1}{\sqrt{M}\log_2\left(\sqrt{M}\right)} \sum_{k=1}^{\log_2\left(\sqrt{M}\right)} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \\ &\quad \cdot \left[(-1)^{\left\lfloor \frac{i \cdot 2^{k-1}}{\sqrt{M}} \right\rfloor} \times \left(2^{k-1} - \left\lfloor \frac{i \cdot 2^{k-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right)\right. \\ &\quad \left. \cdot \operatorname{erfc}\left((2i+1)\sqrt{\frac{3\log_2\left(\sqrt{M}\right)\gamma}{2(M-1)}}\right)\right]. \end{aligned}$$

Therefore, the analysis in our paper can be directly extended to M-QAM by setting

$$\begin{aligned} d &= \frac{1}{\sqrt{M}\log_2\left(\sqrt{M}\right)} \sum_{k=1}^{\log_2\left(\sqrt{M}\right)} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} (-1)^{\left\lfloor \frac{i \cdot 2^{k-1}}{\sqrt{M}} \right\rfloor} \\ &\quad \times \left(2^{k-1} - \left\lfloor \frac{i \cdot 2^{k-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right), \end{aligned}$$

and $a = (2i+1)^2(3\log_2(\sqrt{M})/2(M-1))$.

B. Proof of Lemma 2

Since there are correlations among $\gamma_{k,\min}$ for different k, the calculation of the selected worst E2E BER cannot be done in the same way as that of subset selection. Alternatively, we can calculate the probability terms in Equation (4) in the following way. We first derive the probability terms for the case when there is only one threshold, γ_{thl} $(l=1,2,\ldots,N-1)$. Then we generalize it to multiple thresholds. In the case of one threshold, the quantized CSIs are only distinguished by whether they are greater or less than γ_{thl}. As a consequence, the selected worst E2E SNR, i.e., $\gamma_{k^*,\min}$, can only be either greater or less than γ_{thl}. Let T denote the number of γ_{ij}^q that are greater than γ_{thl}. Note that T is an integer. Since the total number of items in the E2E SNR matrix for a network with two pairs and n relays is $2n$, the probability terms for this case can be written as

$$P_r(\gamma_{k^*,\min} > \gamma_{thl}) = \sum_{t=0}^{2n} P_r\left(\gamma_{k^*,\min} > \gamma_{thl} | T=t\right) P_r(T=t), \tag{33}$$

and

$$P_r(\gamma_{k^*,\min} < \gamma_{thl}) = \sum_{t=0}^{2n} P_r\left(\gamma_{k^*,\min} < \gamma_{thl} | T=t\right) P_r(T=t), \tag{34}$$

where $P_r(T=t)$ represents the probability of having exactly t items greater than γ_{thl}, and $P_r(\gamma_{k^*,\min} > \gamma_{thl}|T=t)$ is the probability that $\gamma_{k^*,\min}$ is greater than γ_{thl} given that there are t items greater than γ_{thl}. Note that the pdf of the E2E SNR is given in (1). The term $P_r(T=t)$ in (33) and (34) can be derived as

$$\begin{aligned} P_r(T=t) &= \frac{2n!}{t!(2n-t)!}\left(\int_{\gamma_{thl}}^{\infty} \frac{1}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}} d\gamma\right)^t \left(\int_0^{\gamma_{thl}} \frac{1}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}} d\gamma\right)^{2n-t} \\ &= \frac{2n!}{t!\,(2n-t)!} e^{-\frac{t\gamma_{thl}}{\overline{\gamma}}} \left(1-e^{-\frac{\gamma_{thl}}{\overline{\gamma}}}\right)^{2n-t}. \end{aligned} \tag{35}$$

For $0 \le T \le 1$, since $\gamma_{k^*,\min}$ is always less than γ_{thl} in this case, we have

$$P_r(\gamma_{k^*,\min} > \gamma_{thl}|0 \le T \le 1) = 0, \tag{36}$$

and

$$P_r(\gamma_{k^*,\min} < \gamma_{thl}|0 \le T \le 1) = 1. \tag{37}$$

For $T=2$, we find that $\gamma_{k^*,\min}$ will be greater than γ_{thl} once the two items that are greater than γ_{thl} are on the same row in the E2E SNR matrix. Since there are a total of $n!/(n-2)!$ rows in the fullset E2E SNR matrix, we have

$$P_r(\gamma_{k^*,\min} > \gamma_{thl}|T=2) = \frac{\frac{n!}{(n-2)!}}{\frac{2n!}{2(2n-2)!}}, \tag{38}$$

and

$$P_r(\gamma_{k^*,\min} < \gamma_{thl}|T=2) = \frac{\frac{2n!}{2(2n-2)!} - \frac{n!}{(n-2)!}}{\frac{2n!}{2(2n-2)!}}. \tag{39}$$

For $2 < T \le n$, we observe that $\gamma_{k^*,\min}$ will be less than γ_{thl} if the items that are greater than γ_{thl} are in the same column of the E2E SNR matrix. Otherwise, $\gamma_{k^*,\min}$ will be greater than γ_{thl} in this case. Since the total number of non identical items in one column is n and there are two columns in the matrix, we have

$$P_r(\gamma_{k^*,\min} > \gamma_{thl}|2 < T \le n) = \frac{\frac{2n!}{t!(2n-t)!} - 2\frac{n!}{t!(n-t)!}}{\frac{2n!}{t!(2n-t)!}}, \tag{40}$$

and

$$P_r(\gamma_{k^*,\min} < \gamma_{thl}|2 < T \le n) = \frac{2\frac{n!}{t!(n-t)!}}{\frac{2n!}{t!(2n-t)!}}. \tag{41}$$

For $n < T \le 2n$, $\gamma_{k^*,\min}$ will always be greater than γ_{thl} in this case. Then we have

$$P_r(\gamma_{k^*,\min} > \gamma_{thl}|n < T \le 2n) = 1, \tag{42}$$

and

$$P_r(\gamma_{k^*,\min} < \gamma_{thl}|n < T \le 2n) = 0. \tag{43}$$

Then plugging (35)–(43) into (33) and (34), we obtain the expressions for $P_r(\gamma_{k^*,\min} < \gamma_{thl})$, which is given in (7) and $P_r(\gamma_{k^*,\min} > \gamma_{thl})$. By replacing γ_{thl} with $\gamma_{th(l+1)}$, we obtain $P_r(\gamma_{k^*,\min} > \gamma_{th(l+1)})$ as expressed in (6). Now we generalize the results of one threshold to multiple thresholds. By observing that

$$\begin{aligned} &P_r\left(\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)}\right) \\ &= 1 - P_r(\gamma_{k^*,\min} < \gamma_{thl}) - P_r\left(\gamma_{k^*,\min} > \gamma_{th(l+1)}\right), \end{aligned} \tag{44}$$

we can indirectly calcuate the probability term in (4) from (44). Note that we can also use $P_r(\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)}) = P_r(\gamma_{k^*,\min} > \gamma_{thl}) - P_r(\gamma_{k^*,\min} > \gamma_{th(l+1)})$ or $P_r(\gamma_{thl} < \gamma_{k^*,\min} < \gamma_{th(l+1)}) = P_r(\gamma_{k^*,\min} < \gamma_{th(l+1)}) - P_r(\gamma_{k^*,\min} < \gamma_{thl})$. Then we obtain a closed-form expression for P_e as expressed in Lemma 2.

C. Proof of Lemma 3

According to the E2E BER given in (3) for subset selection, the E2E BER with one threshold is given as

$$P_e = P_{e0} + P_{e1},$$

where P_{e0} and P_{e1} are the BERs that $\gamma_{k^*,\min}$ is less and greater than γ_{th}, respectively. According to (4), (28), (30), and (31), P_{e0} can be expressed as

$$\begin{aligned} P_{e0} &= \left(1-e^{-\frac{\gamma_{th}}{\rho}}\right)^n \frac{1}{1-e^{-\frac{\gamma_{th}}{\rho}}} \int_0^{\gamma_{th}} \frac{1}{2}\mathrm{erfc}\sqrt{\gamma}\frac{1}{\rho}e^{-\frac{\gamma}{\rho}} d\gamma \\ &\le \left(1-e^{-\frac{\gamma_{th}}{\rho}}\right)^{n-1} \int_0^{\infty} \frac{1}{2}\mathrm{erfc}\sqrt{\gamma}\frac{1}{\rho}e^{-\frac{\gamma}{\rho}} d\gamma \\ &\le \frac{\gamma_{th}^{n-1}}{\rho^{n-1}} \times \int_0^{\infty} \frac{1}{4}e^{-\gamma}\frac{1}{\rho}e^{-\frac{\gamma}{\rho}} d\gamma && (45) \\ &\le \frac{\gamma_{th}^{n-1}}{4\rho^n}, && (46) \end{aligned}$$

where (45) follows from the fact that $1-e^{-x} \le x$ and $\mathrm{erfc}\sqrt{x} \le (1/2)e^{-x}$. Since the asymptotic behaviour will not be changed by the assumption of the symmetric channels and modulation scheme [28], we simply assume $\overline{\gamma}_{\min} = \rho$ by assuming that the equivalent worst E2E channel gains are modeled as zero mean, unit variance complex Gaussian random variables. We also assume BPSK in this part. According to (4), (30), and (31), P_{e1} can be expressed as

$$
\begin{aligned}
P_{e1} &= P_r(\gamma_{k^*,\min} > \gamma_{th}) e^{\frac{1}{\rho}\gamma_{th}} \int_{\gamma_{th}}^{\infty} \frac{1}{2}\mathrm{erfc}\sqrt{\gamma}\frac{1}{\rho}e^{-\frac{\gamma}{\rho}}d\gamma \\
&\leq e^{\frac{1}{\rho}\gamma_{th}} \int_{\gamma_{th}}^{\infty} \frac{1}{2}\mathrm{erfc}\sqrt{\gamma}\frac{1}{\rho}e^{-\frac{\gamma}{\rho}}d\gamma \qquad (47)\\
&\leq e^{\frac{1}{\rho}\gamma_{th}} \int_{\gamma_{th}}^{\infty} \frac{1}{4}e^{-\gamma}\frac{1}{\rho}e^{-\frac{\gamma}{\rho}}d\gamma \qquad (48)\\
&= \frac{1}{4(1+\rho)}e^{-\gamma_{th}} \\
&\leq \frac{1}{4\rho}e^{-\gamma_{th}}, \qquad (49)
\end{aligned}
$$

where (47) follows from knowing that the probability that $\gamma_{k^*,\min} > \gamma_{th}$ is less than one and (48) follows from $\mathrm{erfc}\sqrt{x} \leq (1/2)e^{-x}$. Then plugging (49) and (46) into (3), yields the asymptotic E2E BER for subset selection, which is given by (8). This completes the proof.

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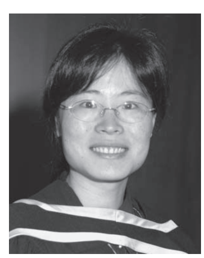

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