An Adaptive Transmission Scheme for Two-Way Relaying With Asymmetric Data Rates

Xuehua Zhang, Mazen O. Hasna, and Ali Ghrayeb

***Abstract*—In this paper, we address the problem of asymmetric data rates in two-way communication systems. In particular, we propose an adaptive transmission scheme that combines network coding (NC) and opportunistic user selection (OUS) with a threshold that determines which transmission mode to use. The underlying system model comprises two source nodes communicating with each other through a relay node. The source nodes are assumed to have different data rate requirements; therefore, they employ different modulation schemes. As per the proposed scheme, if the end-to-end (E2E) signal-to-noise ratio (SNR) of both users are above a specified threshold, both sources transmit over orthogonal channels, and the relay node uses hierarchical modulation and NC to relay the combined signals to both sources in the third time slot. Otherwise, the user with the better E2E SNR transmits, whereas the other user remains silent. The advantage of the proposed scheme is that it compromises between throughput and reliability. That is, when both users transmit, the throughput improves, whereas when the better user transmits, multiuser diversity is achieved. Assuming asymmetric channels, we derive exact closed-form expressions for the E2E bit error rate (BER), access probability, and throughput for this scheme and compare its performance with existing schemes. We also investigate the asymptotic performance of the proposed scheme at high SNRs where we derive the achievable diversity order for both users. We show through analytical and simulation results that the proposed scheme improves the overall system throughput, the fairness between the two users, and the transmission reliability. This all comes while achieving full spatial diversity for both users.**

***Index Terms*—Cooperative diversity, decode-and-forward (DF), hierarchical network coding, network coding (NC), opportunistic user selection (OUS), two-way communication.**

I. Introduction

Cooperative communications has proven to be an effective means to combat wireless fading by allowing devices to share their antennas to achieve spatial diversity [1]–[3]. As we know, for practical reasons, cooperative nodes operate in a half-duplex mode [3], implying a loss in spectral efficiency. To mitigate the spectral efficiency loss and improve the throughput of cooperative communication networks, some efforts have been made to incorporate network coding (NC) into cooperative communication. The concept of NC was first proposed by Ahlswede *et al.* in [4] as a routing method in lossless wireline networks. The key idea of NC is that an intermediate node, normally referred to as a relay node, linearly combines the received data from different sources instead of sending them individually, resulting in improved bandwidth efficiency. Various NC protocols have been proposed for two-way communications, which can be classified into two types: two time-slot NC schemes (i.e., analog NC [5]–[7] and physical-layer NC [8]–[11]) and three time-slot NC schemes [12]. The three time-slot NC scheme was extensively studied (see for example [13]–[18] and references therein). The three-time slot NC scheme combined with threshold-based relaying to control error propagation with maximum ratio combining (MRC) was studied in [13] and [14], and with maximum likelihood (ML) detection in [15]. In [16], the problem of relay assignment for cooperative networks comprising multiple bidirectional transmitting pairs was addressed. The problem of relay selection was addressed in [17] and references therein. In [18], hierarchical modulation was employed at one source to cope with the performance degradation caused by asymmetric relay channels.

However, in most of the work that deals with bidirectional transmission mentioned previously, it is normally assumed that the two transmitting nodes (or users) have the same rate. In many practical scenarios, however (such as having different quality of service (QoS) requirements, different available traffic, and so on), the two users may not have the same transmission rate. Some efforts have been made to solve this data rate mismatch problem at the relay node (see [19]–[21] and references therein). In [19], a mapping codebook-based physical-layer NC was proposed. In [20], Tang *et al.* tried to solve the rate mismatch by reinterpreting NC as a mapping of modulation constellation. However, this joint modulation/NC approach requires considerable changes to the de(modulator) design and increases the detection and demodulation complexity. In contrast, one simple way, without increasing the complexity of demodulation, is to append zeros to the end of the shorter bit sequence to make the two bit sequences have the same length. This zero padding process suggests that the user with the lower rate will have to use a higher order modulation scheme to match that of the user of the higher rate, which deteriorates the performance of that user. To remedy the rate mismatch challenge without increasing the complexity or deteriorating the performance, we have proposed in [21] a hierarchical zero padding/NC (HZPNC), which involves employing hierarchical

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X. Zhang was with Concordia University, Montréal, QC H4B 1R6, Canada (e-mail: xuehua08@gmail.com).

M. O. Hasna is with the Department of Electrical Engineering, Qatar University, Doha, Qatar (e-mail: hasna@qu.edu.qa).

A. Ghrayeb is with the Department of Electrical and Computer Engineering, Texas A&M University, Doha, Qatar (e-mail: ali.ghrayeb@qatar.tamu.edu).

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modulation by the user with the higher data rate while padding zeros at specific positions of the shorter bit sequence at the relay. We also considered another scheme, which is referred to as opportunistic user selection (OUS), where the user with the better end-to-end (E2E) channel quality is given priority for transmission. It was shown that each scheme offers certain advantages over the other. For instance, the HZPNC scheme offers better throughput, but this comes at the expense of a degraded E2E bit error rate (BER) performance as compared with that of OUS. On the other hand, OUS achieves better E2E BER performance, taking advantage of the inherent multiuser diversity. The pitfall of OUS, however, is the lack of fairness between the communicating users. That is, depending on the individual channel quality, one user may enjoy better access probability than the other.

In this paper, we propose an adaptive transmission scheme that aims at taking advantage of both OUS and HZPNC. Specifically, the proposed scheme captures the inherent multiuser diversity offered by OUS and improves the throughput through HZPNC. For simplicity, we consider a relay network comprising two users and one relay. The two users are assumed to transmit at different data rates. To achieve fairness, the proposed scheme allows not only the user with the better E2E SNR to transmit but also the other user if the channel quality of this user is above a predetermined threshold. In addition, to improve the throughput, HZPNC is employed at the relay when both users transmit. Thus, when both users transmit, the relay XORs the signals received from the two users (over two time slots) and forwards the resulting signal in the third time slot. Since the two received sequences have different lengths, i.e., users have different rates, a certain form of zero padding is done at the relay to make the two sequences suitable for XORing.

In the following, we summarize the contributions of this paper.

1) The proposed scheme offers better performance compared with that of OUS in terms of E2E BER, access probability, and throughput. In addition, it has almost the same access probability and throughput as that of HZPNC at a high SNR.
2) We derive the probability density function (pdf) of the instantaneous SNR for OUS and HZPNC. These pdfs are needed to derive the E2E BER performance of the proposed scheme.
3) We derive exact closed-form expressions for the E2E BER performance for our proposed scheme over *asymmetric* channels for *asymmetric* data rates. We also derive expressions for the access probability and throughput for this scheme.
4) We examine the asymptotic E2E BER performance at high SNR for both users and determine the achievable diversity gain. It is shown that the proposed scheme achieves full diversity, which is the number of available users.

The remainder of this paper is organized as follows. The system model is presented in Section II. In Section III, the proposed adaptive transmission scheme is presented. We analyze the E2E BER performance of the proposed scheme in Section IV. The diversity order of the proposed scheme is derived in Section V. We examine its performance in terms of access probability and throughput in Section VI. We present several numerical examples in Section VII and conclude this paper in Section VIII.

II. System Model

We consider a bidirectional cooperative network with two users denoted by S_1 and S_2 and one relay denoted by R, where the users communicate with each other via the relay node over orthogonal subchannels. For simplicity, we assume that there is no direct path between the two users. Both users and the relay are equipped with a single antenna and operate in a half-duplex mode. The two users have different data rates. In particular, we assume that one user uses 4-QAM and the other uses 16-QAM, but the scheme and analytical approach can be extended to other modulation schemes.[1]

The network subchannels are assumed to experience independent slow and frequency nonselective Rayleigh fading. Let h_{ir} and h_{rj} for $i, j = 1, 2$ denote the fading coefficients for hops $S_i \to R$ and $R \to S_j$, respectively. The subchannels are assumed independent and asymmetric, i.e., all the subchannels have different average SNRs, which is the most general case. Let γ_{ir} and γ_{rj} denote the instantaneous SNRs for the links $S_i \to R$ and $R \to S_j$, respectively. To make the presentation simpler, we denote the instantaneous SNRs over different links by γ_{im} for $i, m = 1, 2$ where $\gamma_{11} = \gamma_{1r}$, $\gamma_{12} = \gamma_{r2}$, $\gamma_{21} = \gamma_{2r}$, and $\gamma_{22} = \gamma_{r1}$, i.e., index i refers to the user, and m refers to the mth hop of that user. The pdf of γ_{im} is given as

$$f_{\gamma_{im}}(\gamma_{im}) = \frac{1}{\overline{\gamma}_{im}} e^{-\frac{1}{\overline{\gamma}_{im}}\gamma_{im}} \tag{1}$$

where $\overline{\gamma}_{im} = \rho E[|h_{im}|^2]$ is the average SNR for the pertaining link, and $\rho = E_b/N_0$. For decode-and-forward (DF) relaying where DF refers to uncoded DF, the exact E2E instantaneous SNR of S_i is well approximated as $\gamma_i = \min(\gamma_{i1}, \gamma_{i2})$, and its pdf is expressed as

$$f_{\gamma_i}(\gamma_i) = \frac{1}{\overline{\gamma}_i} e^{-\frac{1}{\overline{\gamma}_i}\gamma_i} \tag{2}$$

where $\overline{\gamma}_i = (\overline{\gamma}_{i1}\overline{\gamma}_{i2}/(\overline{\gamma}_{i1} + \overline{\gamma}_{i2}))$. Thus, the E2E instantaneous SNR of S_i in this paper refers to $\gamma_i = \min(\gamma_{i1}, \gamma_{i2})$.

III. Proposed Adaptive Relaying Scheme

As aforementioned, the proposed scheme aims at improving the fairness between the two users by opportunistically allowing the second user to transmit, although it has a worse channel. In particular, for a given threshold γ_{th}, if both instantaneous E2E SNRs are above γ_{th}, both users transmit, i.e., HZPNC is used; otherwise, the user with the better E2E SNR transmits, whereas the other user remains silent. Here, we describe the mode of operation in each case. Note that, when $\gamma_{\mathrm{th}} = 0$, the proposed adaptive transmission scheme reduces to the case when both

[1] We merely use specific modulations in the development of the proposed adaptive scheme just for ease of presentation. The results obtained in this paper are, in fact, independent of the modulation schemes employed.

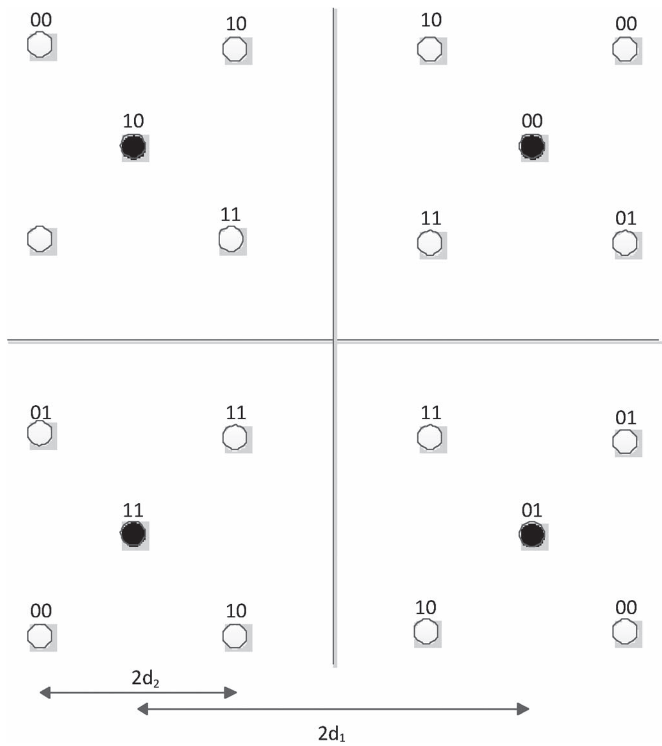

Fig. 1. Hierarchical 4/16-QAM.

users always transmit, i.e., HZPNC, whereas when $\gamma_{\rm th} = \infty$, the proposed scheme reduces to the case when only the best user transmits, i.e., OUS.

A. HZPNC Scheme

When HZPNC is used, in the first two time slots, the two sources transmit their signals in succession. The relay node decodes the two received signals, applies HZPNC to the decoded signals (after applying some form of zero padding if needed), and broadcasts the resulting signal to all nodes. We assume that S_1 uses quadrature phase-shift keying (QPSK) modulation and S_2 uses hierarchical 16-ary quadrature amplitude modulation (16-QAM).

Hierarchical modulation offers different degrees of protection to the transmitted bits according to their relative importance [24]–[26]. In Fig. 1, we illustrate the 4/16 hierarchical constellation. The filled circles represent the fictitious QPSK symbols, and the blank circles represent the actual transmitted 16-QAM symbols. The transmitted bit sequence consists of two subsequences: high-priority (HP) bits and low-priority (LP) bits. The HP bits are assigned to the positions of the fictitious QPSK symbols, whereas the LP bits are assigned to the remaining positions. In the figure, $2d_1$ is the distance between two fictitious QPSK symbol points, and $2d_2$ is the distance between the actual transmitted 16-QAM constellation points within one quadrant. The constellation priority parameter is denoted by $d = d_1/d_2$.

The received bit sequences $\widehat{b}_i$ $(i = 1, 2)$ corresponding to one symbol at the relay will not be equal in length. Since S_2 employs hierarchical 4/16-QAM, $\widehat{b}_2$ consists of two HP bits and two LP bits. Then, the two bits of $\widehat{b}_1$ are XORed with the two HP bits of $\widehat{b}_2$, and the two LP bits of S_2 remain unchanged. The resulting bit sequence is then modulated by hierarchical 4/16-QAM with the NC bits as HP bits and the unchanged bits as LP bits. The modulated signal x_r is then broadcast to all nodes in the third time slot. The signals received at the two users are expressed as $y_{ri} = \sqrt{4\rho} h_{ri} x_r + n_{ri}$ for $i = 1, 2$. These received signals can be decoded at S_2 using ML as

$$\hat{x}_r = \mathop{\arg\min}_{x_r \in \text{fictitious}\, 4-\text{QAM}} \left| y_{r2} - \sqrt{4\rho} h_{r2} x_r \right|$$

and at S_1 as

$$\hat{x}_r = \mathop{\arg\min}_{x_r \in 4/16-\text{QAM}} \left| y_{r1} - \sqrt{4\rho} h_{r1} x_r \right|.$$

As shown earlier, since the data of S_1 at the relay is only involved with the HP bits of x_r, S_2 needs to only decode the HP bits, which correspond to a fictitious QPSK symbol. Since each user knows its own transmitted signal, it can decode the desired signal according to the NC scheme used at the relay.

B. Description of the OUS Scheme

For this scheme, only the user with the better instantaneous E2E SNR transmits at a time. That is, if $\gamma_i > \gamma_j$ $(i, j = 1, 2, \text{s.t. } i \neq j)$, only S_i transmits to S_j with the help of the relay. Let us assume that S_1 is selected as an example. Therefore, in the first time slot, S_1 transmits to the relay. The received signal at the relay is $y_r = \sqrt{2\rho} h_{1r} x_1 + n_{1r}$. Then, the relay decodes the received signal as

$$\hat{x}_1 = \mathop{\arg\min}_{x_1 \in 4-\text{QAM}} \left| y_r - \sqrt{2\rho} h_{1r} x_1 \right|.$$

The resulting sequence is then remodulated by the 4-QAM modulator. The modulated signal, which is denoted by x_r, is transmitted to S_2 in the second time slot. The signal received by S_2 is $y_2 = \sqrt{2\rho} h_{r2} x_r + n_{r2}$. Then, S_2 can decode the received signal as

$$\hat{x}_r = \mathop{\arg\min}_{x_r \in 4-\text{QAM}} \left| y_2 - \sqrt{2\rho} h_{r2} x_r \right|.$$

C. On the Optimal Threshold

As aforementioned, the threshold $\gamma_{\rm th}$ is employed to decide whether to allow both users or the user with the better E2E SNR to transmit. The criterion used to derive $\gamma_{\rm opt}$, which is the optimal threshold, is to minimize the worst E2E BER of the two users. For instance, the optimal threshold for S_i, which is denoted by $\gamma_{\rm opt}$, can be obtained as

$$\gamma_{\rm opt} = \arg \min_{\overline{\gamma}_{im}, \gamma_{\rm th}} (P_i)$$

where the E2E BER of S_i, which is denoted by P_i, is a function of the average SNR and $\gamma_{\rm th}$. Therefore, the thresholds are optimized for the user, which has the worst E2E BER and not for both users. Since the E2E BER expression for S_i is not invertible, the exact optimal thresholds can only be obtained by numerically minimizing the E2E BER with an exhaustive grid search [23], and this is the approach followed in this paper. However, in Section V, we give the optimal threshold function and verify it via simulations in Section VII. As far as implementation is concerned, we assume that a central controller in the network has the CSI of all links. It calculates the optimal thresholds and decides which transmission mode to use.

IV. End-to-End Bit Error Rate Performance Analysis

A. E2E BER Performance

Here, we derive a closed-form expression for the E2E BER for the proposed scheme. We assume that S_1 employs 4-QAM, and S_2 employs 4/16-QAM. However, the performance analysis can be extended to other hierarchical modulation schemes following the results of [27].

Lemma 1: The E2E BER corresponding to user S_i can be expressed as

$$P_{\varepsilon i}^{\text{proposed}} = \frac{2e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\text{th}}} P(\varepsilon_i \,|\min(\gamma_i,\gamma_j) > \gamma_{\text{th}})}{2e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\text{th}}} + 3\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}\left(1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\text{th}}}\right)} + \frac{3\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}\left(1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\text{th}}}\right)P(\varepsilon_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\text{th}})}{2e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\text{th}}} + 3\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}\left(1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\text{th}}}\right)} \tag{3}$$

where $P(\varepsilon_i|\min(\gamma_i,\gamma_j) > \gamma_{\text{th}}.)$ represents the E2E BER of S_i conditioned on $\min(\gamma_i,\gamma_j) > \gamma_{\text{th}}$,, which is the E2E BER for S_i when using HZPNC, and $P(\varepsilon_i|\gamma_i > \gamma_j, \gamma_j < \gamma_{\text{th}})$ denotes the E2E BER of S_i conditioned on $\gamma_i > \gamma_j, \gamma_j < \gamma_{\text{th}}$, which is the E2E BER for S_i when using OUS.

Proof: See Appendix A. ■

In what follows, we derive closed-from expressions for the probabilities in (3).

1) Derivation of $P(\varepsilon_i|\min(\gamma_i,\gamma_j) > \gamma_{\text{th}})$: According to the HZPNC scheme proposed in [21], the bits from S_1 are XORed with the HP bits from S_2 (assuming that S_1 uses 4-QAM and S_2 uses hierarchical 4/16-QAM. Consequently, S_2 decodes only the fictitious 4-QAM constellation of the hierarchical 4/16-QAM constellation. The E2E BER at S_2 conditioned on $\min(\gamma_i,\gamma_j) > \gamma_{\text{th}}$ is given as

$$\begin{aligned} P(\varepsilon_1 \,|\min(\gamma_1,\gamma_2) > \gamma_{\text{th}}) &= \left(1 - P(\varepsilon_{r2}^{\text{HP}} \,|\gamma_{r2} > \gamma_{\text{th}})\right) \\ &\quad\cdot \left[P(\varepsilon_{1r} \,|\gamma_{1r} > \gamma_{\text{th}})\left(1 - P(\varepsilon_{2r}^{\text{HP}} \,|\gamma_{2r} > \gamma_{\text{th}})\right)\right. \\ &\qquad \left. + P(\varepsilon_{2r}^{\text{HP}} \,|\gamma_{2r} > \gamma_{\text{th}})\left(1 - P(\varepsilon_{1r} \,|\gamma_{1r} > \gamma_{\text{th}})\right)\right] \\ &\quad + P(\varepsilon_{r2}^{\text{HP}} \,|\gamma_{r2} > \gamma_{\text{th}}) \\ &\quad\cdot \left\{1 - \left[P(\varepsilon_{1r} \,|\gamma_{1r} > \gamma_{\text{th}})\left(1 - P(\varepsilon_{2r}^{\text{HP}} \,|\gamma_{2r} > \gamma_{\text{th}})\right)\right.\right. \\ &\qquad \left.\left. + P(\varepsilon_{2r}^{\text{HP}} \,|\gamma_{2r} > \gamma_{\text{th}})\left(1 - P(\varepsilon_{1r} \,|\gamma_{1r} > \gamma_{\text{th}})\right)\right]\right\} \end{aligned} \tag{4}$$

where $P(\varepsilon_{2r}^{\text{HP}}|\gamma_{2r} > \gamma_{\text{th}})$ and $P(\varepsilon_{r2}^{\text{HP}}|\gamma_{r2} > \gamma_{\text{th}})$ are the probabilities of making an error over the $S_2 \to R$ and $R \to S_2$ links, which are conditioned on $\gamma_{2r} > \gamma_{\text{th}}$ and $\gamma_{r2} > \gamma_{\text{th}}$, respectively, for the HP bits from S_2; and $P(\varepsilon_{1r}|\gamma_{1r} > \gamma_{\text{th}})$ is the BER over the $S_1 \to R$ link, which is conditioned on $\gamma_{1r} > \gamma_{\text{th}}$, for the bits from S_1. Since $\gamma_i = \min(\gamma_{i1},\gamma_{i2})$, $\min(\gamma_i,\gamma_j) > \gamma_{\text{th}}$ is equivalent to $\gamma_{i1} > \gamma_{\text{th}}$, $\gamma_{i2} > \gamma_{\text{th}}$, $\gamma_{j1} > \gamma_{\text{th}}$, and $\gamma_{j2} > \gamma_{\text{th}}$. The instantaneous SNRs of different links are independent; therefore, the BERs over different links are only related to the instantaneous SNR of their respective links. Thus, we only keep the effective terms in (4).

Lemma 2: The BER over any of the links employing 4-QAM and for the HP bits from S_2 conditioned on $\gamma_{im} > \gamma_{\text{th}}$ can be expressed as

$$\begin{aligned} P(\varepsilon_{im} \,|\gamma_{im} > \gamma_{\text{th}}) &= e^{\frac{1}{\overline{\gamma}_{im}}\gamma_{\text{th}}} I(1,\overline{\gamma}_{im},\gamma_{\text{th}},\infty) \end{aligned} \tag{5}$$

$$\begin{aligned} P(\varepsilon_{im}^{\text{HP}} \,|\gamma_{im} > \gamma_{\text{th}}) &= \frac{1}{2}e^{\frac{1}{\overline{\gamma}_{im}}\gamma_{\text{th}}}\left[I\left(\frac{2(d^2-2d+1)}{1+d^2},\overline{\gamma}_{im},\gamma_{\text{th}},\infty\right)\right. \\ &\qquad \left. + I\left(\frac{2(d^2+2d+1)}{1+d^2},\overline{\gamma}_{im},\gamma_{\text{th}},\infty\right)\right] \end{aligned} \tag{6}$$

where [29]

$$\begin{aligned} I(a,b,\gamma_{\text{th}l},\gamma_{\text{th}(l+1)}) &= \int_{\gamma_{\text{th}l}}^{\gamma_{\text{th}(l+1)}} \frac{1}{2}\text{erfc}\sqrt{a\gamma}\frac{1}{b}e^{-\frac{1}{b}\gamma}d\gamma \\ &= \frac{1}{2}e^{-\frac{1}{b}\gamma_{\text{th}l}}\text{erfc}\sqrt{a\gamma_{\text{th}l}} \\ &\quad - \frac{1}{2}\sqrt{\frac{ab}{1+ab}}\text{erfc}\sqrt{\gamma_{\text{th}l}\left(a+\frac{1}{b}\right)} \\ &\quad - \frac{1}{2}e^{-\frac{1}{b}\gamma_{\text{th}(l+1)}}\text{erfc}\sqrt{a\gamma_{\text{th}(l+1)}} \\ &\quad + \frac{1}{2}\sqrt{\frac{ab}{1+ab}}\text{erfc}\sqrt{\gamma_{\text{th}(l+1)}\left(a+\frac{1}{b}\right)}. \end{aligned}$$

Proof: See Appendix B. ■

Note that $P(\varepsilon_{1r}|\gamma_{1r} > \gamma_{\text{th}}) = P(\varepsilon_{11}|\gamma_{11} > \gamma_{\text{th}})$, $P(\varepsilon_{2r}^{\text{HP}}|\gamma_{2r} > \gamma_{\text{th}}) = P(\varepsilon_{21}^{\text{HP}}|\gamma_{21} > \gamma_{\text{th}})$, and $P(\varepsilon_{r2}^{\text{HP}}|\gamma_{r2} > \gamma_{\text{th}}) = P(\varepsilon_{12}^{\text{HP}}|\gamma_{12} > \gamma_{\text{th}})$. Plugging these expressions into (4) yields a closed-form expression for $P(\varepsilon_1|\min(\gamma_1,\gamma_2) > \gamma_{\text{th}})$.

Now, for the BER at S_1, recall that the bits coming from S_2 consist of HP and LP bits. The HP bits are XORed with the bits from S_1, and the LP bits are relayed without NC. As such, the E2E BER at S_1 is obtained as

$$\begin{aligned} P(\varepsilon_2 \,|\min(\gamma_1,\gamma_2) > \gamma_{\text{th}}) &= \frac{1}{2}\left(P\left(\varepsilon_2^{\text{HP}} \,|\min(\gamma_1,\gamma_2) > \gamma_{\text{th}}\right)\right. \\ &\qquad \left. + P\left(\varepsilon_2^{\text{LP}} \,|\min(\gamma_1,\gamma_2) > \gamma_{\text{th}}\right)\right) \end{aligned} \tag{7}$$

where $P(\varepsilon_2^{\mathrm{HP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})$ and $P(\varepsilon_2^{\mathrm{LP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})$ represent the E2E BER of the HP and LP bits conditioned on $\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}}$, respectively. Now, $P(\varepsilon_2^{\mathrm{HP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})$ can be expressed as [16]

$$\begin{aligned}
&P(\varepsilon_2^{\mathrm{HP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})\\
&\quad=\left(1-P(\varepsilon_{r1}^{\mathrm{HP}}|\gamma_{r1}>\gamma_{\mathrm{th}})\right)\\
&\qquad\cdot\left[P(\varepsilon_{1r}|\gamma_{1r}>\gamma_{\mathrm{th}})\left(1-P(\varepsilon_{2r}^{\mathrm{HP}}|\gamma_{2r}>\gamma_{\mathrm{th}})\right)\right.\\
&\qquad\quad\left.+P(\varepsilon_{2r}^{\mathrm{HP}}|\gamma_{2r}>\gamma_{\mathrm{th}})(1-P(\varepsilon_{1r}|\gamma_{1r}>\gamma_{\mathrm{th}}))\right]\\
&\qquad+P(\varepsilon_{r1}^{\mathrm{HP}}|\gamma_{r1}>\gamma_{\mathrm{th}})\\
&\qquad\cdot\left\{1-\left[P(\varepsilon_{1r}|\gamma_{1r}>\gamma_{\mathrm{th}})\left(1-P(\varepsilon_{2r}^{\mathrm{HP}}|\gamma_{2r}>\gamma_{\mathrm{th}})\right)\right.\right.\\
&\qquad\qquad\left.\left.+P(\varepsilon_{2r}^{\mathrm{HP}}|\gamma_{2r}>\gamma_{\mathrm{th}})(1-P(\varepsilon_{1r}|\gamma_{1r}>\gamma_{\mathrm{th}}))\right]\right\}
\end{aligned}\tag{8}$$

where $P(\varepsilon_{1r}|\gamma_{1r}>\gamma_{\mathrm{th}})$ and $P(\varepsilon_{2r}^{\mathrm{HP}}|\gamma_{2r}>\gamma_{\mathrm{th}})$ have been defined earlier. When $i=m=2$, we have $P(\varepsilon_{r1}^{\mathrm{HP}}|\gamma_{r1}>\gamma_{\mathrm{th}})=P(\varepsilon_{2r}^{\mathrm{HP}}|\gamma_{2r}>\gamma_{\mathrm{th}})$. Having found expressions for all the terms in (8), we can easily find a closed-form expression for $P(\varepsilon_2^{\mathrm{HP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})$.

Concerning the LP bits, since they are relayed without NC, the corresponding E2E BER is given by

$$\begin{aligned}
&P\left(\varepsilon_2^{\mathrm{LP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}}\right)\\
&\quad=P\left(\varepsilon_{2r}^{\mathrm{LP}}|\gamma_{2r}>\gamma_{\mathrm{th}}\right)\left(1-P(\varepsilon_{r1}^{\mathrm{LP}}|\gamma_{r1}>\gamma_{\mathrm{th}})\right)\\
&\qquad+P\left(\varepsilon_{r1}^{\mathrm{LP}}|\gamma_{r1}>\gamma_{\mathrm{th}}\right)\left(1-P(\varepsilon_{2r}^{\mathrm{LP}}|\gamma_{2r}>\gamma_{\mathrm{th}})\right)
\end{aligned}\tag{9}$$

Lemma 3: The BER of the LP bits over any of the links conditioned on $\gamma_{im}>\gamma_{\mathrm{th}}$ can be expressed as

$$\begin{aligned}
&P\left(\varepsilon_{im}^{\mathrm{LP}}|\gamma_{im}>\gamma_{\mathrm{th}}\right)\\
&\quad=e^{\frac{1}{\overline{\gamma}_{im}}\gamma_{\mathrm{th}}}\left[I\left(\frac{2}{1+d^2},\overline{\gamma}_{im},\gamma_{\mathrm{th}},\infty\right)\right.\\
&\qquad+\frac{1}{2}I\left(\frac{2(4d^2-4d+1)}{1+d^2},\overline{\gamma}_{im},\gamma_{\mathrm{th}},\infty\right)\\
&\qquad\left.-\frac{1}{2}I\left(\frac{2(4d^2+4d+1)}{1+d^2},\overline{\gamma}_{im},\gamma_{\mathrm{th}},\infty\right)\right].
\end{aligned}\tag{10}$$

Proof: See Appendix C. ■

By setting $i=m=2$ in (10), we obtain $P(\varepsilon_{r1}^{\mathrm{LP}}|\gamma_{r1}>\gamma_{\mathrm{th}})=P(\varepsilon_{22}^{\mathrm{LP}}|\gamma_{22}>\gamma_{\mathrm{th}})$. We can similarly obtain $P(\varepsilon_{2r}^{\mathrm{LP}}|\gamma_{2r}>\gamma_{\mathrm{th}})=P(\varepsilon_{21}^{\mathrm{LP}}|\gamma_{21}>\gamma_{\mathrm{th}})$. These expressions lead to a closed-form expression for $P(\varepsilon_2^{\mathrm{LP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})$. Having obtained expressions for $P(\varepsilon_2^{\mathrm{HP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})$ and $P(\varepsilon_2^{\mathrm{LP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})$, $P(\varepsilon_2|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})$ is obtained by plugging $P(\varepsilon_2^{\mathrm{HP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})$ and $P(\varepsilon_2^{\mathrm{LP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})$ into (7).

2) Derivation of $P(\varepsilon_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}})$: Here, we derive a closed-form expression for $P(\varepsilon_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}})$ given in (3). This corresponds to the OUS scheme, for which if the instantaneous E2E SNR of S_i is greater than that of S_j, where $i\neq j$, only S_i transmits to S_j. Thus, when either the $S_i\to R$ or $R\to S_j$ link is in error, the received signal at S_j will be in error. Therefore, the E2E BER of S_i given $\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}}$ can be expressed as

$$\begin{aligned}
&P(\varepsilon_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}})\\
&\quad=P(\varepsilon_{im}|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}})\\
&\qquad\times(1-P(\varepsilon_{in}|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}}))\\
&\qquad+P(\varepsilon_{in}|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}})\\
&\qquad\times(1-P(\varepsilon_{im}|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}}))
\end{aligned}\tag{11}$$

where $i,j,n,m=1,2$, and $i\neq j, m\neq n$.

Lemma 4: For M-QAM, the BER over any of the links can be expressed as

$$\begin{aligned}
&P(\varepsilon_{im}|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}})\\
&\quad=\frac{1}{\sqrt{M}\log_2\sqrt{M}}\sum_{k=1}^{\log_2\sqrt{M}}\sum_{i=0}^{(1-2^{-k})\sqrt{M}-1}(-1)^{\left\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}}\right\rfloor}\\
&\qquad\times\left(2^{k-1}-\left\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}}+\frac{1}{2}\right\rfloor\right)\\
&\qquad\cdot\left\{\frac{\overline{\gamma}_{in}(\overline{\gamma}_i+\overline{\gamma}_j)}{\overline{\gamma}_{im}\overline{\gamma}_i(\overline{\gamma}_{in}+\overline{\gamma}_j)\left(1-e^{-(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j})\gamma_{\mathrm{th}}}\right)}\right.\\
&\qquad\cdot\left[\overline{\gamma}_{im}I_1\left(\frac{3\log_2^M(2i+1)^2}{2(M-1)},\overline{\gamma}_{im},\gamma_{\mathrm{th}}\right)\right.\\
&\qquad\left.-\frac{\overline{\gamma}_i\overline{\gamma}_j}{\overline{\gamma}_i+\overline{\gamma}_j}I_1\left(\frac{3\log_2^M(2i+1)^2}{2(M-1)},\frac{\overline{\gamma}_i\overline{\gamma}_j}{\overline{\gamma}_i+\overline{\gamma}_j},\gamma_{\mathrm{th}}\right)\right]\\
&\qquad+\left[\frac{\overline{\gamma}_{in}\left(\overline{\gamma}_i+\overline{\gamma}_j\right)\left(1-e^{-(\frac{1}{\overline{\gamma}_{in}}+\frac{1}{\overline{\gamma}_j})\gamma_{\mathrm{th}}}\right)}{\overline{\gamma}_{im}\overline{\gamma}_i(\overline{\gamma}_{in}+\overline{\gamma}_j)\left(1-e^{-(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j})\gamma_{\mathrm{th}}}\right)}\overline{\gamma}_{im}\right.\\
&\qquad\left.\left.\cdot I\left(\frac{3\log_2^M(2i+1)^2}{2(M-1)},\overline{\gamma}_{im},\gamma_{\mathrm{th}}\right)\right]\right\}.
\end{aligned}\tag{12}$$

Note that, for our case, $M=4$ for S_1 and $M=16$ for S_2.

Proof: See Appendix D. ■

Note that $P_{e,1r}=P(\varepsilon_{11}|\gamma_1>\gamma_2,\gamma_2<\gamma_{\mathrm{th}})$, $P_{e,r2}=P(\varepsilon_{12}|\gamma_1>\gamma_2,\gamma_2<\gamma_{\mathrm{th}})$, $P_{e,2r}=P(\varepsilon_{21}|\gamma_2>\gamma_1,\gamma_1<\gamma_{\mathrm{th}})$, and $P_{e,r1}=P(\varepsilon_{22}|\gamma_2>\gamma_1,\gamma_1<\gamma_{\mathrm{th}})$. Plugging these expressions into (11) yields a closed-form expression for $P(\varepsilon_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}})$.

B. Proposed Scheme Versus OUS

Our proposed scheme switches between HZPNC and OUS. Therefore, it is natural to compare their E2E BER performance. It is shown in [21] that OUS has a better BER performance than that of HZPNC. Therefore, we compare our proposed scheme with OUS. The E2E BER of OUS can be expressed as

$$\begin{aligned}
P_{\varepsilon i}^{\mathrm{OUS}} =& \frac{\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}\left(1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}\right)}{\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}\left(1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}\right)+\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}} \\
&\cdot P(\varepsilon_i\,|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}}) \\
&+\frac{\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}}{\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}\left(1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}\right)+\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}} \\
&\cdot P(\varepsilon_i\,|\gamma_i>\gamma_j,\gamma_j>\gamma_{\mathrm{th}}) \\
=& \left(1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}\right)P(\varepsilon_i\,|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}}) \\
&+e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}P(\varepsilon_i\,|\gamma_i>\gamma_j,\gamma_j>\gamma_{\mathrm{th}}). \qquad (13)
\end{aligned}$$

Note that the exact E2E BER expression of OUS can be obtained directly as shown in [21]. We express it here as a combination of two terms, as shown in (13), just to compare it with that of the proposed scheme. Since $P(\varepsilon_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}})$ is the dominant term at high SNR[2] in (3) and (13), the BER difference between the two schemes, i.e., $\triangle P_{\varepsilon i}$, can be approximated as

$$\begin{aligned}
\triangle P_{\varepsilon i} &\triangleq P_{\varepsilon i}^{\mathrm{OUS}} - P_{\varepsilon i}^{\mathrm{proposed}} \\
&\approx \left[1-\frac{3\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}}{2e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}+3\frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}\left(1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}\right)}\right] \\
&\quad\cdot\left(1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}\right)P(\varepsilon_i\,|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}}). \qquad (14)
\end{aligned}$$

Solving for $\triangle P_{\varepsilon i}>0$, we obtain $(\overline{\gamma}_i/\overline{\gamma}_j)<2$, which is the range for which the proposed scheme can have a better E2E BER than that of OUS for both users. Beyond this range, one user of the OUS scheme can have a better BER performance than those of our proposed scheme, whereas the other one will have a worse E2E BER performance. As we will see, the results are also confirmed by simulations in Section VII.

V. Achievable Diversity Order

While the BER performance expressions derived earlier are exact, they do not yield the diversity order achieved. Here, we derive the corresponding asymptotic expressions at high SNR to show the achievable diversity order. For simplicity, we assume symmetric channels, i.e., all the channel gains are modeled as zero-mean unit-variance complex Gaussian random variables, and $d=2$ since the diversity order does not change with the channel setting and modulation scheme, as demonstrated in [31].

With the assumption made earlier, it is easy to obtain that $\overline{\gamma}_i=\overline{\gamma}_j=(1/2)\rho$. Since S_2 uses a higher modulation scheme than that of S_1, the E2E BER of S_2 is worse than that of S_1. Then, according to (3), we have

$$\begin{aligned}
P_1 &\le P_2 \\
&= \frac{P_2^1+P_2^2}{2e^{-\frac{4}{\rho}\gamma_{\mathrm{th}}}+\frac{3}{2}\left(1-e^{-\frac{4}{\rho}\gamma_{\mathrm{th}}}\right)} \\
&\le \frac{P_2^1+P_2^2}{2} \qquad (15)
\end{aligned}$$

where $P_2^1=2e^{-\frac{4}{\rho}\gamma_{\mathrm{th}}}P(\varepsilon_2|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}})$, and $P_2^2=(3/2)(1-e^{-\frac{4}{\rho}\gamma_{\mathrm{th}}})P(\varepsilon_2|\gamma_2>\gamma_1,\gamma_1<\gamma_{\mathrm{th}})$. Now, we can upper bound P_2^1 as

$$\begin{aligned}
P_2^1 &\le 2\left[\frac{1}{2}\left(P\left(\varepsilon_2^{\mathrm{HP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}}\right)\right.\right. \\
&\qquad\left.\left.+P\left(\varepsilon_2^{\mathrm{LP}}|\min(\gamma_1,\gamma_2)>\gamma_{\mathrm{th}}\right)\right)\right] \\
&\le P\left(\varepsilon_{im}\,|\gamma_{im}>\gamma_{\mathrm{th}}\right)+2P\left(\varepsilon_{im}^{\mathrm{HP}}\,|\gamma_{im}>\gamma_{\mathrm{th}}\right) \\
&\quad+2P\left(\varepsilon_{im}^{\mathrm{LP}}\,|\gamma_{im}>\gamma_{\mathrm{th}}\right) \\
&\le \int_{\gamma_{\mathrm{th}}}^{\infty}\left(\frac{1}{2}\mathrm{erfc}\sqrt{\frac{2}{5}\gamma_{im}}+\mathrm{erfc}\sqrt{\frac{2}{5}\gamma_{im}}+\frac{3}{2}\mathrm{erfc}\sqrt{\frac{2}{5}\gamma_{im}}\right) \\
&\quad\cdot e^{\frac{1}{\rho}\gamma_{\mathrm{th}}}\frac{1}{\rho}e^{-\frac{1}{\rho}\gamma_{im}}d\gamma_{im} \\
&\le e^{\frac{1}{\rho}\gamma_{\mathrm{th}}}\int_{\gamma_{\mathrm{th}}}^{\infty}\frac{3}{2}e^{-\frac{2}{5}\gamma_{im}}\frac{1}{\rho}e^{-\frac{1}{\rho}\gamma_{im}}d\gamma_{im} \qquad (16) \\
&\le \frac{15}{4\rho}e^{-\frac{2}{5}\gamma_{\mathrm{th}}} \qquad (17)
\end{aligned}$$

where (16) follows from $\mathrm{erfc}\sqrt{x}\le(1/2)e^{-x}$. Similarly, P_2^2 can be upper bounded as

$$\begin{aligned}
P_2^2 &\le 3\left(1-e^{-\frac{4}{\rho}\gamma_{\mathrm{th}}}\right)P\left(\varepsilon_{im}\,|\gamma_2>\gamma_1,\gamma_1<\gamma_{\mathrm{th}}\right) \\
&\le \left(1-e^{-\frac{4}{\rho}\gamma_{\mathrm{th}}}\right) \\
&\quad\cdot\int_0^{\infty}\frac{15}{8}\mathrm{erfc}\sqrt{\frac{2}{5}\gamma_{im}}f_{\gamma_{im}|\gamma_i>\gamma_j\gamma_j<\gamma_{\mathrm{th}}}(\gamma_{im})\,d\gamma_{im} \\
&\le \left(1-e^{-\frac{4}{\rho}\gamma_{\mathrm{th}}}\right) \\
&\quad\cdot\int_0^{\infty}\frac{15}{16}e^{-\frac{2}{5}\gamma_{im}}f_{\gamma_{im}|\gamma_i>\gamma_j\gamma_j<\gamma_{\mathrm{th}}}(\gamma_{im})\,d\gamma_{im} \\
&\le P_2^{21}+P_2^{22}
\end{aligned}$$

[2]The exact form of the optimal threshold derived in the succeeding section shows that the optimal threshold is an increasing function of SNR. With γ_{th} approaching infinity, both $P(\varepsilon_i|\min(\gamma_i,\gamma_j)>\gamma_{\mathrm{th}})$ and $P(\varepsilon_i|\gamma_i>\gamma_j,\gamma_j>\gamma_{\mathrm{th}})$ approach zero. Consequently, $P(\varepsilon_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\mathrm{th}})$ is the dominant term at high SNR, and this leads to (14).

where

$$P_2^{21} = \left(1 - e^{-\frac{4}{\rho}\gamma_{\mathrm{th}}}\right) \cdot \int_0^{\gamma_{\mathrm{th}}} \frac{15}{16} e^{-\frac{2}{5}\gamma_{im}} f_{\gamma_{im}|\gamma_i>\gamma_j\gamma_j<\gamma_{\mathrm{th}}}\left(\gamma_{im}\right) d\gamma_{im} \quad (18)$$

$$P_2^{22} = \left(1 - e^{-\frac{4}{\rho}\gamma_{\mathrm{th}}}\right) \cdot \int_{\gamma_{\mathrm{th}}}^{\infty} \frac{15}{16} e^{-\frac{2}{5}\gamma_{im}} f_{\gamma_{im}|\gamma_i>\gamma_j\gamma_j<\gamma_{\mathrm{th}}}\left(\gamma_{im}\right) d\gamma_{im}. \quad (19)$$

Plugging (57) into (18), we have

$$\begin{aligned} P_2^{21} &= \int_0^{\gamma_{\mathrm{th}}} \frac{15}{16}\frac{4}{3\rho} e^{-\frac{2}{5}\gamma_{im}} \left(e^{-\frac{1}{\rho}\gamma_{im}} - e^{-\frac{4}{\rho}\gamma_{im}}\right) d\gamma_{im} \\ &\le \int_0^{\infty} \frac{5}{4\rho} e^{-\frac{2}{5}\gamma_{im}} \left(e^{-\frac{1}{\rho}\gamma_{im}} - e^{-\frac{4}{\rho}\gamma_{im}}\right) d\gamma_{im} \\ &= \frac{15}{4\rho^2\left(\frac{2}{5}+\frac{1}{\rho}\right)\left(\frac{2}{5}+\frac{4}{\rho}\right)} \quad (20) \\ &\le \frac{375}{16\rho^2} \quad (21) \end{aligned}$$

where (20) follows from in [31, Eq. B.1]. Plugging (56) into (19), we have

$$\begin{aligned} P_2^{22} &= e^{\frac{1}{\rho}\gamma_{\mathrm{th}}} \int_{\gamma_{\mathrm{th}}}^{\infty} \frac{15}{16}\frac{4}{3\rho} e^{-\frac{2}{5}\gamma_{im}} \cdot \left(1 - e^{-\frac{3}{\rho}\gamma_{\mathrm{th}}}\right) e^{-\frac{1}{\rho}\gamma_{\mathrm{th}}} e^{-\frac{1}{\rho}\gamma_{im}} d\gamma_{im} \\ &\le e^{\frac{1}{\rho}\gamma_{\mathrm{th}}} \int_{\gamma_{\mathrm{th}}}^{\infty} \frac{5}{4\rho} e^{-\frac{2}{5}\gamma_{im}} e^{-\frac{1}{\rho}\gamma_{im}} d\gamma_{im} \\ &\le \frac{25}{8\rho} e^{-\frac{2}{5}\gamma_{\mathrm{th}}}. \quad (22) \end{aligned}$$

Plugging (17), (21), and (22) into (15), we have

$$\begin{aligned} P_1 &\le \frac{15}{4\rho} e^{-\frac{2}{5}\gamma_{\mathrm{th}}} + \frac{375}{16\rho^2} + \frac{25}{8\rho} e^{-\frac{2}{5}\gamma_{\mathrm{th}}} \\ &= \frac{55}{8\rho} e^{-\frac{2}{5}\gamma_{\mathrm{th}}} + \frac{375}{16\rho^2} \quad (23) \end{aligned}$$

where $(375/16\rho^2)$ achieves diversity order two. Since the diversity order is determined by the term with the lowest diversity order, $(55/8\rho)e^{-\frac{2}{5}\gamma_{\mathrm{th}}}$ should achieve a diversity of at least two. Therefore, to achieve full diversity, $e^{-\frac{2}{5}\gamma_{\mathrm{th}}}$ should be in the form of (c/ρ^m), where $m \ge 1$. That is, $e^{-\frac{2}{5}\gamma_{\mathrm{th}}} = (c/\rho^m)$, where $m \ge 1$. By solving this equation, we have $\gamma_{\mathrm{th}} = n\log(c\rho)$, where $n \ge 2.5$, and c is a constant.

Therefore, we plug $\gamma_{\mathrm{th}} = n\log(c\rho)$ for some constants n and c into (23), which yields

$$\begin{aligned} P_1 &\le \frac{55}{8\rho} e^{-\frac{2}{5}\times n\log(c\rho)} + \frac{375}{16\rho^2} \\ &= \frac{55}{8c^{\frac{2}{5}n}\rho^{\frac{2}{5}n+1}} + \frac{375}{16\rho^2} = O(\rho^{-2}) \quad (24) \end{aligned}$$

for $n \ge 2.5$. Therefore, we conclude that our proposed scheme achieves full diversity, which is the number of available users. It is also confirmed through simulations in Section VII that the exact optimal threshold function for the case when S_1 employs 4-QAM and S_2 employs 4/16-QAM is in fact in the form of $5\log(c\rho)$, which leads to full diversity according to (24).

VI. Access Probability and Throughput

Here, we analyze the proposed adaptive transmission scheme in terms of the access probability and throughput and compare it with those of the HZPNC and OUS schemes individually.

A. Access Probability

According to the HZPNC scheme [21], both users transmit via channel sharing. The two users communicate with each other over three time slots. During the three time slots, each user occupies two time slots with one time-slot overlapping. Then, the access probability is $(2/3)$ for the HZPNC case. As for the OUS scheme, a user transmits once its instantaneous E2E SNR is greater than that of the other one, resulting in an access probability of

$$P_i^{\mathrm{OUS}} = P_r(\gamma_i > \gamma_j) = \frac{\overline{\gamma}_i}{\overline{\gamma}_i + \overline{\gamma}_j}. \quad (25)$$

For symmetric channels, for example, the average E2E SNRs for both users are the same, i.e., $\overline{\gamma}_i = \overline{\gamma}_j$, suggesting that both users will have the same access probability, i.e., $P_i^{\mathrm{OUS}} = 0.5$. According to the proposed scheme, a user transmits once its instantaneous E2E SNR is larger than the other one and the instantaneous E2E SNR of the worse user is below a predetermined threshold γ_{th}, or both instantaneous E2E SNRs of the two users are above γ_{th}. Therefore, the probability that S_i transmits can be expressed as

$$\begin{aligned} P_i^{\mathrm{proposed}} &= P_r(\gamma_i > \gamma_j, \gamma_j < \gamma_{\mathrm{th}}) \times 1 \\ &\quad + P_r\left(\min(\gamma_i, \gamma_j) > \gamma_{\mathrm{th}}\right) \times \frac{2}{3} \\ &= \frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j} + \left[\frac{2}{3} - \frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}\right] e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}}. \quad (26) \end{aligned}$$

Now, define $\triangle P_i$ as

$$\begin{aligned} \triangle P_i &\triangleq P_i^{\mathrm{proposed}} - P_i^{\mathrm{OUS}} \\ &= \left[\frac{2}{3} - \frac{\overline{\gamma}_i}{\overline{\gamma}_i + \overline{\gamma}_j}\right] e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\mathrm{th}}} \quad (27) \end{aligned}$$

which represents the access probability difference between the two schemes. When the channels are symmetric, the average E2E SNRs are the same, i.e., $\overline{\gamma}_i = \overline{\gamma}_j$. Therefore, our proposed scheme offers a higher access probability than that of OUS for each user by $(1/6)e^{-(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j})\gamma_{\mathrm{th}}}$. As for asymmetric channels, by solving $(2/3) - (\overline{\gamma}_i/(\overline{\gamma}_i + \overline{\gamma}_j)) > 0$, we obtain $(\overline{\gamma}_i/\overline{\gamma}_j) < 2$, which is the range for which the proposed scheme has a higher access probability compared with that of the OUS scheme for both users. Beyond this range, one user of the OUS scheme will have a higher access probability than those of our proposed scheme, whereas the other one will have a lower access probability.

B. Throughput

We do the throughput analysis for the general case where it is assumed that S_1 uses 2^{2s}-QAM and S_2 uses $2^{2s}/2^{2r}$-QAM hierarchical modulation for $s = 1, 2, \ldots, r-1$, and $r = 2, 3, \ldots, R$. The throughput of our proposed scheme can be written as

$$\begin{aligned} T^{\text{proposed}} =& T^{\text{HZPNC}} \times P_r\left(\min(\gamma_1, \gamma_2) > \gamma_{\text{th}}\right) \\ &+ T^{\text{OUS}} \times \left[P_r(\gamma_1 > \gamma_2, \gamma_2 < \gamma_{\text{th}}) \right. \\ &\left. + P_r(\gamma_2 > \gamma_1, \gamma_1 < \gamma_{\text{th}})\right] \end{aligned} \tag{28}$$

where T^{OUS} and T^{HZPNC} are the throughputs of OUS and HZPNC, respectively, which are given as [21]

$$T^{\text{OUS}} = \frac{\overline{\gamma}_1}{\overline{\gamma}_1 + \overline{\gamma}_2} s + \frac{\overline{\gamma}_2}{\overline{\gamma}_1 + \overline{\gamma}_2} r \tag{29}$$

$$T^{\text{HZPNC}} = \frac{2s + 2r}{3} \tag{30}$$

where we assume that S_1 uses 2^{2s}-QAM, and S_2 uses $2^{2s}/2^{2r}$-QAM hierarchical modulation. Plugging (29), (30), (35), and (36) into (28), we obtain

$$\begin{aligned} T^{\text{proposed}} =& e^{-\left(\frac{1}{\overline{\gamma}_1} + \frac{1}{\overline{\gamma}_2}\right)\gamma_{\text{th}}} \frac{2s + 2r}{3} \\ &+ \left(1 - e^{-\left(\frac{1}{\overline{\gamma}_1} + \frac{1}{\overline{\gamma}_2}\right)\gamma_{\text{th}}}\right) \\ &\cdot \left(\frac{\overline{\gamma}_1}{\overline{\gamma}_1 + \overline{\gamma}_2} s + \frac{\overline{\gamma}_2}{\overline{\gamma}_1 + \overline{\gamma}_2} r\right). \end{aligned} \tag{31}$$

We know that, in general, HZPNC has a higher throughput than that of OUS [21]. Thus, the throughput of the proposed scheme has a higher throughput than that of OUS. However, in the extreme case when one user has a much higher order modulation scheme and a higher access probability than those of the other user, OUS will have a higher throughput than HZPNC. In this unlikely case, OUS will also have a higher throughput than that of our proposed scheme.

VII. Simulation Results

We present here numerical examples that aim at validating the E2E BER expressions derived for the proposed scheme. In addition, we compare our proposed scheme with HZPNC and OUS in terms of the E2E BER, access probability and throughput. Throughout the simulations, we assume that S_1 uses 4-QAM and S_2 uses 4/16-QAM hierarchical modulation, and $d = 2$. According to our analysis, the behavior of both BER and access probability can differ in the range of $(\overline{\gamma}_i/\overline{\gamma}_j) < 2$ and $(\overline{\gamma}_i/\overline{\gamma}_j) > 2$. Therefore, we use three values of $\overline{\gamma}_1/\overline{\gamma}_2$, namely 1, 1.25, and 4.

In Fig. 2, we compare the derived theoretical results to the simulation results over symmetric channels with different thresholds for S_1. In our simulations, we set the variances of the channel coefficients to $(1/4)$. We consider the cases: $\gamma_{\text{th}} = 5, 10$. We also plot three particular cases, namely HZPNC (i.e., $\gamma_{\text{th}} = 0$), OUS (i.e., $\gamma_{\text{th}} = \infty$) and $\gamma_{\text{th}} = \gamma_{\text{opt}}$ where $\gamma_{\text{opt}} = [4\ 6.9\ 13.2\ 18.5]$ for $\rho = [15\ \ 20\ \ 25\ \ 30]$, which is obtained numerically by minimizing the worst E2E BER of the two users. We also include the curve obtained by using $(0.1/\rho^2)$, which is used as a benchmark. We can see from the figure that the performance improves as the threshold increases from 0 to 10. We also see the perfect match between theory and simulations, which validates the derived BER for symmetric channels. Furthermore, we can see that the performance of our proposed scheme with $\gamma_{\text{th}} = \gamma_{\text{opt}}$ is slightly worse than the case $\gamma_{\text{th}} = 10$ at low SNRs. This is due to fact that the optimal thresholds are optimized for S_2, which has the worst E2E BER. However, we observe that our proposed scheme is slightly better than OUS at high SNR, which is expected. Finally, by comparing the slopes of the curves, we observe that our proposed scheme achieves a diversity gain of two.

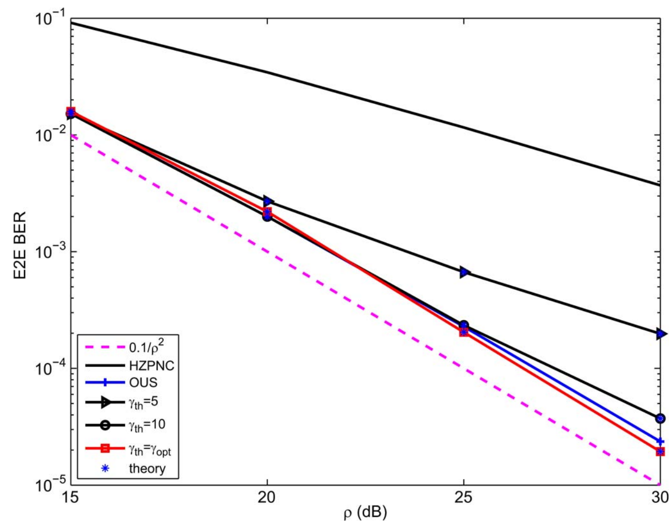

Fig. 2. E2E BER performance (simulated and theoretical) of our proposed adaptive transmission scheme for symmetric channels for S_1.

The E2E BER performance for S_2 is shown in Fig. 3, with similar observations. As shown in the figure, the E2E BER performance for S_2 with γ_{opt} is also slightly better than that of OUS and achieves a diversity gain of two. Therefore, both users have a better BER performance than that of OUS for symmetric channels, i.e., $\overline{\gamma}_1/\overline{\gamma}_2$ is 1.

In Fig. 4, we present the access probability for HZPNC, OUS, and our proposed HNCOUS scheme with the optimal thresholds corresponding to Figs. 2 and 3. Since the channels are symmetric, the two users have the same access probability for OUS, which is 0.5. We can see that the access probability of our proposed HNCOUS scheme falls between that of HZPNC and OUS, as expected. It is also shown, however, that the access probability for the proposed scheme is close to that of OUS at low SNRs and approaches that of HZPNC as SNR increases. This is attributed to the fact that, as SNR increases, it is more likely that both channels are in a good state, leading the use of HZPNC more often.

In Fig. 5, we present the throughput for HZPNC, OUS, and our proposed HNCOUS scheme with the optimal thresholds

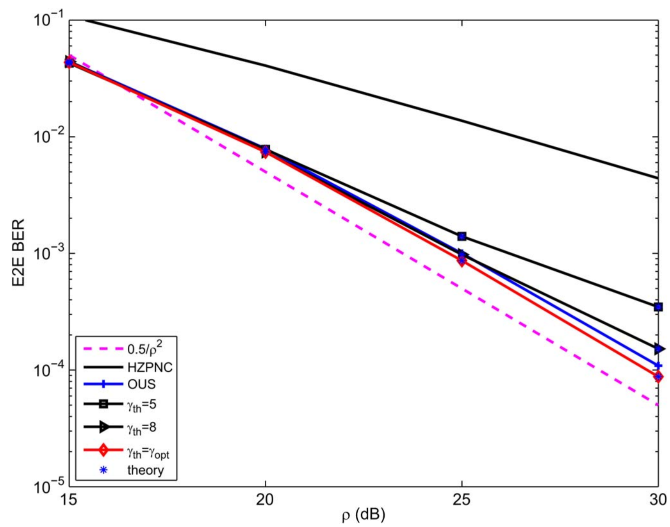

Fig. 3. E2E BER performance (simulated and theoretical) of our proposed adaptive transmission scheme for symmetric channels for S_2.

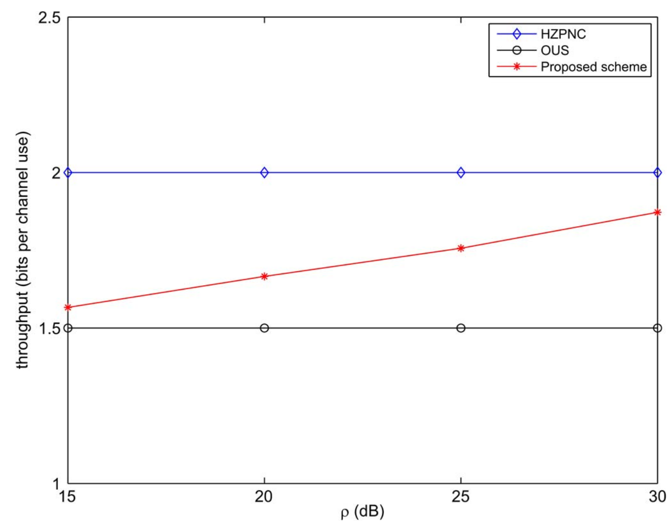

Fig. 5. Throughput of HZPNC, OUS, and our proposed adaptive transmission scheme corresponding to Figs. 2 and 3.

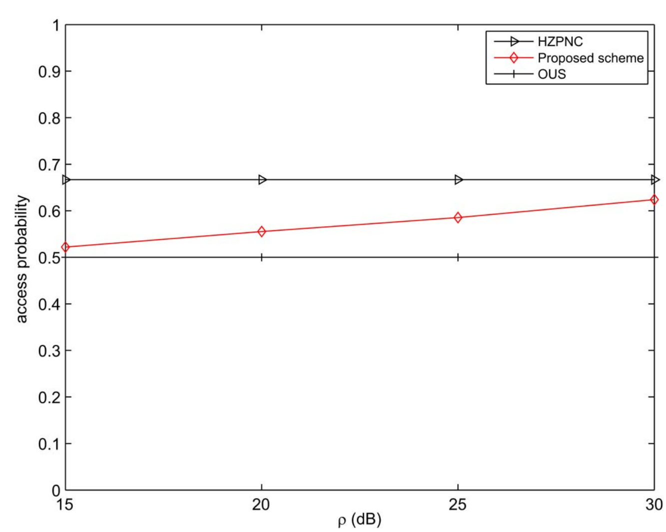

Fig. 4. Access probability of HZPNC, OUS, and our proposed adaptive transmission scheme corresponding to Figs. 2 and 3.

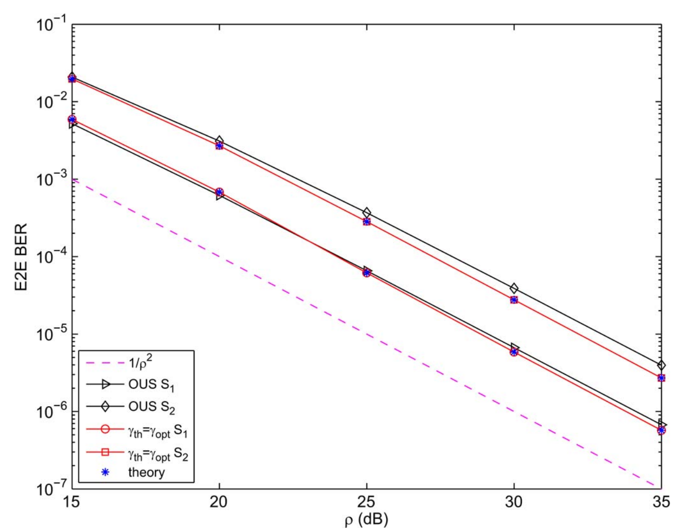

Fig. 6. E2E BER performance of our proposed adaptive transmission scheme for asymmetric channels.

corresponding to Figs. 2 and 3. It is shown that our proposed scheme has a higher throughput than that of OUS, as expected, but this throughput approaches that of HZPNC at high SNRs, which follows the behavior of the access probability reported in Fig. 4.

In Fig. 6, we compare the performance of our proposed scheme with OUS for asymmetric channels. In our simulations, we set the variances of the channel coefficients for the $S_1 - R$, $R - S_2$, $S_2 - R$, and $R - S_1$ links to $(1/2)$, $(1/2)$, $(1/4)$, and 1, respectively. We numerically obtain the optimal thresholds. The optimal thresholds are $\gamma_{\rm opt} = [6\ 9\ 15.3\ 21]$ for $\rho = [15\ 20\ 25\ 30]$. We see the perfect match between theory and simulations, which validates the derived BER for asymmetric channels. In addition, we observe that the performance of our proposed HNCOUS scheme with $\gamma_{\rm th} = \gamma_{\rm opt}$ is slightly better than those of OUS and achieves a diversity gain of two for both users.

In Fig. 7, we plot the access probabilities of HZPNC, OUS, and our proposed HNCOUS scheme with the optimal thresholds $\gamma_{\rm opt}$ corresponding to Fig. 6. The access probability of OUS for S_1 is 0.5556 and 0.4444 for S_2. It is shown that the access probability of our proposed scheme falls between that of HZPNC and OUS for both users. As the SNR increases, however, the access probability of our proposed scheme for both users increases and approaches that of HZPNC. We can also observe that the gap between the access probability of our proposed scheme for the two users is less than that of OUS and decreases as the SNR increases. This clearly shows how our proposed scheme improves fairness between the users as compared with OUS.

In Fig. 8, we illustrate the throughput of HZPNC, OUS, and our proposed scheme with the optimal thresholds $\gamma_{\rm opt}$ corresponding to Fig. 6. Similar to the observations of Fig. 5, our proposed scheme also achieves a higher throughput than that of OUS in this case.

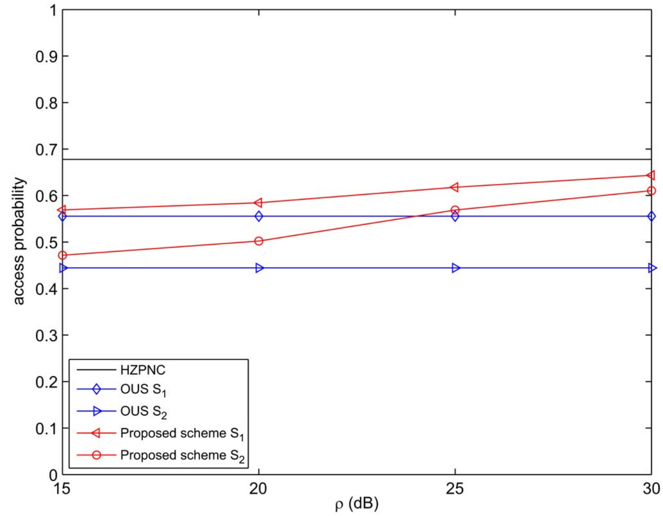

Fig. 7. Access probability of HZPNC, OUS, and our proposed adaptive transmission scheme corresponding to Fig. 6.

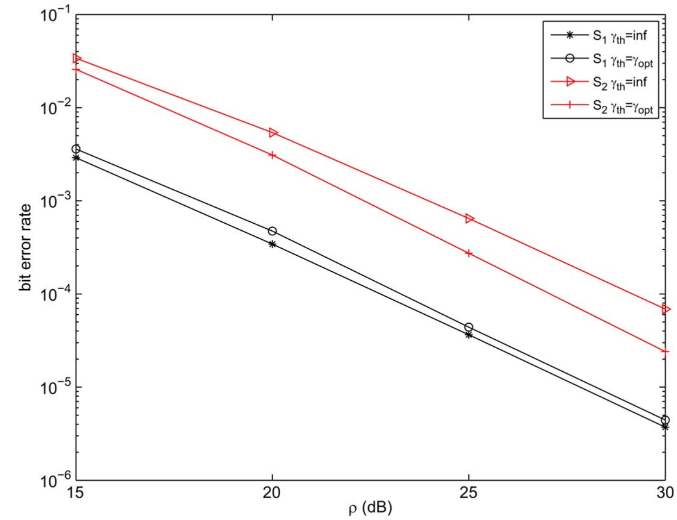

Fig. 9. E2E BER performance of our proposed adaptive transmission scheme for asymmetric channels.

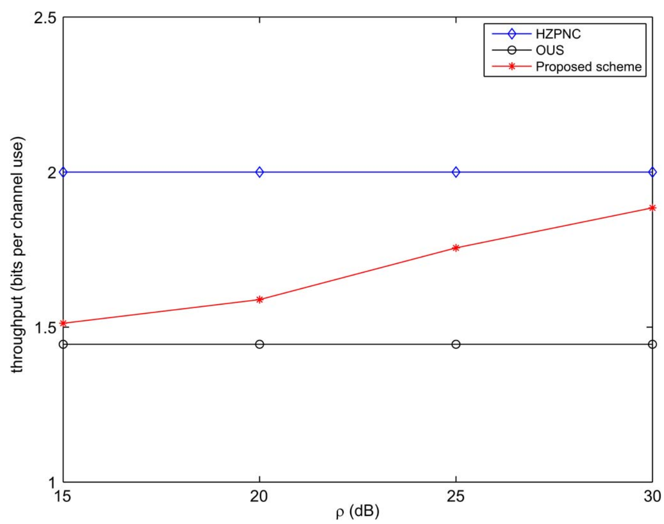

Fig. 8. Throughput of HZPNC, OUS, and our proposed adaptive transmission scheme corresponding to Fig. 6.

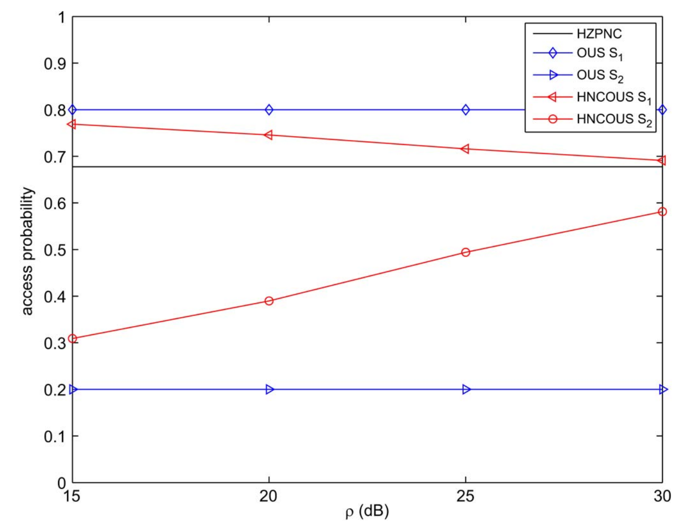

Fig. 10. Access probability of HZPNC, OUS, and our proposed adaptive transmission scheme for asymmetric channels corresponding to Fig. 9.

In Fig. 9, we consider another case of asymmetrical channels. We set the variances of the channel coefficients for the $S_1 - R$, $R - S_2$, $S_2 - R$, and $R - S_1$ links to 1, 1, (1/4), and (1/4), respectively. The value of $\overline{\gamma}_1/\overline{\gamma}_2$ is 4, which is greater than that corresponding to Fig. 6 (there, it was 1.25). The optimal thresholds are $\gamma_{\rm opt} = [4.9\ 8.3\ 14.6\ 20.4]$. As shown in the figure, the performance of S_1 with $\gamma_{\rm opt}$ is slightly worse than that of OUS, whereas the performance of S_2 with $\gamma_{\rm opt}$ is much better than that of OUS. As a whole, our proposed scheme has a better sum BER performance (the average performance of both users) than that of OUS.

In Fig. 10, we plot the access probabilities for the asymmetric channel case with optimal thresholds $\gamma_{\rm opt}$ that correspond to Fig. 9. The access probability of OUS for S_1 is 0.8 and 0.2 for S_2. Unlike Fig. 7, S_1 using our proposed scheme has a lower access probability than that of OUS. However, we observe that the access probability gap between the two users is smaller than that of OUS and decreases with increasing SNR, which indicates that our proposed scheme improves the fairness between the two users.

In Fig. 11, we show the throughput with $\gamma_{\rm opt}$ corresponding to Fig. 9. We find that our proposed scheme has a higher throughput than OUS for this case. It is also shown that the throughput of NC and OUS remain unchanged during all the range, whereas the throughput of our proposed scheme increases with SNR.

In Fig. 12, we plot the values of $\gamma_{\rm opt}$ that correspond to Fig. 2 (symmetric channels) and Figs. 6 and 9 (asymmetric channels) versus ρ. In the same figure, we plot the function $5\log(0.1\rho)$. We observe from the figure that all curves have the same behavior, suggesting that $\gamma_{\rm opt} = 5\log(c\rho)$, which was assumed in obtaining (24).

VIII. Concluding Remarks

We have proposed an HNCOUS scheme for two-way communication with asymmetric data rates that aims at improving

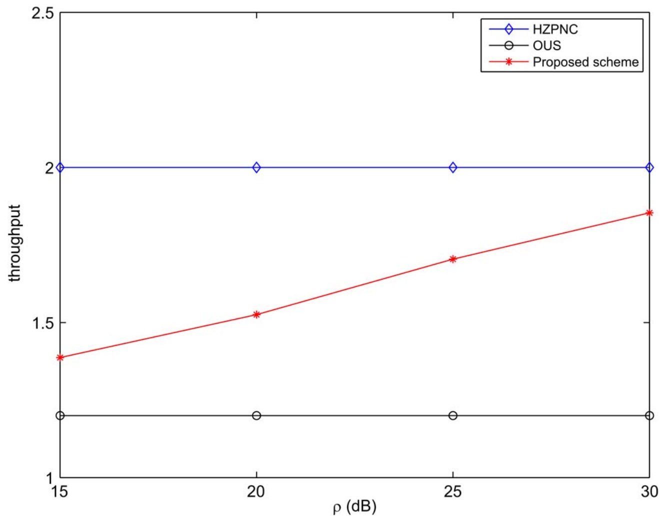

Fig. 11. Throughput of HZPNC, OUS, and our proposed adaptive transmission scheme corresponding to Fig. 9.

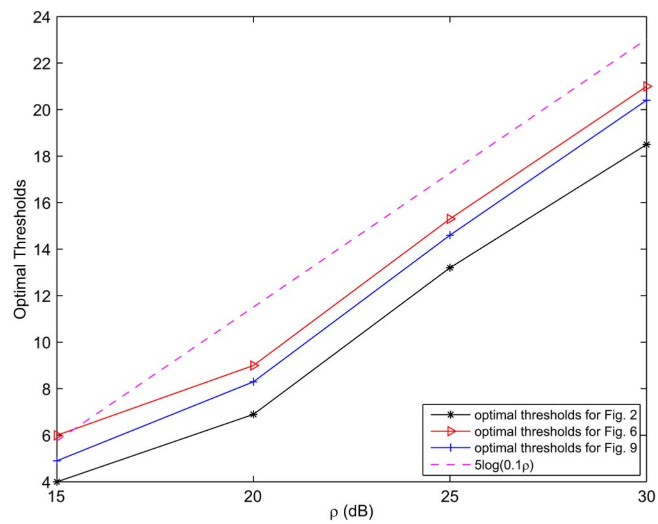

Fig. 12. Optimal threshold values as a function of ρ for Figs. 2, 6, and 9.

fairness and still exploiting multiuser diversity gain. The proposed scheme either allows only the best user to transmit or both users transmit using NC, depending on the user channels quality. If both users have good channels, both users transmit using NC; otherwise, the better user transmits, whereas the other user remains silent. We analyzed the proposed adaptive transmission scheme in terms of the E2E BER, access probability, and throughput. We examined the performance of the proposed schemes over asymmetric fading channels. We showed that the proposed scheme retains the diversity achieved through multiuser selection and offers throughput that approaches the best possible throughput at high SNRs. The implication here is that the two users enjoy better transmission fairness and throughput while in general achieving better BER performance.

Appendix A
Proof of Lemma 1

According to the proposed scheme, if both instantaneous E2E SNRs of the two users are above $\gamma_{\rm th}$, both users transmit using HZPNC. In this case, three time slots are needed for two new transmissions. In contrast, if either instantaneous E2E SNRs of the two users is below a threshold $\gamma_{\rm th}$, only the better user transmits. Then, two time slots are needed per transmission. To elaborate, let us assume, without loss of generality, that the coherence time is six time slots (of course, the coherence time is normally much larger than this; this is used merely for illustration purposes.) When HZPNC is used, two new transmissions are completed per coherence time, whereas three new transmissions are completed for the same duration for the OUS case.

Given the hybrid nature of our proposed scheme, errors can occur when HZPNC or OUS is used. Let $N^e_{\rm OUS}$ and $N^b_{\rm OUS}$ represent the number of decoded errors and number of transmitted bits when OUS is used, respectively. Similarly, let $N^e_{\rm NC}$ and $N^b_{\rm NC}$ represent the number of decoded errors and number of transmitted bits when HZPNC is used, respectively. As such, the E2E BER corresponding to user S_i can be expressed as

$$\begin{aligned} P_{\varepsilon i}^{\rm proposed} &= \frac{N^e_{\rm OUS}+N^e_{\rm NC}}{N^b_{\rm OUS}+N^b_{\rm NC}} \\ &= \frac{N^b_{\rm NC}\times P(\varepsilon_i\,|\min(\gamma_i,\gamma_j)>\gamma_{\rm th})}{N^b_{\rm OUS}+N^b_{\rm NC}} \\ &\quad + \frac{N^b_{\rm OUS}\times P(\varepsilon_i\,|\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th})}{N^b_{\rm OUS}+N^b_{\rm NC}} \end{aligned} \tag{32}$$

where $P(\varepsilon_i|\min(\gamma_i,\gamma_j)>\gamma_{\rm th})$ is the BER corresponding to S_i, which is conditioned on $\min(\gamma_i,\gamma_j)>\gamma_{\rm th}$, and $P(\varepsilon_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th})$ is the BER corresponding to S_i, which is conditioned on $\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}$.

Let us assume that the number of coherent time slots is A and the number of bits transmitted during this coherence time is B_i per transmission for S_i. Thus, $N^b_{\rm NC}$ and $N^b_{\rm OUS}$ for S_i can be written as

$$N^b_{\rm NC} = 2AB_iP_r\left(\min(\gamma_i,\gamma_j)>\gamma_{\rm th}\right) \tag{33}$$

$$N^b_{\rm OUS} = 3AB_iP_r(\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}) \tag{34}$$

respectively. $P_r(\min(\gamma_i,\gamma_j)>\gamma_{\rm th})$ and $P_r(\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th})$ are the probabilities of using NC and OUS, respectively, which are derived as

$$\begin{aligned} &P_r(\min(\gamma_i,\gamma_j)\gamma_{\rm th}) \\ &\quad = \int_{\gamma_{\rm th}}^{\infty}\frac{1}{\overline{\gamma}_i}e^{-\frac{\gamma_i}{\overline{\gamma}_i}}d\gamma_i\int_{\gamma_{\rm th}}^{\infty}\frac{1}{\overline{\gamma}_j}e^{-\frac{\gamma_j}{\overline{\gamma}_j}}d\gamma_j \\ &\quad = e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}} \end{aligned} \tag{35}$$

$$\begin{aligned} &P_r(\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}) \\ &\quad = \int_{0}^{\gamma_{\rm th}}\frac{1}{\overline{\gamma}_j}e^{-\frac{\gamma_j}{\overline{\gamma}_j}}d\gamma_j\int_{\gamma_j}^{\infty}\frac{1}{\overline{\gamma}_i}e^{-\frac{\gamma_i}{\overline{\gamma}_i}}d\gamma_i \\ &\quad = \frac{\overline{\gamma}_i}{\overline{\gamma}_i+\overline{\gamma}_j}\left(1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}\right). \end{aligned} \tag{36}$$

Plugging (33) and (34) into (32), we obtain (3).

Appendix B
Proof of Lemma 2

The BER over any of the links can be expressed as

$$P\left(\varepsilon_{im} \,|\gamma_{im} > \gamma_{\text{th}}\right) = \int P_e^{4-\text{QAM}}(\gamma_{im}) f_{\gamma_{im}|\gamma_{im}>\gamma_{\text{th}}}\left(\gamma_{im}\right) d\gamma_{im} \quad (37)$$

$$P\left(\varepsilon_{im}^{\text{HP}} \,|\gamma_{im} > \gamma_{\text{th}}\right) = \int P_{e,\text{HP}}^{4/16-\text{QAM}}(\gamma_{im}) f_{\gamma_{im}|\gamma_{im}>\gamma_{\text{th}}}\left(\gamma_{im}\right) d\gamma_{im} \quad (38)$$

where $P_e^{4-\text{QAM}}(\gamma_{im})$ and $P_{e,\text{HP}}^{4/16-\text{QAM}}(\gamma_{im})$ are the exact conditional BER for 4-QAM and the HP bits of 4/16-QAM, which are conditioned on the instantaneous SNR, and are given by [28]

$$P_e^{4-\text{QAM}}(\gamma_{im}) = \frac{1}{2}\text{erfc}\sqrt{\gamma_{im}} \quad (39)$$

and [27]

$$P_{e,\text{HP}}^{4/16-\text{QAM}}(\gamma_{im}) = \frac{1}{2}\left[\frac{1}{2}\text{erfc}\sqrt{\frac{2(d^2-2d+1)}{1+d^2}\gamma_{im}} + \frac{1}{3}\text{erfc}\sqrt{\frac{2(d^2+2d+1)}{1+d^2}\gamma_{im}}\right] \quad (40)$$

respectively, where d is the constellation priority parameter, and $f_{\gamma_{im}|\gamma_{im}>\gamma_{\text{th}}}(\gamma_{im})$ is the conditional pdf of γ_{im} conditioned on $\gamma_{im} > \gamma_{\text{th}}$, which is derived as

$$f_{\gamma_{im}|\gamma_{im}>\gamma_{\text{th}}}\left(\gamma_{im}\right) = e^{\frac{1}{\overline{\gamma}_{im}}\gamma_{\text{th}}} \frac{1}{\overline{\gamma}_{im}} e^{-\frac{1}{\overline{\gamma}_{im}}\gamma_{im}}. \quad (41)$$

By plugging (39) and (41) into (37), we obtain (5). Similarly, by plugging (40) and (41) into (38) and integrating by parts, we obtain (6).

Appendix C
Proof of Lemma 3

The BER over any of the links for LP bits can be expressed as

$$P\left(\varepsilon_{im}^{\text{LP}} \,|\gamma_{im} > \gamma_{\text{th}}\right) = \int P_{e,\text{LP}}^{4/16-\text{QAM}}(\gamma_{im}) f_{\gamma_{im}|\gamma_{im}>\gamma_{\text{th}}}\left(\gamma_{im}\right) d\gamma_{im} \quad (42)$$

where $P_{e,\text{LP}}^{4/16-\text{QAM}}(\gamma_{im})$ is the exact conditional BER for the LP bits, which is conditioned on the instantaneous SNR, for the 4/16-QAM and is given by [27]

$$P_{e,\text{LP}}^{4/16-\text{QAM}}(\gamma_{im}) = \frac{1}{2}\text{erfc}\sqrt{\frac{2}{1+d^2}\gamma_{im}} + \frac{1}{4}\text{erfc}\sqrt{\frac{2(4d^2-4d+1)}{1+d^2}\gamma_{im}} + \frac{1}{2}\text{erfc}\sqrt{\frac{2(4d^2+4d+1)}{1+d^2}\gamma_{im}}. \quad (43)$$

Then, plugging (41) and (43) into (42) and carrying out the integration, we obtain (10).

Appendix D
Proof of Lemma 4

For the M-QAM, the BER over any of the links can be expressed as

$$P\left(\varepsilon_{im} \,|\gamma_i > \gamma_j, \gamma_j < \gamma_{\text{th}}\right) = \int_0^\infty P_e^{M-\text{QAM}}(\gamma_{im}) f_{\gamma_{im}|\gamma_i>\gamma_j\gamma_j<\gamma_{\text{th}}}\left(\gamma_{im}\right) d\gamma_{im} \quad (44)$$

where $i, j, n, m = 1, 2$, and $i \neq j, m \neq n$, and $P_e^{M-\text{QAM}}(\gamma_{im})$ is the exact conditional BER, conditioned on the instantaneous SNR, and is given by [28]

$$P_e^{M-\text{QAM}}(\gamma_{im}) = \frac{1}{\sqrt{M}\log_2\sqrt{M}} \sum_{k=1}^{\log_2\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \times \left[(-1)^{\left\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}}\right\rfloor} \times \left(2^{k-1} - \left\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}} + \frac{1}{2}\right\rfloor\right) \cdot \text{erfc}\left((2i+1)\sqrt{\frac{3\log_2^M \gamma_{im}}{2(M-1)}}\right)\right] \quad (45)$$

and $f_{\gamma_{im}|\gamma_{im}>\gamma_{\text{th}}}(\gamma_{im})$ is the conditional pdf of γ_{im} conditioned on $\gamma_{im} > \gamma_{\text{th}}$, which is derived as (56) and (57) in Lemma 5, shown below. Note that the pdfs $f_{\gamma_{im}|\gamma_i>\gamma_j\gamma_j<\gamma_{\text{th}}}(\gamma_{im})$ have different expressions for $\gamma_{im} < \gamma_{\text{th}}$ and $\gamma_{im} > \gamma_{\text{th}}$. Therefore, (44) can be rewritten as

$$P\left(\varepsilon_{im} \,|\gamma_i > \gamma_j, \gamma_j < \gamma_{\text{th}}\right) = \int_0^{\gamma_{\text{th}}} P_e^{M-\text{QAM}}(\gamma_{im}) f_{\gamma_{im}|\gamma_i>\gamma_j\gamma_j<\gamma_{\text{th}}}\left(\gamma_{im}\right) + \int_{\gamma_{\text{th}}}^\infty P_e^{M-\text{QAM}}(\gamma_{im}) f_{\gamma_{im}|\gamma_i>\gamma_j\gamma_j<\gamma_{\text{th}}}\left(\gamma_{im}\right). \quad (46)$$

By plugging (45), (56), and (57) into (46) and carrying out the integration, we obtain (12).

Appendix E
Proof of Lemma 5

The pdf of γ_{im}, given $\gamma_i > \gamma_j \ \gamma_j < \gamma_{\text{th}}$, can be expressed as

$$f_{\gamma_{im}|\gamma_i>\gamma_j,\gamma_j<\gamma_{\text{th}}}(\gamma_{im}) = \int_0^\infty f_{\gamma_{im}|\gamma_i=\gamma}\left(\gamma_{im}\right) f_{\gamma_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\text{th}}}(\gamma) d\gamma \quad (47)$$

where $\gamma_i = \min(\gamma_{im}, \gamma_{in})$ for $i, j, n, m = 1, 2$ and $i \neq j, m \neq n$. The pdfs of γ_{im} and γ_i are given in (1) and (2), respectively. Then, (47) can be further expressed as [30]

$$
\begin{aligned}
&f_{\gamma_{im}|\gamma_i>\gamma_j \gamma_j<\gamma_{\rm th}}(\gamma_{im}) \\
&\quad = \int_0^{\infty} \frac{\frac{1}{\overline{\gamma}_{im}} e^{-\frac{\gamma_{im}}{\overline{\gamma}_{im}}} \frac{1}{\overline{\gamma}_{in}} e^{-\frac{\gamma_{in}}{\overline{\gamma}_{in}}}}{\frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma_i}{\overline{\gamma}_i}}} f_{\gamma_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}}(\gamma) d\gamma \\
&\qquad + \frac{\frac{1}{\overline{\gamma}_{im}} e^{-\frac{\gamma_{im}}{\overline{\gamma}_{im}}} e^{-\frac{\gamma_{in}}{\overline{\gamma}_{in}}}}{\frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma_{im}}{\overline{\gamma}_i}}} f_{\gamma_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}}(\gamma_{im}). \qquad (48)
\end{aligned}
$$

The conditional cumulative distribution function (cdf) of γ_i, which is conditioned on $\gamma_i > \gamma_j$ and $\gamma_j < \gamma_{\rm th}$, is defined as

$$
F_{\gamma_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}}(\gamma) = \frac{P_r(\gamma_i < \gamma, \gamma_i > \gamma_j, \gamma_j < \gamma_{\rm th})}{P_r(\gamma_i > \gamma_j, \gamma_j < \gamma_{\rm th})}. \qquad (49)
$$

By using the law of total probability, the numerator in (49) is given by

$$
\begin{aligned}
&P_r(\gamma_i < \gamma, \gamma_i > \gamma_j, \gamma_j < \gamma_{\rm th}) \\
&\quad = P_r(\gamma_i < \gamma, \gamma_i > \gamma_j, \gamma_i < \gamma_{\rm th}) \\
&\qquad + P_r(\gamma_i < \gamma, \gamma_{\rm th} > \gamma_j, \gamma_i > \gamma_{\rm th}) \\
&\quad = \begin{cases} P_r(\gamma_i < \gamma, \gamma_i > \gamma_j), & \gamma < \gamma_{\rm th} \\ P_r(\gamma_i < \gamma_{\rm th}, \gamma_i > \gamma_j) & \\ +P_r(\gamma_i < \gamma, \gamma_i > \gamma_{\rm th}, \gamma_j < \gamma_{\rm th}), & \gamma > \gamma_{\rm th}. \end{cases} \qquad (50)
\end{aligned}
$$

where

$$
\begin{aligned}
&P_r(\gamma_i < \gamma, \gamma_i > \gamma_j) \\
&\quad = \int_0^{\gamma} \frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma_i}{\overline{\gamma}_i}} d\gamma_i \int_0^{\gamma_i} \frac{1}{\overline{\gamma}_j} e^{-\frac{\gamma_j}{\overline{\gamma}_j}} d\gamma_j \\
&\quad = 1 - e^{-\frac{1}{\overline{\gamma}_i}\gamma} - \frac{\overline{\gamma}_j}{\overline{\gamma}_i + \overline{\gamma}_j} \left(1 - e^{-\left(\frac{1}{\overline{\gamma}_i} + \frac{1}{\overline{\gamma}_j}\right)\gamma}\right) \qquad (51)
\end{aligned}
$$

$$
\begin{aligned}
&P_r(\gamma_i < \gamma_{\rm th}, \gamma_i > \gamma_j) \\
&\quad = \int_0^{\gamma_{\rm th}} \frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma_i}{\overline{\gamma}_i}} d\gamma_i \int_0^{\gamma_i} \frac{1}{\overline{\gamma}_j} e^{-\frac{\gamma_j}{\overline{\gamma}_j}} d\gamma_j \\
&\quad = 1 - e^{-\frac{1}{\overline{\gamma}_i}\gamma_{\rm th}} - \frac{\overline{\gamma}_j}{\overline{\gamma}_i + \overline{\gamma}_j} \left(1 - e^{-\left(\frac{1}{\overline{\gamma}_i} + \frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}\right) \qquad (52)
\end{aligned}
$$

$$
\begin{aligned}
&P_r(\gamma_i < \gamma, \gamma_i > \gamma_{\rm th}, \gamma_j < \gamma_{\rm th}) \\
&\quad = \int_{\gamma_{\rm th}}^{\gamma} \frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma_i}{\overline{\gamma}_i}} d\gamma_i \int_0^{\gamma_{\rm th}} \frac{1}{\overline{\gamma}_j} e^{-\frac{\gamma_j}{\overline{\gamma}_j}} d\gamma_j \\
&\quad = \left(e^{-\frac{1}{\overline{\gamma}_i}\gamma_{\rm th}} - e^{-\frac{1}{\overline{\gamma}_i}\gamma}\right) \left(1 - e^{-\frac{1}{\overline{\gamma}_j}\gamma_{\rm th}}\right). \qquad (53)
\end{aligned}
$$

The denominator in (49) is given by

$$
\begin{aligned}
P_r(\gamma_i \gamma_j, \gamma_j < \gamma_{\rm th}) &= \int_0^{\gamma_{\rm th}} \frac{1}{\overline{\gamma}_j} e^{-\frac{\gamma_j}{\overline{\gamma}_j}} d\gamma_j \int_{r_j}^{\infty} \frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma_i}{\overline{\gamma}_i}} d\gamma_i \\
&= \frac{\overline{\gamma}_i}{\overline{\gamma}_i + \overline{\gamma}_j} \left(1 - e^{-\left(\frac{1}{\overline{\gamma}_i} + \frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}\right). \qquad (54)
\end{aligned}
$$

Plugging (51)–(53) and (54) into (49), we can derive the conditional cdf $F_{\gamma_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}}(\gamma)$. The conditional pdf $f_{\gamma_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}}(\gamma)$ is given by taking derivative $F_{\gamma_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}}(\gamma)$, i.e.,

$$
\begin{aligned}
&f_{\gamma_i|\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}}(\gamma_i) \\
&\quad = \begin{cases} \dfrac{\frac{\overline{\gamma}_i+\overline{\gamma}_j}{\overline{\gamma}_i^2}\left(e^{-\frac{1}{\overline{\gamma}_i}\gamma_i} - e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_i}\right)}{1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}}, & \gamma_i < \gamma_{\rm th} \\ \dfrac{\frac{\overline{\gamma}_i+\overline{\gamma}_j}{\overline{\gamma}_i^2} e^{-\frac{1}{\overline{\gamma}_i}\gamma_i}\left(1-e^{-\frac{1}{\overline{\gamma}_j}\gamma_{\rm th}}\right)}{1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}}, & \gamma_i > \gamma_{\rm th}. \end{cases} \qquad (55)
\end{aligned}
$$

Then, for $\gamma_{im} > \gamma_{\rm th}$, (48) can be further expressed as

$$
\begin{aligned}
&f_{\gamma_{im}|\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}}(\gamma_{im}) \\
&\quad = \int_0^{\gamma_{\rm th}} \frac{\frac{1}{\overline{\gamma}_{im}} e^{-\frac{\gamma_{im}}{\overline{\gamma}_{im}}} \frac{1}{\overline{\gamma}_{in}} e^{-\frac{\gamma_{in}}{\overline{\gamma}_{in}}}}{\frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma_i}{\overline{\gamma}_i}}} \\
&\qquad \times \frac{\frac{\overline{\gamma}_i+\overline{\gamma}_j}{\overline{\gamma}_i^2}\left(e^{-\frac{1}{\overline{\gamma}_i}\gamma} - e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma}\right)}{1 - e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}} d\gamma \\
&\qquad + \int_{\gamma_{\rm th}}^{\gamma_{im}} \frac{\frac{1}{\overline{\gamma}_{im}} e^{-\frac{\gamma_{im}}{\overline{\gamma}_{im}}} \frac{1}{\overline{\gamma}_{in}} e^{-\frac{\gamma_{in}}{\overline{\gamma}_{in}}}}{\frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma_i}{\overline{\gamma}_i}}} \\
&\qquad \times \frac{\frac{\overline{\gamma}_i+\overline{\gamma}_j}{\overline{\gamma}_i^2} e^{-\frac{1}{\overline{\gamma}_i}\gamma}\left(1 - e^{-\frac{1}{\overline{\gamma}_j}\gamma_{\rm th}}\right)}{1 - e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}} d\gamma \\
&\qquad + \frac{\frac{1}{\overline{\gamma}_{im}} e^{-\frac{\gamma_{im}}{\overline{\gamma}_{im}}} e^{-\frac{\gamma_{in}}{\overline{\gamma}_{in}}}}{\frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma_i}{\overline{\gamma}_i}}} \frac{\frac{\overline{\gamma}_i+\overline{\gamma}_j}{\overline{\gamma}_i^2} e^{-\frac{1}{\overline{\gamma}_i}\gamma_{im}}\left(1 - e^{-\frac{1}{\overline{\gamma}_j}\gamma_{\rm th}}\right)}{1 - e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}} \\
&\quad = \frac{\overline{\gamma}_{in}\left(\overline{\gamma}_i + \overline{\gamma}_j\right)\left(1 - e^{-\left(\frac{1}{\overline{\gamma}_{in}}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}\right)}{\overline{\gamma}_{im}\overline{\gamma}_i(\overline{\gamma}_{in} + \overline{\gamma}_j)\left(1 - e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}\right)} e^{-\frac{1}{\overline{\gamma}_{im}}\gamma_{im}} \qquad (56)
\end{aligned}
$$

and for $\gamma_{im} < \gamma_{\rm th}$, (48) can be further expressed as

$$
\begin{aligned}
&f_{\gamma_{im}|\gamma_i>\gamma_j,\gamma_j<\gamma_{\rm th}}(\gamma_{im}) \\
&= \int_0^{\gamma_{im}} \frac{\frac{1}{\overline{\gamma}_{im}} e^{-\frac{\gamma_{im}}{\overline{\gamma}_{im}}} \frac{1}{\overline{\gamma}_{in}} e^{-\frac{\gamma_{in}}{\overline{\gamma}_{in}}}}{\frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma_i}{\overline{\gamma}_i}}} \\
&\quad \cdot \frac{\frac{\overline{\gamma}_i+\overline{\gamma}_j}{\overline{\gamma}_i^2}\left(e^{-\frac{1}{\overline{\gamma}_i}\gamma} - e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma}\right)}{1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}} d\gamma \\
&\quad + \frac{\frac{1}{\overline{\gamma}_{im}} e^{-\frac{\gamma_{im}}{\overline{\gamma}_{im}}} e^{-\frac{\gamma_{in}}{\overline{\gamma}_{in}}}}{\frac{1}{\overline{\gamma}_i} e^{-\frac{\gamma_{im}}{\overline{\gamma}_i}}} \\
&\quad \cdot \frac{\frac{\overline{\gamma}_i+\overline{\gamma}_j}{\overline{\gamma}_i^2}\left(e^{-\frac{1}{\overline{\gamma}_i}\gamma_{im}} - e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{im}}\right)}{1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}} \\
&= \frac{\overline{\gamma}_{in}(\overline{\gamma}_i+\overline{\gamma}_j)\left[e^{-\frac{1}{\overline{\gamma}_{im}}\gamma_{im}} - e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{im}})\right]}{\overline{\gamma}_{im}\overline{\gamma}_i(\overline{\gamma}_{in}+\overline{\gamma}_j)\left(1-e^{-\left(\frac{1}{\overline{\gamma}_i}+\frac{1}{\overline{\gamma}_j}\right)\gamma_{\rm th}}\right)} \qquad (57)
\end{aligned}
$$

where $i, j, n, m = 1, 2$, and $i \neq j, m \neq n$.

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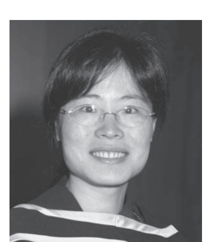

Xuehua Zhang received the Ph.D. degree in electrical engineering from Concordia University, Montreal, QC, Canada, in 2014.

Her current research interests include wireless communications, cooperative communications, and network coding.

Dr. Zhang received the Alexander Graham Bell Canada Graduate Scholarships-Doctoral and the Michael Smith Foreign Study Supplements.

Mazen O. Hasna received the B.S. degree from Qatar University, Doha, Qatar, in 1994; the M.S. degree from the University of Southern California, Los Angeles, CA, USA, in 1998; and the Ph.D. degree from the University of Minnesota Twin Cities, Minneapolis, MN, USA, in 2003, all in electrical engineering.

In 2003, he joined the Department of Electrical Engineering, Qatar University, as an Assistant Professor, where he currently serves as the Vice President and the Chief Academic Officer. His main research interests include digital communication theory and its application to performance evaluation of wireless communication systems over fading channels. His current research interests include cooperative communications, ad hoc networks, cognitive radio, and network coding.

Ali Ghrayeb received the Ph.D. degree in electrical engineering from The University of Arizona, Tucson, AZ, USA, in 2000.

He is currently a Professor with the Department of Electrical and Computer Engineering, Texas A&M University at Qatar, Doha, Qatar (on leave from Concordia University, Montreal, QC, Canada). He is the coauthor of the book *MIMO Communication Systems* (Wiley, 2008). His research interests include wireless and mobile communications, error correcting coding, multiple-input–multiple-output systems, wireless cooperative networks, and cognitive radio systems.

Dr. Ghrayeb has instructed/coinstructed technical tutorials at several major IEEE conferences. He served as the Technical Program Committee (TPC) Cochair for the Communications Theory Symposium of the 2011 IEEE Global Communications Conference (IEEE Globecom). He will serve as the TPC Cochair for the 2016 IEEE Wireless Communications and Networking Conference. He served as an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING, the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, *Elsevier Physical Communications*, and *Wiley Wireless Communications and Mobile Computing*. He currently serves as an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and the IEEE TRANSACTIONS ON COMMUNICATIONS. He coreceived the IEEE Globecom Best Paper Award in 2010.