Asynchronous Network Coding for Multiuser Cooperative Communications

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***Abstract*—In this paper, we present an asynchronous network coding (ANC) transmission strategy for multiuser co-operative networks. In particular, we consider a relay network comprised of multiple sources and mutiple relays. The sources all transmit simultaneously. The relay receives a sum of the signals with different delays. The decoded signals at the relay are network-coded by asynchronous delays and transmitted to the destination. We first present a decoding strategy for our ANC. We also compare our proposed ANC to complex field network coding and show the superiority of our proposed scheme in terms of decoding complexity and bit error rate (BER) performance. Next, we develop three relay selection strategies with different complexity in conjunction with the proposed ANC scheme. The proposed full selection and joint selection (JS) schemes consider both channel quality and delay effect in the selection. But the proposed individual selection scheme only considers channel quality in the selection. We derive an end-to-end (E2E) BER expression in terms of worst E2E SNR for the JS scheme. We also theoretically show that the proposed schemes achieve full diversity. In addition, we compare the performance of the three proposed relay selection schemes by simulations.**

***Index Terms*—Asynchronous transmission, cooperative diversity, decode-and-forward, network coding, relay selection.**

I. Introduction

THE incorporation of cooperative communication and network coding (NC) [1]–[3] in wireless networks has attracted considerable attention. This is attributed to the fact that the throughput of cooperative networks can be improved by utilizing NC at relay nodes, in which multiple transmissions of multiple sources are reduced to only one transmission. Different network coding schemes have been proposed for either two-way relay networks or multiuser cooperative networks. For a decode-and-forward (DF) relaying cooperative network, the proposed schemes include physical-layer network coding [5], exclusive-OR network coding [6], [7] and complex field network coding (CFNC) [8], [9].

Among these schemes, CFNC has the advantage of higher throughput, better bit error rate (BER) performance, and is more suitable for multiuser cooperative networks [8]. The key idea of CFNC is to create a one-to-one mapping between individual signals and their mixed signal by multiplying the individual signals with complex field coefficients. The transmissions of multiple sources to their destination can be completed in only two time slots instead of $2M$ time slots without network coding, where M is the number of the sources. This greatly improves the network throughput. However, CFNC has two disadvantages. First, joint maximum-likelihood (ML) is required for the decoding, which has a very high complexity. Second, it requires perfect symbol-level synchronization among the network nodes. However, the nodes in the cooperative networks are distributed, so it is difficult to synchronize the received signals at the relays and the destination in real world conditions [10].

Different techniques have been proposed to deal with the issue of imperfect timing synchronization in wireless communication networks [11]–[18]. The capacity region of multiple-access channels with symbol asynchronous transmission is derived in [11]. Various decoding strategies for multiple-input-multiple-output (MIMO) systems with timing delays are illustrated in [12], [13] and [14]. It is shown that asynchronous transmission can bring performance improvement. Recently, asynchronous signaling has been also studied in the context of DF cooperative networks. In [15], the authors derive the diversity-multiplexing trade-off of asynchronous cooperative networks with one source and multiple DF relays. Asynchronous two-way relaying has been considered in [16] and [17], focusing on how to decode a linear synchronous sum of transmitted signals from random asynchronous combinations. In [18], we incorporate analog network coding with asynchronous signaling in multiuser cooperative networks. However, little work has been done on asynchronous multiuser cooperative networks with DF relaying.

Motivated by the above observations, we propose an asynchronous NC (ANC) transmission strategy for DF multiuser cooperative networks. Compared to CFNC, the signals are network-coded by different delays instead of complex field coefficients. Therefore, the delays are added intentionally to different signals after decoding at the relay before forwarding. The other delays in the system may come from either imperfect timing synchronization among different sources or be added in a perfectly synchronized cooperative network. We can also add delays on top of imperfect timing delays to achieve optimal delay values [14] to have better performance.

Besides NC, relay selection has been shown to be another effective way to improve the performance of cooperative networks [19]. Thus, some research work has been done to

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study the combination of NC and relay selection in a multiuser cooperative network. In [8], the authors study the performance of CFNC with relay selection and show that full diversity is achieved. In [20], we propose two relay assignment schemes for a cooperative network with multiple two-way channels where exclusive-OR NC is used. We prove that the proposed schemes provide full diversity. However, due to the different system models and network coding schemes, the proposed schemes in [20] are not suitable for the case in this work since they do not provide full diversity as shown in Section VII.

In the following, we summarize the contributions of the paper:

1) We propose a novel ANC transmission strategy for multiuser cooperative networks which has better performance than the CFNC scheme. The received and transmitted signals at a relay are network-coded by adding intentional asynchronous delays. To the best of our knowledge, we are the first to propose this delay-based network coding scheme for both uplink and downlink of a multiuser cooperative network [14]–[18].
2) To further improve the performance, we investigate relay selection in conjunction with the proposed ANC scheme. In particular, we propose three relay selection schemes, i.e., FS, JS and IS scheme.
3) We derive an E2E BER expression in terms of the worst E2E SNR for the JS scheme. In addition, we analyze the diversity achieved by all proposed schemes.
4) We present simulation results to validate the performance analysis. In addition, we compare the performance of ANC to that of CFNC and we also compare the performance of FS, JS and IS.

The remainder of the paper is organized as follows. The system model is illustrated in Section II. The ANC scheme and its decoding strategies are presented in Section III. In Section IV, we present three relay selection schemes. The BER expression in terms of the worst E2E SNR for the JS scheme is derived in Section V. We analyze the achievable diversity of JS, FS and IS schemes in Section VI. Simulation results are provided in Section VII. We conclude the paper in Section VIII.

II. System Model

We consider a cooperative network with M sources denoted by S_m $(m = 1, \ldots, M)$, L relays denoted by R_l $(l = 1, \ldots, L)$, and one common destination as shown in Fig. 1. All nodes in the network are equipped with a single antenna and operate in a half-duplex mode. We assume that there is no direct path between the source nodes and the destination.

The network subchannels are assumed to experience independent slow and frequency non-selective Rayleigh fading. Let $h_{S_m R_l}$ $(m = 1, \ldots, M,\ l = 1, \ldots, L)$ and $h_{R_l D}$ denote the fading coefficients for the following hops $S_m \to R_l$ and $R_l \to D$, respectively. We assume that the channel coefficients that correspond to different links are perfectly known to the destination. The destination collects the channel state information (CSI) of all the links by pilot training [23]. Let $\gamma_{S_m R_l} = \rho \left|h_{S_m R_l}\right|^2$ and $\gamma_{R_l D} = \rho \left|h_{R_l D}\right|^2$, where $\rho = \frac{E_b}{N_0}$,

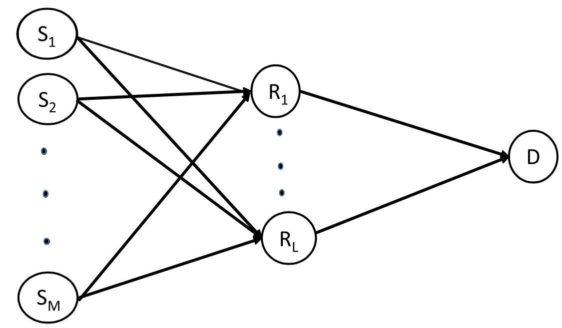

Fig. 1. A cooperative network with M sources, L relays and one destination.

denotes the instantaneous SNRs for the hops $S_m \to R_l$ and $R_l \to D$. Each hop has the same ρ indicating that all the nodes in the network use the same power which is a general assumption in the literature. Then $\overline{\gamma}_{S_m R_l} = \rho E\left[\left|h_{S_m R_l}\right|^2\right]$ and $\overline{\gamma}_{R_l D} = \rho E\left[\left|h_{R_l D}\right|^2\right]$ are the average SNR for the corresponding links. The pdf of $\gamma_{S_m R_l}$ and $\gamma_{R_l D}$ are given as

$$f_{\gamma_{S_m R_l}}(\gamma) = \frac{1}{\overline{\gamma}_{S_m R_l}} e^{-\frac{1}{\overline{\gamma}_{S_m R_l}}\gamma}, \tag{1}$$

and

$$f_{\gamma_{R_l D}}(\gamma) = \frac{1}{\overline{\gamma}_{R_l D}} e^{-\frac{1}{\overline{\gamma}_{R_l D}}\gamma}. \tag{2}$$

III. Proposed Asynchronous Network Coding

A. ANC Transmission Strategy

In this subsection, we introduce the ANC-based transmission strategy for the multiuser cooperative network. The transmission process of ANC is illustrated in Fig. 2. The transmission process is divided into two phases. In the first phase, all the sources broadcast N information symbols to the relays with different delays. Without loss of generality, we assume that the relay and destination are synchronized with the first source. The signals from the mth source, $m = 1, 2, \ldots, M$, arrive $\tau_{S_m R_l}$ seconds later with respect to that of the first source where $\tau_{S_1 R_l} = 0$ and $0 < \tau_{S_2 R_l} < \cdots < \tau_{S_m R_l} < T_s$. In this work, we adopt the commonly used delay setting [1], [10]–[16], [18] in the literature, where the delay values are assumed to be known and fixed. The transmitted signal from the mth source is

$$s_m(t) = \sum_{n=1}^{N} x_m(n)\, \psi\left(t - (n-1)T_s - \tau^1_{S_m R_l}\right) \tag{3}$$

where $x_m(n)$ is the nth transmitted symbol by source m in one coherent time slot, $\psi(t)$ is the rectangular shaping waveform being used by the sources, T_s is the symbol duration and $\tau^1_{S_m R_l}$ is the intentionally added delay value for Source m at Relay l. Note that we have included $\tau^1_{S_m R_l}$ to account for the added delay values for the perfectly synchronized case. In other words, if $\tau^1_{S_m R_l} = 0$, our system will still work for an

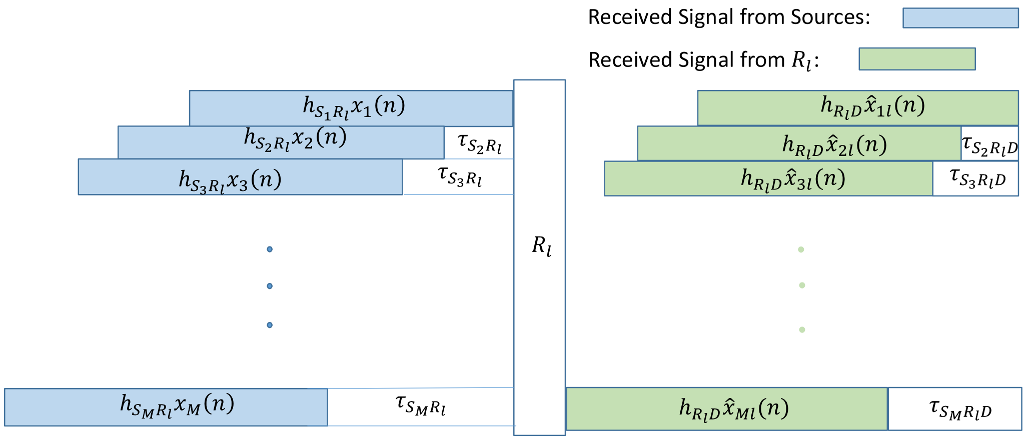

Fig. 2. The transmission process of ANC.

imperfect timing synchronized system. The received signal at the l th relay is given by

$$y_{R_l}(t) = \sqrt{\rho}\sum_{m=1}^{M} h_{S_mR_l}\sum_{n=1}^{N} x_m(n) \cdot \psi\left(t-(n-1)T_s-\tau_{S_mR_l}\right)+n_{R_l}(t), \quad (4)$$

where n_{R_l} is an additive white complex Gaussian noise (AWGN) sample, with zero mean and unit variance and $\tau_{S_mR_l} = \tau^1_{S_mR_l} + \tau^2_{S_mR_l}$, where $\tau^2_{S_mR_l}$ is the imperfect timing delay value for Source m at Relay l. As will be discussed later, the relay will only need to know $\tau_{S_mR_l}$ and not its components. In other words, the relay does not need to know $\tau^1_{S_mR_l}$ and $\tau^2_{S_mR_l}$ separately and its operation is the same when for example $\tau^1_{S_mR_l} = 0$.

Then the relays decode the transmitted symbols from different sources. The decoded nth symbol for Source m at Relay l are denoted by $\widehat{x}_{ml}(n)$. Finally, the decoded symbols are added together after inserting intentional different delays. The transmitted signal from the lth relay is

$$s_{R_l}(t) = \sum_{m=1}^{M}\sum_{n=1}^{N}\widehat{x}_{ml}(n)\,\psi\left(t-(n-1)T_s-\tau^1_{S_mR_lD}\right), \quad (5)$$

where $\tau^1_{S_mR_lD}$ is the intentionally added delay value for Source m at Relay l.

Thus, the received signal at the destination from the lth relay can be written as follows:

$$y_D(t) = \sqrt{\rho}h_{R_lD}\sum_{m=1}^{M}\sum_{n=1}^{N}\widehat{x}_{ml}(n) \cdot \psi\left(t-(n-1)T_s-\tau_{S_mR_lD}\right)+n_D(t), \quad (6)$$

where $\tau_{S_mR_lD} = \tau^1_{S_mR_lD} + \tau^2_{S_mR_lD}$, where $\tau^2_{S_mR_lD}$ is the imperfect timing delay value for Source m and Relay l at Destination.

B. Decoding Strategies

The proposed ANC can be decoded by various decoding methods for asynchronous signaling in the literature [12]–[14].

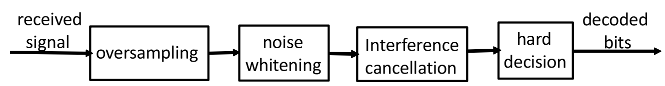

Fig. 3. ZF decoding steps.

In this work, we employ zero-forcing (ZF) and Viterbi (VB) decoding. In what follows, we only describe the ZF decoding process at the relays since the decoding process at the destination is the same as that at the relays. The details of VB decoding can be found in [14] and [18]. The decoding steps of ZF are illustrated in Fig. 3, which include oversampling, noise whitening, interference cancellation and hard decision. The details of the steps are as follows.

In order to decode successfully, oversampling is exploited first to acquire a sufficient number of samples of the received signal. To this end, the received signals at the lth relay are passed through M matched filters and sampled at $t = (j-1)\,T_s+\tau_{S_iR_l}$ ($i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, N$) [21]. The output of the i th matched filter at the jth instant is given as:

$$y^i_{R_l}(j) = \int_{(j-1)T_s+\tau_{S_iR_l}}^{jT_s+\tau_{S_iR_l}} y_{R_l}(t)\psi^*\left(t-(j-1)T_s-\tau_{S_iR_l}\right)dt, \quad (7)$$

for $i = 1, \ldots, M$ and $j = 1, 2, \ldots, N$.

Then, the sampled output vector of matched filters can be written in a matrix form as [13]

$$\mathbf{y}_{SR_l} = \sqrt{\rho}\mathbf{R}_{SR_l}\mathbf{H}_{SR_l}\mathbf{x}+\mathbf{n}_{SR_l}, \quad (8)$$

where $\mathbf{R}_{SR_l}$ is an $MN \times MN$ matrix and is defined as

$$\mathbf{R}_{SR_l} = \begin{pmatrix} \mathbf{R}_0 & \mathbf{R}_1^T & 0 & \ldots & 0 & 0 & 0 \\ \mathbf{R}_1 & \mathbf{R}_0 & \mathbf{R}_1^T & 0 & \ldots & 0 & 0 \\ 0 & \mathbf{R}_1 & \mathbf{R}_0 & \mathbf{R}_1^T & 0 & \ldots & 0 \\ \vdots & & & \ddots & & & \vdots \\ \vdots & & & & \ddots & & \vdots \\ 0 & 0 & \ldots & 0 & \mathbf{R}_1 & \mathbf{R}_0 & \mathbf{R}_1^T \\ 0 & 0 & 0 & \ldots & 0 & \mathbf{R}_1 & \mathbf{R}_0 \end{pmatrix}, \quad (9)$$

where

$$\mathbf{R}_0(p,\ q)=\begin{cases}1, & \text{if } p=q,\\ 1-\left(\tau_{S_qR_l}-\tau_{S_pR_l}\right), & \text{if } q>p,\\ 1-\left(\tau_{S_pR_l}-\tau_{S_qR_l}\right), & \text{if } p>q,\end{cases}$$

and

$$\mathbf{R}_1(p,\ q)=\begin{matrix}\tau_{S_pR_l}-\tau_{S_qR_l}, & \text{if } p>q,\\ 0, & \text{otherwise},\end{matrix}\quad p=1,\ldots,M,$$

$q=1,\ldots,M$; $\mathbf{x}=(\mathbf{x(1)},\ \mathbf{x(2)},\cdots,\mathbf{x(N)})^T$, where $\mathbf{x(n)}=(x_1(n)\ \ x_2(n),\cdots,x_M(n))^T$, $n=1,\cdots,N$; $\mathbf{y}_{SR_l}=(\mathbf{y}_{SR_l}\mathbf{(1)},\ \mathbf{y}_{SR_l}\mathbf{(2)},\cdots,\mathbf{y}_{SR_l}\mathbf{(N)})^T$, where $\mathbf{y}_{SR_l}\mathbf{(j)}=(y^1_{SR_l}(j)\ \ y^2_{SR_l}(j),\cdots,y^M_{SR_l}(j))^T$, $j=1,\cdots,N$; $\mathbf{H}_{SR_l}=\text{diag}\left\{\underbrace{\bar{\mathbf{H}}_{SR_l},\bar{\mathbf{H}}_{SR_l}\cdots,\bar{\mathbf{H}}_{SR_l}}_{N}\right\}$, where $\bar{\mathbf{H}}_{SR_l}=\text{diag}\left\{h_{S_1R_l},h_{S_2R_l},\cdots,h_{S_MR_l}\right\}$ and $\mathbf{n}_{SR_l}=(\mathbf{n}_{SR_l}\mathbf{(1)},\ \mathbf{n}_{SR_l}\mathbf{(2)},\cdots,\mathbf{n}_{SR_l}\mathbf{(N)})^T$, where $\mathbf{n}_{SR_l}\mathbf{(j)}=(n^1_{SR_l}(j)\ \ n^2_{SR_l}(j),\cdots,n^M_{SR_l}(j))^T$, $j=1,\cdots,N$. The noise covariance matrix is given by $E[\mathbf{n}_{SR_l}\mathbf{n}^H_{SR_l}]=\mathbf{R}_{SR_l}$.

The noise can be whitened by Cholesky factorization. Therefore, we have $\mathbf{R}_{SR_l}=\mathbf{U}^H_{SR_l}\mathbf{U}_{SR_l}$ in which $\mathbf{U}_{SR_l}$ is an upper triangular matrix. Multiplying $\mathbf{y}_{SR_l}$ in (8) by $\mathbf{U}^{-H}_{SR_l}$ results in independent noise samples and we have

$$\widetilde{\mathbf{y}}_{SR_l}=\sqrt{\rho}\mathbf{U}_{SR_l}\mathbf{H}_{SR_l}\mathbf{x}+\widetilde{\mathbf{n}}_{SR_l},\tag{10}$$

where $\widetilde{\mathbf{n}}_{SR_l}=\mathbf{U}^{-H}_{SR_l}\mathbf{n}_{SR_l}$.

Followed by this noise whitening process, we can cancel the interference by multiplying the output of the noise whitening filter by $\mathbf{U}^{-1}_{SR_l}$. As a result, we will have

$$\widehat{\mathbf{y}}_{SR_l}=\sqrt{\rho}\mathbf{H}_{SR_l}\mathbf{x}+\widehat{\mathbf{n}}_{SR_l},\tag{11}$$

where $E[\widehat{\mathbf{n}}_{SR_l}\widehat{\mathbf{n}}^H_{SR_l}]=\mathbf{R}^{-1}_{SR_l}$. Finally, the signals are decoded by making a hard decision on $\mathbf{H}^*_{SR_l}\widehat{\mathbf{y}}_{SR_l}$.

IV. Relay Selection Strategies

A. Selection Criteria

From (11), the equation for the jth sample of the ith source at the relay can be written as

$$\widehat{\mathbf{y}}_{SR_l}(M(j-1)+i)=\sqrt{\rho}h_{S_iR_l}\times x_i(j)+\widehat{n}_{SR_l}(M(j-1)+i),\tag{12}$$

where $i=1,\ldots,M,\ j=1,\ 2,\ldots,N$ and $l=1,2,\ldots,L$.

Therefore, the instantaneous SNRs for the jth sample of the ith source at the relay is

$$\widehat{\gamma}_{S_iR_l}(j)=\frac{\gamma_{S_iR_l}}{\mathbf{R}^{-1}_{SR_l}(k,k)},\tag{13}$$

where $\gamma_{S_iR_l}=P\left|h_{S_iR_l}\right|^2$, $k=M(j-1)+i$, $i=1,\ldots,M$, $j=1,\ 2,\ldots,N$, $l=1,\ 2,\ldots,L$ and $\mathbf{A}(m,n)$ denotes the (m,n)th entry of matrix $\mathbf{A}$. Similarly, the instantaneous SNRs for the jth sample of the ith source at the destination is

$$\widehat{\gamma}_{R_lD}(j)=\frac{\gamma_{R_lD}}{\mathbf{R}^{-1}_{R_lD}(k,k)},\tag{14}$$

where $\gamma_{R_lD}=P\left|h_{R_lD}\right|^2$ and k, i, and j are similarly defined as (13).

The values of $\gamma_{S_iR_l}$ and γ_{R_lD} are varying from one block to another and their pdfs are shown in (1) and (2) as exponential distribution. It is shown in (9) that the values of $\frac{1}{\mathbf{R}^{-1}_{SR_l}(k,k)}$ and $\frac{1}{\mathbf{R}^{-1}_{R_lD}(k,k)}$ are related to the values of delays. Since $k=M(j-1)+i$, $\frac{1}{\mathbf{R}^{-1}_{SR_l}(k,k)}$ and $\frac{1}{\mathbf{R}^{-1}_{R_lD}(k,k)}$ are functions of j and have different values at j ($j=1,\ 2,\ldots,N$). As a result, the instantaneous SNRs with respect to different samples in one block are different. However, it is impossible to perform relay selection per sample based on the instantaneous SNRs. In practice, relay selection is usually performed per block [19], [20], [23]. Therefore, in order to take the effect of the delays into account in selection, we define a new variable which is average instantaneous SNR. The average instantaneous SNR for the total N samples of the ith source at relay and destination in one block is

$$\widetilde{\gamma}_{S_iR_l}=\alpha_{S_iR_l}\gamma_{S_iR_l},\tag{15}$$

and

$$\widetilde{\gamma}_{S_iR_lD}=\alpha_{S_iR_lD}\gamma_{R_lD},\tag{16}$$

where $\alpha_{S_iR_l}=\frac{1}{N}\sum_{j=1}^{N}\frac{1}{\mathbf{R}^{-1}_{SR_l}(k,k)}$ and $\alpha_{S_iR_lD}=\frac{1}{N}\sum_{j=1}^{N}\frac{1}{\mathbf{R}^{-1}_{R_lD}(k,k)}$ $(k=M(j-1)+i,\ i=1,\ldots,M,\ j=1,\ 2,\ldots,N)$.

B. ANC With Relay Selection

For relay selection, each source selects one relay. With ANC scheme, one relay can help multiple sources by adding different delays at different sources. Thus, for relay selection with ANC, several sources can select the same relay which is different from [20] where one relay can only help one pair. Moreover, with ANC scheme, different sources can select different relays and different relays transmit at the same time by having different delays among the received signals at the destination.

Thus, for relay selection with ANC, we have the flexibility of selecting any number of relays to help all the sources. We can also perform relay selection with or without considering the delay effect. In what follows, we propose three relay selection schemes [1] which are based on maximizing the worst E2E SNRs.

C. Full Selection (FS)

This selection scheme finds the best relay selection choice by searching over all possible relay assignment choices. There is no constraint on the total number of the selected relays. Let k^* be the index of the selected choice. Then we have

$$k^*=\arg\max_k\left\{\widetilde{\gamma}_{k,\min},\ k=1,2,\ldots,K\right\},\tag{17}$$

[1] For the proposed schemes, we assume that the destination obtains the required channel information and delay information by training. Then, it performs relay selection and sends the selection results to the relays.

where

$$\widetilde{\gamma}_{k,\min} = \min\left(\widetilde{\gamma}_{S_1 R_{s_1} D}, \widetilde{\gamma}_{S_2 R_{s_2} D}, \ldots, \widetilde{\gamma}_{S_M R_{s_M} D}\right), \quad (18)$$

where R_{s_i} represents the selected relay for the ith source and $\widetilde{\gamma}_{S_i R_{s_i} D} = \min(\widetilde{\gamma}_{S_i R_{s_i}}, \widetilde{\gamma}_{R_{s_i} D})$(for $i = 1, \ldots, M$ and $l = 1, 2, \ldots, L$), where $\widetilde{\gamma}_{S_i R_{s_i}}$ and $\widetilde{\gamma}_{R_{s_i} D}$ are given in (15) and (16). The number of choices, K, is greater than $\frac{L!}{(L-M)!}$ which is the size of the fullset selection scheme in [20]. This is because for the fullset selection in [20], there is the constraint that one relay can only help one source. But there is no such constraint for full selection. This scheme is too complex for implementation. In what follows, we propose two other schemes with lower complexity.

D. Joint Selection (JS)

In this scheme, all the sources jointly select one best relay to help the transmissions of all the sources using ANC. The index of the selected relay for all the sources is

$$l_i^* = \arg\max_l \left\{\widetilde{\gamma}_{il,\ \min},\ l = 1, 2, \ldots, L\right\}, \quad (19)$$

where

$$\widetilde{\gamma}_{il,\ \min} = \min\left(\widetilde{\gamma}_{\min,\ S_1 R_l D}, \widetilde{\gamma}_{\min,\ S_2 R_l D}, \ldots,\ \widetilde{\gamma}_{\min,\ S_M R_l D}\right), \quad (20)$$

and $\widetilde{\gamma}_{\min,\ S_i R_l D} = \min(\widetilde{\gamma}_{S_i R_l},\ \widetilde{\gamma}_{S_i R_l D})$ (for $i = 1, \ldots, M$ and $l = 1,\ 2, \ldots, L$), where $\widetilde{\gamma}_{S_i R_l}$ and $\widetilde{\gamma}_{S_i R_l D}$ are given in (15) and (16). The best choice is selected from only L choices, which is a subset of the choices of the FS scheme. Thus, the complexity is much lower than that of the FS scheme.

E. Individual Selection (IS)

The ith source selects the best relay which has the best worst E2E channel quality. The selected relay for the ith source is

$$l^* = \arg\max_l \left\{\gamma_{l,\min},\ l = 1, 2, \ldots, L\right\}, \quad (21)$$

where

$$\gamma_{l,\min} = \min\left(\gamma_{S_i R_l},\ \gamma_{R_l D}\right), \quad (22)$$

where $\gamma_{S_i R_l} = \rho \left|h_{S_i R_l}\right|^2$ and $\gamma_{R_l D} = \rho \left|h_{R_l D}\right|^2$ (for $i = 1, \ldots, M$ and $l = 1, 2, \ldots, L$).

In the IS scheme, each source selects its own best relay from L relays solely based on the channel quality and independent of the other signals and their time delays. The delay effect is not considered in the selection for this scheme. The complexity of this scheme is ML which is much lower than that of FS scheme but higher than the complexity of the JS scheme.

V. E2E BER Performance

In this section, we derive the E2E BER expression in terms of the worst E2E SNR for the JS scheme. We use a general conditional error probability form in the derivation to simplify the presentation. In particular, we use the form of $u \cdot \mathrm{erfc}\sqrt{\upsilon\gamma}$ where γ is the instantaneous SNR, and $u = 1$ and $\upsilon = 2$ for binary phase shift keying (BPSK). At the end, we show how to extend the derived results for BPSK to M-ary quadrature amplitude modulation (M-QAM).

The E2E BER in terms of the worst E2E SNR can be expressed as

$$P_e^b = P_{e,S_{i^*} R_{l^*}}\left(1 - P_{e,R_{l^*} D}\right) + P_{e,R_{l^*} D}\left(1 - P_{e,S_{i^*} R_{l^*}}\right), \quad (23)$$

where $P_{e,S_{i^*} R_{l^*}}$ and $P_{e,R_{l^*} D}$ are the average BERs of the first hop and second hop which has the worst SNR, respectively, i^* represents the index of the selected source which has the worst SNR over the first hop and l^* represents the index of the selected best relay.

From Equations (13) and (14), we observe that the instantaneous SNRs with respect to different samples in one block are different. Thus, $P_{e,S_{i^*} R_{l^*}}$ and $P_{e,R_{l^*} D}$ are expressed as [14]

$$P_{e,S_{i^*} R_{l^*}} = \frac{1}{N} \sum_{j=1}^{N} P_{e,S_{i^*} R_{l^*}}(j), \quad (24)$$

and

$$P_{e,R_{l^*} D} = \frac{1}{N} \sum_{j=1}^{N} P_{e,R_{l^*} D}(j), \quad (25)$$

where N is the number of samples in one block and $P_{e,S_{i^*} R_{l^*}}(j)$ and $P_{e,R_{l^*} D}(j)$ represent the probabilities that the jth sample (for $j = 1, \ldots, N$) makes an error over the selected first and second hop, respectively. $P_{e,S_{i^*} R_{l^*}}(j)$ and $P_{e,R_{l^*} D}(j)$ can be expressed as

$$P_{e,S_{i^*} R_{l^*}}(j) = \int_0^\infty u\ \mathrm{erfc}\sqrt{\upsilon \widehat{\gamma}_{S_{i^*} R_{l^*}}(j)} f_{\widehat{\gamma}_{S_{i^*} R_{l^*}}(j)}(\gamma)\, d\gamma, \quad (26)$$

and

$$P_{e,R_{l^*} D}(j) = \int_0^\infty u\ \mathrm{erfc}\sqrt{\upsilon \widehat{\gamma}_{R_{l^*} D}(j)} f_{\widehat{\gamma}_{R_{l^*} D}(j)}(\gamma)\, d\gamma, \quad (27)$$

which are functions of $\mathbf{R}_{SR_{l^*}}^{-1}(k,k)$ and $\mathbf{R}_{R_{l^*} D}^{-1}(k,k)$, respectively.

Lemma 1: For asymmetric channels and delay settings, where the variances of the channels and delay settings in the network are all different, $P_{e,S_{i^*} R_{l^*}}(j)$ and $P_{e,S_{i^*} R_{l^*}}(j)$ are obtained as

$$\begin{aligned} P_{e,S_{i^*} R_{l^*}}(j) = {} & \sum_{l=1}^{L} \sum_{\varsigma \in T_l^L} \frac{\mathrm{sign}(\varsigma)\beta_{S_{i^*} R_l}\beta_{R_l D}}{\varsigma + \beta_{R_l D}} \\ & \cdot \left(\begin{array}{l} \frac{1}{\beta_{S_{i^*} R_l}} I\left(\frac{1}{\alpha_{S_{i^*} R_{l^*}} \mathbf{R}_{SR_{l^*}}^{-1}(k,k)\beta_{S_{i^*} R_l}}\right) \\ -\frac{1}{(\varsigma + \beta_{R_l D})} I\left(\frac{1}{\alpha_{S_{i^*} R_{l^*}} \mathbf{R}_{SR_{l^*}}^{-1}(k,k)(\varsigma + \beta_{R_l D})}\right) \end{array} \right) \\ & + \sum_{l=1}^{L} \sum_{\varsigma \in T_l^L} \frac{\mathrm{sign}(\varsigma)\beta_{S_{i^*} R_l}}{\beta_{l,\ \min} + \varsigma} \\ & \cdot I\left(\frac{1}{\alpha_{S_{i^*} R_{l^*}} \mathbf{R}_{SR_{l^*}}^{-1}(k,k)\left(\beta_{l,\ \min} + \varsigma\right)}\right), \end{aligned} \quad (28)$$

and

$$P_{e,R_{l^*}D}(j) = \sum_{l=1}^{L}\sum_{\varsigma\in T_l^L}\frac{\operatorname{sign}(\varsigma)\beta_{S_{i^*}R_l}\beta_{R_lD}}{\varsigma+\beta_{S_{i^*}R_l}} \cdot\left(\begin{array}{c}\frac{1}{\beta_{R_lD}}I\left(\frac{1}{\alpha_{R_{l^*}D}\mathbf{R}_{R_{l^*}D}^{-1}(k,k)\beta_{R_lD}}\right)\\ -\frac{1}{\varsigma+\beta_{S_{i^*}R_l}}I\left(\frac{1}{\alpha_{R_{l^*}D}\mathbf{R}_{R_{l^*}D}^{-1}(k,k)\left(\varsigma+\beta_{S_{i^*}R_l}\right)}\right)\end{array}\right) + \sum_{l=1}^{L}\sum_{\varsigma\in T_l^L}\frac{\operatorname{sign}(\varsigma)\beta_{R_lD}}{\beta_{l,\ \min}+\varsigma} \cdot I\left(\frac{1}{\alpha_{R_{l^*}D}\mathbf{R}_{R_{l^*}D}^{-1}(k,k)\left(\beta_{l,\ \min}+\varsigma\right)}\right), \quad (29)$$

where $I(c) = u\left(1-\sqrt{\frac{\upsilon c}{1+\upsilon c}}\right)$, the other items are defined as $\beta_{S_iR_l} = \frac{1}{\alpha_{S_iR_l}\overline{\gamma}_{S_iR_l}}$, $\beta_{R_lD} = \frac{1}{\alpha_{R_lD}\overline{\gamma}_{R_lD}}$ $\beta_{S_{i^*}R_l} = \sum_{i=1}^{M}\beta_{S_iR_l}$ and $\beta_{l,\min} = \beta_{S_{i^*}R_l}+\beta_{R_lD}$.

Proof: See Appendix A. ■

Lemma 2: For semi-symmetric channels and delay settings, where sources-relays have the same channel variances and delay setting and relays-destination have the same channel variances and delay settings, $P_{e,S_{i^*}R_{l^*}}(j)$ and $P_{e,R_{l^*}D}(j)$ are obtained as

$$P_{e,S_{i^*}R_{l^*}}(j) = \sum_{j=0}^{L-1}(-1)^j\binom{n-1}{j}\frac{\beta_{S_{m^*}R}}{(j+1)\beta_{\min}} \cdot I\left(\frac{1}{\alpha_{S_{i^*}R}\mathbf{R}_{SR}^{-1}(k,k)(j+1)\beta_{\min}}\right) + \sum_{j=0}^{L-1}(-1)^j\binom{n-1}{j}\frac{\beta_{S_{m^*}R}\beta_{RD}}{\beta_{RD}+j\beta_{\min}} \cdot\left(\begin{array}{c}\frac{1}{\beta_{S_{i^*}R}}I\left(\frac{1}{\alpha_{S_{i^*}R}\mathbf{R}_{SR}^{-1}(k,k)\beta_{S_{i^*}R}}\right)\\ -\frac{1}{(\varsigma+\beta_{R_lD})}\\ \cdot I\left(\frac{1}{\alpha_{S_{i^*}R}\mathbf{R}_{SR}^{-1}(k,k)\left(\beta_{S_{m^*}R}+\beta_{R_lD}+j\beta_{\min}\right)}\right)\end{array}\right) \quad (30)$$

and

$$P_{e,R_{l^*}D}(j) = \sum_{j=0}^{L-1}(-1)^j\binom{n-1}{j}\frac{\beta_{RD}}{(j+1)\beta_{\min}} \cdot I\left(\frac{1}{\alpha_{RD}\mathbf{R}_{RD}^{-1}(k,k)(j+1)\beta_{\min}}\right) + \sum_{j=0}^{L-1}(-1)^j\binom{n-1}{j}\frac{\beta_{S_{i^*}R}\beta_{RD}}{\beta_{RD}+j\beta_{\min}} \cdot\left(\begin{array}{c}\frac{1}{\beta_{RD}}I\left(\frac{1}{\alpha_{RD}\mathbf{R}_{RD}^{-1}(k,k)\beta_{RD}}\right)\\ -\frac{1}{\beta_{S_{i^*}R}+\beta_{R_lD}+j\beta_{\min}}\\ \cdot I\left(\frac{1}{\alpha_{RD}\mathbf{R}_{RD}^{-1}(k,k)\left(\beta_{S_{i^*}R}+\beta_{R_lD}+j\beta_{\min}\right)}\right)\end{array}\right), \quad (31)$$

where $\beta_{S_iR} = \frac{1}{\alpha_{S_iR}\overline{\gamma}_{S_iR}}$, $\beta_{RD} = \frac{1}{\alpha_{RD}\overline{\gamma}_{RD}}$, $\beta_{S_{i^*}R} = \sum_{i=1}^{M}\beta_{S_iR}$ and $\beta_{\min} = \beta_{S_{i^*}R}+\beta_{RD}$.

Proof: See Appendix B. ■

Then plugging (28) and (29) or (30) and (31) into (24) and (25) and then plugging the resulting expressions into (23), we obtain the E2E BER expression in terms of the worst E2E SNR for the JS scheme with asymmetric or semi-symmetric channels and delay settings. The obtained expressions are functions of $\mathbf{R}_{SR_{l^*}}^{-1}(k,k)$ and $\mathbf{R}_{R_{l^*}D}^{-1}(k,k)$. The BER averaging over $\mathbf{R}_{SR_{l^*}}^{-1}(k,k)$ and $\mathbf{R}_{R_{l^*}D}^{-1}(k,k)$ for B blocks can be obtained as $P_e = \frac{\sum_{b=1}^{B}P_e^b}{B}$.

In what follows, we show how to extend the above results to M-QAM. For M-QAM, the conditional error probability of the cth bit can be represented in the summation form by $\sum_{i=0}^{g(c)}u(i,c)\operatorname{erfc}\left(\sqrt{\upsilon_i\gamma}\right)$ according to Eq. (14) in [22]. Since $u(i,c)$ and υ_i are not functions of γ, when we integrate over γ, they can be considered as constants. Then, one can use the above results derived from the expression $u\cdot\operatorname{erfc}\sqrt{\upsilon\gamma}$ and the corresponding summation form for M-QAM to calcutate $P_{e,S_{i^*}R_{l^*}}(j,c)$ and $P_{e,R_{l^*}D}(j,c)$. The results will be a function of both j and c. As a result, (24) and (25) should be replaced by $P_{e,S_{i^*}R_{l^*}} = \frac{1}{NC}\sum_{j=1}^{N}\sum_{c=1}^{C}P_{e,S_{i^*}R_{l^*}}(j,c)$ and $P_{e,R_{l^*}D} = \frac{1}{NC}\sum_{j=1}^{N}\sum_{c=1}^{C}P_{e,R_{l^*}D}(j,c)$.

VI. Achievable Diversity Order

In this section, we derive an upper bound on the JS scheme's error rate to derive its achievable diversity order. For simplicity of presentation, we assume that channel variances are equal to one, that is, $\overline{\gamma}_{S_mR_l} = \overline{\gamma}_{R_lD} = \rho$. But this assumption does not change the diversity order.

Lemma 3: For a cooperative network with M sources and L relays, the JS selection scheme achieves diversity order L.

Proof: The E2E BER expression in (23) can be upper bounded as

$$P_e \le P_{e,S_{i^*}R_{l^*}} + P_{e,R_{l^*}D}, \quad (32)$$

where $P_{e,S_{i^*}R_{l^*}} = \frac{1}{N}\sum_{j=1}^{N}P_{e,S_{i^*}R_{l^*}}(j)$ and $P_{e,R_{l^*}D} = \frac{1}{N}\sum_{j=1}^{N}P_{e,R_{l^*}D}(j)$. The upper bound is obtained by assuming $0 \le P_{e,S_{i^*}R_{l^*}} \ll 1$ and $0 \le P_{e,R_{l^*}D} \ll 1$. $P_{e,S_{i^*}R_{l^*}}(j)$ can be expressed as

$$P_{e,S_{i^*}R_{l^*}}(j) = \int_0^{\infty} u\ \operatorname{erfc}\sqrt{\frac{\upsilon\rho\gamma}{\mathbf{R}_{SR_{l^*}}^{-1}(k,k)}}f_{\gamma_{S_{i^*}R_{l^*}}}(\gamma)\,d\gamma \quad (33)$$

Since $\widetilde{\gamma}_{S_{i^*}R_{l^*}} = \alpha_{S_{i^*}R}\gamma_{S_{i^*}R_{l^*}}$, then $f_{\gamma_{S_{i^*}R_{l^*}}}(\gamma) = \alpha_{S_{i^*}R}f_{\widetilde{\gamma}_{S_{i^*}R_{l^*}}}\left(\alpha_{S_{i^*}R}\gamma\right)$. Using (56), we have

$$\begin{aligned} f_{\gamma_{S_{i*}R_{l*}}}(\gamma) &= \alpha_{S_{i*}R}\beta_{S_{i*}R}e^{-\alpha_{S_i R}\beta_{\min}\gamma}\left(1-e^{-\alpha_{S_{i*}R}\beta_{\min}\gamma}\right)^{L-1} \\ &\quad + \alpha_{S_{i*}R}\beta_{S_{i*}R}e^{-\beta_{S_{i*}R}\gamma} \\ &\quad \cdot \left[\int_0^{\alpha_{S_{i*}R}\gamma} \beta_{RD}e^{-\beta_{RD}\widetilde{\gamma}_{R_l D}}\left(1-e^{-\beta_{\min}\widetilde{\gamma}_{R_l D}}\right)^{L-1} d\widetilde{\gamma}_{R_l D}\right] \end{aligned} \tag{34}$$

Then plugging (34) into (33), we have $P_{e,S_{i*}R_{l*}}(j) = P_1 + P_2$, where

$$\begin{aligned} P_1 = \int_0^{\infty} & uerfc\sqrt{\frac{\upsilon\rho\gamma}{\mathbf{R}_{SR_{l*}}^{-1}(k,k)}} \\ & \cdot \alpha_{S_{i*}R}\beta_{S_{i*}R}e^{-\alpha_{S_i R}\beta_{\min}\gamma} \\ & \cdot \left(1-e^{-\alpha_{S_{i*}R}\beta_{\min}\gamma}\right)^{L-1} d\gamma \end{aligned}$$

and

$$\begin{aligned} P_2 = \int_0^{\infty} & uerfc\sqrt{\frac{\upsilon\rho\gamma}{\mathbf{R}_{SR_{l*}}^{-1}(k,k)}} \\ & \cdot \alpha_{S_{i*}R}\beta_{S_{i*}R}e^{-\beta_{S_{i*}R}\gamma}\int_0^{\alpha_{S_{i*}R}\gamma}\beta_{RD}e^{-\beta_{RD}\widetilde{\gamma}_{R_l D}} \\ & \cdot \left(1-e^{-\beta_{\min}\widetilde{\gamma}_{R_l D}}\right)^{L-1} d\widetilde{\gamma}_{R_l D}d\gamma. \end{aligned}$$

P_1 can be upper bounded as

$$\begin{aligned} P_1 &\leq C_{11}\int_0^{\infty} e^{-\left(\frac{\upsilon\rho}{\mathbf{R}_{SR}^{-1}(k,k)}+\alpha_{S_i R}\beta_{\min}\right)\gamma} \\ &\qquad \cdot \left(1-e^{-\alpha_{S_{i*}R}\beta_{\min}\gamma}\right)^{L-1} d\gamma && (35) \\ &\leq C_{12}\int_0^{\infty} e^{-\left(\frac{\upsilon\rho}{\mathbf{R}_{SR}^{-1}(k,k)}+C_{14}\right)\gamma}\gamma^{L-1}d\gamma && (36) \\ &= \frac{C_{13}}{\left(\frac{\upsilon\rho}{\mathbf{R}_{SR}^{-1}(k,k)}+C_{14}\right)^{L}} && (37) \\ &= o(\rho^{-L}), && (38) \end{aligned}$$

where $C_{11} = 2u\alpha_{S_{i*}R}\beta_{S_{i*}R}$, $C_{12} = 2u\alpha_{S_{i*}R}^{L}\beta_{\min}^{L-1}\beta_{S_{i*}R}$, $C_{13} = 2u\alpha_{S_{i*}R}^{L}\beta_{\min}^{L-1}\beta_{S_{i*}R}(L-1)!$ and $C_{14} = \alpha_{S_i R}\beta_{\min}$. Inequality (35) is followed from $\text{erfc}\sqrt{x} \leq 2e^{-x}$, (36) is followed from $1-e^{-x} \leq x$ and (37) is derived from integration. For the assumption of fix delay values in this work, C_{11}, C_{12}, C_{13}, C_{14} and $\frac{1}{\mathbf{R}_{SR}^{-1}(k,k)}$ are constants. Therefore, (38) shows that P_1 achieves diversity order L.

Similarly, P_2 can be upper bounded as

$$\begin{aligned} P_2 &\leq C_{21}\int_0^{\infty} e^{-\left(\frac{\upsilon\rho}{\mathbf{R}_{SR}^{-1}(k,k)}+\beta_{S_{i*}R}\right)\gamma} \\ &\qquad \cdot \int_0^{\alpha_{S_{i*}R}\gamma} e^{-\beta_{RD}\widetilde{\gamma}_{R_l D}} \\ &\qquad \cdot \left(1-e^{-\beta_{\min}\widetilde{\gamma}_{R_l D}}\right)^{L-1} d\widetilde{\gamma}_{R_l D}d\gamma && (39) \\ &\leq C_{22}\int_0^{\infty} e^{-\left(\frac{\upsilon\rho}{\mathbf{R}_{SR}^{-1}(k,k)}+\beta_{S_{i*}R}\right)\gamma} \\ &\qquad \cdot \int_0^{\alpha_{S_{i*}R}\gamma} e^{-\beta_{RD}\widetilde{\gamma}_{R_l D}}\left(\widetilde{\gamma}_{R_l D}\right)^{L-1} d\widetilde{\gamma}_{R_l D}d\gamma && (40) \\ &\leq C_{23}\int_0^{\infty} e^{-\left(\frac{\upsilon\rho}{\mathbf{R}_{SR}^{-1}(k,k)}+C_{25}\right)\gamma}\gamma^{L}d\gamma && (41) \\ &= \frac{C_{24}}{\left(\frac{\upsilon\rho}{\mathbf{R}_{SR}^{-1}(k,k)}+C_{25}\right)^{L+1}} && (42) \\ &= o(\rho^{-(L+1)}), && (43) \end{aligned}$$

where $C_{21} = u\alpha_{S_{i*}R}\beta_{RD}\beta_{S_{i*}R}$, $C_{22} = u\alpha_{S_{i*}R}\beta_{\min}^{L-1}\beta_{RD}\beta_{S_{m*}R}$, $C_{23} = u\alpha_{S_{i*}R}^{L+1}\beta_{\min}^{L-1}\beta_{RD}\beta_{S_{i*}R}$, $C_{24} = 2u\alpha_{S_{i*}R}^{L}\beta_{\min}^{L-1}\beta_{S_{i*}R}(L)!$ and $C_{25} = \beta_{S_{i*}R} + \beta_{RD}\alpha_{S_{i*}R}$. C_{21}, C_{22}, C_{23}, C_{24} and $\frac{1}{\mathbf{R}_{SR}^{-1}(k,k)}$ are constants for fix delay values. For the purpose of diversity analysis, P_2 can be ignored since its exponent, $L+1$, is larger than L, the exponent of P_1. Since the overall diversity order is dominated by the term P_1, we conclude that $P_{e,S_{i*}R_{l*}}(j)$ achieves diversity L. Following the similar steps from (33) to (43), we can also show that $P_{e,R_{l*}D}(j)$ achieves diversity L. Finally, we conclude that the JS scheme achieves a diversity of L. This completes the proof. ∎

Lemma 4: For a cooperative network with M sources and L relays, the FS selection scheme achieves diversity order L.

Proof: Since the selection choices of the JS scheme are a subset of those of the FS scheme, the performance of the JS scheme is an upper bound of that of the FS scheme. Therefore, we conclude that the FS scheme also achieves full diversity. ∎

Lemma 5: For a cooperative network with M sources and L relays, the IS selection scheme achieves diversity order L.

Proof: The proof is similar to that of the JS scheme and is omitted. ∎

VII. Simulation Results

In this section, we present simulation results to validate the performance analysis of ANC. In addition, we compare the performance of ANC to that of CFNC, and compare the performance of the three proposed relay selection schemes for different delay settings. Throughout this section, we assume all nodes use BPSK, $N = 16$, fixed delays where the symbol time is equally divided to M parts and each part is used as

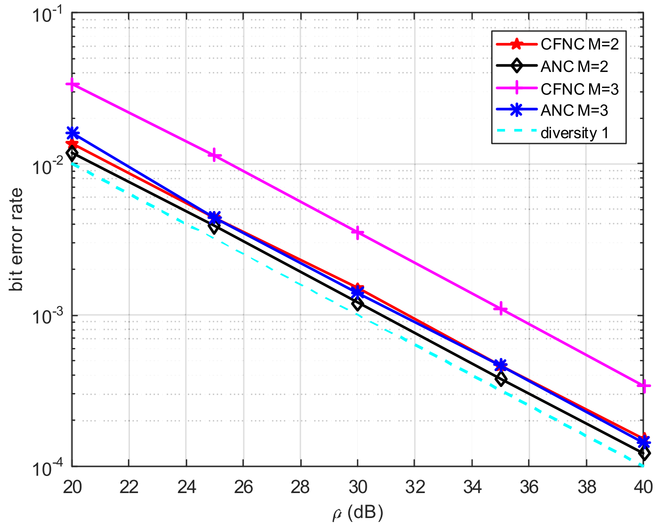

Fig. 4. Bit error rate performance of ANC and CFNC.

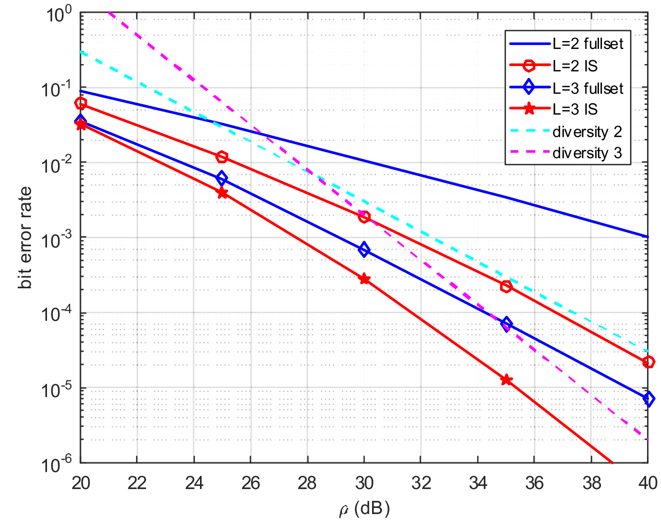

Fig. 6. Bit error rate performance comparison between the IS scheme and fullset selection.

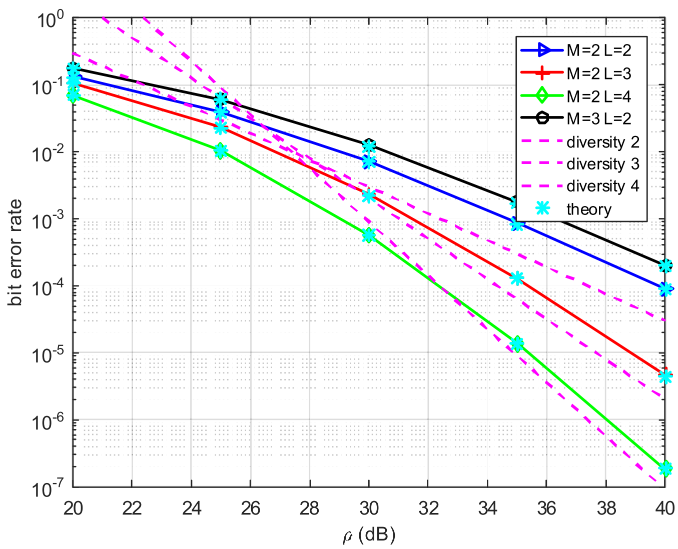

Fig. 5. Bit error rate performance in terms of worst E2E SNR (simulated and theoretical) of JS scheme.

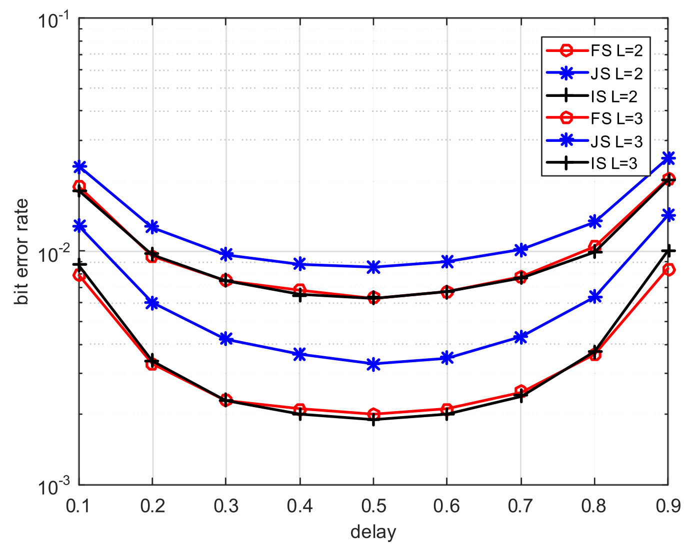

Fig. 7. Bit error rate performance of the FS, JS and IS scheme for semi-symmetric delay setting.

the delay of one source. and all the channel variances are set to 0.5 unless otherwise stated.

In Fig. 4, we show the performance results for ANC with VB decoding and compare them to those of CFNC with ML decoding [8]. We consider the following cases: $M = 2$, $L = 1$ and $M = 3$, $L = 1$. We also include the diversity reference curve which achieves diversity 1. It is shown that both ANC and CFNC achieve diversity 1 which is full diversity. However, ANC provides a better BER performance compared to that of CFNC. The improvement is about 0.5 dB for $M = 2$ and 5 dB for $M = 3$ at a BER of 10^{-3}.

In Fig. 5, we present the BER performance of the JS scheme in terms of the worst E2E SNR for the following cases: $M = 2, L = 2$; $M = 2, L = 3$ and $M = 3, L = 2$. The channel variances for the first hop and the second hop are 0.5 and 0.33, respectively. For $M = 2$, the delays are set to 0.1 and 0.5 at the relays and destination, respectively. For $M = 3$, the delay values are [0.1 0.4] and [0.33 0.66] at the relays and destination, respectively. We also include the reference curves which achieve diversity 2 and 3. It is shown that the diversity is 2 for $L = 2$ and 3 for $L = 3$, which is full diversity. In addition, the performance degrades by increasing M from 2 to 3, while the diversity order is the same. Finally, we observe a match between theory and simulations, which validates the derived BER expression for the JS scheme in Lemma 1.

In Fig. 6, we compare the performance of the IS scheme to that of the fullset selection in [23]. We consider $M = 2, L = 2$ and $M = 2, L = 3$. We also include the reference diversity curves which achieve diversity 2 and 3. By comparing the slopes of the curves, the diversity order of the IS scheme is 2 for $L = 2$ and 3 for $L = 3$, which is full diversity. However, the fullset selection in [23] only achieves diversity 1 for $L = 2$ and 2 for $L = 3$.

In Fig. 7, we compare the three proposed relay selection schemes with semi-symmetric delays settings for $M = 2$ and

$L = 2, 3$. In particular, the delay values at the destination are fixed to 0.5. The delay values at the relays increase from 0.1 to 0.9. The value of ρ is fixed at 25dB. First, the performance of the IS scheme is almost the same to that of the FS scheme for the whole range, which is better than that of the JS scheme. Second, we observe that the performance of the three schemes are varying with the delays. Finally, we find that the performance gap between the JS scheme and the other two schemes increases when L increases from 2 to 3.

VIII. Conclusions

In this work, we proposed a novel ANC scheme for multiuser cooperative networks and illustrated its decoding strategy. From our study, the advantages of our proposed ANC scheme over CFNC can be summarized as follows: (1) our proposed ANC scheme with VB decoding has a better E2E BER performance compared to that of the CFNC with ML decoding; (2) our proposed ANC can be decoded by linear decoding such as ZF, while CFNC can only be decoded by ML decoding; and (3) ANC has almost the same spectral efficiency as that of CFNC. Since the delays are within one symbol, the spectral loss caused by the ANC delays is negligible. We also studied the relay selection schemes in conjunction with the proposed ANC. In particular, we proposed three relay selection schemes, FS, JS and IS, with different performance and complexity. In this work, we only study the case of perfect CSI due to the page limitation. Analyzing the effects of noisy channel estimation, like the study in [25], is an interesting future work.

Appendix

A. Proof of Lemma 1

After some manipulations, (20) can be rewritten as

$$\widetilde{\gamma}_{l,\ \min} = \min(\widetilde{\gamma}_{S_{i^*}R_l}, \widetilde{\gamma}_{R_lD}), \tag{44}$$

where $\widetilde{\gamma}_{S_{i^*}R_l} = \min(\widetilde{\gamma}_{S_1R_l}, \widetilde{\gamma}_{S_2R_l} \ldots, \widetilde{\gamma}_{S_MR_l})$ and $\widetilde{\gamma}_{R_lD} = \min(\widetilde{\gamma}_{S_1R_lD}, \widetilde{\gamma}_{S_2R_lD} \ldots, \widetilde{\gamma}_{S_MR_lD})$.

Based on (1) and (15), the pdf of $\widetilde{\gamma}_{S_iR_l}$ is derived as

$$f_{\widetilde{\gamma}_{S_iR_l}}(\gamma) = \beta_{S_iR_l} e^{-\beta_{S_iR_l}\gamma}, \tag{45}$$

where $\beta_{S_iR_l} = \frac{1}{\alpha_{S_iR_l}\overline{\gamma}_{S_iR_l}}$. Similarly, the pdf of $\widetilde{\gamma}_{R_lD}$ is

$$f_{\widetilde{\gamma}_{R_lD}}(\gamma) = \beta_{R_lD} e^{-\beta_{R_lD}\gamma}, \tag{46}$$

where $\beta_{R_lD} = \min(\frac{1}{\alpha_{S_1R_lD}\overline{\gamma}_{R_lD}}, \frac{1}{\alpha_{S_2R_lD}\overline{\gamma}_{R_lD}} \ldots, \frac{1}{\alpha_{S_MR_lD}\overline{\gamma}_{R_lD}})$.

Since $\widetilde{\gamma}_{S_{i^*}R_l} = \min(\widetilde{\gamma}_{S_1R_l}, \widetilde{\gamma}_{S_2R_l} \ldots, \widetilde{\gamma}_{S_MR_l})$, the pdf of $\widetilde{\gamma}_{S_{i^*}R_l}$ can be derived as

$$f_{\widetilde{\gamma}_{S_{i^*}R_l}}(\gamma) = \beta_{S_{i^*}R_l} e^{-\beta_{S_{i^*}R_l}\gamma}, \tag{47}$$

where $\beta_{S_{i^*}R_l} = \sum_{i=1}^{M} \beta_{S_iR_l}$.

Since $\widetilde{\gamma}_{l,\ \min} = \min(\widetilde{\gamma}_{S_{i^*}R_l}, \widetilde{\gamma}_{R_lD})$, the pdf of $\widetilde{\gamma}_{l,\ \min}$ is

$$f_{\widetilde{\gamma}_{l,\ \min}}(\gamma) = \beta_{l,\min} e^{-\beta_{l,\min}\gamma}, \tag{48}$$

where $\beta_{l,\min} = \beta_{S_{i^*}R_l} + \beta_{R_lD}$.

With R_{l^*} as the selected relay, the CDF of $\widetilde{\gamma}_{S_{i^*}R_{l^*}}$ can be expressed as

$$F_{\widetilde{\gamma}_{S_{i^*}R_{l^*}}}(\gamma) = P_r\left(\widetilde{\gamma}_{S_{i^*}R_{l^*}} \le \gamma\right) = \sum_{l=1}^{L} P_r\left(\widetilde{\gamma}_{S_{i^*}R_{l^*}} \le \gamma,\ l^* = l\right), \tag{49}$$

where

$$\begin{aligned} &P_r\left(\widetilde{\gamma}_{S_{i^*}R_l} \le \gamma,\ l^* = l\right) \\ &= P_r\left(\widetilde{\gamma}_{S_{i^*}R_l} \le \gamma,\ \min(\widetilde{\gamma}_{S_{i^*}R_l},\ \widetilde{\gamma}_{R_lD}) > \widetilde{\gamma}_{\upsilon,\ \min}\,(l \ne \upsilon)\right) \\ &= P_{r1} + P_{r2}, \end{aligned} \tag{50}$$

where

$$P_{r1} = P_r\begin{pmatrix} \widetilde{\gamma}_{S_{i^*}R_l} \le \gamma, \widetilde{\gamma}_{S_{i^*}R_l} > \widetilde{\gamma}_{R_lD}, \\ \widetilde{\gamma}_{R_lD} > \widetilde{\gamma}_{\upsilon,\ \min}\,(l \ne \upsilon) \end{pmatrix}$$

and

$$P_{r2} = P_r\begin{pmatrix} \widetilde{\gamma}_{S_{i^*}R_l} \le \gamma, \widetilde{\gamma}_{S_{i^*}R_l} < \widetilde{\gamma}_{R_lD}, \\ \widetilde{\gamma}_{S_{i^*}R_l} > \widetilde{\gamma}_{\upsilon,\ \min}\,(l \ne \upsilon) \end{pmatrix}.$$

Then using the results of (46), (47), and (48), the first term of (50) can be calculated as

$$\begin{aligned} P_{r1} &= \int_0^{\gamma} \beta_{S_{i^*}R_l} e^{-\beta_{S_{i^*}R_l}\widetilde{\gamma}_{S_{i^*}R_l}} \\ &\quad \cdot \left[\begin{array}{c} \int_0^{\widetilde{\gamma}_{S_{i^*}R_l}} \beta_{R_lD} e^{-\beta_{R_lD}\widetilde{\gamma}_{R_lD}} \\ \cdot \prod_{\substack{\upsilon=1 \\ \upsilon \ne l}}^{L} \left(1 - e^{-\beta_{\upsilon,\min}\widetilde{\gamma}_{R_lD}}\right) d\widetilde{\gamma}_{R_lD} \end{array} \right] d\widetilde{\gamma}_{S_{i^*}R_l} \\ &= \sum_{\varsigma \in T_j^n} \frac{\mathrm{sign}(\varsigma)\beta_{S_{i^*}R_l}\beta_{R_lD}}{\varsigma + \beta_{R_lD}} \int_0^{\gamma} e^{-\beta_{S_{i^*}R_l}\widetilde{\gamma}_{S_{i^*}R_l}} \\ &\quad \cdot \left(1 - e^{-(\varsigma + \beta_{R_lD})\widetilde{\gamma}_{S_{i^*}R_l}}\right) d\widetilde{\gamma}_{S_{i^*}R_l}, \end{aligned} \tag{51}$$

where the set T_j^n is obtained by expanding the product $\prod_{\substack{\upsilon=1 \\ \upsilon \ne l}}^{L} \left(1 - e^{-\beta_{\upsilon,\min}\widetilde{\gamma}_{R_lD}}\right)$ and then taking the logarithm of each term, and $\mathrm{sign}(\varsigma)$ is the sign function [24]. Similarly, the second term of (50) can be expressed as

$$\begin{aligned} P_{r2} &= \beta_{S_{i^*}R_l} \int_0^{\gamma} e^{-\beta_{l,\min}\widetilde{\gamma}_{S_{i^*}R_l}} \\ &\quad \cdot \prod_{\substack{\upsilon=1 \\ \upsilon \ne l}}^{L} \left(1 - e^{-\beta_{\upsilon,\min}\widetilde{\gamma}_{S_{i^*}R_l}}\right) d\widetilde{\gamma}_{S_{i^*}R_l} \\ &= \sum_{\varsigma \in T_j^n} \mathrm{sign}(\varsigma)\beta_{S_{i^*}R_l} \int_0^{\gamma} e^{-(\beta_{l,\min} + \varsigma)\widetilde{\gamma}_{S_{i^*}R_l}} d\widetilde{\gamma}_{S_{i^*}R_l}. \end{aligned} \tag{52}$$

Then plugging (51) and (52) into (50) and taking the derivative, we have the pdf of $f_{\widetilde{\gamma}_{S_{i^*}R_{l^*}}}(\gamma)$. Since

$\widehat{\gamma}_{S_{i^*}R_{l^*}}(j) = \frac{1}{\alpha_{S_{i^*}R_{l^*}}\mathbf{R}^{-1}_{SR_{l^*}}(k,k)}\widetilde{\gamma}_{S_{i^*}R_{l^*}}$, the pdf of $\widehat{\gamma}_{S_{i^*}R_{l^*}}(j)$ can be obtained as

$$f_{\widehat{\gamma}_{S_{i^*}R_{l^*}(j)}}(\gamma) = \alpha_{S_{i^*}R_{l^*}}\mathbf{R}^{-1}_{SR_{l^*}}(k,k) \cdot f_{\widetilde{\gamma}_{S_{i^*}R_{l^*}}}\left(\alpha_{S_{i^*}R_{l^*}}\mathbf{R}^{-1}_{SR_{l^*}}(k,k)\gamma\right). \quad (53)$$

Finally, we obtain the pdf of $\widehat{\gamma}_{S_{i^*}R_{l^*}}(j)$

$$\begin{aligned} f_{\widehat{\gamma}_{S_{i^*}R_{l^*}}(j)}(\gamma) &= \alpha_{S_{i^*}R_{l^*}}\mathbf{R}^{-1}_{SR_{l^*}}(k,k) \\ &\cdot \left[\begin{array}{c} \sum_{l=1}^{L}\sum_{\varsigma\in T_j^n}\mathrm{sign}(\varsigma)\beta_{S_{i^*}R_l} \\ e^{-\left(\beta_{\widetilde{\gamma}_{l,\ \min}}+\varsigma\right)\alpha_{S_{i^*}R_{l^*}}\mathbf{R}^{-1}_{SR_{l^*}}(k,k)\gamma} \\ +\sum_{l=1}^{L}\sum_{\varsigma\in T_j^n}\frac{\mathrm{sign}(\varsigma)\beta_{S_{i^*}R_l}\beta_{R_lD}e^{-\beta_{S_{i^*}R_l}\mathbf{R}^{-1}_{SR_{l^*}}(k,k)\gamma}}{\varsigma+\beta_{R_lD}} \\ \alpha_{S_{i^*}R_{l^*}}\left(1-e^{-(\varsigma+\beta_{R_lD})\alpha_{S_{i^*}R_{l^*}}\mathbf{R}^{-1}_{SR_{l^*}}(k,k)\gamma}\right) \end{array}\right], \end{aligned} \quad (54)$$

Following the same steps from (49)-(52), we can obtain the pdf of $f_{\widehat{\gamma}_{R_{l^*}D}(j)}(\gamma)$ as

$$\begin{aligned} f_{\widehat{\gamma}_{R_{l^*}D}(j)}(\gamma) &= \alpha_{R_{l^*}D}\mathbf{R}^{-1}_{R_{l^*}D}(k,k) \\ &\cdot \left[\begin{array}{c} \sum_{l=1}^{L}\sum_{\varsigma\in T_j^n}\mathrm{sign}(\varsigma)\beta_{R_lD} \\ \cdot e^{-\left(\beta_{\widetilde{\gamma}_{l,\ \min}}+\varsigma\right)\alpha_{R_{l^*}D}\mathbf{R}^{-1}_{R_{l^*}D}(k,k)\gamma} \\ +\sum_{l=1}^{L}\sum_{\varsigma\in T_j^n}\frac{\mathrm{sign}(\varsigma)\beta_{S_{i^*}R_l}\beta_{R_lD}e^{-\alpha_{R_{l^*}D}\mathbf{R}^{-1}_{R_{l^*}D}(k,k)\gamma}}{\varsigma+\beta_{S_{i^*}R_l}} \\ \cdot\alpha_{R_{l^*}D}\left(1-e^{-(\varsigma+\beta_{R_lD})\alpha_{R_{l^*}D}\mathbf{R}^{-1}_{R_{l^*}D}(k,k)\gamma}\right) \end{array}\right], \end{aligned} \quad (55)$$

Then plugging (54) and (55) into (26) and (27) and carrying out the integration, we have the results of Lemma 1. This completes the proof.

B. Proof of Lemma 2

For semi-symmetric channels and delay setting, we have $\beta_{S_iR} = \frac{1}{\alpha_{S_iR}\overline{\gamma}_{S_iR}}$, $\beta_{RD} = \frac{1}{\alpha_{RD}\overline{\gamma}_{RD}}$, $\beta_{S_{i^*}R} = \sum_{i=1}^{M}\beta_{S_iR}$ and $\beta_{\min} = \beta_{S_{i^*}R} + \beta_{RD}$. Then following the similar steps from (45) to (54), we have the pdf of $f_{\widetilde{\gamma}_{S_{i^*}R_{l^*}}}(\gamma)$ and as

$$\begin{aligned} f_{\widetilde{\gamma}_{S_{i^*}R_{l^*}}}(\gamma) &= \beta_{S_{i^*}R}e^{-\beta_{\min}\gamma}\left(1-e^{-\beta_{\min}\gamma}\right)^{L-1} \\ &\quad+\beta_{S_{i^*}R}e^{-\beta_{S_{i^*}R}\gamma}\int_0^{\gamma}\beta_{RD}e^{-\beta_{RD}\widetilde{\gamma}_{R_lD}} \\ &\quad\left(1-e^{-\beta_{\min}\widetilde{\gamma}_{R_lD}}\right)^{L-1}d\widetilde{\gamma}_{R_lD} \end{aligned} \quad (56)$$

$$\begin{aligned} &= \sum_{j=0}^{L-1}(-1)^j\binom{n-1}{j}\beta_{S_{i^*}R_l}e^{-(j+1)\beta_{\min}\gamma} \\ &\quad+\sum_{j=0}^{L-1}(-1)^j\binom{n-1}{j} \\ &\quad\cdot\frac{\beta_{S_{i^*}R}\beta_{RD}e^{-\beta_{S_{i^*}R}\gamma}\left(1-e^{-(\beta_{R_lD}+j\beta_{\min})\gamma}\right)}{\beta_{RD}+j\beta_{\min}} \end{aligned} \quad (57)$$

We can also obtain the pdf of $f_{\widetilde{\gamma}_{R_{l^*}D}}(\gamma)$ as

$$\begin{aligned} f_{\widetilde{\gamma}_{R_{l^*}D}}(\gamma) &= \sum_{j=0}^{L-1}(-1)^j\binom{n-1}{j}\beta_{RD}e^{-(j+1)\beta_{\min}\gamma} \\ &\quad+\sum_{j=0}^{L-1}(-1)^j\binom{n-1}{j}\frac{\beta_{S_{i^*}R}\beta_{RD}}{\beta_{S_{i^*}R}+j\beta_{\min}} \\ &\quad\cdot e^{-\beta_{RD}\gamma}\left(1-e^{-\left(\beta_{S_{i^*}R}+j\beta_{\min}\right)\gamma}\right) \end{aligned} \quad (58)$$

Then using (53), we can have $f_{\widehat{\gamma}_{S_{i^*}R_{l^*}}(j)}(\gamma)$ and $f_{\widehat{\gamma}_{R_{l^*}D}(j)}(\gamma)$. Finally plugging $f_{\widehat{\gamma}_{S_{i^*}R_{l^*}}(j)}(\gamma)$ and $f_{\widehat{\gamma}_{R_{l^*}D}(j)}(\gamma)$ into (26) and (27) and carrying out the integration, we have the results of Lemma 2. This completes the proof.

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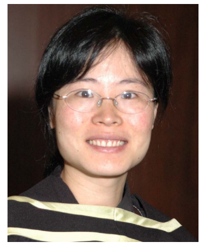

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