Mini Project: Monte Carlo Simulation for Financial Risk Assessment

Objective

This mini-project will guide students through the implementation of a Monte Carlo simulation to estimate financial risk using advanced random number generation techniques. By completing this project, students will apply pseudo-random and quasi-random numbers, variance reduction methods, and numerical approximations to assess the Value at Risk (VaR) of a portfolio.

Problem Statement

You are a quantitative analyst working for a financial institution. Your task is to estimate the **Value at Risk (VaR)** of a stock portfolio using Monte Carlo simulations. The risk estimation should incorporate techniques such as:

- Pseudo-random and quasi-random number generation
- Antithetic variance reduction
- Taylor series approximations
- Convergence analysis

Steps to Complete the Project

Step 1: Data Initialization

1. Simulate the daily return of a stock using a normal distribution with a given mean and standard deviation.
2. Implement a pseudo-random number generator (PRNG) to generate these stock returns.
3. Compare the output with a quasi-random number generator (QRNG) using a Sobol sequence.

Step 2: Monte Carlo Simulation for Stock Price Evolution

1. Simulate stock price paths using the **Geometric Brownian Motion (GBM)** model:

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right)$$

where W_t is a Wiener process.

2. Generate multiple stock price paths using both PRNG and QRNG methods.
3. Visualize the simulated stock price paths.

Step 3: Estimating Value at Risk (VaR)

1. Compute portfolio returns from the simulated stock prices.
2. Estimate the **95% and 99% VaR** using both **historical simulation** and **Monte Carlo methods**.
3. Implement **antithetic variance reduction** to improve estimation accuracy.
4. Compare VaR estimates using pseudo-random and quasi-random methods.

Step 4: Taylor Series Approximation for Risk Adjustment

1. Use a **second-order Taylor series expansion** to approximate the portfolio's risk exposure.
2. Analyze the effect of approximation on risk estimation.
3. Compare results with the direct computation approach.

Step 5: Convergence Analysis and Final Report

1. Study the convergence behavior of Monte Carlo estimates by increasing the number of simulations.
2. Plot error vs. number of simulations to assess efficiency.
3. Write a summary report discussing findings, methodology, and computational performance.

Rubric (Total: 100 Points)

Criteria	Excellent (20)	Good (15)	Fair (10)	Poor (5)
Step 1: Data Initialization	PRNG and QRNG correctly implemented and analyzed	Minor issues in implementation	Incomplete or incorrect implementation	Little to no effort
Step 2: Monte Carlo Stock Simulation	GBM correctly simulated and visualized	Small inaccuracies	Major issues in simulation	No proper implementation
Step 3: VaR Estimation	VaR correctly computed with all methods	Minor computational errors	Missing one key method	Poor implementation

Criteria	Excellent (20)	Good (15)	Fair (10)	Poor (5)
Step 4: Taylor Series Approximation	Taylor expansion correctly used and compared	Some approximation issues	Incorrect implementation	Not attempted
Step 5: Convergence Analysis & Report	Convergence studied with clear insights	Minor inconsistencies in analysis	Weak discussion	No analysis provided

Submission Guidelines

- Submit a Jupyter Notebook (.ipynb) with well-commented code and visualizations.
- Include a short report summarizing findings and observations.
- Ensure reproducibility by setting random seeds where applicable.

Bonus Challenge

- Implement a risk-adjusted portfolio optimization strategy using Monte Carlo results.
- Compare performance with standard VaR estimation.

This project provides a structured, hands-on experience in financial risk modeling using Monte Carlo simulations. It integrates multiple numerical techniques, ensuring practical application and computational efficiency.