$$\int f(x)dx = F(x) + C, \quad \text{kur } \left(F(x)+C\right)^{/} = f(x)$$

$$\int \left[f_1(x) \pm f_2(x) \right] dx = \int f_1(x)\,dx \pm \int f_2(x)\,dx$$

Integrēšanas pamatīpašības

$$\left(\int f(x)\,dx\right)^{/} = f(x)$$

$$d\left(\int f(x)\,dx\right) = f(x)\,dx$$

$$\int f^{/}(x)\,dx = f(x) + C$$

$$\int df(x) = f(x) + C$$

$$\int Cf(x)dx = C\int f(x)dx$$

$$dx = d(x+a) \text{ ,ja } a - const$$

$$dx = \frac{1}{a} d(ax) \text{ ,ja } a - const$$

$$f^{/}(x)\,dx = d\left[f(x) \right]$$

$$\int 0dx = C \qquad \int 1dx = x + C$$

$$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$$

Integrēšanas pamatformulas

$$\int u^a du = \frac{u^{a+1}}{a+1} + C\,(a \neq -1)$$

$$\int \cos u du = \sin u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \frac{du}{\cos^2 u} = tgu + C$$

$$\int \frac{du}{\sin^2 u} = -ctgu + C$$

$$\int \frac{du}{u} = \ln|u| + C\,(u \neq 0)$$

$$\int a^u du = \frac{a^u}{\ln a} + C\,(0 < a \neq 1)$$

$$\int e^u du = e^u + C$$

$$\int chudu = shu + C$$

$$\int shudu = chu + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \begin{cases} \arcsin \dfrac{u}{a} + C \\ -\arccos \dfrac{u}{a} + C \end{cases}$$

$$\int \frac{du}{a^2 + u^2} = \begin{cases} \dfrac{1}{a} arctg \dfrac{u}{a} + C \\ -\dfrac{1}{a} arcctg \dfrac{u}{a} + C \end{cases}$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln\left|\frac{a+u}{a-u}\right| + C$$

$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left|u + \sqrt{u^2 \pm a^2}\right| + C$$

$$\int \frac{du}{\cos u} = \ln\left|tg\left(\frac{u}{2} + \frac{\pi}{4}\right)\right| + C$$

$$\int \frac{du}{\sin u} = \ln\left|tg\frac{u}{2}\right| + C$$

Atvasināšanas pamatformulas

$$\left(u^a\right)^{/} = au^{a-1}u'$$

$$(\sin u)' = \cos u \cdot u'$$

$$(\cos u)' = -\sin u \cdot u'$$

$$(tgu)' = \frac{1}{\cos^2 u} u'$$

$$(ctgu)' = -\frac{1}{\sin^2 u} u'$$

$$(\ln u)' = \frac{1}{u} u'$$

$$\left(a^u\right)' = a^u \ln a \cdot u'$$

$$\left(e^u\right)' = e^u u'$$

$$(shu)' = chu \cdot u'$$

$$(chu)' = shu \cdot u'$$

$$\begin{cases} (\arcsin u)' = \dfrac{1}{\sqrt{1-u^2}} u' \\ (\arccos u)' = -\dfrac{1}{\sqrt{1-u^2}} u' \end{cases}$$

$$\begin{cases} (arctgu)' = \dfrac{1}{1+u^2} u' \\ (arcctgu)' = -\dfrac{1}{1+u^2} u' \end{cases}$$

Trigonometriskas substitūcija

$$\int R\left(x, \sqrt{a^2 - x^2}\right) dx$$
$$x = a\cos t \text{ vai } x = a\sin t$$

$$\int R\left(x, \sqrt{a^2 + x^2}\right) dx$$
$$x = a \cdot tgt$$

$$\int R\left(x, \sqrt{x^2 - a^2}\right) dx$$
$$x = \frac{a}{\cos t} \text{ vai } x = \frac{a}{\sin t}$$

Iracionālu funkciju integrēšana

$$\int R\left(x, \sqrt[n]{\frac{ax+b}{cx+d}}\right) dx$$
$$t^n = \frac{ax+b}{cx+d}$$

$$\int R\left(x, \sqrt[n]{\frac{ax+b}{cx+d}}, \sqrt[m]{\frac{ax+b}{cx+d}}\right) dx$$
$$t^p = \frac{ax+b}{cx+b}$$

$$\int \frac{dx}{(Mx+N)\sqrt{ax^2+bx+c}}$$
$$(Mx+N) = \frac{1}{t}$$

Parciālās integrēšanas formula : $\int u dv = uv - \int v du$

$$\left\{\begin{array}{l}\int P_n(x)e^{\alpha x}dx \\ \int P_n(x)\sin(\alpha x)dx \\ \int P_n(x)\cos(\alpha x)dx\end{array}\right. \Rightarrow \quad u = P_n(x)$$

$$\alpha \in \mathrm{R}$$

$$\left\{\begin{array}{l}\int x^{\alpha}\ln x dx \\ \int x^{\alpha}\arcsin x dx \\ \int x^{\alpha} arctgx dx\end{array}\right. \Rightarrow u = \left\{\begin{array}{l}\ln x \\ \arcsin x \\ arctgx\end{array}\right.$$

$\left\{\begin{array}{l}\int e^{\alpha x}\sin\beta x dx \\ \int e^{\alpha x}\cos\beta x dx\end{array}\right.$ un $\left\{\begin{array}{l}\int \sin(\ln x)dx \\ \int \cos(\ln x)dx\end{array}\right. \Rightarrow$ konstruēsim lineāru vienādojumu attiecībā pret doto integrāli.

Daļveida racionālu funkciju integrēšana :

$$\int \frac{Q_m(x)}{P_n(x)}dx \quad \left\{\begin{array}{l} m < n \Rightarrow (\mathrm{I}) \text{ īsta dalveida racionāla funkcija} \\ m \geq n \Rightarrow (\mathrm{II}) \text{ neīsta dalveida racionāla funkcija}\end{array}\right.$$

(I): 1. $m = 0, n = 2 \Rightarrow$

saucēju papildināt līdz pilnajām kvadrātam

2. $m = 1, n = 2 \Rightarrow$

aprēķināt saucēja atvasinājumu

3. a.- jāsadala polinoms $P_n(x)$ reizinātājos

b.- jāsadala elementārdaļās ==========⇒

c.- jāaprēķina koeficienti

d.- jāintegrē katra veida elementārdaļas

(II): Jāizdala polinomi ➔ (I)

$$a \Rightarrow (x-a) \Rightarrow \frac{A}{x-a}$$

$$a^k \Rightarrow (x-a)^k \Rightarrow \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \ldots + \frac{A_k}{(x-a)^k}$$

$$(D<0) \Rightarrow x^2 + px + g \Rightarrow \frac{Ax+B}{x^2+px+g}$$

Trigonometrisko funkciju integrēšana.

1. $\int \sin\alpha x \sin\beta x dx \qquad \int \sin\alpha x \cos\beta x dx \qquad \int \cos\alpha x \cos\beta x dx$

$$\sin x \cdot \sin y = -0.5(\cos(x+y) - \cos(x-y)) \qquad \cos x \cdot \cos y = 0.5(\cos(x+y) + \cos(x-y))$$

$$\sin x \cdot \cos y = 0.5(\sin(x+y) + \sin(x-y))$$

2. $\int \sin^m x \cos^n x dx$ **a.** (m – nepāra) ➔ $t = \cos x$ **b.** (n – nepāra) ➔ $t = \sin x$

c. (m un n – pāra m≥0 , n≥0) ➔ $\sin^2 x = \frac{1-\cos 2x}{2}$ $\cos^2 x = \frac{1+\cos 2x}{2}$

d. (m un n – pāra m<0 vai n<0) ➔ $t = tgx$ $\sin^2 x = \frac{tg^2 x}{1+tg^2 x}$ $\cos^2 x = \frac{1}{1+tg^2 x}$

3. $\int R(tgx)dx$ ➔ $t = tgx$

4. $\int R(\sin x, \cos x)dx$ **a.** $R(\sin x, \cos x)$ ir nepāra attiecībā pret $\sin x$ ➔ $t = \cos x$

b. $R(\sin x, \cos x)$ ir nepāra attiecībā pret $\cos x$ ➔ $t = \sin x$

c. $R(\sin x, \cos x)$ ir pāra attiecībā pret $\sin x$ un $\cos x$ ➔ $t = tgx$

d. $R(\sin x, \cos x)$ nav pāra , nav nepāra ➔ $t = tg\frac{x}{2}$

$$\sin x = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}$$

$$\int \sqrt{a^2-x^2}dx = \frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2\arcsin\frac{x}{a}\right) + C$$

$$\int \sqrt{x^2+a^2}dx = \frac{1}{2}\left(x\sqrt{x^2+a^2} + a^2\ln\left|x+\sqrt{x^2+a^2}\right|\right) + C$$