



PINQ

FRANK MC SHERRY

What are we trying to solve?

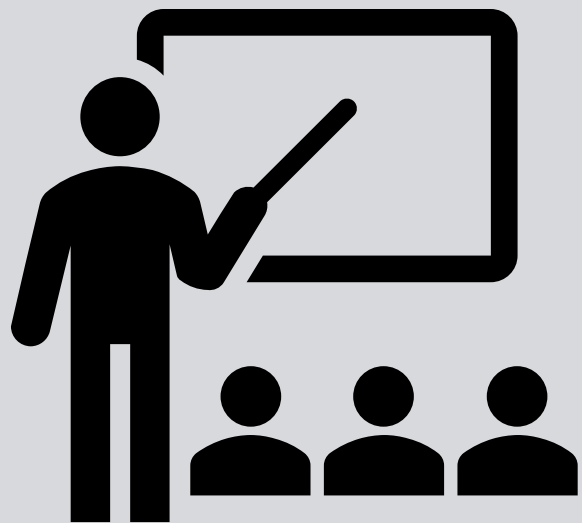
Why Differential Privacy?

What is Differential Privacy?

What is PINQ?

What functionality does PINQ offer?

How does PINQ operate?



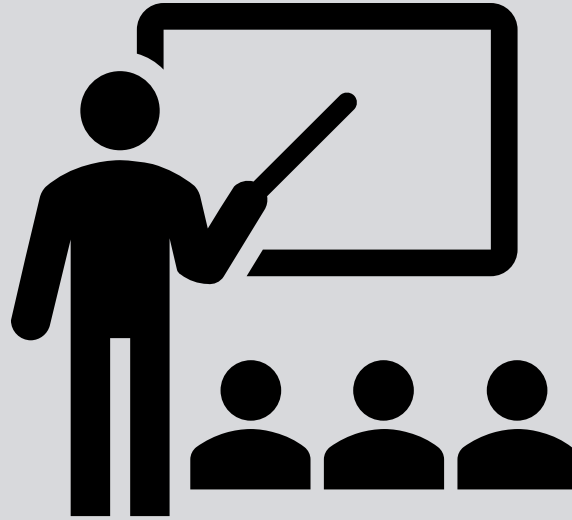


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- Release aggregate statistics



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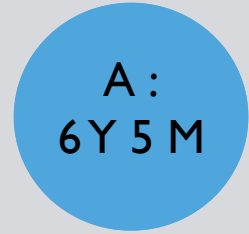
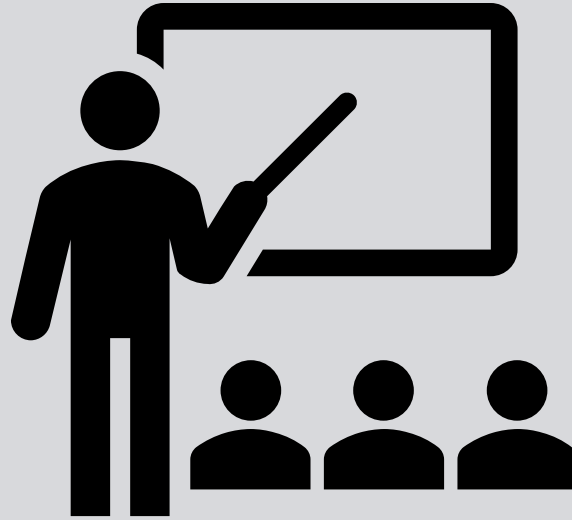
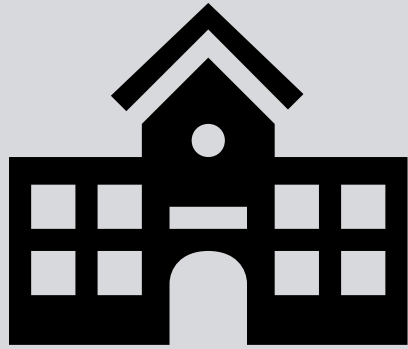




AVERAGE AGE

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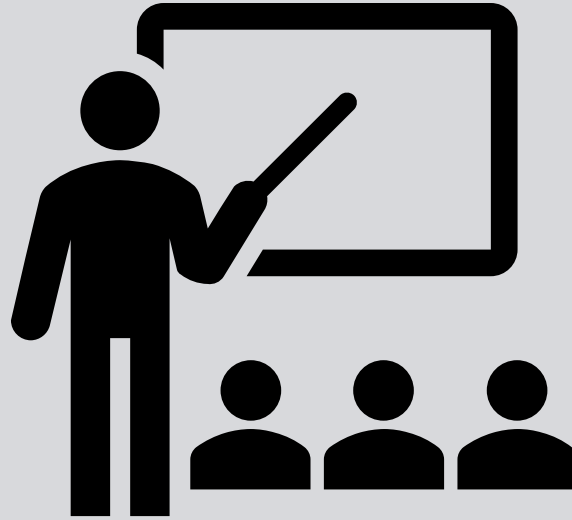
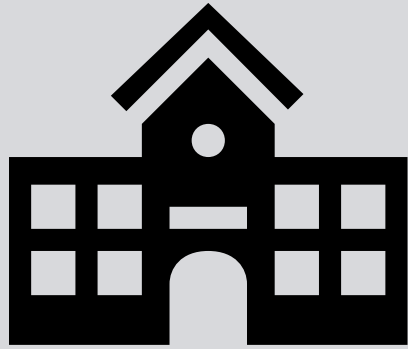




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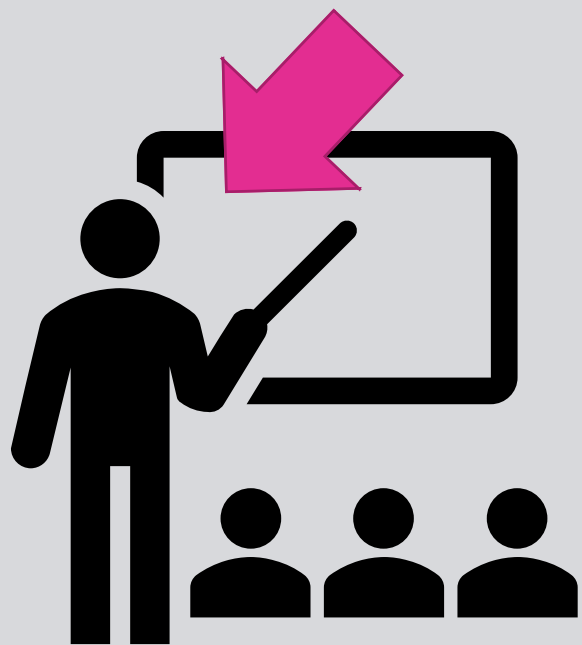
A:
6Y 5 M

AVERAGE AGE

B:
12Y 2 M

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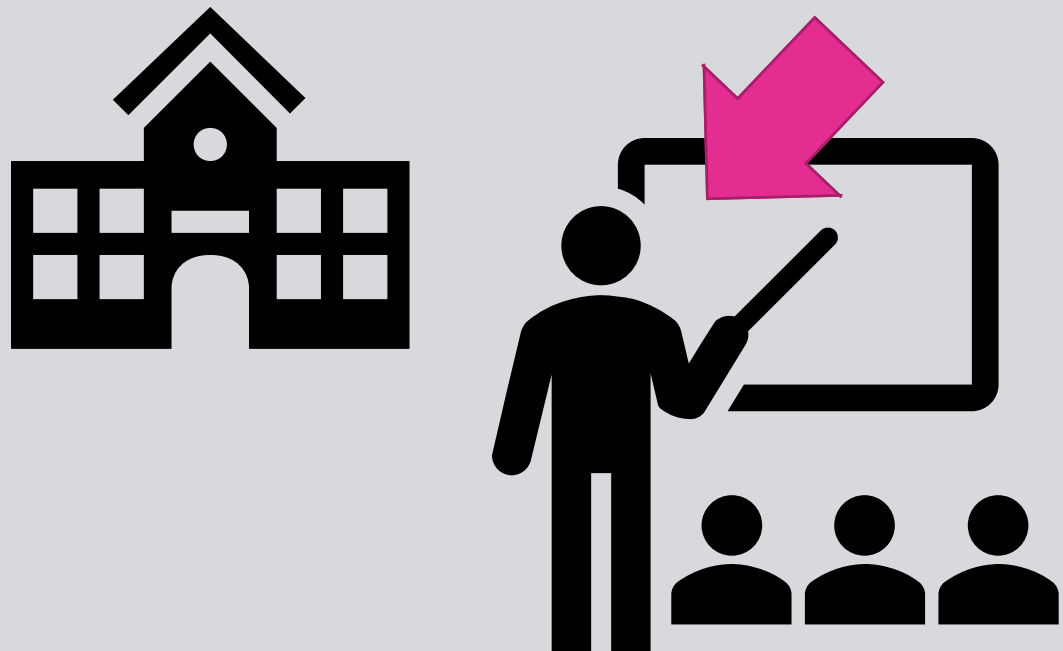
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Given that we need to reveal something about the dataset...

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ACCURACY

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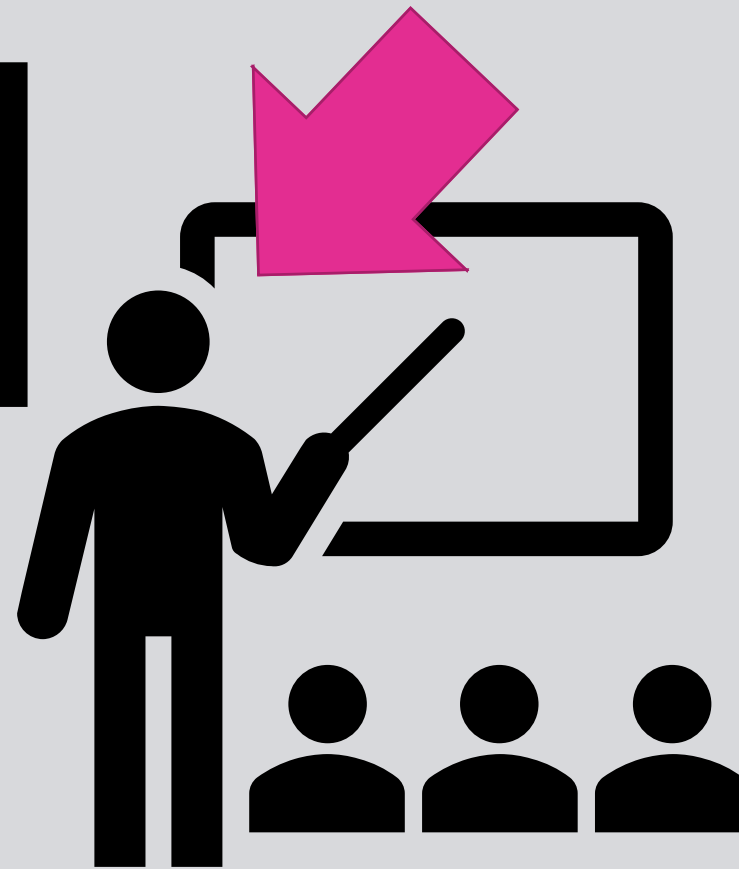
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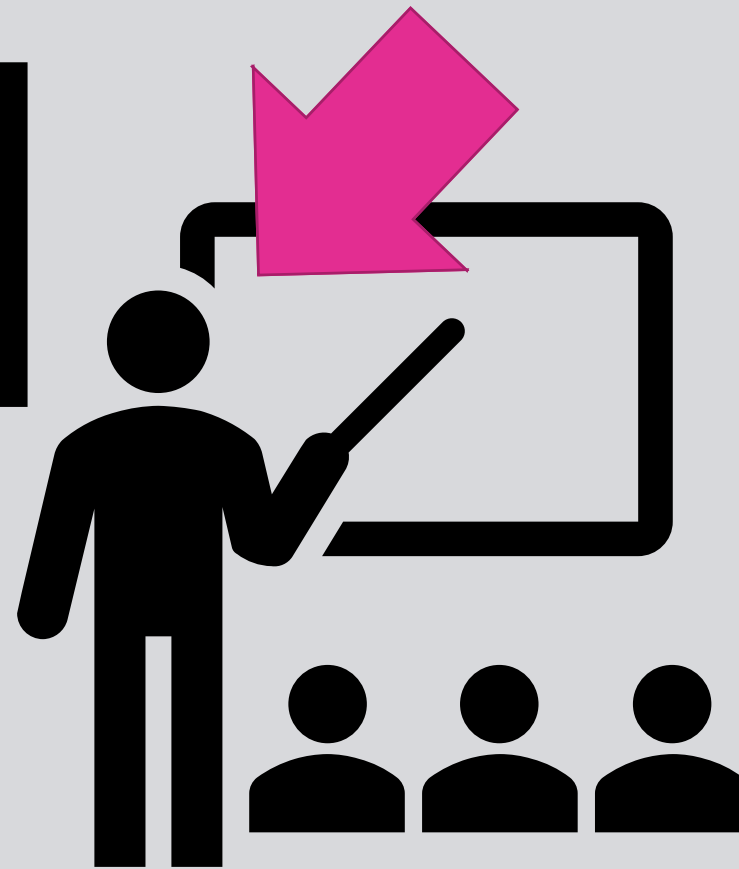
- ✓ Want 'reasonable' numbers
- ✓ Be able to give some 'privacy guarantee' to a user – no deanonymization



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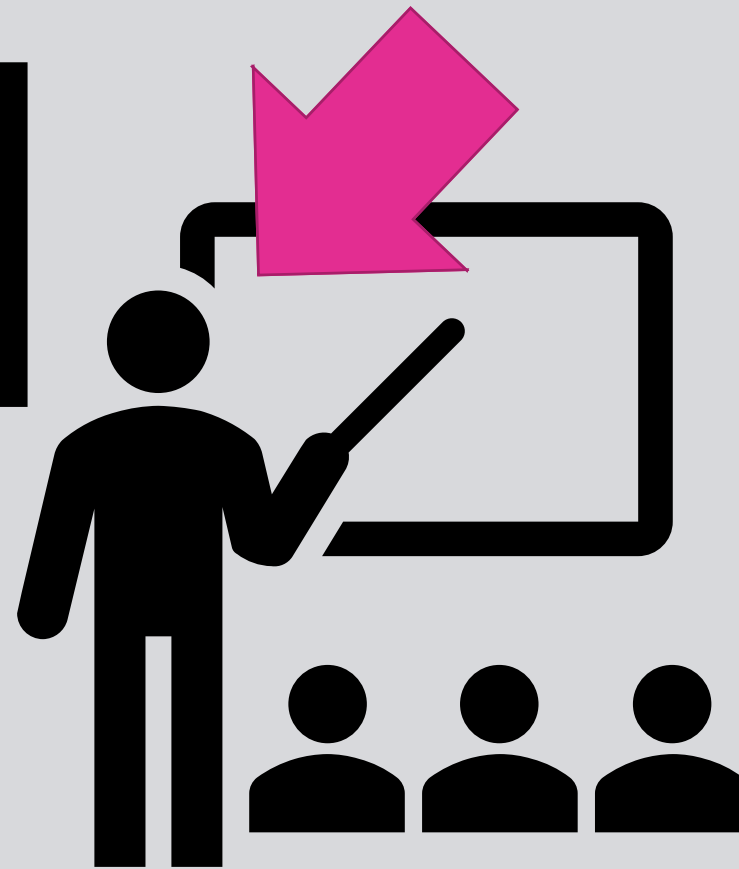
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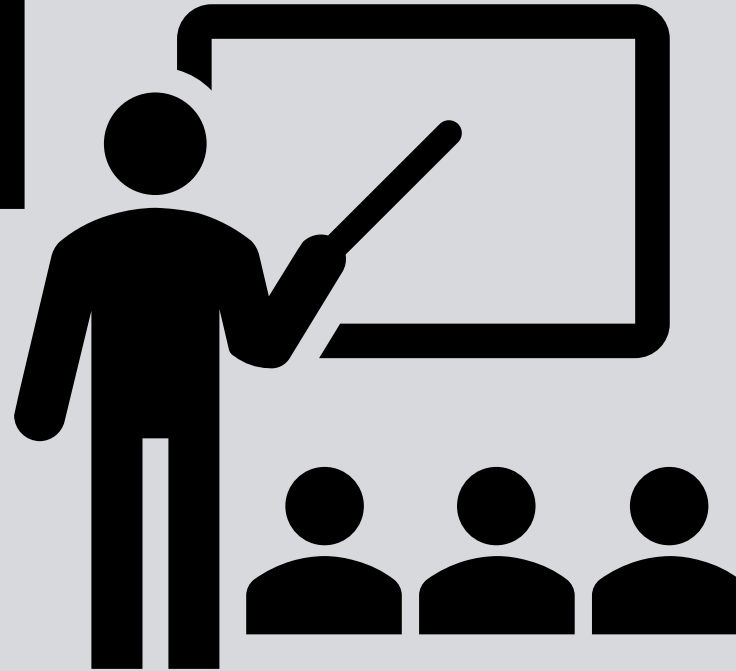
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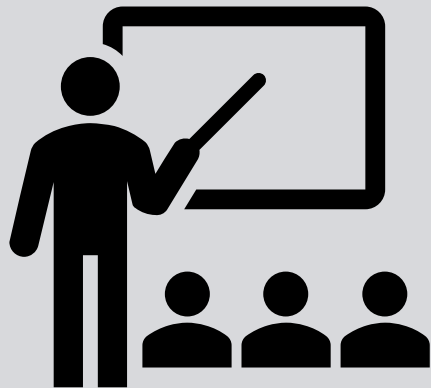


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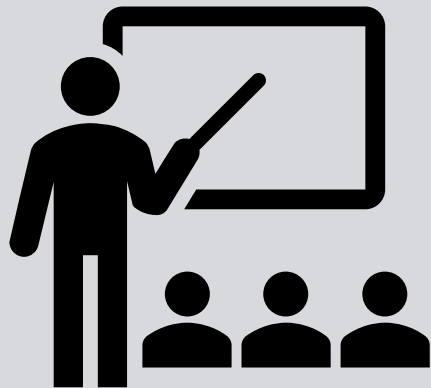
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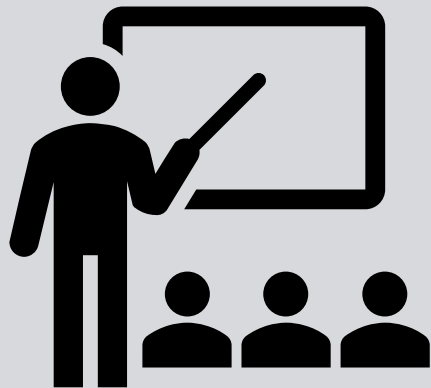


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Why is it called
differential
privacy?



DEFINITION (DWORK ET AL.)

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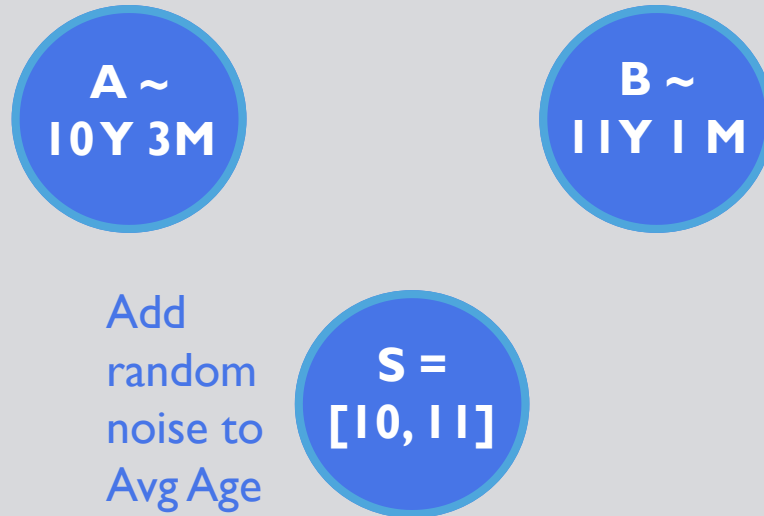
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For one
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For one
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What happens as ϵ becomes greater?

BUT...



BUT...



Can we add **ANY** noise?

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Can we add **ANY** noise?

Can we allow **ANY** number of queries?

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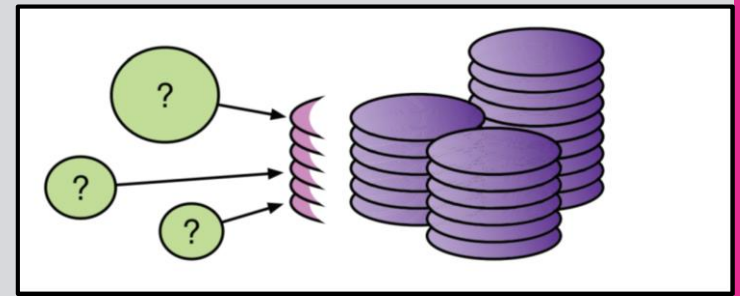
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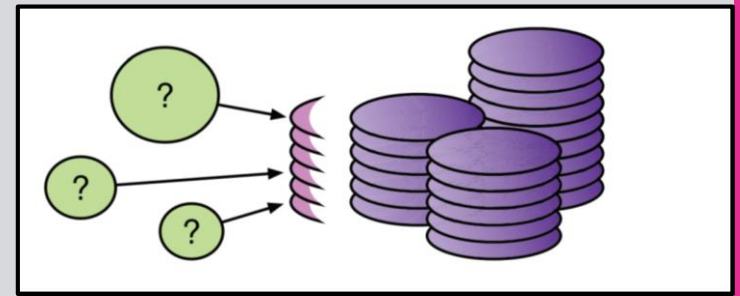
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ENTER PINQ (PRIVACY INTEGRATED QUERIES)



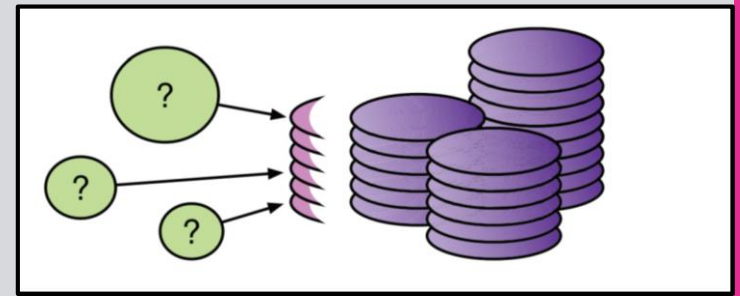
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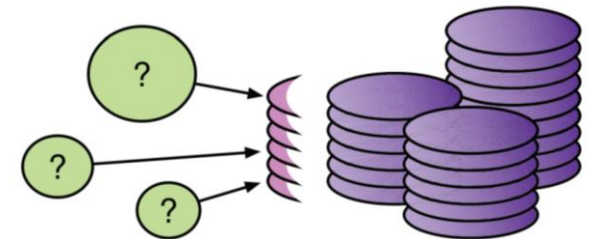
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Example 1 Counting searches from distinct users in PINQ.

```
var data = new PINQueryable<SearchRecord>( ... );

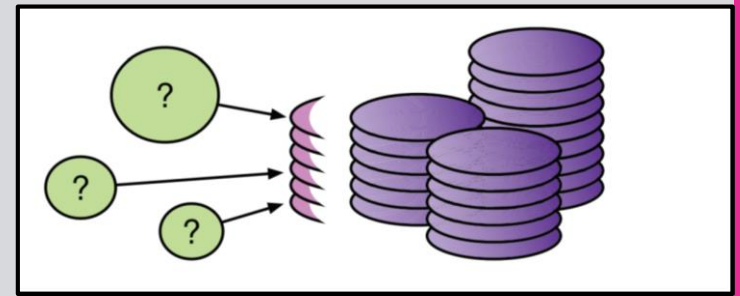
var users = from record in data
            where record.Query == argv[0]
            groupby record.IPAddress

Console.WriteLine(argv[0] + ": " + users.NoisyCount(0.1));
```



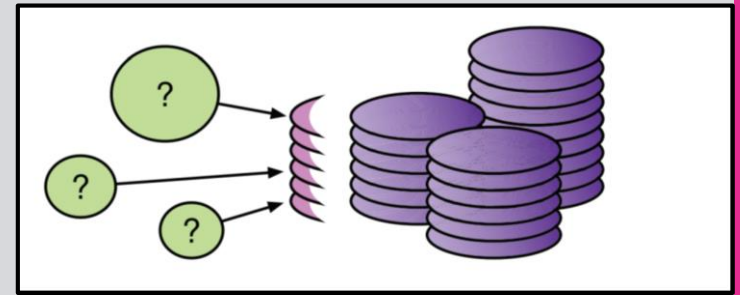
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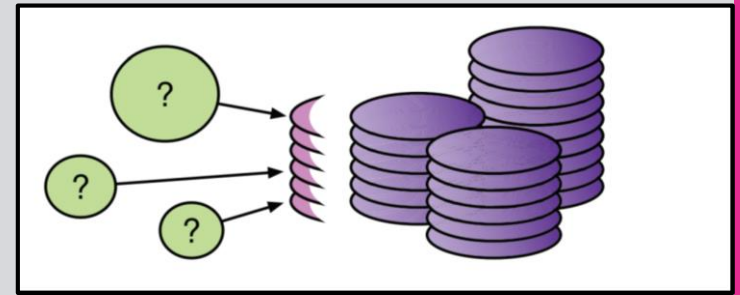
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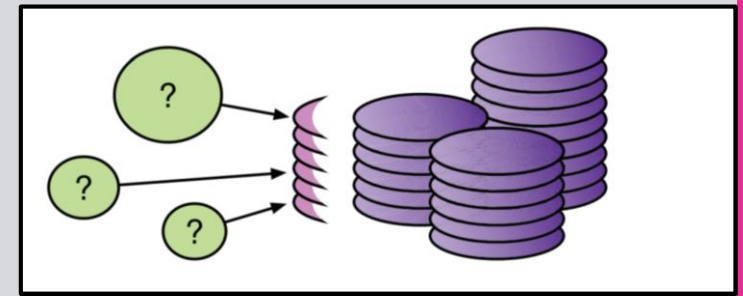
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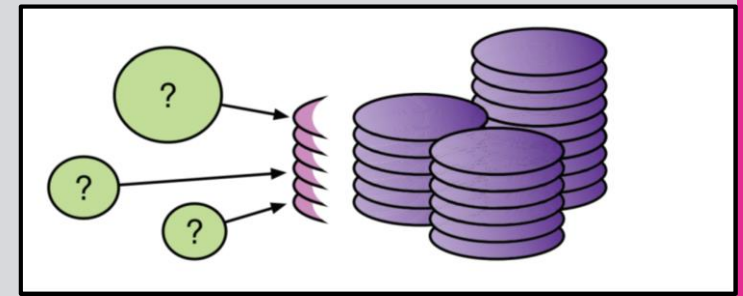
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- PINQ manages this **privacy budget**



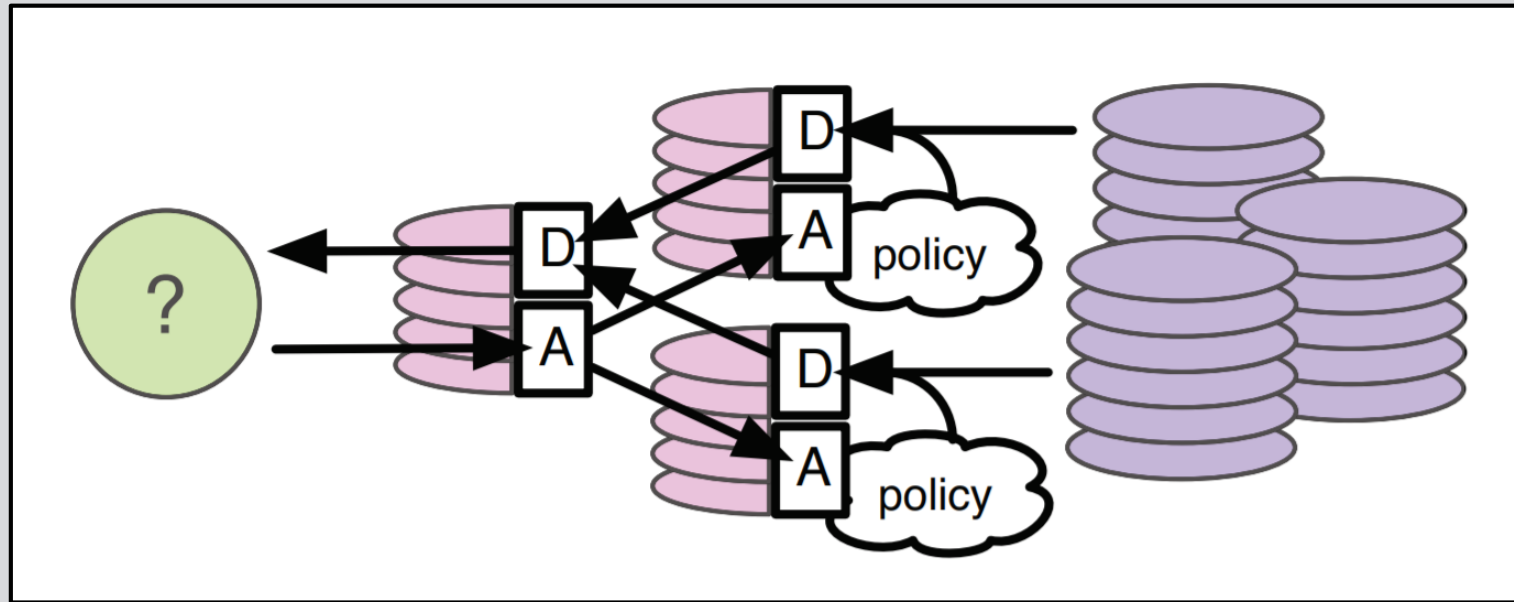
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- PINQ manages this **privacy budget**
- Privacy guarantees come from differential privacy



PINQ SYSTEM

- Central Type – PINQueryable
- **PINQueryable** = Iqueryable (unprotected) + **PINQAgent** (privacy)



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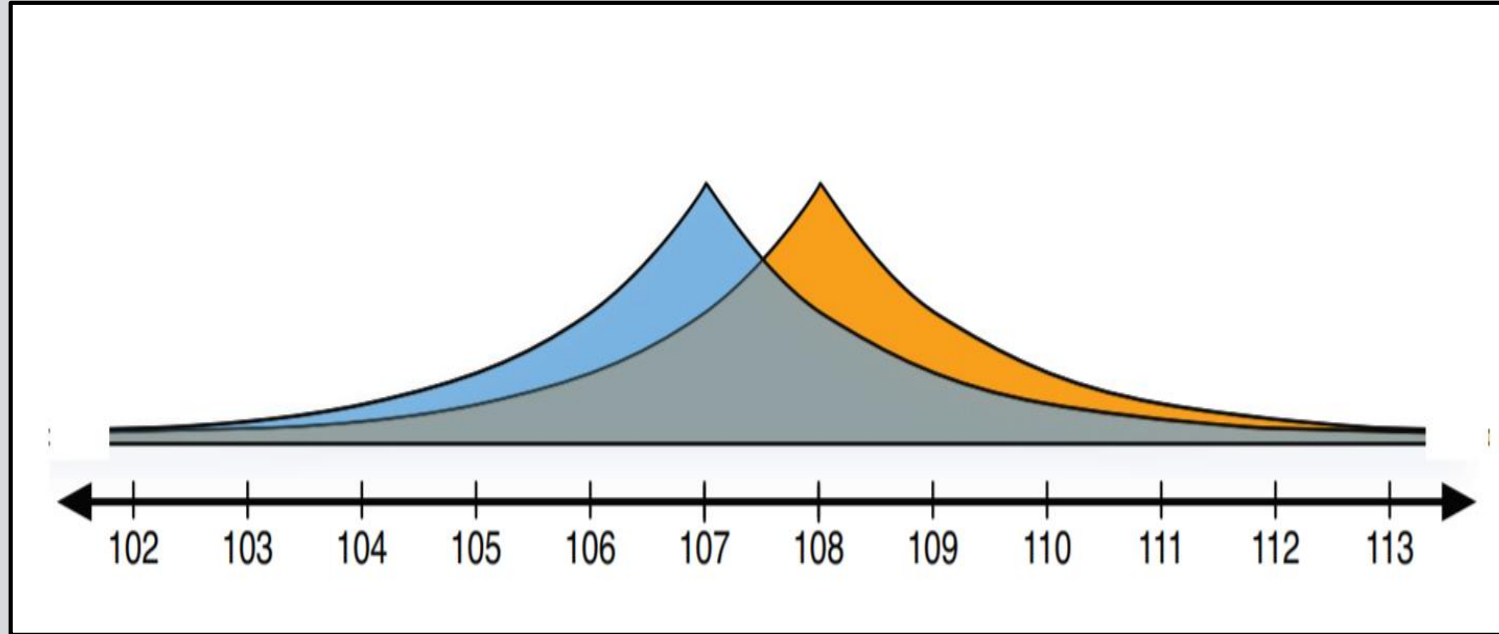
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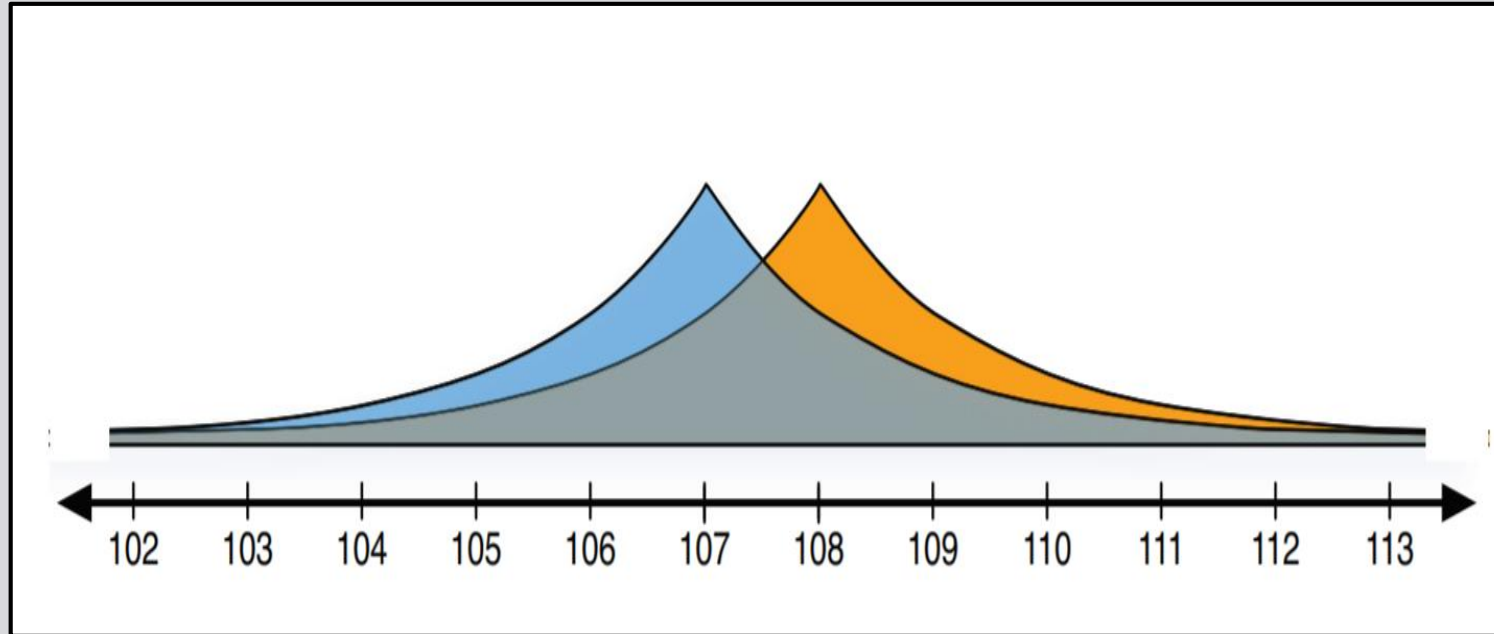
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NOISY COUNT

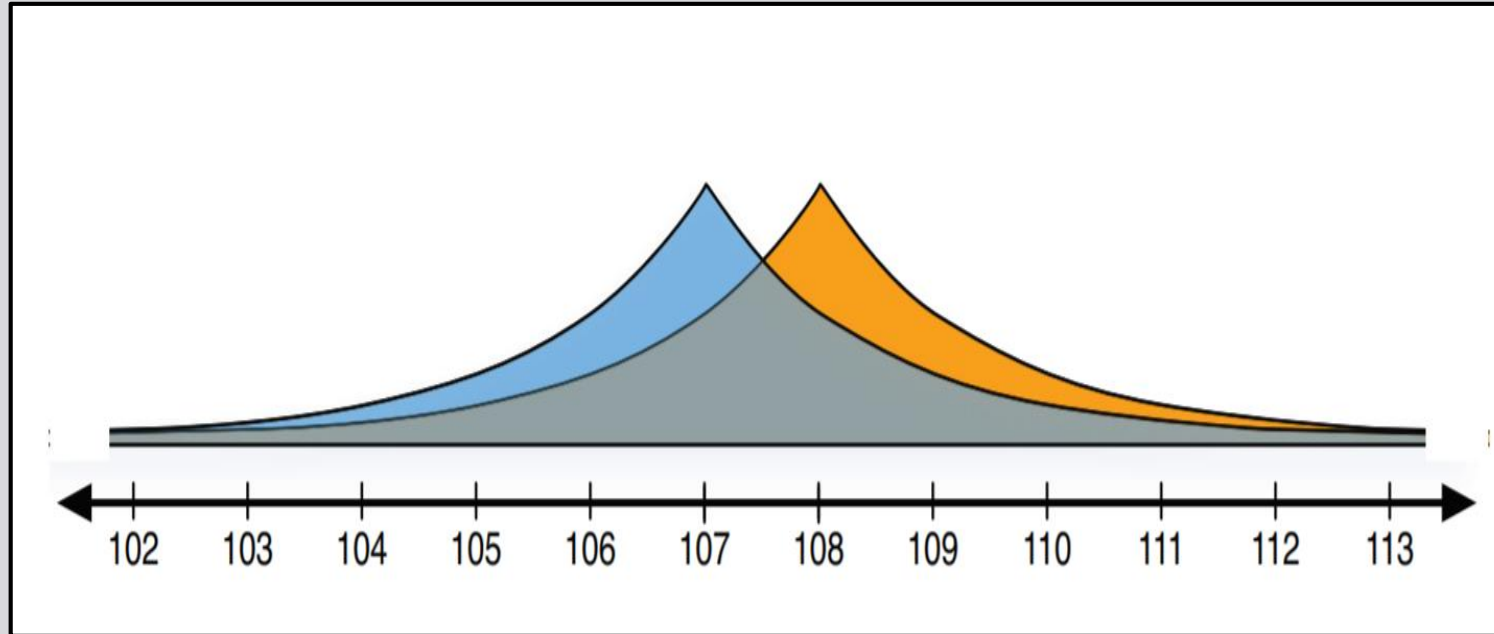


NOISY COUNT



Count = 107 + Laplace noise – would only shift by a multiplicative factor if Count = 108 instead – if the Laplace noise parameter is $1/\epsilon$ you get ϵ -differentially private counts

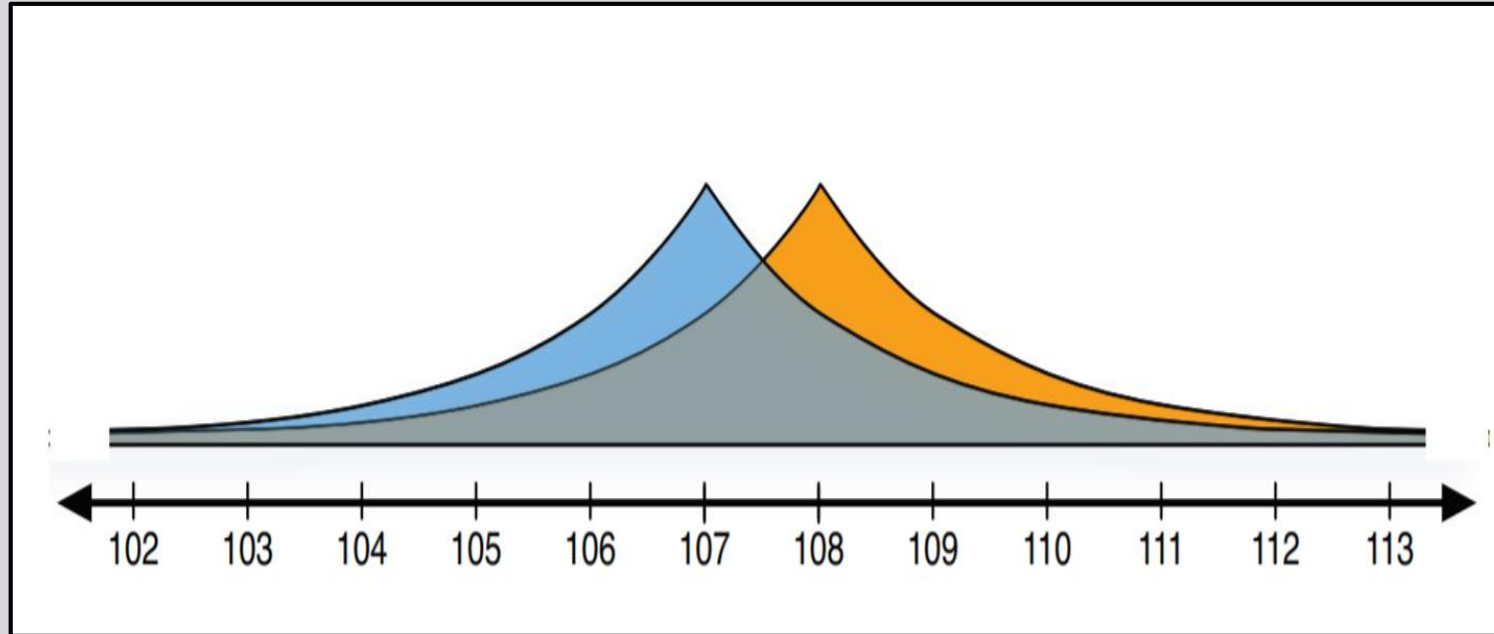
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What happens as ϵ becomes greater?

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Is this enough?

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C = 1
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C = 2

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C = 1

SELECT

C = 1

GROUP BY

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JOIN*

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WHERE	$C = 1$
SELECT	$C = 1$
GROUP BY	$C = 2$
JOIN*	$C = 2$

An ϵ -DP aggregation on a c-stable transformation gives $c\epsilon$ -DP

HOW DOES IT WORK?

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Example 5 Measuring query frequencies in PINQ.

```
// prepare data with privacy budget
var agent = new PINQAgentBudget(1.0);
var data  = new PINQQueryable<string>(rawdata, agent);

// break out fields, filter by query, group by IP
var users = data.Select(line => line.Split(','))
                .Where(fields => fields[20] == args[0])
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// output the count to the screen, or anywhere else
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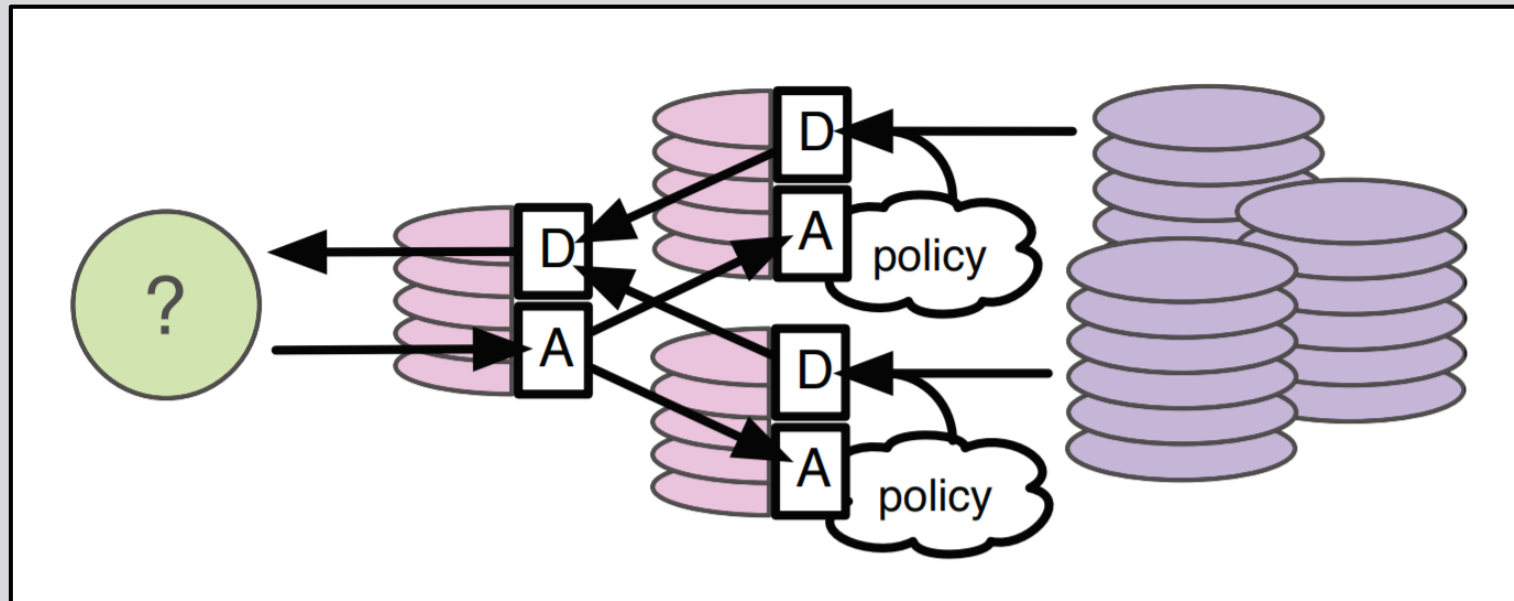
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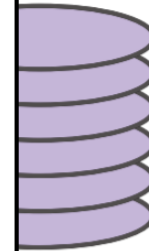
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public class PINQAgentBudget : PINQAgent
{
    private double budget;

    public override bool Alert(double epsilon)
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        budget = budget - epsilon;
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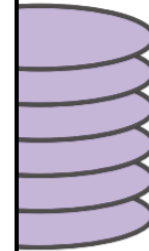
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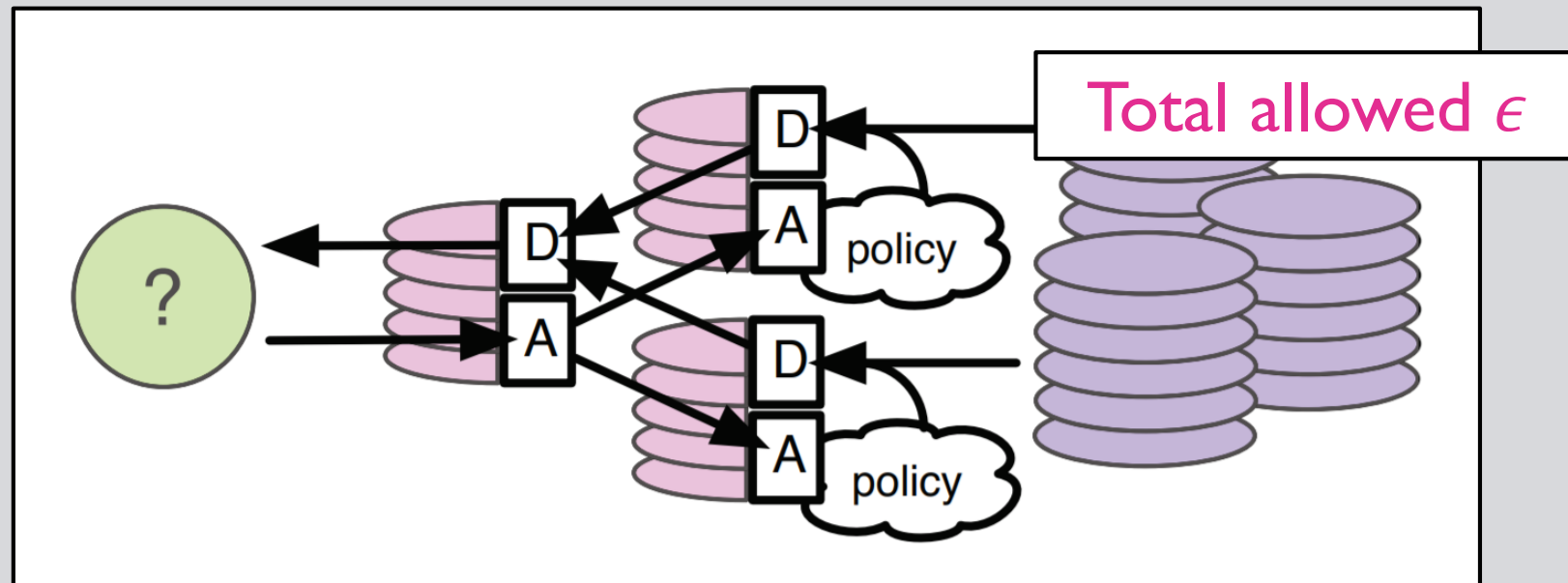
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Total allowed ϵ



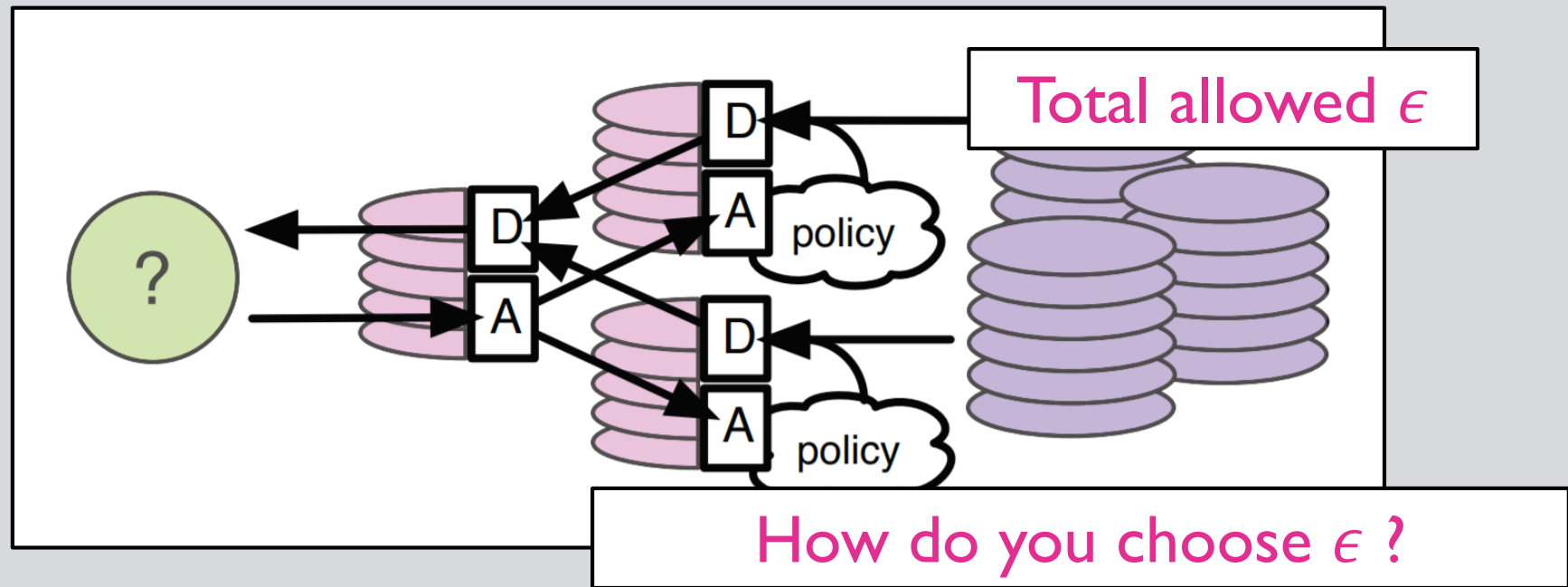
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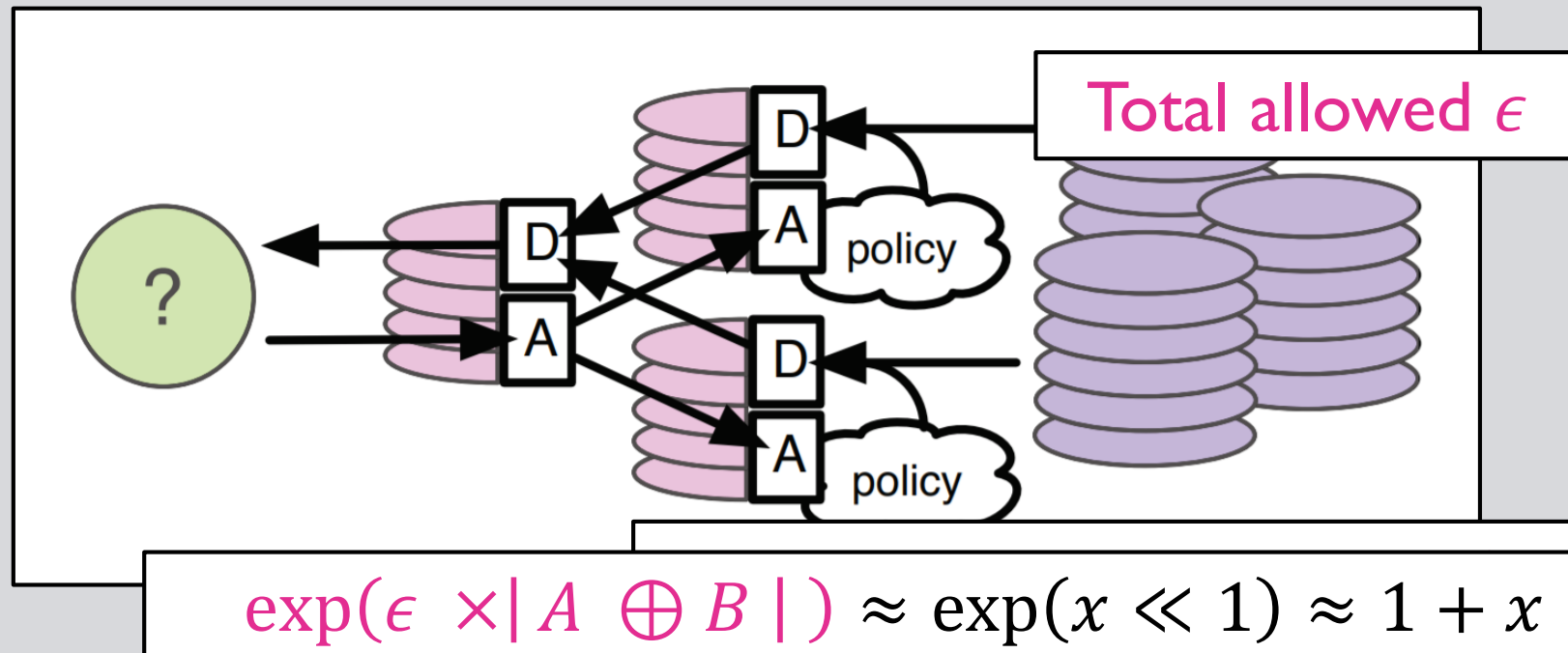
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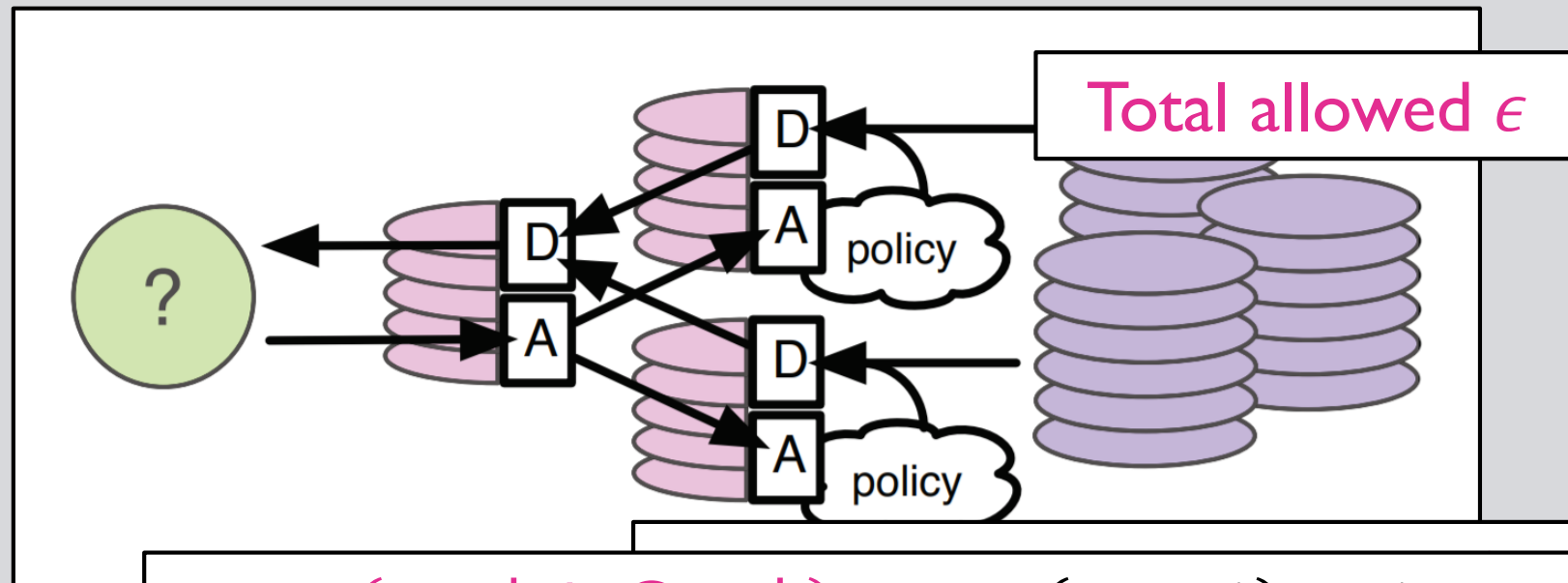
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$$\exp(\epsilon \times |A \oplus B|) \approx \exp(x \ll 1) \approx 1 + x \approx 1$$

What $|A \oplus B|$ are you protecting?

PARTITION (PARALLEL COMPOSITION)

PARTITION_(PARALLEL COMPOSITION)

Parallel (Disjoint) ϵ -DP analyses take MAX ϵ for total effective ϵ

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Example 5 Measuring query frequencies in PINQ.

```
// prepare data with privacy budget
var agent = new PINQAgentBudget(1.0);
var data  = new PINQQueryable<string>(rawdata, agent);

// break out fields, filter by query, group by IP
var users = data.Select(line => line.Split(','))
                .Where(fields => fields[20] == args[0])
                .GroupBy(fields => fields[0]);

// output the count to the screen, or anywhere else
Console.WriteLine(args[0] + ": " + users.NoisyCount(0.1));
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0.2

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What if you needed
the counts for 10
queries?

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0.2

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Use PARTITION

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0.2

Example 6 Measuring many query frequencies in PINQ.

```
// prepare data with privacy budget
var agent = new PINQAgentBudget(1.0);
var data = new PINQueryable<string>(rawdata, agent);

// break out fields, but partition rather than filter
var parts = data.Select(line => line.Split(','))
    .Partition(args, fields => fields[20]);

foreach (var query in args)
{
    // use the searches for query, grouped by IP address
    var users = parts[query].GroupBy(fields => fields[0]);

    // further partition by the frequency of searches
    var freqs = users.Partition(new int[] {1,2,3,4,5},
        group => group.Count());

    // output the counts to the screen, or anywhere else
    Console.WriteLine(query + ":");
    foreach (var count in new int[] {1,2,3,4,5})
        Console.WriteLine(freqs[count].NoisyCount(0.1));
}
```

PARTITION_(PARALLEL COMPOSITION)

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0.2

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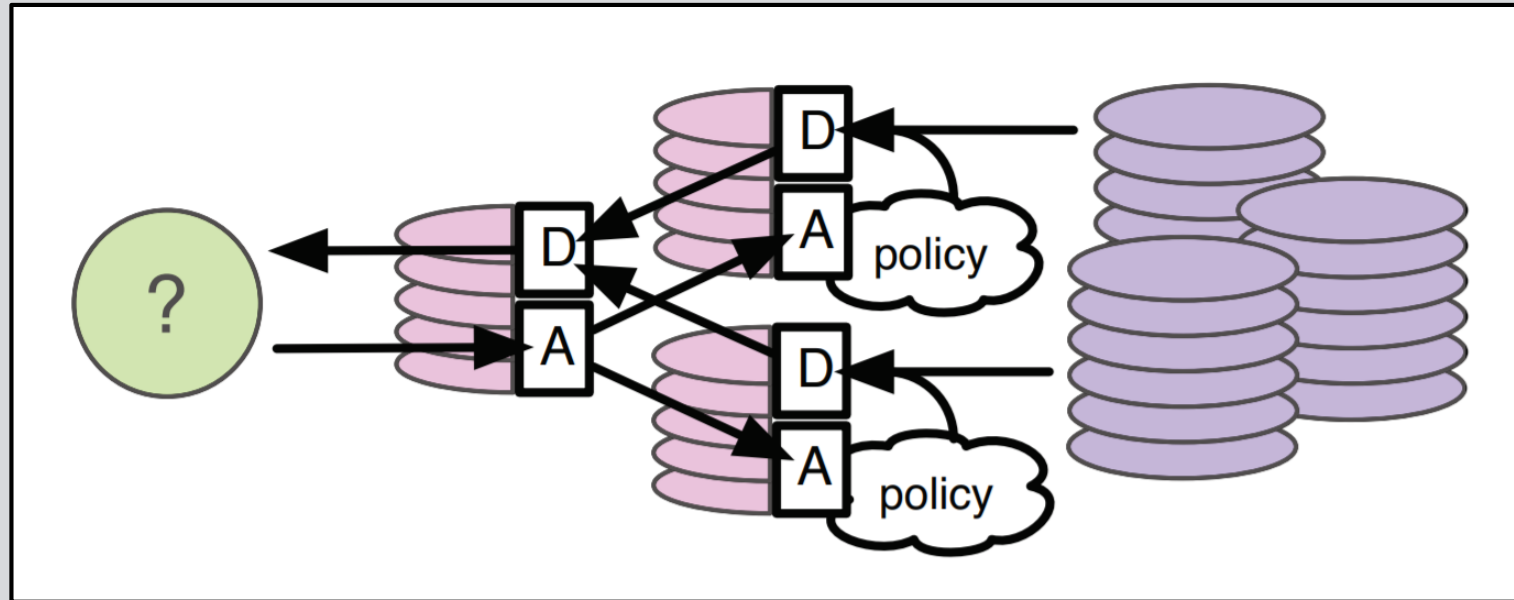
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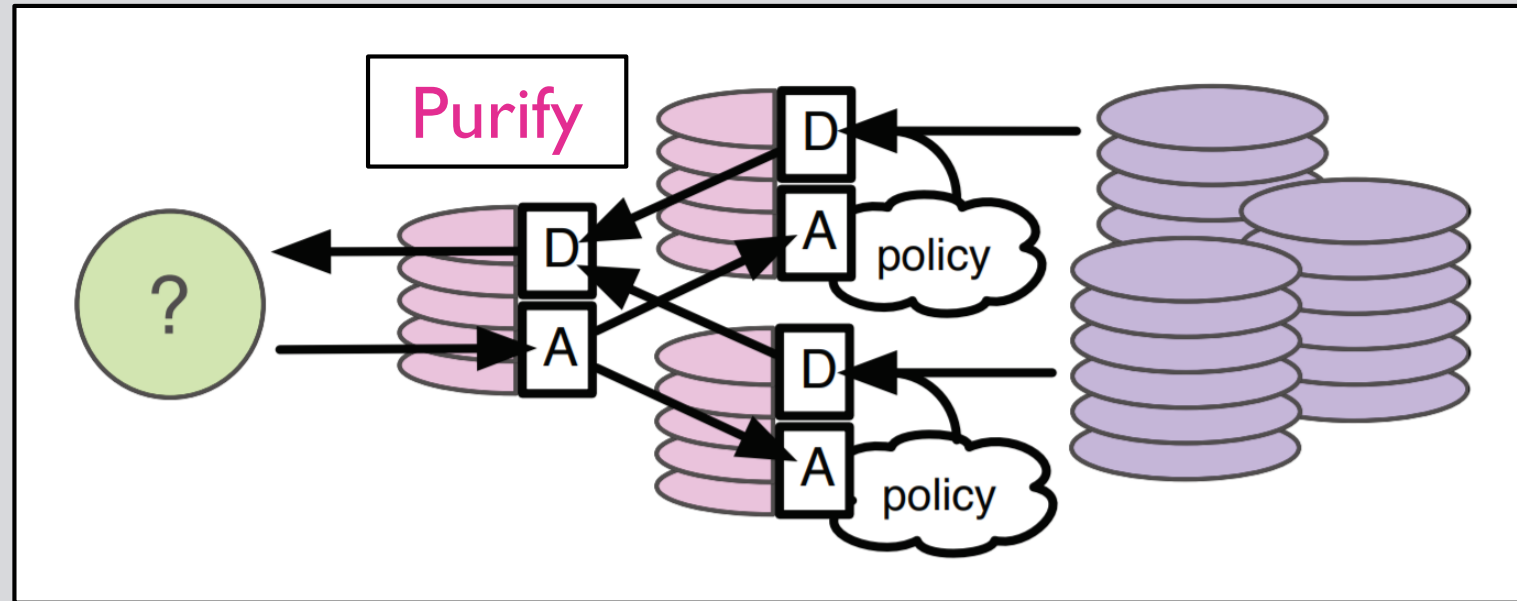
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0.2

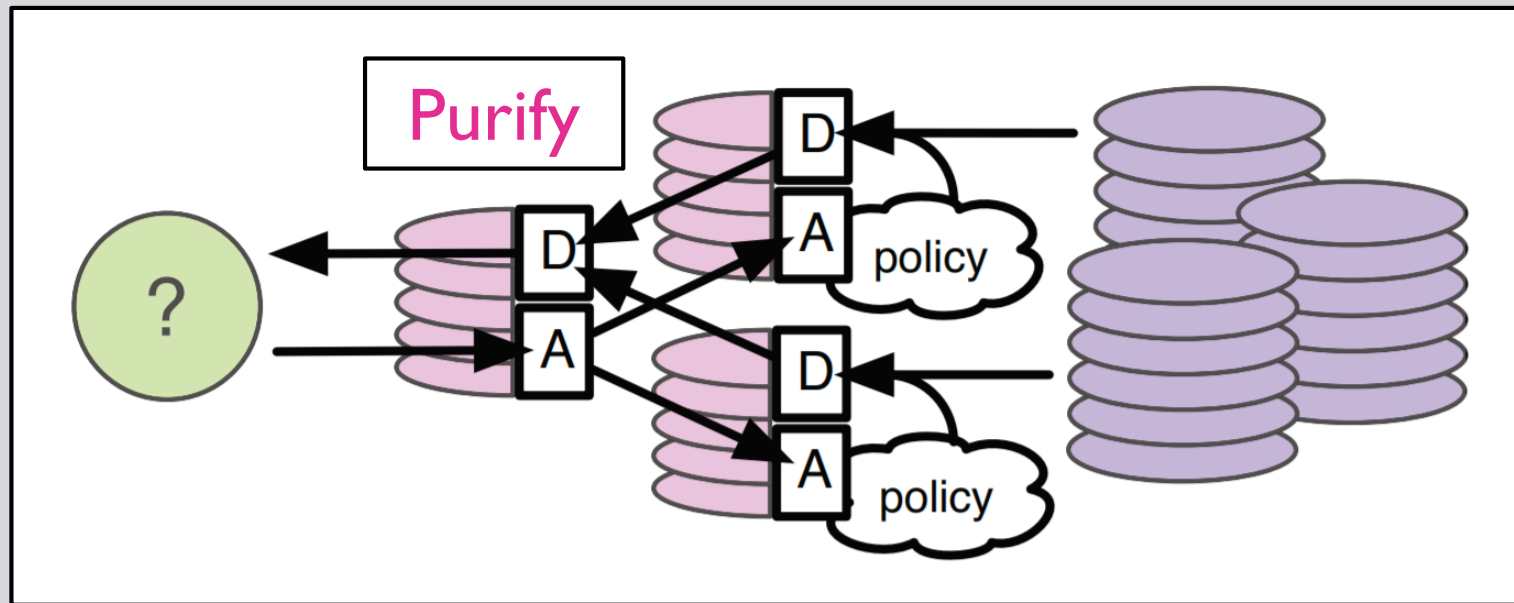
FINALLY...



FINALLY...



FINALLY...



Performance?

DISCUSSION



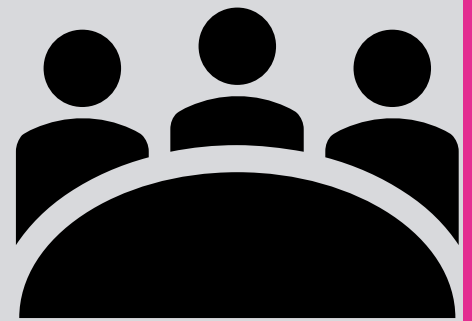
DISCUSSION

- Accessible
- Not much overhead
- Mathematical guarantee
- Allows richer queries



DISCUSSION

- Accessible
 - Not much overhead
 - Mathematical guarantee
 - Allows richer queries
- Is it really?
 - Budget is finite (DP)
 - One user can use up
 - Data source available always
 - Note: Works with static data



SOURCES

- PINQ 1 <https://dl.acm.org/citation.cfm?id=1559850>
- PINQ 2 <https://www.microsoft.com/en-us/research/wp-content/uploads/2010/09/pinq-CACM.pdf>
- PINQ talk – Frank McSherry - <https://www.youtube.com/watch?v=GnIB7KJ5kVg>
- <https://www.refinery29.com/en-us/2017/10/179039/this-is-fine-meme-halloween-costume>
- <https://www.vulture.com/2019/06/this-is-fine-dog-meme-comic-kc-green-interview.html>
- <https://knowyourmeme.com/memes/math-lady-confused-lady>