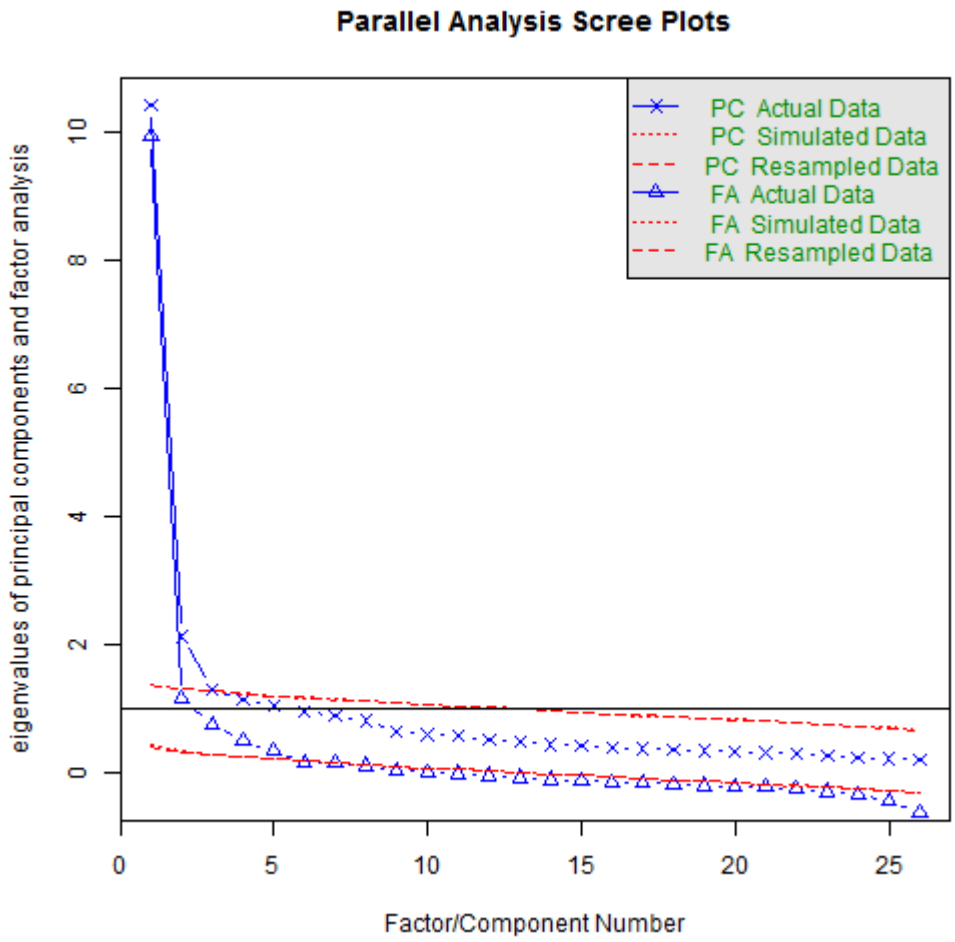
Exploratory Factor Analysis – Solutions

Below are the solutions to these exercises on exploratory factor analysis.

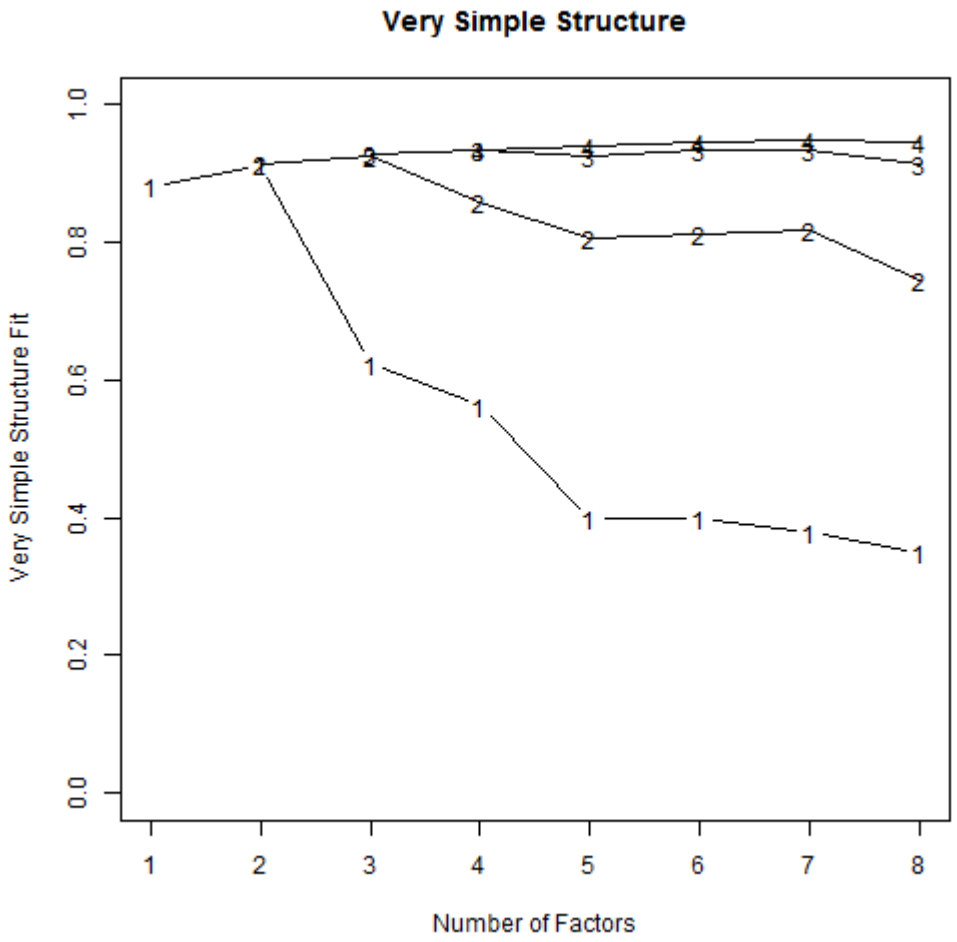
```
####################
#                  #
#    Exercise 1    #
#                  #
####################

install.packages(c("psych", "GPArotation"))
library(psych)
data <- read.file("efa.csv")
describe(data)

##     vars   n mean   sd median trimmed  mad min max range
skew kurtosis
## V1     1 649 4.79 0.47      5    4.89 0.00   3   5     2
-2.16     3.96
## V2     2 649 4.77 0.53      5    4.89 0.00   2   5     3
-2.63     7.59
## V3     3 649 4.62 0.72      5    4.79 0.00   1   5     4
-2.13     4.73
## V4     4 649 4.84 0.45      5    4.96 0.00   2   5     3
-3.13    10.87
## V5     5 649 4.85 0.44      5    4.97 0.00   2   5     3
-3.31    12.40
## V6     6 649 4.83 0.48      5    4.95 0.00   2   5     3
-3.31    12.40
## V7     7 649 4.71 0.61      5    4.85 0.00   2   5     3
-2.20     4.59
## V8     8 649 4.70 0.62      5    4.85 0.00   2   5     3
-2.22     4.70
## V9     9 649 4.50 0.85      5    4.69 0.00   1   5     4
-1.97     3.86
## V10   10 649 4.69 0.72      5    4.86 0.00   1   5     4
```

```
-2.81    8.78
## V11   11 649 4.61 0.89      5    4.86 0.00   1   5     4
-2.65    6.63
## V12   12 649 4.39 1.12      5    4.67 0.00   1   5     4
-1.86    2.39
## V13   13 649 4.68 0.62      5    4.80 0.00   1   5     4
-2.27    5.96
## V14   14 649 4.37 0.80      5    4.50 0.00   1   5     4
-1.17    0.93
## V15   15 649 4.61 0.62      5    4.71 0.00   2   5     3
-1.60    2.54
## V16   16 649 4.71 0.60      5    4.85 0.00   1   5     4
-2.23    5.09
## V17   17 649 4.71 0.62      5    4.85 0.00   1   5     4
-2.42    6.32
## V18   18 649 4.72 0.60      5    4.85 0.00   1   5     4
-2.50    7.29
## V19   19 649 4.56 0.73      5    4.72 0.00   1   5     4
-1.70    2.68
## V20   20 649 4.42 0.90      5    4.60 0.00   1   5     4
-1.73    2.80
## V21   21 649 3.30 0.96      3    3.29 1.48   1   5     4
-0.16   -1.03
## V22   22 649 2.98 1.10      3    2.89 1.48   1   5     4
0.41   -0.98
## V23   23 649 3.27 1.23      3    3.29 1.48   1   5     4
-0.10   -1.11
## V24   24 649 2.69 1.01      3    2.68 1.48   1   5     4
0.33   -0.64
## V25   25 649 2.85 1.07      3    2.85 1.48   1   5     4
0.14   -0.92
## V26   26 649 3.69 0.83      4    3.75 0.00   1   5     4
-0.74    0.53
##       se
## V1  0.02
## V2  0.02
## V3  0.03
## V4  0.02
## V5  0.02
## V6  0.02
## V7  0.02
```

```
## V8  0.02
## V9  0.03
## V10 0.03
## V11 0.03
## V12 0.04
## V13 0.02
## V14 0.03
## V15 0.02
## V16 0.02
## V17 0.02
## V18 0.02
## V19 0.03
## V20 0.04
## V21 0.04
## V22 0.04
## V23 0.05
## V24 0.04
## V25 0.04
## V26 0.03

######################
#                    #
#    Exercise 2      #
#                    #
######################

fa.parallel(data)
```




```
## Parallel analysis suggests that the number of factors =  5
and the number of components =  3

######################
#                    #
#    Exercise 3      #
#                    #
######################

vss(data)
```



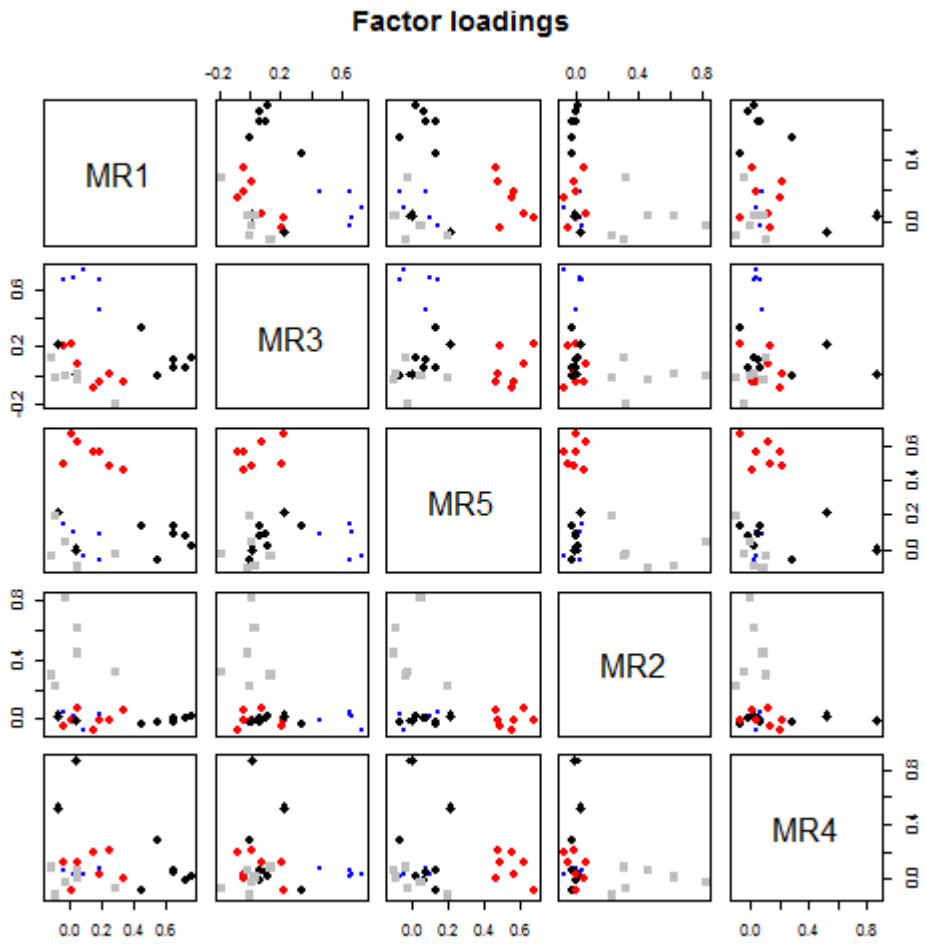

```
##
## Very Simple Structure
## Call: vss(x = data)
## VSS complexity 1 achieves a maximimum of 0.91  with  2
factors
## VSS complexity 2 achieves a maximimum of 0.93  with  3
factors
##
## The Velicer MAP achieves a minimum of 0.01  with  3
factors
## BIC achieves a minimum of  -742.22  with  5  factors
## Sample Size adjusted BIC achieves a minimum of  -160.17
with  8  factors
##
## Statistics by number of factors
##   vss1 vss2   map dof chisq     prob sqresid  fit RMSEA
BIC SABIC complex
## 1 0.88 0.00 0.014 299  2003 1.9e-249    14.6 0.88 0.095
67  1016     1.0
```

```
## 2 0.91 0.91 0.014 274  1546 3.3e-176    10.5 0.91 0.086
-228   642     1.0
## 3 0.62 0.93 0.013 250  1095 4.9e-106     9.0 0.93 0.073
-524   270     1.4
## 4 0.56 0.86 0.015 227   783  2.9e-62     8.1 0.93 0.062
-687    34     1.7
## 5 0.40 0.81 0.019 205   585  2.8e-38     7.3 0.94 0.054
-742   -91     1.9
## 6 0.40 0.81 0.022 184   502  2.4e-31     6.4 0.95 0.053
-689  -105     2.0
## 7 0.38 0.82 0.024 164   425  4.8e-25     5.7 0.95 0.050
-637  -116     2.1
## 8 0.35 0.75 0.027 145   318  5.0e-15     5.3 0.96 0.044
-621  -160     2.3
##   eChisq  SRMR eCRMS  eBIC
## 1   2201 0.072 0.075   265
## 2   1093 0.051 0.055  -682
## 3    665 0.040 0.045  -953
## 4    461 0.033 0.040 -1009
## 5    320 0.028 0.035 -1007
## 6    237 0.024 0.031  -955
## 7    162 0.020 0.028  -900
## 8    115 0.017 0.025  -824

####################
#                  #
#    Exercise 4    #
#                  #
####################

sapply(data, shapiro.test)

##           V1                             V2
## statistic 0.4880158                      0.4818245
## p.value   1.790874e-39                   1.218162e-39
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                       "X[[i]]"
##           V3                             V4
## statistic 0.5837358                      0.4039763
## p.value   1.193739e-36                   1.287584e-41
```

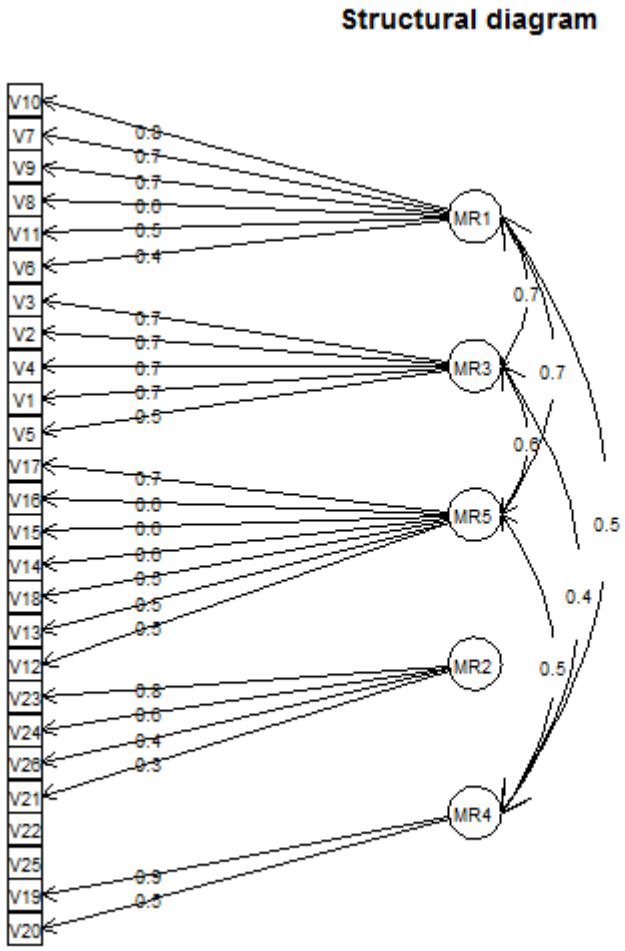
```
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                       "X[[i]]"
##           V5                             V6
## statistic 0.386337                       0.4010764
## p.value   4.917495e-42                   1.097388e-41
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                       "X[[i]]"
##           V7                             V8
## statistic 0.5352881                      0.5343438
## p.value   3.876222e-38                   3.636428e-38
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                       "X[[i]]"
##           V9                             V10
## statistic 0.631653                       0.4956688
## p.value   4.943564e-35                   2.898898e-39
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                       "X[[i]]"
##           V11                            V12
## statistic 0.4952979                      0.6048789
## p.value   2.83163e-39                    5.900295e-36
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                       "X[[i]]"
##           V13                            V14
## statistic 0.5630808                      0.7476871
## p.value   2.666746e-37                   2.854301e-30
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                       "X[[i]]"
##           V15                            V16
## statistic 0.6405669                      0.5328645
## p.value   1.031309e-34                   3.290914e-38
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                       "X[[i]]"
##           V17                            V18
## statistic 0.5253764                      0.5207042
```

```
## p.value   1.993172e-38                 1.462449e-38
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                     "X[[i]]"
##           V19                          V20
## statistic 0.6448004                    0.6737761
## p.value   1.469911e-34                 1.828638e-33
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                     "X[[i]]"
##           V21                          V22
## statistic 0.8599259                    0.8613114
## p.value   1.369292e-23                 1.744914e-23
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                     "X[[i]]"
##           V23                          V24
## statistic 0.8992078                    0.8899322
## p.value   3.080095e-20                 4.157814e-21
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                     "X[[i]]"
##           V25                          V26
## statistic 0.8948388                    0.8292828
## p.value   1.17986e-20                  9.748275e-26
## method     "Shapiro-Wilk normality test" "Shapiro-Wilk
normality test"
## data.name "X[[i]]"                     "X[[i]]"

####################
#                  #
#    Exercise 5    #
#                  #
####################

f.solution  <-  fa(data,  nfactors=5,  rotate="oblimin",
fm="minres")
print(f.solution$loadings, cutoff=0.3)

##
## Loadings:
```

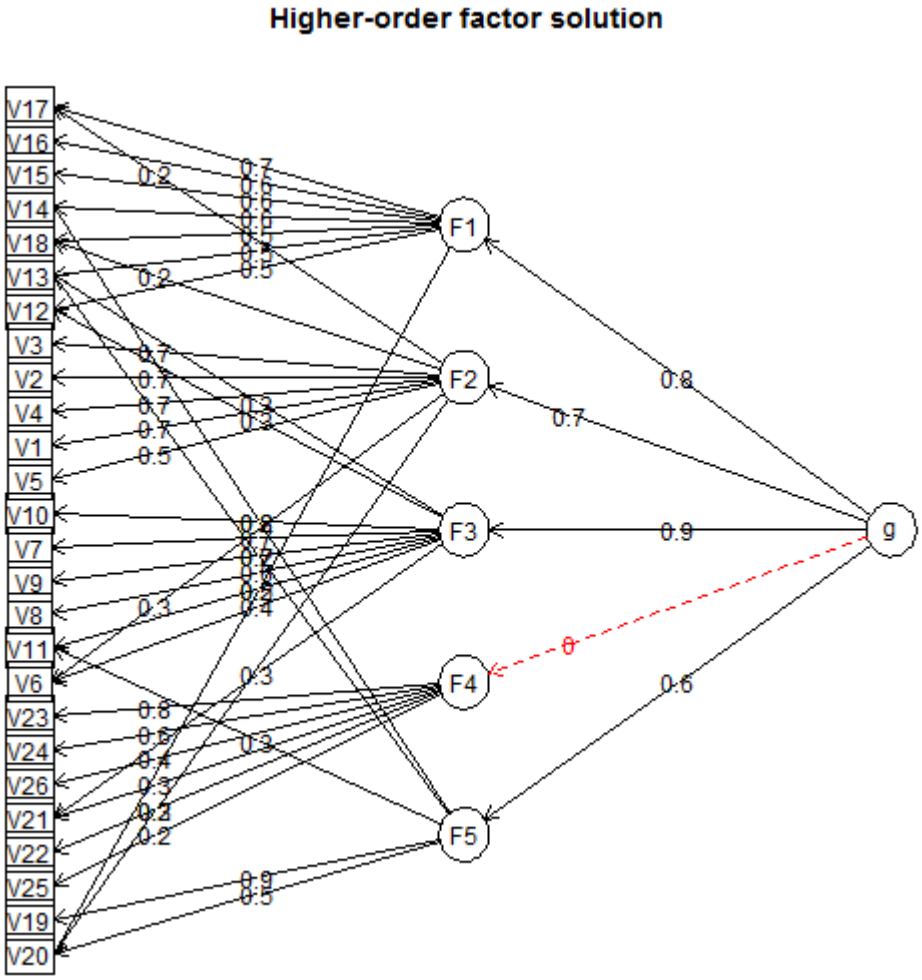
```
##     MR1    MR3    MR5    MR2    MR4
## V1           0.662
## V2           0.675
## V3           0.740
## V4           0.664
## V5           0.460
## V6   0.448  0.336
## V7   0.722
## V8   0.648
## V9   0.652
## V10  0.759
## V11  0.546
## V12  0.344         0.465
## V13                0.483
## V14                0.563
## V15                0.566
## V16                0.621
## V17                0.676
## V18                0.495
## V19                                0.866
## V20                                0.524
## V21                       0.316
## V22
## V23                       0.820
## V24                       0.616
## V25
## V26                       0.448
##
##                  MR1   MR3   MR5   MR2   MR4
## SS loadings    2.887 2.452 2.381 1.522 1.283
## Proportion Var 0.111 0.094 0.092 0.059 0.049
## Cumulative Var 0.111 0.205 0.297 0.355 0.405

####################
#                  #
#    Exercise 6    #
#                  #
####################

plot(f.solution, title="Factor loadings")
```




```
#####################
#                   #
#    Exercise 7     #
#                   #
#####################

fa.diagram(f.solution, main="Structural diagram")
```




```
#####################
#                   #
#    Exercise 8     #
#                   #
#####################

omega(data, nfactors = 5, sl=FALSE, title="Higher-order factor
solution")
```

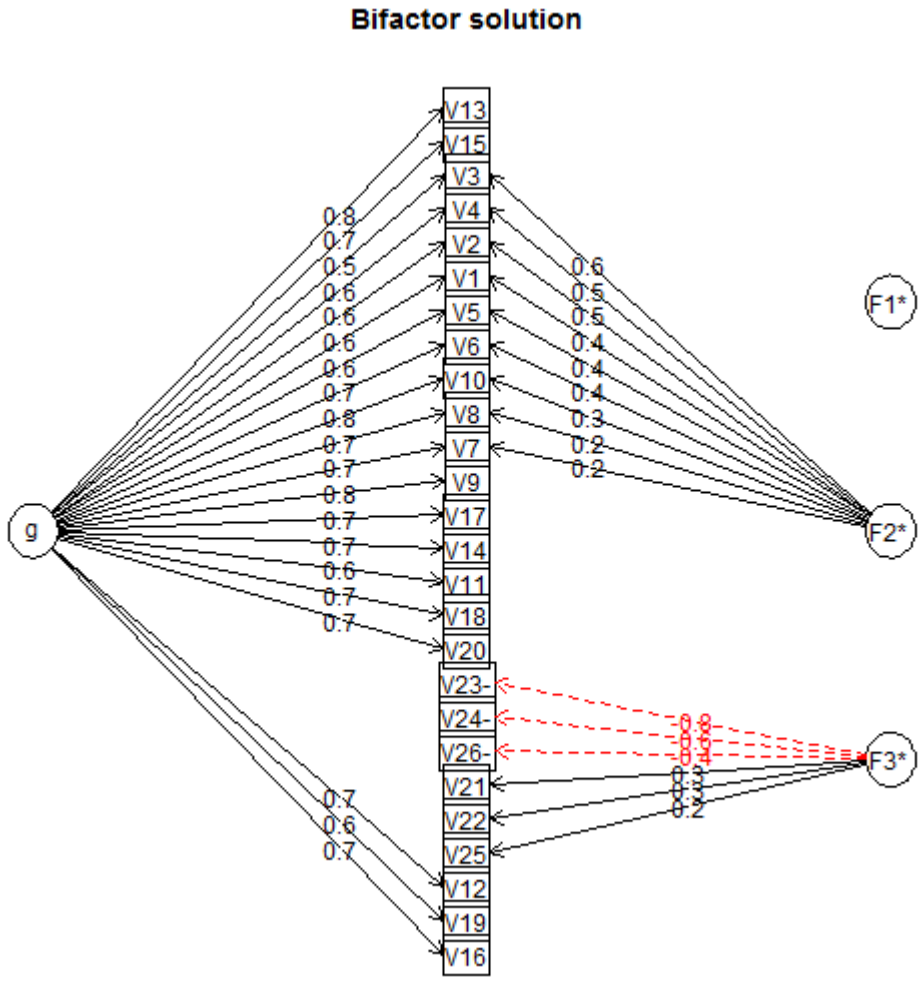



```
## Higher-order factor solution
## Call: omega(m = data, nfactors = 5, title = "Higher-order
factor solution",
##     sl = FALSE)
## Alpha:                 0.91
## G.6:                   0.94
## Omega Hierarchical:    0.81
## Omega H asymptotic:    0.86
## Omega Total            0.94
##
## Schmid Leiman Factor loadings greater than  0.2
##            g   F1*   F2*   F3*   F4*   F5*   h2   u2   p2
## V1    0.62        0.45                     0.58 0.42 0.65
## V2    0.62        0.46                     0.59 0.41 0.65
## V3    0.61        0.50                     0.63 0.37 0.58
## V4    0.62        0.45                     0.60 0.40 0.63
## V5    0.62        0.31                     0.50 0.50 0.77
## V6    0.70        0.23  0.23               0.62 0.38 0.80
## V7    0.72              0.37               0.65 0.35 0.79
```

```
## V8    0.74              0.33              0.66 0.34 0.82
## V9    0.76              0.33              0.69 0.31 0.83
## V10   0.77              0.38              0.76 0.24 0.79
## V11   0.59              0.28        0.22  0.46 0.54 0.74
## V12   0.64  0.27                          0.52 0.48 0.79
## V13   0.75  0.28                          0.69 0.31 0.82
## V14   0.65  0.33                          0.57 0.43 0.73
## V15   0.62  0.33                          0.50 0.50 0.76
## V16   0.67  0.36                          0.60 0.40 0.75
## V17   0.68  0.39                          0.63 0.37 0.72
## V18   0.60  0.29                          0.48 0.52 0.75
## V19   0.57                        0.69  0.79 0.21 0.41
## V20   0.60                        0.41  0.57 0.43 0.63
## V21                        0.32         0.15 0.85 0.01
## V22                        0.30         0.10 0.90 0.00
## V23-                      -0.82         0.68 0.32 0.00
## V24-                      -0.62         0.38 0.62 0.00
## V25                        0.23         0.08 0.92 0.00
## V26-                      -0.45         0.20 0.80 0.00
##
## With eigenvalues of:
##    g  F1*  F2*  F3*  F4*  F5*
## 8.68 0.81 1.12 0.74 1.52 0.80
##
## general/max  5.71   max/min =   2.06
## mean percent general =  0.55    with sd =  0.32 and cv of
0.58
## Explained Common Variance of the general factor =  0.64
##
## The degrees of freedom are 205  and the fit is  0.92
## The number of observations was  649  with Chi Square =
585.24  with prob <  2.8e-38
## The root mean square of the residuals is  0.03
## The df corrected root mean square of the residuals is  0.03
## RMSEA index =  0.054  and the 10 % confidence intervals are
0.048 0.059
## BIC =  -742.22
##
## Compare this with the adequacy of just a general factor and
no group factors
## The degrees of freedom for just the general factor are 299
```

```
and the fit is  3.3
## The number of observations was  649  with Chi Square =
2103.59  with prob <  3.5e-268
## The root mean square of the residuals is  0.09
## The df corrected root mean square of the residuals is  0.09
##
## RMSEA index =  0.097  and the 10 % confidence intervals are
0.093 0.1
## BIC =  167.43
##
## Measures of factor score adequacy
##                                                   g   F1*
F2*   F3*  F4*
## Correlation of scores with factors             0.92  0.68
0.77  0.64 0.88
## Multiple R square of scores with factors       0.84  0.46
0.59  0.41 0.77
## Minimum correlation of factor score estimates 0.67 -0.08
0.19 -0.19 0.55
##                                                  F5*
## Correlation of scores with factors            0.82
## Multiple R square of scores with factors      0.68
## Minimum correlation of factor score estimates 0.35
##
##  Total, General and Subset omega for each subset
##                                                    g  F1*
F2*  F3*  F4*
## Omega total for total scores and subscales    0.94 0.88
0.87 0.88 0.21
## Omega general for total scores and subscales  0.81 0.71
0.62 0.72 0.00
## Omega group for total scores and subscales    0.07 0.17
0.25 0.16 0.21
##                                                  F5*
## Omega total for total scores and subscales    0.78
## Omega general for total scores and subscales  0.42
## Omega group for total scores and subscales    0.37

####################
#                  #
#    Exercise 9    #
```

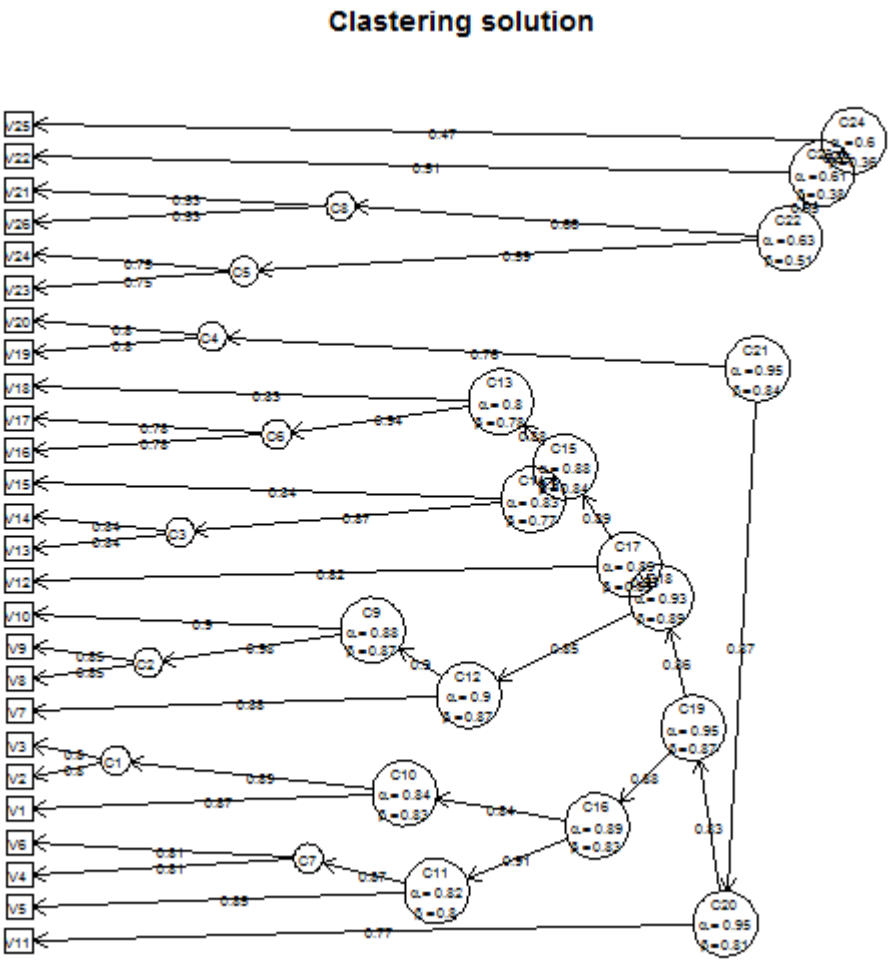
```
#                     #
#####################

omega(data, title="Bifactor solution")
```




```
## Bifactor solution
## Call: omega(m = data, title = "Bifactor solution")
## Alpha:                 0.91
## G.6:                   0.94
## Omega Hierarchical:    0.86
## Omega H asymptotic:    0.92
## Omega Total            0.94
##
## Schmid Leiman Factor loadings greater than  0.2
##           g   F1*   F2*   F3*   h2   u2   p2
## V1     0.59        0.44       0.54 0.46 0.64
## V2     0.58        0.48       0.56 0.44 0.59
## V3     0.54        0.56       0.63 0.37 0.47
## V4     0.56        0.54       0.60 0.40 0.52
## V5     0.61        0.37       0.50 0.50 0.73
```

```
## V6    0.68        0.35       0.59 0.41 0.79
## V7    0.72        0.21       0.56 0.44 0.91
## V8    0.74        0.22       0.61 0.39 0.92
## V9    0.77                   0.64 0.36 0.94
## V10   0.77        0.26       0.66 0.34 0.89
## V11   0.61                   0.38 0.62 0.97
## V12   0.70                   0.50 0.50 0.98
## V13   0.83                   0.70 0.30 1.00
## V14   0.75                   0.58 0.42 0.97
## V15   0.69                   0.48 0.52 0.99
## V16   0.74                   0.56 0.44 0.99
## V17   0.71                   0.53 0.47 0.97
## V18   0.65                   0.44 0.56 0.97
## V19   0.63                   0.41 0.59 0.98
## V20   0.66                   0.44 0.56 0.98
## V21                 0.34 0.13 0.87 0.02
## V22                 0.27 0.07 0.93 0.00
## V23-               -0.81 0.66 0.34 0.00
## V24-               -0.61 0.37 0.63 0.00
## V25                 0.23 0.06 0.94 0.00
## V26-               -0.44 0.19 0.81 0.00
##
## With eigenvalues of:
##    g  F1*  F2*  F3*
## 9.28 0.03 1.55 1.53
##
## general/max  5.98   max/min =   57.78
## mean percent general =  0.66    with sd =  0.4 and cv of
0.6
## Explained Common Variance of the general factor =  0.75
##
## The degrees of freedom are 250  and the fit is  1.72
## The number of observations was  649  with Chi Square =
1095.18  with prob <  4.9e-106
## The root mean square of the residuals is  0.04
## The df corrected root mean square of the residuals is  0.05
## RMSEA index =  0.073  and the 10 % confidence intervals are
0.068 0.077
## BIC =  -523.68
##
## Compare this with the adequacy of just a general factor and
```

```
no group factors
## The degrees of freedom for just the general factor are 299
and the fit is  3.37
## The number of observations was  649  with Chi Square =
2150.99  with prob <  4.8e-277
## The root mean square of the residuals is  0.09
## The df corrected root mean square of the residuals is  0.09
##
## RMSEA index =  0.099  and the 10 % confidence intervals are
0.094 0.102
## BIC =  214.83
##
## Measures of factor score adequacy
##                                                  g   F1*
F2*  F3*
## Correlation of scores with factors             0.97  0.08
0.83 0.87
## Multiple R square of scores with factors       0.93  0.01
0.69 0.76
## Minimum correlation of factor score estimates 0.86 -0.99
0.38 0.53
##
##  Total, General and Subset omega for each subset
##                                                   g  F1*
F2*  F3*
## Omega total for total scores and subscales    0.94 0.74
0.94 0.51
## Omega general for total scores and subscales  0.86 0.74
0.82 0.43
## Omega group for total scores and subscales    0.07 0.00
0.13 0.08


####################
#                  #
#    Exercise 10   #
#                  #
####################

iclust(data, title="Clastering solution")
```




```
## ICLUST (Item Cluster Analysis)
## Call: iclust(r.mat = data, title = "Clastering solution")
##
## Purified Alpha:
##  C21  C24
## 0.95 0.60
##
## G6* reliability:
## C21 C24
##   1   1
##
## Original Beta:
##  C21  C24
## 0.84 0.36
##
## Cluster size:
## C21 C24
##  20   6
##
```

```
## Item by Cluster Structure matrix:
##       O   P   C21   C24
## V1  C21 C21  0.69 -0.01
## V2  C21 C21  0.69 -0.01
## V3  C21 C21  0.67 -0.13
## V4  C21 C21  0.68  0.02
## V5  C21 C21  0.68 -0.02
## V6  C21 C21  0.75 -0.01
## V7  C21 C21  0.75  0.03
## V8  C21 C21  0.78  0.03
## V9  C21 C21  0.80  0.01
## V10 C21 C21  0.81  0.05
## V11 C21 C21  0.62 -0.03
## V12 C21 C21  0.67  0.08
## V13 C21 C21  0.80  0.01
## V14 C21 C21  0.69 -0.07
## V15 C21 C21  0.65  0.04
## V16 C21 C21  0.72  0.08
## V17 C21 C21  0.72  0.01
## V18 C21 C21  0.65 -0.05
## V19 C21 C21  0.61 -0.05
## V20 C21 C21  0.66 -0.03
## V21 C24 C24  0.03  0.36
## V22 C24 C24  0.02  0.31
## V23 C24 C24 -0.02  0.72
## V24 C24 C24 -0.02  0.57
## V25 C24 C24  0.00  0.27
## V26 C24 C24 -0.03  0.51
##
## With eigenvalues of:
##  C21  C24
## 10.0  1.4
##
## Purified scale intercorrelations
##  reliabilities on diagonal
##  correlations corrected for attenuation above diagonal:
##      C21   C24
## C21 0.95 -0.01
## C24 0.00  0.60
##
## Cluster fit =  0.91   Pattern fit =  0.99  RMSR =  0.05
```