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Abstract. *Spectral analysis belongs to widely used method for biomedical data exploration. In order to interpret spectrum correctly, prior knowledge how various phenomena affect the spectrum of analyzed signal is desirable. Spectrum of ECG signal is affected by waveform component shapes, their time positions within cardiac cycle as well as by regularity of heart period. This paper deals with effect of heart rate variability on ECG signal spectrum. Three representations are proposed to model variable heart timing in the case of normal sinus rhythm (pulse process with time jitter, random inter-pulse interval and integral pulse frequency modulation model). Properties of corresponding spectra are theoretically studied and compared with real-world signals from PhysioBank data archive.*

Keywords

ECG spectrum, heart rate variability, random pulse process, integral pulse frequency modulation model.

1. Introduction

Electrocardiographic signal (ECG) of healthy human (normal sinus rhythm) resembles periodic signal consisting of typical wavelets (P, Q, R, S, T, U waves). However, it is known that cardiac period of normal heart is not constant, but exhibit fluctuations according to sinus node activation. Mean heart rate is determined by sympatho-vagal balance and its modulation reflects short-term changes in autonomic nervous system activity related to blood pressure control and another phenomena. Therefore, analysis of heart rate variability is nowadays of great interest.

The heart rate analysis is conventionally performed on RR interval data derived from raw ECG based on fiducial point detection. Since RR interval data represents unevenly sampled sequence, various methods for spectral analysis can be found in literature dealing with this problem. As pointed in [7], large amount of original ECG data are unused and some information may be lost by this reduction. In next sections, we will explore how information about variable heart rate is embedded in the frequency spectrum of ECG signal instead of usual analysis based on cardiotachogram (RR interval sequences) derived from measured ECG.

2. Signal Models

Impulse sequence, whose parameters are random variables, is called a random impulse process. These impulse processes can be divided into two classes in dependency on statistic attributes of the impulse formation:

- Pulse process with time jitter
- Pulse process with random interbeat interval (for ECG)

Random impulse process is described by various parameters, which can be stochastic. Impulse sequences with different properties are analyzed on the basis of these parameters. In the next, we consider only sequences with constant width and amplitude of the impulse.

2.1 Pulse Process with Time Jitter

In this class we can evaluate the moment $t_{2n}^{(k)}$, in which arbitrary n-th impulse of process realization occurs, in the form

$$t_{2n}^{(k)} = nT + v_n^{(k)} \tag{1}$$

where T is impulse repeating period and v_n is random variable, which mean value equals zero. In this case, there is only one impulse in the repetition interval, thus the absolute value of v_n don't exceed $T/2$. Statistic properties of stochastic variables v_n will be expressed by their characteristic functions $\Theta_V(\omega)$. Because in the considered case the amplitude dispersion is $\sigma = 0$ and probability of impulse width $w_\tau = \delta(t-\tau_0)$, we obtain from the general form [7]:

$$F(\omega) = \frac{2\tau_0^2 a^2}{T} \left| g(\omega\tau_0) \right|^2 \left\{ 1 - \left| \Theta_V(\omega) \right|^2 + \right.$$
$$\left. + \frac{2\pi}{T} \left| \Theta_V(\omega) \right|^2 \sum_{r=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi r}{T} \right) \right\} \tag{2}$$

where τ_0 is impulse width, a is impulse amplitude, $g(\omega\tau_0)$ is the Fourier transform of one pulse.

2.2 Random Interbeat Interval

Time interval between two adjacent pulses of one realisation is identified by a symbol

$$\mu_n^{(k)} = t_{2n+2}^{(k)} - t_{2n}^{(k)} \tag{3}$$

We assume, that intervals are mutually indpendent and we have a characteristic function $\Theta_\mu(\omega)$ of a random variable μ_n, which does not depend on n; characteristic function is thus independent on location of interval μ_n on time axis.

Stationariness of the random impulse process can be achieved, if these conditions are fulfilled. In the next we restrict on the case, when impulse width and interbeat intervals are independent random variables. Special case is sequence of pulses with constant width τ_o, which arise independently from each other in random moments. Following analytic expression describes stationary random pulse process spectrum:

$$F(\omega) = \frac{2a^2\tau_0^2}{T}|g(\omega\tau_0)|^2\left\{1+\left(\frac{\sigma}{a}\right)^2 + +2\,\mathrm{Re}\frac{\Theta_\mu(\omega)}{1-\Theta_\mu(\omega)} + \frac{\delta(\omega)}{T}\right\} \tag{4}$$

where σ^2 is dispersion, a is mean value of impulse amplitudes. This form is power spectrum of sequence of pulses with constant width and random amplitude. It depends only on dispersion σ^2 and mean amplitude value a, but not on their distribution.

2.3 IPFM Model

Heart rate variability is commonly used tool for mechanism of cardiovascular system control analysis. The integral pulse frequency modulation model is a model for the generation of an event series, such as the series of heart beat occurrences, and assumes the existence of a continuous-time input signal which possesses a particular physiological interpretation. In the problems of IPFM model, we will assume, that sino-atrial node influences can be expressed with the modulation signal, which is integrated until it reaches the set limit. Then the impulse is generated.

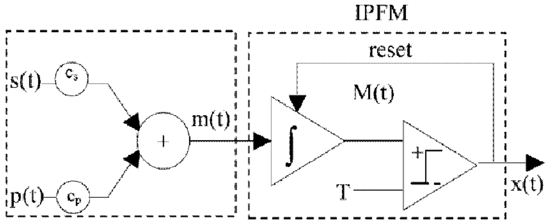

Fig. 1. Principle schematics of IPFM model.

Time courses, which depict moments of impulse formation, can be generated using IPFM model [6] as

$$k = \int_0^{t_k} \frac{1+m(t)}{T}dt \tag{5}$$

where k is an integer value, which represent number of impulses and t_k is the moment of k-th impulse formation. The term $(1+m(t))/T$ represents the instantaneous heart rate. T is the mean value of R-R intervals and $m(t)/T$ reflects the dynamic component with zero mean value. This component is usually negligible, compared to the heartbeat frequency ($m(t)$<<1).

Formerly expressed IPFM model can be rewritten as follows:

$$p(t_k) = kT - t_k = \int_0^{t_k} m(\tau)d\tau \tag{6}$$

where symbol $p(t_k)$ expresses a non uniform distribution and can be easily calculated through the moment of impulse formation. Every $p(t_k)$ represents the location shift of a given impulse, situated in the R-R interval. Within the sampled signal we can find the continuous shape of the signal. In general, using the form:

$$x = \int_0^{t(x)} \frac{1+m(t)}{T}dt \tag{7}$$

in which we transformed discrete signal to continuous signal, it is possible to transform also $p(t_k)$ to *continuous modulating* signal:

$$p(t) = x(t).T - t = \int_0^t m(\tau)d\tau \tag{8}$$

which modulates the position of signal formation. Therefore, this type of modulation is called PPM (Pulse Position Modulation).

The following terms constitute a jumping-off point to determine the spectrum of described signal, modulating the position of an impulse. It is useful to depict the beginning of beat formation t_k with Dirac delta functions, positioned in time t_k. Then, the time continuous function, representing a sequence of discrete events is

$$spc(t) = \sum_{k=-\infty}^{\infty} \delta(t-t_k) \tag{9}$$

Fourier transform of this signal shows its spectrum [3]

$$SPC(\omega) = \sum_{k=-\infty}^{\infty} e^{-j\omega t_k} \tag{10}$$

This signal represents the PPM discrete modulation in frequency domain, but there is need to evaluate a term, which includes also continuous modulation of pulse position. Time shift of k-th impulse to $t = 0s$ will be expressed with respect to previous term $p(t)$ as $(kT - p(t_k))$, or, generalized to the continuous function, $(kT - p(t))$. Then the term (9) transforms into

$$spc(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT + p(t_k)) = \\ = (1 + p'(t)) \sum_k \delta(t - kT + p(t)) \tag{11}$$

This term expresses a sequence of Dirac delta functions $\sum_{k=-\infty}^{\infty} \delta(t - kT)$ in time moment $(t - p(t))$. Evaluation of the signal spectrum is equivalent to PPM and thus $p'(t) = m(t)$. When we realize this substitution in the term (11), we obtain

$$spc(t) = \frac{1 + m(t)}{T} \left\{ 1 + 2 \sum_{n_0=1}^{\infty} \cos\left(\frac{2\pi n_0}{T} (t + p(t)) \right) \right\} \tag{12}$$

as a final expression of PPM in time domain. Spectrum of signal $spc(t)$ using frequency convolution is:

$$SPC(\omega) = \frac{1}{T}(2\pi \cdot \delta(\omega) + M(\omega)) * \\ * \left\{ 2\pi \cdot \delta(\omega) + 2 \sum_{n_0=1}^{\infty} FT\left\{ \cos\left(\frac{2\pi n_0}{T}(t + p(t)) \right) \right\} \right\} \tag{13}$$

3. Simulation Results

3.1 Spectrum of pulse processes with time jitter and random interbeat interval

As a reference realization for spectral analysis of random pulse process was a sequence of triangular impulses with constant height (a = 1) and width (τ = 0,1s). This shape of impulse produces spectral function, similar to square of $si(x)$ function.

While evaluating the final spectrum of a triangular impulse sequence, spectrum of one pulse will take place as a function of angular frequency ω:

$$g(\omega) = \frac{\tau}{2} si^2\left(\frac{\tau\omega}{4} \right) \tag{14}$$

or, after conversion to the frequency, measured in Hertz:

$$g(f) = \frac{\tau}{2} si^2\left(\frac{\pi f \tau}{2} \right) \tag{15}$$

Spectrum of impulse sequence with time jitter contains the characteristic function $\Theta_\mu(\omega)$. In this case it is considered as a Gaussian function [3] in the form

$$\Theta_\mu(\omega) = e^{-\frac{\omega^2}{4a}} \cdot e^{-j\omega T} \tag{16}$$

where $a = 1/2\sigma^2$ and σ is standard deviation.

Spectrum of random interbeat interval impulse sequence also contains the characteristic Gaussian function

$$\Theta_V(\omega) = e^{-\frac{\omega^2}{4a}} \tag{17}$$

where $a = 1/2\sigma'^2$. It is necessary to realize, that σ' is not equal to standard deviation σ, which was used in previous case. It's because the addition of dispersions of two adjacent pulses positions would be doubled. To achieve equal deviations in both cases, they must be related in the manner $\sigma' = \sigma/\sqrt{2}$. The impulse repetition period in both random pulse processes was set to T = 1s.

The following figures display power spectra of impulse sequences. They correspond to the terms for random pulse processes with time jitter (2) and random interbeat interval (4). The changing parameter was the standard deviation σ.

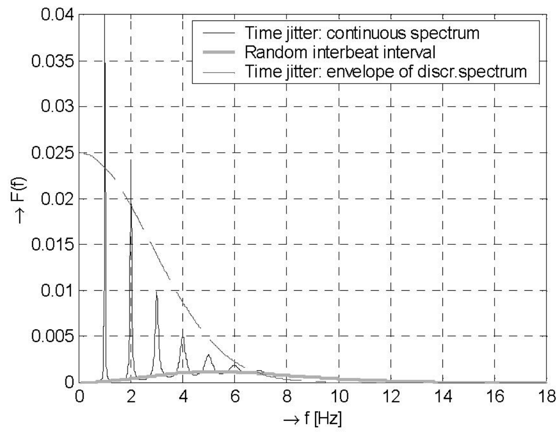

Fig. 2. Power spectra of sequences of impulses with time jitter and random interbeat interval for standard deviation σ = 0.05s.

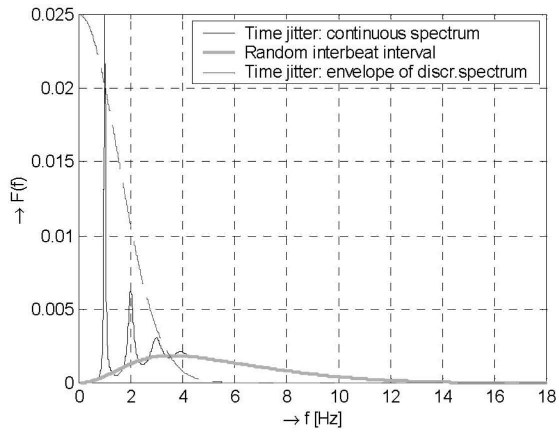

Fig. 3. Power spectra of sequences of impulses with time jitter and random interbeat interval for standard deviation σ = 0,1s.

3.2 Spectrum of signal generated by IPFM model

The form (11) consists of first term, representing AM modulation $(1+m(t))$, which multiplies the second term, which is the sum of frequency modulated (FM) elements. These are overtones with carrier frequency $2\pi n_0/T$ i.e. multiples of middle frequency of pulses (See Fig.4). The modulation itself is represented by expression $\frac{2\pi}{T}n_0 \cdot p(t)$. It is the change of instantaneous phase for n_0-th carrier wave. Derivation of instantaneous phase equals to the deviation of instantaneous frequency f:

$$\Delta\omega(t) = \frac{2\pi}{T}n_0 \cdot p'(t) = \frac{2\pi}{T}n_0 \cdot m(t) \tag{18}$$

where $\Delta\omega(t)$ is index of FM modulation.

In the following we will assume, that the modulating signal $m(t)$ is harmonic. Index of FM modulation depends only on n_0. The bandwidth of modulated signal increases proportionally to n_0.

The influence of AM modulation exhibit as the AM modulation term in time domain is multiplied by FM term (12). That is the convolution of modulating signal spectrum with particular FM spectra. Because $m(t) << 1$ and instantaneous frequency $f << 2\pi/T$, the bandwidth slightly expands, but this change is negligible in general in cases, when $n_0 > 1$.

In the spectrum (13) are Dirac delta functions $\delta(\omega)$ present too, but these spectral parts are not interesting for our analysis. We obtain the final formula for spectrum of triangular pulse sequence as multiplication of PPM modulated Dirac delta functions (13) and spectrum of one pulse (14).

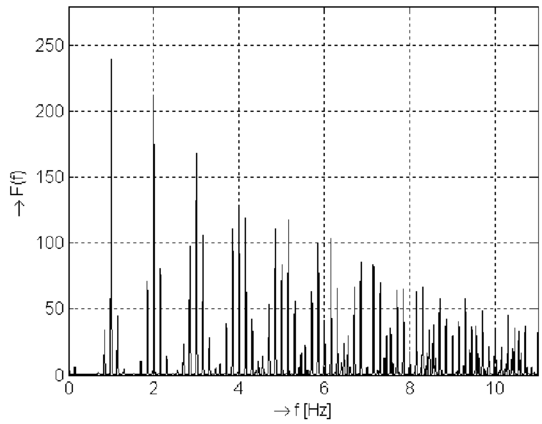

Fig. 4. Frequency spectrum of signal generated by IPFM model.

4. Discussion

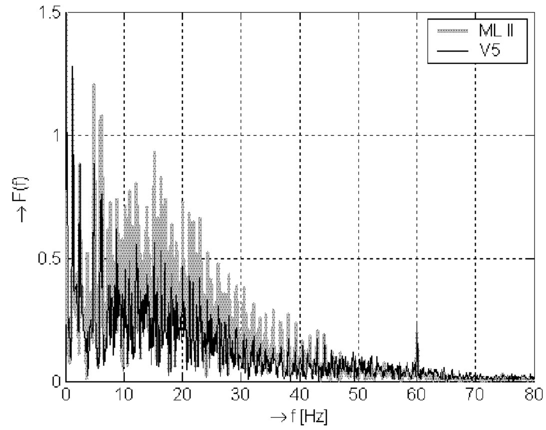

Fig. 5. Frequency spectrum of two ECG signals present in the record 100 [1].

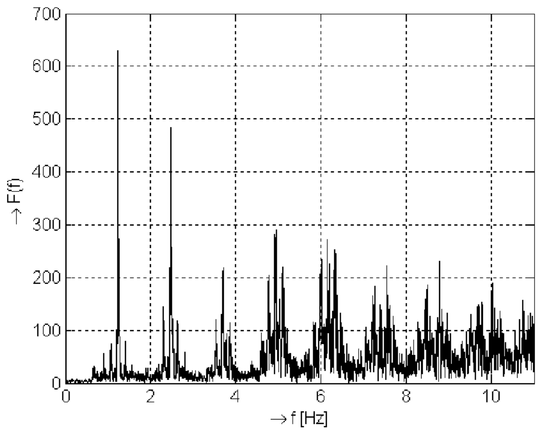

Fig. 6. Frequency spectrum of segment of ECG signal present in the record 100 [1].

In order to compare features of modeled spectra with spectra of real-world ECG we have analyzed selected records from PhysioBank data archive [1]. Spectra have been computed as periodograms with Hanning windows without segment averaging that contributed to variance of spectral estimation but kept good frequency resolution. Spectrum shown in Fig. 6 resembles power spectral density corresponding to the model with random RR interval (Fig. 2 and Fig.3). In Fig. 6 we can see spectrum of different segment extracted from the same record. In this case RR tachogram exhibited oscillating pattern, which can be detected as sidelobes near dominant spectral peaks - harmonics corresponding to mean hear rate. Depicted spectrum closely resembles spectrum derived from IPFM model (Fig. 4).

Both abovementioned spectra correspond to model representing normal sinus rhythm. Different spectral patterns can be found in the cases of arrhythmia (Fig. 7, ventricular fibrillation) or paced rhythm (Fig. 8).

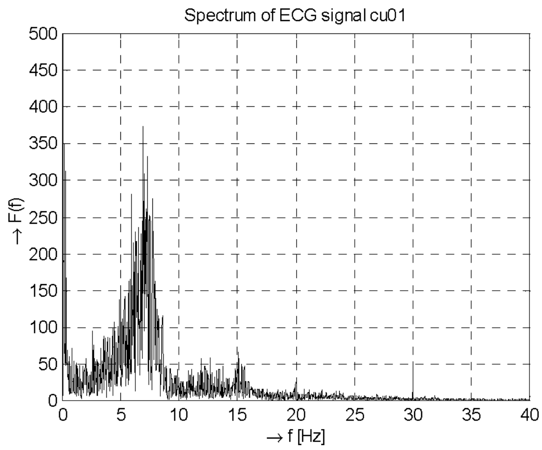

Fig. 7. Frequency spectrum of ECG signal cu01.

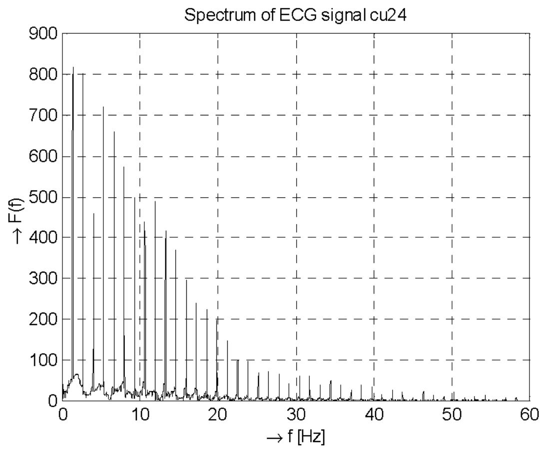

Fig. 8. Frequency spectrum of ECG signal cu24.

Conclusion

In the paper we gave insight into structure of ECG signal spectrum. Spectra of measured ECG records have been confronted with spectra produced by three signal models. The model with time jitter is probably not realistic, because it assumes only limited deviation from exactly periodic timing in long-time scale. IPFM model is widely used in sinus node modeling.

Frequency spectrum of ECG reflects heart rate variability, potentially retaining more information than usual analysis based on RR interval. However, this information is embedded in ECG spectrum in complicated way, as we shown by analysis of spectra resulting from considered models.

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