

The Probability Distribution of Daily Rainfall in the United States

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Abstract: Choosing a probability distribution to represent the precipitation depth at various durations has long been a topic of interest in hydrology. Early study into the distribution of wet-day daily rainfall has identified the 2-parameter Gamma (G2) distribution as the most likely candidate distribution based on traditional goodness of fit tests. This paper uses probability plot correlation coefficient test statistics and L-moment diagrams to examine the complete series and wet-day series of daily precipitation records at 237 U.S. stations. The analysis indicates that the Pearson Type-III (P3) distribution fits the full record of daily precipitation data remarkably well, while the Kappa (KAP) distribution best describes the observed distribution of wet-day daily rainfall. We also show that the G2 distribution performs poorly in comparison to either the P3 or KAP distributions.

1. Introduction

Establishing a probability distribution that provides a good fit to daily rainfall depth has long been a topic interest in the fields of hydrology, meteorology, and others. The investigations into the daily rainfall distribution are primarily spread over three main research areas, namely, (1) stochastic precipitation models, (2) frequency analysis of precipitation, and (3) precipitation trends related to global climate change.

The first research area of interest is stochastic precipitation modeling. The purpose of such models is not so much to investigate the properties of rainfall, but instead to be able to produce artificially generated rainfall sequences that can be used as inputs in other models to explore the behavior of hydrologic systems (Buishand, 1978). A wide range of types of stochastic rainfall generators exist (see the introduction of Mehrotra et al., 2006 for a nice review). We are only concerned with the class of “two-part” stochastic daily precipitation models that utilize a probability distribution function to describe rainfall amounts on wet-days, while rainfall occurrence is separately described using a Markov model or alternating renewal process (Gabriel and Neumann, 1962; Buishand, 1978; Geng et al., 1986; among

many others). This study is concerned only with the distribution chosen to model the daily rainfall depths.

Thom's 1951 suggestion of the two parameter Gamma (G2) distribution function for wet-day amounts seems to carry considerable weight. Buishand (1978) lends support to the suggestion of the Gamma distribution by showing that for the wet-day series at six stations, the ratio of the empirical Coefficient of Variation to Coefficient of Skewness was quite close to the theoretical value of two for a Gamma distribution. Geng et al. (1986) use a simple regression to show that the beta parameter of the Gamma distribution for a given month can be predicted reasonably well by the average rainfall per wet-day in that month.

While the 2 parameter Gamma is by far the most preferred distribution for wet-day rainfall amounts, other distributions have been suggested. Woolhiser and Roldan (1982) and Wilks (1998) both suggest the use of a three-parameter mixed exponential distribution instead of the Gamma. The three-parameter exponential distribution fits wet-day amounts by mixing two distinct exponential distributions (each with its own mean parameter) with a probability parameter that chooses which one to use. Through a variety of goodness of fit tests and log-likelihood analyses, the mixed exponential is shown as being preferred to the Gamma by Wilks. The Weibull (W2) and to a lesser extent the exponential distribution have also been suggested for modeling daily rainfall amounts (Duan et al., 1995; Burgueno et al., 2005).

The second group of literature is related to precipitation frequency analyses. A key step in frequency analysis of precipitation involves selection of a suitable distribution for representing precipitation depth to investigate the extremes. While these analyses can be conducted for multiple rainfall durations, we focus on those that investigate the 1-day duration.

For many years, the most common approach to summarizing precipitation frequency analyses in the U.S. was the work of Hershfield (1962), which is commonly referred to as TP-40. Hershfield (1962) fit a Gumbel distribution to the AMS series of 24-hour rainfall. More recently, several authors completing these types of analyses use the method of L-moments and other methods that are more powerful than more traditional goodness of fit measures. In the context of a national revision to the TP-40 rainfall frequency atlas and after the application of L-moment goodness-of-fit evaluations, Bonnin et al. (2006) fit a GEV distribution to the AMS of rainfall.

Bonnin et al. (2006) performed a very comprehensive national assessment of rainfall frequency by applying the most up-to-date developments in regional frequency analysis to series of annual maximum n-minute precipitation. Using both at-site and regional L-moment goodness-of-fit results, climatic considerations and sensitivity testing, the GEV distribution is selected to best represent the underlying distributions of all daily and hourly AMS rainfall data. GEV is also selected for the 5-, 10-, and 15-minute AMS rainfall data. Naghavi and Yu (1995) also chose the GEV for a study of rainfall extremes in Louisiana.

While the results of Bonnin et al. (2006) apply to the United States, other authors have found similar results using similar methods in other parts of the world. For example, Pilon et al. (1991) select the GEV in Ontario, Canada, Park and Jung (2002) successfully use the Kappa distribution to generate extreme precipitation

quantile maps in Korea. Deidda and Puliga (2006) find some weak evidence for the Generalized Pareto distribution for left-censored records in Sardinia.

Interestingly, while a great deal of attention is given to fitting distributions to the relatively short AMS series of rainfall depth, very few studies directly explore the probability distribution of the complete series of daily rainfall (including zeros) or the wet-day series of daily rainfall (zeros excluded). Shoji and Kitaura (2006) investigate both full-record and wet-day daily precipitation series, but include only the normal, lognormal, exponential, and Weibull distributions as candidate distributions, and do not employ modern regional hydrologic methods such as the method of L-moments.

The third body of research includes precipitation trend literature, where we find a reliance on previous studies of the probability distribution of daily rainfall in selecting candidate distributions. Almost universally, the 2 parameter Gamma distribution is accepted without serious consideration of alternative distributions. For instance, Groisman et al. (1999) write simply, "It is widely recognized that the distribution of daily precipitation totals, P , can be approximated by the gamma distribution."

This is an interesting contrast to the precipitation frequency analysis literature where a Gamma distribution is fit to wet-day series for the purpose of examining extreme rainfalls instead of using the AMS series fitted by a GEV or other distribution. Yoo et al. (2005) explain that conventional frequency analysis (using AMS) cannot expect to predict precipitation changes resulting from climate change, while an examination of the differences in the gamma distribution's parameters (fitted to the whole wet-day record) might. They find that modifying the parameters of the daily gamma distribution can explain changes in rainfall quantiles predicted by General Circulation Models (GCM) under various climate change scenarios. Wilby and Wigley (2002) plot the expected 100-year changes in the shape and scale parameter of the G2 distribution according to two GCM models' predictions.

The precipitation trend and climate change literature use the G2 distribution as a powerful tool to examine not only the possible changes in rainfall patterns, but also the relative rate of change in a geospatial context through mapping. In summary, there are a wide variety of previous studies which have explored the probability distribution of daily rainfall for the purposes of rainfall frequency analysis, stochastic rainfall modeling and for trend detection. There seems to be a consensus that annual maxima follow either a GEV, Gumbel or Gamma probability density function (pdf) and that series of wet-day daily rainfall totals may follow a Gamma pdf, or in some cases a mixed Exponential. However, we are unaware of any studies that have used recent developments in regional hydrologic frequency analysis such as L-moment diagrams or probability plot goodness of fit evaluations to evaluate the probability distribution of the *complete series* of daily rainfall. This paper also seeks to re-examine the question of which continuous distribution best fits wet-day, observed daily precipitation data.

Our primary objective is to determine a suitable distribution of full-record series and wet-day series of daily rainfall using L-moment diagrams and probability plot correlation coefficient goodness of fit statistics. These evaluations yield very different conclusions than previous research on this subject.

2. Dataset

We employ a dataset comprised of daily rainfall depths at 237 first-order stations well distributed across 49 U.S. states. The mean record length is 24,657 days (67.5 years). In addition to complete series of daily rainfalls, a wet-day series was constructed for each station by excluding zero and “Trace” values (those with less than 0.01” recordable rainfall). Wilks (1990) discusses other ways to treat trace rainfall and left-censored data, but for convenience, they are simply excluded. The mean wet-day record length is 7219 days (equivalent to nearly 20 years).

The data were quality controlled to remove null values. When greater than 6 null values occurred in a given year or greater than 3 in a given month, the full year of data was removed. When fewer than these numbers of null values were present, they were treated as zeroes.

3. Methodology:

This section describes the methods of analysis used in distributional hypothesis evaluations, namely, L-Moment diagrams and probability plot correlation coefficient analysis.

3.1 L-Moment Diagrams

L-moment analysis is a widely accepted approach for evaluating the goodness of fit of alternative distributions to observations. The theory and application of L-moments introduced by Hosking (1990) is now widely available in the literature (see Hosking and Wallis, 1997 and Stedinger et al., 1993) hence it is not reproduced here.

The distribution of daily rainfall totals is highly skewed due to the large proportion of days with zero rainfall (rarely less than 60%). L-moment ratios are approximately unbiased in comparison to conventional moment ratios, which can exhibit enormous downward bias, even for very large samples (Vogel and Fennessey, 1994). Higher order conventional moment ratios such as skewness and kurtosis are very sensitive to extreme values and can exhibit enormous downward bias even for large sample sizes. (Vogel and Fennessey, 1994).

L-moment ratio diagrams provide a convenient visual way to view the characteristics of sample data compared to theoretical statistical distributions. The L-moment diagrams: L-Kurtosis (τ_4) vs L-Skew (τ_3) and L-Cv (τ_2) vs L-Skew (τ_3) enable us to compare the goodness of fit of a range of three-parameter, two parameter, and one parameter (or special case) distributions. Table 1 displays distributions analyzed by means of the τ_4 vs τ_3 L-moment ratio diagrams.

While L-moment ratio diagrams have been used before to examine the distribution of series of annual maximum precipitation data (Lee and Maeng, 2003; Park and Jung, 2002; Pilon et al., 1991) and left-censored records (Deidda and Puliga, 2006), this may be the first time a set of uncensored daily precipitation records have been subjected to such an analysis. L-moment estimators were chosen in this study for a variety of reasons: (1) they are easily computed and nicely summarized by Hosking and Wallis (1997) for all the cases considered in this study, (2) the sample

sizes considered are extremely large, and as a result it is unlikely that parameter estimation issues will be nearly as big an issue as distribution choice.

Table 1: Theoretical Probability Distributions presented on the L-Kurtosis, τ_4 , vs L-Skew, τ_3 , L-moment diagram.

Distribution	Abbreviation	Parameters	L-moment diagram	PPCC Test?
Kappa	KAP	4		Yes
Generalized Extreme Value Type III	GEV	3	τ_4 vs τ_3	Yes
Generalized Logistic	GLO	3	τ_4 vs τ_3	Yes
Generalized Pareto	GPA	3	τ_4 vs τ_3	Yes
Lognormal	LN3	3	τ_4 vs τ_3	Yes
Pearson Type III	P3	3	τ_4 vs τ_3	Yes
Gamma	G2	2	τ_2 vs τ_3	Yes
Generalized Pareto	GP2	2	τ_2 vs τ_3	No
Lognormal	LN2	2	τ_2 vs τ_3	No
Weibull	W2	2	τ_2 vs τ_3	No
<i>Exponential</i>	E	2	τ_4 vs τ_3	No
<i>Gumbel</i>	G	2	τ_4 vs τ_3	No
<i>Normal</i>	N	2	τ_4 vs τ_3	No
<i>Logistic</i>	L	2	τ_4 vs τ_3	No
<i>Uniform</i>	U	1	τ_4 vs τ_3	No

3.2 Probability Plot Correlation Coefficient Goodness-of-fit Evaluation

Probability plots are constructed for each of the full record and wet-day series using L-moment estimators of the distribution parameters (see Hosking and Wallis, 1997) for the distributions indicated in last column of Table 1.

The goodness of fit of each probability plot is summarized using a probability plot correlation coefficient (PPCC, or simply, r). The PPCC has been shown to be a powerful statistic for evaluating alternative distributional hypotheses (Stedinger et al. 1993).

4. Results

Figure 1 displays the L-Cv vs L-Skew L-moment diagram. The various curves represent the theoretical relationship between L-Cv and L-Skew for the distributions indicated. Each plotted point represents the empirical relationship between L-Cv and L-Skew for a single precipitation station. By comparing the empirically derived points with the theoretical curves, it is possible to see the degree to which the statistical character of the data record matches those of the candidate distributions.

Figure 1 illustrates a nearly linear relationship between the L-Skew and L-Cv for full-record series. This line of points, however, does not fall along **any** of the theoretical curves, demonstrating the 2-parameter Gamma distribution does not describe the tail behavior of complete series of rainfall as has often been assumed in the past.

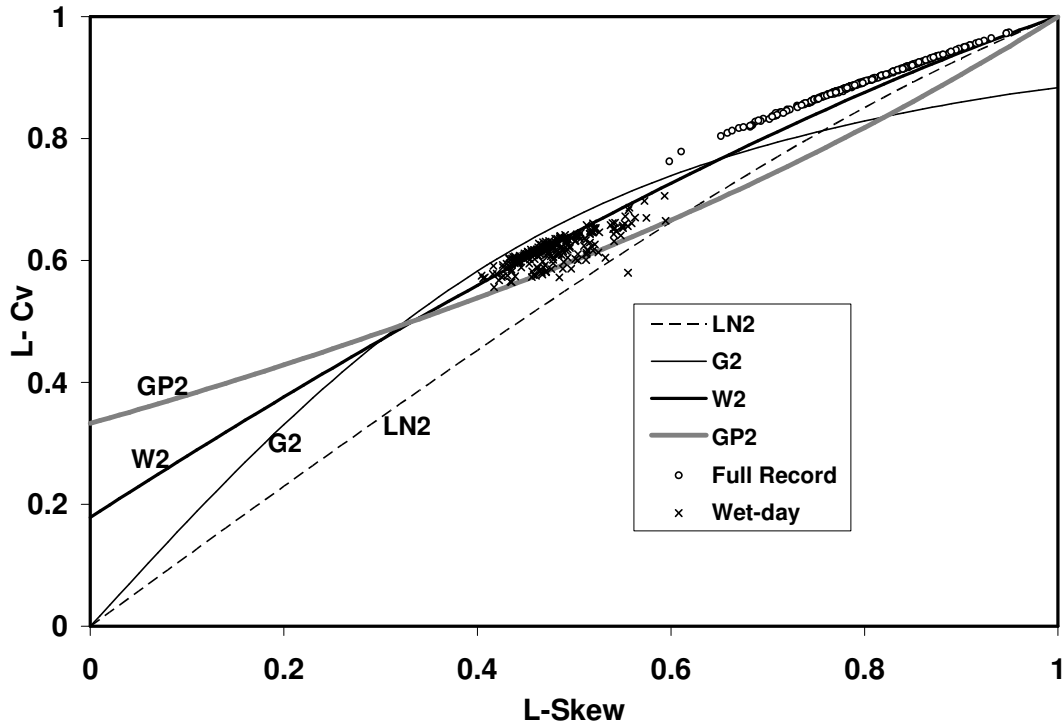


Figure 1: L-Cv vs L-Skew L-moment ratio diagram. L-moments of the full-record series (o) and wet-day series (x) are plotted.

The wet-day series' points fall primarily in a region bounded by the G2 and GP2 theoretical curves, with the W2 passing through some of the points. The pattern does not indicate a clearly preferred distribution, especially considering that the large sample sizes associated with these series result in negligible sampling variability. Archfield et al. (2007, *Figure 3*) used L-moment diagrams for complete series of daily streamflow observations to demonstrate that the sampling variability in L-moment ratios is negligible for the sample sizes considered in this study. Thus, the scatter shown in Figure 1 is likely due to real distributional differences rather than due to sampling variability as is often the case when one constructs L-moment diagrams for short AMS rainfall records.

Figure 2 compares theoretical distributional relationships between L-Skew vs L-Kurtosis with their estimated values from the stations. Similar to Figure 1, the plotted points for the full record series seem to follow a very clear relationship, but this time that relationship is remarkably similar to the theoretical curve for a Pearson Type-III (P3) distribution. In fact, the P3 seems to be the only 3-parameter distribution that can possibly fit the full record data. It is worth noting that the overall lower bound of all distributions falls not far below the P3 curve at high L-Skew values.

Again as in Figure 1, the wet-day series shows more scatter on the plot than the complete series. In this case, the closest theoretical curve to the wet-day points is also the P3, but the fit is less striking. Nevertheless, the data fall into space that can be well represented by the Kappa distribution, which occupies not a curve, but a

whole region of the L-Kurtosis vs L-Skew plot. See Hosking (1994) for a very complete description of the 4-parameter Kappa distribution.

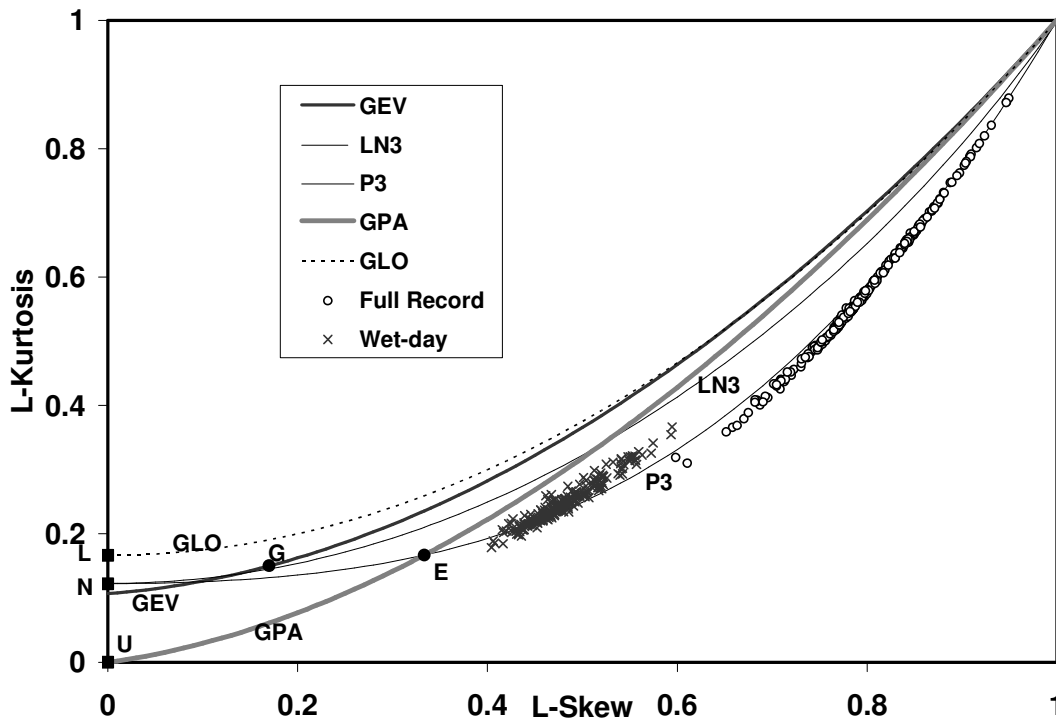


Figure 2: L-Skew vs L-Kurtosis plot for the Full-record (o) and Wet-day (x) data series. L,N,U,G, and E appear as a single point.

The L-moment diagrams successfully identify several potential candidate distributions for both full-record and wet-day data series. The probability plot correlation coefficient goodness of fit statistic offers another quantitative method for comparing the goodness of fit of different distributions to the observations. Table 2 summarizes the central tendency and spread of the values of the PPCC for each of the distributions for both the full-record and wet-day series. The highest values for the mean, median, 95th percentile, and 5th percentile are shown in bold.

Table 2: Central Tendency and Spread of values of PPCC for the 237 stations.

Distribution	Full Record		Percentiles		Wet Day		Percentiles	
	Mean	Median	95th	5th	Mean	Median	95th	5th
P3	0.996	0.996	0.9991	0.9892	0.995	0.997	0.9995	0.9872
GEV	0.595	0.593	0.6755	0.5166	0.933	0.938	0.9609	0.8944
GPA	0.619	0.618	0.7145	0.5339	0.979	0.983	0.9949	0.9500
GLO	0.594	0.592	0.6708	0.5172	0.912	0.915	0.9423	0.8734
LN3	0.798	0.808	0.8731	0.7055	0.984	0.986	0.9924	0.9727
G2	0.995	0.995	0.9988	0.9876	0.993	0.995	0.9990	0.9789
KAP	0.978	0.978	0.9926	0.9644	0.997	0.999	0.9997	0.9915

Table 2 indicates that for the full-record series, only the G2, P3, and KAP distributions perform well, and are again the best performing distribution for the wet day series. Figures 2a and b show detailed views of the plots for the G2, P3, and KAP distributions. We conclude from these boxplots that the P3 is the best performing distribution on average for the full-record series, but the KAP distribution shows the highest PPCCs on average for the wet-day series.

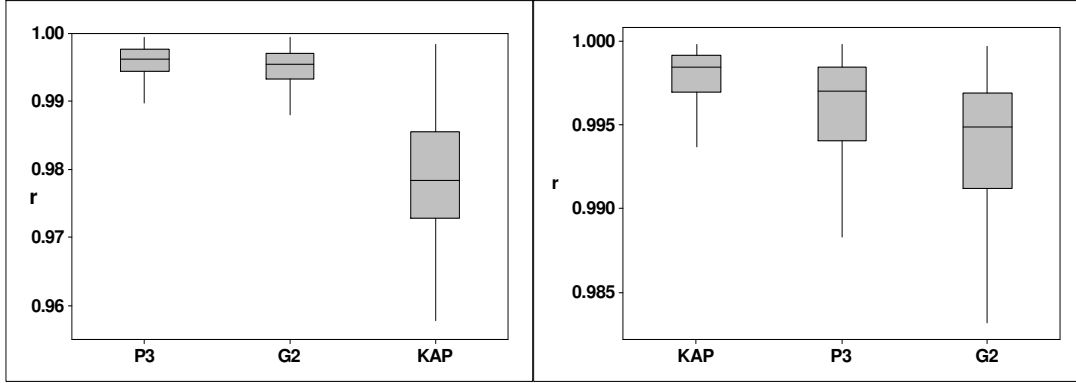


Figure 2: PPCC for KAP, P3 and G2 distributions for a) (left) Full-record, and b) (right) Wet-day data series

The box plots of the PPCC values compare the relative performance of each distribution, but in cases where the ranges of the box plots are overlapping, they do not differentiate on a station by station basis which distribution has the highest PPCC. In such a case, pairwise comparisons of the PPCC values of two highly performing distributions for all the stations may be highly instructive. A simple graphical method can accomplish this goal.

Figures 3a and b compare the PPCC values of the P3 (vertical axis) and G2 (horizontal axis) for the full record series and wet day series, respectively. Points lying above the diagonal line mean that the P3 distribution had a higher PPCC for that particular station, and points lying below indicate the G2 resulted in a higher PPCC. The full-record plot shows that in nearly every case, the P3 outperforms the G2. The wet-day plot shows that the P3 distribution in fact performs significantly better than the G2 distribution in many cases. Thus, we conclude the P3 better represents wet-day rainfall in nearly every case than the more commonly used G2 distribution.

Figures 4a and b display similar plots comparing the KAP (vertical axis) and P3 (horizontal axis) distribution, for the full-record and wet-day series, respectively. For the full-record series the 3 parameter P3 distribution unexpectedly outperforms the 4 parameter KAP distribution. This is somewhat surprising in that the extra information contained in the 4th parameter (essentially a second shape parameter in the case of the Kappa distribution) would be expected to lead to a better goodness-of-fit. The L-moment diagram (Figure 3), however, shows that the fit of the full record data to the P3 theoretical curve is so good that a 4th parameter could be extraneous. Additionally, it should be noted that the pattern of the full record stations on the L-Cv vs L-Skew plot approach the overall lower bound for all distributions, a place where the Kappa distribution parameter estimates may become less accurate. The “h” shape

parameter, for example, approaches infinity in this region. (Hosking and Wallis, 1997, *Figure A.1*) For the wet-day records, the KAP distribution outperforms the P3 for a large majority of stations.

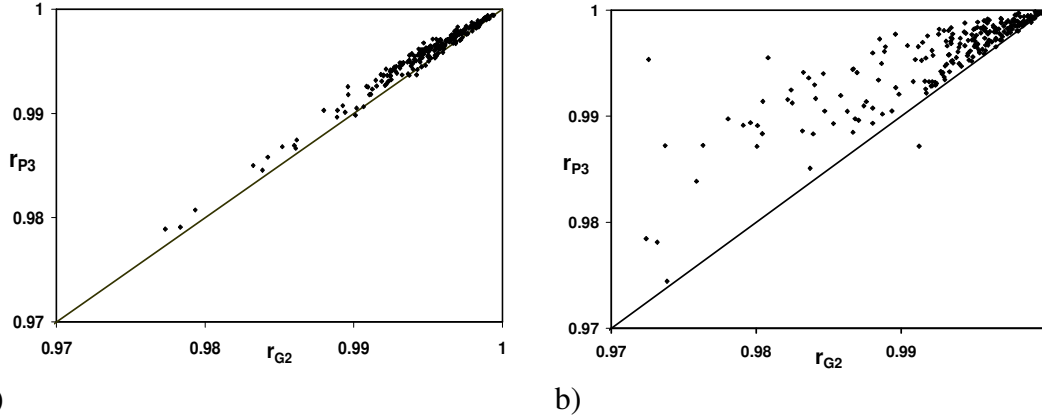


Figure 3a,b Comparison of PPCC (r) values for the P3 (vertical axis) and G2 (horizontal axis) distributions for the a) full-record, and b) wet-day series.

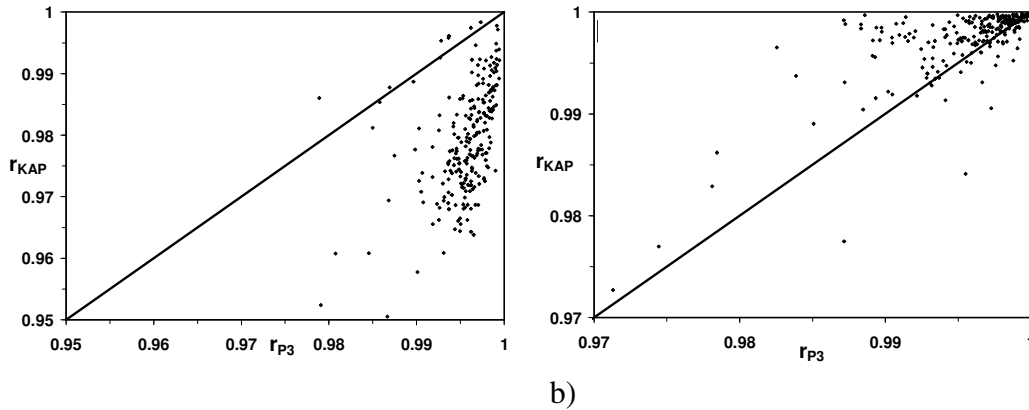


Figure 4a, b: Comparison of r values for P3 (horizontal axis) and KAP (vertical axis) distributions. Full-record (a) and wet-day (b) values are plotted.

5. Conclusions

This study has demonstrated that L-moment diagrams and probability plot goodness of fit evaluations provide new insight into the distribution of very long series of daily rainfall. Though the commonly used 2-parameter Gamma distribution performs fairly well on the basis of traditional goodness-of-fit tests, L-moment diagrams and probability plot correlation coefficient goodness of fit evaluations reveal that very long series of uncensored daily rainfall observations are better approximated by a Pearson-III distribution and importantly, they do not resemble any of the other commonly used distributions.

We conclude that for representing uncensored, full record daily rainfall, the 3-parameter Pearson-III distribution performs remarkably well. For cases in which only wet-day precipitation amounts are required, the 4-parameter Kappa distribution

should be the distribution of choice when only continuous distributions are considered.

Identifying the distribution of daily rainfall could have a wide range of applications in hydrology, engineering design, and climate research.

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