Exam Prep Section 6 - CS61A Spring 2018

Worksheet 6: Orders of Growth & Linked Lists

1. Interpretation (Fa14 Mock Final Q5e)

```
def g(n):
    if n % 2 == 0 and g(n + 1) == 0:
        return 0
    return 5
```

Circle the correct order of growth for a call to `g(n)`:

$\Theta(1)$ $\Theta(\log n)$ $\Theta(n)$ $\Theta(n^2)$ $\Theta(b^n)$

Solution: Θ(1)

2. The weakest link (Su15 Midterm 2 Q5d)

(2 pt) Consider the following linked list functions:

```
def append(link, value):
    """Mutates link by adding value to the end of link."""
    if link.rest is Link.empty:
        link.rest = Link(value)
    else:
        append(link.rest, value)

def extend(link1, link2):
    """Mutates link1 so that all elements of link2 are added to the end
    of link1.
    """
    while link2 is not Link.empty:
        append(link1, link2.first)
        link2 = link2.rest
```

Circle the order of growth that best describes the runtime of calling **append**, where n is the number of elements in the input `link`.

$O(1)$ $O(\log n)$ $O(n)$ $O(n^2)$ $O(2^n)$

Assuming the two input linked lists to **extend** both contain n elements, circle the order of growth that best describes the runtime of calling **extend**.

$O(1)$ $O(\log n)$ $O(n)$ $O(n^2)$ $O(2^n)$

Solution 1: O(n)
Solution 2: O(n^2)

3. Not with a fizzle, but with a bang (Su13 Midterm 2 Q2b)

(2 pt) Now consider the following function definitions.

```
def boom(n):
    if n == 0:
        return "BOOM!"
    return boom(n - 1)

def explode(n):
    if n == 0:
        return boom(n)
    i = 0
    while i < n:
        boom(n)
        i += 1
    return boom(n)
```

Circle the correct order of growth for a call to `explode(n)`:

$\Theta(1)$ $\Theta(\log n)$ $\Theta(\sqrt{n})$ $\Theta(n)$ $\Theta(n^2)$ $\Theta(n^3)$ $\Theta(2^n)$

Solution: Θ(n^2)

4. Not with a fizzle, but with a bang (Su13 Midterm 2 Q2c)

(2 pt) Now consider the following function definition.

```
def dreams(n):
    if n <= 0:
        return n
    if n > 0:
        return n + dreams(n // 2)
```

Circle the correct order of growth for a call to `dreams(n)`:

$\Theta(1)$ $\Theta(\log n)$ $\Theta(\sqrt{n})$ $\Theta(n)$ $\Theta(n^2)$ $\Theta(n^3)$ $\Theta(2^n)$

Solution: Θ(log n)

5. Various Programs (Sp14 Final Q5c)

(2 points) Give worst-case asymptotic $\Theta(\cdot)$ bounds for the running time of the following code snippets. (Note: although we haven't explicitly talked about it, it is meaningful to write things with multiple arguments like $\Theta(a+b)$, which you can think of as "$\Theta(N)$ where $N = a + b$.")

```
def a(m, n):
    for i in range(m):
        for j in range(n // 100):
            print("hi")
```

Bound: _____

```
def b(m, n):
    for i in range(m // 3):
        print("hi")
    for j in range(n * 5):
        print(bye")
```

Bound: _____

```
def d(m, n):
    for i in range(m):
        j = 0
        while j < i:
            print("hi")
            j = j + 100
```

Bound: _____

```
def f(m):
    i = 1
    while i < m:
        i = i * 2
    return i
```

Bound: _____

Solution: Θ(mn)

Solution: Θ(m+n)

Solution: Θ(m^2)

Solution: Θ(log m)

6. OOG Potpourri

What is the order of growth of each of the following functions?

a. Weighted

```
def weighted_random_choice(lst):
      temp = []
      for i in range(len(lst)):
            temp.extend([lst[i]] * (i + 1))
      return random.choice(temp)
```

Solution: Θ(n^2)

b. Iceskate

```
def ice(n):
      skate = n
      def rink(n):
            nonlocal skate
            print(n)
            if skate > 0:
                  skate -= 1
                  rink(skate)
            return skate
      return rink(n//2)
```

Solution: Θ(n)

c. Olympics

```
def olym(pics):
    total, counter = 0, 0
    for i in range(pics):
        while counter == 0:
            total += (i + counter)
            counter += 1
        return total
```

Solution: Θ(1)

d. Palindrome

```
def is_palindrome(s):
    if len(s) <= 1:
        return True
    return s[0] == s[-1] and is_palindrome(s[1:-1])
```

Solution: Θ(n^2)

e. More Palindrome

```
def is_palindrome2(s):
    for i in range(len(s) // 2):
        if s[i] != s[-i-1]:
            return False
    return True
```

Solution: Θ(n)

f. Havana

```
def camila(m, n):
     if n <= 1:
          return 0
     cabello = 0
     for i in range(3 ** m):
          cabello += i // n
     return cabello + camila(m - 5, n // 3)
```

Solution: Θ(3^m log n)

g. Barbados

```
def ri(na):
     if na < 1:
          return na
     def han(na):
          i = 1
          while i < na:
               i *= 2
          return i
     return ri(na / 2) + ri(na / 2) + han(na - 2)
```

Solution: O(nlogn)

7. Conserve Links (Challenge Linked List problem)
Implement conserve_links, as described below.

```
def conserve_links(a, b):
      """Makes Linked List a share as many Link instances as possible
      with Linked List b.a can use b's i-th Link instance as its i-th
      Link instance if a and b have the same element at position i.

      Should mutate a. b is allowed to be destroyed. Returns the new
      first Link instance of a.
      """
       if a.first == b.first:
            b.rest = conserve_links(a.rest, b.rest)
           return b
      else:
           return a
```

8. Slice Reverse (Challenge Linked List problem)

Implement `slice_reverse` which takes a linked list `s` and mutatively reverses the elements on the interval, $[i, j)$ (including i but excluding j). Assume `s` is zero-indexed, $i > 0$, $i < j$, and that `s` has at least j elements.

```
def slice_reverse(s, i, j):
    """
    >>> s = Link(1, Link(2, Link(3)))
    >>> slice_reverse(s, 1, 2)
    >>> s
    Link(1, Link(2, Link(3)))
    >>> s = Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> slice_reverse(s, 2, 4)
    >>> s
    Link(1, Link(2, Link(4, Link(3, Link(5)))))
    """
    start = s

    for _ in range(i - 1):

        start = start.rest

    reverse = Link.empty

    current = start.rest

    for _ in range(j - i):

        rest = current.rest

        current.rest = reverse

        reverse = current

        current = rest

    start.rest.rest = current

    start.rest = reverse
```