Image Segmentation Using Markov Random Field Model Learning Feature and Parallel Hybrid Algorithm

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Abstract

In this paper, a new notion of image segmentation using Markov Random Field (MRF) model learning feature is addressed. The segmentation problem is formulated as pixel labeling problem in a supervised framework. MRF model is employed to model the class labels. This model learns a given training image derived from a class of images. The model having learnt is validated for other images of the same class. The learning problem is formulated using Conditional Pseudo Likelihood (CPL) approach and the parameters are estimated using homotopy continuation method. This learning attribute is exploited to obtain the MAP estimates of the class labels using proposed Hybrid Tabu Search (HTS) and Parallel Hybrid Tabu Search (PHTS) algorithms with a view to reduce the computational burden. The performance of these algorithms is compared with that of the Simulated Annealing (SA) algorithm. This learning feature avoids the parameter estimation of each and every individual image of a class of images.

1. Introduction

Image segmentation is one of the early vision problem and has wide application domain. The problem becomes more compounded while segmenting noisy scenes. The model based approaches for such problems are very popular for the last three decades. Stochastic models, specifically, Markov Random Field (MRF) model provides a better framework for many complex problems in image segmentation [1,2,3]. The potentiality of the model was enhanced further by incorporation of line fields that attributes to the edge preserving property [1]. Often, the segmentation problem is formulated as pixel labeling problem in a supervised framework and the pixel labels are estimated using Maximum a-Posterior Estimation (MAP) [1,2,4,5,6]. By and large Simulated Annealing (SA) algorithm has been used to obtain the MAP estimates [4,5,6].

Tabu Search proposed by Glover [7] can be viewed as a strategy to solve combinatorial optimization problem and is an adaptive procedure which overcomes the limitations of local optimality. Here Tabu search has been used to obtain the MAP estimate of the image labels and a parallel Tabu search algorithm is proposed to reduce the computational burden. The performance of the Tabu search is compared with that of the simulated annealing (SA) algorithm. The globally convergent homotopy continuation algorithm based algorithms are also successfully used for obtaining optimal model parameter estimation [5,8,9,10].

It is believed that images possessing similar intrinsic characteristics could belong to the same class. Keeping the same in view, a new concept of image segmentation using MRF

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model learning feature is introduced here. It is assumed that the observed image is derived from a set, known as *class of images*. From this given *class of images*, one image is selected as the training data for the MRF model learning. The MRF model learns this image. The learning is viewed as estimation of MRF model parameters as it is assumed to have the knowledge of the structure of the model. The problem is formulated using Conditional Pseudo Likelihood (CPL) approach and the parameters are estimated using homotopy continuation method. Once the model learns one image, the same model is used as the image model for all the images of a class from which the training image is derived. This model is used to obtain the MAP estimate of the class labels using proposed Hybrid Tabu Search (HTS) and Parallel Hybrid Tabu Search (PHTS) algorithms. The performance of these algorithms are compared with that of the Simulated Annealing algorithm.

2. Image Model

Let X denotes the label process associated with the true but unknown labels and x be a realization of the same. The label process is modeled as MRF. Y denotes the observed random field corresponding to observed degraded image. Degradation model considered is

$$Y_{ij} = X_{ij} + W_{ij}, \ \forall (i,j) \in (N \times N) \tag{1}$$

which with a lexicographical ordering will be $Y = X + N$ is assumed to be a Markov random field with respect to a neighborhood system $\eta = \eta_{ij}; (i,j) \in L$ and is described in terms of its local characteristics.

$$\begin{aligned} &P(X_{ij} = x_{ij} \mid X_{kl} = x_{kl}, kl \in (N \times N), (k,l) \neq (i,j)) \\ &\quad = P(X_{ij} = x_{ij} \mid X_{kl} = x_{kl}, k, l \in \eta) \end{aligned}$$

Since X is MRF or equivalently Gibbs distributed, the joint distribution can be expressed as $P(X = x) = \frac{1}{Z} e^{-U(x)}$ where $Z = \sum_{z} e^{-U(x)}$ is the partition function. In general, the parameter vector $\theta = [\alpha, \beta, \sigma^2]^T$.

3. Problem Statement

The problem is formulated in a supervised framework where *a priori* knowledge of the number of classes are assumed to be known. We also assume to have *a priori* knowledge of noise variance of the noisy image. The optimal estimate of the class label x^* is to be obtained based on the noisy random field realization y and hence achieve proper segmentation. This label estimation is formulated using MAP criterion and optimality criterion is as follows.

$$\hat{x} = \arg \max_{x} P(X = x \mid Y = y, \hat{\theta}) \tag{2}$$

where $\hat{\theta}$ denotes the estimates of the parameter vector. The problem of determining $\hat{\theta}$, is referred to as the learning of the model for a class to which the given realization x belongs. Estimates of the model parameters are obtained using the notion of Maximum Conditional Pseudo Likelihood (MCPL) principle and homotopy continuation algorithm.. We consider the following optimality criterion.

$$\hat{\theta} = \arg \max_{\theta} P(X = x \mid Y = y, \theta) \tag{3}$$

Having determined $\hat{\theta}$, we use it in the MAP estimation problem of (2) for estimating the image labels and thus segment the image.

3.1. MRF Model Learning using homotopy continuation method

Model learning is obtained by selecting initial image model parameters arbitrarily. MAP estimation criterion is used to estimate X from Y. The following optimality criterion is considered.

$$\theta^* = \arg\max_{\theta} P(X = x \mid Y = y, \theta) \tag{4}$$

Using Bayes rule the conditional probability can be written as

$$P(X = x \mid Y = y,\theta) = \frac{P(Y = y \mid X = x,\theta)P(X = x \mid \theta)}{P(Y = y \mid \theta)} \tag{5}$$

Since, noise is assumed to be white Gaussian,

$$P(Y = y \mid X = x,\theta) = \frac{1}{(2\pi\sigma^2)^{\frac{N^2}{2}}} e^{\frac{-\|y-x\|^2}{2\sigma^2}} \tag{6}$$

By definition, the marginal conditional probability is given by,

$$P(Y = y,\theta) = \sum_{\xi} \frac{1}{(2\pi\sigma^2)^{N^2/2}} e^{\frac{-\|y-x\|^2}{2\sigma^2}} \frac{1}{Z} e^{-u(\xi,\varphi)} \tag{7}$$

We approximate (7) using the CPL function.

$$P(X = x \mid Y = y,\theta) \approx$$

$$\prod_{(i,j)} P(X_{(i,j)} = x_{(i,j)} \mid X_{(k,l)} = x_{(k,l)}, (k,l) \in \eta, Y = y, \theta) \stackrel{\Delta}{=} \hat{P}(X = x \mid Y = y, \theta) \tag{8}$$

This is the CPL function, where we are approximating the posterior probability distribution instead of the *a priori* probability distribution. We maximize (8) to obtain the MCPL estimate of the parameter vector θ. Now the parameter estimation problem is recast as

$$\theta^* = \arg\max_{\theta} \hat{P}(X = x \mid Y = y,\theta) \tag{9}$$

3. 2. Parameter Estimation Using Homotopy Continuation Method

It is clear from previous section that the parameter estimation problem has been reduced to maximization of (9) with respect to θ. Towards this end let

$$f(\theta) = \frac{\partial}{\partial\theta}\{\log[\hat{P}(X = x \mid Y = y,\theta)]\} = 0 \tag{10}$$

Now the homotopy method is employed to solve $f(\theta) = 0$, where θ is the unknown parameter vector to be determined. The following weak membrane is used as *a priori* energy function

$$U(x,\phi,h,v) = \sum_{(i,j)} \alpha \begin{Bmatrix} (x_{(i,j)} - x_{(i,j-1)})^2(1 - h_{(i,j)}) + \\ (x_{(i,j)} - x_{(i-1,j)})^2(1 - v_{(i,j)}) \end{Bmatrix} + \beta(h_{(i,j)} + v_{(i,j)}) \tag{11}$$

and the corresponding *a posteriori* energy function is expressed as

$$U_p(x,\varphi,h,v) = \frac{1}{2\sigma^2}(y_{(i,j)} - x_{(i,j)})^2 + U(x,\varphi,h,v) \tag{12}$$

where α, β are the external field and internal field model parameters respectively, $h_{(i,j)}$ and $v_{(i,j)}$ are horizontal and vertical line fields respectively. Using the above energy functions and carrying out the maximization process, we land up in the three nonlinear equations that need to be solved. Since the equations are highly nonlinear and there is no *a priori* knowledge of the solutions are known, the notion of homotopy continuation method is used to obtain the solution. In this regard, we have considered the fixed point homotopy map [9] which offers the advantage of arbitrary starting point for the path.

4. Tabu Search

Tabu Search (TS) is a general heuristic search procedure devised for finding a global minimum of a function which may be linear or nonlinear. Our algorithm is designed by exploiting the notion of Tabu search. The basic steps of the proposed Hybrid Tabu Search algorithm to obtain the MAP estimates are as follows.

4.1 Hybrid Tabu Search (HTS) Algorithm

1. Initialize the initial temperature T_{in}.
2. The initial image for the algorithm is the degraded image.
3. A Tabu array, i.e. Tabu image set is created to store the recent moves, i.e. the image estimates of the algorithm. The array is of fixed length.
4. From the current move or image the next Tabu image is generated.
   i) Perturb $z_{ij}(t)$ with a zero mean Gaussian Distribution with a suitable variance.
   ii) Evaluate the energy $U_p(z_{ij}(t+1))$ & $U_p(z_{ij}(t))$. If $\Delta f = (U_p(z_{ij}(t+1)) - U_p(z_{ij}(t))) < 0$,assign the modified value as the new value. If $\Delta f > 0$, accept the $z_{ij}(t+1)$ with a probability (if $\exp(-\Delta f/T(z))>\mathrm{random}(0,1)$) .
   iii) Repeat step 2 for all the pixels of the image.
5. Compute the power of the updated image z(t+1) as $P_{z(t+1)}$ and compare it with the powers of the tabu list named as Tabu energy. If $P_{z(t+1)} < P_{Tabu}$ then z(t+1) is a Tabu image.
6. Aspiration condition: If $P_{z(t+1)} > P_{Tabu}$, accept z(t+1) as Tabu image with probability
7. Update the Tabu list.
8. Decrease the Temperature according to the logarithmic cooling schedule.
9. Repeat step 4 – 8 for a fixed number of iterations

4.2. Parallel Hybrid Tabu Search (PHTS) algorithm

The computational burden incurred by the proposed HTS algorithm could further be reduced by parallelizing the algorithm. The image is partitioned into a set of sub-images, of size (16X16) and (32X32). The energy can be computed over each sub-image simultaneously. The energy for the whole image is the sum of the energies of individual sub-images. The computation of energy of each sub-image can be achieved by submitting each job to individual processor. The total energy can be computed in parallel machine.

5. Map Estimation of Image Labels

The segmentation is formulated as pixel labeling problem. Let x* denote the true but unknown labeling configuration and $\hat{x}$ denote the estimate for x*. x* is the realization of random field X, which is modeled as MRF. The following optimality criterion is considered,

$$\hat{x} = \arg\max_{x} P(X = x \mid Y = y, \hat{\theta}) \tag{13}$$

The above can be solved as a posterior MAP estimation problem using Bayesian approach. The white Gaussian noise is independent of X. Therefore given the parameter vector θ the problem reduces to

$$\hat{x} = \arg\min_{x} \left[\frac{\| y - x \|^2}{2\sigma^2} + U(x, \phi) \right] \tag{14}$$

We solve (14) using proposed HTS and PHTS algorithm and the performance is compared with that of SA algorithm.

6. Simulations and Results

In simulation, the model learning is carried out for a *class of images*. We have considered two classes namely, the indoor (object) images and outdoor images. The learning is carried out with a training image derived from a given class and the learning attribute is validated with the test images derived from the same class. The first training image considered is an object image as shown in Figure 1(a). The corresponding noisy image of SNR 20dB is shown in Figure 1(b). Both the images are considered while learning the models using homotopy continuation method. The model learning algorithm starts from an arbitrary set of parameter vector and eventually converge to a value of α = 0.0096, β = 2.517 and σ = 21.037. Since Gaussian noise of a known variance σ^2 is added, the estimated σ serves as an indicator of the estimate of model parameters as well. It is observed that the estimated σ value of 21.037 is close to the actual σ value of 20.358. Thus, the image model is known. This image model together with the known noise variance σ^2 are now used to obtain the segmented image from a noisy image of SNR 20dB as shown in Figure 1(b). The segmented images obtained by SA algorithm, proposed HTS and PHTS algorithms are shown in Figures 1(c), 1(d) and 1(e) respectively. It is observed that segmentation of the objects could be obtained and also the edges could be preserved in all the cases. The HTS and PHTS algorithms converge faster than SA albeit visually similar segmented images. The convergence of posterior energy for Figure 1. is shown in Figure 3(a).

The model thus trained is validated with other object image as shown in Figure 2(a). The noisy image of SNR 20 dB is shown in Figure 2(b). Segmented images obtained are shown in Figures 2(c), 2(d) and 2(e). The convergence of HTS and PHTS algorithm are faster than that of SA. The above results imply that the model trained for one object image is valid for other object test images. This in turn implies that the test indoor image belong to the same class from which the training image is derived. Hence, this avoids the estimation of parameters for all the images of the class from which the training image is derived. The above findings are corroborated with an noisy outdoor scene of SNR 20dB as shown in Figure 4(a). The converged values of model parameters are α = 0.031, β = 4.568, and σ = 16.399. The additive white Gaussian noise of σ = 17.106 has been added and hence serves as a pointer to the learning accuracy. The segmented images using three algorithms are shown in Figures 4(b), 4(c) and 4(d) respectively. The results obtained are satisfactory, but the rate of convergence of HTS and PHTS are found to be faster than that of SA algorithm which is observed from the posterior energy plots as shown in Figure 3(b). Having learnt the training image, the model is now validated for another outdoor noisy image of SNR 20dB as shown in Figure 5(a). The segmented images obtained by SA, HTS and PHTS algorithms are shown in Figures 5(b), 5(c) and 5(d) respectively. The trained model is valid for this image and thus implies that the test image belongs to a class from which the training image is derived.

7. Conclusion

The problem of Markov Random Field model learning is addressed in a supervised image segmentation framework. The model learning is accomplished using homotopy continuation algorithm. This proposed notion is validated for real indoor and outdoor images as well. The model is trained for a given image derived from a class and the model, having learnt, is employed for segmentation of images supposed to be members of the same class as that of the training image. It could be perceived that the trained model yielded satisfactory results for indoor as well as outdoor scenes. Hence in the supervised image segmentation mode, once the training phase is over, the model can be employed for a set of images from which the training image is derived. This concept avoids the estimation of model parameters for each and every individual image of a given class. The only bottleneck is the computational burden due to the learning by homotopy continuation method

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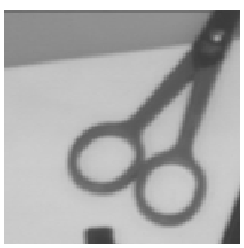
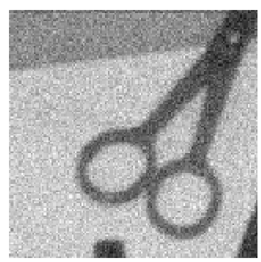


(a)



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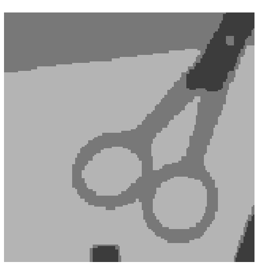


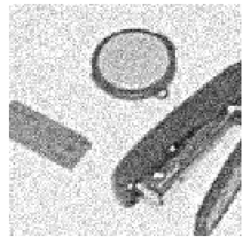
(c)

(d)

(e)

Figure 1. Training indoor (object) image : (a) original image, (b) noisy image of SNR 20dB ; (c), (d) and (e) shows the segmented images using SA, HTS and PHTS algorithms respectively.

(a)



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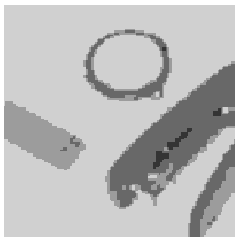

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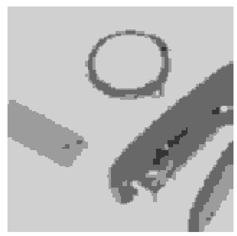

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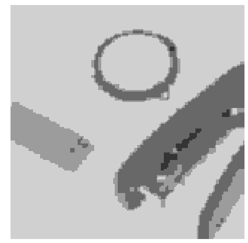


(e)

Figure 2. Test indoor (object) image : (a) original image, (b) noisy image of SNR 20dB ; (c), (d) and (e) shows the segmented images using SA, HTS and HTS algorithms respectively.

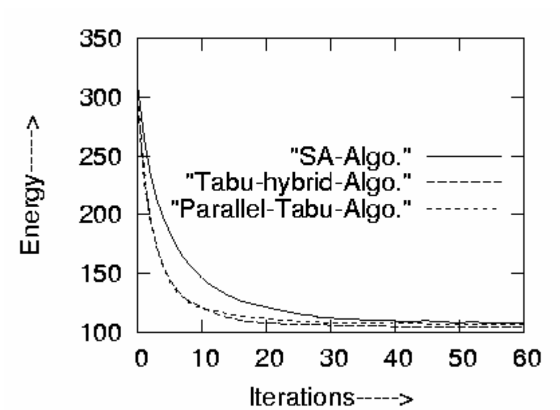


Figure 3(a)

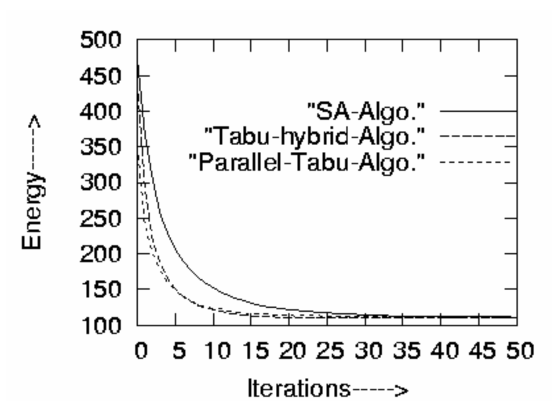
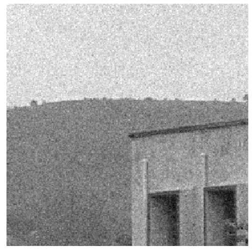


Figure 3(b)

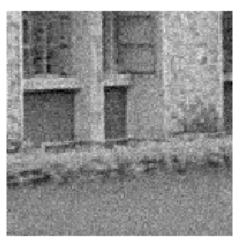


(a)

(b) (c) (d)

Figure 4. Training outdoor image : (a) noisy image of SNR 20dB ; (b), (c), (d) shows the segmented images using SA, HTS and HTS algorithms respectively.



(a)

(b) (c) (d)

Figure 5. Test outdoor image : (a) noisy image of SNR 20dB ; (b), (c), (d) shows the segmented images using SA, HTS and HTS algorithms respectively.