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EECS 545 Winter 2018 — Problem Set 3

Problem 1 Problem 1

Proof:

1. For this problem the error rate was 10.57%
2. For this problem, we show the derivation that the binary naive bayes classifier is a linear classifier in the following picture.

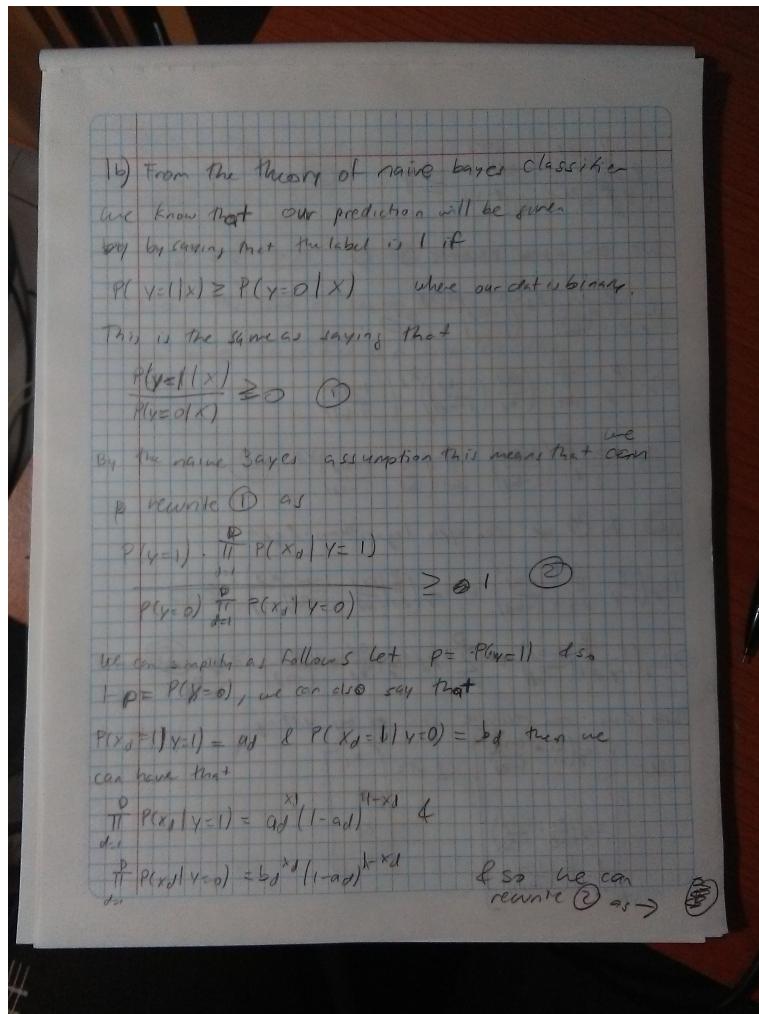


Figure 1: **Problem 1 part b:** Image showing the work done to show that the binary naive bayes classifier is a linear classifier

Problem 2 Problem 2

$$\begin{aligned}
 & \frac{P \prod_{d=1}^D a_d^{x_d} (1-a_d)^{1-x_d}}{1 - P \prod_{d=1}^D b_d^{x_d} (1-b_d)^{1-x_d}} \geq 1 \quad (3) \\
 \text{we can again rewrite (3) as} \\
 & \frac{P}{1-P} \prod_{d=1}^D \left(\frac{a_d}{b_d} \right)^{x_d} \left(\frac{1-a_d}{1-b_d} \right)^{1-x_d} \geq 0 \\
 \Rightarrow & \frac{P}{1-P} \prod_{d=1}^D \left(\frac{a_d}{b_d} \left(\frac{1-b_d}{1-a_d} \right) \right)^{x_d} \left(\frac{1-a_d}{1-b_d} \right) \geq 0 \\
 \Rightarrow & \frac{P}{1-P} \prod_{d=1}^D \left(\frac{1-a_d}{1-b_d} \right) \cdot \prod_{d=1}^D \left(\frac{a_d}{b_d} \cdot \frac{1-b_d}{1-a_d} \right)^{x_d} \quad (4) \\
 \text{Now to (4) we can apply logarithms and get} \\
 & \log \left(\frac{P}{1-P} \prod_{d=1}^D \left(\frac{1-a_d}{1-b_d} \right) \right) + \log \left(\prod_{d=1}^D \left(\frac{a_d}{b_d} \cdot \frac{1-b_d}{1-a_d} \right)^{x_d} \right) \geq 0 \\
 \Rightarrow & \log \left(\frac{P}{1-P} \prod_{d=1}^D \left(\frac{1-a_d}{1-b_d} \right) \right) + \sum_{d=1}^D x_d \log \left(\frac{a_d}{b_d} \cdot \frac{1-b_d}{1-a_d} \right) \geq 0 \quad (5) \\
 \text{and this can rewrite this as by letting} \\
 & W = \begin{bmatrix} \log \left(\frac{a_1}{b_1} \cdot \frac{1-b_1}{1-a_1} \right) \\ \vdots \\ \log \left(\frac{a_D}{b_D} \cdot \frac{1-b_D}{1-a_D} \right) \\ \hline \frac{P}{1-P} \prod_{d=1}^D \left(\frac{1-a_d}{1-b_d} \right) \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ \vdots \\ x_D \\ 1 \end{bmatrix}
 \end{aligned}$$

Figure 2: **Problem 1 part b:** Image showing the work done to show that the binary naive bayes classifier is a linear classifier

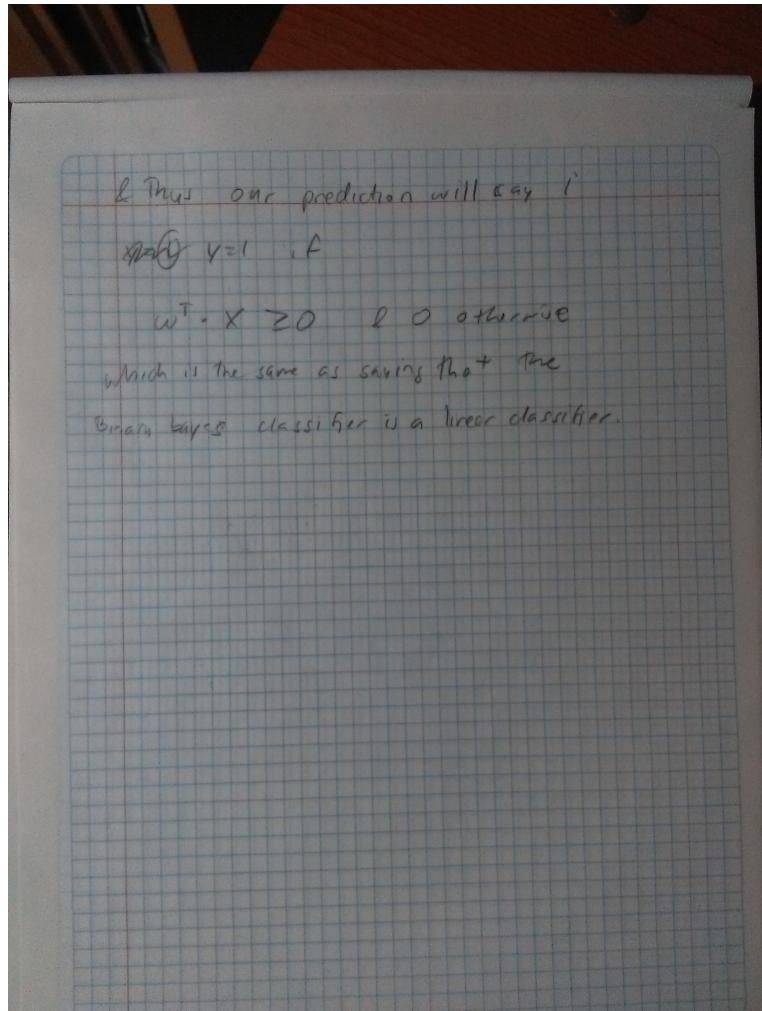


Figure 3: **Problem 1 part b:** Image showing the work done to show that the binary naive bayes classifier is a linear classifier

Proof: The solution to this problem is presented in the following pictures.

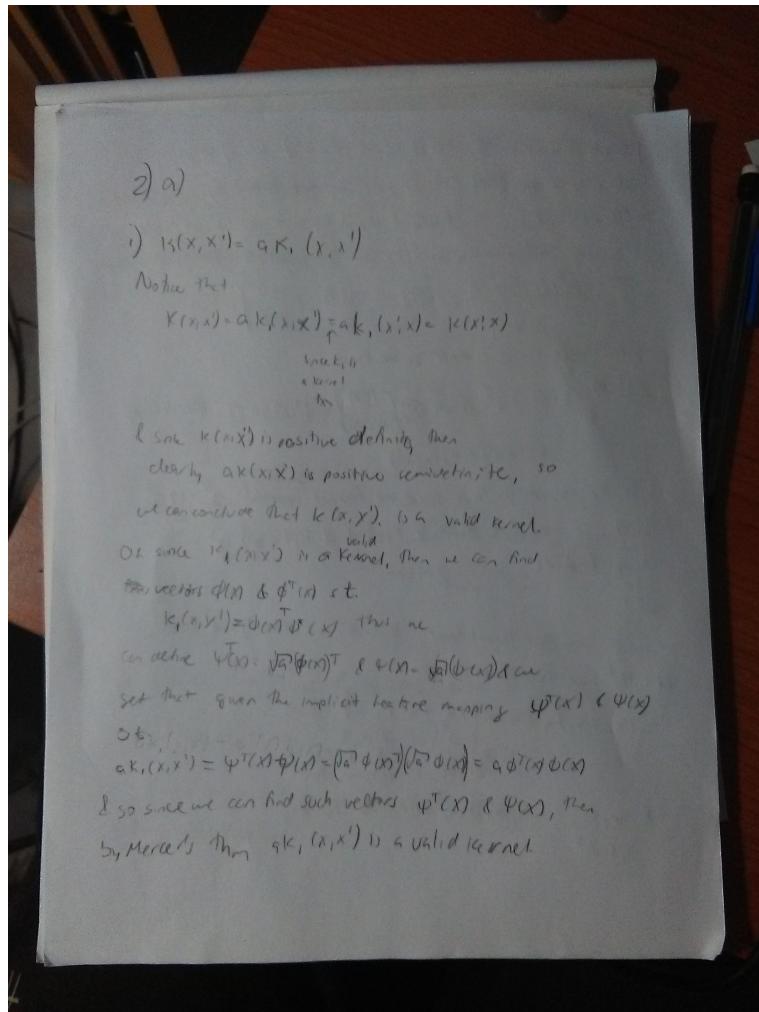


Figure 4: **Problem 2a part 1:** Image showing the work for part 1 of problem 2a

Now, we show the solution for **problem 2 part b**

Problem 3 Problem 3

Proof: For this problem, first we show our work for **part a** in the following picture.

Now, we show the images obtained for part b. The first image shown is just the plot of the data, separated by color. In this plot blue represents label -1 and red represents label 1 . The next two pictures show a kernel perceptron classifier for when $\sigma = 0.1$ (the second figure below) and $\sigma = 1$ the third figure below.

Problem 4 Problem 4

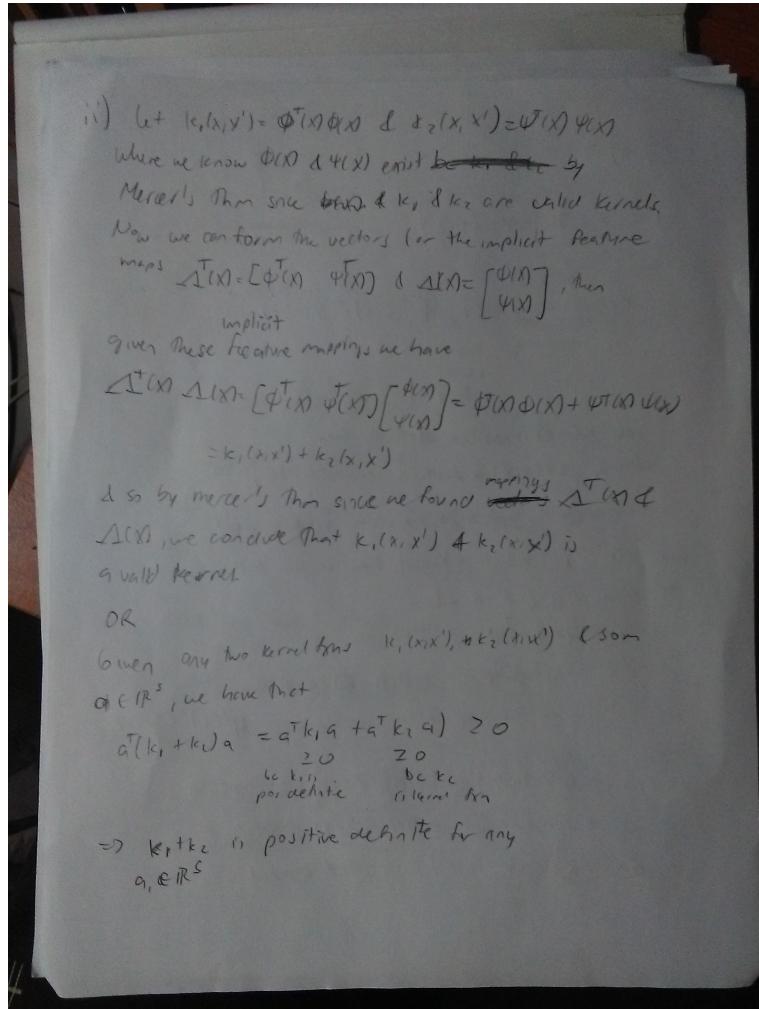


Figure 5: **Problem 2a part 2:** Image showing the work for part 2 of problem 2a

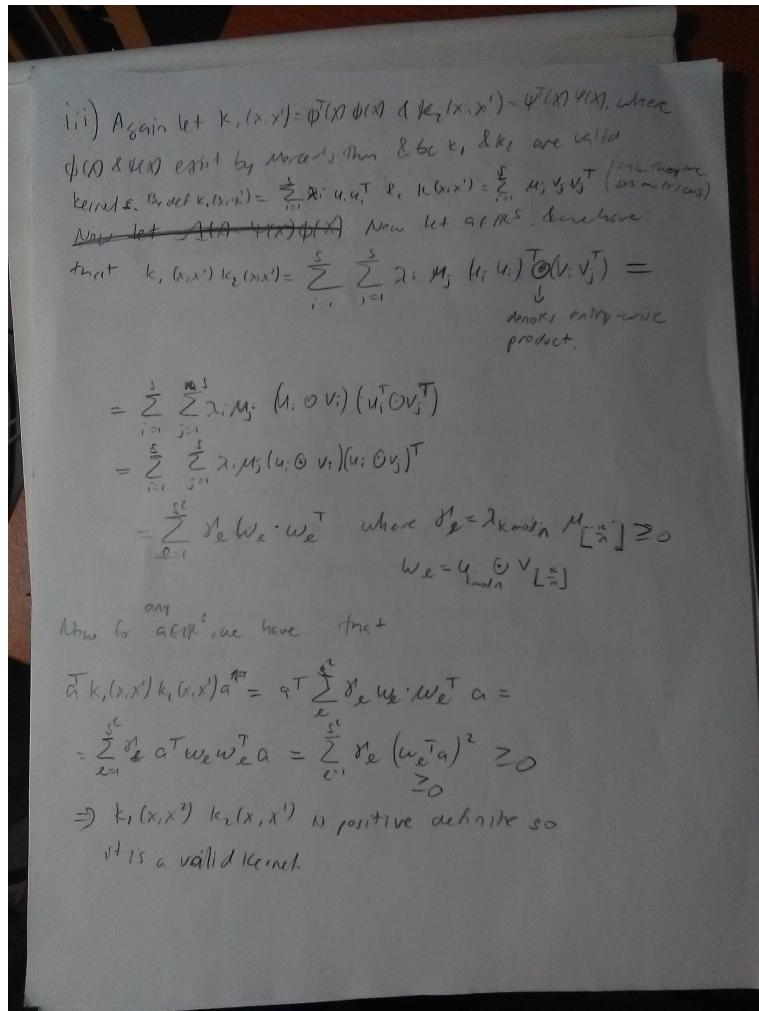


Figure 6: **Problem 2a part 3:** Image showing the work for part 3 of problem 2a

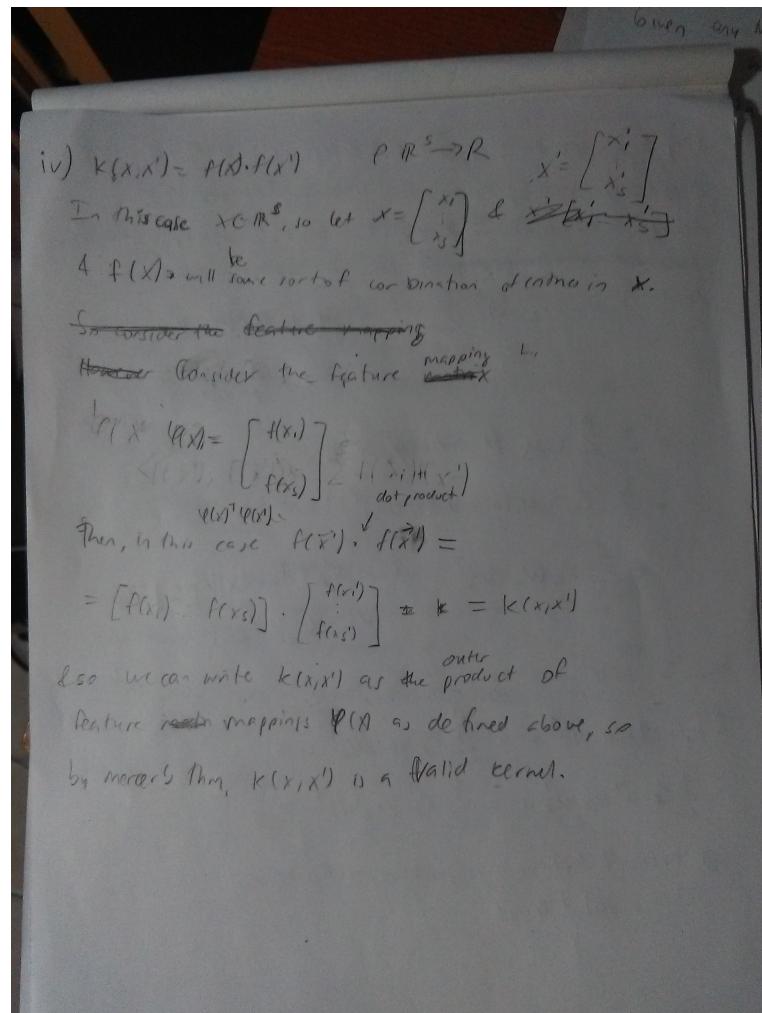


Figure 7: **Problem 2a part 4:** Image showing the work for part 4 of problem 2a

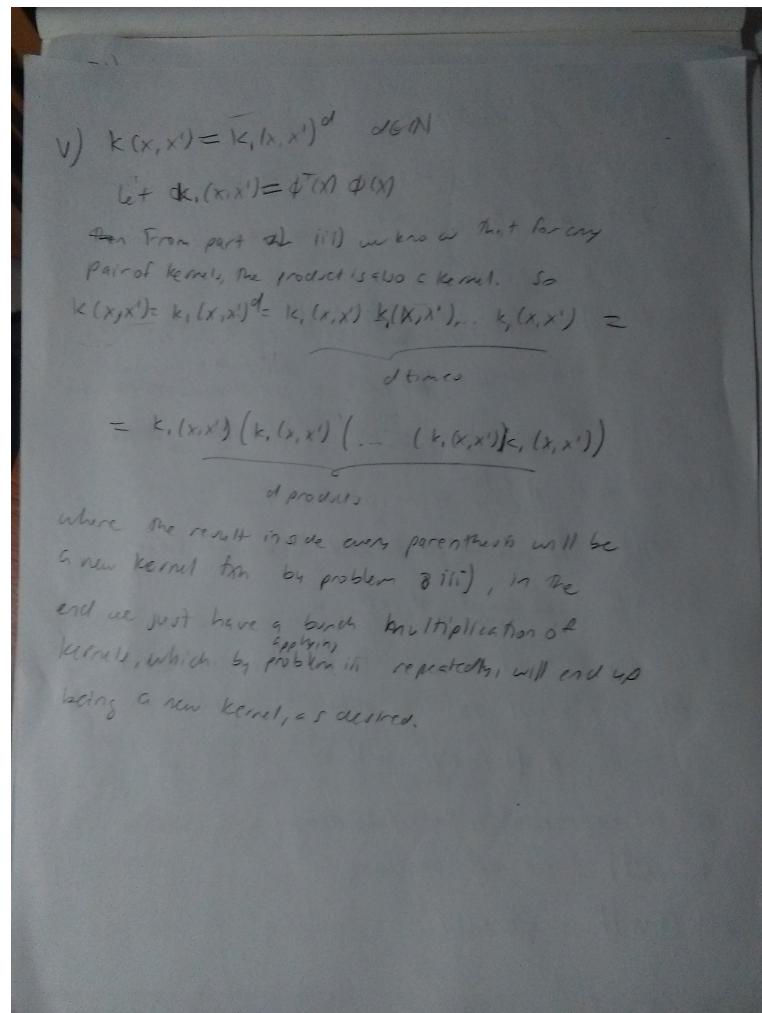


Figure 8: **Problem 2a part 5:** Image showing the work for part 5 of problem 2a

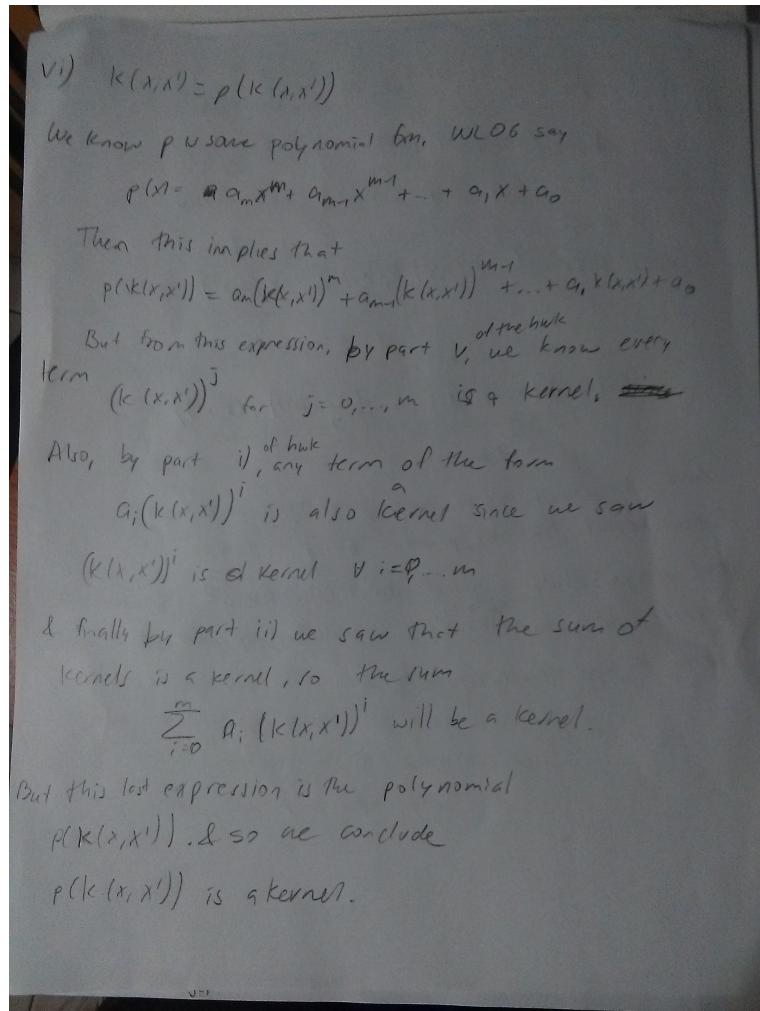


Figure 9: **Problem 2a part 6:** Image showing the work for part 6 of problem 2a

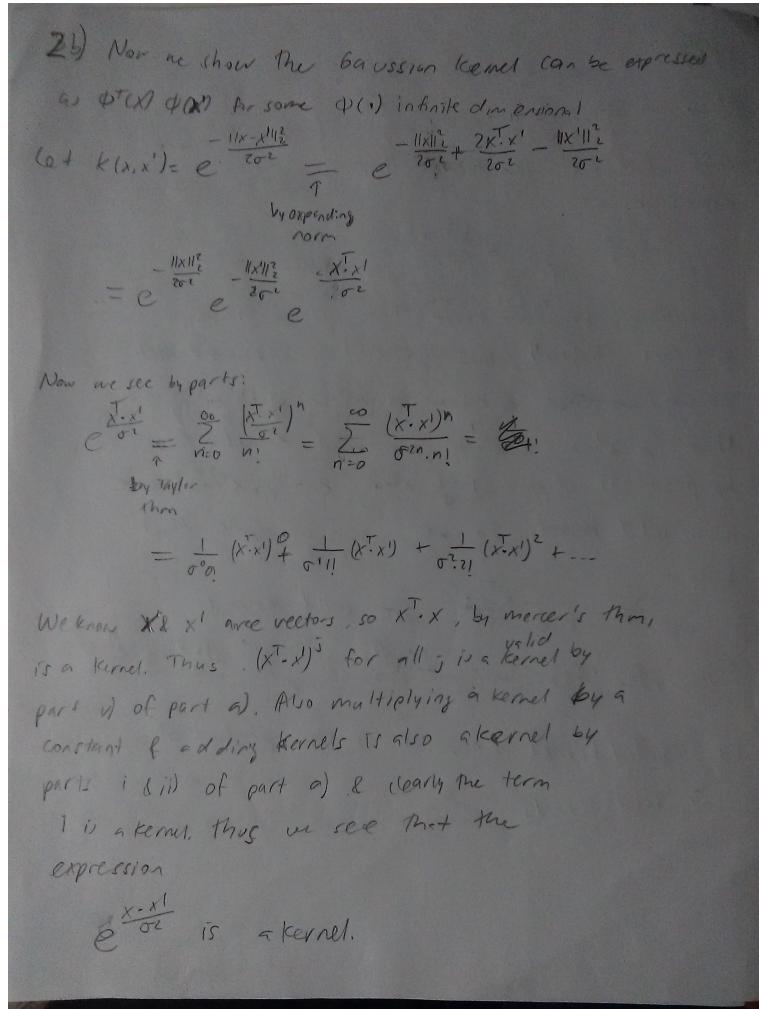


Figure 10: Problem 2b part 1: Image showing the work for part b of problem 2

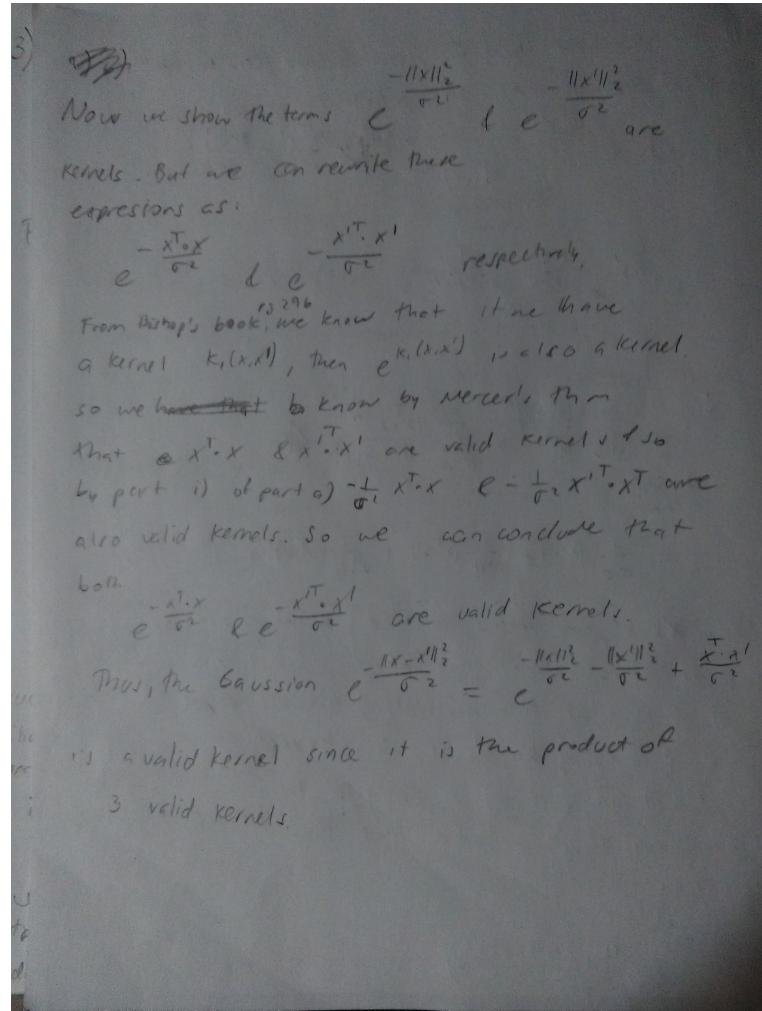


Figure 11: **Problem 2b part 2:** Image showing the work for part b of problem 2

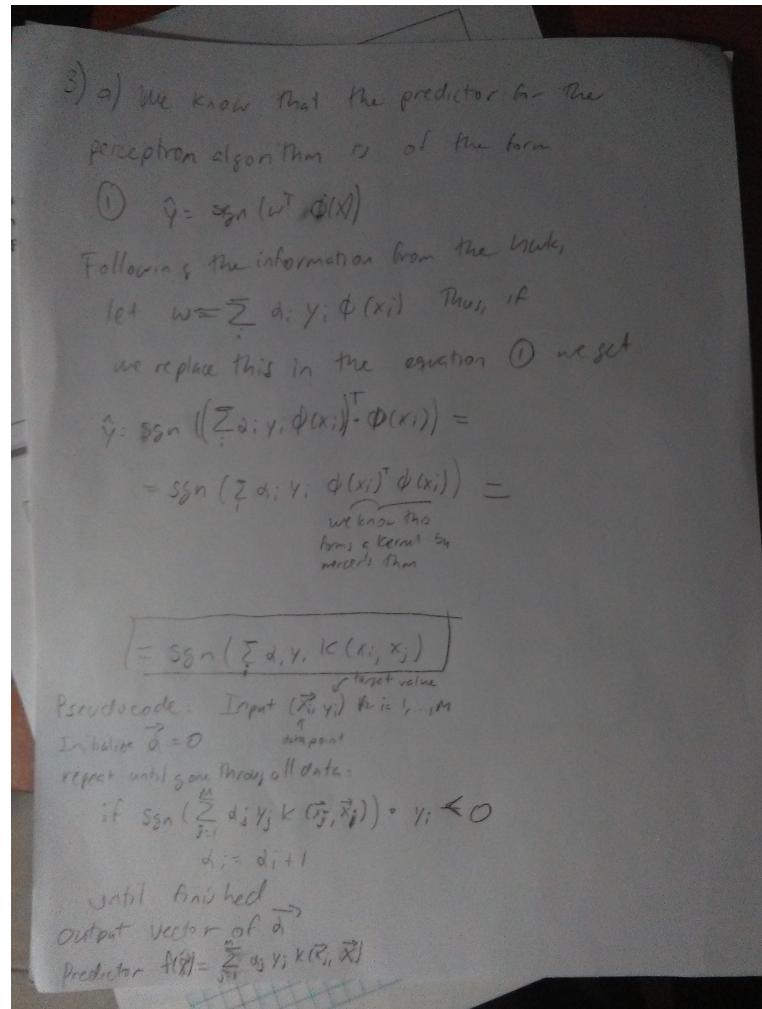


Figure 12: **Problem 3 part a:** Image showing the work for part a of problem 3

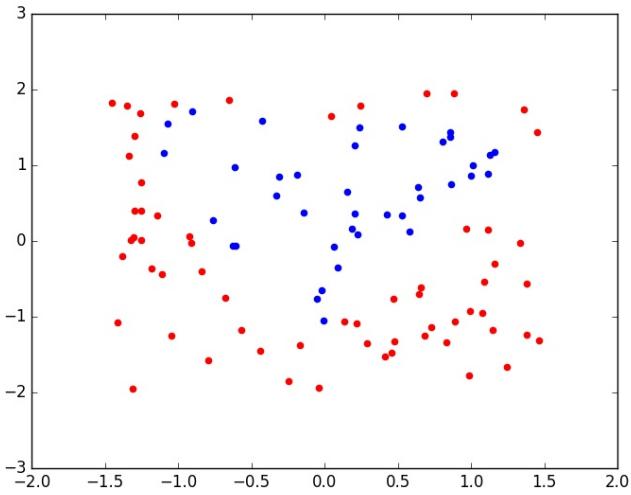


Figure 13: **Problem 3 part b:** Image showing the scatter plot of the data with red color meaning label 1 and blue color meaning label -1

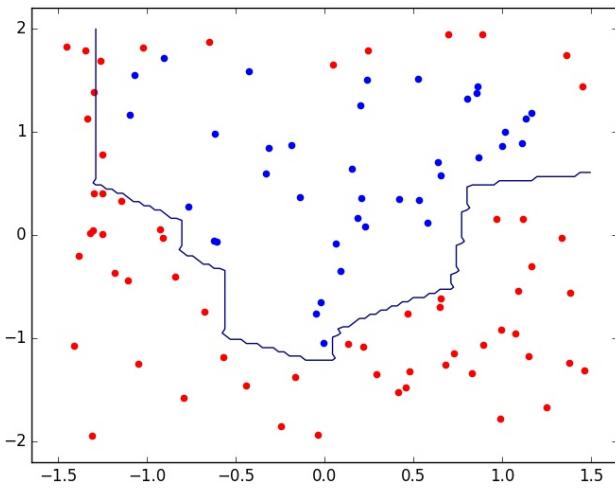


Figure 14: **Problem 3 part b:** Image showing the scatter plot of the data with a kernel perceptron classifier with $\sigma = 0.1$

Proof: For this problem, we present 2 plots. The **first plot** we present corresponds to the **LDA analysis** and the **second plot** we present corresponds to the **QDA analysis**.

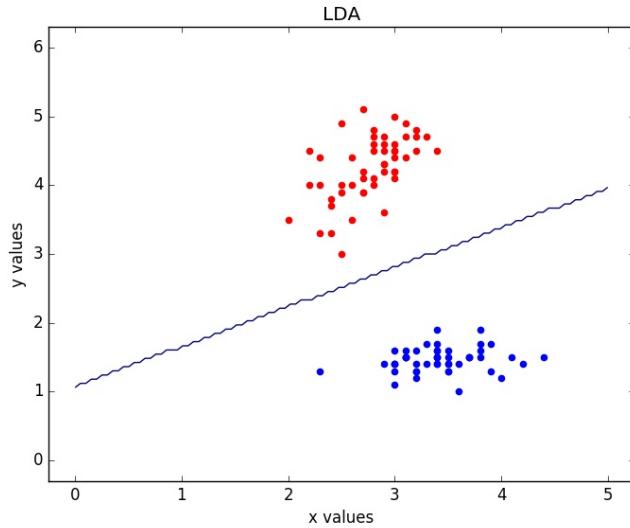


Figure 15: **Problem 4 part LDA:** Image showing the scatter plot of the data with a LDA classifier

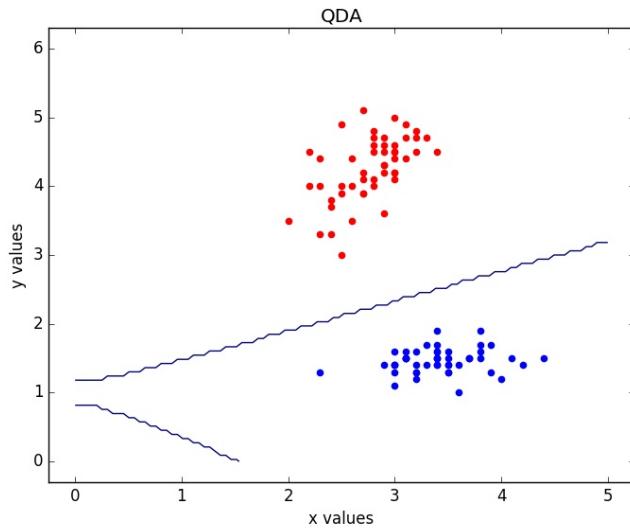


Figure 16: **Problem 4 part QDA:** Image showing the scatter plot of the data with a QDA classifier

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