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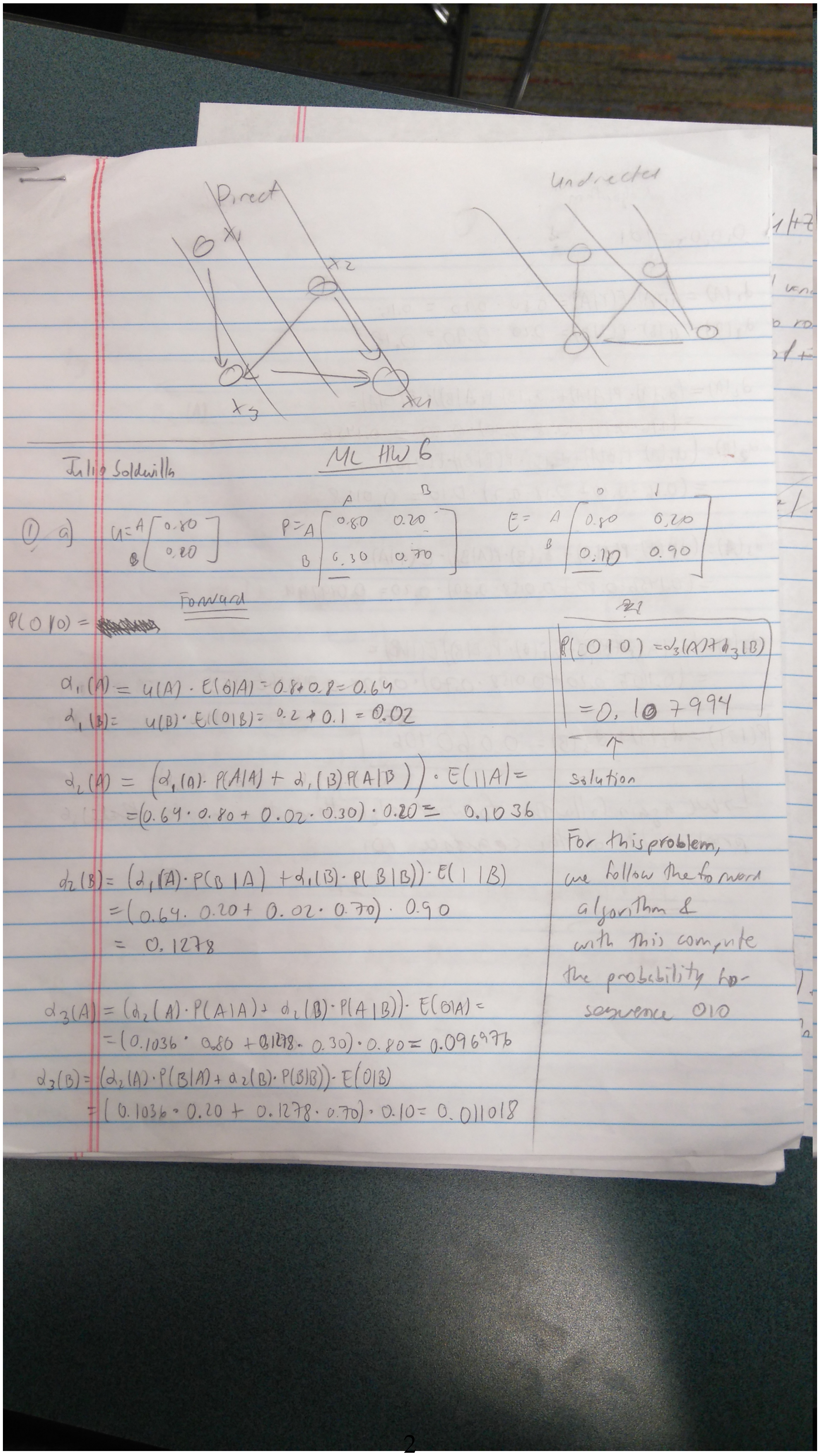
EECS 545 Winter 2018 — Problem Set 5

Problem 1 Problem 1

Proof:



Direct
x_1
x_2
x_3
x_4

Undirected

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ML HW 6

① a) $u = \begin{array}{c} A \\ B \end{array}\begin{bmatrix} 0.80 \\ 0.20 \end{bmatrix}$ $P = \begin{array}{c|cc} & A & B \\ \hline A & 0.80 & 0.20 \\ B & 0.30 & 0.70 \end{array}$ $E = \begin{array}{c|cc} & 0 & 1 \\ \hline A & 0.80 & 0.20 \\ B & 0.10 & 0.90 \end{array}$

Forward

$P(010) =$

$\alpha_1(A) = u(A) \cdot E(0|A) = 0.8 * 0.8 = 0.64$
$\alpha_1(B) = u(B) \cdot E(0|B) = 0.2 * 0.1 = 0.02$

$\alpha_2(A) = (\alpha_1(A) \cdot P(A|A) + \alpha_1(B) P(A|B)) \cdot E(1|A) =$
$= (0.64 \cdot 0.80 + 0.02 \cdot 0.30) \cdot 0.20 = 0.1036$

$\alpha_2(B) = (\alpha_1(A) \cdot P(B|A) + \alpha_1(B) \cdot P(B|B)) \cdot E(1|B)$
$= (0.64 \cdot 0.20 + 0.02 \cdot 0.70) \cdot 0.90$
$= 0.1278$

$\alpha_3(A) = (\alpha_2(A) \cdot P(A|A) + \alpha_2(B) \cdot P(A|B)) \cdot E(0|A) =$
$= (0.1036 \cdot 0.80 + 0.1278 \cdot 0.30) \cdot 0.80 = 0.096976$

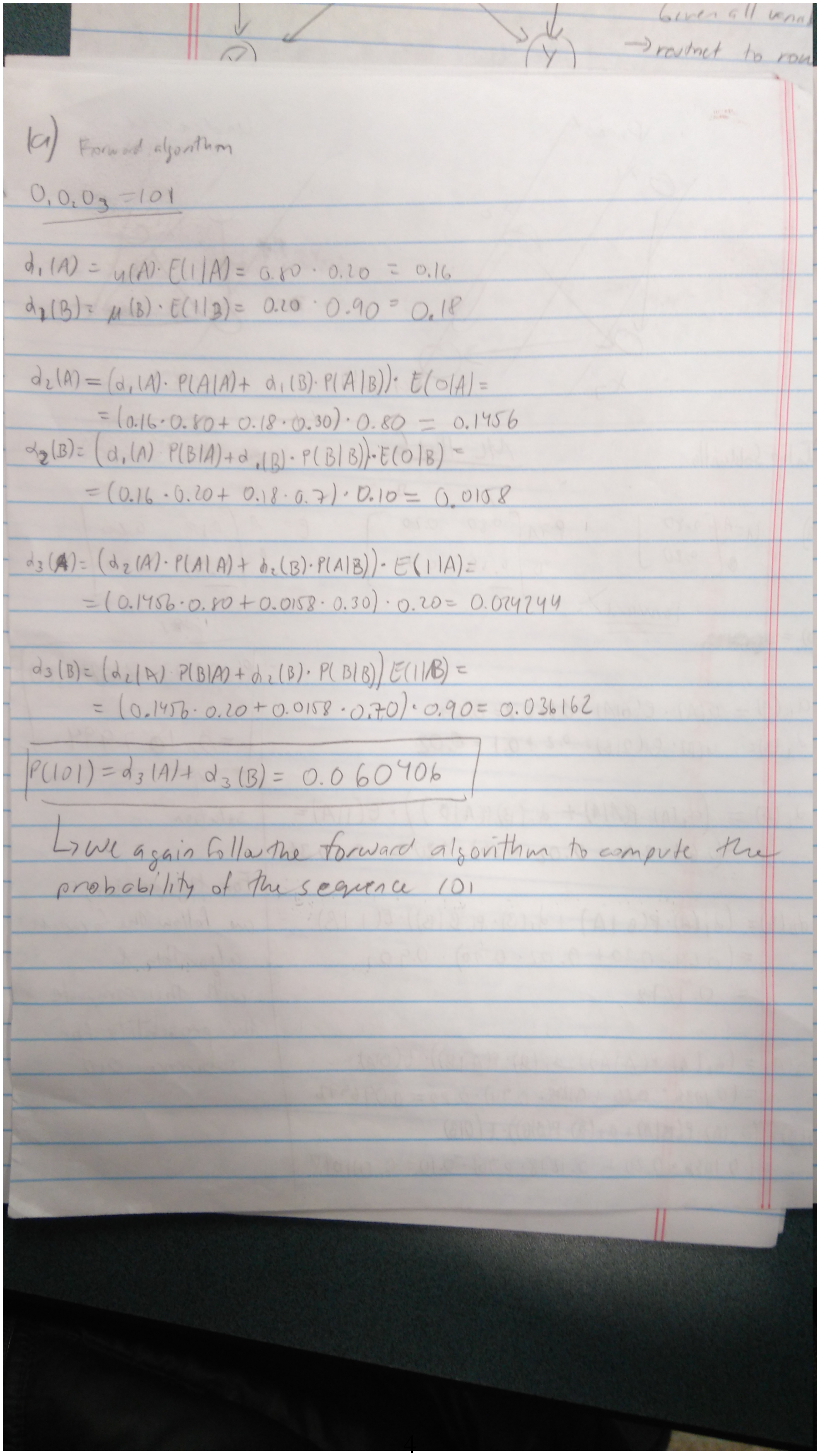
$\alpha_3(B) = (\alpha_2(A) \cdot P(B|A) + \alpha_2(B) \cdot P(B|B)) \cdot E(0|B)$
$= (0.1036 \cdot 0.20 + 0.1278 \cdot 0.70) \cdot 0.10 = 0.011018$

$\boxed{P(010) = \alpha_3(A) + \alpha_3(B) = 0.107994}$

solution

For this problem, we follow the forward algorithm & with this compute the probability for sequence 010

Figure 1: **Problem 1 part a :** Image showing the work for 1 sequence 010



(a) Forward algorithm

$O_1 O_2 O_3 = 101$

$\alpha_1(A) = \mu(A) \cdot E(1|A) = 0.80 \cdot 0.20 = 0.16$

$\alpha_1(B) = \mu(B) \cdot E(1|B) = 0.20 \cdot 0.90 = 0.18$

$\alpha_2(A) = (\alpha_1(A) \cdot P(A|A) + \alpha_1(B) \cdot P(A|B)) \cdot E(0|A) =$

$= (0.16 \cdot 0.80 + 0.18 \cdot 0.30) \cdot 0.80 = 0.1456$

$\alpha_2(B) = (\alpha_1(A) \cdot P(B|A) + \alpha_1(B) \cdot P(B|B)) \cdot E(0|B) =$

$= (0.16 \cdot 0.20 + 0.18 \cdot 0.7) \cdot 0.10 = 0.0158$

$\alpha_3(A) = (\alpha_2(A) \cdot P(A|A) + \alpha_2(B) \cdot P(A|B)) \cdot E(1|A) =$

$= (0.1456 \cdot 0.80 + 0.0158 \cdot 0.30) \cdot 0.20 = 0.024244$

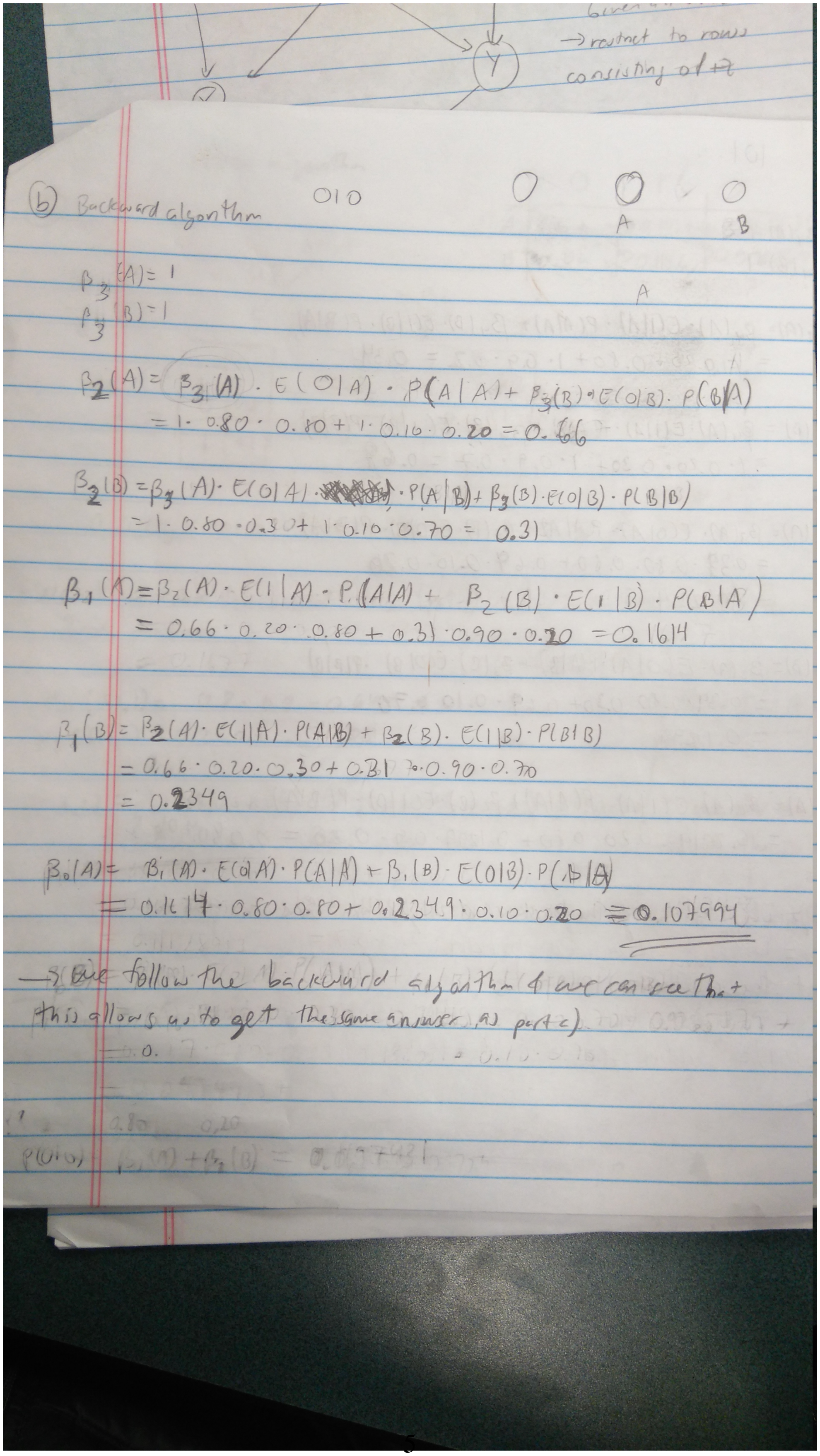
$\alpha_3(B) = (\alpha_2(A) \cdot P(B|A) + \alpha_2(B) \cdot P(B|B)) \cdot E(1|B) =$

$= (0.1456 \cdot 0.20 + 0.0158 \cdot 0.70) \cdot 0.90 = 0.036162$

$\boxed{P(101) = \alpha_3(A) + \alpha_3(B) = 0.060406}$

↳ we again follow the forward algorithm to compute the probability of the sequence 101

Figure 2: **Problem 1 part a :** Image showing the work for 1 sequence 101



(b) Backward algorithm 010

$\beta_3(A) = 1$

$\beta_3(B) = 1$

$\beta_2(A) = \beta_3(A) \cdot E(0|A) \cdot P(A|A) + \beta_3(B) \cdot E(0|B) \cdot P(B|A)$
$= 1 \cdot 0.80 \cdot 0.80 + 1 \cdot 0.10 \cdot 0.20 = 0.66$

$\beta_2(B) = \beta_3(A) \cdot E(0|A) \cdot P(A|B) + \beta_3(B) \cdot E(0|B) \cdot P(B|B)$
$= 1 \cdot 0.80 \cdot 0.30 + 1 \cdot 0.10 \cdot 0.70 = 0.31$

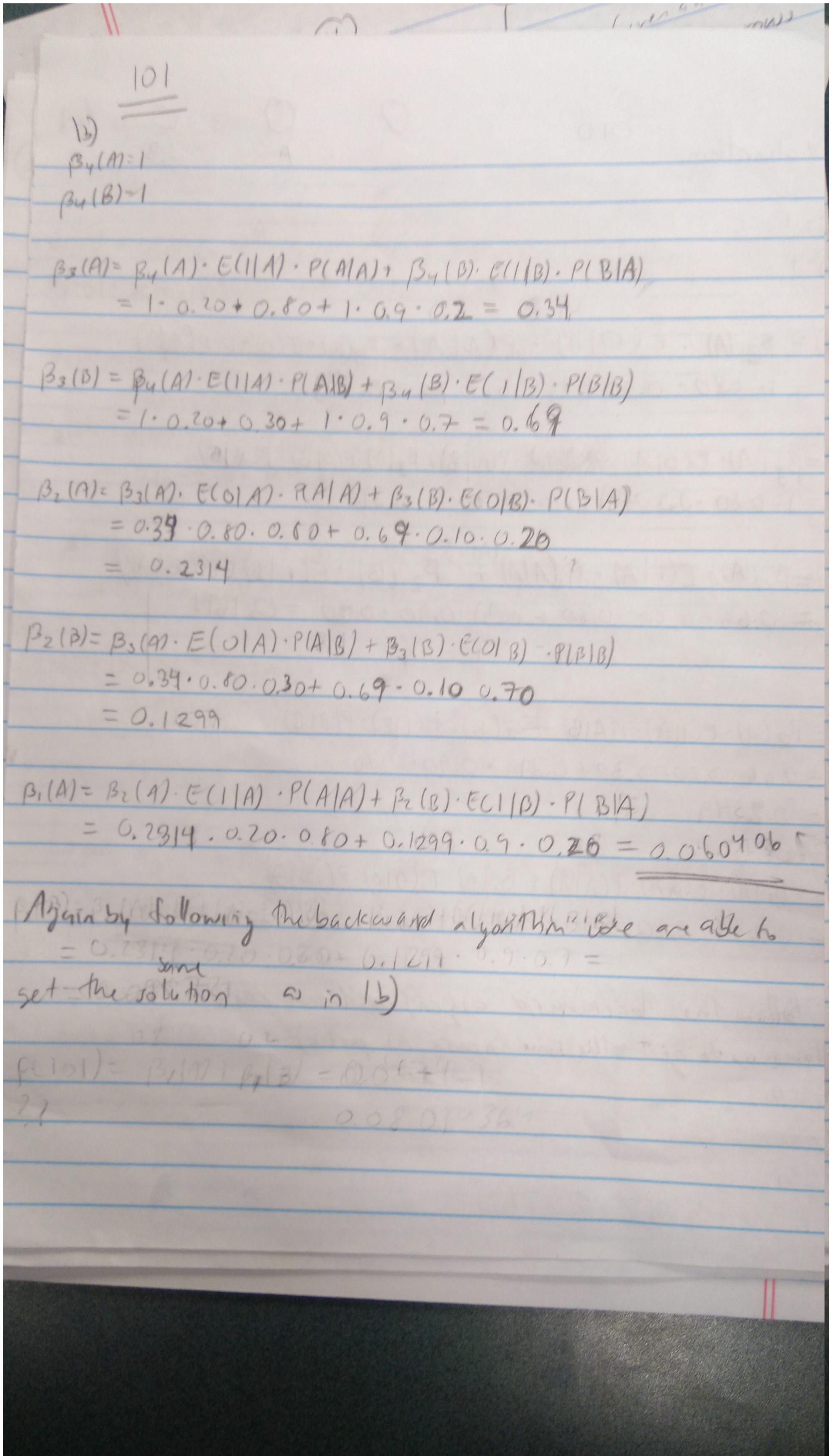
$\beta_1(A) = \beta_2(A) \cdot E(1|A) \cdot P(A|A) + \beta_2(B) \cdot E(1|B) \cdot P(B|A)$
$= 0.66 \cdot 0.20 \cdot 0.80 + 0.31 \cdot 0.90 \cdot 0.20 = 0.1614$

$\beta_1(B) = \beta_2(A) \cdot E(1|A) \cdot P(A|B) + \beta_2(B) \cdot E(1|B) \cdot P(B|B)$
$= 0.66 \cdot 0.20 \cdot 0.30 + 0.31 \cdot 0.90 \cdot 0.70$
$= 0.2349$

$\beta_0(A) = \beta_1(A) \cdot E(0|A) \cdot P(A|A) + \beta_1(B) \cdot E(0|B) \cdot P(B|A)$
$= 0.1614 \cdot 0.80 \cdot 0.80 + 0.2349 \cdot 0.10 \cdot 0.20 = 0.107994$

→ we follow the backward algorithm & we can see that this allows us to get the same answer as part c).

Figure 3: **Problem 1 part b:** Image showing the work for 1 sequence 010 backward alg



101

1b)

$\beta_4(A)=1$

$\beta_4(B)=1$

$\beta_3(A) = \beta_4(A)\cdot E(1|A)\cdot P(A|A) + \beta_4(B)\cdot E(1|B)\cdot P(B|A)$

$= 1\cdot 0.20 + 0.80 + 1\cdot 0.9\cdot 0.2 = 0.34$

$\beta_3(B) = \beta_4(A)\cdot E(1|A)\cdot P(A|B) + \beta_4(B)\cdot E(1|B)\cdot P(B|B)$

$= 1\cdot 0.20 + 0.30 + 1\cdot 0.9\cdot 0.7 = 0.69$

$\beta_2(A) = \beta_3(A)\cdot E(0|A)\cdot P(A|A) + \beta_3(B)\cdot E(0|B)\cdot P(B|A)$

$= 0.34\cdot 0.80\cdot 0.60 + 0.69\cdot 0.10\cdot 0.20$

$= 0.2314$

$\beta_2(B) = \beta_3(A)\cdot E(0|A)\cdot P(A|B) + \beta_3(B)\cdot E(0|B)\cdot P(B|B)$

$= 0.34\cdot 0.80\cdot 0.30 + 0.69\cdot 0.10\; 0.70$

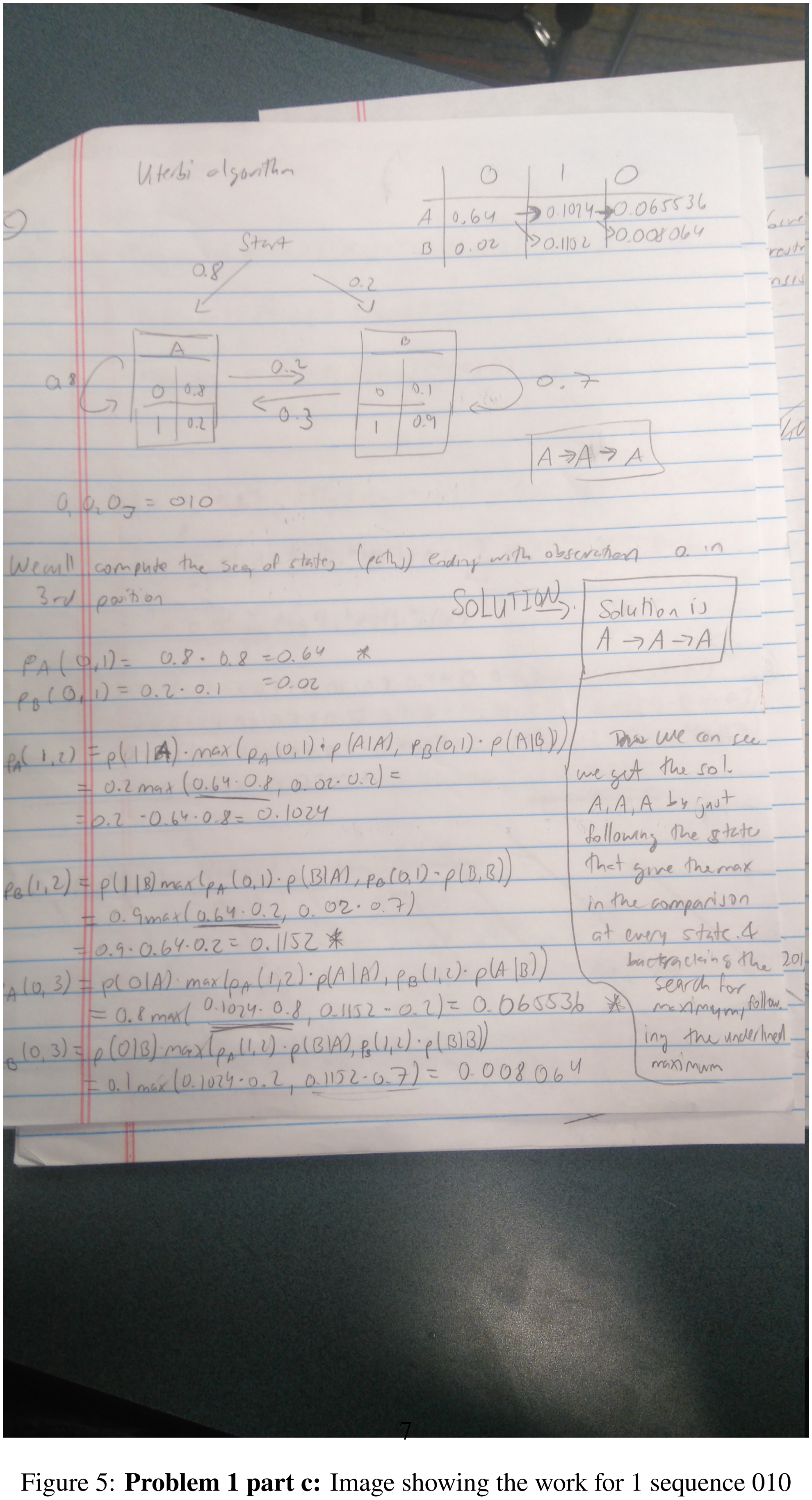
$= 0.1299$

$\beta_1(A) = \beta_2(A)\cdot E(1|A)\cdot P(A|A) + \beta_2(B)\cdot E(1|B)\cdot P(B|A)$

$= 0.2314\cdot 0.20\cdot 0.80 + 0.1299\cdot 0.9\cdot 0.20 = 0.060406$

Again by following the backward algorithm we are able to get the same solution as in 1b)

Figure 4: **Problem 1 part b:** Image showing the work for 1 sequence 101 backward alg



Viterbi algorithm

	0	1	0
A	0.64	→ 0.1024	→ 0.065536
B	0.02	↗ 0.1152	↗ 0.008064

$\boxed{A \to A \to A}$

$O_1 O_2 O_3 = 010$

We'll compute the seq. of states (paths) ending with observation 0 in 3rd position

SOLUTION → $\boxed{\text{Solution is } A \to A \to A}$

$p_A(0,1) = 0.8 \cdot 0.8 = 0.64$ *

$p_B(0,1) = 0.2 \cdot 0.1 = 0.02$

$p_A(1,2) = p(1|A) \cdot \max(p_A(0,1) \cdot p(A|A),\ p_B(0,1) \cdot p(A|B))$
$= 0.2 \max(\underline{0.64 \cdot 0.8},\ 0.02 \cdot 0.2) =$
$= 0.2 \cdot 0.64 \cdot 0.8 = 0.1024$

$p_B(1,2) = p(1|B) \max(p_A(0,1) \cdot p(B|A),\ p_B(0,1) \cdot p(B|B))$
$= 0.9 \max(\underline{0.64 \cdot 0.2},\ 0.02 \cdot 0.7)$
$= 0.9 \cdot 0.64 \cdot 0.2 = 0.1152$ *

$p_A(0,3) = p(0|A) \cdot \max(p_A(1,2) \cdot p(A|A),\ p_B(1,2) \cdot p(A|B))$
$= 0.8 \max(\underline{0.1024 \cdot 0.8},\ 0.1152 \cdot 0.2) = 0.065536$ *

$p_B(0,3) = p(0|B) \cdot \max(p_A(1,2) \cdot p(B|A),\ p_B(1,2) \cdot p(B|B))$
$= 0.1 \max(0.1024 \cdot 0.2,\ \underline{0.1152 \cdot 0.7}) = 0.008064$

Thus we can see we get the sol. A, A, A by just following the state that give the max in the comparison at every state & backtracking the search for maximum, following the underlined maximum

Figure 5: **Problem 1 part c:** Image showing the work for 1 sequence 010

Problem 2 Problem 2

Proof: ■

Problem 3 Problem 3

Proof:

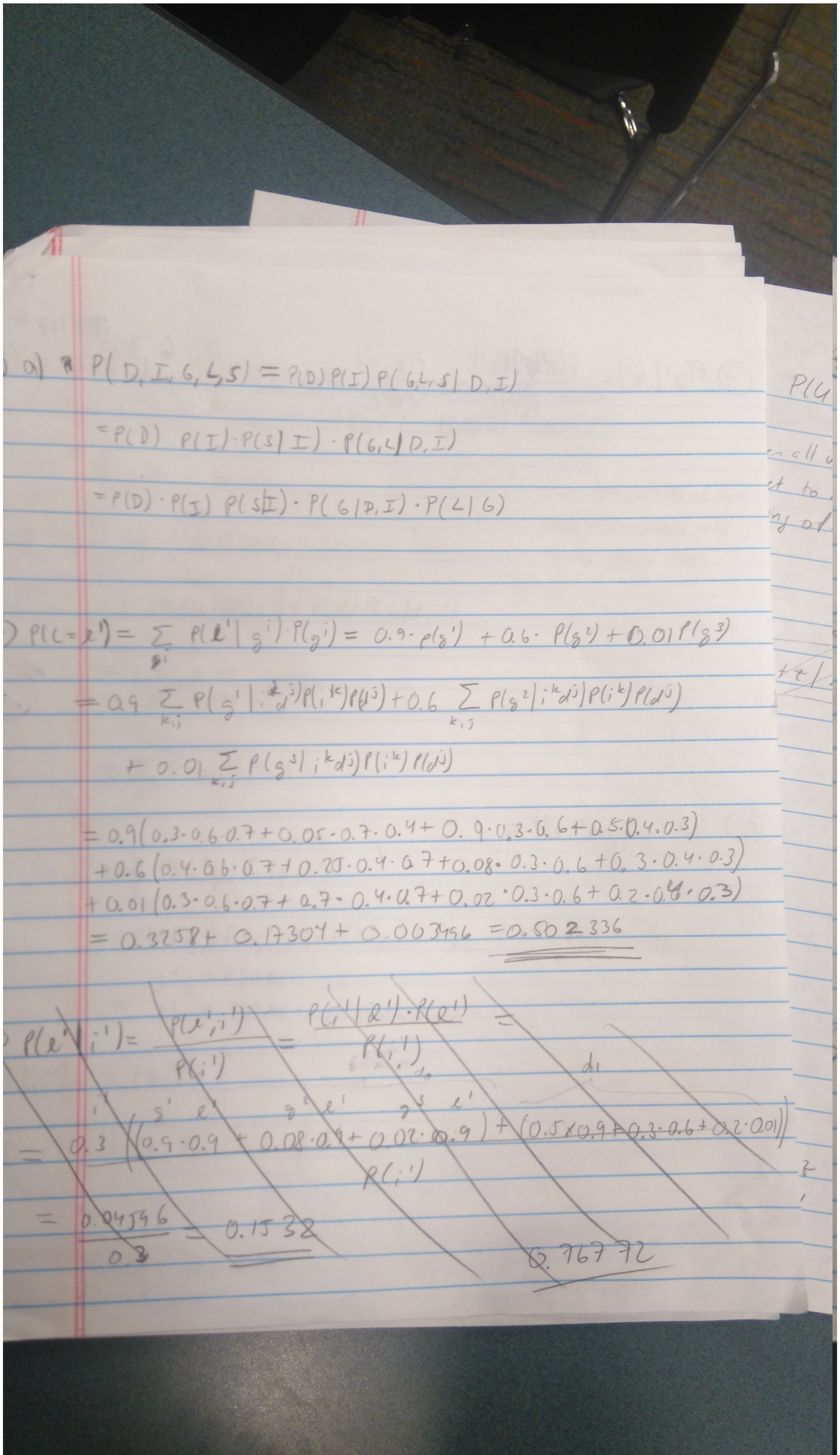
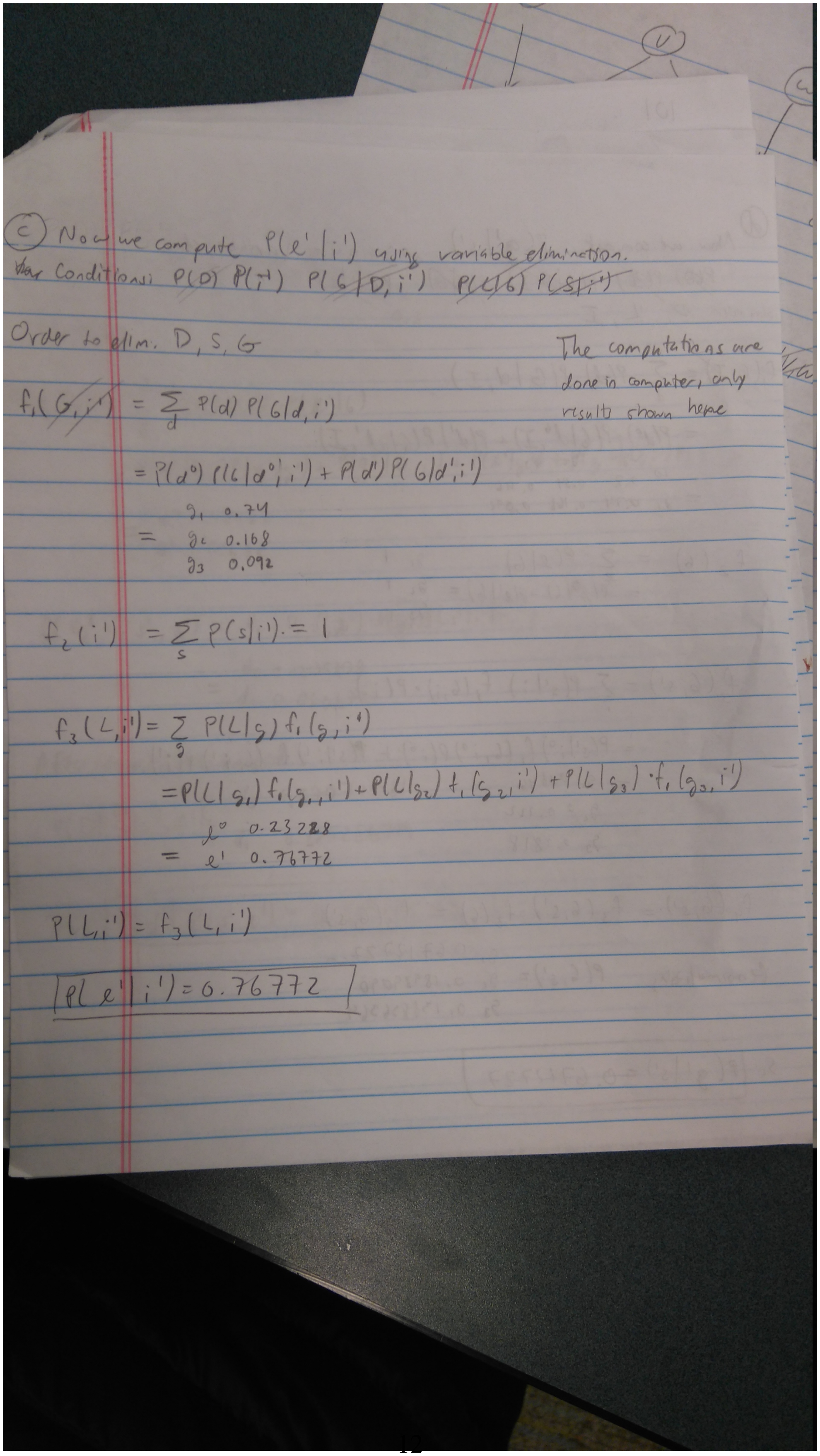


Figure 6: **Problem 3** Image showing the work for problem 3 part a and b



(c) Now we compute $P(e^1|i^1)$ using variable elimination.

Conditionals: $P(D)$ $P(i^1)$ $P(G|D,i^1)$ $P(L|G)$ $P(S|i^1)$

Order to elim. D, S, G

The computations are done in computer, only results shown here

$$f_1(G,i^1) = \sum_d P(d)\,P(G|d,i^1)$$

$$= P(d^0)\,P(G|d^0,i^1) + P(d^1)\,P(G|d^1,i^1)$$

$$= \begin{array}{ll} g_1 & 0.74 \\ g_2 & 0.168 \\ g_3 & 0.092 \end{array}$$

$$f_2(i^1) = \sum_s P(s|i^1) = 1$$

$$f_3(L,i^1) = \sum_g P(L|g)\,f_1(g,i^1)$$

$$= P(L|g_1)\,f_1(g_1,i^1) + P(L|g_2)\,f_1(g_2,i^1) + P(L|g_3)\,f_1(g_3,i^1)$$

$$= \begin{array}{ll} l^0 & 0.23228 \\ l^1 & 0.76772 \end{array}$$

$$P(L,i^1) = f_3(L,i^1)$$

$$\boxed{P(l^1|i^1) = 0.76772}$$

Figure 7: **Problem 3** Image showing the work for problem 3c

(d) Now we compute $P(g^1|s^1)$, again use variable elim.

$\cancel{P(D)}\ \cancel{P(I)}\ \cancel{P(G|D,I)}\ \cancel{P(L|G)}\ \cancel{P(s^1|I)}$

elim order: D, L, I

$\cancel{f_1(G,I)} = \sum_d P(d)\, P(G|d,I)$

$= P(d^0)\cdot P(G|d^0,I) + P(d^1)\, P(G|d^1,I)$

		g^1	g_2	g_3
	i_0	0.2	0.34	0.46
=	i_1	0.74	0.168	0.092

$f_2(G) = \sum_l P(l|G)$

$= P(l^0|G) + P(l^1|G) =$ g_1 1, g_2 1, g_3 1

$f_3(G,s^1) = \sum_i P(s^1|i)\ f_1(G,i)\cdot P(i)$

$= P(s^1|i^0)\, f_1(G,i^0)\, P(i^0) + P(s^1|i^1)\, f_1(G,i^1)\, P(i^1)$

= g_1 0.1846

g_2 0.05222

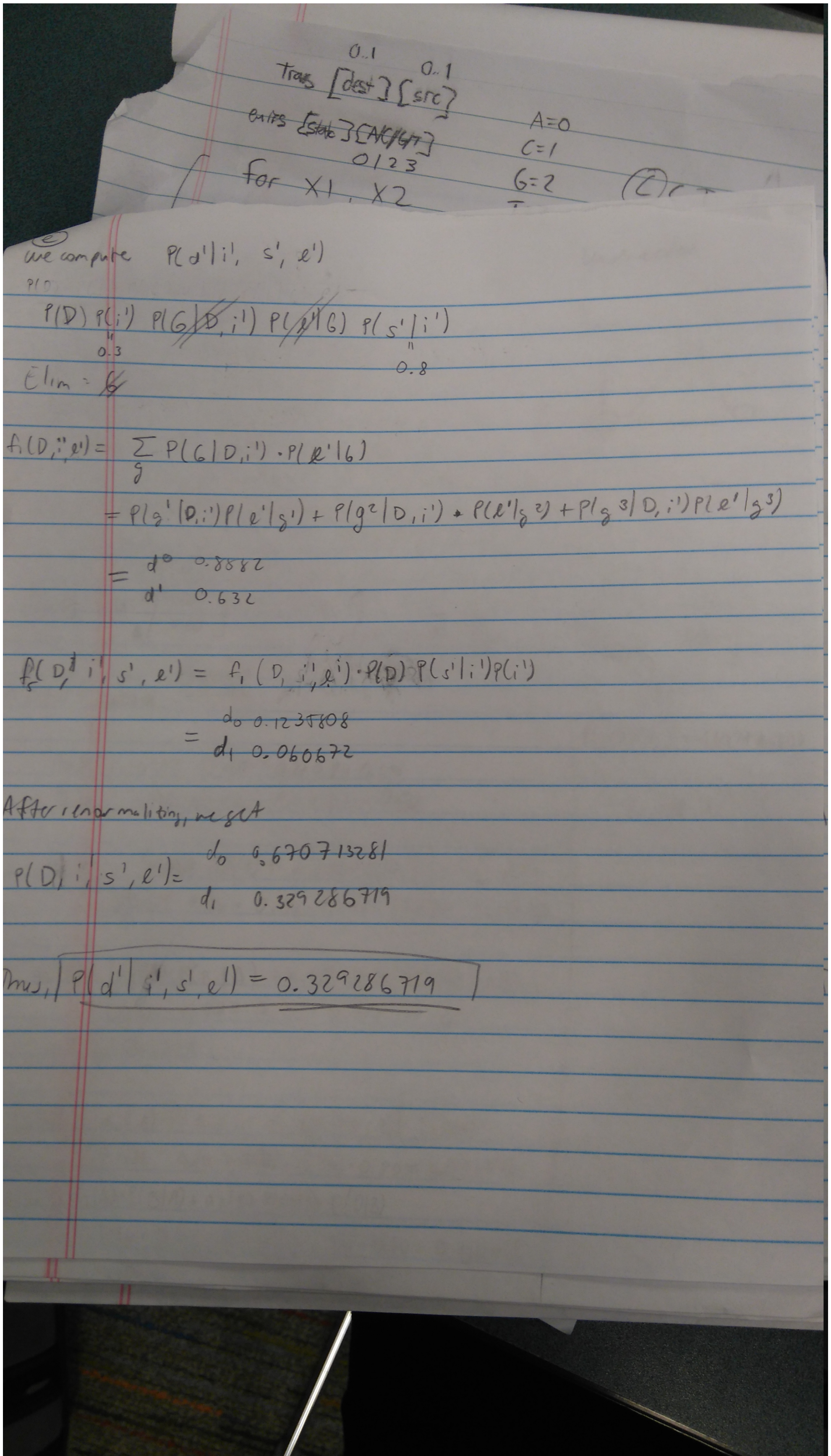
g_3 0.3818

$f_4(G,s^1) = f_3(G,s^1)\cdot f_2(G) = f_3(G,s^1)$

Renormalizing $P(G,s^1) =$ g_1 0.6712727..., g_2 0.18989090, g_3 0.138836364

So $\boxed{P(g^1|s^1) = 0.6712727}$

Figure 8: **Problem 3** Image showing the work for problem 3d



0..1 0..1
trans [dest] [src]
emits [state] [A/C/G/T]
0 1 2 3
for X1, X2

A=0
C=1
G=2

(e) we compute $P(d^1 | i^1, s^1, e^1)$

$P(D)\, P(i^1)\, P(G | D, i^1)\, P(e^1 | G)\, P(s^1 | i^1)$

$P(i^1) = 0.3$, $P(s^1 | i^1) = 0.8$

Elim = G

$$f_1(D, i^1, e^1) = \sum_g P(G | D, i^1) \cdot P(e^1 | G)$$

$$= P(g^1 | D, i^1) P(e^1 | g^1) + P(g^2 | D, i^1) \cdot P(e^1 | g^2) + P(g^3 | D, i^1) P(e^1 | g^3)$$

$$= \begin{array}{ll} d^0 & 0.8882 \\ d^1 & 0.632 \end{array}$$

$$f_2(D, i^1, s^1, e^1) = f_1(D, i^1, e^1) \cdot P(D)\, P(s^1 | i^1) P(i^1)$$

$$= \begin{array}{ll} d_0 & 0.1235808 \\ d_1 & 0.060672 \end{array}$$

After renormalizing, we get

$$P(D, i^1, s^1, e^1) = \begin{array}{ll} d_0 & 0.670713281 \\ d_1 & 0.329286719 \end{array}$$

Thus, $\boxed{P(d^1 | i^1, s^1, e^1) = 0.329286719}$

Figure 9: **Problem 3** Image showing the work for problem 3e

Problem 4 Problem 4

Proof: The two principal directions are:

$$\text{vector 1} = \left\{\begin{bmatrix} -0.24959319 \\ 0.31318631 \\ -0.24705298 \\ -0.06642483 \\ 0.07998801 \\ 0.24796042 \\ -0.77360385 \\ -0.14534989 \\ -0.04552843 \\ -0.25537919 \\ 0.0881295 \\ 0.10648068 \\ -0.01737437 \end{bmatrix}\right\}$$

$$\text{and vector 2} = \left\{\begin{bmatrix} 0.25652131 \\ 0.32130825 \\ -0.29855754 \\ -0.12862481 \\ 0.32209839 \\ 0.31167125 \\ 0.28754911 \\ 0.39962744 \\ 0.08089873 \\ -0.36040796 \\ -0.07174148 \\ -0.26369745] \end{bmatrix}\right\}$$

Also, the scatter plot of this first two principal components is:

■

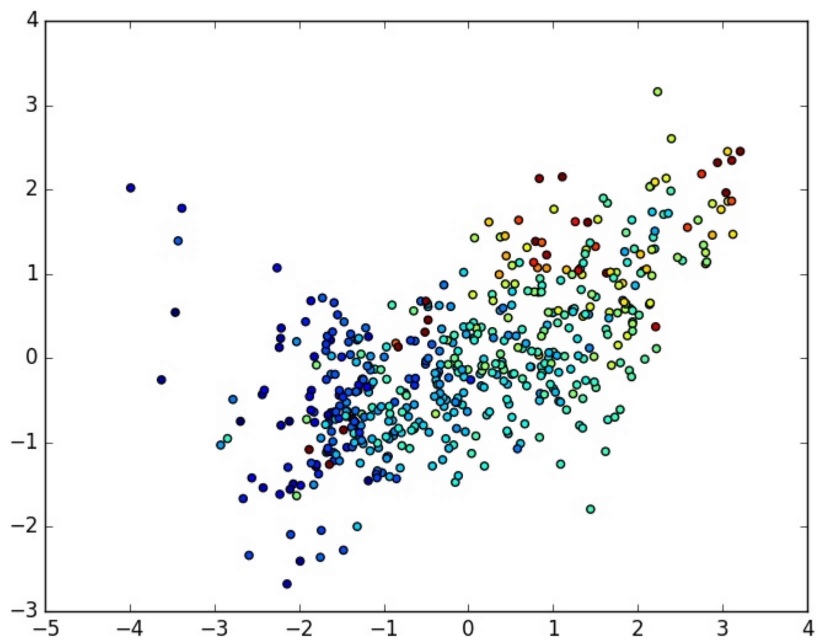


Figure 10: **Problem 4** Image showing scatter plot with color