Contents

Chapter 1

Some PL topology

In this chapter we will collect some basic facts from the piecewise-linear category. Most of the proofs are omitted and can be found in standard references like [1]. If the reader is already familiar with basics from PL topology, the chapter may be skipped without loss of continuity.

Definition 1. *Let $v_0, ..., v_k \in \mathbb{R}^n$ be points in some affine space such that $\{v_1 - v_0, ..., v_k - v_0\}$ is a set of linearly independent vectors. We call*

$$[v_0, ..., v_k] := \left\{ \lambda_0 v_0 + ... + \lambda_k v_k \,\middle|\, \sum_{i=0}^{k} \lambda_i = 1 \text{ and } \lambda_i \geq 0 \text{ for all } i \right\} \tag{1.1}$$

*the simplex spanned by $\{v_0, ..., v_k\}$. Its dimension is k and we call it a k-**simplex** for short. The points that span a simplex are called vertices. For a simplex σ we say that τ is a **face** of σ if τ is a simplex spanned by a nonempty subset of the vertices of σ and we abbreviate this by writing $\tau < \sigma$.*

Definition 2. *A **simplicial complex** K is a set of simplices that satisfies the following conditions:*

- *Every face of a simplex in K is also contained in K.*
- *The intersection of any two simplices $\sigma, \tau \in K$ is either empty or a face of both σ and τ.*

*We define the **geometric realization** of K by $|K| := \bigcup\{\sigma | \sigma \in K\}$.*

Definition 3. *Suppose that there are two simplicial complexes K and K' such that $|K| = |K'|$. If every simplex of K' is contained in some simplex of K, we say that K' is a **subdivision** of K and write $K' \lhd K$.*

Definition 4. *A topological space X is said to be* **triangulable** *if there exists a simplicial complex T and a homeomorphism $\phi : |T| \to X$. The triple (T, X, ϕ) is called a* ***triangulation*** *of X. In this situation we will simply say that T is a triangulation of X, by abuse of notation.*

Definition 5. *A* ***PL space*** *is a pair $(X, \mathcal{T})$ consisting of a topological space X and a class $\mathcal{T}$ of locally finite triangulations of X which satisfies the following conditions:*

- *If $T \in \mathcal{T}$ then $T' \in \mathcal{T}$ for any subdivision $T' \lhd T$.*
- *If $T, T' \in \mathcal{T}$ then there exists $T'' \in \mathcal{T}$ such that both $T'' \lhd T$ and $T'' \lhd T'$.*

We will simply write X for a PL space $(X, \mathcal{T})$ if there is no danger of confusion.

Bibliography

[1] Colin Patrick Rourke, Brian Joseph Sanderson, *Introduction to piecewise-linear topology.*