IBM Ponder This - March 2021

While the real Perseverance rover is having fun on Mars, we imagine an alternative version that scouts out an $N \times N$ grid of Mars according to the following rules:

(1) Surveying a cell is possible only if all its upper neighbors were already explored. The upper neighbors of (i, j) are defined as $(i-1, j-1)$, $(i-1, j)$, $(i-1, j+1)$. Cells that are not on the $N \times N$ grid do not need to be surveyed first.

(2) Each cell has a "score" $s_{ij} \in [s_{\min}, s_{\max}]$ points, indicating how valuable it is to explore it.

(3) Exploring a cell also requires rover maintenance, equivalent to a "cost" of c points.

The goal of the rover is to earn the maximum score possible from the grid. This means choosing which cells to explore that satisfy condition (1), such that the total score gained, considering rules (2) and (3), is the maximum score possible.
We define the set of indices as $[N] = \{1, \ldots, N\}$. Further, we define the predecessors of cell (i, j) as

$$P(i,j) = \begin{cases} \emptyset & \text{for } i = 1 \\ \{j-1, j, j+1\} & \text{for } 1 < i \leq N, 1 < j < N \\ \{1, 2\} & \text{for } 1 < i \leq N, j = 1 \\ \{N-1, N\} & \text{for } 1 < i \leq N, j = N \end{cases}$$

Then, define the binary decision variable x_{ij} to indicate whether or not cell (i, j) is explored. Then the problem can be stated as the following Integer Program:

$$\max \quad \sum_{i \in [N]} \sum_{j \in [N]} (s_{ij} - c) x_{ij} \tag{1}$$

$$\text{s.t.} \qquad x_{i,j} \leq x_{i-1,k} \qquad \forall i \in [N], j \in [N], k \in P(i,j) \tag{2}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in [N], j \in [N] \tag{3}$$