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Stochastic Resonance

Stochastic resonance is a phenomenon of an optimal response of a noise-driven system to a periodic forcing, at a specific noise intensity. See the article in Scholarpedia
`http://www.scholarpedia.org/article/Stochastic_resonance`
for an introduction, and a review paper [L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, Rev. Mod. Phys. 70, 223 (1998)] for more details.

In the simplest setup one considers a noise- and periodic-driven bistable system

$$\frac{dx}{dt} = x - x^3 + \sigma\xi(t) + A\cos(\omega t)$$

with white Gaussian noise $\langle\xi(t)\xi(t')\rangle = 2\delta(t - t')$. In the absence of the periodic force ($A = 0$) and relatively small noise the "oscillator" demonstrates random switches between states ± 1, according to the statistics given by the Kramers' formula [see Scholarpedia article above]; with small periodic forcing these switches may be "synchronized" by the periodic forcing and become highly regular.

(1) Implement numerically the stochastic differential equation above using the Euler-Maruyama method.

(2) Check the validity Kramers formula (dependence of the switching rate on the noise)

(3) With periodic forcing with fixed period (take a value $T = 100$ or close), find sin and cos Fourier components in $x(t)$ at the frequency of the forcing, and calculate (with errorbars) the amplitude and the phase shift of the response in dependence on the noise intensity σ

(4) By varying A check nonlinear properties of the stochastic resonance

(5) How the results depend on the time step?