Evapotranspiration Modeling in ECHSE: Documentation

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1 Introduction

This is a documentation of my work done in exploring some aspects of the way how evapotranspiration is simulated in the Eco-Hydrological Simulation Environment (ECHSE, Kneis 2015) with methods, classes, and engines written by Tobias Pilz. ECHSE is a model framework, i.e. a software providing calculation methods that can be composed modularly to form model engines. These engines can be compiled and be run as independent models with input data. ECHSE is used in the field of (eco)hydrology and therefore comes with a collection of methods especially relevant for issues in this field. The goal of the bigger project is to apply with ECHSE different ways of determining the potential (et_{pot}) and the actual evapotranspiration (et_{act}) . These are, as of the date of this documentation:

- the simple model of Makkink (1957),
- the equation of Penman & Monteith (Monteith, 1965),
- the FAO Penman-Monteith equation ("reference evaporation"),
- the equation of Shuttleworth & Wallace (Shuttleworth and Wallace, 1985).

The engine-specific names of variables and parameters are written in fixed-width whereas the physical symbols of variables, constants and parameters are written in *italics*. E.g., extraterrestrial radiation = R_{ex} = radex. Terms that describe parts of the ECHSE architecture are written in sans-serif.

1.1 Model overview – parameter overview

The engine parameters can be grouped into

- aerodynamic parameters,
- geographical parameters,
- radiation parameters,
- soil hydraulic parameters,
- vegetation parameters,

according to their physical role, and will be discussed in this order in chapter 3. Within the ECHSE engines, parameters are grouped into the types

- paramNum (object-specific scalar parameters),
- sharedParamNum (group-specific scalar parameters),
- inputExt (group-specific time-dependent scalar parameters treated as external input variables),

according to their role in the computation process. Tab. 1.1 contains an overview of the variables and parameters and whether they are used in the different et models (Makk = Makkink, PM = Penman-Monteith, FAO = FAO Penman-Monteith, SW = Shuttleworth-Wallace model). The parameters in this overview are grouped into the types paramNum, sharedParamNum. Parameters of the inputExt type can be found together with the actual external input variables.

In chapter 7, an overview of the estimated parameter values is given, grouped into the paramNum, sharedParamNum, and inputExt parameter types.

1.2 Study areas – available data

1.2.1 Portugal

The Machoqueira do Grou area is a 2.500 ha sized woodland in the Santarém district, between the towns of Coruche and Foros do Arrão, Portugal. The vegetation is dominated by cork oak trees and grasses. A part of the data comes from two measurement stations within the woodland, which were set up for measuring eddy covariance data with measurement towers and other meteorological variables. One station is located between trees under the open sky (Hauptstation, HS), the other was set up under a cork oak (Nebenstation A, NSA). The data include net radiation, air temperature, soil moisture content, sensible heat flux, latent heat flux, vapor pressure deficit, vapor pressure, and wind speed; they were recorded hourly. An evaporation flux was determined from energy balance considerations.

Additional meteorological data come from a nearby weather station. Those include photosynthetically active radiation (PAR), incoming short-wave radiation, outgoing long-wave radiation, air temperature, relative humidity, rainfall, and atmospheric pressure; they were recorded half-hourly.

1.2.2 Morocco

The study area is a citrus orchard near the village Ait Cheikh, southwest of Marrakesh, lying in the Haouz plain north of the Atlas mountains (Mroos, 2014). The available data contain relative humidity, global radiation, air temperature, wind speed, and latent heat; they were recorded (latent heat: calculated?) half-hourly. An evaporation flux was determined from energy balance considerations. Additionally, soil data were taken from Mroos (2014).

Table 1.1: External input variables, parameters and their usage in the et models. For abbreviations, see text. •: required by engine primarily, •: required only in cases when primary input is missing.

	et_p	oot			et_{act}		
Makk	PM	FAO	SW	PM	FAO	SW	ECHSE name
							inputExt
	0	0	0				alb
•	•	•	•				apress
	•		•				cano_height
0	0	0	0				cloud*
0	0	0	0				doy
•	•	0	•				glorad
	0	0	0				glorad_max
0	0	0	0				hour
	•		•				lai
	0	0	0				rad_long
	•	•	•				rad_net
	0	0	•				rad_net_soil
	•	•	•				rhum
_	•	_	•				soilheat
0	0	0	0				sundur
0	0	0	0				temp_max
0	0	0	0				temp_min
•	•	•	•				temper
0	0	0	•				totalheat utc_add
0	0	0	0				wc_vol_root
							wc_vol_top
							wc_voi_top
	_						paramNum
							bubble
	•	•	•				crop_faoref
•		_					crop_makk
0	0	0	0				elev
	•		•				glo_half
0	0	0	0				lat
	0	0	0				lon
				•		•	par_stressHum
				•		•	pores_ind
	•		•				res_leaf_min
			•				soil_dens
							wc_etmax
							wc_pwp
							wc_res
							wc_sat
							wstressmax
							wstressmin
							sharedParamNum
•	•	•	•				choice_et
	0	0	0				choice_gloradmax
	•		•				choice_plantDispl
	•		•				choice_rcs
	•		•				choice_roughLen
	•		•				drag_coef
			•				eddy_decay
	0	0	0				emis_a
	0	0	0				emis_b
	•	_	•				ext
	0	0	•				f_day
	0	0	•				f_night
	0	0	0				fcorr_a
	0	0	0				fcorr_b
	•						h_humMeas h_tempMeas
	•	_	_				h_tempMeas h_windMeas
0	•	•	•				n_windmeas radex_a
0	0	0					radex_a radex_b
O	U	J	0				radex_b res_b
			•				rough_bare
	-		•				rss_a
			•				rss_b
							*currently not used
				I			I carrong not asca

2 Aims

- Estimation of engine parameters through transfer functions and model calibration (chapter 3),
- comparison of the different et models (chapter 4),
- sensitivity analysis of engine parameters (chapter 5),
- evaluation of single methods employed in the et models (chapter 6).
- Which input data do we still need? (chapter 8)

3 Parameters: methods and results

3.1 Aerodynamic parameters: Portugal

Below the canopy toward the ground, wind speed is assumed to decrease exponentially with a scaling coefficient n, called the **eddy diffusivity decay constant** (eddy_decay); more precisely: The shearing stress of wind on a horizontal plane is proportional to $\rho \partial u/\partial z$ (where u is the horizontal wind component, z the vertical coordinate, and ρ the density of air) with the proportionality factor K. n relates the magnitude of K at the canopy height to that at the ground. Shuttleworth and Wallace (1985) use a value of n = 2.5, arguing that it results from the crop specification made by Monteith (1973) in deriving the above relation. Although I couldn't reconstruct this value from the second edition (Monteith, 1990), Shuttleworth and Wallace (1985) concluded from an sensitivity analysis that the resulting evapotranspiration was hardly influenced by changing n. Therefore, n has been set to 2.5.

The measurement heights of relative humidity (h_humMeas), temperature (h_tempMeas), and wind speed (h_windMeas) are known from the measurement setup and each equal to 2 m.

The aerodynamic **mean boundary layer resistance** r_b (res_b) was taken as 25 s m⁻¹ by Shuttleworth and Wallace (1985) based on field measurements by Denmead (1976) and Uchijima (1976). Like for n, the models seem to be quite insensitive for variations of r_b (Shuttleworth and Wallace, 1985), and r_b was set to 25 s m⁻¹.

In the calculation of the aerodynamic resistance between canopy and reference level, the displacement height of the vegetation and the roughness lengths of the crop for latent and sensible heat fluxes are used (see section 3.5 for these parameters). Following Shuttleworth and Gurney (1990), both of these are dependent on the roughness length of the bare substrate z_0 (rough_bare) and the effective mean drag coefficient of the vegetation c_d (drag_coef) as well as on the leaf area index and the canopy height (see section 3.5). The values of z_0 and c_d were numerically estimated as 0.01 m and 0.07, respectively, by the authors and adopted here.

3.2 Geographical parameters

The models employ 3 geographical parameters for calculating the radiation balance: latitude φ (lat), longitude L_m (lon), elevation h (elev). For the sites in Portugal, the locations were given as GIS data from which I could derive $\varphi = 39.14^{\circ}\text{N}$ and $L_m = 8.33^{\circ}\text{W}$ for both field stations using Google Maps. A common value for h of the Portugal sites was estimated from local elevation maps (floodmap.net). For Morocco, φ and L_m were derived from the location descriptions in Mroos (2014) while h was also estimated from floodmap.net.

3.3 Radiation parameters

3.3.1 alb

3.3.2 emis_a, emis_b: estimation methods

3.3.3 emis_a, emis_b: Portugal

I chose default values of emis_a = 0.34 and emis_b = -0.14, as suggested for average conditions by Maidment (1993). The values could not be estimated individually because the data from Portugal were not sufficient.

3.3.4 emis_a, emis_b: Morocco

I chose default values of emis_a = 0.34 and emis_b = -0.14, as suggested for average conditions by Maidment (1993). The values could not be determined because the data from Morocco were not sufficient.

3.3.5 f_day, f_night: estimation methods

Within the ECHSE engines, the sub-daily soil heat flux is currently calculated as

$$G_{soil} = f_{day} R_{net}$$
 during daytime, (3.1)

$$G_{soil} = f_{night} R_{net}$$
 during nighttime. (3.2)

with

 G_{soil} : soil heat flux (soilheat), in W m⁻²,

 f_{day} , f_{night} : soil heat factors (f_day, f_night), no unit,

 R_{net} : net incoming short-wave and long-wave radiation (rad_net), in W m⁻².

3.3.6 f_day, f_night: Portugal

Since the data didn't include measurements of R_{net} , I used the results of internally calculated values of R_{net} and measurements of G_{soil} for calculating f_{day} , f_{night} hourly. The distinction into daytime and nighttime was made using the RAtmosphere::suncalc procedure by Gionata Biavati. Averaging over all hours gave the results.

3.3.7 f_day, f_night: Morocco

3.3.8 fcorr_a, fcorr_b: estimation methods

3.3.9 fcorr_a, fcorr_b: Portugal

The cloudiness correction factors could not be determined because the data from Portugal were not sufficient.

3.3.10 fcorr_a, fcorr_b: Morocco

The cloudiness correction factors could not be determined because the data from Morocco were not sufficient.

3.3.11 glo_half

3.3.12 radex_a, radex_b: estimation methods

The estimation of the Angström parameters is based on the equation

$$\langle R_{inS} \rangle = \left(a_s + b_s \frac{n}{N} \right) \langle R_{ex} \rangle,$$
 (3.3)

with

 R_{inS} : incoming short-wave radiation (global radiation, glorad), in W m⁻²,

 R_{ex} : extraterrestrial short-wave radiation (radex), in W m⁻²,

 a_s , b_s : Ångström parameters (radex_a, radex_b), no unit,

n: sunshine duration of current day (time for which $\langle R_{inS} \rangle \geq 120 \text{ W m}^2$, sundur), in hours,

N: maximum possible sunshine duration, in hours.

Since R_{ex} and N are calculated internally, the uncertainty of the estimation should only depend on the observed global radiation (by definition, n follows from R_{inS}). Note that the equation as written represents only daily mean values of R_{inS} : The parameters are weighted depending on the daily ratio n/N in order to account for cloudiness. Thus,

$$a_s + b_s = \frac{R_{inS}}{R_{ex}}$$
 on days when $n = N$, (3.4)
 $a_s = \frac{R_{inS}}{R_{ex}}$ on days when $n = 0$. (3.5)

$$a_s = \frac{R_{inS}}{R_{er}}$$
 on days when $n = 0$. (3.5)

In both cases the parameters can be found for any given day with known values of n and R_{inS} . Averaging over all days would then determine the parameters. Problem: It is difficult to find either days when n = N or days when n = 0.

Another way of estimating a_s and b_s based on shorter time intervals (in our case, hourly) would be

$$a_s + b_s = \max_{h=1,\dots,24} \left\{ \frac{R_{inS}(h)}{R_{ex}(h)} \right\},$$
 (3.6)

$$a_s = \min_{h=1,\dots,24} \left\{ \frac{R_{inS}(h)}{R_{ex}(h)} \right\}. \tag{3.7}$$

For finding unique solutions, both (3.6) and (3.7) need to be determined because the equations are no longer dependent on n. The biggest problem of estimating the parameters with subdaily data is that extreme values (max, min) can result from measuring errors and statistical outliers. Taking very low quantiles of the R_{inS}/R_{ex} distribution and applying an upper limit for max $(a_s + b_s)$ can't be greater than 1) would probably avoid estimating a_s as too low and b_s as too high.

3.3.13 radex_a, radex_b: Portugal

Global radiation was measured hourly at the local weather station in Portugal. Therefore I chose to estimate radex_a and radex_b with (3.6) and (3.7). The R_{inS}/R_{ex} ratio was calculated for daytime hours between 6:00 and 18:00 local time. This assured that only radiation between sunrise and sunset was taken into account for all of the simulation period. For the determination of reasonable values I chose the lower 5-percentile of the R_{inS}/R_{ex} distribution as the minimum and the maximum value of $\{R_{inS}/R_{ex} < 1\}$ as the maximum over the whole period in which R_{inS} observations were available. These choices proved successful in computing clear-sky radiation that didn't exceed the observed global radiation and therefore fulfilled the physical conditions.

3.3.14 radex_a, radex_b: Morocco

3.4 Soil hydraulic parameters

wc_sat, wc_res, wc_pwp, wc_etmax, bubble, pores_ind, wstressmin, wstressmax, soil_dens, rss_a, rss_b.

3.5 Vegetation parameters

ext, par_stressHum, crop_makk, crop_faoref, res_leaf_min, lai, cano_height.

The Makkink crop factor crop_makk was estimated from the leaf area index by the affine relation

Makkink crop factor
$$\approx 0.14 \ LAI + 0.4$$
 (3.8)

with

LAI: leaf area index, in m² m⁻².

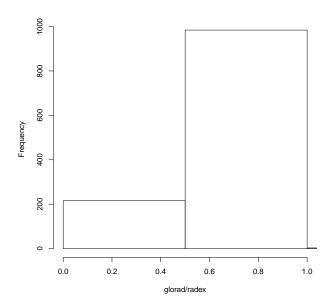


Figure 3.1: Histogram of the ratio R_{inS}/R_{ex} for values less than 1 in Portugal.

This relation was derived in the ECHSE documentation from data of Feddes (1987) and Ludwig and Bremicker (2006).

The FAO crop factor was set to 1. This value corresponds to the reference crop (well-watered grass of 0.12 m height, 70 s m $^{-1}$ surface resistance, 0.23 albedo). Since the FAO Penman-Monteith equation is just a simplification of the Penman-Monteith equation, I preferred using the original Penman-Monteith model for the study cases and using the FAO reference evaporation only for a subsumption of the different models.

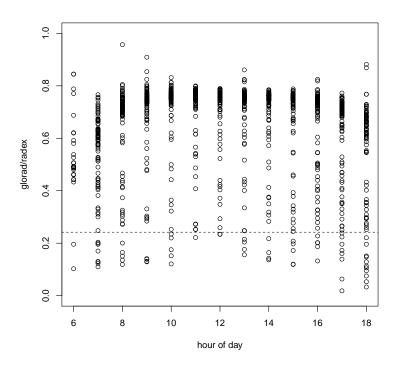


Figure 3.2: R_{inS}/R_{ex} ratio dependent on hour of day. The shift of higher values toward early hours may indicate either a small temporal deviation between the simulated and the actual R_{ex} , or a local effect that weakens R_{inS} during afternoon, or both. The dashed line marks the 5-percentile of the total distribution.

4	Comparison	of	evapotranspiration	models
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5 Sensitivity analysis

6 Evaluation of ECHSE methods

6.1 Global radiation glorad

6.1.1 glorad: Portugal

glorad_portugal

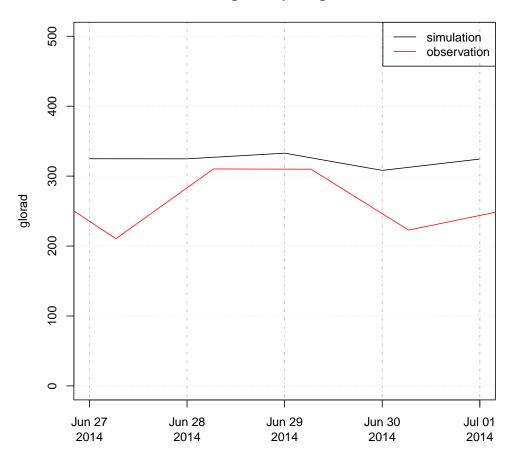


Figure 6.1: .

6.1.2 glorad: Morocco

6.2 Net incoming radiation rad_net

6.2.1 rad_net: Portugal

6.3 Soilheat flux soilheat

6.3.1 soilheat: Portugal

glorad_portugal

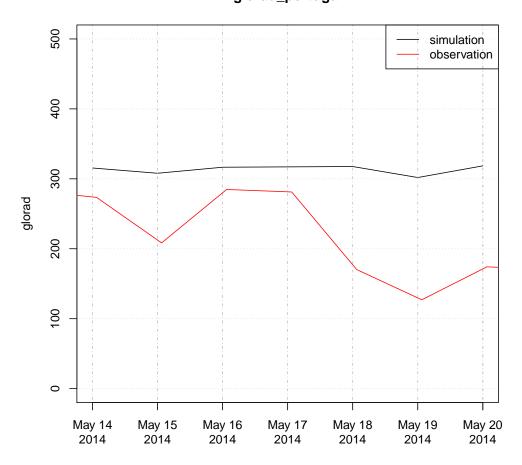


Figure 6.2: .

rad_net_portugal

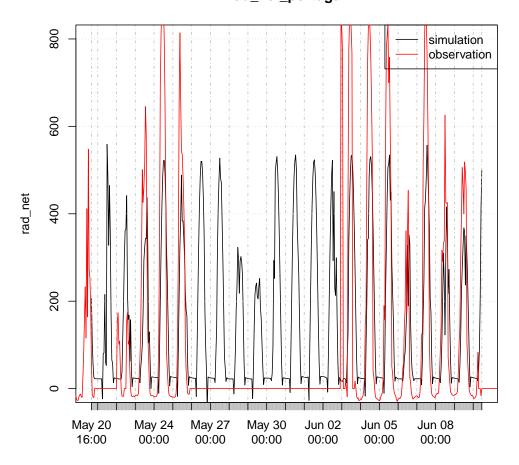


Figure 6.3:.

rad_net_portugal

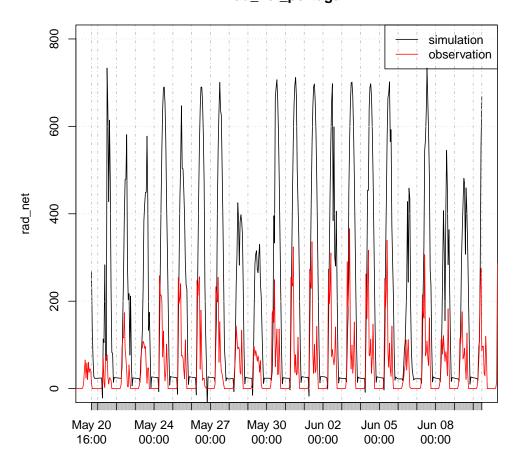


Figure 6.4: .

soilheat_portugal

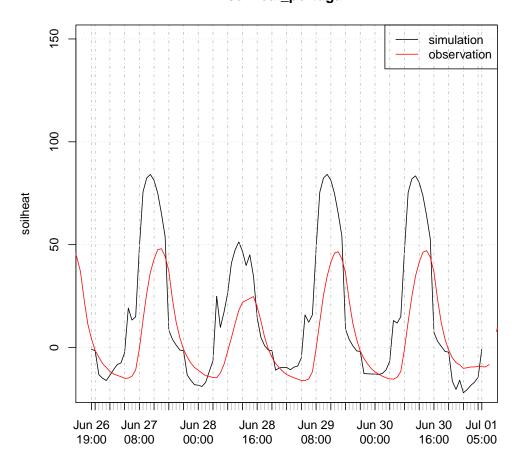


Figure 6.5: .

soilheat_portugal

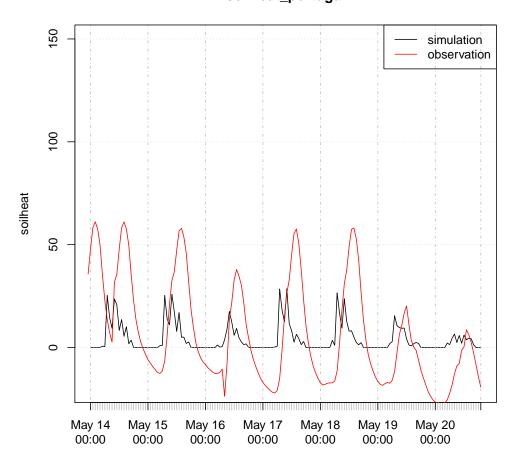


Figure 6.6: .

7 Results: overview

7.1 Parameters

An overview of the parameters is given for the paramNum group (object-specific scalar parameters, HS Portugal: Tab. 7.1, NSA Portugal: Tab. 7.2, Morocco: Tab.), the sharedParamNum group (group-specific scalar parameters, HS Portugal: Tab. 7.3, NSA Portugal: Tab. 7.4, Morocco: Tab.), and the inputExt group (group-specific scalar parameters given as time series, HS Portugal: Tab. 7.5, NSA Portugal: Tab. 7.6, Morocco: Tab.).

Table 7.1: Object-specific scalar parameters (paramNum), HS Portugal

Table 7.1. Object-specific scalar parameters (paramitum), 115 1 ortugar					
Parameter	Value	Unit	Comment		
bubble	8.08	hPa	PTF by Rawls and Brakensiek (1985)		
crop_faoref	1.00	_	evaporation of reference crop		
crop_makk	0.80	_	Eq. 3.8		
elev	160.00	m	local elevation map		
glo_half	200.00	${ m W~m^{-2}}$	guessed		
lat	39.14		GIS data		
lon	8.33		ditto		
par_stressHum	0.03	hPa^{-1}	guessed		
pores_ind	0.45	_	PTF by Rawls and Brakensiek (1985)		
res_leaf_min	50.00	${ m s~m^{-1}}$	guessed		
soil_dens	1500.00	${\rm kg~m^{-3}}$	guessed		
wc_etmax	0.13	_	calibration		
wc_pwp	0.07	_	PTF by Rawls and Brakensiek (1985)		
wc_res	0.05	_	(PTF by Rawls and Brakensiek (1985))		
wc_sat	0.39	_	PTF by Wösten et al. (1999)		
wstressmax	10000.00	hPa	wilting point		
wstressmin	100.00	hPa	field capacity		

Table 7.2: Object-specific scalar parameters (paramNum), NSA Portugal

	0 1		(, , , , , , , , , , , , , , , , , , ,
Parameter	Value	Unit	Comment
bubble	8.08	hPa	PTF by Rawls and Brakensiek (1985)
crop_faoref	1.00	_	evaporation of reference crop
crop_makk	0.80	_	Eq. 3.8
elev	160.00	m	local elevation map
glo_half	200.00	${ m W~m^{-2}}$	guessed
lat	39.14		GIS data
lon	8.33		ditto
par_stressHum	0.03	hPa^{-1}	guessed
pores_ind	0.45	_	PTF by Rawls and Brakensiek (1985)
res_leaf_min	50.00	${ m s~m^{-1}}$	guessed
soil_dens	1500.00	${\rm kg~m^{-3}}$	guessed
wc_etmax	0.13	_	calibration
wc_pwp	0.07	_	PTF by Rawls and Brakensiek (1985)
wc_res	0.05	_	(PTF by Rawls and Brakensiek (1985))
wc_sat	0.39	_	PTF by Wösten et al. (1999)
wstressmax	10000.00	hPa	wilting point
wstressmin	100.00	hPa	field capacity

Table 7.3: Group-specific scalar parameters (sharedParamNum), HS Portugal

			C
Parameter	Value	Unit	Comment
drag_coef	0.07	_	calibration
eddy_decay	2.50	_	as used by Shuttleworth and Wallace (1985) from Monteith (1973)
emis_a	0.34	_	as used by Maidment (1993) for average conditions
emis_b	-0.14	_	ditto
ext	0.40	_	guessed
f_day	0.10	_	estimation from soil heat data
f_night	0.70	_	ditto
fcorr_a	1.35	_	as used by Maidment (1993)
fcorr_b	-0.35	_	ditto
h_humMeas	2.00	m	
h_tempMeas	2.00	\mathbf{m}	
h_windMeas	2.00	\mathbf{m}	
radex_a	0.14	_	estimation from radiation data
radex_b	0.67	_	ditto
res_b	25.00	${ m s~m^{-1}}$	as used by Shuttleworth and Wallace (1985)
rough_bare	0.01	\mathbf{m}	ditto
rss_a	37.50	_	
rss_b	-1.23	_	

Table 7.4: Group-specific scalar parameters (sharedParamNum), NSA Portugal

Parameter	Value	Unit	Comment
drag_coef	0.07	_	calibration
eddy_decay	2.50	_	as used by Shuttleworth and Wallace (1985) from Monteith (1973)
emis_a	0.34	_	as used by Maidment (1993) for average conditions
emis_b	-0.14	_	ditto
ext	0.40	_	guessed
f_day	0.10	_	estimation from soil heat data
f_night	0.70	_	ditto
fcorr_a	1.35	_	as used by Maidment (1993)
fcorr_b	-0.35	_	ditto
h_humMeas	4.84	m	
$h_tempMeas$	4.84	m	
$h_windMeas$	4.84	\mathbf{m}	
radex_a	0.14	_	estimation from radiation data
radex_b	0.67	_	ditto
res_b	25.00	${ m s~m^{-1}}$	as used by Shuttleworth and Wallace (1985)
rough_bare	0.01	m	ditto
rss_a	37.50	_	
rss_b	-1.23	_	

Table 7.5: Time-dependent parameters (inputExt), HS Portugal

		` •
Parameter	Value	Unit
alb	0.30	_
cano_height	0.20	\mathbf{m}
lai	0.78	_

Table 7.6: Time-dependent parameters (inputExt), NSA Portugal

Parameter	Value	Unit
alb	0.30	_
cano_height	4.84	\mathbf{m}
lai	1.40	_

8 Conclusion

- Soil heat flux needs a better model.

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