NOTES ON RADIATION IN THE ATMOSPHERE. I.

By D. BRUNT, M.A., B.Sc.

[Manuscript received February 18—read April 20, 1932.]

1. Introduction and Summary

The present note arose out of an effort to find a simple means of estimating the net loss of heat by radiation from the ground at night; but as this required the use of detailed information as to the form of the absorption spectrum of water vapour, it was found necessary to discuss in detail the differences in absorptive power of water vapour for long-waved radiation of different wave-lengths. A curve is reproduced in Fig. 1 which gives the coefficient of absorption for a range of wave-lengths from 0 to 34μ, based on the experimental work of Hettner.

Several writers have claimed that the work of Eva von Bahr indicates that the absorption by water vapour depends not only on the total amount of water vapour present, but also on the total pressure to which it is subjected. Arguments are adduced which appear to show conclusively that the crude correction for total pressure is unjustifiable; and that the absorption by water vapour, at least in the lowest layers of the atmosphere, may be evaluated by the direct use of the coefficients shown in Fig. 1.

The absorption spectrum of liquid water is shown graphically in Fig. 2, and the possibility of treating cloud and fog as black-body radiators is discussed.

A simple formula connecting the downward radiation R from the atmosphere, σT^4 the total black-body radiation at temperature T, and e the vapour pressure, is given in the form

$$R=\sigma T^4(a+b\sqrt{e})$$

where a and b are constants. The formula appears to give a very close fit to all the available series of data. It is also shown that observations at different heights above sea level also fit the same formula, thus indicating that the net radiation from high ground is the same as from low ground under the same conditions of temperature and vapour pressure.

The variation of surface temperature during the night, when radiation alone is taken into account, is shown to give a parabolic thermograph curve from sunset to sunrise, and a formula is derived which can be used to forecast the extreme minimum to which the temperature can fall during a calm night with clear skies.

The radiation from the ground when the sky is overcast is shown to fit in with the very small diurnal variation of temperature observed at the ground.

The radiation from and absorption by cloud sheets are discussed, the theoretical conclusions being tested by comparison with the observations of Ångström and Asklöf. It is shown that any cloud sheet should be cooled at the top and heated at the bottom. Both effects increase with height, the heating of the base being very slight in low cloud. This appears to offer an explanation of the more frequent occurrence of clouds broken up into cloudlets or strips at middle and high levels than at low levels. In the daytime, with the sun shining on the top of the cloud, the cooling may be diminished, and it is suggested that the more frequent occurrence of ascending central motion in cloudlets observed by day may be

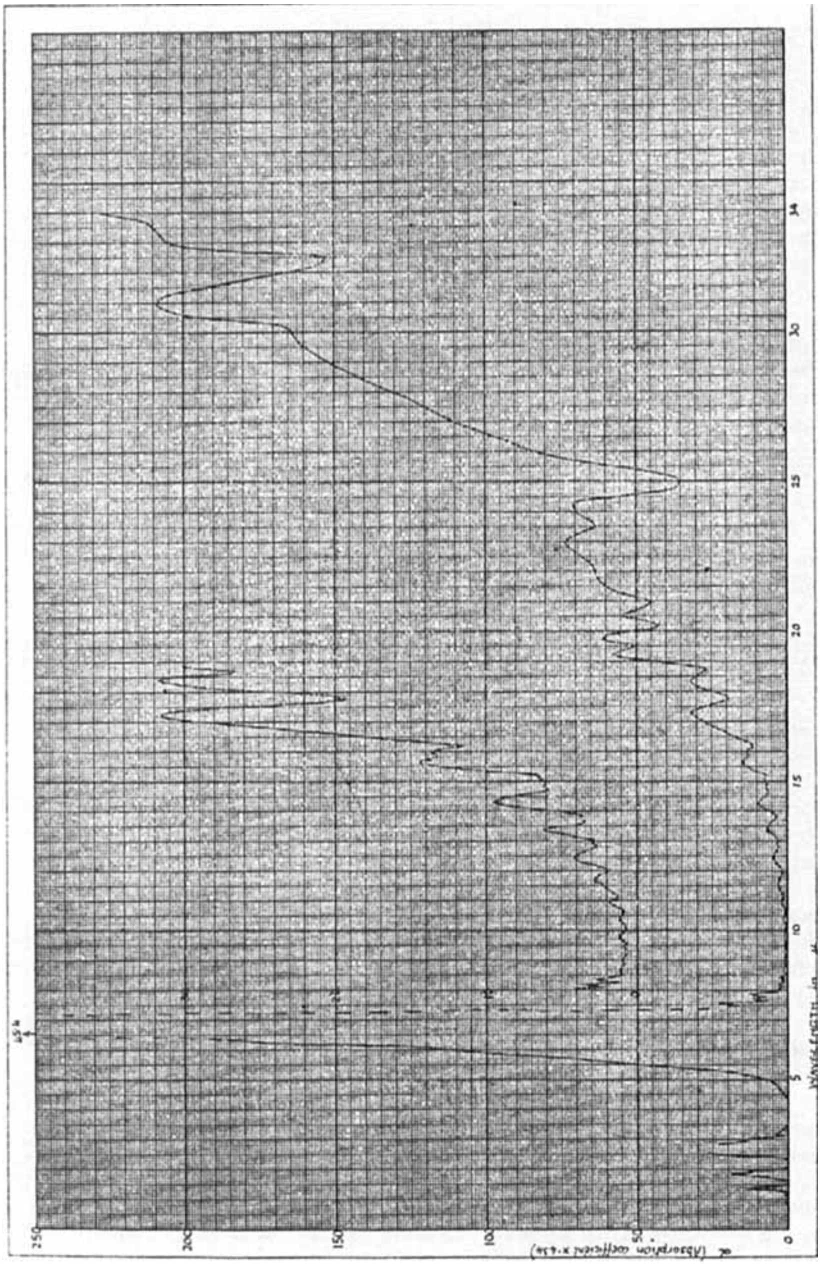

FIG. 1.—The absorption spectrum of water vapour. The ordinate gives a the decimal absorption coefficient for each wave-length. The transmission through m centimetres of precipitable water is 10^{-am}

thus explained as due to the base being the centre of activity. It is shown that radiation may cause the formation of inversions both at the top and below the base of a cloud sheet, the inversion at the top forming slightly more readily by night than by day, but the inversion below the cloud being independent of the time of day.

2. The Absorption Spectrum of Water Vapour

The absorption spectrum of water vapour has been the subject of many researches, of which the most notable is that of Hettner (1), which we shall take as the basis of our discussion in the following pages. Earlier writers had, however, found marked absorption bands centred at $2{\cdot}7\mu$ and $6{\cdot}7\mu$ and a wide band extending from about 14μ upwards. (The unit of wave-length, μ, is one-thousandth of a millimetre.) Rubens and Hettner (2) mention the occurrence of very strong absorption at 50μ, $58{\cdot}5\mu$, 66μ and 79μ. In the earth's atmosphere we shall in general not be concerned with radiation of wave-lengths above about 60μ, so that it is not necessary to consider the nature of the absorption spectrum of water vapour beyond this limit. Radiation from a black body at a temperature of 300°A. is within the limits 4μ to 40μ, with a maximum intensity at 10μ. At a temperature of 200°A. the limits are 5μ to 30μ, with the maximum intensity at 15μ.

In the paper referred to, Hettner gives a series of curves showing the total absorption at different wave-lengths by a column of water vapour at a temperature of 127°C., or 400°A. On account of the enormous variation in absorption with wave-length, Hettner used columns of steam of different lengths, 109 cm. and 104 cm. for regions of relatively slight absorption, and 32·4 cm. for regions of great absorption, while in regions of very intense absorption he used a mixture of air and water vapour. All the columns, whether of steam or of air and steam, were at atmospheric pressure, and corresponded to an amount of precipitable water varying from ·45 mm. to ·017 mm. per cm.^{2} of cross-section.

Hettner's curves give the total absorption in a column of water vapour. His results have been used by Simpson (3) to give the total absorption by ·3 mm. of precipitable water. Simpson found that above about 14μ and within the band $5\frac{1}{2}\mu$ to 7μ the absorption by this amount of precipitable water was effectively complete. In any actual computation, involving any quantity of water vapour other than those used by Hettner, or those discussed by Simpson, what we actually require for each wave-length is the coefficient of absorption for unit mass of water vapour. If the coefficient of absorption for radiation of wave-length λ is k, the fraction of the incident radiation which is transmitted through an amount of water vapour m is ϵ^{-km}, or 10^{-am}, where

$$a = {\cdot}4343\, k.$$

For our immediate purposes the second form is more convenient, and consequently this is adopted in the present paper. From time to time we may require to use either a or k. No specific name has ever been given to the quantity a and it is suggested that it would be useful to call it the "decimal coefficient of absorption" while k might be called the "Naperian coefficient of absorption." The use of these two names would avoid any possible confusion in the use of the two coefficients. The value of a has been evaluated for a large number of separate wave-lengths, by taking from Hettner's curves the logarithm of the transmission, and dividing by the appropriate mass of precipitable water in the column of water vapour. The results are shown in Fig. 1, for wave-lengths up to 34μ. The inset figure is a reproduction, with an enlarged vertical scale, of the portion between 9μ and 19μ. In this diagram the vertical ordinate measures a, or the decimal coefficient of absorption; here a measures the transmission referred to 1 cm. of precipitable water. The use of the diagram can be readily seen from a few examples. At $19{\cdot}5\mu$, $a=50$, and the fraction of the incident radiation of this wave-length transmitted through a column containing 1 mm. of precipitable water is 10^{-5}, and the fraction transmitted through a column containing 2 mm. of precipitable water is 10^{-10}. At about 28μ, $a=130$, and the fraction of the incident radiation transmitted through a column containing 1 mm. of precipitable water is 10^{-13}.

The main regions of absorption shown by the diagram of Fig. 1, apart from some very narrow lines around 1μ are:—

(a) bands centred at $1{\cdot}37\mu$, $1{\cdot}84\mu$, and $2{\cdot}66\mu$,
(b) a very intense band centred at $6{\cdot}26\mu$,
(c) a wide band beginning at about 9μ, rising rapidly, though with some oscillations, and becoming continuously more intense with increasing wave-length, up to the limit of 34μ at which Hettner's observations terminate. There is reason to suppose that this intense absorption continues up to at least 60μ.

Bands in the infra-red spectra are due in general to oscillation, rotation, or the combination of oscillation and rotation of the molecules. Rubens and Hettner (*loc. cit.*) have shown that the frequencies of lines due to oscillation and rotation of the molecules are given by the formula

$$s_1v_1+s_2v_2$$

where v_1 and v_2 are the frequency numbers of two fundamental lines, and s_1s_2, are integers or zero. If v_1 and v_2 correspond to the lines at $6{\cdot}26\mu$ and $2{\cdot}66\mu$, then by giving s_1 and s_2 appropriate integral values, we obtain the frequencies of the lines or bands at $3{\cdot}15\mu$, $2{\cdot}00\mu$, $1{\cdot}84\mu$, $1{\cdot}46\mu$, $1{\cdot}37\mu$, $1{\cdot}16\mu$, $1{\cdot}13\mu$, $0{\cdot}94\mu$, $0{\cdot}77\mu$ and $0{\cdot}66\mu$. The lines or bands of absorption which are shown in Fig. 1, at wave-lengths up to $6{\cdot}26\mu$ are therefore accountable as belonging to the family of lines whose frequencies are of the form $s_1v_1+s_2v_2$. The members of this family omitted from the diagram are too narrow or too slight in effect to be of meteorological importance, and cannot in any case be represented on a diagram with this vertical scale.

The bands at $1{\cdot}37\mu$, $1{\cdot}84\mu$, $2{\cdot}66\mu$, and a subsidiary band at $3{\cdot}15\mu$ have been studied in great detail by numerous writers. Sleator and Phelps (4), using a ruled grating, measured 126 lines in the band centred at $6{\cdot}26\mu$, thus showing this band to be of an extremely complicated nature.

From $3\frac{1}{2}\mu$ to $4\frac{1}{2}\mu$ no appreciable absorption has been measured. Also from $8\frac{1}{2}\mu$ to $9\frac{1}{2}\mu$ the absorption measured by Hettner and others is so small that it is questionable whether it is to be regarded as real. Simpson (*loc. cit.*) treated the absorption in the region $8\frac{1}{2}\mu$ to 11μ as negligible. In view of the importance of establishing the reality or otherwise of the absorption in this region, I have considered in some detail the nature of the curve of absorption beyond 9μ. It is seen from Fig. 1 that from about $9\frac{1}{2}\mu$ there is a marked rise in the curve, but with an oscillation about a smooth curve. The maxima of the curve at about $9{\cdot}8\mu$, $10{\cdot}4\mu$, 11μ, $11{\cdot}7\mu$, etc., appear to correspond to the positions of lines of absorption superposed on a background of continuous absorption. The difference in wave-length between successive maxima increases with increasing wave-length, but it is found that when the frequencies corresponding to these maxima are evaluated, the frequency difference between successive maxima has the uniform value $\Delta v = 1{\cdot}7 \times 10^{12}$. The bands of lines with equal intervals of frequency are a common feature of infra-red spectra; they are ascribed to the rotation of the molecules, the bands being usually referred to as *rotation* spectra. Sommerfeld (5) gives the theory of the formation of such spectra, and deduces a formula connecting the frequency interval Δv with Planck's quantum number h, and the moment of inertia J of the molecule about its axis of rotation—

$$\Delta v = \frac{h}{4\pi^2 J}$$

Inserting the values of h and Δv, we find $J = {\cdot}98 \times 10^{-40}$ in c.g.s. units. This is a very low value for J, when compared with the values obtained for J for other molecules in a similar manner; but the order of magnitude is the same as for other molecules, J being always measured in units of 10^{-40}. It can only be explained on the supposition that the molecule of water vapour is almost a linear structure, the oxygen atom being only slightly displaced from the centre of the line joining the hydrogen atoms.

If we treat the water-vapour molecule as consisting of 3 atomic masses concentrated at 3 points, and rotating about a line parallel to the line joining the centres of the hydrogen atoms we find that the value of the moment of inertia leads to the result that the oxygen atom is displaced from the centre of the line joining the hydrogen atoms through a distance of ${\cdot}21 \times 10^{-8}$ cm. Watson (6) interprets the observations of Bragg (7) on the structure of ice as indicating that the separation of adjacent hydrogen atoms is $1{\cdot}38 \times 10^{-8}$ cm. and that the distance between the oxygen atom and the centre of the line joining adjacent hydrogen atoms is ${\cdot}79 \times 10^{-8}$ cm. In the water-vapour molecule the atoms of hydrogen appear to be displaced further apart than in ice, and the oxygen atom approaches more nearly to the centre of the line joining the hydrogen atoms.

Sleator and Phelps (4) observed three series of lines in the band at 6μ, from which they deduced three values of J all near 3×10^{-40}. From this figure we deduce that the separation of the hydrogen atoms in the water-vapour molecule is about $1{\cdot}5 \times 10^{-8}$ cm.

The above digression into atomic physics is here included only in order to indicate that the wavy portion of the curve in Fig. 1 between $9\frac{1}{2}\mu$ and 18μ has a physical meaning. From this we deduce that the peak at $9{\cdot}8\mu$ and a barely perceptible peak at about $9{\cdot}25\mu$ are both real, and we also obtain a striking confirmation of the careful nature of the work carried out by Hettner.

The background underlying the rotation band and the absorption at greater wave-lengths can perhaps be regarded as continuous absorption. A continuous absorption spectrum arises if, say, an originally neutral atom becomes ionised and sets an electron free. The frequency of the radiation absorbed is measured by the amount of work involved in setting the electron free, and continuous absorption is only possible if there is a practically infinite gradation of configurations which the electrons may take up.

3. The Effect of Pressure on Water-Vapour Absorption

It was found by K. Ångström (8) that the absorption by a given amount of CO_2 increased with increasing pressure. If air or hydrogen was introduced into the tube containing CO_2 so as to increase its pressure, the amount of absorption was increased, not by the widening of the lines, but by intensifying them. Eva von Bahr (9) describes briefly the work of K. Ångström, and explains that she continued this work. In the course of two papers she gives a number of measurements of absorption by water vapour at different pressures. There is, however, no specific mention of the method by which the amount of the total pressure was varied, and it is not possible to interpret with certainty the meaning of these observations. The observations were confined to absorption in the neighbourhood of $2{\cdot}7\mu$. In the second paper referred to the author gives the following table of percentage absorption in a tube 150 cm. long.

Pressure in mm. ...	12	100	235	370	405	570	755
Percentage absorption...	3·0	4·7	7·2	8·6	8·5	10·6	12·1

There is no definite statement in either paper of the method by which the pressure is varied, nor is it stated whether the total amount of water vapour in the tube is kept constant throughout the experiment. It is therefore not possible to interpret the above table with certainty. Several Continental writers, notably Albrecht (10) and Ångström (11) have interpreted this table as giving the total absorption by a fixed mass of water vapour at varying pressures, so that in order to calculate the amount of absorption by a given mass of water vapour we must take into account the total pressure to which the water vapour is subjected. This is in contradiction with the law of absorption usually employed (Beer's Law), which states that the transmission is $10^{-\alpha m}$ where α is constant for any wave-length, and m is the mass of water vapour in the path, per unit cross-section.

The interpretation of Eva von Bahr's observations has become a matter of importance, since they have been assumed to mean that the absorptive power of a given mass of water vapour decreases with decrease of total pressure, so that in order to use the coefficients of absorption shown in Fig. 1 to discuss absorption at a height where the pressure is, say, 400 mm., we should in effect have to diminish the assumed amount of water vapour in the ratio $8{\cdot}5/12{\cdot}1$, or roughly ·7, while at a height where the pressure is about one-fourth the surface pressure, we should have to take one-half the amount of water vapour actually present. Simpson has shown (*loc. cit.*) that the amount of water vapour present in the stratosphere probably amounts to about ·3 mm. of precipitable water; and so, with Hettner's coefficients, is sufficient to produce the radiation necessary to send back into space the energy received from the sun, allowing for the radiation sent out in the transparent bands. If we have to allow for a decrease of absorptive and radiative power with total pressure, then it becomes clear that the effective radiating layer of the atmosphere is in the upper troposphere as suggested by Albrecht (10).

There is no evidence of any effect due to other gases in the work of Hettner. In the regions of most intense absorption Hettner used a mixture of air and water vapour, at a total pressure of 750 mm. The results thus derived agree perfectly with those derived with columns of water vapour alone, for all wave-lengths which overlap in the separate determinations. It is, however, to be noted that the total pressure was constant in all Hettner's experiments.

The observations of Eva von Bahr on the oscillation spectrum of HCl were repeated by G. Becker (12) in Hettner's laboratory at Berlin. Becker showed that the effect of the change of total pressure upon the form of an absorption band could be explained by assuming that higher pressure widened the band, making it less intense in the centre. The form of the absorption band may be represented graphically by plotting the coefficient of absorption k as abscissa against the frequency v as ordinate. Becker assumed that this diagram could be represented by a Gaussian or error curve, which was lowered and flattened at higher pressures, and showed that his observations and those of Eva von Bahr could be satisfactorily explained in this manner. Eva von Bahr's observations of absorption in water vapour summarised in the table above can be interpreted in the same manner as due to the change in the form of the band at $2{\cdot}6\mu$ with changing pressure. It is, therefore, clear that the crude application of Eva von Bahr's measurements to correct Hettner's measured coefficients of absorption by Albrecht, and by Ångström is not justified.

We are, however, faced with an established result that the form of the separate lines in the band which starts at about 9μ depends upon the physical condition of the absorbing medium, and will, therefore, vary with pressure and temperature. Before any estimate can be made of the form which Fig. 1 will take at low pressures and temperatures, we must find an explanation of the form of Fig. 1 as shown. The peaks of the lines in the rotation band correspond to definite frequencies which are dynamically determined, so that the lines should appear as narrow lines of almost infinitesimal width. The fact that they do not appear as narrow lines is to be explained by the interference of adjacent molecules with one another.

Dennison (13) has considered four factors which may be effective in this connection:—

(*a*) Doppler effect due to motion of the molecules.

(*b*) An effect corresponding to the damping of a classical oscillator.

(*c*) A resonance effect between adjacent molecules.

(*d*) An effect due to the limitation of the length of the wave trains which may be absorbed by the molecules.

With regard to (*a*) since the molecules are moving with a velocity comparable with that of sound, the effect is a proportional change of wave-length comparable with the ratio of the velocities of sound and light, and is therefore negligibly small. Dennison shows that effects (*b*) and (*c*) are also small, but that (*d*) gives effects of the right order of magnitude, giving an equation

$$k=\frac{A}{a+(v-v_0)^2}$$

connecting k and v. Dennison uses two curves of this form, adopting modified values of A and a for the outer parts of the band. His theory enables him to derive a plausible value for the diameter of the molecule of HCl, defining the diameter as the distance of nearest approach of two molecules before they begin to interfere with one another's phase. Dennison's equation replaces the Gaussian equation as used by Becker, but the general form of the two curves is similar and the flattening and widening with increased pressure follows from Dennison's theory as it does from Becker's.

The widening of the lines as shown in the experimental curves such as Fig. 1 is in part due to the width of slit employed. If the main feature of the absorption curve of water vapour were represented by a continuation of the oscillation band which shows up so definitely between 9μ and 19μ, the distance between successive lines would increase with increasing wave-length, and the slit widths used by Hettner were sufficiently small to permit the clear resolution of the lines. The fact that the lines cease to be clearly defined beyond 20μ is not satisfactorily explicable by the width of slit, and it is therefore probable that the intense absorption shown in the region 25μ to 34μ is, in part at least, continuous absorption. But it must be admitted that neither theory nor experimental observation is as yet sufficiently advanced to enable us to interpret rigorously the curve in Fig. 1, and we cannot with certainty say whether for low temperatures and pressures Fig. 1 should be replaced by a series of narrow intense lines reaching down to the axis (or in other words having transparent regions between them), or whether it should still have the same general outline except for the narrowing of the lines of the rotation band. The subject of infra-red spectroscopy is still in its infancy, and workers in the subject have been hitherto more interested in the positions of the lines in the band than in their form. Further work, both theoretical and experimental, will be necessary before the very complicated structure of the infra-red spectrum of water vapour can be completely disentangled. Further work with high dispersion and narrow slit widths is required to establish the true nature of the infra-red spectrum in the regions in which we are interested.

In subsequent portions of this paper we shall only be concerned with radiation and absorption in the lower troposphere, where the changes in the form of the absorption bands are only of secondary importance, the main features of the absorption and radiation being deducible from the data plotted in Fig. 1.

Since this paper was written Albrecht (14) has published a preliminary account of some work on somewhat similar lines to the above discussion. He claims that it is now possible to deduce theoretically the true nature of the absorption spectrum at low pressures and temperatures, and apparently comes to the conclusion that in such conditions the spectrum becomes a series of fine lines, with transparent regions between them. No final opinion can be formed of the results until fuller details are available.

4. The Absorption Spectrum of Liquid Water

The absorption spectrum of liquid water has been investigated in some detail by Rubens and Ladenburg (15) up to wave-lengths of 18μ, using films of water and glycerine, and by Aschkinass (16), using pure water, up to a wave-length of 7μ. The curve of absorption coefficients plotted against wave-length has the same general form as the curve in Fig. 1, except that the values are much greater. Using the same notation as is used above, referring to a unit of 1 cm. of liquid water we obtain the values of a plotted in the continuous curve in Fig. 2, which are therefore directly comparable with those in Fig. 1. The only striking difference of form between Figs. 1 and 2 are the intensity of the band at 3μ in the liquid water spectrum, and the relatively smaller intensity of the band at 6μ by comparison with the intensity in the region beyond 12μ. It is seen that even for those wave-lengths for which water vapour is most transparent to long waves, liquid water of a thickness of 1 mm. only transmits 10^{-20} of the incident unreflected radiation, and that a layer of water of ·1 mm. thickness only transmits 1/100th. In the regions of greater absorption such as the band at 6μ a layer of thickness ·02 mm. (20 microns) only transmits 1/100th of the unreflected radiation, and even a layer of 10 microns only transmits 1/10th of the incident radiation.

The broken curve in Fig. 2 gives the reflecting power of a water surface for normal incidence. In the wave-lengths at which atmospheric radiation is most intense, say 5μ to 15μ, the value is everywhere small, so that only a few per cent of the incident rays are reflected. Hence we may assume that films or drops of water of a thickness of ·1 mm. or more are within a few per cent of being black-body radiators.

The absorption curve of Fig. 2 is based on observations on thin plane films of water, but presumably the results may be applied to water drops also. Thus individual water drops of 1/10 mm. in diameter may be treated as black-body radiators for all practical purposes. Even individual small drops of 10μ in diameter such as are found in fogs and clouds, may be regarded as almost equivalent to black-body radiators, and a layer of fog or mist containing the equivalent of 1/10 mm. of water per cm.^{2} of cross section, may be treated as a perfect black body within the limits of a few per cent. It is, however, clear that in a fog near the ground there will be a certain amount of diffuse scattering or reflection of long-wave radiation produced in the same manner as the scattering of light in the sky, by particles whose dimensions are less than the wave-lengths of the radiation. W. H. Dines (17) found that the radiation from a fog was equivalent to the complete black-body radiation at the same temperature, but his measurements could not separate the true radiation from the fog particles from the radiation originating at the ground and undergoing diffuse scattering in the fog.

According to the "Meteorological Glossary," the amount of suspended water in a cloud or fog is of the order of a few grammes of water per (metre)^3. Let us assume 1 (metre)^3 to contain say 2 cc. of liquid water. The length of path which contains 1 cm. of liquid water per cm.^{2} of cross section is therefore 5,000 metres, and the length of path which contains ·1 mm. of liquid water per cm.^{2} of cross section is 50 metres. Thus a layer of cloud or fog 50 metres thick can be treated as a black-body radiator, to a high degree of approximation.

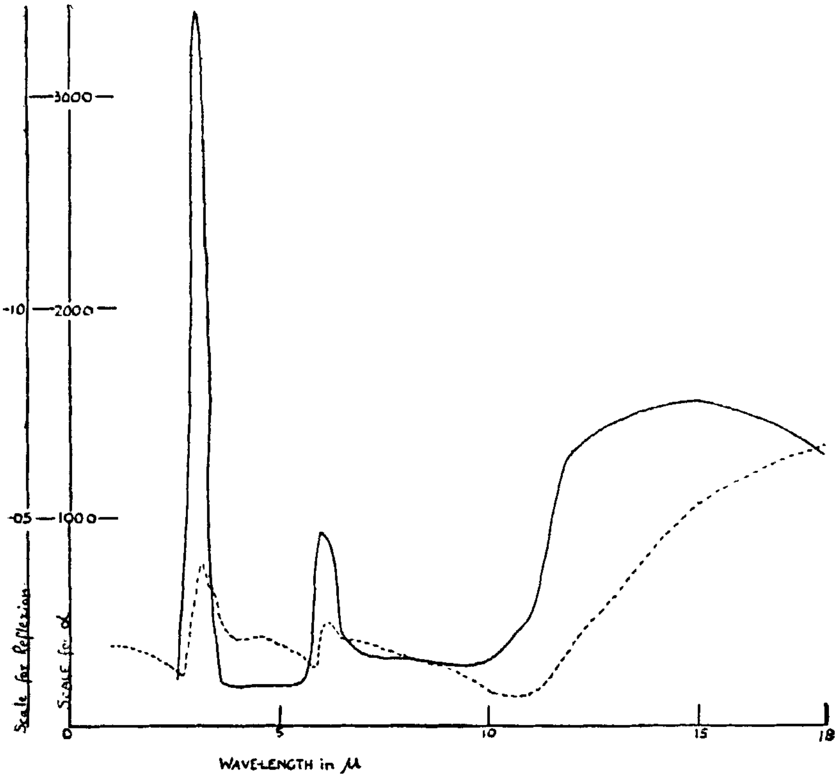

Fig. 2.—Absorption and reflection coefficients of liquid water. The broken line gives the coefficient of reflection for normal incidence, and the continuous line the decimal absorption coefficient or α.

The computation of the thickness of drop-laden air which is capable of functioning as a black body neglects the effect of reflection and scattering by the drops. A figure of a few per cent was quoted above for the reflecting power of water (in sheets), but it is not strictly justifiable to use the same figure for a layer of a few metres in thickness, containing small water drops, since any portion of a beam of radiation passing through these few metres would meet a large number of reflecting surfaces, and the number would be greater the smaller the drops. Thus a cloud or fog is nearer to being a black body when the drops are large than when the drops are small. There appears to be no method of estimating the effect mentioned. Moreover, some of the observations of W. H. Dines previously referred to appear to justify the assumption that cloud and fog can be treated as black bodies so far as their absorption and radiation of long-wave radiation is concerned; we shall therefore neglect the possible effects of reflection and scattering in fog and cloud in the discussion which follows.

The coefficient of absorption of liquid water for wave-lengths less than 2μ such as occur in the direct solar beam are everywhere small, and the values derived for wave-lengths less than $\cdot 5\mu$ can be attributed to Rayleigh scattering by the molecules. The following table gives the value of a for a range of wave-lengths below $1\cdot 2\mu$ referred to 1 cm. of water.

Wave-length μ	·779	·865	·945	1·19
a	·118	·128	·234	·842

Thus the absorptive power of water is very small for light waves. In a cloud or fog, the incident solar rays are scattered by irregular reflection by the drops, and the total path through the water drops in the upper layers of the cloud or fog will be many times greater than the equivalent depth of liquid water. It is, therefore, possible that there is an appreciable amount of absorption of solar rays in the upper region of a cloud sheet or fog.

5. Penetration of the Atmosphere by Long Waves

We can use the information shown graphically in Fig. 1, to estimate the height from which radiation reaches the ground. The mass m of water vapour measured in cm. of precipitable water which will allow one-tenth of the incident radiation to pass through it, is given by

$$10^{-am} = 10^{-1} \text{ or } am = 1.$$

Having found m we have next to find the depth of the layer containing this quantity of water. We are not here concerned to obtain an absolutely accurate estimate of this depth, but only to obtain an approximate idea of the magnitudes involved. We shall therefore assume with Simpson (*loc. cit.* p. 13) that the depth of the layer containing 1 mm. of precipitable water is $380/e$ metres, where e is the vapour pressure in millibars. Further, we shall assume $e = 10$ *mb.*, but for other vapour pressures the heights are inversely proportional to the assumed vapour pressure. The density of the water vapour is assumed not to vary with height. In the following table μ is the wave-length in microns, and h, measured in metres, is a height such that not more than one-tenth of the radiation originating above h can penetrate down to the surface.

μ	5·0	5·3	6·4	6·55	7·6	7·75	8·0	9·0	9·8	10·0	10·4	11·0	11·1	11·7
h	230	60	7	$2\frac{1}{2}$	50	110	420	1260	1000	1800	1000	650	1000	450

μ	12·0	12·5	13·0	13·4	13·8	14·3	15·0	17·3	20	25	28	30	34
h	700	300	450	210	350	125	125	40	20	36	10	8	$5\frac{1}{2}$

The larger values are probably in all cases underestimates, since the vapour density diminishes with height, and in order to take into account the required total mass of water vapour we should then increase the height. Even allowing for the fact that the figures, particularly the larger ones, are very rough approximations, the table of h is of considerable interest. It gives the height at which the radiation from the ground will have been diminished to one-tenth of its original value. For all wave-lengths above 19μ, h is not more than 25 metres except in a narrow range about 25μ.

Simpson assumed that the absorption in the region $8\frac{1}{2}\mu$ to 11μ was negligible, and that this band could be treated as transparent. The computations given above show that, using Hettner's coefficients of absorption, there are narrow bands at $9{\cdot}25\mu$, $9{\cdot}8\mu$, and $10{\cdot}4\mu$, within which the absorption is real though small in amount. These bands are very narrow, and under atmospheric conditions are probably much narrower than would appear from Fig. 1. Moreover, Fowle (18) found no trace of absorption in the atmosphere at these wave-lengths, in the course of a series of very careful measurements, and Simpson's assumption of complete transparency in the region $8\frac{1}{2}\mu$ to 11μ is thus justified. It is possible that there is a slight absorption at certain points within this range, but the amount of energy thus absorbed must be extremely small.

The present writer was led to consider the questions discussed in the preceding portions of this paper from a consideration of the magnitude of nocturnal radiation. If we imagine the conditions in the atmosphere as regards temperature to be specified by a constant lapse rate β, and an assigned surface temperature T_s, the temperature at a height h will be $T_s - \beta h$. This specification may be in error in the lowest layer at night when an inversion forms, but for the moment we shall neglect this possibility of error. The radiation from the atmosphere reaching the surface of the earth will originate at heights which are determined largely by the vapour pressure of the atmosphere, the temperature of the layer of origin being highest when the vapour pressure is greatest, and when the lapse rate β is least. Unfortunately, it is not possible to compute the amount of the downcoming radiation from a theoretical standpoint on account of the enormous range of variation of the coefficient of absorption with wave-length. Simpson (*loc. cit.* Table V) has fixed certain limits within which the radiation from the atmosphere must be, and on strictly theoretical grounds it is not possible to narrow these limits.

6. Nocturnal Radiation of the Atmosphere with Clear Skies

It will be seen from equation (8) of §8 that in a medium in which the coefficient of thermal conductivity is κ the expression for the flow of heat involves $\sqrt{\kappa}$. The present writer has shown (19) that in the atmosphere the transfer of heat by radiation is analogous to molecular conduction, the coefficient which replaces κ being inversely proportional to the vapour pressure. The theory on which this is based is not strictly applicable to the lowest layer of the atmosphere in immediate contact with the ground. It does, however, suggest that the radiation from the atmosphere should be a function of the square root of the vapour pressure. In order to test this assumption the monthly mean values of atmospheric radiation from clear skies made by W. H. Dines (20), at Benson, were taken and compared with the square root of vapour pressure for Kew. Vapour pressures for Benson are not available, and the

Kew data are used, with a reservation that there may be slight differences involved. Dines gives for each month of the year, and for each of six zones of mean zenith distances $7\frac{1}{2}°$, $22\frac{1}{2}°$, $37\frac{1}{2}°$, $52\frac{1}{2}°$, $67\frac{1}{2}°$ and $82\frac{1}{2}°$, as well as for the whole hemisphere of sky, the amount of incoming radiation in gramme calories per day per cm.^{2} of the earth's surface. The figures for any zone give the amount of radiation which would reach the earth's surface if the whole sky radiated at the same uniform rate as the zone in question. Dines also gives for each month the radiation from a black body at the mean surface temperature, *i.e.*, σT^4 where σ is Stefan's constant and T the mean absolute temperature of the surface. Calling the incoming radiation R_1, R_2, R_3, R_4, R_5, R_6 and R from each of the six zones and the whole sky, the ratio $R_i/\sigma T^4$ for each of the seven series of data was formed for each month and $\sqrt{e}$ was taken for comparison with these ratios, e being the corresponding monthly mean vapour pressure for Kew. Considering the whole hemisphere it was found that $R/\sigma T^4$ was almost exactly a linear function of $\sqrt{e}$. The coefficient of correlation between $R/\sigma T^4$ and $\sqrt{e}$ was found to be ·97, the line of regression being

$$R/\sigma T^4 = {\cdot}52\ (1 + {\cdot}125\sqrt{e}).$$

The results derived for the six separate zones were equally striking, as will be seen from the table below.

	Correlation Coefficient	$R/\sigma T^4$
Zone 1	·96	·47 $(1+{\cdot}137\sqrt{e})$
,, 2	·96	·48 $(1+{\cdot}136\sqrt{e})$
,, 3	·97	·51 $(1+{\cdot}118\sqrt{e})$
,, 4	·97	·61 $(1+{\cdot}106\sqrt{e})$
,, 5	·94	·54 $(1+{\cdot}140\sqrt{e})$
,, 6	·96	·75 $(1+{\cdot}075\sqrt{e})$
Hemisphere	·97	·52 $(1+{\cdot}125\sqrt{e})$

The uniformly high values of the correlation coefficients is striking. It will also be seen that except for the fifth zone, which is curiously out of step with the others, there is a regular increase of the first coefficient in the regression equation, and a regular decrease of the coefficient of $\sqrt{e}$ inside the bracket. This is in accordance with what we should anticipate from theoretical considerations. If, for example, we compare zone 1 with zone 6, the former is centred about the zenith, while the latter is near the horizon. Radiation in zone 1 will have originated at considerably greater heights than the radiation in zone 6. Moreover, much of the radiation in zone 6 will have originated at very low levels, no matter how low the vapour pressure may be, and if we could measure the radiation in a still narrower zone, only just above the horizon, we should expect to find the amount of radiation practically independent of vapour pressure, since a horizontal cylinder of the atmosphere will always contain enough water vapour to radiate effectively as a black body in the appropriate wave-lengths.

The next step was to consider the applicability of the formula

$$R/\sigma T^4 = a + b\sqrt{e}$$

to individual observations instead of to mean values for the months such as were given by Dines. A series of 28 individual observations for clear nights at Upsala has been given by Sten Asklöf (21). He gives a table of observations of net radiation from the earth's surface, together with the temperature and vapour pressure. If we call the net outward radiation from the earth's surface R_N, then

$$R_N = \sigma T^4 - R$$

where R is the downward radiation from the atmosphere, so that R is readily computed. For the 28 observations the coefficient of correlation between $R/\sigma T^4$ and $\sqrt{e}$, where e is in millibars, was found to be 0·83, with the regression equation,

$$R/\sigma T^4 = \cdot 43\ (1 + \cdot 19\sqrt{e}).$$

Asklöf quotes a formula suggested by A. Ångström (11), of the form

$$R/\sigma T^4 = A - B\ 10^{-\gamma e}$$

and gives in the last column of his table of observations the values computed for R_N from this formula. At first sight the agreement appears to be good, but the correlation coefficient between the observed and computed values was found to be only 0·46, a value much inferior to that of 0·83 found by using the formula suggested in the present paper.

Ångström (*loc. cit.*) has made numerous series of observations of nocturnal radiation at Bassour in Algeria, and at various mountain stations in California. In Table I of his paper he gives details of 38 observations on clear nights at Bassour, at a height of 1,160 m. above mean sea level, the mean atmospheric pressure being about 665 m. The regression equation giving atmospheric radiation as a function of vapour pressure e is

$$R/\sigma T^4 = \cdot 48\ (1 + \cdot 12\sqrt{e})$$

the correlation coefficient being 0·73. The dot diagram of $R/\sigma T^4$ against $\sqrt{e}$ showed a linear distribution, but with considerable scatter of the points about the regression line. There was, however, no indication of a departure from linearity. In view of the less satisfactory fit to the equation, a diagram was constructed with temperature as ordinate and vapour pressure as abscissa. A point was marked in this diagram to indicate each observation, the value of the net radiation being written opposite the dot. If the observed radiation were a clearly defined function of T and e it should be possible to draw in this diagram isopleths which would fit the observations closely. This was found to be impossible, adjacent points in the diagram, indicating similar values of T and e often corresponding to widely-varying values of the net radiation. It was therefore concluded that the less perfect fit to the equation

$$R/\sigma T^4 = \cdot 48\ (1 + \cdot 12\sqrt{e})$$

was not to be ascribed to any imperfection of the formula, but rather to the nature of the observations. This statement is not to be interpreted as in any sense a criticism of the observations. The observations of Dines and Asklöf were made at low-level stations, and the high correlations obtained between $R/\sigma T^4$ and $\sqrt{e}$ indicate that the factor $\sqrt{e}$ is capable in some manner of representing the effects of the vertical distribution of water vapour and temperature. Ångström's observations at Bassour were made at a height of 1,160 metres above mean sea level, and it is probable that the surface vapour pressure at mountain stations is not in the same sense representative of conditions at higher level. The cooling of the surface layers of air leads to katabatic flow down the slopes of the mountain, and it is not obvious what effects on the distribution of water vapour and temperature will be produced by this flow. The irregularity of the relationship of radiation, temperature, and vapour pressure is still more strikingly shown by Ångström's observations on Mount Whitney in California, at a height of 4,420 metres above mean sea level (*loc. cit.* Table XIX).

The mean monthly vapour pressures at Kew which were correlated with Dines's observations of radiation at Benson, vary between 7·7 mb. and 14·4 mb. Ångström's Bassour observations of vapour pressure vary between about 5 mb. and 15·3 mb. To test the formula for a wider range of vapour pressures, we shall apply it to a series of grouped observations of radiation at Lindenberg given by Robitsch (22), in which the vapour pressure varies between 2·5 mm. and 16 mm. (*i.e.* about 4 mb. to 21 mb.). The groupings are based on a total of about 1,350 individual observations. The first four columns of the table below give the absolute temperature T, the vapour pressure e in mm., the atmospheric radiation R, and the black-body radiation σT^4, taken from Robitsch's table. Subsequent columns give $\sqrt{e}$, $R/\sigma T^4$ and $(R/\sigma T^4)_c$, the last being the value computed by using the formula given below.

T	e	R	σT^4	$\sqrt{e}$	$R/\sigma T^4$	$(R/\sigma T^4)_c$
303·2	16·0	·601	·700	4·0	·859	·851
301·2	14·5	·561	·682	3·8	·823	·825
299·6	13·5	·633	·668	3·6	·798	·800
294·4	12·3	·486	·622	3·5	·782	·787
290·4	11·6	·455	·590	3·4	·771	·774
287·8	10·2	·423	·570	3·2	·743	·749
285·6	9·6	·407	·552	3·1	·738	·736
281·0	7·5	·354	·517	2·7	·685	·685
279·3	5·5	·321	·505	2·3	·636	·634
276·6	4·3	·294	·485	2·1	·607	·609
269·1	2·5	·246	·435	1·6	·566	·545

The correlation coefficient between $R/\sigma T^4$ and $\sqrt{e}$ is 1·0, the regression line being

$$R/\sigma T^4 = \cdot 342 + \cdot 127\sqrt{e} = \cdot 342\ (1 + \cdot 37\sqrt{e}).$$

The values of $R/\sigma T^4$ derived by using this equation are shown in the last column. The agreement is extraordinarily close in view of the fact that $\sqrt{e}$ has only been evaluated to two significant figures; and the empirical equation is therefore applicable over the wide range of variation of temperature and vapour pressure shown in the table.

Ångström gives in Tables V, VI, VII and VIII (*loc. cit.*) series of observations on Mount Whitney (4,420 m.) and San Gorgonio (3,500 m.); at Mount San Antonio (3,000 m.) and Lone Pine Canyon (2,500 m.); at Lone Pine (1,150 m.); and at Indio (0 m.). The tables give a quantity which Ångström denotes by E_a, which is the observed atmospheric radiation corrected to a standard absolute temperature of 290°. Thus

$$E_a = R\left(\frac{T}{290}\right)^4$$

The equation which fits the Bassour observations can be converted readily to give E_a instead of $R/\sigma T^4$. It then reads,

$$E_a = \cdot 274 + \cdot 037\sqrt{e}$$

where $\sqrt{e}$ is measured in millimetres. This equation was used to compute the mean value of E_a which should correspond to the observed mean values of $\sqrt{e}$, and the following table gives the observed mean value of E_a and of $\sqrt{e}$, while the last column gives the computed value of E_a derived by using the Bassour equation.

	E_a	$\sqrt{e}$	E_a (compd.)
Mount Whitney and San Gorgonio ...	·314	1·54	·329
San Antonio and Lone Pine Canyon ...	·353	2·11	·352
Lone Pine	·384	2·20	·356
Indio	·405	3·16	·391

The agreement between E_a and the computed value is only moderate except at San Antonio and Lone Pine Canyon. Ångström (23) pointed out that the observations at Mount Whitney and San Gorgonio did not fit his curves, and the lack of agreement in the first line of the table above is probably attributable to the different instrumental constants involved in the observations at these two stations and at Bassour. Ångström (24) gives in a later paper revised values for Mount Whitney from which we find the mean values $\sqrt{e}=·73$; $E_a=·294$, whereas the computed value derived from the Bassour regression equation is ·301, in reasonably close agreement with the observed value.

The test applied above is not altogether satisfactory, since the observations at different levels were made with individual instruments, whose constants were not equally reliable.

The only other series of observations at different levels which I have been able to trace are those carried out by Boutaric (25) at Montpellier and Pic du Midi (2,859 m.). Boutaric gives the following table of mean values:—

			R	σT^4	$R/\sigma T^4$	$\sqrt{e}$	$(R/\sigma T^4)c$
Montpellier:	May	1914	·405	·554	·731	2·90	·736
,,	June	1914	·450	·597	·754	3·27	·754
,,	Nov.	1914	·354	·490	·723	2·60	·721
,,	Dec.	1914	·329	·467	·705	2·26	·705
,,	Jan.	1915	·298	·434	·688	1·90	·687
Pic du Midi:	Aug.	1919	·330	·483	·684	1·82	·684

The first two columns give the values of R and σT^4. In the next two columns are given the corresponding values of $R/\sigma T^4$ and $\sqrt{e}$, e being in millimetres. Plotting these values we find a line of close fit drawn by inspection to be

$$R/\sigma T^4 = ·596 + ·048\sqrt{e} = ·596\,(1 + ·08\sqrt{e}).$$

In order to test the goodness of fit of this equation, there are given in the last column the values of $R/\sigma T^4$ computed from this equation using the values of $\sqrt{e}$ given in the fourth column. Comparing the third and fifth columns we find a remarkable agreement between the observations and the formula, the agreement extending even to the third place of decimals, to within two units except for May, 1914. The agreement again confirms the view previously expressed that the same formula fits observations at different heights without any correction for the supposed variation of absorption with total pressure.

Thus the observational data indicate that the net loss of heat by radiation from mountain tops is strictly comparable with the net loss from the surface at lower levels, under the same conditions of temperature and humidity. In practice the net loss from high ground is usually large by comparison with that from low ground, on account of the lower vapour pressure at the higher levels.

7. Meaning of the Formula for Atmospheric Radiation

In §6 of this paper a formula

$$R/\sigma T^4 = a + b\sqrt{e}$$

where a and b are numerical constants, has been suggested as giving a close fit to the observational data available. This formula is here regarded as an empirical formula, which fits the data of observation, and no strictly theoretical justification for its use has been suggested. It has been shown that this formula gives a better fit to Asklöf's observations than the formula suggested by Ångström,

$$R/\sigma T^4 = A - B\ 10^{-\gamma e}$$

Ångström suggested that when $e=0$ his formula gives the radiation from absolutely dry air. The present writer does not agree with this suggestion, and does not regard the value obtained when $e=0$ as representing the radiation from dry air. The term in $\sqrt{e}$ is regarded as in some manner taking account of the variation of conditions with height. In an infinite isothermal atmosphere the total radiation coming down from the atmosphere should amount to the black-body radiation at the same temperature within the effective range of wave-lengths, no matter what the vapour pressure might be provided there was a finite vapour pressure, and the term in $\sqrt{e}$ should disappear yielding an approximate formula

$$R = A'\sigma T^4.$$

The values of the constants a, b, and b/a vary from one set of observations to another. This may be in part due to real differences in the rate of variation of temperature and vapour pressure with height, but is probably largely due to the differences in methods of observation, involving different instrumental constants which at best are somewhat uncertain, and to differences in methods of determination of vapour pressure. The estimation of vapour pressure involves factors dependent on the type of instruments used and their exposure, as well as the hygrometric tables used.

In the following table are collected the values of a, b, and b/a for the series of observations discussed in §6. Where the original values were expressed in millimetres, the constant b has been corrected so that the results as given in the table shall all apply to vapour pressures in millibars. The column headed r gives the coefficient of correlation between $R/\sigma T^4$ and $\sqrt{e}$.

	a	b	b/a	r
Dines' figures (Benson)	·52	·065	·125	·97
Asklöf (Upsala)	·43	·082	·19	·83
Ångström (Bassour)	·48	·058	·12	·73
Boutaric (France)	·60	·042	·07	
Robitsch (Lindenberg)	·34	·110	·32	1·0
Mean	·47	·072		

The range of variation of the constants is rather wide, but further consideration of the meaning of this variation must be deferred to another occasion.*

* Since the above was written a paper by Ramanathan and Desai on Nocturnal radiation measurements at Poona (*Beitr. Geophysik, Leipzig*, **35**, h.1, p. 68) has come to hand. From the monthly values the following are computed—

$$b = \cdot 12, \quad r = \cdot 93.$$

8. NOCTURNAL RADIATION: CONDITIONS WITHIN THE SURFACE LAYERS OF THE GROUND

Since the earth's surface radiates effectively as a black body, the amount of heat which it sends into the atmosphere is independent of the nature of the ground. The amount of energy which the earth's surface gains from the atmosphere depends upon the distribution of temperature and humidity in the lower layers of the atmosphere. Thus the net outward radiation at night from the earth's surface depends only on atmospheric conditions, and on the temperature of the earth's surface. But the temperature changes which are produced in the earth by a given amount of radiation will depend upon the readiness with which the loss of heat from the surface is compensated by conduction of heat upwards from lower layers of the earth. The precise nature of this dependence can be readily ascertained.

We shall neglect the effect of the transfer of heat by conduction from the air to the earth's surface, as there is no obvious method of allowing for this. Further, we are here concerned mainly with conditions during night inversions, and in such conditions the transfer of heat is from air to ground, tending to prevent the fall of temperature of the ground. Thus any formula which we derive for estimating the drop of temperature at the ground will give an overestimate, or a limit to the maximum fall of temperature at the surface.

The surface layers of the earth will be assumed to have a specific conductivity of heat κ_1. Values of κ_1 have been derived by Johnson and Davies (26) and by Wright (27), the mean of their determinations being $\kappa_1 = 4{\cdot}7\ 10^{-3}$ in c.g.s. units. The equation of conduction of heat through the ground is then

$$\frac{\partial T}{\partial t} = \kappa_1 \frac{\partial^2 T}{\partial z^2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where z is the depth measured positive downwards.

If the net loss of heat by radiation from the ground to the atmosphere is R_N, then

$$R_N = \kappa_1 \rho_1 c_1 \left(\frac{\partial T}{\partial z}\right)_{z=0} \quad . \quad . \quad . \quad . \quad (2)$$

where ρ_1 and c_1 are respectively the density and specific heat of the ground. For the loss of heat from the surface outwards is equal to the flow of heat from below to the surface. Further, we have seen that to a first approximation the net radiation R_N remains constant throughout the night, since

$$R_N = \sigma T^4 (1 - a - b\sqrt{e}).$$

The vapour pressure has only a slight diurnal variation, and since the fall of temperature during the night is only a relatively small fraction of T, we may assume as a first approximation that R_N is constant. By equation (2) this involves a constant value of $\partial T/\partial z$ at $z=0$.

Differentiating equation (1) once with respect to z and replacing $\partial T/\partial z$ by S we find

$$\frac{\partial S}{\partial t} = \kappa_1 \frac{\partial^2 S}{\partial z^2}$$

This now has to be solved with the boundary condition

$$S = \frac{R_N}{\rho_1 c_1 \kappa_1} \quad \text{at } z = 0.$$

The appropriate solution is given in any textbook on the subject, in the form

$$S = \frac{2}{\sqrt{\pi}} \frac{R_N}{\rho_1 c_1 \kappa_1} \int_{\frac{z}{2\sqrt{\kappa t}}}^{\infty} e^{-u^2} du \quad . \quad . \quad . \quad (3)$$

Integrating equation (3) we readily find

$$T = T_1 - \frac{2}{\sqrt{\pi}} \frac{R_N}{\rho_1 c_1 \kappa_1} \left\{ \sqrt{(\kappa_1 t)} \cdot e^{-\frac{z^2}{4\kappa t}} - z \int_{\frac{z}{2\sqrt{\kappa t}}}^{\infty} e^{-u^2} du \right\} . \quad . \quad (4)$$

Thus the temperature at $z = 0$ is given by

$$T = T_1 - \frac{2}{\sqrt{\pi}} \frac{R_N}{\rho_1 c_1 \sqrt{\kappa_1}} \sqrt{t} \quad . \quad . \quad . \quad . \quad (5)$$

In this equation R_N is to be measured in gramme-calories per second and t in seconds. It is readily seen that we can take R_N to indicate the net radiation in gramme-calories per minute, and t the time in hours.

Before we consider the application of this result to any practical problem, we must consider the conditions under which it has been derived. Referring back to equation (3) we find that at $z = 0$, S or $\partial T/\partial z$ is constant at all times except at $t = 0$, when it is zero, through the lower limit of the integral becoming infinite. Thus the solutions represented by equations (4) and (5) correspond to the case when the outward radiation is initially zero at $t = 0$, and then instantaneously jumps up to the value R_N. Such a change in physical conditions cannot happen in nature, but something approaching it probably happens at sunset, on a clear evening. It is known that the incoming short-waved radiation from sun and sky becomes negligibly small just before sunset, and that just before this happens there is a very rapid fall in the amount of the incoming radiation. Thus while the change from a net flow of radiation inwards through a complete balance, to a net flow outwards at the rate R_N, does not take place instantaneously, it takes only a very short time to be accomplished, and we should expect to find that equation (5) should give a reasonable approximation to the fall of temperature of the ground during a clear night, t being measured from sunset, except for small values of t, when the infinite rate of fall of temperature given by equation (5) at $t = 0$ would in practice be replaced by a steep but finite rate of fall.

A nearer approximation to the conditions pre-supposed in the deduction of the above equations occurs on an occasion when the sky, after being overcast with low cloud, suddenly clears. It is shown later in §9 of the present paper that there is only a very slight loss of heat from the ground when the sky is overcast with low cloud, and so the clearing of the cloud brings suddenly on the net loss R_N appropriate to a clear night. On such occasions, therefore, we may use equation (5) with fair confidence to forecast the fall of temperature in a time t after the disappearance of the cloud, assuming that there is no wind and therefore no convection of heat from the ground.

There do not appear to be any published observations available which make it possible to test the hypotheses adopted above with regard to the lapse-rate of temperature in the ground, and some doubt inevitably remains as to the possible effects of the conditions prevailing during the daytime upon the temperature changes during the night. We can test this by adopting a plausible expression for the inward flow of short-waved radiation during the day. For a clear day at the equinox, with sunrise at 6 a.m. and sunset at 6 p.m., we shall assume the incoming net radiation to be represented by $-I \sin qt$, where the time t is measured from sunset, and q is a constant whose value is $2\pi/24 \times 60 \times 60$, so that $qt = -\pi$ at sunrise, and $qt = +\pi$ at the succeeding sunrise. We shall further assume that the net outward radiation is constant during the whole 24 hours. This assumption might be improved upon by assuming during the daytime that the net outward radiation follows a harmonic law,

$$R_N - P \sin qt,$$

but it will be seen later that P would only enter the equations as a correction to I, and the final result is unaltered.

The next step is to represent the net flow of radiation by a Fourier Series. Let $\theta = qt$ and let $f(\theta)$ represent the net *inward* radiation as a function of θ.

Then we have

$$f(\theta) = -I \sin\theta - R_N \text{ for } -\pi < \theta < 0$$
$$f(\theta) = -R_N \text{ for } 0 < \theta < \pi.$$

Let

$$f(\theta) = A_0 + A_1 \cos\theta + A_2 \cos 2\theta + A_3 \cos 3\theta + \dots$$
$$+ B_1 \sin\theta + B_2 \sin 2\theta + B_3 \sin 3\theta + \dots$$

Then

$$\pi A_n = \int_{-\pi}^{+\pi} f(\theta) \cos n\theta d\theta$$
$$= -\int_{-\pi}^{0} I \sin\theta \cos n\theta d\theta.$$

It is readily found that

$$A_1 = A_3 = A_5 = \dots = 0$$
$$A_2 = -\frac{2I}{3\pi},\ A_4 = -\frac{2I}{15\pi},\text{ and for } n \text{ even } A_n = -\frac{2I}{(n^2-1)\pi}$$

Also since

$$\pi B_n = \int_{-\pi}^{+\pi} f(\theta) \sin n\theta d\theta$$
$$= -I\int_{-\pi}^{0} \sin\theta \sin n\theta d\theta$$

we readily find

$$B_1 = -\tfrac{1}{2}I,\ B_2 = B_3 = B_4 = \dots = 0.$$

Further,

$$A_0 = \frac{I}{\pi} - R_N.$$

It is necessary to assume that A_0 is zero, and $I = \pi R_N$, in other words, that the net gain or loss of heat by the ground during 24 hours is zero. This assumption is unavoidable if the problem is to be discussed by the method of using Fourier Series, since the use of harmonic terms whose period is the day or sub-multiples of a day cannot take account of a secular change from one day to the next.

The net inward radiation is then given by

$$f(qt) = -\tfrac{1}{2} I \sin qt - \frac{2I}{\pi}\left\{ \tfrac{1}{3}\cos 2qt + \frac{1}{15}\cos 4qt + \dots \right\}. \qquad (6)$$

Let the temperature at the surface of the ground be represented by the Fourier Series.

$$T = T_0 + P_1 \cos(qt - \epsilon_1) + P_2 \cos(2qt - \epsilon_2) + \dots, \text{ etc.} \qquad (7)$$

The solution of equation (1) which satisfies the boundary condition in equation (7) at $z = 0$ is

$$T = T_0 + P_1 e^{-\sqrt{\frac{q}{2\kappa_1}}z} \cos\left(qt - \epsilon_1 - \sqrt{\frac{q}{2\kappa_1}}z\right)$$
$$+ P_2 e^{-\sqrt{\frac{2q}{2\kappa_1}}z} \cos\left(2qt - \epsilon_2 - \sqrt{\frac{2q}{2\kappa_1}}z\right) + \dots, \text{ etc.}$$

Differentiating this and putting $z = 0$ we find that

$$f(qt) = -\rho_1 c_1 \kappa_1 \frac{\partial T}{\partial z} = -\rho_1 c_1 \sqrt{\kappa_1} \sum_{n=1}^{\infty} \sqrt{nq} P_n \cos\left(nqt - \epsilon_n + \frac{\pi}{4}\right). \qquad (8)$$

Series (8) and (6) must be identical, so that

$$P_1 = \frac{T}{2\rho_1 c_1 \sqrt{q\kappa_1}} \quad \epsilon_1 = -\frac{\pi}{4}$$

All the other P's with odd suffixes vanish.

$$P_2 = -\frac{2I}{3\pi}\frac{1}{\rho_1 c_1 \sqrt{2q\kappa_1}} \quad \epsilon_2 = \epsilon_4 = \epsilon_6 = \dots = \frac{\pi}{4}$$

and for n even

$$P_n = -\frac{2I}{(n^2 - 1)\pi\rho_1 c_1 \sqrt{nq\kappa_1}}$$

Thus series (7) reduces to

$$T - T_0 = \frac{2R}{\rho_1 c_1 \sqrt{q\kappa_1}} \left\{ \frac{\pi}{4}\cos\left(qt + \frac{\pi}{4}\right) - \frac{1}{3\sqrt{2}}\cos\left(2qt - \frac{\pi}{4}\right) \right.$$
$$\left. - \frac{1}{30}\cos\left(4qt - \frac{\pi}{4}\right) - \dots - \frac{1}{(n^2 - 1)\sqrt{n}}\cos\left(nqt - \frac{\pi}{4}\right) \dots \right\} \qquad (9)$$

Equation (9) gives the final solution of the problem in the form of an infinite series, which converges so rapidly that only three or four terms need be considered.

In Fig. 3 the series inside the brackets on the right hand side of equation (9) has been represented graphically by the continuous curve. This figure thus represents the diurnal variation of temperature within a period of 24 hours, assuming that radiation only is effective.

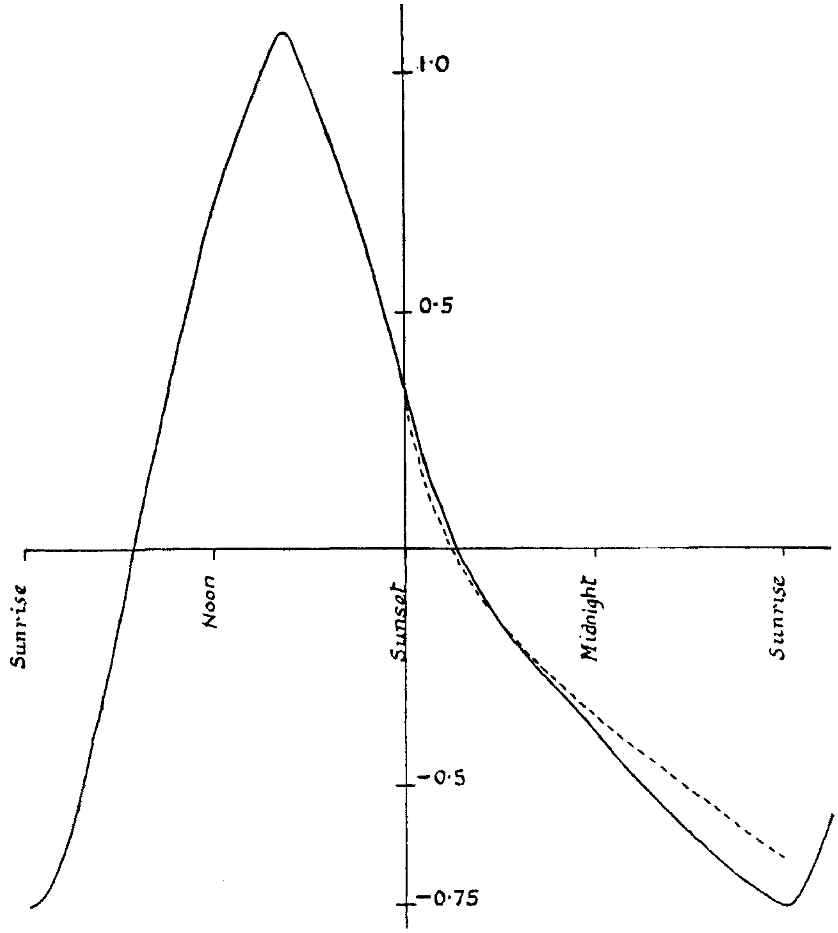

FIG. 3.—Theoretical curve of diurnal variation of temperature. The unit is $\frac{2}{\sqrt{\pi}} \frac{R_N}{\rho_1 c_1 \sqrt{\kappa_1}} \sqrt{12}$. The broken curve represents the parabola of equation 5, $T - T_0 = \frac{2}{\sqrt{\pi}} \frac{R_N}{\rho_1 c_1 \sqrt{\kappa_1}} \sqrt{t}$ where t is measured in hours from sunset.

The general form of this curve bears quite a striking resemblance to the curves of observed temperatures. The sharp rise of temperature at sunrise is well brought out in the curve. This should be compared with the diagrams for clear days and nights in June and December respectively reproduced by N. K. Johnson (28) in his Fig. 15, where the sharp rise at sunrise is clearly seen. The curve in Fig. 3 indicates that maximum temperature occurs about $2\frac{1}{4}$ hours after noon.

The dotted line in the diagram represents the parabola of equation (5). It is seen that the deviation between the parabola and the continuous curve is never great, and at its maximum only amounts to about 10 per cent of the fall of temperature since sunset. Thus Fig. 3 may be held to justify the use of equation (5) as a reasonable approximation to the effects of nocturnal radiation. The discussion leading up to equation (9), graphically represented in Fig. 3, is theoretically sounder than the derivation of equation (5); but the fact that equation (5) leads at night to a close approximation to the curve of Fig. 3 is of considerable practical importance in view of the simplicity of its application to the forecasting of night minimum temperatures.

Returning to equation (5) we find that the fall of temperature after sunset is (approximately)

$$\frac{2}{\sqrt{\pi}}\frac{R_N}{\rho_1 c_1\sqrt{\kappa_1}}\sqrt{t}=\frac{2}{\sqrt{\pi}}\frac{\sigma T^4(1-a-b\sqrt{e})}{\rho_1 c_1\sqrt{\kappa_1}}\sqrt{t} \quad . \quad (10)$$

The minimum temperature of the night is derived by making t here equal to the number of hours from sunset to sunrise. For dry soil we can adopt as approximate values

$$\rho_1=2{\cdot}5,\ c_1=0{\cdot}2,\ \kappa_1=4{\cdot}7\times10^{-3}.$$

Johnson gives (*loc. cit.*, Fig. 15a) the mean curve for clear days and nights in June, and taking the mean value of T at 287°A., and assuming the factor $1-a-b\sqrt{e}$ to be equal to its mean value at Benson in June, ·225, we find the fall of temperature in the seven hours from sunset to sunrise should be

$$\frac{2\times{\cdot}56\times{\cdot}225\times2{\cdot}65}{\sqrt{\pi}\times2{\cdot}5\times0{\cdot}2\times\sqrt{4{\cdot}7\times10^{-3}}},\ \text{or } 11^\circ\text{C}.$$

The mean fall of temperature shown by Johnson's curve is about 9°C., which is in very close agreement with the theoretically derived value, but is lower than the latter, as we should expect, since the effect of wind will be to diminish the fall of surface temperature by the downward transfer of heat through the inversion to the ground.

In the application of the above equation to the practical problem of forecasting night minimum temperatures care must be taken to ensure that the source of supply of air is not likely to change during the night. The fuller discussion of the problem of forecasting night minimum temperatures is deferred to another occasion; but in the meantime it is worthy of note that on clear nights radiation from the ground is capable of accounting for the observed changes of temperature.

Equations (5) and (9) both indicate that the amplitude of the variation of temperature is inversely proportional to $\rho_1 c_1\sqrt{\kappa_1}$. This factor therefore indicates the nature of the dependence of the variations of temperature upon the type of ground surface. For snow rough values of the constants are

$$\rho_1=0{\cdot}1,\ c_1=0{\cdot}5,\ \rho_1 c_1\kappa_1={\cdot}00025.$$

For snow $\rho_1 c_1\sqrt{\kappa_1}={\cdot}004$; for soil $\rho_1 c_1\sqrt{\kappa_1}={\cdot}035$.

The temperature variation over the surface of snow may thus be many times that over a surface of soil, if radiation and conduction are the only factors which are active. This accords with experience, the lowest temperatures which are measured being associated with a snow covering over the ground. While snow causes extremely low temperatures at its surface it is at the same time a protection to anything which it covers, in that it conducts heat so slowly that any object covered by it retains its own heat. During any prolonged spell of clear weather with snow on the ground the temperature does not recover during the day, since 80 per cent of the solar radiation is reflected upwards at the snow surface, leaving only a small fraction to be absorbed and to become available for raising the surface temperature.

This is in accordance with the extraordinary results shown by Simpson (29) in his discussions of the diurnal range of temperature on the Barrier in the Antarctic, where the small diurnal change in daily insolation corresponding to the sun oscillating between 10° and 35° above the horizon produced a diurnal variation of temperature with an average amplitude of 20°F.

A point of interest in equation (9) is that it represents the variations of temperature very accurately by means of two terms only, the first and second harmonic, the fourth harmonic being very small, and all subsequent harmonics negligible; and this in spite of the fact that the nocturnal variation of temperature is very closely represented by a parabola.

The late occurrence of maximum surface temperature in Fig. 3 was referred to above. This is not in very good agreement with observation, which shows that the temperature of the surface of the ground attains its maximum temperature about half an hour after noon. A comparison of the first harmonic terms in equations (6) and (9) shows that if the net radiation reaching the ground were a pure sine curve during day and night, the maximum temperature should occur three hours after the maximum of radiation, and it is the effect of the second harmonic which displaces the time of maximum temperature back towards noon.

Wright (*loc. cit.*, Table V) shows that the ratio of the amplitudes of the first and second harmonics of temperature at the equinox is about 3 : 1, which agrees very closely with equation (9), but the phase of maximum is different, the time of maximum of the first harmonic being about four hours after noon, and of the second harmonic $2\frac{1}{2}$ hours after noon and after midnight. We cannot expect the theoretical curve derived on the simple assumptions made above to fit the daytime observations accurately. For the increase of wind by day leads to a conduction of heat from the ground which is not likely to have its maximum at the same time as the inward radiation. It is true that the function $f(qt)$ which we defined above as the balance of radiation flow into the ground per unit time may be defined as the net loss or gain of heat from the ground by radiation + conduction + convection, without any change in the subsequent analysis. The process could be reversed, the constants P_1, P_2, ϵ_1, ϵ_2, etc., in equation (7) being derived by harmonic analysis of observed temperature, and equation (8) then used to derive the rate of net loss or gain of heat by the ground in unit time, but this process is only valid for surface temperatures, which may differ appreciably from screen temperatures during the day.

9. Nocturnal Radiation with Cloudy Skies

When the sky is overcast with cloud, the radiation conditions are enormously modified, on account of the fact that the cloud sheet absorbs and radiates effectively as a black body. The effect can be readily understood from a consideration of the rate of change of intensity of the beam of radiation of one wave-length λ in the vertical direction.

Let the upward and downward beams of radiation be A and B at the level where the temperature is T, and let E be the black-body radiation of the same wave-length at temperature T. We shall use the forms ΣA, ΣB and ΣE to denote the total upward beam, the total downward beam, and the complete black-body radiation, the summation being for all wave-lengths. The equations connecting A, B and E, are

$$\frac{dA}{d\mu}=k\,(A-E)$$

$$\frac{dB}{d\mu}=k\,(E-B)$$

where μ is the total water-vapour content of the atmosphere from the top down to the level under consideration, and k is the coefficient of absorption for the wave-length λ. In a thin layer containing $d\mu$ of water vapour, the change in A is $kd\mu\,(A-E)$, and the change in B is $kd\mu(E-B)$. This is equivalent to saying that either beam of radiation is brought nearer to E by an amount equal to $kd\mu$ times its difference from E. If k is large the change is great, and the intensity A or B follows closely the value of E at each level. If k is small, the upward beam A, which equals E at the ground, will be greater than E at higher levels, while B, which equals E just below the cloud level, will become less than E at lower levels. Hence the total upward beam at any intermediate level will exceed the black-body radiation at the temperature of that level, while the total downward beam will be less than the black-body radiation at the temperature of that level. The upward beam of radiation will only differ appreciably from the corresponding black-body radiation at the same level in those wave-lengths for which the atmosphere is most transparent, the difference being negligible in those wave-lengths which are readily absorbed. The same is true of the downward beam from a cloud sheet. The argument is based on the assumption that the temperature of the atmosphere diminishes steadily from the ground up to the base of the cloud sheet. If we assume a constant lapse-rate and a constant density of water vapour within this range of height, there should be a rough proportionality between the gain of heat by the base of the cloud sheet and the height of the cloud, and a still closer relation between this gain of heat and the difference of the fourth powers of the temperatures of the ground and the cloud sheet. Similarly there should be a rough proportionality between the net radiation from the ground and the height of the cloud sheet. A closer approximation is obtained if we assume the net radiation from the ground to be proportional to the difference of the fourth powers of the temperatures of the ground and the base of the cloud sheet. Both of these statements can be to some extent checked by observation. We shall consider them in turn.

Ångström (24) has given the results of observations of (*a*) net radiation received from the ground and the intervening atmosphere, by a horizontal black body, and (*b*) the net radiation lost by a horizontal black body to the atmosphere above, the observations being carried out in a free balloon. The figures for the first of these were as follows:—

Height (metres)	975	1350	2000	3000	4000
Net gain of radiation ...	·015	·000	·008	·019	·036

The net radiation is in gramme-calories per cm.[2] The observations were commenced at about 10 p.m. on July 3, 1922, so that there was in all probability an inversion at the ground, and the radiation leaving the ground would be initially less than the equivalent black-body radiation at the temperature of air a short distance above the ground. Thus the conditions were not similar to those we have supposed, and the net radiation measured is at all levels less than would have been measured if there had been no inversion. From 1,350 m. upwards there is, however, a very rough proportionality of net radiation to excess of height above 1,350 m. Similar results were obtained during the night of June 5-6, 1923, when probably with an inversion at the ground negative values of net gain of radiation were observed up to nearly 1,200 m., above which the values were positive, becoming ·036 at 2,000 m. and ·055 at 2,750 m., the greatest height at which measurements were obtained. On this occasion there was a varying amount of cloud about 2,600 m. to 2,850 m. Above this level observations were made of the net loss of radiation of a black body to the atmosphere above it. This value was 0·226 at 3,300 m., diminishing to 0·216 at 4,400 m. We shall make further use of these results in discussing the effects of radiation on cloud sheets later.

Ångström's observations are sufficient to show that our conclusion of a rough proportionality between $(\Sigma A - \Sigma E)$ and the height is justified as to order of magnitude; and we therefore conclude that in a similar way $(\Sigma E - \Sigma B)$ is proportional to depth below the cloud level. The special form of the last relation which we require is that the net loss of radiation from the ground is roughly proportional to the height of the cloud. Asklöf's (21) observations on cloudy nights can be applied to check this conclusion roughly. Asklöf's observations on clear nights, and those on cloudy nights, were made in the period March to June, 1918. The net radiation from the ground on clear nights was between about ·15 and ·20 gramme-calories per cm.[2] The net radiation on overcast nights gave the following average values.

Cloud.	Net radiation.	Average height, Upsala.
Nb, St., or St.-Cu.	·023	1·5 km.
A.-Cu.	·039	2·8 ,,
Ci.-St.	·135	6·4 ,,
Clear sky	·169	—

Thus the net loss of heat from the ground with a sky covered with high cloud is almost as great as with clear skies, while when the sky is covered with low cloud, the net loss of heat from the ground is only about one-seventh the value observed for clear skies. Consequently the fall of temperature during the night with a sky overcast with high cloud should be nearly as great as with a clear sky, but should be only of the order of one-seventh of this amount, if the sky is overcast with low cloud. N. K. Johnson (29) reproduces a chart showing temperature variations during the night of March 15-16, 1923, in which the drop of temperature during the night is shown to be about 1°F., the sky being overcast with St.-Cu. and Nb. This is of about the right order of magnitude to agree with Asklöf's observations.

Ångström has also suggested that the net loss of radiation from the ground may be represented by

$$R_m = (1 - \cdot 09\, m)\, R_0$$

where R_m is the observed net loss of radiation from the ground when m tenths of the sky are covered with cloud, R_0 being the net loss of radiation to a clear sky in the same circumstances of temperature and humidity; but it is clear that such a rule cannot deal with clouds of different height, and a different formula, with constant appropriate to the height, should be employed for each type of cloud.

During the daytime the net loss of heat from the ground by long-wave radiation is practically the same as it would be at night with the same atmospheric conditions of temperature and humidity. But the surface of the earth gains heat by absorption of the short-wave radiation in the direct beam of sunlight, or in the diffuse solar radiation from the sky. When the sky is overcast, the amount of diffuse radiation which reaches the earth's surface is very much diminished, but it is still sufficient to outweigh the loss by the balance of long-wave radiation and absorption of the ground. W. H. Dines and L. H. G. Dines (20) give tables of mean monthly values of diffuse radiation from overcast skies which can be directly compared with their mean values of net loss by long waves, and for each month the gain from short waves outweighs the loss by long waves. We therefore conclude that the surface temperature should rise during the day, even with overcast skies, but the rise will be small when the sky is overcast with a thick sheet of low cloud. The record for an overcast day and night, March 15-16, 1923, reproduced by Johnson (*loc. cit.*) indicates a rise of only about 2°F. during the day on the 15th.

10. Effect of Radiation on Cloud Sheets and Fog

We have seen in the preceding section that theoretical considerations, borne out by the observations of Ångström, indicate that at night the base of a sheet of cloud, in virtue of its possessing the radiative properties of a black body, should absorb more heat than it radiates downward, while the top of a cloud sheet should lose more heat by radiation upward than it gains by absorption of the downward moving radiation. Within the cloud sheet the net flow of heat upward or downward is very slight, on account of its black-body properties, so that the upper part of the cloud radiates downward almost exactly the same amount of heat as it receives by absorption from the upward-moving radiation. Similarly the lower part of the cloud gains or loses only very slightly by exchange with the parts of the cloud above it. Hence we need only consider the exchanges of heat between the upper surface of the cloud and the atmosphere above it, and between the lower surface of the cloud and the atmosphere and earth beneath it.

There is a net loss of heat at the upper surface and a net gain of heat at the lower surface of a cloud sheet, except in so far as the diffuse scattering of radiation by the water drops interferes with the black-body character of the cloud. We shall disregard the latter effect, as there is no means of ascertaining how great it is. Since the upper surface of a cloud radiates as a black body, the net loss of heat by radiation is the same as that of an elevated surface of ground at the same level. The net loss from the upper surface of the cloud is always therefore an appreciable fraction of the black-body radiation at the temperature of the upper surface, and Ångström's observations in a free balloon indicate that this fraction has a value of about one-third for low cloud, nearly one-half for medium cloud, and a slightly greater fraction for high cloud. This is in accordance with our formula for net loss by radiation, $\sigma T^4 \{ 1 - a - b\sqrt{e} \}$ as $\sqrt{e}$ diminishes with height. The net gain of heat at the lower surface is roughly proportional to the height of the cloud, and when the cloud is low (St., St.-Cu., or Nb.), this gain is very small.

We thus arrive at the conclusion that any cloud sheet tends to become unstable as a direct result of radiation and absorption. The tendency will be more marked in high than in low cloud, and it appears to provide a satisfactory explanation for the frequency of instability types of cloud such as have been described by the present writer (30), by S. Mal (31), and by G. T. Walker (32).

A thick sheet of cloud of, say, 500 metres or more, might become unstable at the top and bottom separately, the central portions remaining stable; and, at least initially, one cellular structure might be formed in the top and another at the bottom of the cloud sheet, separated by a stable layer in the middle of the cloud.

The gain of heat at the base of a sheet of low cloud is so slight that instability does not often arise at the base, though the top may become unstable and form a cellular structure which is not visible from the ground. But the net loss from the upper surface is also less in low cloud than in high or medium cloud.

The writer makes no claim that the views expressed above are in any way novel. They have been put forward from time to time by almost every writer who has considered the physics of cloud formation. But the estimates of the exchanges of heat by radiation and absorption which are given in §9 are possibly new, and appear to show that radiation and absorption are of greater importance in the physics of clouds than has yet been conceded.

The heat gained at the base of a cloud is partly used up in evaporating cloud drops, while the cooling of the top tends to produce increased condensation there. The net effect is that the cloud is partly destroyed from below, but renewed at the top. When the cloud is low the cooling of the top is more potent than the heating of the base, but with high cloud the heating at the base is more important.

One effect of the cooling of the upper surface of a cloud sheet by radiation is to produce an inversion above it in precisely the same manner as the nocturnal inversion is produced at the ground. I am not aware of any observations on the history of inversions at the top of cloud sheets, but it would be of great interest to know whether these inversions increase with time in the same manner as inversions at the ground. If such an increase were observed it would confirm the view expressed above that the inversion is due to radiative exchanges. The heating of the base of the cloud should also tend to form an inversion below the base of the cloud as a result of transfer of heat downward by radiation and turbulence. Like the inversion at the top, the inversion below should tend to become more marked with increasing cloud height.

During the daytime the effects mentioned are all in part affected by the absorption of direct solar radiation. It is, however, estimated that cloud sheets reflect about four-fifths of the incident solar radiation. A portion of the remainder undergoes diffuse reflection and scattering, but some of it must be absorbed by the water drops. If the cloud is at, say, 3,000 m., the net loss of heat from the upper surface is of the order of ·2 calories per cm.^{2} per minute. The incoming solar beam will be of about one calorie, and if less than one-fifth of this is absorbed by the water drops at the top of the cloud, there will still be a net loss of heat from the top. The short-waved radiation which diffuses through the cloud will produce a general slight warming whose magnitude will diminish with increasing depth.

The main difference between conditions by day and those by night will be the rather uncertain diminution in the net loss of heat at the top of the stratum. We should, perhaps, expect to find the inversion at the top, and the extent of the instability within the cloud, less marked by day than by night. This is in accordance with some figures given by C. S. Durst (33), relating to the frequency of alto-cumulus cloud by day and by night, the percentage of sky covered being considerably greater by night than by day.

It is also probable that the greater frequency of ascending motion in the centre of the cloudlets formed in sheet clouds is to be explained by the greater activity of the lower surface, the loss of heat at the top being checked by direct sunshine. It is possible that downward motion by day is associated with the existence of still higher cloud sheets, and that at night-time downward motion is far more frequent than by day.

In practice the radiational effects are not easily separable from dynamical effects due to the ascent or descent of adjacent layers of saturated and unsaturated air. The presence of haze at other levels may also complicate the phenomena. It is hoped, however, that some of the estimates of the net exchanges of heat by radiation and absorption at the upper and lower surfaces of cloud sheets may be of assistance in helping to disentangle the phenomena observed in individual cases.

Fog layers are formed at the ground when the sky is clear, the wind light, and the air moist. As soon as the fog has attained a depth sufficient to radiate effectively as a black body, the fall of temperature at the ground is checked, since heat only leaks very slowly through the fog. The top of the fog loses heat by radiation at almost exactly the same rate as the ground loses heat before the fog is formed, so that on a perfectly still night the fog will steadily increase in thickness. The checking of the fall of temperature at the ground by fog is borne out by observation.

BIBLIOGRAPHY

1. G. Hettner: *Ann. Physik, Leipzig*, **55**, 1918, p. 476.
2. H. Rubens and G. Hettner: *Berlin, SitzBer. Ak. Wiss.*, 1916, p. 167.
3. G. C. Simpson: *Mem. R. Meteor. Soc.*, **3**, No. 21.
4. W. W. Sleator and E. R. Phelps: *Astrophysical Journal*, **62**, 1925, p. 28.
5. A. Somerfeld: Atomic structure and spectral lines, Chapter VII.
6. W. W. Watson: *Astrophysical Journal*, **60**, 1924, p. 145.
7. W. H. Bragg: *London, Proc. Physic. Soc.*, **34**, 1922, p. 98.
8. K. Ångström: *Arkiv. f. Mat. Ast. och Fysik*, **4**, No. 30, 1908.
9. E. von Bahr: *Ann. Physik, Leipzig*, **29**, 1909, p. 780; *ibid.*, **33**, 1910, p. 585.
10. F. Albrecht: *Met. Zs.*, Nov. 1931.
11. A. Ångström: *Washington, D.C., Smithsonian Inst., Misc. Coll.*, **65**, No. 3, 1918.
12. G. Becker: *Zs. Physik, Berlin*, **34**, 1925, p. 255.
13. D. M. Dennison: *Physic. Rev., Minneapolis*, **31**, 1928, p. 503.
14. F. Albrecht: *Met. Zs.*, Dec. 1931.
15. H. Rubens and E. Ladenburg: *Berlin, Verhandl. D. Physik. Ges.*, **11**, 1909, p. 16.
16. E. Aschkinass: *Wied. Ann.*, **55**, 1895, p. 401.
17. W. H. Dines: *London, Meteor. Off., Geophys. Mem.*, No. 18, 1921.
18. F. E. Fowle: *Washington, D.C., Smithsonian Inst., Misc. Coll.*, **68**, 1917, p. 41.
19. D. Brunt: *London, Proc. R. Soc.*, **124** (A), 1929, p. 201.
20. W. H. Dines and L. H. G. Dines: *Mem. R. Meteor. Soc.*, **2**, No. 11.
21. S. Asklöf: *Geogr. Ann., Stockholm*, **2**, 1920, p. 253.
22. M. Robitsch: *Lindenberg, Arbeit. Preuss. Aeron. Obs.*, **15**, p. 203.
23. A. Ångström: *Met. Zs.*, **33**, 1916, p. 532.
24. A. Ångström: *Beitr. Physik. Atmosph.*, **14**, h.1/2, 1928.
25. A. Boutaric: *La Météorologie*, **4**, 1928, p. 289.
26. N. K. Johnson and E. Ll. Davies: *Q.J.R. Meteor. Soc.*, **53**, 1927, p. 45.
27. H. L. Wright: *Mem. R. Meteor. Soc.*, **4**, No. 31.
28. N. K. Johnson: *London, Meteor. Off., Geophys. Mem.*, No. 46.
29. G. C. Simpson: British Antarctic Expedition 1910-13. Meteorology, Vol. I, p. 56.
30. D. Brunt: *Nature*, **115**, 1925, p. 300; *Meteor. Mag.*, **60**, 1925, p. 1.
31. S. Mal: *Beitr. Physik. Atmosph.*, **17**, h.1, 1930.
32. G. T. Walker and A. C. Phillips: *Q.J.R. Meteor. Soc.*, **58**, 1932, p. 23.
33. C. S. Durst: *Meteor. Mag.*, Jan. 1932.

DISCUSSION.

Dr. G. C. SIMPSON said that this was a most valuable paper and that the Society could congratulate itself on having received it. Radiation is one of the most important problems in meteorology and also one of the most difficult. We could not expect that any one worker will solve all the problems of radiation in the atmosphere. Each worker must nibble at a small part and in time the whole may be reduced, but that is not likely to be in our time. The meteorologist is faced with the absence of physical data about the radiation from water vapour, which is practically the only constituent in the atmosphere which absorbs and emits radiation. At present we have only the absorption coefficients determined by Hettner and as these were obtained with water vapour in the state of steam at atmospheric pressure, that is at about 100°C., it is too great an extrapolation to use these in the condition of the upper atmosphere with its low temperature and low pressure. He expressed the hope that physicists would soon give us more data, but he recognised the difficulty of measuring the long-wave absorption coefficients at low pressures. Dr. Simpson said that there was no time to discuss many of the interesting problems treated in the paper, but he would like to refer to one interesting and important point. Mr. Brunt had discovered the empirical relationship between the radiation received from the sky on a clear night (R) and the temperature (T) and the vapour pressure (e) at the place of observation. This relationship $R = \sigma T^4 (a + b\sqrt{e})$ indicates that if there is no water vapour in the atmosphere then $R = a\sigma T^4$. Now a is practically $\frac{1}{2}$ and σT^4 is the radiation from a black body at the temperature of the place of observation. Therefore the relationship says that if there is no water vapour there will be half as much radiation from the sky as if it were a black body. This contradicts all we know about radiation, for dry air cannot radiate, therefore where does this radiation come from? He suggested that as there is little correlation between the humidity at the surface and the humidity in the upper atmosphere, there would still be a lot of radiation from the upper atmosphere even if the air were dry near the surface, and it is this fact which comes out in the empirical relationship.

Sir NAPIER SHAW wished to associate himself with Dr. Simpson in congratulating the Society and Mr. Brunt on the achievements of the paper which, he said, were really too numerous for one evening's discussion. Incidentally he remarked that the effect upon the liquid itself of the great absorption of radiation of a film of liquid water, as compared with vapour, deserved consideration; and he noted also that on the first page of the proof e was chosen as the symbol for vapour pressure and on the second page was used for a different purpose. Relying on the President to deal with the mathematical side of the paper, he remarked that the scientific importance of a successful mathematical formula was that its success established, as laws, the principles, hypotheses and assumptions upon which the proof of the formula was based. The principles and hypotheses on which Mr. Brunt relied were, first, Stefan's law of radiation according to the fourth power of the temperature of the radiating body measured from absolute zero and, secondly, the numerical value of the constant based on experimental observation. The numerical verification of the formula places Stefan's law on a footing similar to that of universal gravitation—very amazing considering the great variety of radiating bodies and the highly specialised thermodynamics by which the law is theoretically established. The third principle is that of similarity of the conduction of molecular energy and the transference of radiant energy; and the last hypothesis is that of the identity of the vapour pressure as measured at Kew with that at Benson, combined with the disregard of its variation with height. The verification in the paper was so accurate that it should be acclaimed as essentially phenomenal. It was a great bag, for the cast of one paper, of which Mr. Brunt might justly be proud.

Dr. B. A. KEEN said that Mr. Brunt deserved credit for the ingenious adaptation of his treatment of radiation to the problem of soil surface temperatures. The parabolic prediction formula for temperature fall after sunset was attractively simple. Since the experimental curves have, in general, no point of inflexion over this range, they can be fitted reasonably well by conic curves or exponentials by suitable adjustment of the constants, and it might be of interest to compare the physical meaning of the terms in Mr. Brunt's equation with those of an exponential equation quoted by Pernter in Vol. 42 (1914) of the *Monthly Weather Review*. One other point, to which Mr. Brunt also alluded, was of considerable interest and importance. The thermal conductivity of the soil varies with the moisture content, and Patten's experimental values for a sandy soil show that the thermal conductivity in c.g.s. units may increase from ·001 at a moisture content of about 5 per cent to ·005 when the moisture content is 20 per cent. If we take these two moisture contents as roughly representing summer and winter conditions respectively, and remember that the square root of the conductivity occurs in the denominator of Mr. Brunt's formula, we see, that, so far as this effect is concerned, the predicted fall of surface temperature should be over twice as great in summer as in winter. While this is in accordance with expectation it would be of interest to test the point if data are available. Finally, the upward flow of heat through the soil at night would produce a certain movement of water vapour from the warmer to the cooler surface layers of the soil, and distillation into the atmosphere. The latter effect is implicitly included in the equation by the term $\sqrt{e}$, but the former might result in an accession of heat to the soil surface from the heat of condensation of the water vapour thereon. It is probable that the amount would not be very great, however, since the vapour pressure of moist soil rises rapidly towards its upper value with increase of moisture content from dryness. The effect would be most pronounced in summer conditions, and its tendency would be to reduce the predicted fall of surface temperature, partly directly in the fashion just mentioned, and partly indirectly, owing to the increase in specific heat of the surface layer.

In reply (partly communicated later) Mr. D. Brunt agreed with Dr. Simpson that the value of R obtained when $\sqrt{e}=0$ does not represent the radiation from dry air, but probably represents the radiation from layers removed from the influence of surface variations of vapour pressure. In reply to Sir Napier Shaw he was unable to say what happened to the water film which absorbed so much radiation in the laboratory experiments which he had quoted. In the course of the paper he had, however, referred to the probable effect of absorption at the base of a cloud leading to evaporation of the cloud particles. With reference to the second point raised by Dr. Keen, he had found from Patten's figures, that the factor $\rho_1 c_1 \sqrt{\kappa_1}$ was five times as great for a light soil containing 20 per cent of water as for the same soil when dry. He had used values corresponding to dry soil for June. If we take 20 per cent as representing December conditions, a comparison of the results derived by using the formula, with actual observations as quoted by N. K. Johnson (*loc. cit.*), is readily made. The formula predicts a fall of the surface temperature by 3·3°C. between sunset and sunrise in December, while Johnson's diagram indicates a mean fall of about 2·9°C. It should again be noted that no allowance is made for the difference between the screen and surface soil temperatures; a correction for this would bring the computed value still nearer the observed value. The formula given by Trabert in the *Monthly Weather Review,* Vol. 42, p. 655, 1914, is quoted from Maurer. It is of the form

$$T = T_0 + Cb^t$$

where C and b are constants, and b is stated to be $e^{-\sigma/c}$ where σ is the "radiative constant" and c the specific heat of air. There is no very clear physical argument behind the derivation of this equation, and its good fit to observations is due to the general form of the exponential curve and the arbitrary constant C. With reference to the final point raised by Dr. Keen, the flow of water vapour through the soil is difficult to compute, but it should be noted that to some extent this phenomenon is allowed for if we use values of κ_1 derived from observations of temperature in the soil.