Potentiel energi og energibevarelse, Y & F kap. 7

DTU Fotonik
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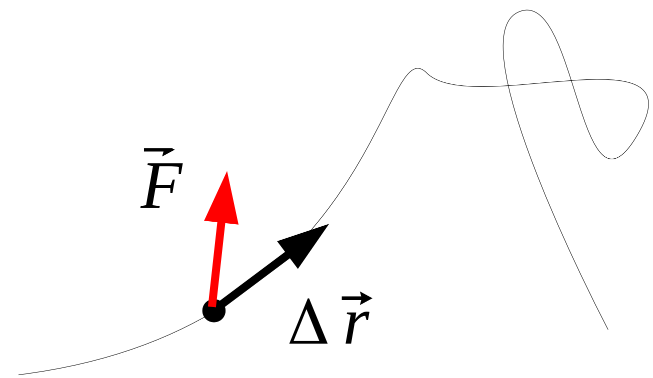
Fra sidste gang

$W=\int \vec{F}\cdot d\vec{r}$ *Arbejde* på en partikel i bevægelse under påvirkning af resulterende kraft $\vec{F}$



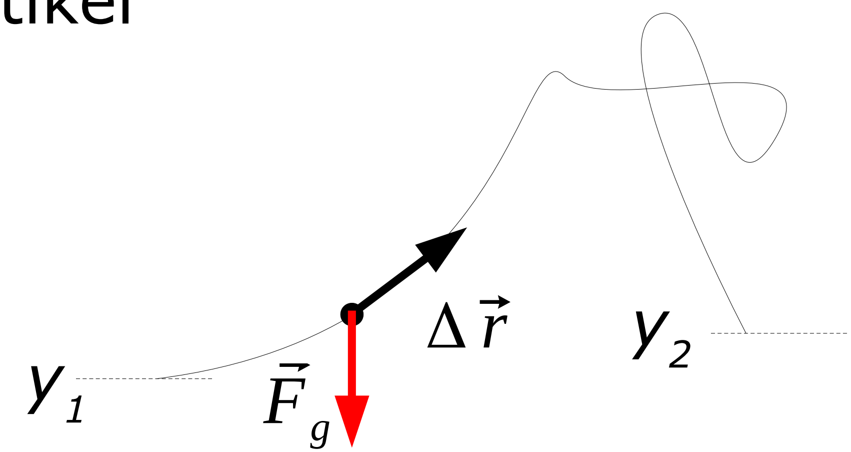
$K=\frac{1}{2}mv^2$ *Kinetisk energi*

$\boxed{W=\Delta K}$ *Arbejdssætningen* for kinetisk energi

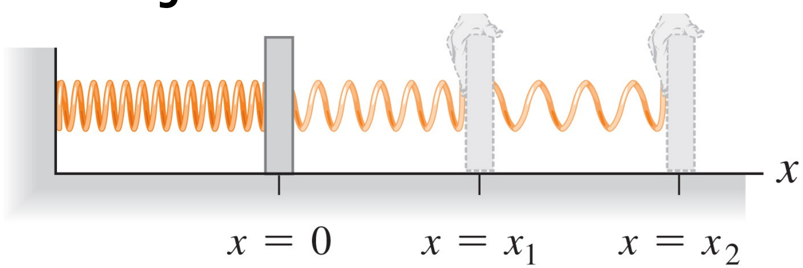
$P=\frac{dW}{dt}=\vec{F}\cdot\vec{v}$ *Effekt* – arbejde pr. tid

Fra sidste gang

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_g \cdot d\vec{r} = -mg\left(y_2 - y_1\right)$$ - tyngdekraftens arbejde på partikel



Fjederkraft:



$$\vec{F} = -kx\,\hat{\boldsymbol{i}}$$

$$W_{x_1 \to x_2} = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

Denne uges læringsmål

-Forstå begreberne potentiel energi og potentialfunktion

-Skelne mellem *konservative* og *ikke-konservative* kræfter

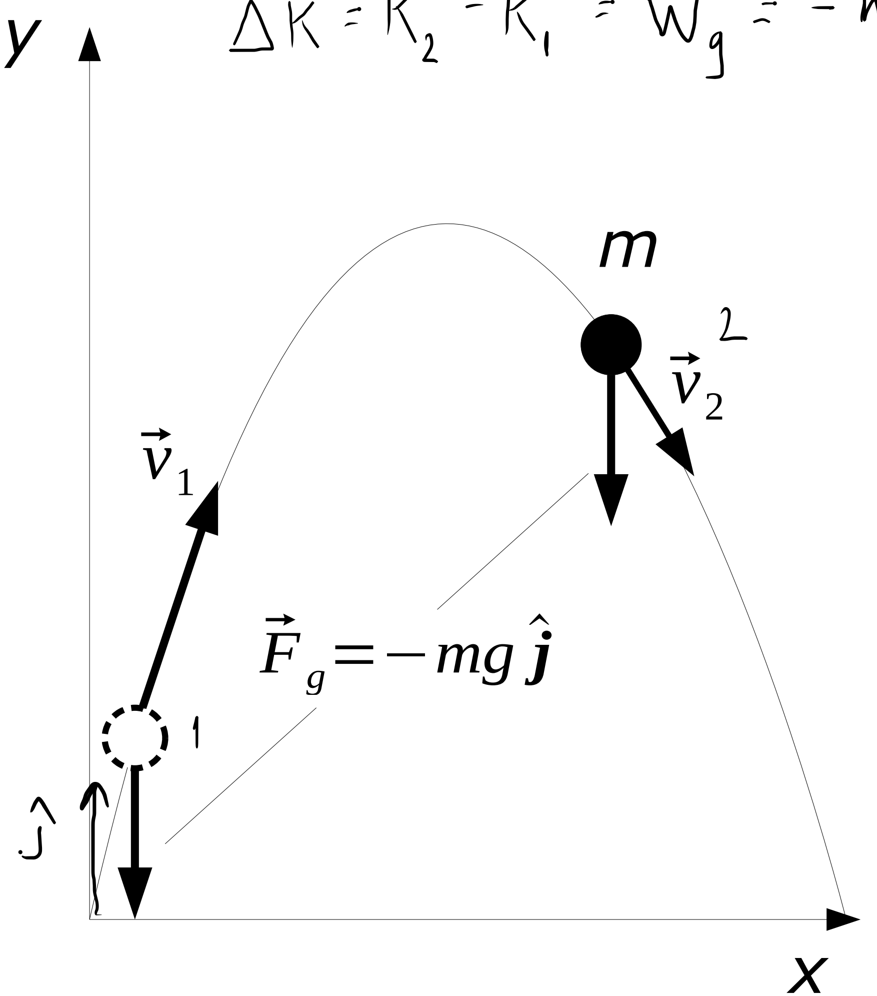
-Forstå og benytte *mekanisk energibevarelse* og betingelserne herfor

-Forstå hvad der menes med (total) *energibevarelse*

-Skelne mellem *stabile* og *ustabile ligevægtspunkter*

Bevægelse i tyngdefelt (skrått kast)

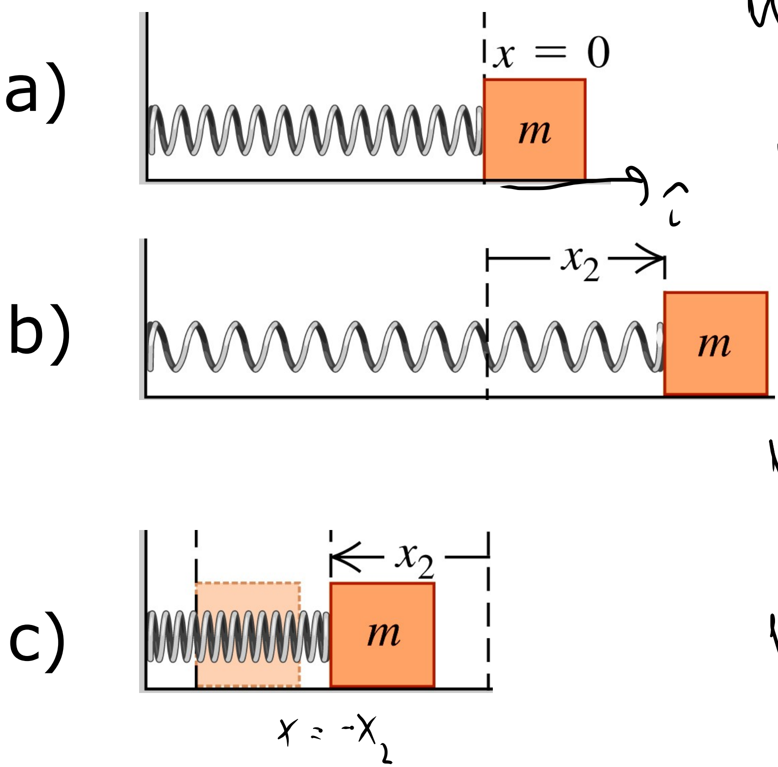
$$\Delta K = K_2 - K_1 = W_g = -mg(y_2 - y_1) = -(\underbrace{U_g(y_2)}_{U_2} - \underbrace{U_g(y_1)}_{U_1}) \quad \boxed{U_g(y) = mgy}$$



$$K_2 - K_1 = -(U_2 - U_1) \Rightarrow K_2 + U_2 = K_1 + U_1$$

$$\bar{F}_g = -mg\hat{j} = -\frac{dU_g}{dy}\hat{j}$$

Bevægelse i fjeder



$$W_{a-b} = -\frac{1}{2}k\left(x_2^2 - 0^2\right) = -\frac{1}{2}kx_2^2$$

$$U = \frac{1}{2}kx^2$$

$$W_{b-c} = -\frac{1}{2}k\left((-x_2)^2 - x_2^2\right) = 0$$

$$W_{a-b} = -\left(U(x_2) - U(0)\right)$$

$$W_{a-c} = -\frac{1}{2}k\left((-x_2)^2 - 0^2\right) = -\frac{1}{2}kx_2^2$$

$$W_{a-c} = W_{a-b} + W_{b-c}$$

$$\bar{F} = -kx\,\hat{i} = -\frac{dU}{dx}\,\hat{i}$$

Generelt – vejuafhængig arbejdsfunktion

$$W_F = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$ uafhængigt af vejen fra start til slut:

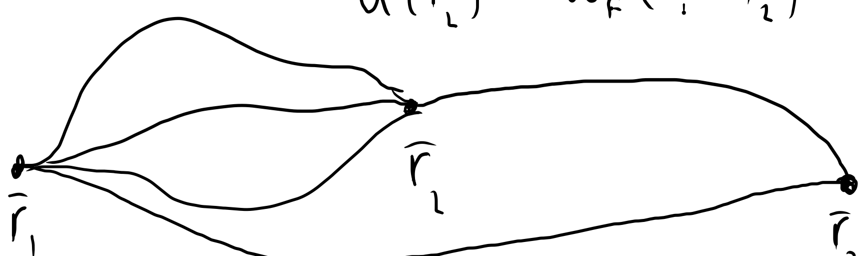
$$W_F(\bar{r}_1 \to r_3) = W_F(\bar{r}_1 \to r_2) + W_F(\bar{r}_2 \to r_3)$$

$$\Downarrow$$

$$-U_3 = -U_2 + W_F(\bar{r}_2 \to \bar{r}_3) \Rightarrow W_F(\bar{r}_2 \to \bar{r}_3) = U_2 - U_3 = -(U_3 - U_2)$$

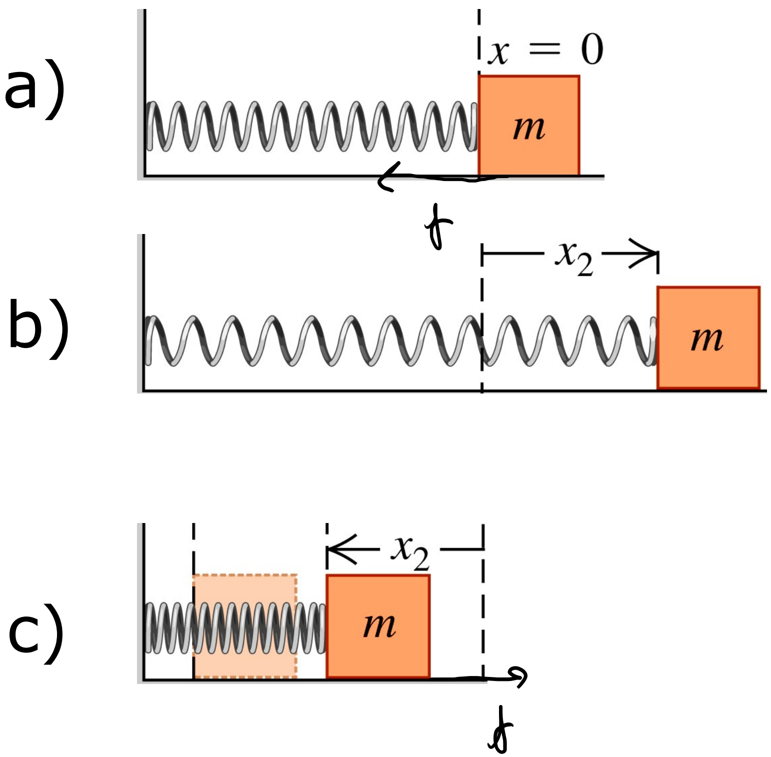
$$U(\bar{r}_2) = -W_F(\bar{r}_1 \to r_2)$$

$$U(\bar{r}_1) \equiv 0$$



$$U(\bar{r}_3) = -W_F(\bar{r}_1 \to \bar{r}_3)$$

Bevægelse i fjeder med friktion



$$W_{f\,a-b} = -f(x_2 - 0) = -f x_2$$

$$W_{f\,b-c} = -2f x_2$$

$$W_{f\,a-c} = -f x_2 + W_{f\,a-b} + W_{f\,b-c} = -3f x_2$$

Summa summarum...

Tyngdekraft og *fjederkraft*: $W(\vec{r}_1 \rightarrow \vec{r}_2) = -(U(\vec{r}_2) - U(\vec{r}_1))$

Arbejdet uafhængigt af hvordan vi kommer fra x_1 til x_2

Størrelsen $K+U$ er *bevaret* under bevægelsen.

U kaldes *potentiel energi.*

Friktionskraft: Arbejdet fra x_1 til x_2 afhænger af vejen mellem x_1 og x_2

Vi kan ikke etablere en potentialfunktion og en bevaret størrelse $K+U$

Mekanisk energi og konservative kræfter

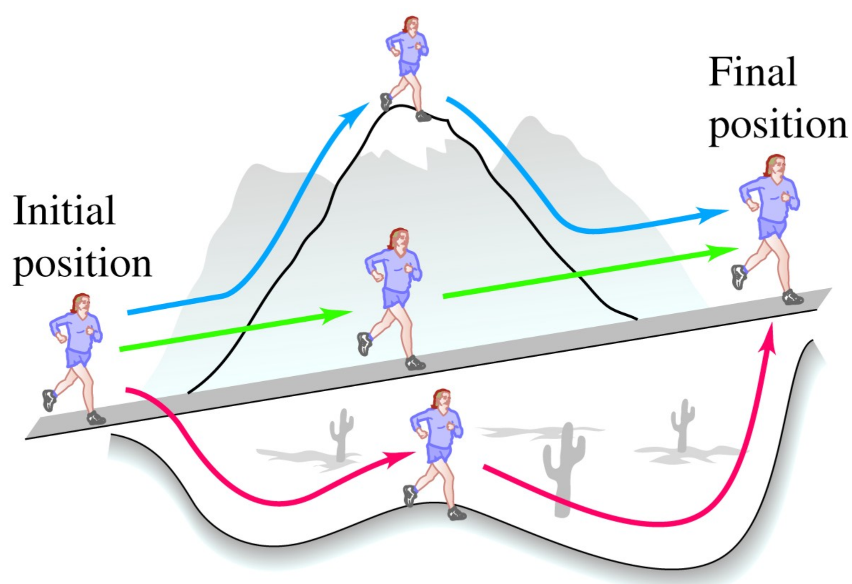
En kraft kaldes *konservativ* hvis dens arbejde mellem to punkter er uafhængig af vejen imellem dem.

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} = -\left(U\left(\vec{r}_2\right) - U\left(\vec{r}_1\right)\right) \Rightarrow \vec{F} = -\vec{\nabla} U = -\left(\hat{\boldsymbol{i}}\frac{dU}{dx} + \hat{\boldsymbol{j}}\frac{dU}{dy} + \hat{\boldsymbol{k}}\frac{dU}{dz}\right)$$

1D: $F(x) = -\dfrac{dU}{dx}$

$K(\vec{r}) + U(\vec{r})$ kaldes *mekanisk energi*

Mekanisk energi *bevaret* for konservative kræfter



Nulpunkt for potentiel energi

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = -\left(U\left(\vec{r}_2\right) - U\left(\vec{r}_1\right)\right) \qquad \vec{F} = -\vec{\nabla} U = -\left(\hat{\boldsymbol{i}}\frac{dU}{dx} + \hat{\boldsymbol{j}}\frac{dU}{dy} + \hat{\boldsymbol{k}}\frac{dU}{dz}\right)$$

$U\left(\vec{r}\right) \rightarrow U\left(\vec{r}\right) + C$ ændrer hverken kraften eller arbejdet.

Potentiel energi er entydig på nær en konstant C.

$\Downarrow$

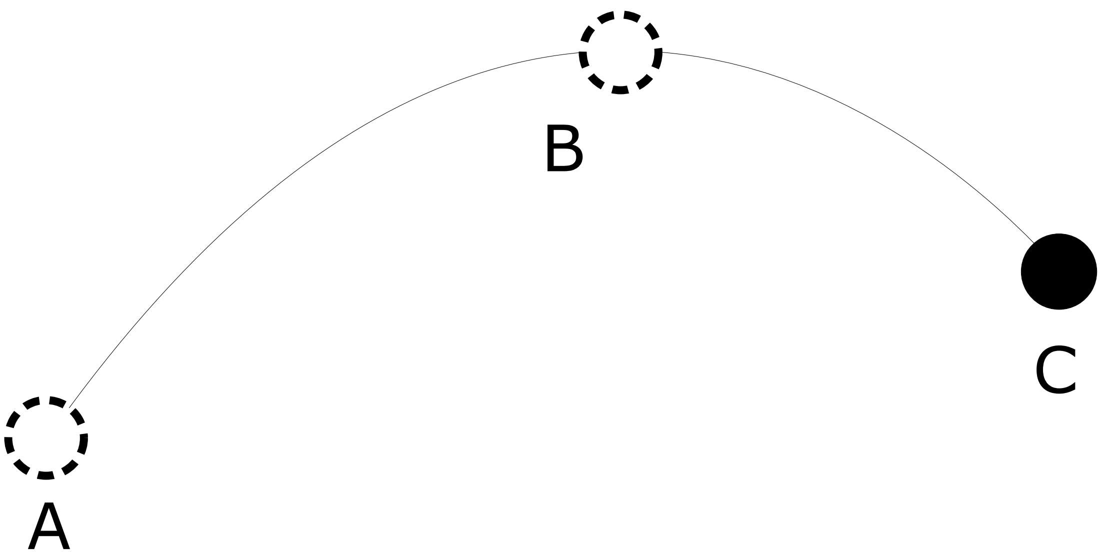
Vi kan frit vælge nulpunktet for potentiel energi.

Quiz: Skråt kast

Bolden påvirkes kun af tyngdekraften (potentialfunktion U). U_A=0. K=kinetisk energi. Sandt/falsk?

1: Tyngdekraften udfører positivt arbejde på bolden fra A-C

2: Boldens potentielle energi vokser fra A-C



3: $U_B=K_A$

4: $K_A+U_A=K_C+U_C$

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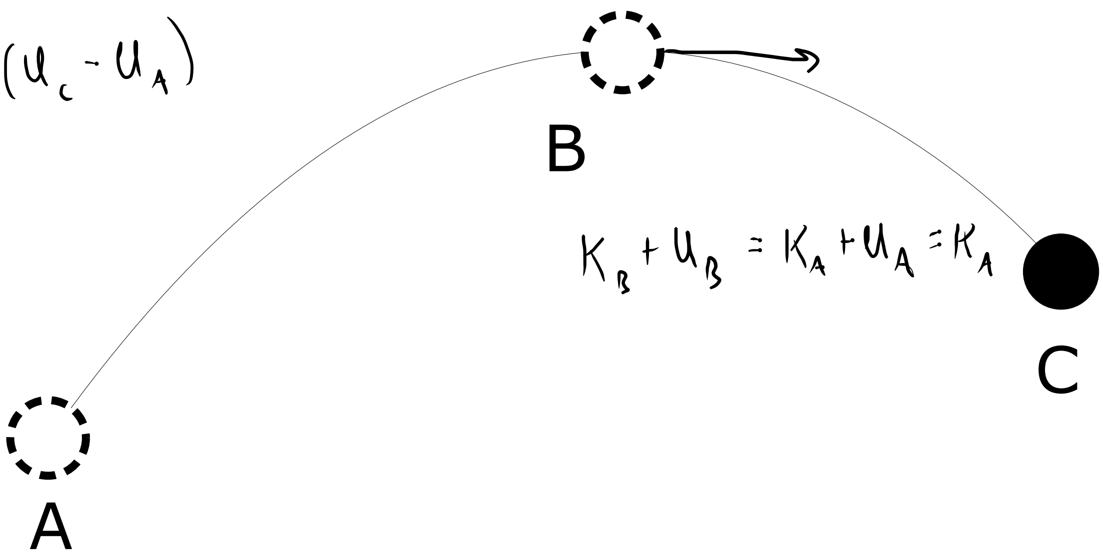
1: Tyngdekraften udfører positivt arbejde på bolden fra A-C

2: Boldens potentielle energi vokser fra A-C

$W_{A-C} = -(U_C - U_A)$



$K_B + U_B = K_A + U_A = K_A$

3: $U_B = K_A$

4: $K_A + U_A = K_C + U_C$

Eksempel: Samlet potentiel energi

Hvad er loddets potentielle energi som funktion af *y*?

$U = U_g + U_s$

$$W_{tot} = \int (\bar{F}_g + \bar{F}_s) \cdot d\bar{r} = W_g + W_s = -\Delta U_g - \Delta U_s = -\Delta U$$

$$U(y) = mgy + \frac{1}{2} k y^2 = \frac{1}{2} k (y - y_0)^2 + U_0 \; !$$

$$mgy + \cancel{\frac{1}{2} k y^2} = \frac{1}{2} k (y^2 - 2 y y_0 + y_0^2) + U_0 = \cancel{\frac{1}{2} k y^2} - k y_0 y + \frac{1}{2} k y_0^2 + U_0$$

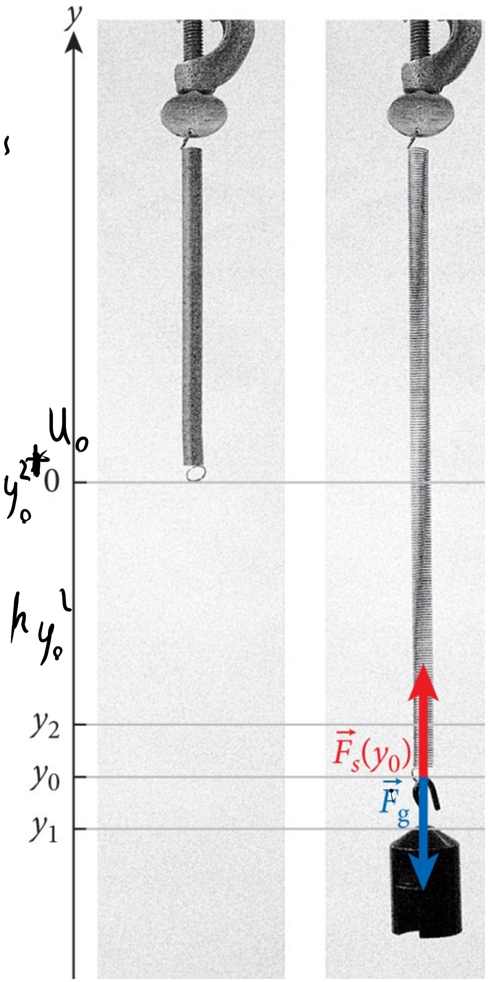
$$\Downarrow$$

$$mg = -k y_0 \Rightarrow y_0 = -\frac{mg}{k} \; ; \quad U_0 + \frac{1}{2} k y_0^2 = 0 \Rightarrow U_0 = -\frac{1}{2} k y_0^2$$

$F_s \uparrow -k y_0$

mg

$$F_s - mg = 0 \Rightarrow -k y_0 - mg = 0 \Rightarrow y_0 = -\frac{mg}{k}$$



Totalenergibevarelse

Kun konservative kræfter: $\Delta E_{mek} = \Delta K + \Delta U = 0$

U sum af alle potentialfunktioner.

$W_{tot} = -\Delta U + W_{other}$

Generelt: $\Delta E_{mek} = \overbrace{\Delta K}^{} + \Delta U = W_{other}$

W_{other} arbejde fra ikke-konservative kræfter.

Definér $E_{total} = E_{mek} + E_{other}$ og $\Delta E_{other} = -W_{other}$

$$\Downarrow$$

$$\boxed{\Delta E_{total} = \Delta E_{mek} + \Delta E_{other} = W_{other} - W_{other} = 0}$$

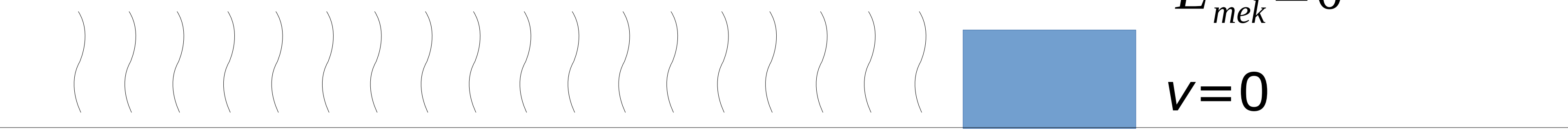
Energibevarelse - varmeenergi

v

$$E_{mek}=\frac{1}{2}mv^2 \qquad E_{varme}=E_{v0}$$

μ_k

$$E_{mek}=0$$



$v=0$

μ_k

$$E_{varme}=E_{v0}+\frac{1}{2}mv^2$$

Varme=uordnet mikroskopisk bevægelse

Quiz: Friktionsvarme

Hvilken udspringer genererer den største varmeeffekt fra luftmodstanden? Antag de vejer det samme og bevæger sig med terminalfarten (dvs. konstant fart).

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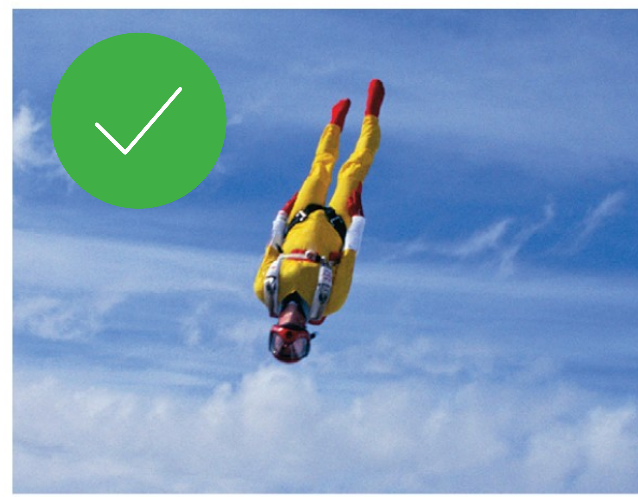
Hvilken udspringer genererer den største varmeeffekt fra luftmodstanden? Antag de vejer det samme og bevæger sig med terminalfarten (dvs. konstant fart).

$$\Delta E_{tot} = \cancel{\Delta K} + \Delta U + \Delta E_v = 0 \Rightarrow \frac{\Delta E_v}{\Delta t} = -\frac{\Delta U}{\Delta t}$$



Quiz – mekanisk energi

Ingen kinetisk friktion/luftmodstand.
Hvad er kinetisk energi K_1 i slutsituationen?

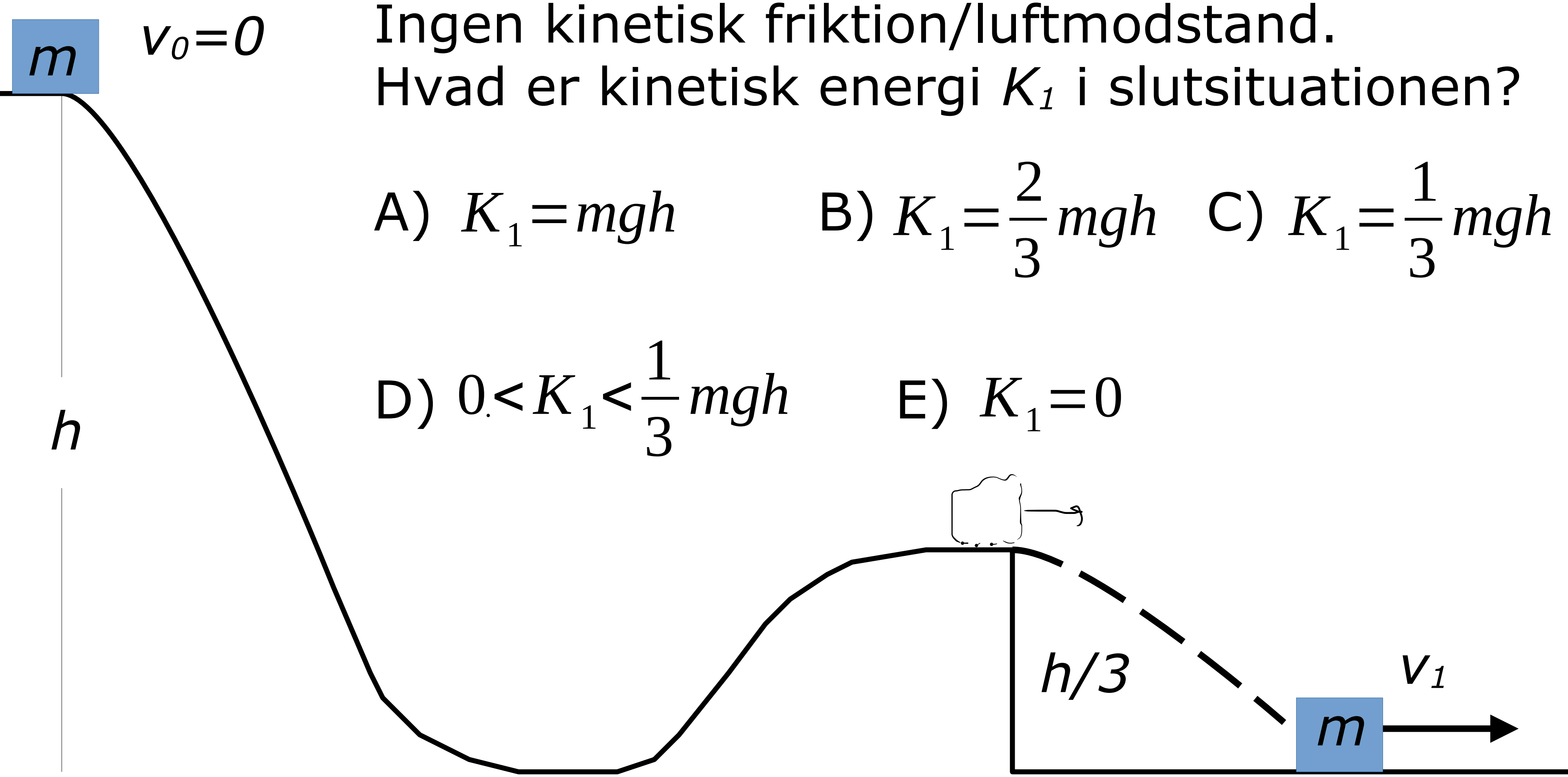
A) $K_1 = mgh$

B) $K_1 = \frac{2}{3} mgh$

C) $K_1 = \frac{1}{3} mgh$

D) $0 < K_1 < \frac{1}{3} mgh$

E) $K_1 = 0$



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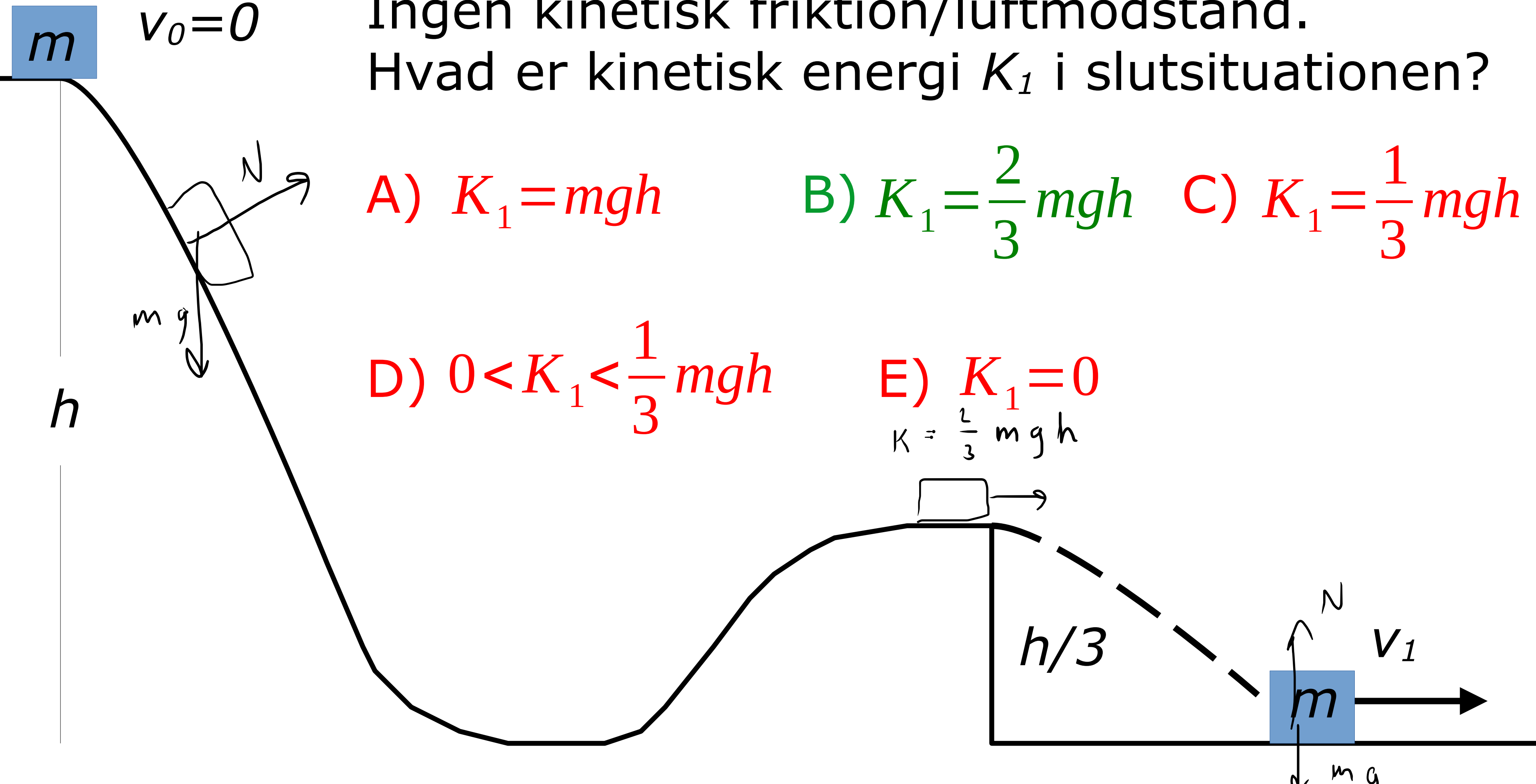
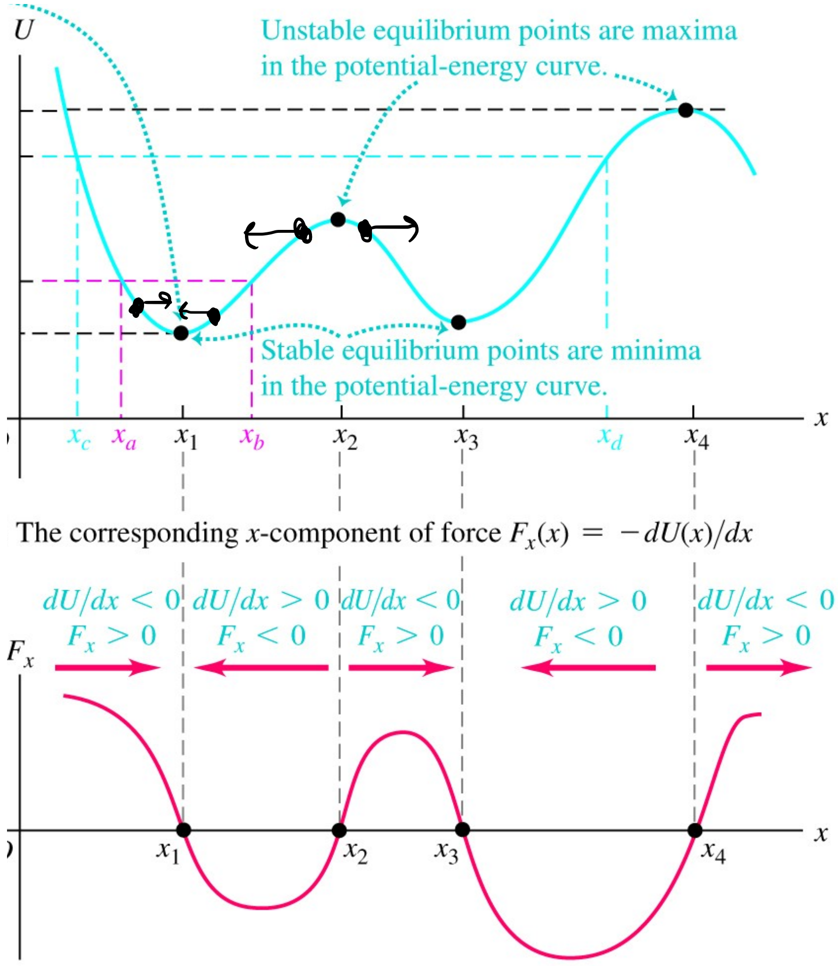


Ligevægtspunkter

$$F(x) = -\frac{dU}{dx}$$

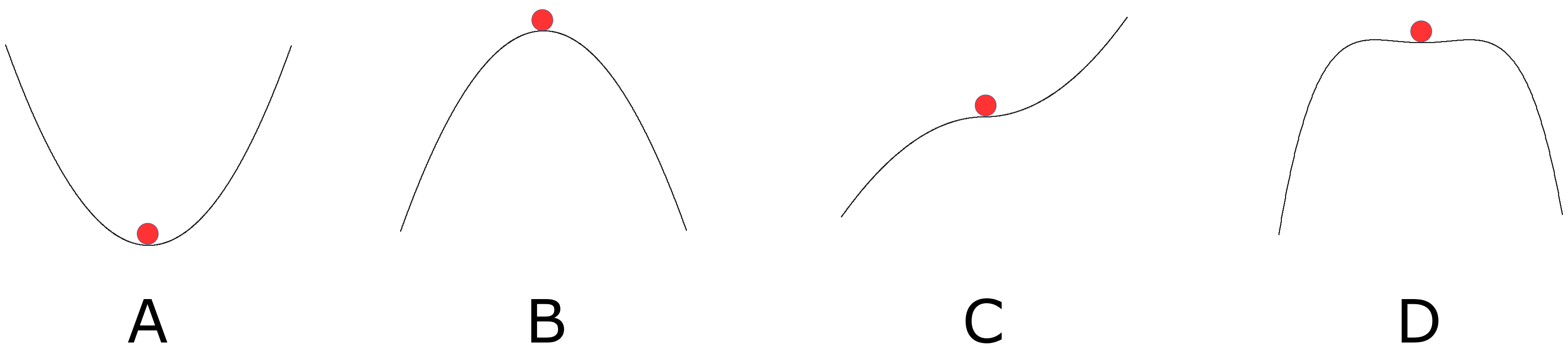
I et *ligevægtspunkt, x_{min}* er $F(x_{min})=0$.

I et *stabilt ligevægtspunkt* vil kræfterne omkring x_{min} pege ind mod x_{min}



Quiz: Ligevægtspunkter

Hvilke punkter er stabile ligevægtspunkter?



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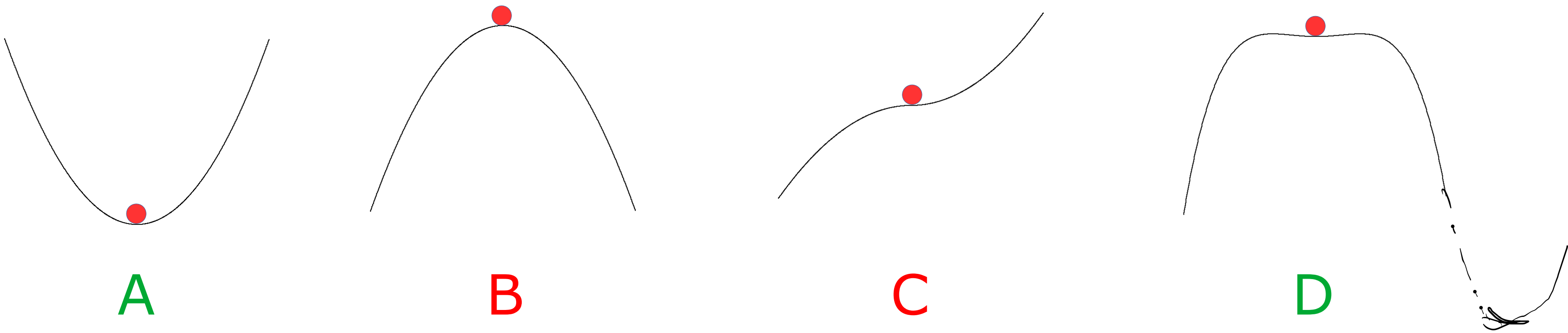
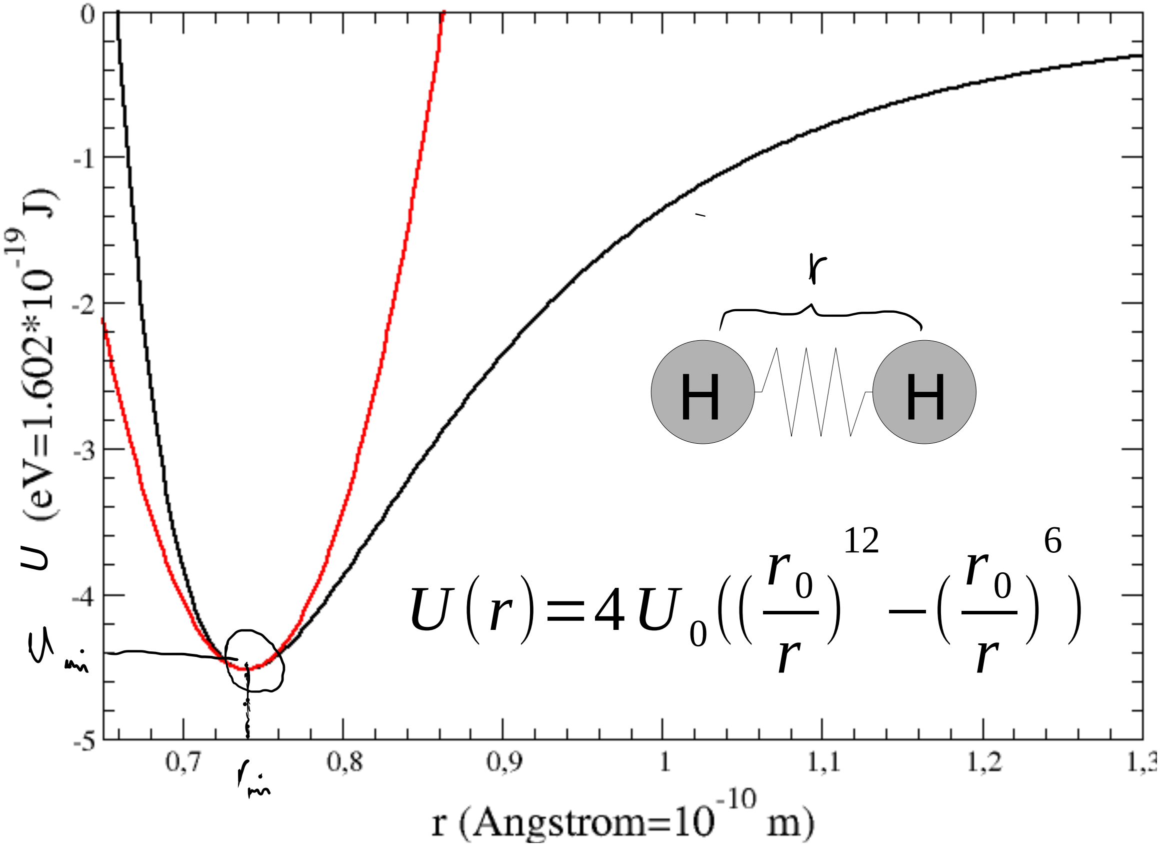


Ligevægtspunkt og fjederpotential

$$U(r) = U(r_{min}) + \cancel{\frac{dU}{dr}\Big|_{r=r_{min}}(r-r_{min})} + \frac{1}{2}\frac{d^2U}{dr^2}(r-r_{min})^2 + \frac{1}{6}\frac{d^3U}{dr^3}(r-r_{min})^3 + \ldots$$



$$U(r) = U(r_{min}) + \frac{1}{2}\frac{d^2U}{dr^2}(r-r_{min})^2 + \ldots$$

Opsummering

Arbejdet fra en *konservativ* kraft ved bevægelse fra $\vec{r}_1$ til $\vec{r}_2$ kan udtrykkes ved en *potentialfunktion U:*

$$W = -\left(U\left(\vec{r}_2\right) - U\left(\vec{r}_1\right)\right)$$

Nulpunkt for *U* kan vælges frit.

Arbejdet er uafhængigt af vejen fra $\vec{r}_1$ til $\vec{r}_2$

Kraften er givet ved $\vec{F} = -\vec{\nabla} U = -\left(\hat{\boldsymbol{i}}\frac{dU}{dx} + \hat{\boldsymbol{j}}\frac{dU}{dy} + \hat{\boldsymbol{k}}\frac{dU}{dz}\right)$

Tyngdekraft og *fjederkraft* er eksempler på konservative kræfter.

Opsummering

Mekanisk energi $E_{mek} = K(\vec{r}) + U(\vec{r})$ hvor K= kinetisk energi og U er summen af potentialfunktioner for alle konservative kræfter der virker på partiklen

Mekanisk energi er *bevaret* for en partikel der kun påvirkes af konservative kræfter.

Friktionskræfter er ikke konservative, og bevarer ikke E_{mek}.

Den *totale* energi E_{total} er altid bevaret.

Fysik 1 intro, Y&F kap. 1

DTU Fotonik
Institut for Fotonik

Om kurset

Indhold: Klassisk mekanik og elektromagnetisme

Bog: Young & Freedman selected chapters.. kun mekanik
Fuld e-bog med elektromagnetisme

Øvelser: Dansksprogede opgaver+evt. nogle fra bogen.
Løsninger lægges op fredag efter øvelserne.

Eksperimenter: Ingen eksperimenter i denne udgave.

Om kurset

Eksamen: Skriftlig, sommer 2024. Ingen obl. opgaver eller anden forudgående evaluering af teoristof.

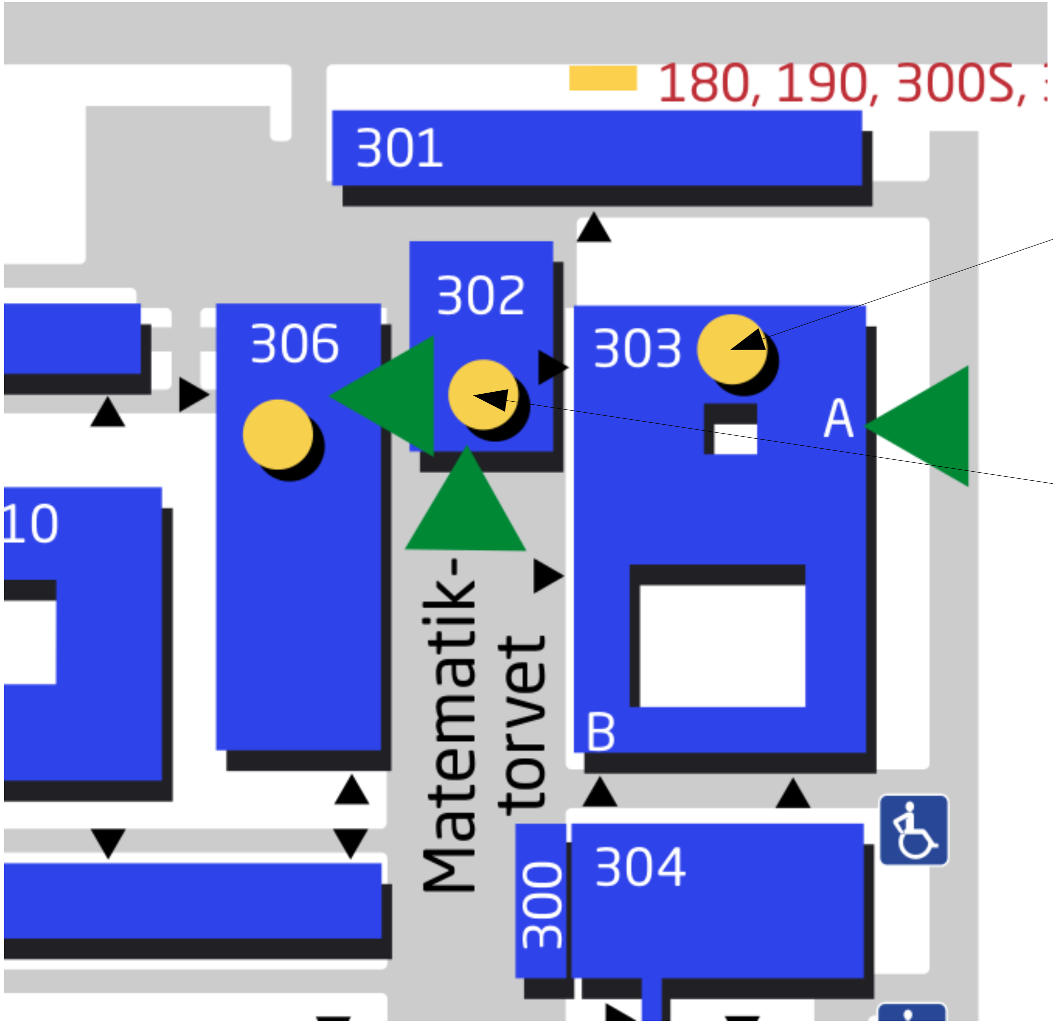
Gruppearbejde er fint, men husk eksamen er individuel.

Fora: DTU Learn primær information/fildeling.
DTU Inside gruppe bruges ikke!

Tag meget gerne indledende mekanik-quiz på Learn!

Øvrige quizzer er forberedelse til hver forelæsning, og giver mig en ide om jeres niveau inden undervisningen.

DTU-geografi



Du er (måske) her

Øvelser hele stuen
samt 1. sal nordøst

Denne uges læringsmål

-Genopfriske begreber om *fysiske enheder* og *præcision*

-Problemløsningsteknik, <u>kvalitetskontrol</u>

-Kort repetition af grundbegreber fra vektorregning

Tal og enheder

Jordens befolkningstal i 2015:

7,256,490,011 (US census Bureau)
7,336,435,000 (Pop. reference bureau)
7,349,472,000 (FN)

Eksponent - ”antal nuller”

Vi skriver $7.3 \cdot 10^9$

2 betydende cifre: Vi mener tallet er mellem $7.25 \cdot 10^9$ og $7.35 \cdot 10^9$

Decimalkomma

Tal og enheder

Jordens befolkningstal i 2015:

7,256,490,011 (US census Bureau)
7,336,435,000 (Pop. reference bureau)
7,349,472,000 (FN)

$7.\cdot 10^9$ -> 6,500,000,000 – 7,500,000,000 

$7.3\cdot 10^9$ -> 7,250,000,000 – 7,350,000,000 

$7.30\cdot 10^9$ -> 7,295,000,000 – 7,305,000,000 

$7.300\cdot 10^9$ -> 7,299,500,000 – 7,300,500,000 

Regning med endelig præcision

Addition

$$113.3+4.2=117.5=1.175\cdot 10^2$$

$$113.3+42.=155.=1.55\cdot 10^2$$

$$113.3+4.235=117.5=1.175\cdot 10^2$$

Multiplikation

$$113.3\cdot 4.2=1.133\cdot 10^2\cdot 4.2=1.133\cdot 4.2\cdot 10^2=4.7586\cdot 10^2\approx 4.8\cdot 10^2$$

$$113.3\cdot 4.235=1.133\cdot 10^2\cdot 4.235=1.133\cdot 4.235\cdot 10^2=4.798255\cdot 10^2\approx 4.798\cdot 10^2$$

Tal og enheder

Højden af Rundetårn=
$41.55\ \text{m} = 4.155\cdot10^{1}\ \text{m} =$
$415.5\cdot10^{1}\ \text{cm} = 4.155\cdot10^{3}\ \text{cm} =$
$0.04155\ \text{km} = 4.155\cdot10^{-2}\ \text{km}$

Faktor	Præfix	Symbol	Faktor	Præfix	Symbol
10^{15}	peta	P	10^{-15}	femto	f
10^{12}	tera	T	10^{-12}	pico	p
10^{9}	giga	G	10^{-9}	nano	n
10^{6}	mega	M	10^{-6}	mikro	μ
10^{3}	kilo	k	10^{-3}	milli	m
10^{2}	hekto	h	10^{-2}	centi	c
10^{1}	deka	da	10^{-1}	deci	d

Tal og enheder – SI enhedssystemet

Fundamentale SI-enheder

Enhed	Forkortelse	Enhed for
Meter	m	Længde
Sekund	s	Tid
Kilogram	kg	Masse
Ampere	A	Strømstyrke
Kelvin	K	Temperatur
Mol	mol	Antal
Candela	cd	Luminositet

Afledte SI-enheder

Navn	Enhed	Enhed for
m/s	m/s	Hastighed/fart
m/s^2	m/s^2	Acceleration
Newton (N)	$kg\ m/s^2$	Kraft
Joule (J)	$kg\ m^2/s^2$	Energi
Watt (W)	$kg\ m^2/s^3$	Effekt
Pascal (Pa)	$N/m^2=kg/(ms^2)$	Tryk

Tal og enheder – SI enhedssystemet

Fundamentale SI-enheder

Enhed	Forkortelse	Enhed for
Meter	m	Længde
Sekund	s	Tid
Kilogram	kg	Masse
Ampere	A	Strømstyrke
Kelvin	K	Temperatur
Mol	mol	Antal
Candela	cd	Luminositet

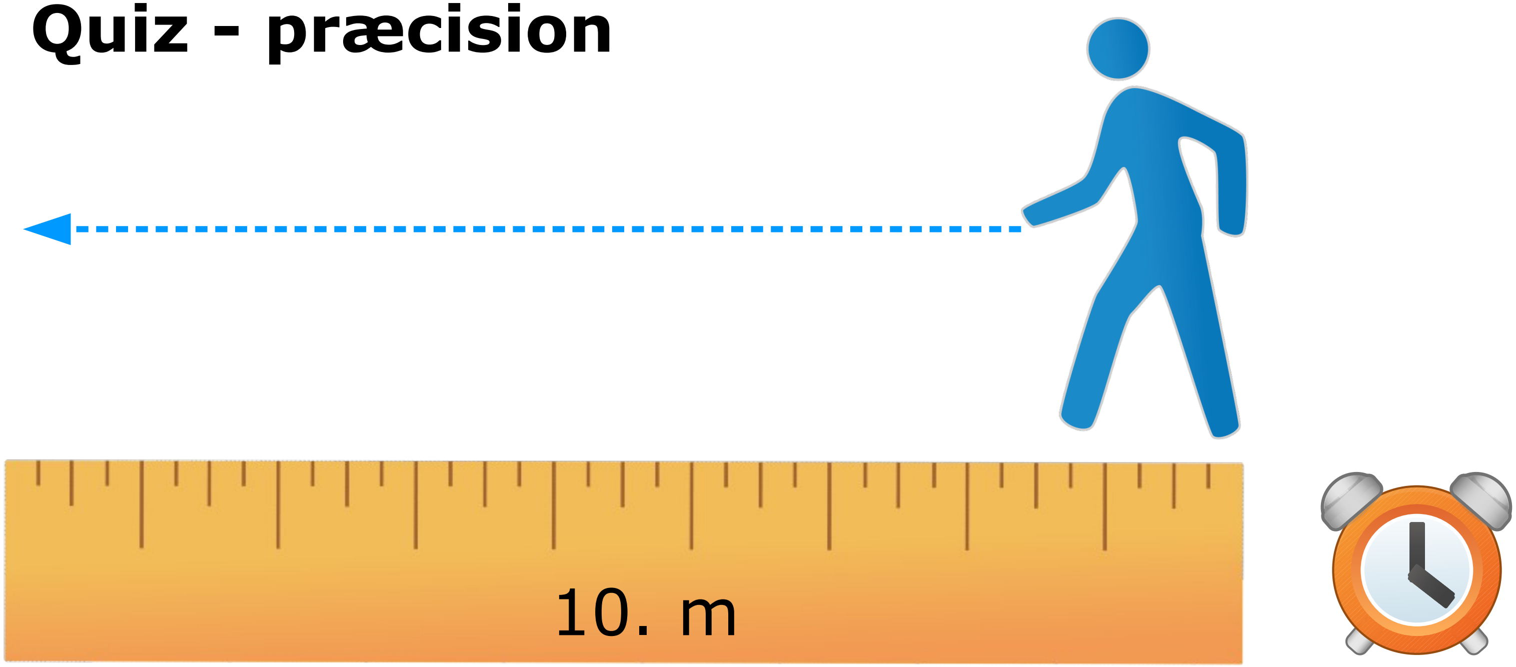
1 s=9,192,631,770 svingninger af mikrobølger fra et bestemt kvantespring i Cs-atomet

1 m=1/299,792,458 af den afstand lyset rejser på 1 s

1 kg=  ~~normalkilogram i Paris~~

Definition baseret på naturkonstant (Planck’s konstant) siden 2018.

Quiz - præcision

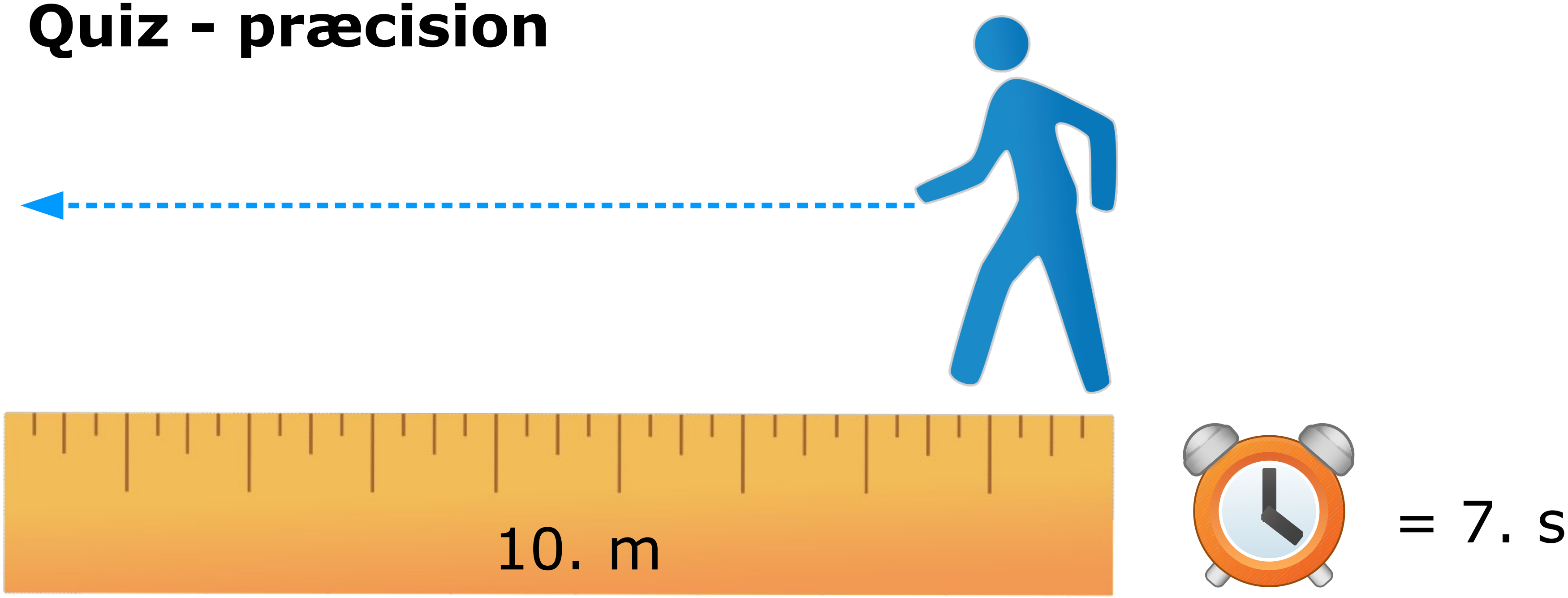


= 7. s

Hvordan kan vi angive farten?

A: 1.4 m/s   B: 1.428571 m/s   C: 1. m/s   D: 1.0 m/s

Quiz - præcision



Hvordan kan vi angive farten?

A: 1.4 m/s B: 1.428571 m/s C: 1. m/s D: 1.0 m/s

Problemløsningsteknik

4-trinsmodel

Analyse - Hvilken slags problem?
Hvordan kan det gribes an?

Planlægning – Opstil relevante formler og identiteter
Hvilke variable er kendte/ubekendte?

Udførelse - Løs ligningerne

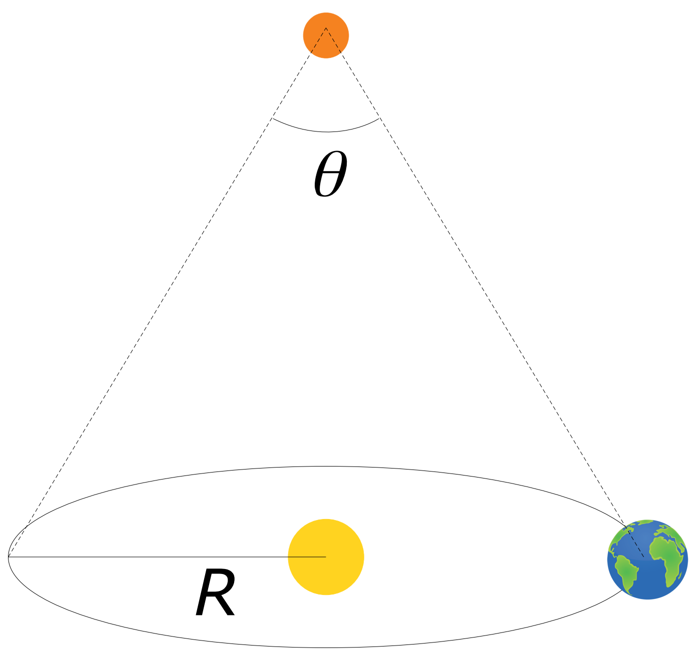
Kontrol – Check om resultatet kan være korrekt
Enheder, grænser..

Eksempel

Stjernen Alpha Centauri's position på himlen varierer halvårligt med ca. $4.3 \cdot 10^{-4}$ grader. Estimér dens afstand til jorden/solen.

Eksempel

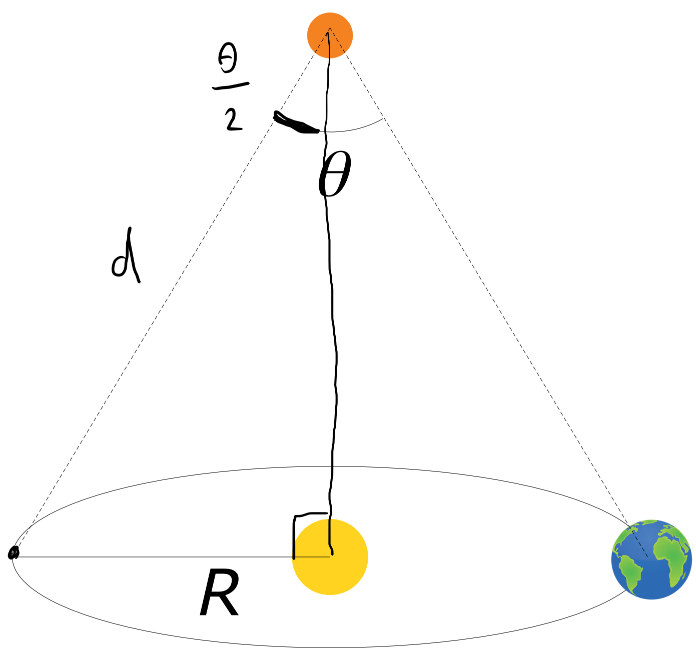
Stjernen Alpha Centauri's position på himlen varierer halvårligt med ca. $4.3 \cdot 10^{-4}$ grader. Estimér dens afstand til jorden/solen.



Analyse: Observeret position varierer pga. jordens bevægelse om solen.

Eksempel

Stjernen Alpha Centauri's position på himlen varierer halvårligt med ca. $4.3 \cdot 10^{-4}$ grader. Estimér dens afstand til jorden/solen.

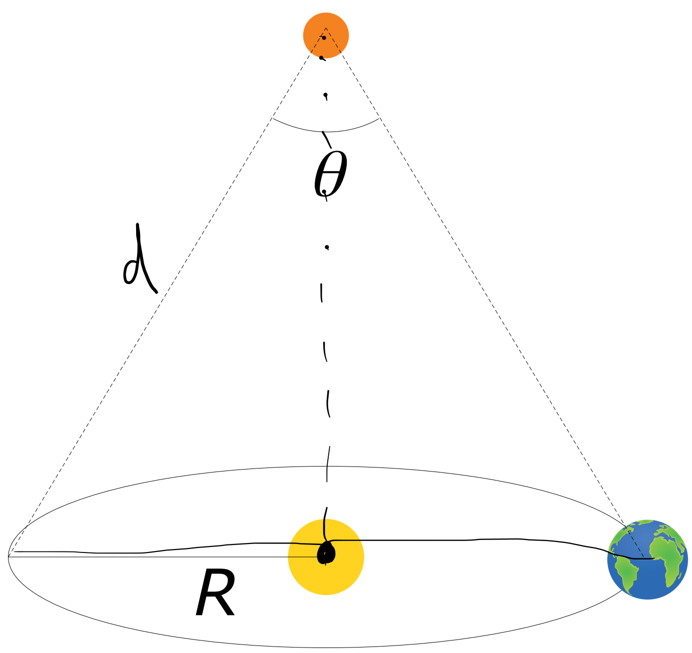


Planlægning: Trigonometri, tabelværdi for $R \approx 1.5 \cdot 10^{8}$ km

$$\sin\frac{\theta}{2} = \frac{R}{d}$$

Eksempel

Stjernen Alpha Centauri's position på himlen varierer halvårligt med ca. $4.3\cdot10^{-4}$ grader. Estimér dens afstand til jorden/solen.



Udførelse: $R \approx 1.5\cdot10^{8}$ km

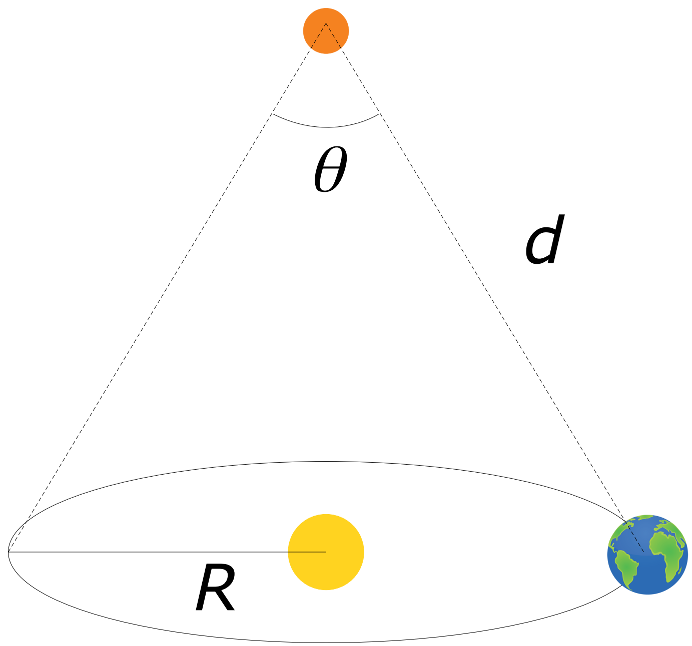
$$\sin\frac{\theta}{2} = \frac{R}{d} \Rightarrow d = \frac{R}{\sin\frac{\theta}{2}} \simeq$$

$$4.0\cdot10^{13}\ \text{km} \simeq 4.2\ \text{lysår}$$

$$\theta = 180^\circ \Rightarrow d = \frac{R}{\sin 90^\circ} = R$$

Eksempel

Stjernen Alpha Centauri's position på himlen varierer halvårligt med ca. $4.3 \cdot 10^{-4}$ grader. Estimér dens afstand til jorden/solen.



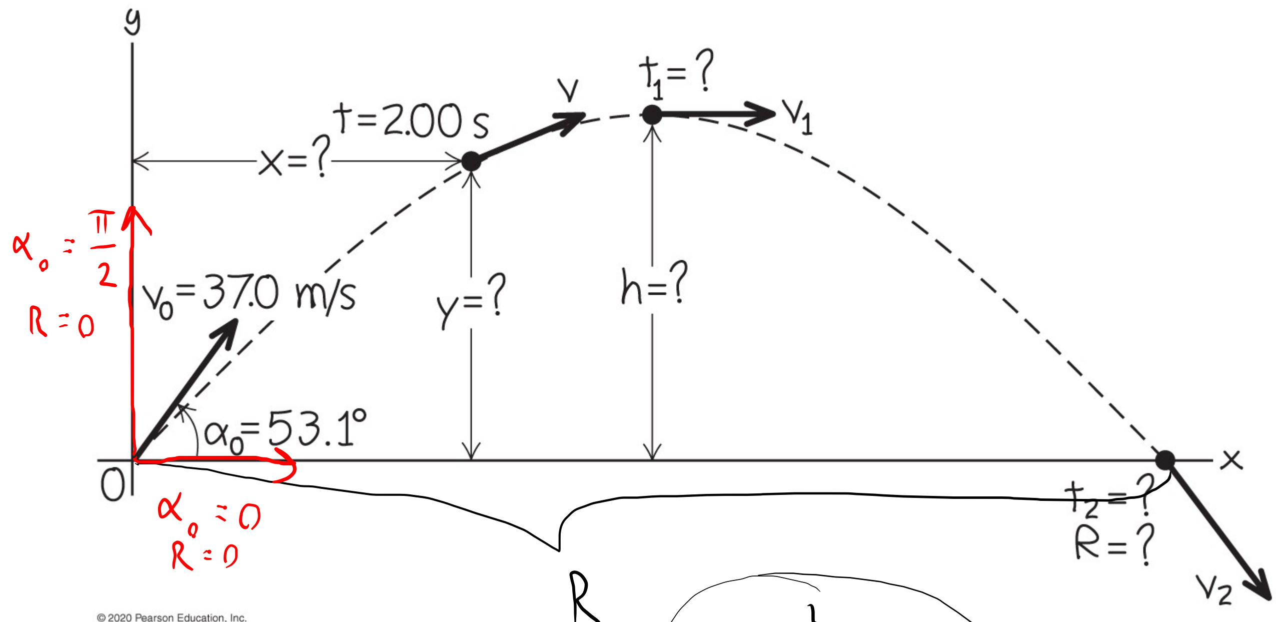
Kontrol: Stor d → lille θ
Lille d → stor θ

Kontrol – enheder og grænser

Skråt kast

g=9.8 m/s^2

$$R=\frac{v_0^2}{g}\cos(2\alpha_0)?$$



$$\frac{(m/s)^2}{m/s^2} \sim m \ \checkmark$$

$$\frac{v_0^2}{g}\underbrace{\cos 0}_{1} = \frac{v_0^2}{g} \div$$

$$R = \frac{v_0^2}{g}\sin 2\alpha_0$$

$$v_0 \to \infty \Rightarrow R \to \infty \ \checkmark$$

$$\frac{v_0^2}{g}\cos\pi = -\frac{v_0^2}{g} \div$$

$$g \to 0 \Rightarrow R \to \infty \ \checkmark$$

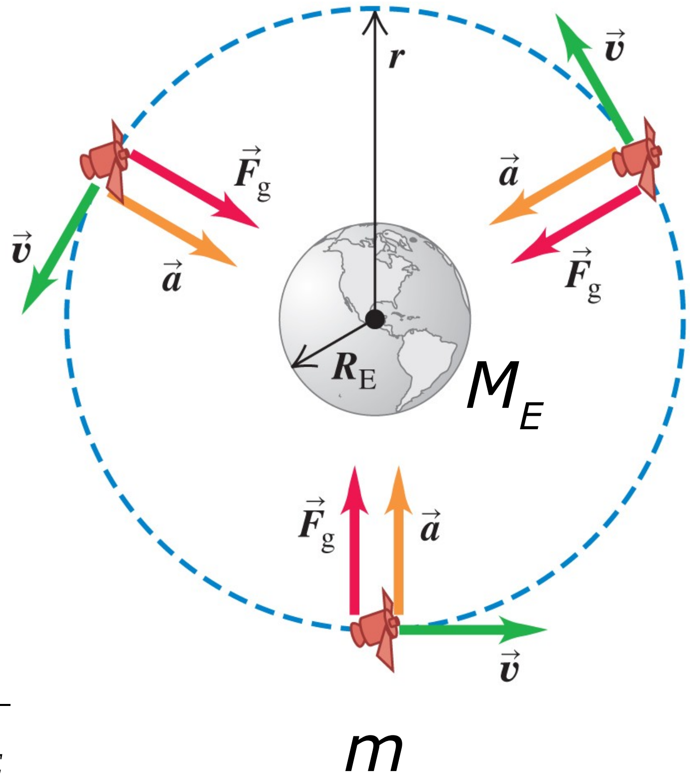
Urimelig quiz – enheder

Gravitationskraften mellem

satellit og jord er $\dfrac{GmM_E}{r^2}$

G=6.67·10^{-11} m^3 kg^{-1} s^{-2}

Farten i satellitbanen (m/s) er?



A: $r\dfrac{M_E}{G}$ B: $\dfrac{GM_E}{r}$ C: $\sqrt{\dfrac{GM_E}{r}}$

D: $\sqrt{r\dfrac{M_E}{G}}$ E: $\sqrt{GM_E r}$

Urimelig quiz – enheder

Gravitationskraften mellem

satellit og jord er $\dfrac{GmM_E}{r^2}$

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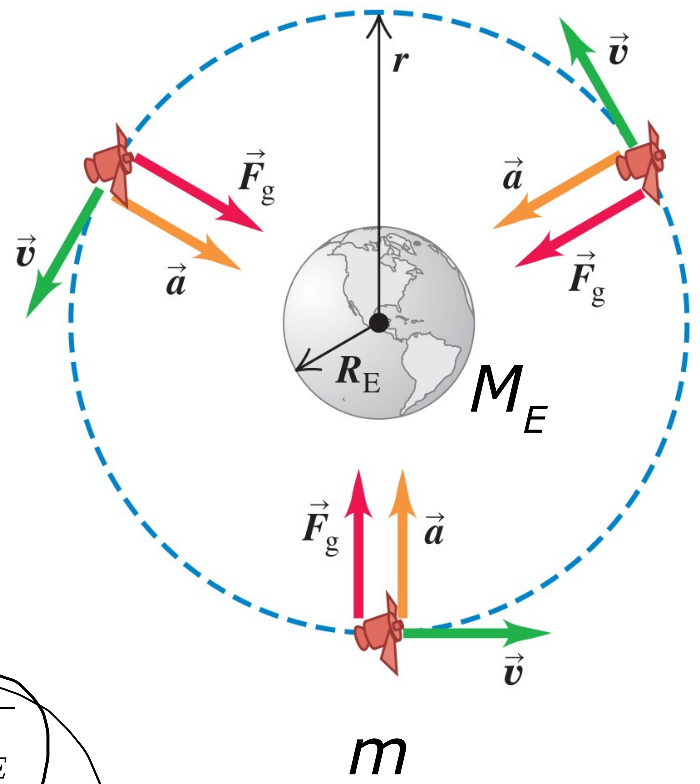


A: $r\dfrac{M_E}{G}$

B: $\dfrac{GM_E}{r}$

C: $\sqrt{\dfrac{GM_E}{r}}$

D: $\sqrt{r\dfrac{M_E}{G}}$

E: $\sqrt{GM_E r}$

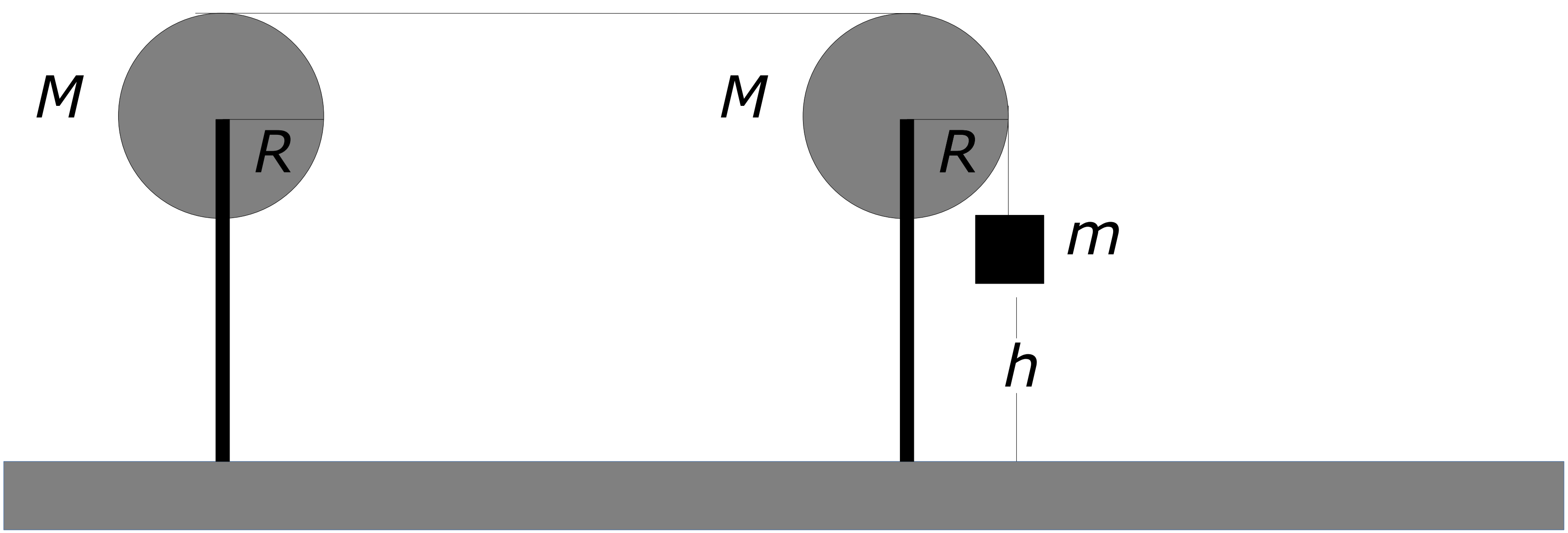
$GM_E \sim \dfrac{m^3}{s^2}$ $\dfrac{GM_E}{r} \sim \dfrac{m^2}{s^2}$

$\sqrt{\dfrac{GM_E}{r}} \sim \dfrac{m}{s}$

Urimelig quiz - grænser

Hvad er loddets fart når det rammer jorden? g=9.8 m/s^2

A: $\sqrt{2gh}$ B: $\sqrt{\frac{2mgh}{M}}$ C: $\sqrt{\frac{2mgh}{M+m}}$ D: $\sqrt{\frac{2(m+M)gh}{m}}$



Urimelig quiz - grænser

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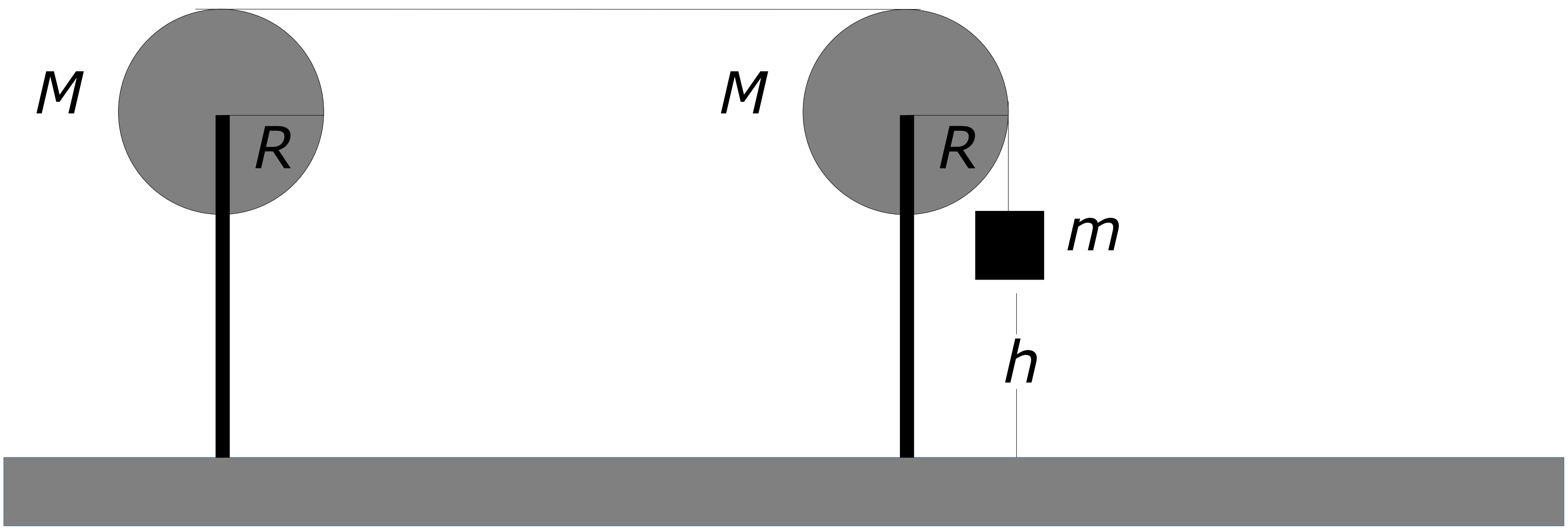


Urimelig quiz - grænser

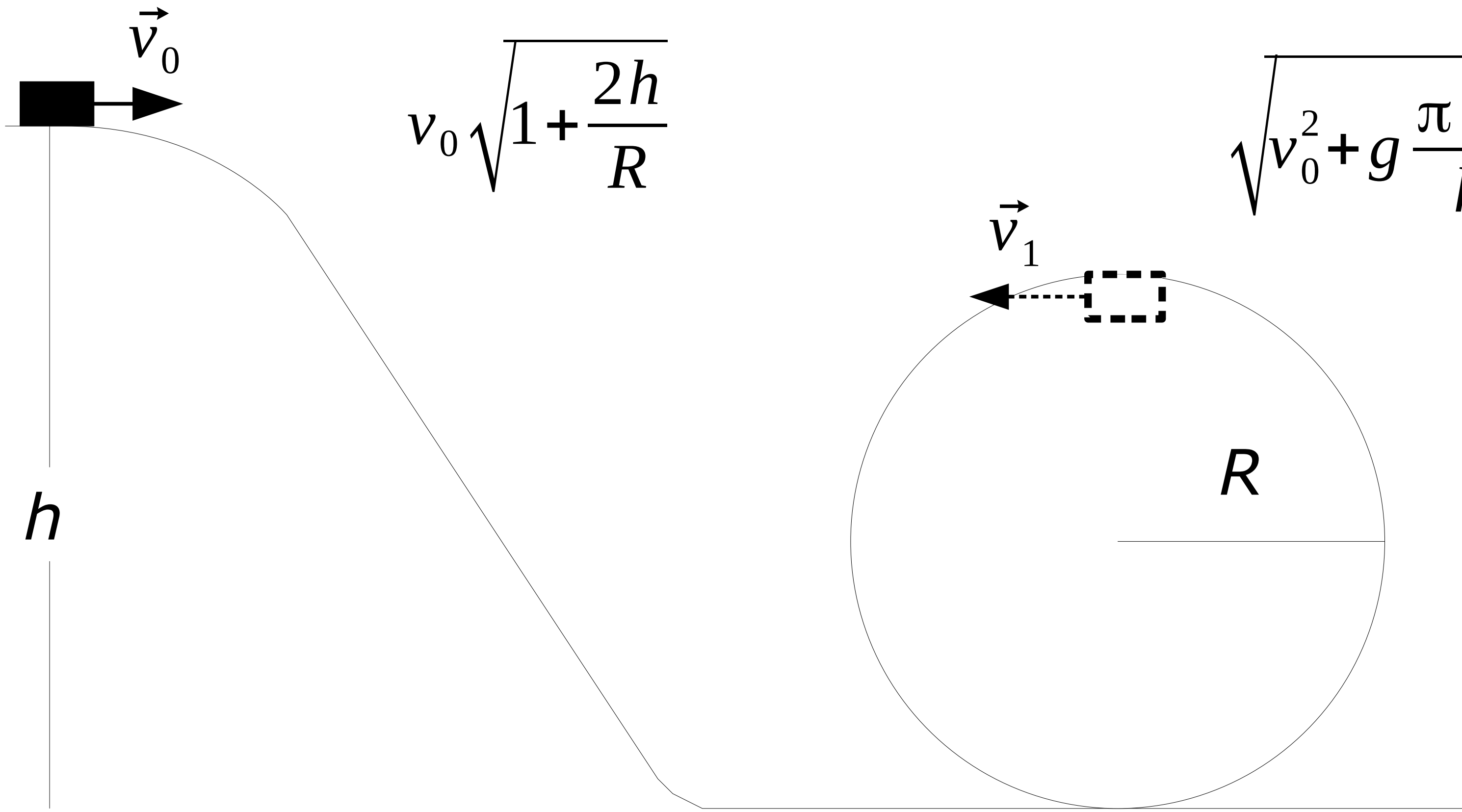
Hvad bliver v_1?

$$\sqrt{v_0^2+2gh}$$

$$\sqrt{v_0^2+2g(h-2R)}$$

$$v_0\sqrt{1+\frac{2h}{R}}$$

$$\sqrt{v_0^2+g\frac{\pi R^2}{h}}$$



Urimelig quiz - grænser

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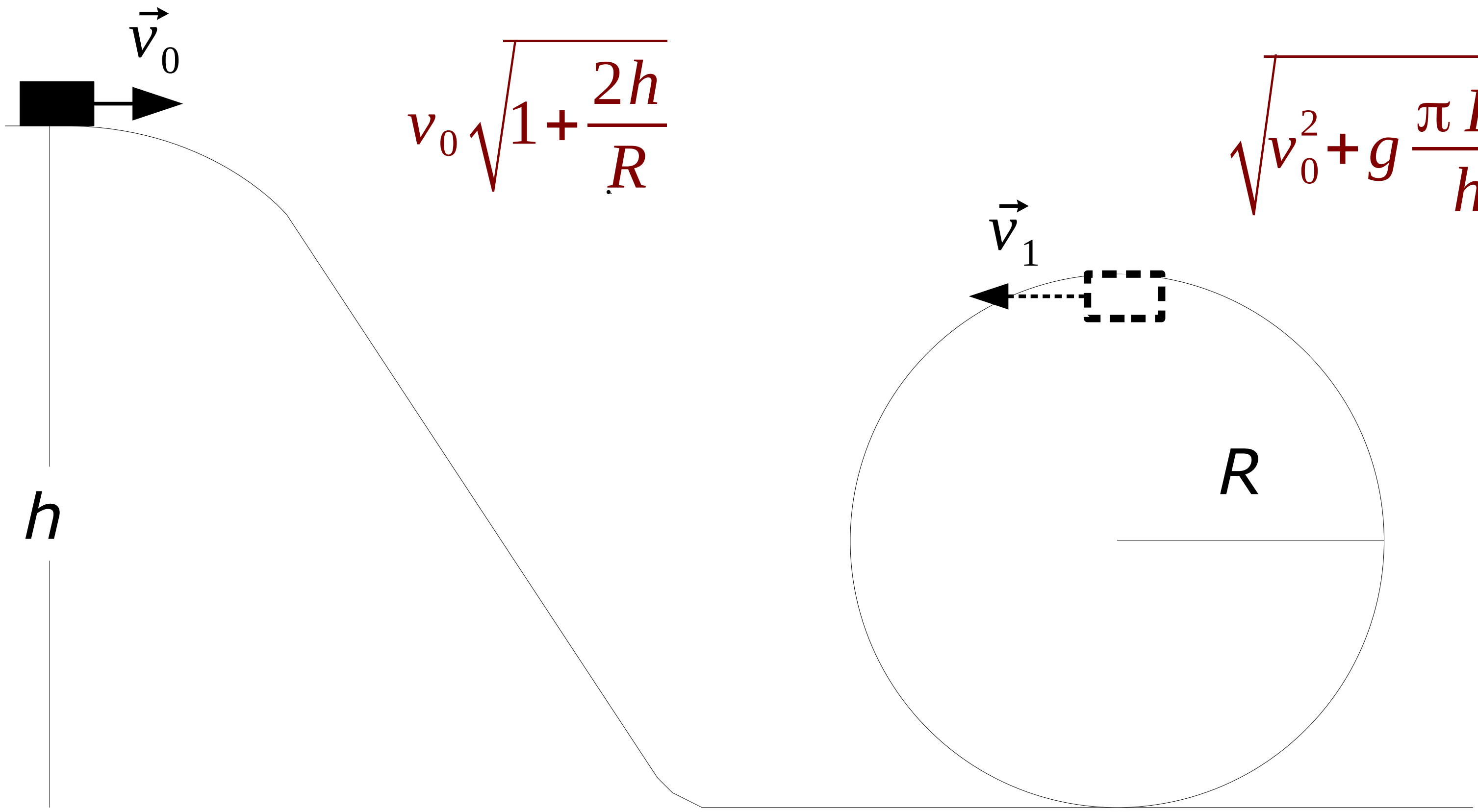
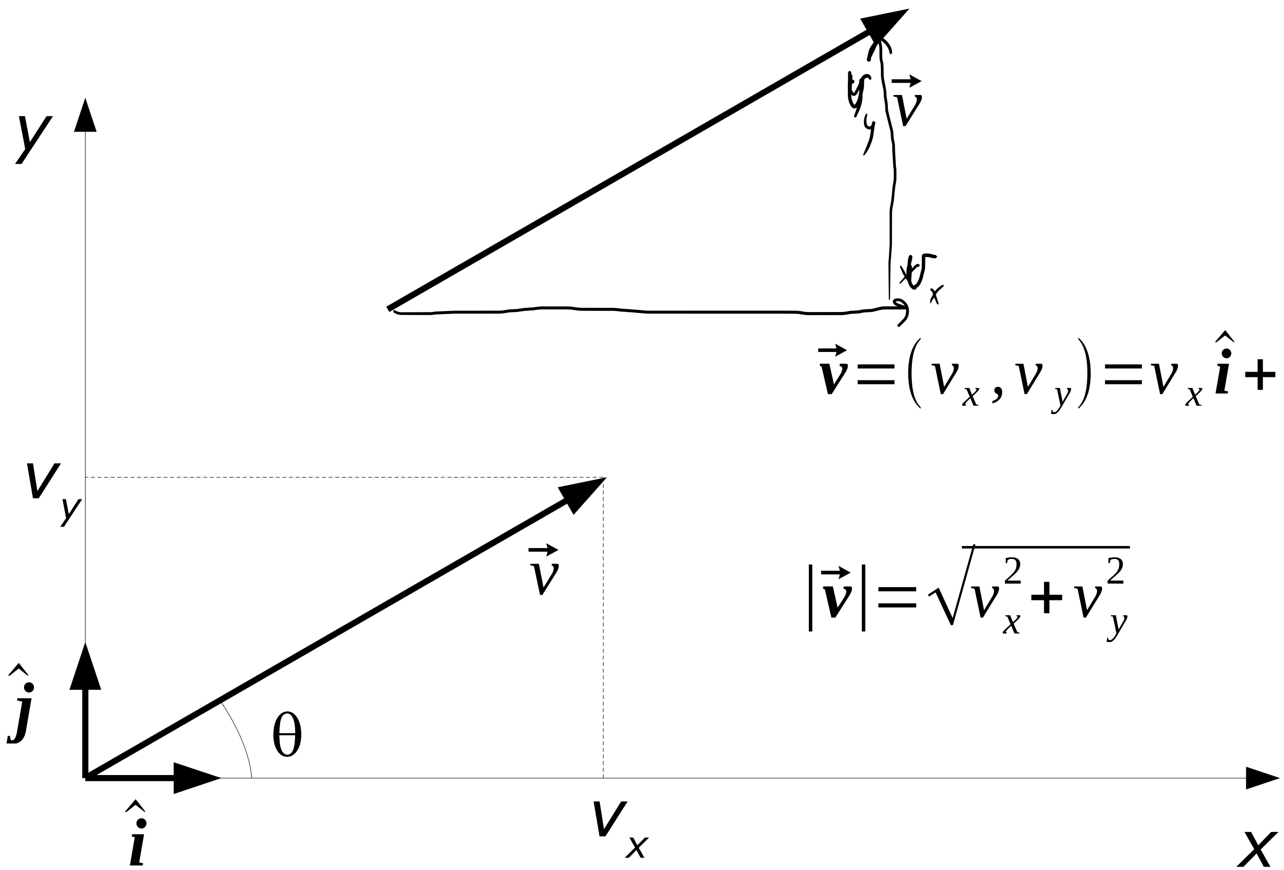


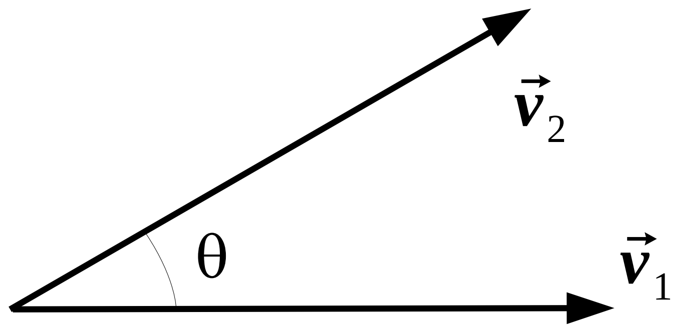
Vektorer



$$\vec{v} = (v_x, v_y) = v_x \hat{i} + v_y \hat{j}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

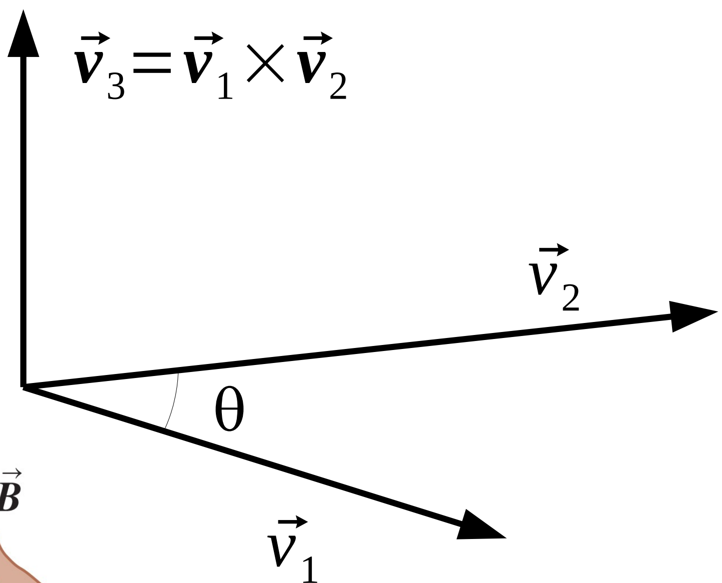
Vektorer - skalarprodukt



$$\vec{v}_1 \cdot \vec{v}_2 = v_{1x} v_{2x} + v_{1y} v_{2y} = |\vec{v}_1||\vec{v}_2| \cos\theta$$

I 3D: $$\vec{v}_1 \cdot \vec{v}_2 = v_{1x} v_{2x} + v_{1y} v_{2y} + v_{1z} v_{2z} = |\vec{v}_1||\vec{v}_2| \cos\theta$$

Vektorer – krydsprodukt (vektorprodukt)



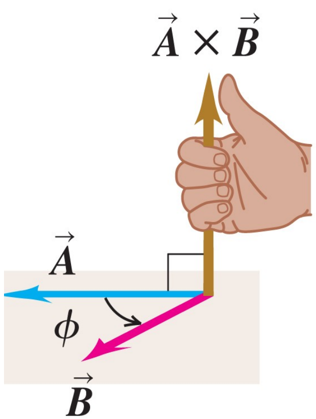
$$\vec{v}_3 \perp \vec{v}_1 \qquad \vec{v}_3 \perp \vec{v}_2$$

$$|\vec{v}_3| = |\vec{v}_1||\vec{v}_2| \sin\theta$$

$$v_{3x} = v_{1y} v_{2z} - v_{1z} v_{2y}$$

$$v_{3y} = v_{1z} v_{2x} - v_{1x} v_{2z}$$

$$v_{3z} = v_{1x} v_{2y} - v_{1y} v_{2x}$$



Højrehåndsregel:

$$\vec{C} = \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

Quiz - vektorer

Hvilke udtryk er meningsfulde?

$$\vec{v}_3 = \vec{v}_1 \cdot \vec{v}_2$$

$$\vec{v}_3 = \vec{v}_1 \times \vec{v}_2$$

$$a\,\vec{v}_3 + b = \vec{v}_1 \times \vec{v}_2 + \vec{v}_1 \cdot \vec{v}_2$$

$$\vec{v}_3 = \vec{v}_1 \times \vec{v}_2 + \vec{v}_1 \left(\vec{v}_1 \cdot \vec{v}_2 \right)$$

Quiz - vektorer

Hvilke udtryk er meningsfulde?

$$\vec{v}_3 = \vec{v}_1 \cdot \vec{v}_2$$

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$$\vec{v}_3 = \vec{v}_1 \times \vec{v}_2 + \vec{v}_1 \left(\vec{v}_1 \cdot \vec{v}_2 \right)$$

Quiz - vektorer

Hvilke udtryk kunne være korrekte? $\vec{v}_1 \text{og } \vec{v}_2 \neq \vec{0}$

$$-\vec{v}_2 = \vec{v}_2 \left(\vec{v}_1 \cdot \vec{v}_1 \right)$$

$$-\vec{v}_2 = \vec{v}_2 \left(\vec{v}_1 \cdot \vec{v}_2 \right)$$

$$\vec{v}_1 = \vec{v}_1 \times \vec{v}_2 - \vec{v}_2$$

$$\vec{v}_1 = \vec{v}_1 \times \vec{v}_2 - \vec{v}_1$$

Quiz - vektorer

Hvilke udtryk kunne være korrekte?

$$-\vec{v}_2 = \vec{v}_2(\vec{v}_1 \cdot \vec{v}_1)$$

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Lineær bevægelse (*kinematik*), Y & F kap. 2

Fra sidste gang

Tal og enheder -regning med endelig præcision

$$113.3 + 42. \approx 155.$$

$$113.3 \cdot 4.2 \approx 4.8 \cdot 10^2$$

Problemløsning – *Analyse, planlægning, udførelse,* ***kontrol***

Kvalitetskontrol – check af enheder og velvalgte grænsetilfælde

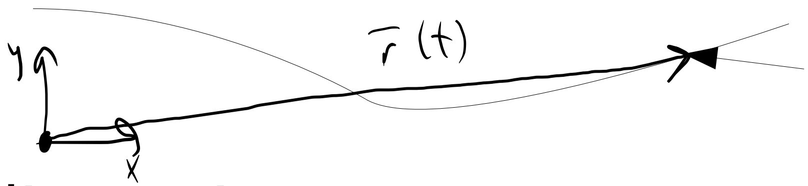
Denne uges læringsmål

-Forstå definition af *stedfunktion, hastighed, fart, acceleration*, samt sammenhænge mellem disse

-Beskrive bevægelse i én dimension med (stykkevis) konstant acceleration

-Angribe konkrete kinematiske problemstillinger

Stedfunktion, lineær bevægelse

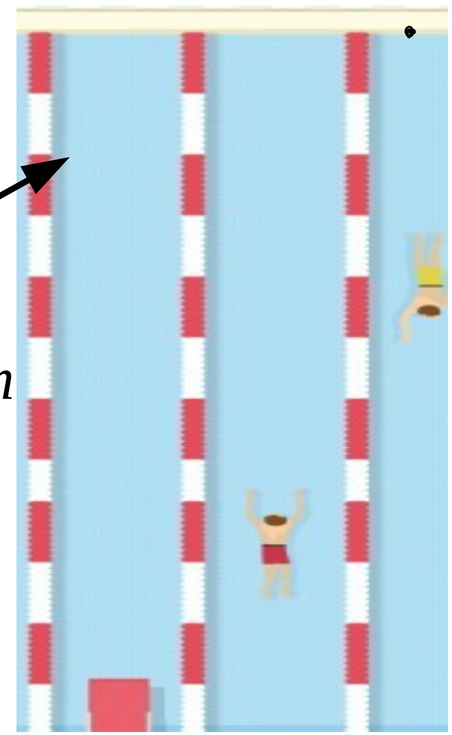


$\vec{\boldsymbol{r}}(t)$ position af legeme som funktion af tiden.

1 dimension:

$\vec{\boldsymbol{r}}(t) = r(t)$ (evt. negativ)

$\vec{\boldsymbol{r}}(t)$



$x_S = 0$ m

50 m

$x_M = 50$ m

Vejlængde $= |x_M - x_S| + |x_S - x_M| = 100\,m$

Forskydning $= |x_S - x_S| = 0\,m$

Hastighed og acceleration i endelige intervaller

$$\bar{v}_x = \frac{\Delta x}{\Delta t} \qquad \bar{a}_x = \frac{\Delta v}{\Delta t}$$

Interval	Samlet	Split	v_x
0-10 m	1.85 s	1.85 s	5.41 m/s
10-20 m	2.87 s	1.02 s	9.80 m/s
20-30 m	3.78 s	0.91 s	11.0 m/s
30-40 m	4.65 s	0.87 s	11.5 m/s
40-50 m	5.50 s	0.85 s	11.8 m/s
50-60 m	6.32 s	0.82 s	12.2 m/s
60-70 m	7.14 s	0.82 s	12.2 m/s
70-80 m	7.96 s	0.82 s	12.2 m/s
80-90 m	8.79 s	0.83 s	12.0 m/s
90-100 m	9.69 s	0.90 s	11.1 m/s

a_x

$3.05\ \mathrm{m/s^2} \longleftarrow \dfrac{v_2 - v_1}{(\Delta t_2 + \Delta t_1)/2}$

$1.24\ \mathrm{m/s^2}$

$0.6\ \mathrm{m/s^2}$

$0.4\ \mathrm{m/s^2}$

$0.5\ \mathrm{m/s^2}$

$0.\ \mathrm{m/s^2}$

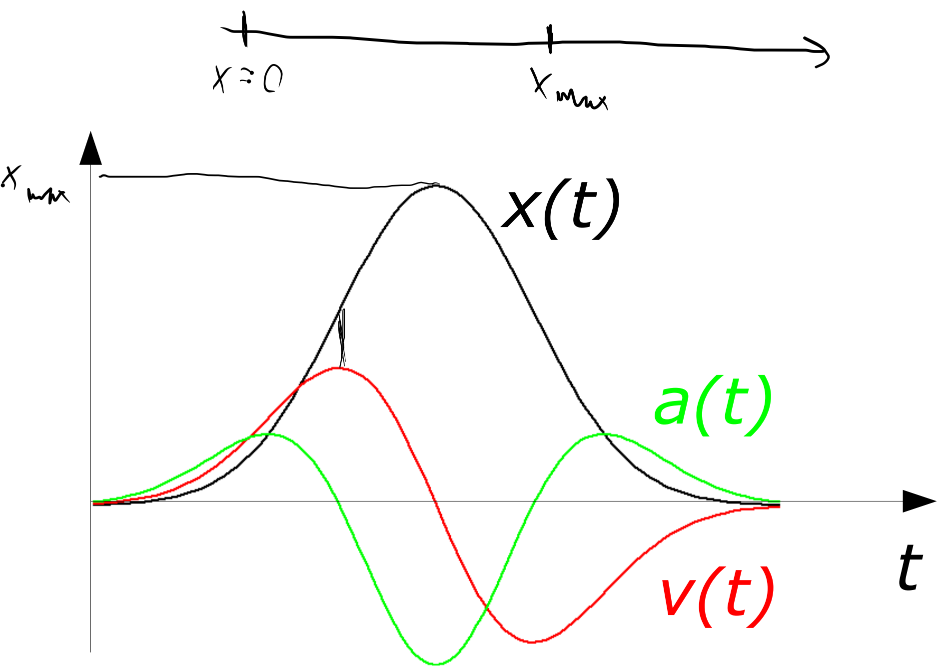
$0.\ \mathrm{m/s^2}$

$-0.2\ \mathrm{m/s^2}$

$-1.\ \mathrm{m/s^2}$

Hastighed og acceleration som differentialer

$$\bar{v}_x = \frac{\Delta x}{\Delta t} \qquad \bar{a}_x = \frac{\Delta v}{\Delta t} \qquad \Delta t \rightarrow 0 \Rightarrow v_x = \frac{dx}{dt} \qquad a_x = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

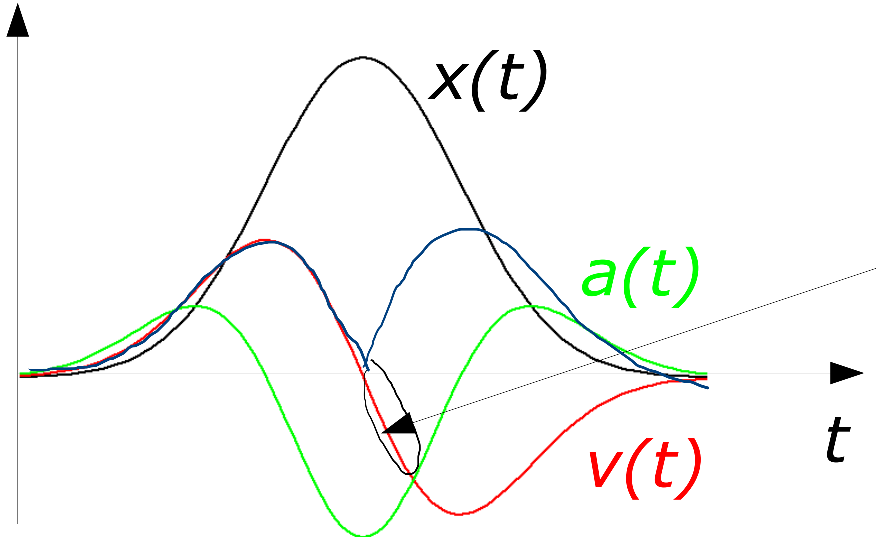


Farten defineres som den absolutte værdi af hastigheden, $|v(t)|$

Hastighed, fart og acceleration - quiz

Hvis accelerationen af et legeme er negativ så aftager legemets fart.

True		10 (66.67 %)
⇨ False		5 (33.33 %)

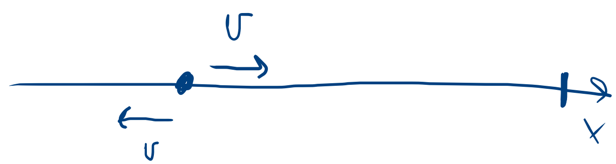


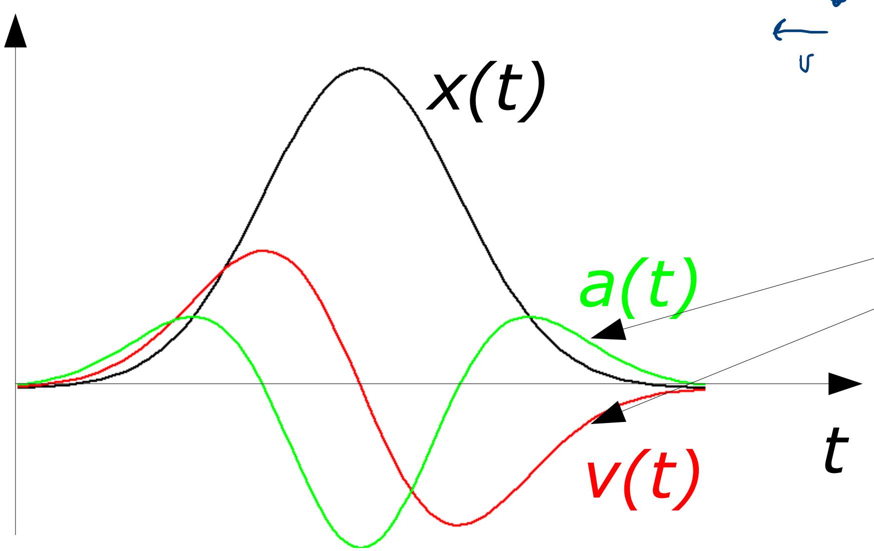
Farten |v(t)| stiger her.

Hastighed, fart og acceleration - quiz

Hvis et legeme accelererer mod et punkt så må legemet bevæge sig tættere på punktet.

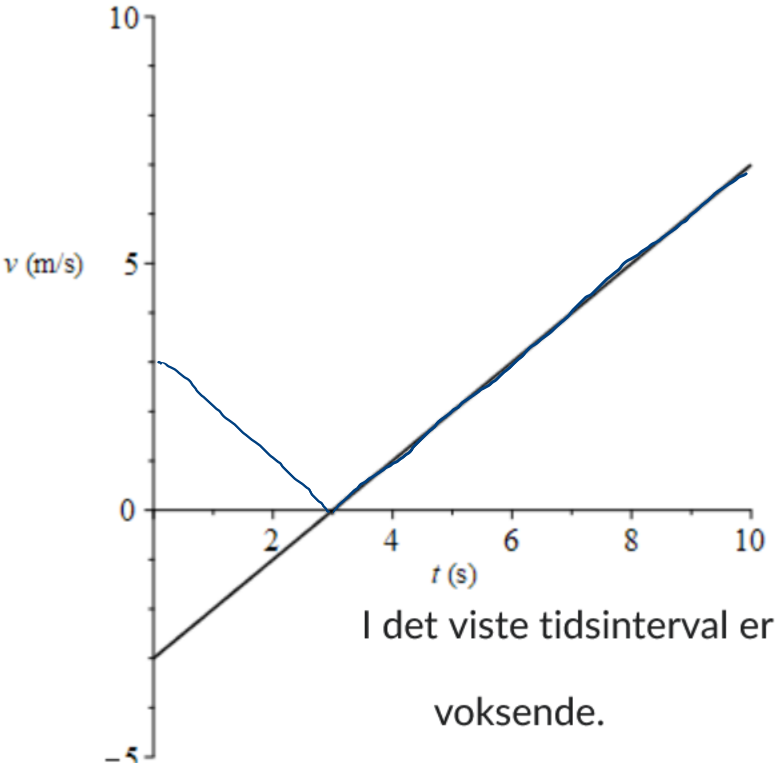
True	13 (86.67 %)
⇨ False	2 (13.33 %)





v og *a* i hver sin retning.

Hastighed, fart og acceleration - quiz

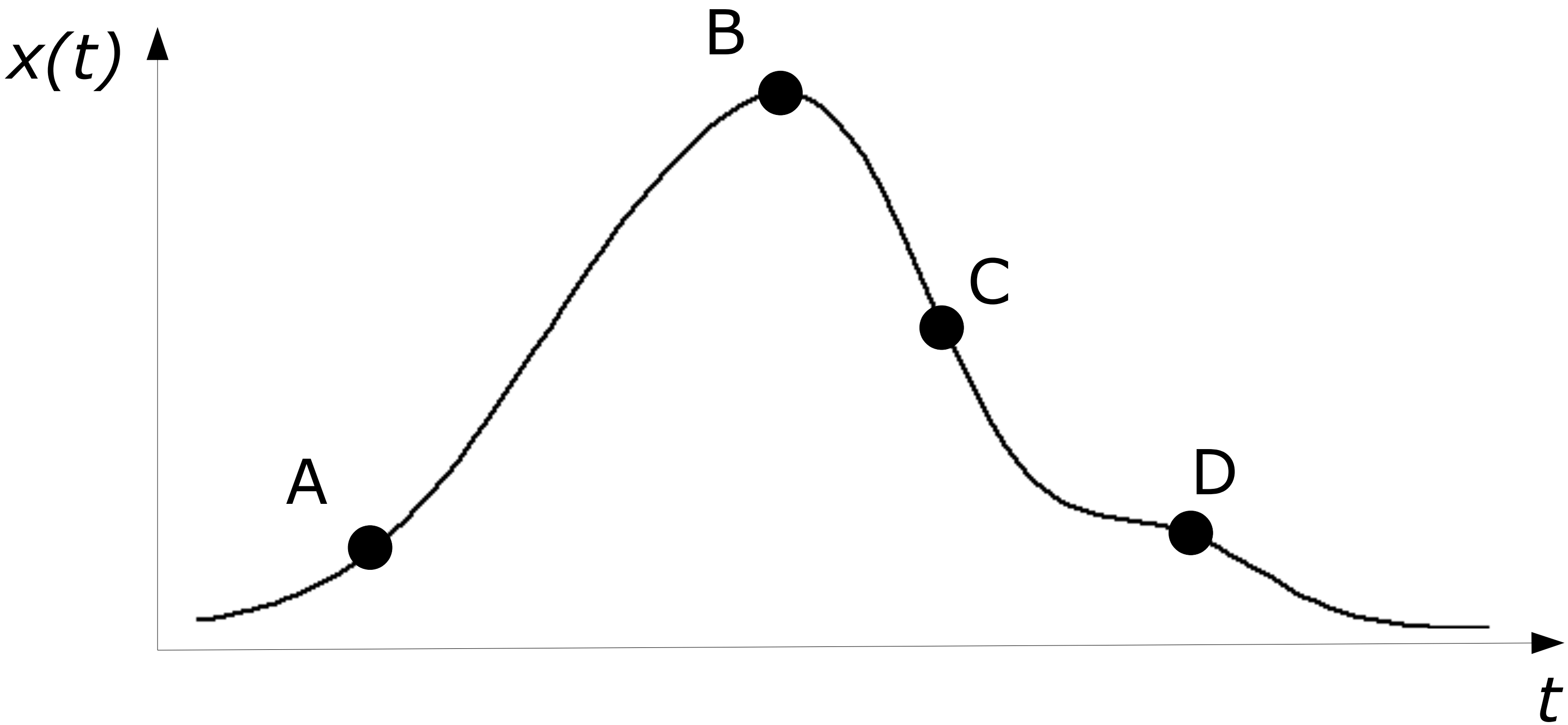


I det viste tidsinterval er farten af bilen

voksende.		10 (66.67 %)
aftagende.		0 (0 %)
først voksende og dernæst aftagende.		0 (0 %)
første aftagende og dernæst voksende.		5 (33.33 %)

Quiz: Stedfunktion

I hvilket af de fire punkter er farten mindst?



Quiz: Stedfunktion

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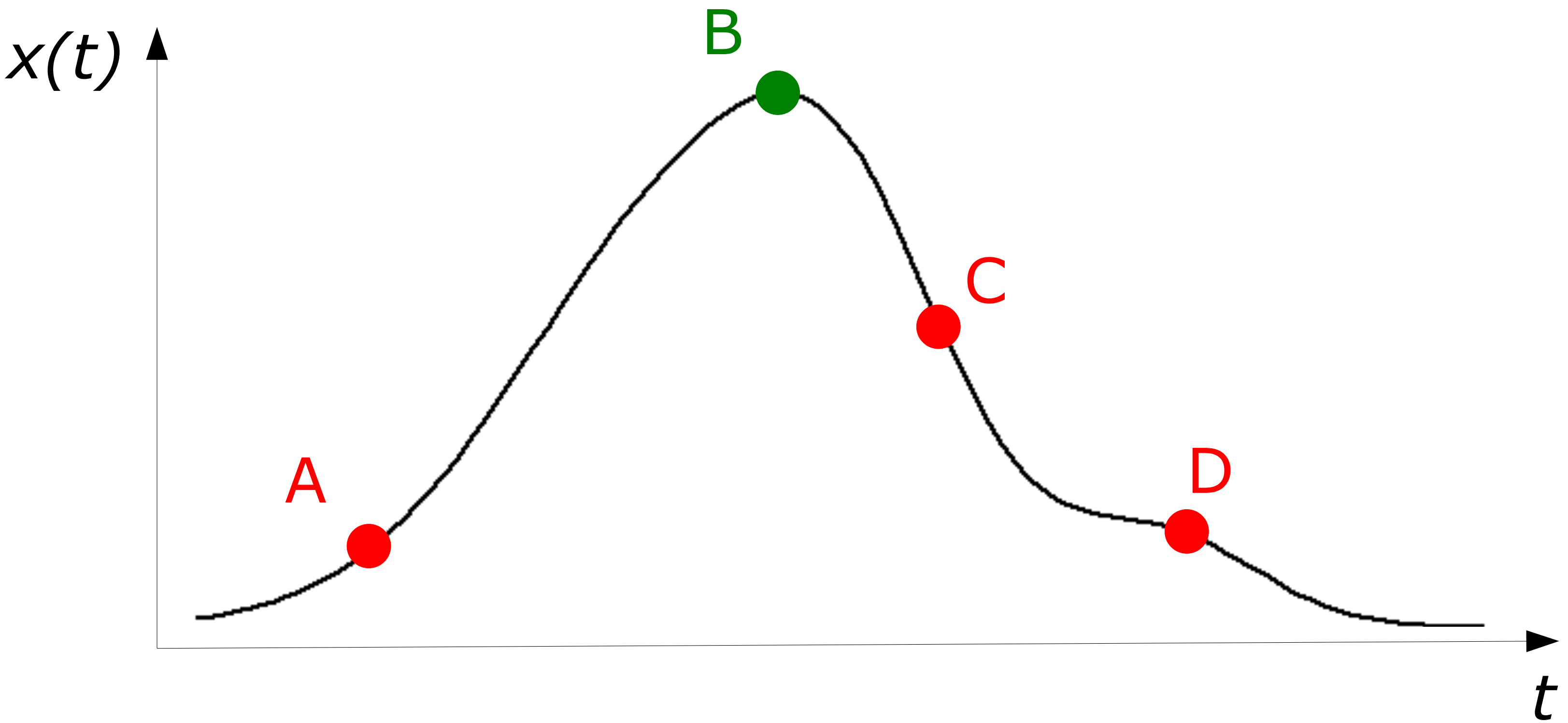
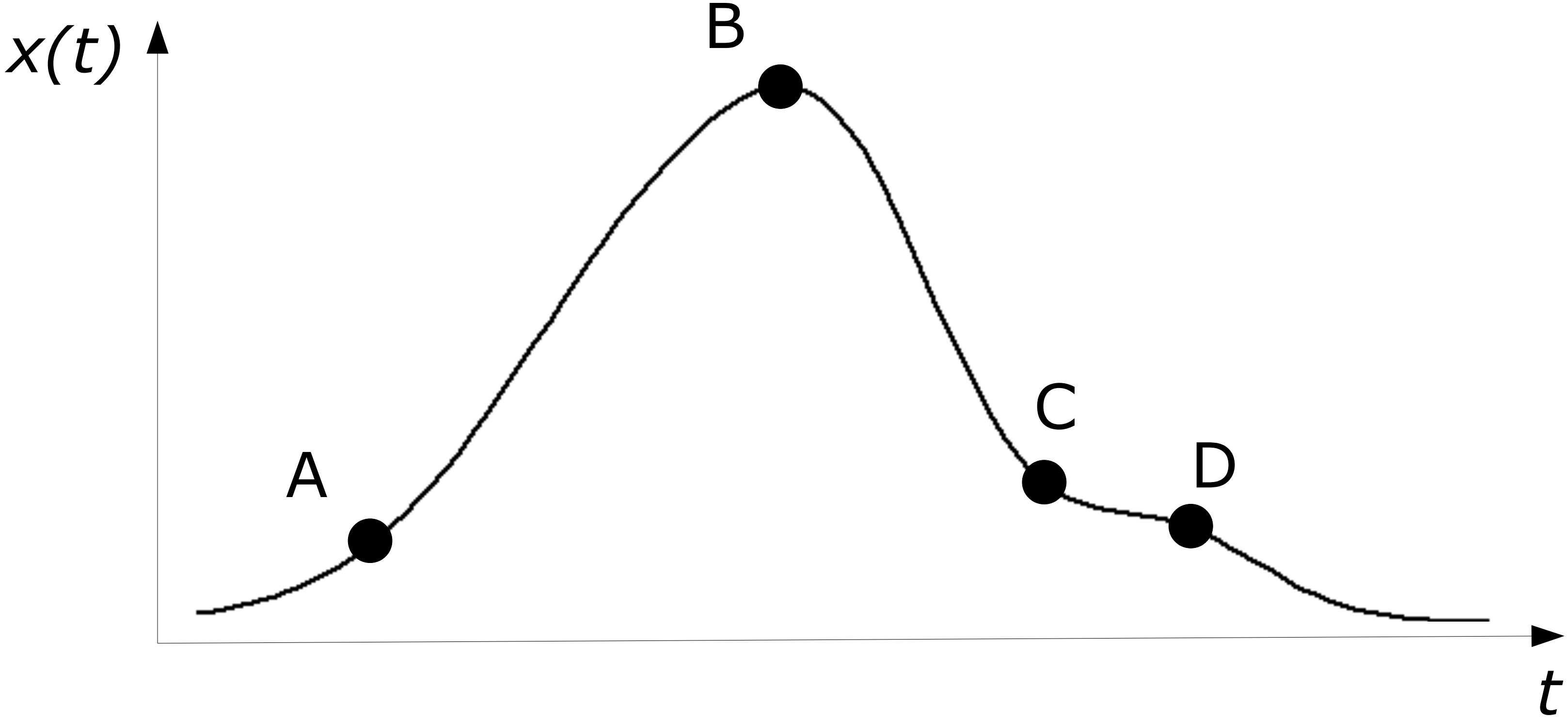


Quiz: Stedfunktion

I hvilke af de fire punkter er accelerationen positiv?



Quiz: Stedfunktion

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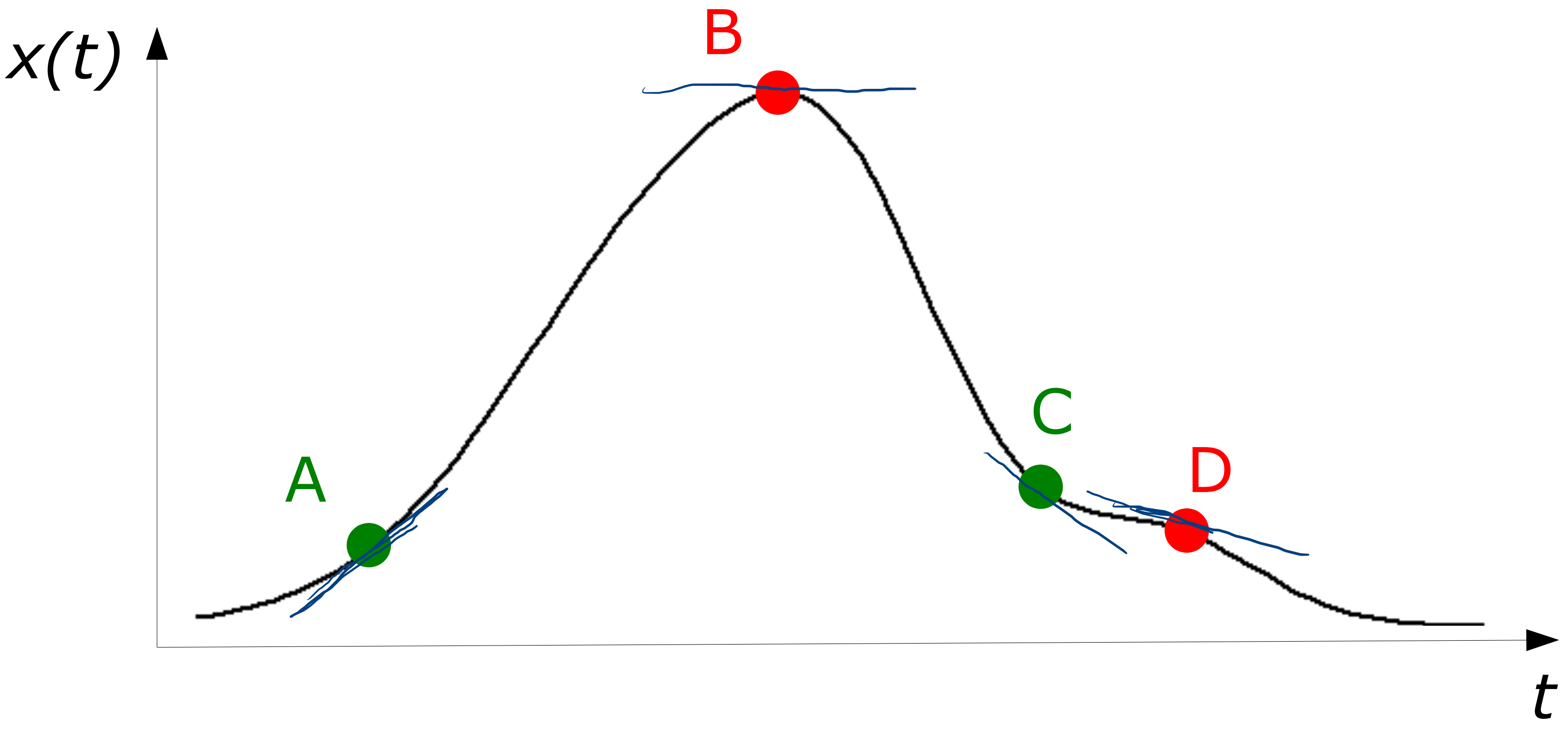


Integralrelationer

Sammenhæng mellem *x, v, a* på integralform:

$$a_x(t) = \frac{dv_x}{dt} \Rightarrow v_x(t) = v_x(0) + \int_0^t a_x(t')\,dt'$$

$$v_x(t) = \frac{dx}{dt} \Rightarrow x(t) = x(0) + \int_0^t v_x(t')\,dt'$$

$a_x(t) = a_x$

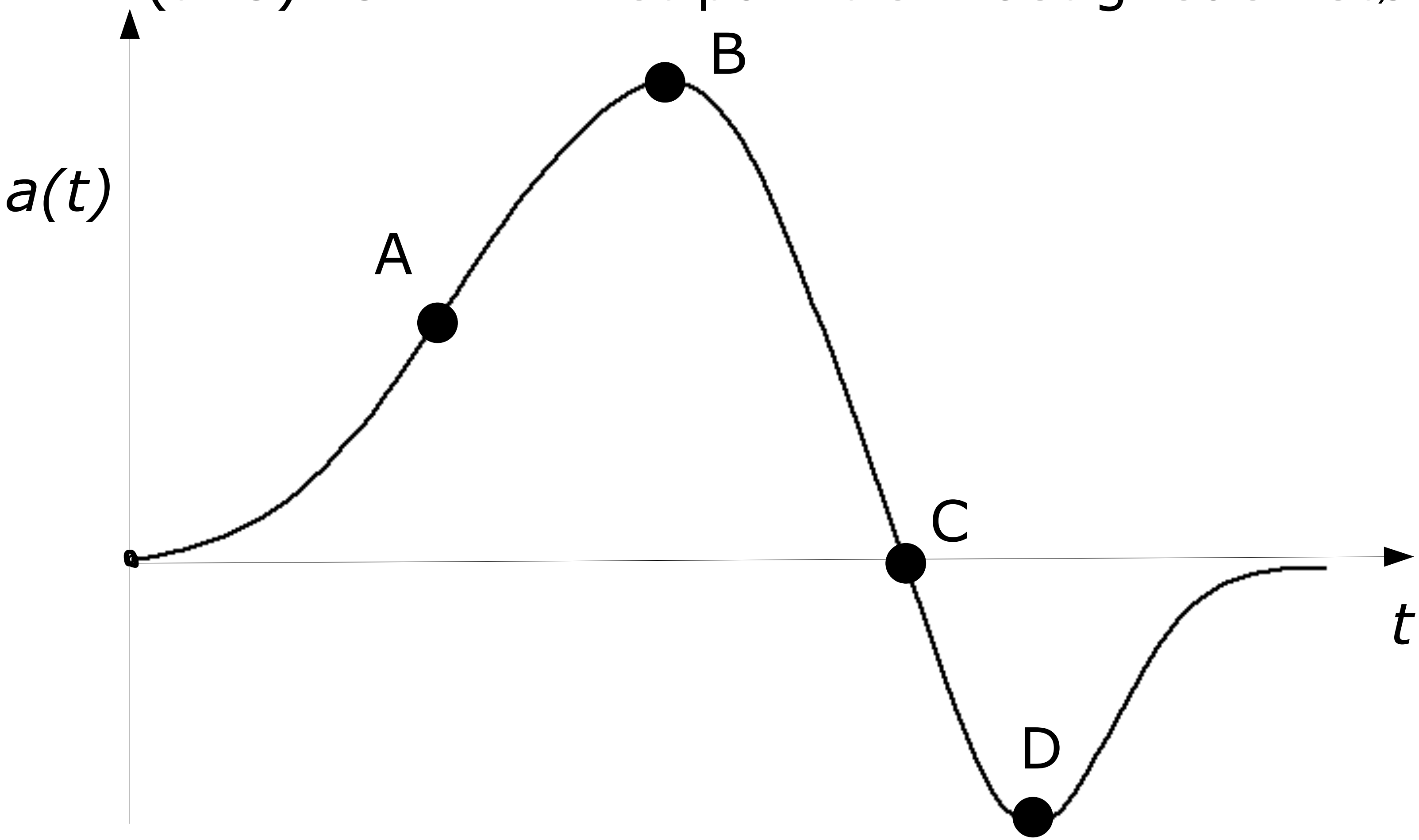
$\Downarrow$

$v_x(t) = v_x(0) + at \Rightarrow x(t) = x(0) + \int_0^t (v_x(0) + a_x t')\,dt' =$

$x(0) + v_x(0)t + \frac{1}{2}[a_x t'^2]_0^t = \underbrace{x(0) + v_x(0)t + \frac{1}{2}a_x t^2}$

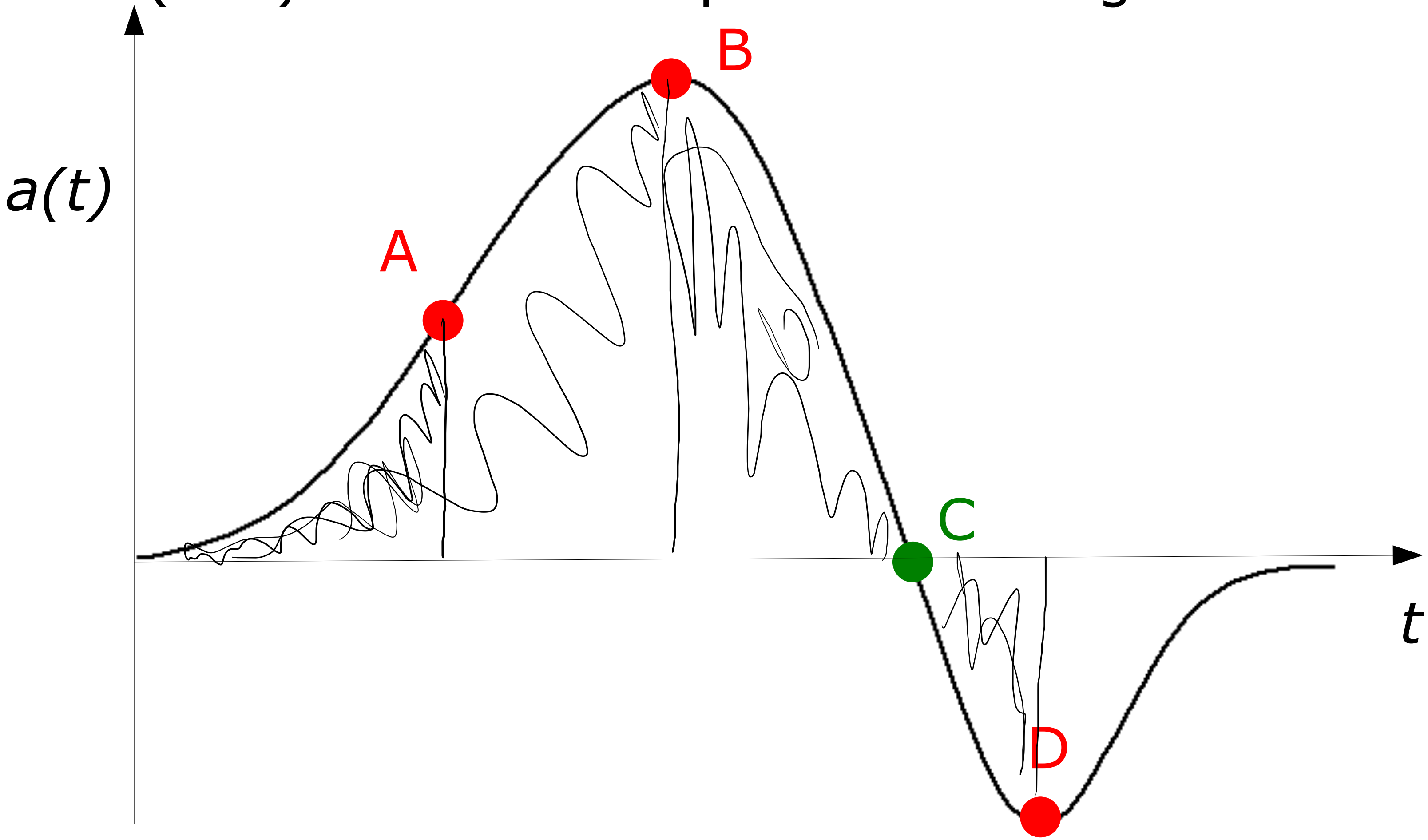
Quiz: Hastighedsfunktion

$v(t=0)=0$. I hvilket punkt er hastigheden størst?



Quiz: Hastighedsfunktion

$v(t\!=\!0)\!=\!0$. I hvilket punkt er hastigheden størst?



Bevægelse med konstant acceleration

Sammenhæng mellem *x*, *v*, *a* på integralform:

$$a_x(t) = \frac{dv_x}{dt} \Rightarrow v_x(t) = v_x(0) + \int_0^t a_x(t')\,dt'$$

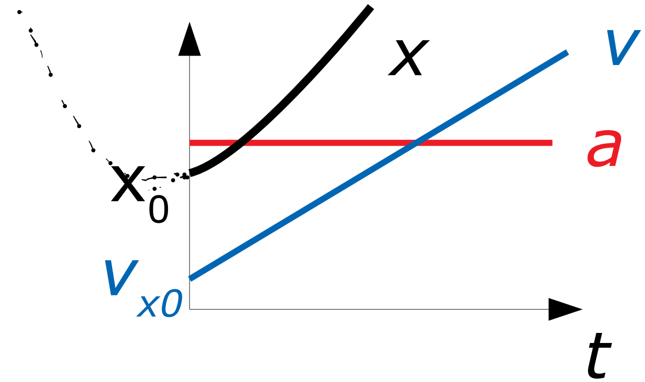
$$v_x(t) = \frac{dx}{dt} \Rightarrow x(t) = x(0) + \int_0^t v_x(t')\,dt'$$

$$a_x(t) = a_x \Rightarrow v_x(t) = v_x(0) + a_x t \qquad x(t) = x(0) + v_x(0)\,t + \frac{1}{2} a_x t^2$$

Endvidere (udledning i Y&F):

$$x(t) = x(0) + \overbrace{\frac{1}{2}\left(v_x(t) + v_x(0)\right)}\,t \qquad v_x^2(t) = v_x^2(0) + 2a_x\left(x(t) - x(0)\right)$$

Bevægelse med konstant acceleration



$$x_0 = x(0) \qquad v_{x0} = v_x(0)$$

$$\boxed{\begin{aligned} v_x(t) &= v_{x0} + a_x t \\ x(t) &= x_0 + v_{x0} t + \frac{1}{2} a_x t^2 \\ x(t) &= x_0 + \frac{1}{2}\left(v_x(t) + v_{x0}\right) t \\ v_x^2(x) &= v_{x0}^2 + 2 a_x (x - x_0) \end{aligned}}$$

Uden x

Uden v(t)

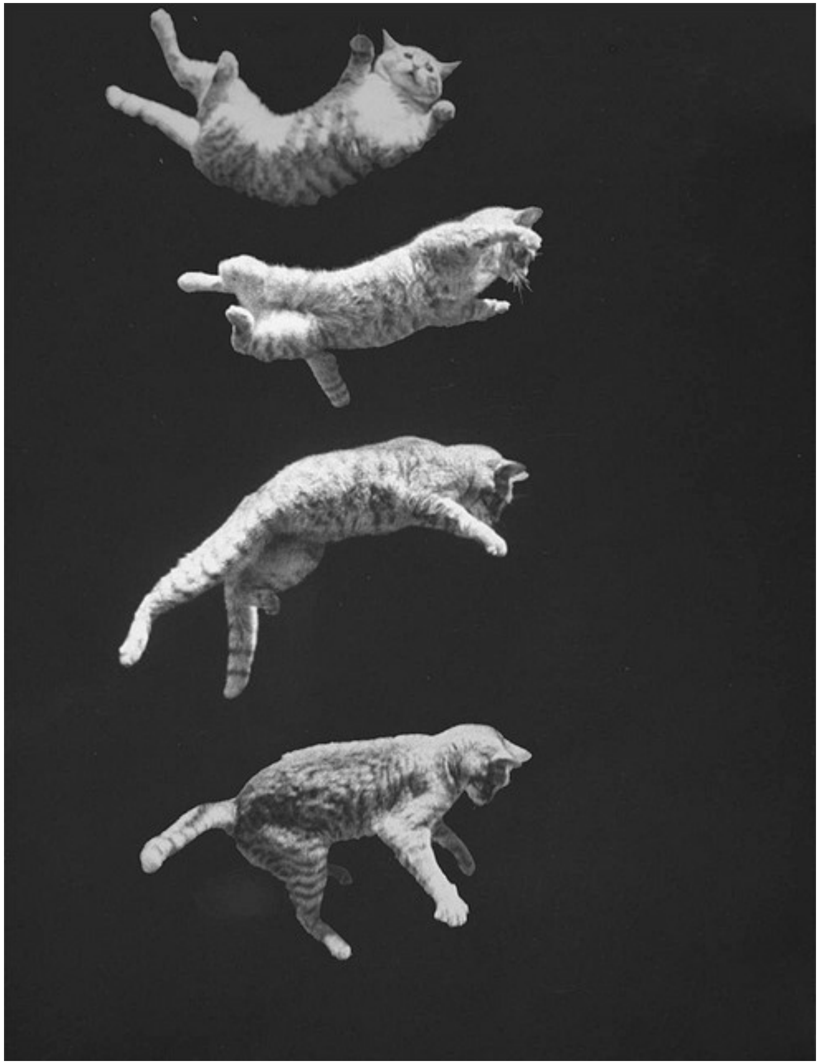
Uden a

Uden t

Eksempel: Frit fald

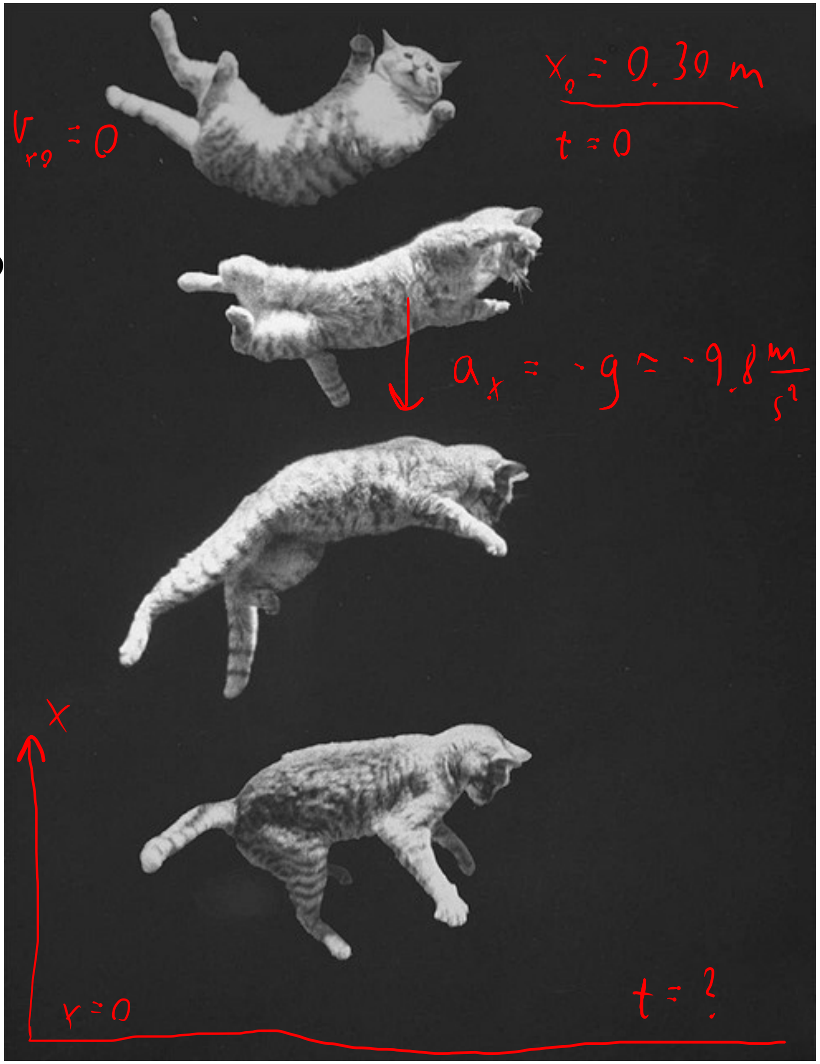
Et legeme i frit fald nær jordoverfladen har konstant nedadrettet acceleration af størrelsen g=9.8 m/s^2

En kat kan lande på fødderne fra en minimumshøjde af ca. 30 cm. Hvor hurtigt kan katten dreje sig?



Eksempel: Frit fald

En kat kan lande på fødderne fra en minimumshøjde af 30. cm. Hvor hurtigt kan katten dreje sig?



$$v_x(t) = v_{x0} + a_x t$$ *Uden x*

$$x(t) = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$ *Uden v(t)*

$$x(t) = x_0 + \frac{1}{2}\left(v_x(t) + v_{x0}\right) t$$ *Uden a*

$$v_x^2(x) = v_{x0}^2 + 2 a_x (x - x_0)$$ *Uden t*

Eksempel: Frit fald

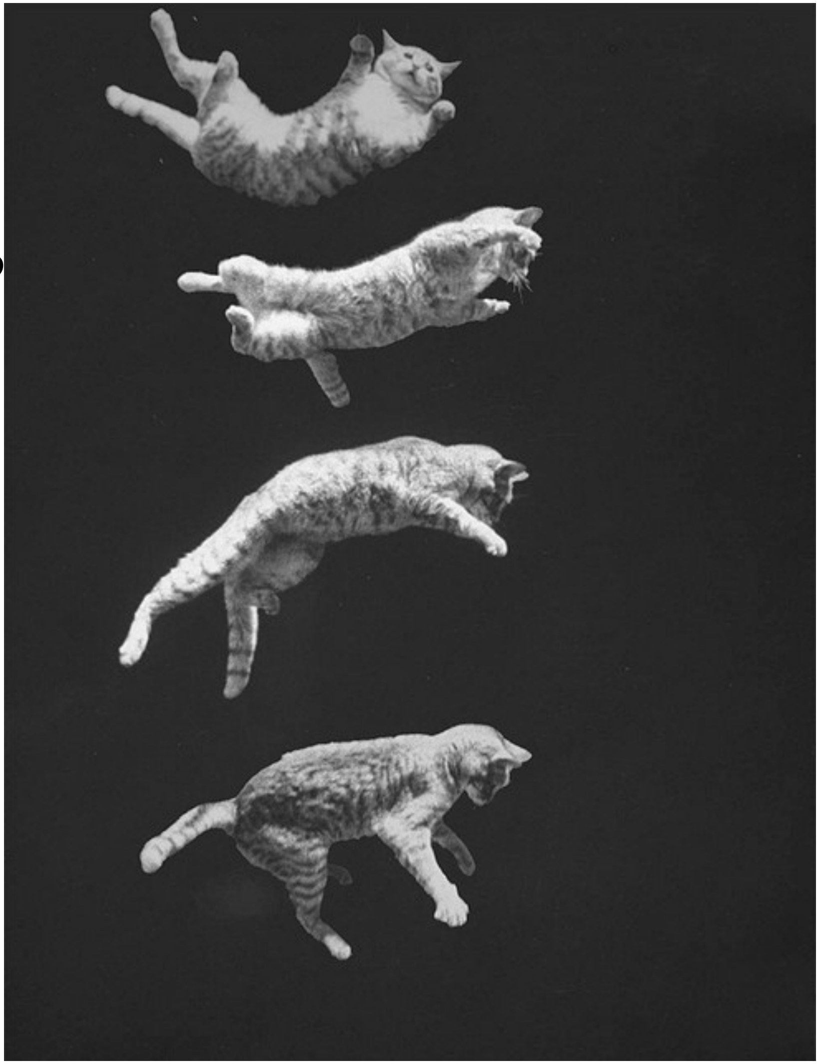
En kat kan lande på fødderne fra en minimumshøjde af 30. cm. Hvor hurtigt kan katten dreje sig?

$$x(t) = x_0 + \cancel{v_{x0}}t + \frac{1}{2}a_x t^2 =$$

$$x_0 - \frac{1}{2}gt^2 = 0 \Rightarrow x_0 = \frac{1}{2}gt^2 \Rightarrow t^2 = \frac{2x_0}{g} \Rightarrow$$

$$t = \sqrt{\frac{2x_0}{g}} \simeq 0.25\,s$$

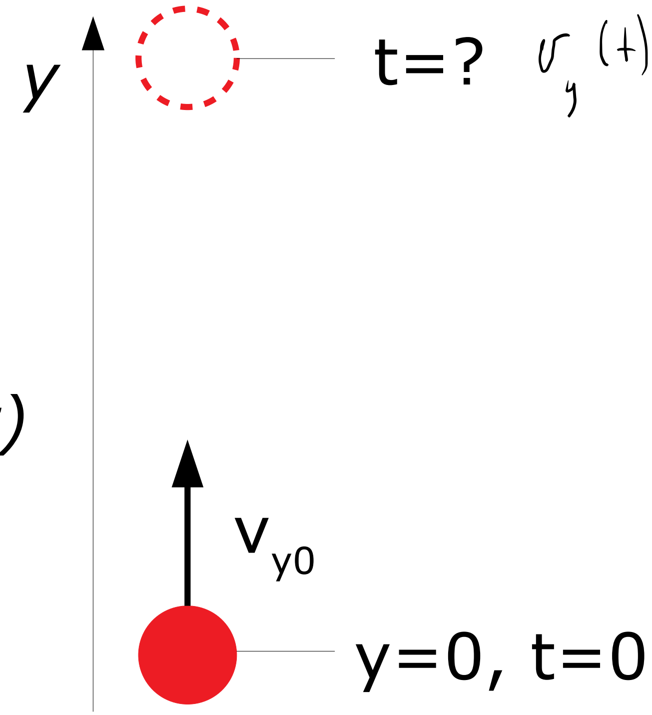
$$\sqrt{\frac{\cancel{m}}{\cancel{m}/s^2}} \simeq \sqrt{s^2} \sim s$$



Eksempel: Lodret kast

Hvor lang tid er bolden om at nå max. højde?

$a_y = -g$



$v_y(t) = 0$

$$v_y(t) = v_{y0} + a_y t$$ *Uden y*

$$y(t) = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$$ *Uden v(t)*

$$y(t) = y_0 + \frac{1}{2}\left(v_y(t) + v_{y0}\right) t$$ *Uden a*

$$v_y^2(y) = v_{y0}^2 + 2 a_y (y - y_0)$$ *Uden t*

Eksempel: Lodret kast

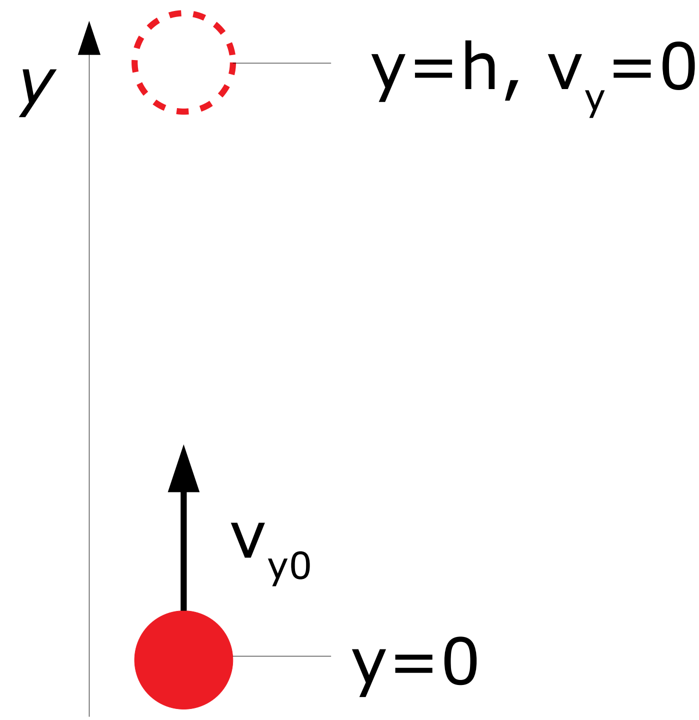
Hvor lang tid er bolden om at nå max. højde?

$$v_y(t) = v_{y0} + a_y t = v_{y0} - gt = 0$$

$$\Downarrow$$

$$v_{y0} = gt \Rightarrow t = \frac{v_{y0}}{g}$$

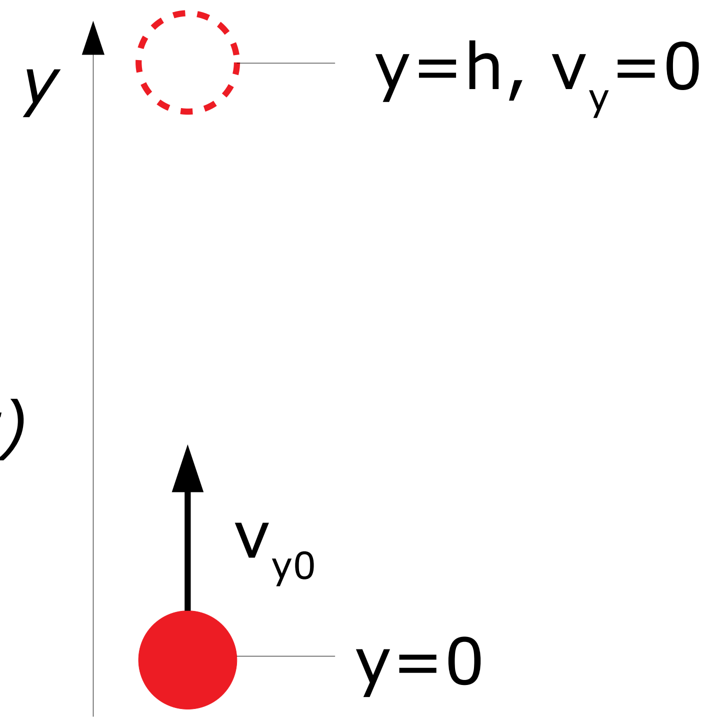
$$\frac{v_{y0}}{g} \sim \frac{\cancel{m}/\cancel{s}}{\cancel{m}/s^{\cancel{2}}} \sim \frac{1}{1/s} \sim s$$



Eksempel: Lodret kast

Hvor højt kommer bolden op?

$v_y(t) = v_{y0} + a_y t$	*Uden y*
$y(t) = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$	*Uden v(t)*
$y(t) = y_0 + \frac{1}{2}\left(v_y(t) + v_{y0}\right) t$	*Uden a*
$v_y^2(y) = v_{y0}^2 + 2 a_y (y - y_0)$	*Uden t*



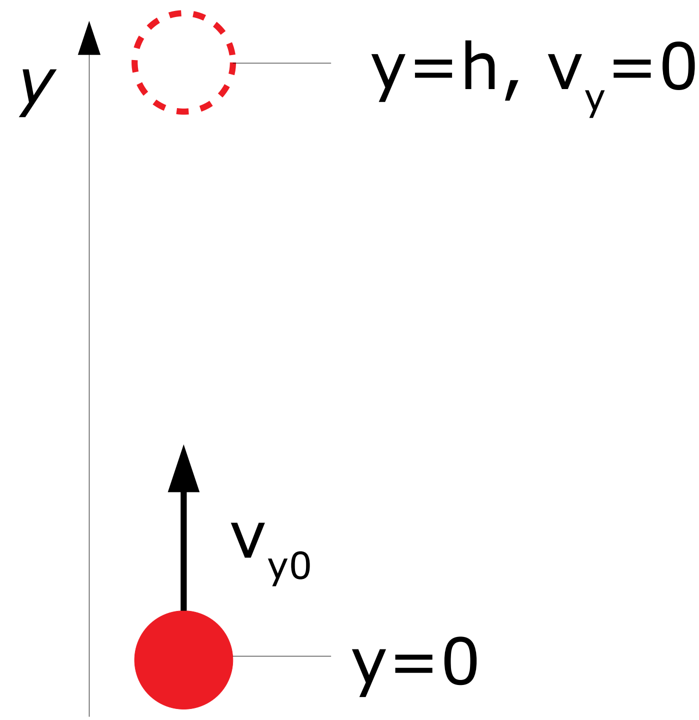
Eksempel: Lodret kast

Hvor højt kommer bolden op?

$$v_{y}^2(h) = v_{y0}^2 + 2a_y(y - \cancel{y_0}) =$$

$$v_{y0}^2 - 2gh = 0 \Rightarrow v_{y0}^2 = 2gh \Rightarrow h = \frac{v_{y0}^2}{2g}$$

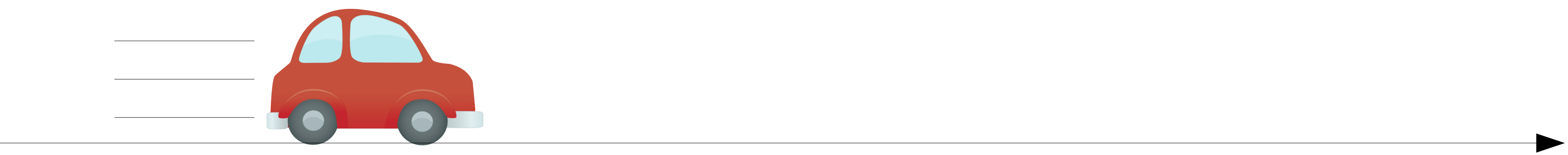
$$\frac{v_{y0}^2}{2g} \sim \frac{(m/\cancel{s})^2}{m/\cancel{s^2}} \sim m$$



Eksempel: Acceleration

Bilen accelererer 0-100 km/t på 6.0 s

Hvis konstant acceleration, hvor langt kommer den?

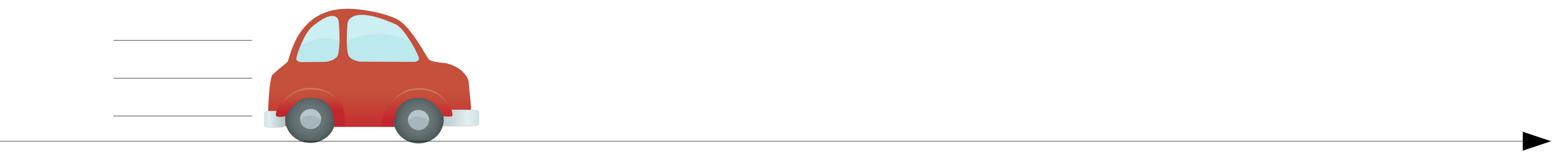


$v_x(t) = v_{x0} + a_x t$	*Uden x*
$x(t) = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	*Uden v(t)*
$x(t) = x_0 + \frac{1}{2}\left(v_x(t) + v_{x0}\right) t$	*Uden a*
$v_x^2(x) = v_{x0}^2 + 2 a_x (x - x_0)$	*Uden t*

Eksempel: Acceleration

Bilen accelererer 0-100 km/t på 6.0 s

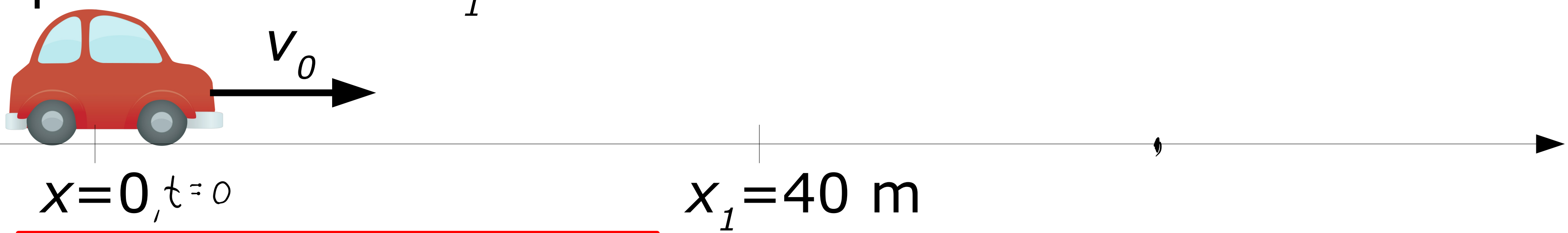
Hvis konstant acceleration, hvor langt kommer den?



$$x(t) - x_0 = \frac{1}{2}\left(v_{x0} + v_x(t)\right)t = \frac{1}{2} v_x(t)\, t \simeq 83.\ \mathrm{m}$$

Eksempel: Nedbremsning

v_0=30 m/s, a=-9. m/s² fra x=0, t=0. Til hvilken tid passerer bilen x_1=40 m?



$$v_x(t)=v_{x0}+a_x t$$

$$x(t)=x_0+v_{x0}t+\frac{1}{2}a_x t^2$$

$$x(t)=x_0+\frac{1}{2}\left(v_x(t)+v_{x0}\right)t$$

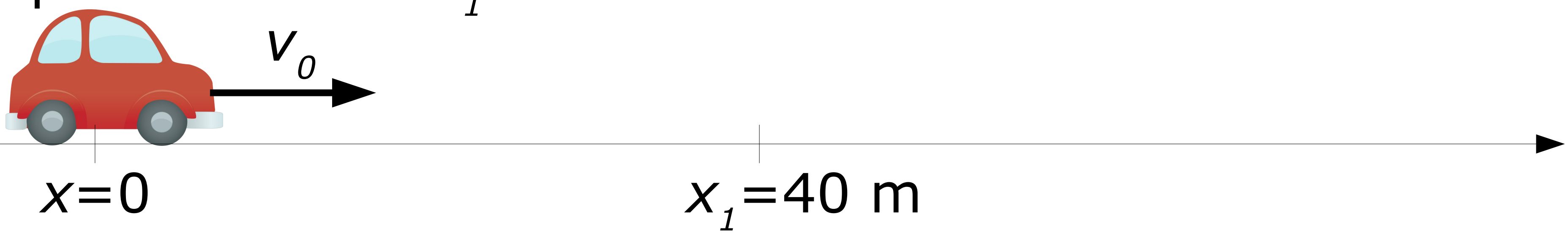
$$v_x^2(x)=v_{x0}^2+2a_x(x-x_0)$$

$$x_1 = v_0 t + \frac{1}{2} a t^2 \Rightarrow \frac{1}{2} a t^2 + v_0 t - x_1 = 0$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 + 2 a x_1}}{a} \simeq \begin{cases} 1.8\ \mathrm{s} \\ 4.8\ \mathrm{s} \end{cases}$$

Eksempel: Nedbremsning

v_0=30 m/s, a=-9. m/s^2 fra x=0, t=0. Til hvilken tid passerer bilen x_1=40 m?



Opsummering

Gennemsnitlig hastighed, acceleration

$$\bar{v}_x = \frac{\Delta x}{\Delta t} \qquad \bar{a}_x = \frac{\Delta v}{\Delta t}$$

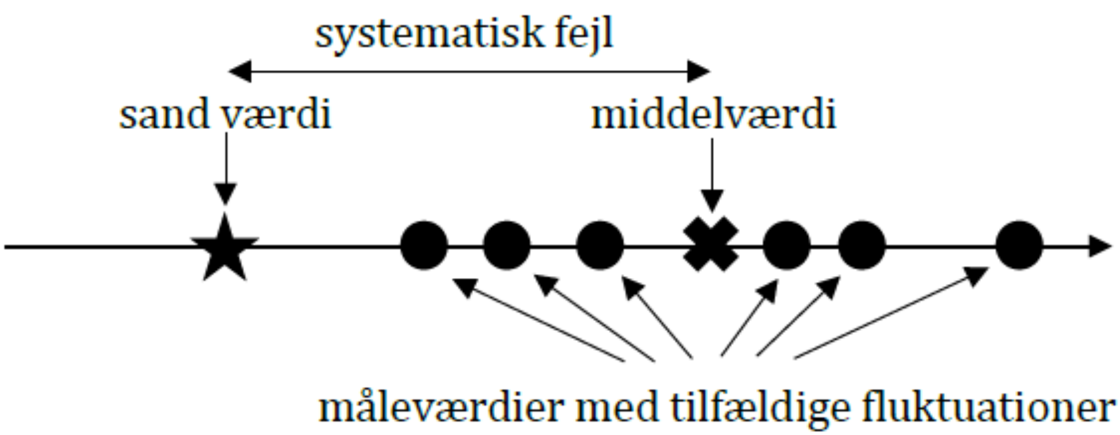
Øjeblikkelig hastighed, acceleration

$$v_x = \frac{dx}{dt} \qquad a_x = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

Ligninger for lineær bevægelse med konstant acceleration

$$\boxed{\begin{aligned} v(t) &= v_0 + at \\ x(t) &= x_0 + v_0 t + \frac{1}{2} a t^2 \\ x(t) &= x_0 + \frac{1}{2}\left(v(t) + v_0\right) t \\ v^2(x) &= v_0^2 + 2a\left(x - x_0\right) \end{aligned}}$$

Usikkerheder og fejlophobning



Fra sidste gang

Hastighed og acceleration fra stedfunktion *x(t)*

$$v_x = \frac{dx}{dt} \qquad a_x = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Farten er |*v(t)*|

Ligninger for lineær bevægelse med konstant acceleration - f.eks. frit fald med a_y=-*g*

$$\boxed{\begin{aligned} v(t) &= v_0 + at \\ x(t) &= x_0 + v_0 t + \frac{1}{2}at^2 \\ x(t) &= x_0 + \frac{1}{2}\left(v(t) + v_0\right)t \\ v^2(x) &= v_0^2 + 2a(x - x_0) \end{aligned}}$$

Denne uges læringsmål

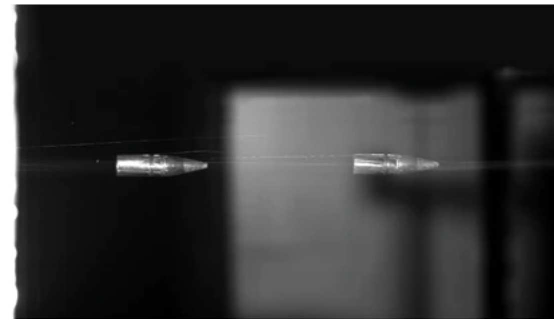
-Forstå begreberne *tilfældig* og *systematisk* usikkerhed

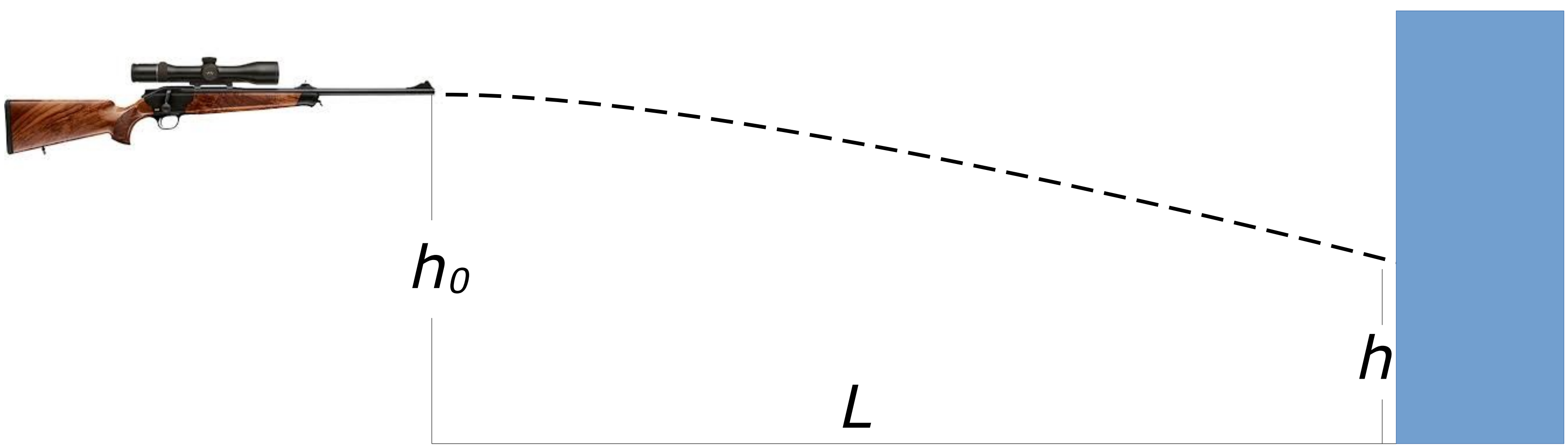
-Forstå og benytte *fejlophobningsloven* for *uafhængige* og *afhængige* usikkerheder.

-Forstå betydningen af *standardafvigelse*

-Lineært og kvadratisk fit (regression)

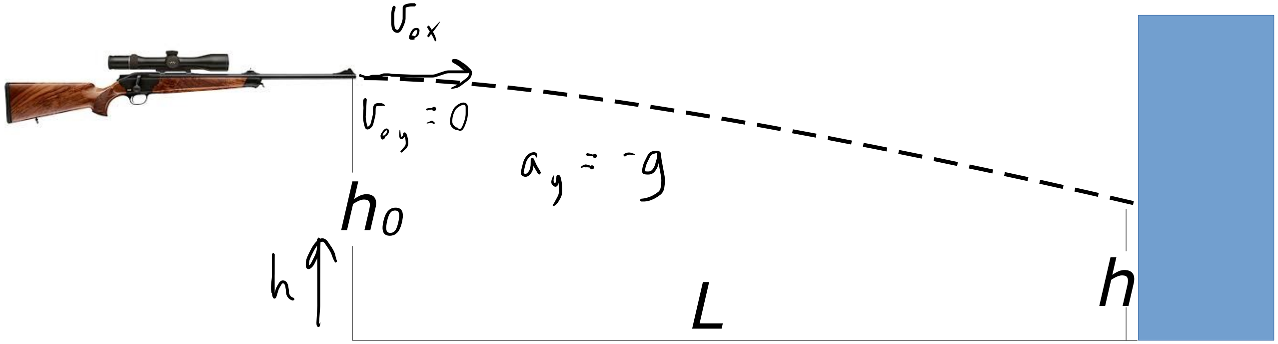
Eksempel: Måling af en stor hastighed





Eksempel: Måling af en stor hastighed

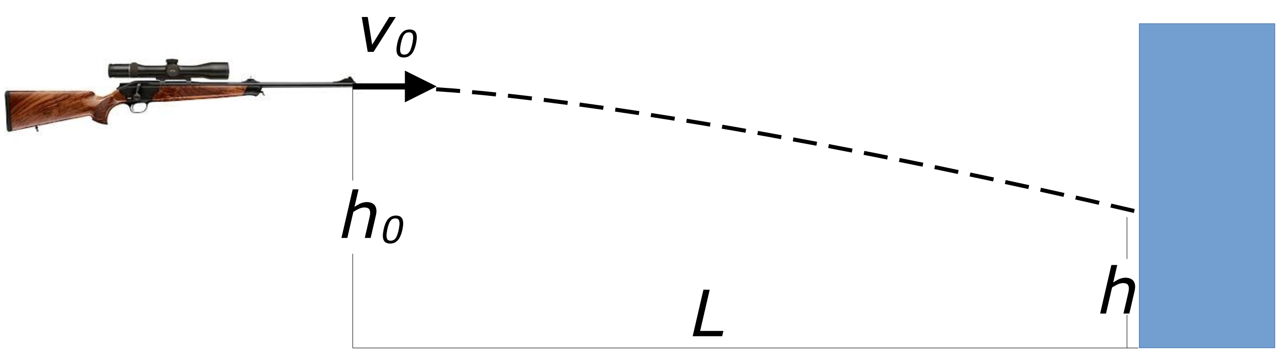
$$L = v_0 t \Rightarrow v_0 = \frac{L}{t} \quad (1)$$



$$h(t) = h_0 + \cancel{v_{0y}}\, t + \frac{1}{2} a_y t^2 = h_0 - \frac{1}{2} g t^2 = h \Rightarrow h_0 - h = \frac{1}{2} g t^2 \Rightarrow$$

$$\frac{2(h_0 - h)}{g} = t^2 \Rightarrow t = \sqrt{\frac{2(h_0 - h)}{g}} \overset{(1)}{\Rightarrow} v_0 = \frac{L}{\sqrt{\frac{2(h_0 - h)}{g}}} = L\sqrt{\frac{g}{2(h_0 - h)}}$$

Eksempel: Måling af en stor hastighed



$$\boxed{v_0 = L\sqrt{\frac{g}{2(h_0-h)}}}$$

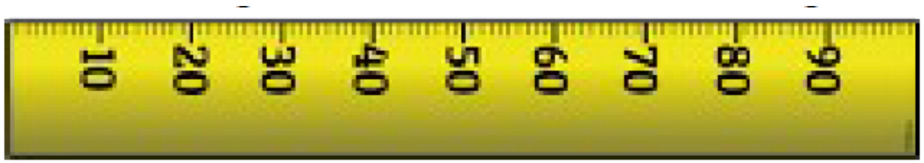
$L = L \pm \delta L$ m $h_0 = h_0 \pm \delta h_0$ m $h = h \pm \delta h$ m $g = 9.81$ m/s^2

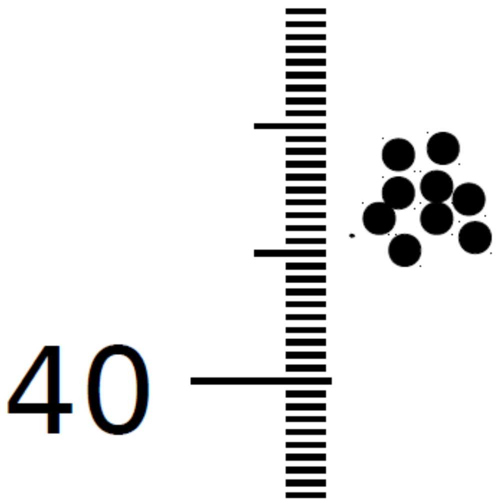
$$\downarrow$$

$v_0 = v_0 \pm \delta v_0$ m/s

Længdemåling

Tilfældig usikkerhed: Aflæsning, tilfældige variationer i v_0





n målinger:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\delta x_t = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}$$

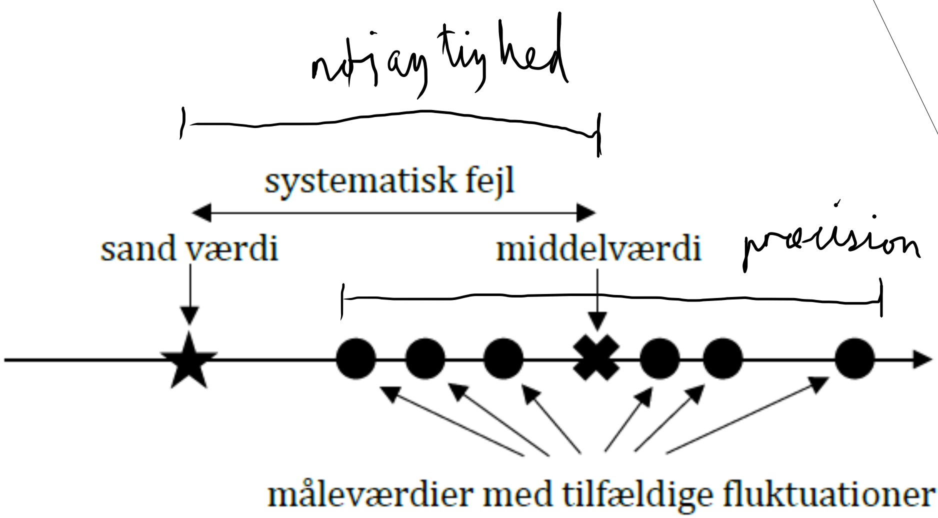
$$\delta \bar{x} = \frac{\delta x_t}{\sqrt{n}}$$

Eks. $h = 0.414 \pm 0.003$ m

Længdemåling

$l = 1.000 \pm 0.005$ m

Tilfældig og systematisk usikkerhed:



$$\delta h = \sqrt{(\delta h_t)^2 + (\delta h_s)^2}$$

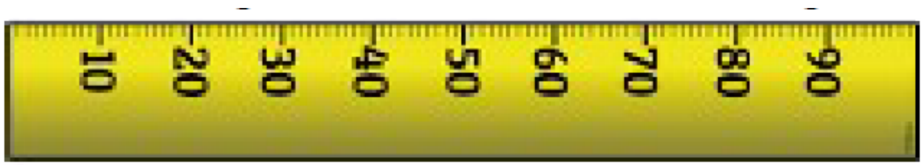
Tilfældig Systematisk

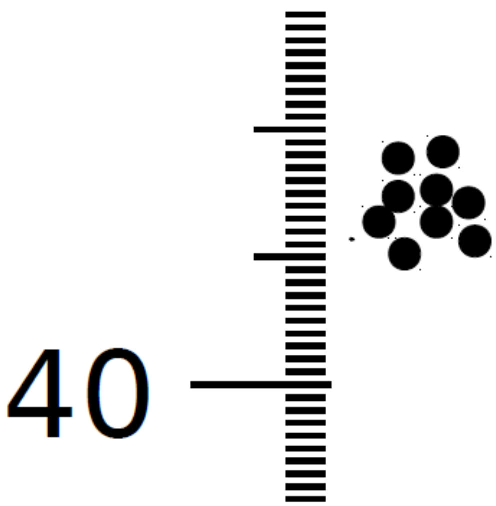
$$\delta h_s = h \cdot 0.005$$

Relativ usikkerhed

Lændgemåling

$l = 1.000 \pm 0.005$ m





Samlet usikkerhed på *h*:

$$\delta h = \sqrt{(\delta h_t)^2 + (\delta h_s)^2}$$

$$\delta h = \sqrt{(0.003\, m)^2 + (0.002\, m)^2} \approx 0.004\, m$$

$h = 0.414 \pm 0.004$ m

Lændgemåling kalibrering

l=1.003±0.00012 m

Egen lineal

Mere nøjagtig
lånt lineal

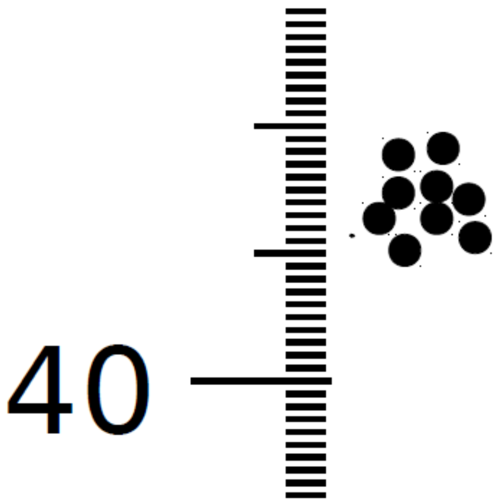
l=1.0000±0.00012 m

To cifre hvis 1. ciffer er 1 eller 2.

Lændgemåling

$l = 1.003 \pm 0.00012$ m

$h \to h \cdot 1.003$



Samlet usikkerhed på h:

$$\delta h = \sqrt{(\delta h_t)^2 + (\delta h_s)^2}$$

$$\delta h = \sqrt{(0.003\,m)^2 + (0.00005\,m)^2} \approx 0.003\,m$$

$h = 0.415 \pm 0.003$ m

Digression - kvadratsummer

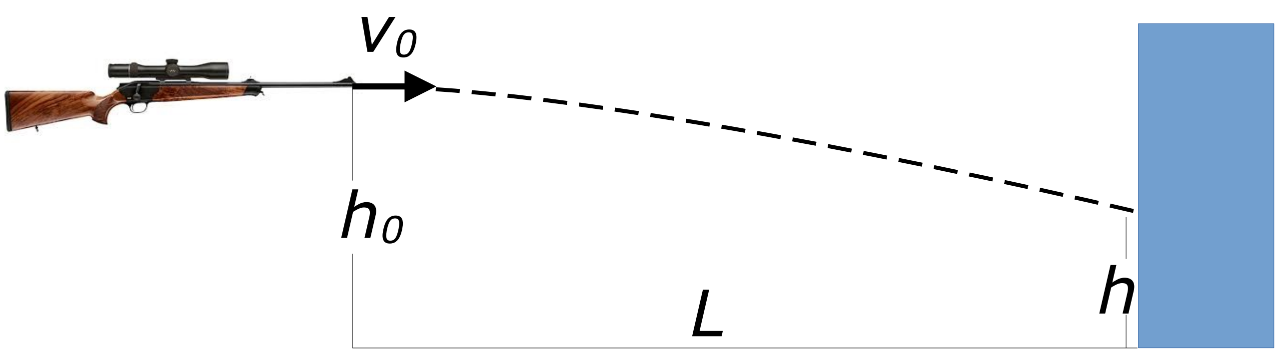
- Hvis de uafhængige, relative usikkerheder af tre størrelser i et produkt er 3%, 6% og 9% henholdsvis, kan vi ignorere usikkerheden på de 3%.

0 (0 %)

$$\delta x = \overset{0.01}{}\sqrt{9^2 + 6^2 + 3^2} \simeq 0.11\cancel{2}$$

$$\delta x = 0.01\sqrt{9^2 + 6^2} \simeq 0.\overset{11}{\cancel{108}}$$

Eksempel: Måling af en stor hastighed



$$v_0 = L\sqrt{\frac{g}{2(h_0 - h)}}$$

L=100.00±0.012 m h_0=1.0000±0.0005 m
h=0.415±0.003 m g=9.81 m/s^2

$$v_0 = v_0 \pm \delta v_0 \text{ m/s ??}$$

Fejlophobningsloven

$$y=f\left(x_1,x_2,..,x_N\right)$$

Målte

Eksempel: $v_0=L\sqrt{\dfrac{g}{2\left(h_0-h\right)}}$

$$\delta y=\sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2\delta x_1^2+..+\left(\frac{\partial f}{\partial x_N}\right)^2\delta x_N^2}$$

δx_i uafhængige

$$\delta y=\left|\frac{\partial f}{\partial x_1}\delta x_1\right|+..+\left|\frac{\partial f}{\partial x_N}\delta x_N\right|$$

δx_i afhængige

Quiz – hvad for usikkerheder har vi?

Hvilke (hvis nogen) af vore usikkerheder er afhængige?

A: Tilfældig og systematisk usikkerhed på h_0

B: Tilfældig usikkerhed på h og h_0

C: Systematiske usikkerheder på L, h, h_0

D: Tilfældig usikkerhed på h_0 og L

Quiz – hvad for usikkerheder har vi?

Hvilke (hvis nogen) af vore usikkerheder er afhængige?

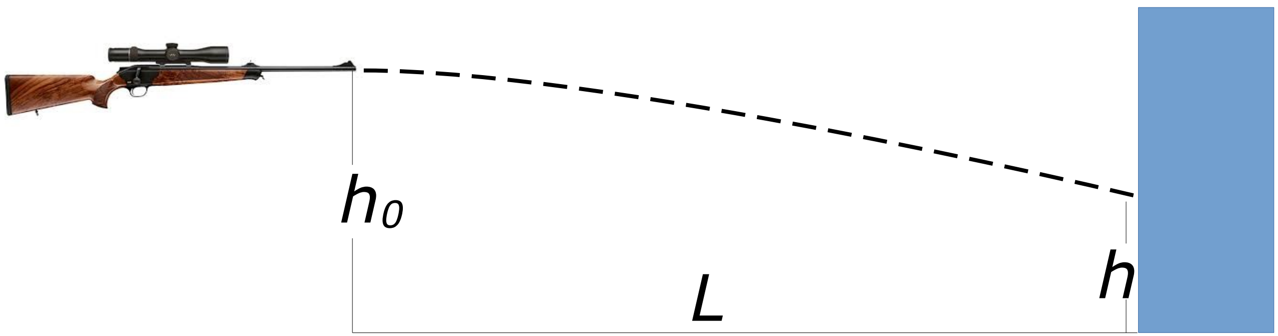
A: Tilfældig og systematisk usikkerhed på h_0

B: Tilfældig usikkerhed på h og h_0

C: Systematiske usikkerheder på L, h, h_0

D: Tilfældig usikkerhed på h_0 og L

Eksempel: Måling af en stor hastighed



$$v_0 = L\sqrt{\frac{g}{2(h_0 - h)}}$$

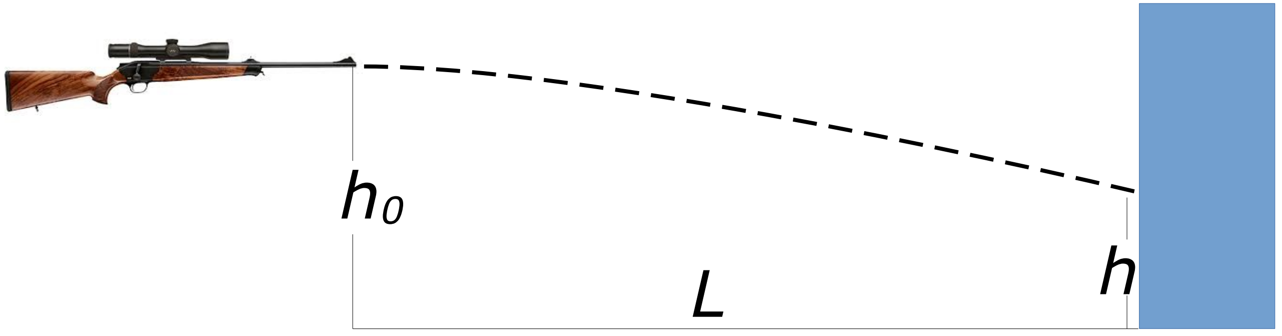
$$\frac{\partial v_0}{\partial L} = \sqrt{\frac{g}{2(h_0-h)}} = \frac{v_0}{L} \qquad \frac{\partial v_0}{\partial h_0} = L\sqrt{\frac{g}{2}}\frac{\partial}{\partial h_0}(h_0-h)^{-1/2} = L\sqrt{\frac{g}{2}}\left(-\frac{1}{2}\right)(h_0-h)^{-3/2} =$$

$$L\sqrt{\frac{g}{2}}\frac{1}{\sqrt{h_0-h}}\;\frac{-\frac{1}{2}}{h_0-h} = -\frac{v_0}{2(h_0-h)}\;; \qquad \frac{\partial v_0}{\partial h} = -\frac{\partial v_0}{\partial h_0} = \frac{v_0}{2(h_0-h)}$$

$$\delta v_0 = \sqrt{\left(\frac{\partial v_0}{\partial L}\right)^2\delta L^2 + \left(\frac{\partial v_0}{\partial h_0}\right)^2\left(\delta h_0^2 + \delta h^2\right)} = \sqrt{\left(\frac{v_0}{L}\right)^2\delta L^2 + \frac{v_0^2}{4(h_0-h)^2}\left(\delta h_0^2 + \delta h^2\right)} \Rightarrow$$

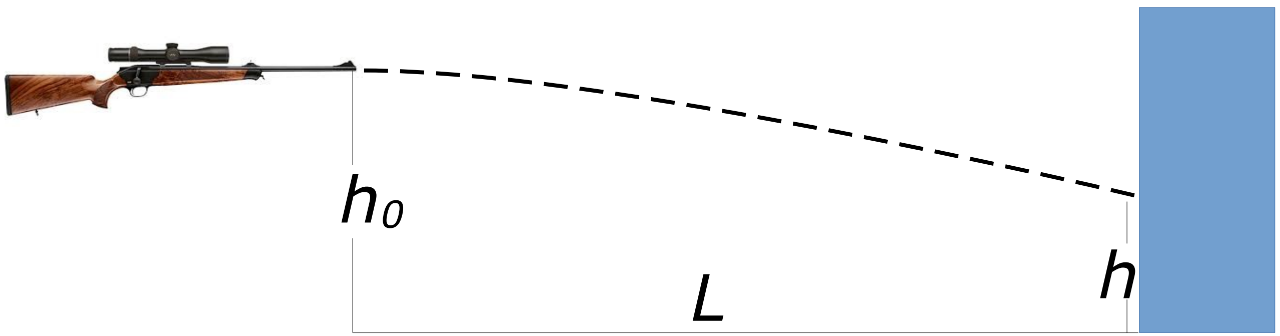
$$\frac{\delta v_0}{v_0} = \sqrt{\left(\frac{\delta L}{L}\right)^2 + \frac{\delta h_0^2 + \delta h^2}{4(h_0-h)^2}}$$

Eksempel: Måling af en stor hastighed



$$v_0 = L\sqrt{\frac{g}{2(h_0 - h)}}$$

Eksempel: Måling af en stor hastighed



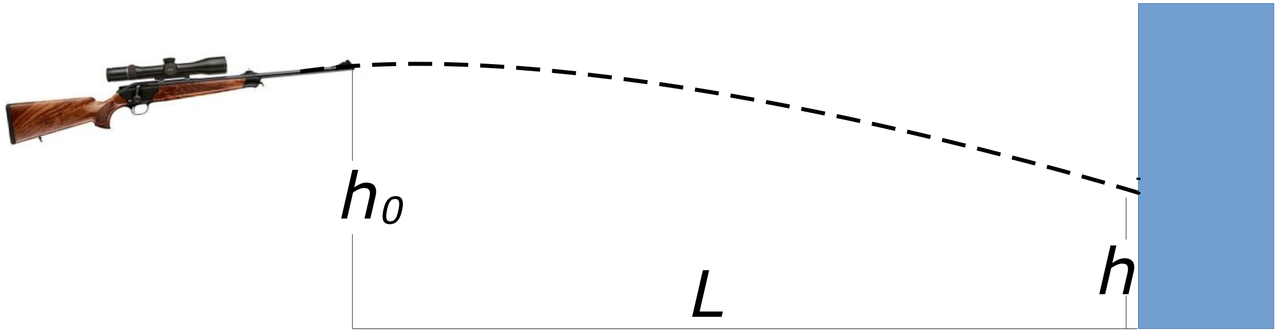
$$v_0 = L\sqrt{\frac{g}{2(h_0 - h)}} \approx 289.562..\,\mathrm{m/s}$$

$$\delta v_0 = v_0\sqrt{\left(\frac{\delta L}{L}\right)^2 + \frac{\delta h_0^2 + \delta h^2}{4(h - h_0)^2}}$$

$$\frac{\delta v_0}{v_0} \approx \sqrt{(0.00012)^2 + \frac{(0.003\,m)^2 + (0.0005\,m)^2}{1.37\,m^2}} \approx 0.0026$$

$$\boxed{v_0 = 289.6 \pm 0.8\,\mathrm{m/s}}$$

Eksempel: Måling af en stor hastighed



Andre fejlkilder:

-Usikkerhed på skudvinkel

-Usikkerhed på g

-Luftmodstand

Ophobningsloven – summer og produkter

Uafhængige usikkerheder: $y=\sum_i x_i \Rightarrow \delta y=\sqrt{\sum_i \delta x_i^2}$

$$y=\frac{x_1 x_2 .. x_N}{z_1 z_2 .. z_M} \Rightarrow \frac{\delta y}{|y|}=\sqrt{\sum_{i=1}^{N}\left(\frac{\delta x_i}{x_i}\right)^2+\sum_{j=1}^{M}\left(\frac{\delta z_j}{z_j}\right)^2}$$

Afhængige usikkerheder: $y=\sum_i x_i \Rightarrow \delta y=\sum_i \delta x_i$

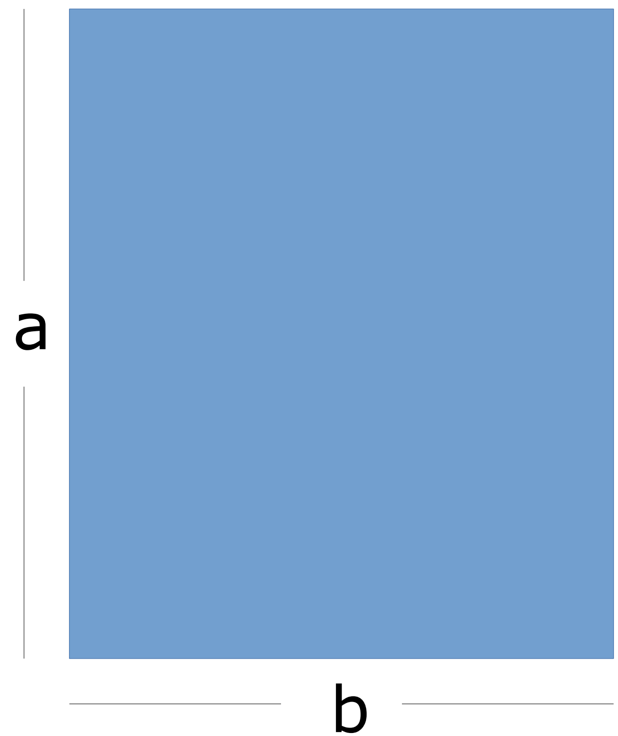
$$y=\frac{x_1 x_2 .. x_N}{z_1 z_2 .. z_M} \Rightarrow \frac{\delta y}{|y|}=\sum_{i=1}^{N}\frac{\delta x_i}{|x_i|}+\sum_{j=1}^{M}\frac{\delta z_j}{|z_j|}$$

Eksempel: Måling af areal

$a = 0.62 \pm 0.04$ m

$A = ab$

$b = 0.79 \pm 0.02$ m



Uafhængige usikkerheder:

$$\frac{\delta A}{A} = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2} \approx 0.07$$

$$A \approx 0.49 \pm 0.03\,\mathrm{m}^2$$

Afhængige usikkerheder:

$$\frac{\delta A}{A} = \frac{\delta a}{a} + \frac{\delta b}{b} \approx 0.09$$

$$A \approx 0.49 \pm 0.04\,\mathrm{m}^2$$

I Maple

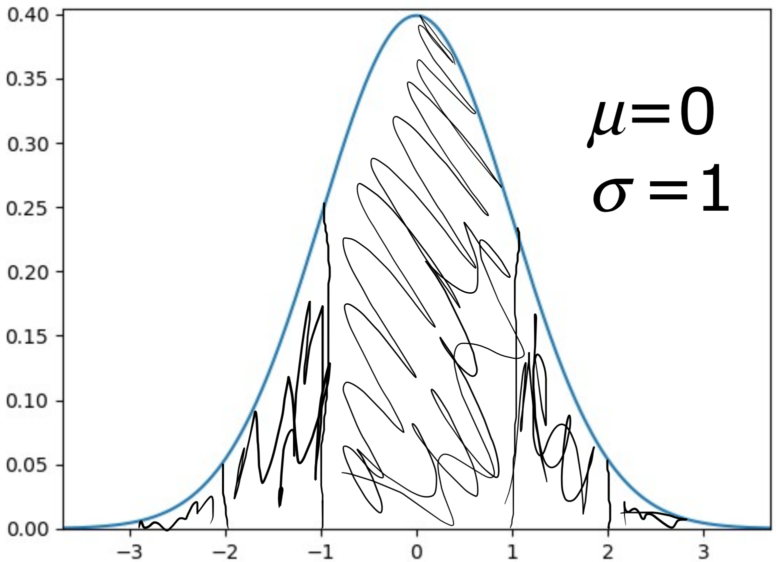
Addition

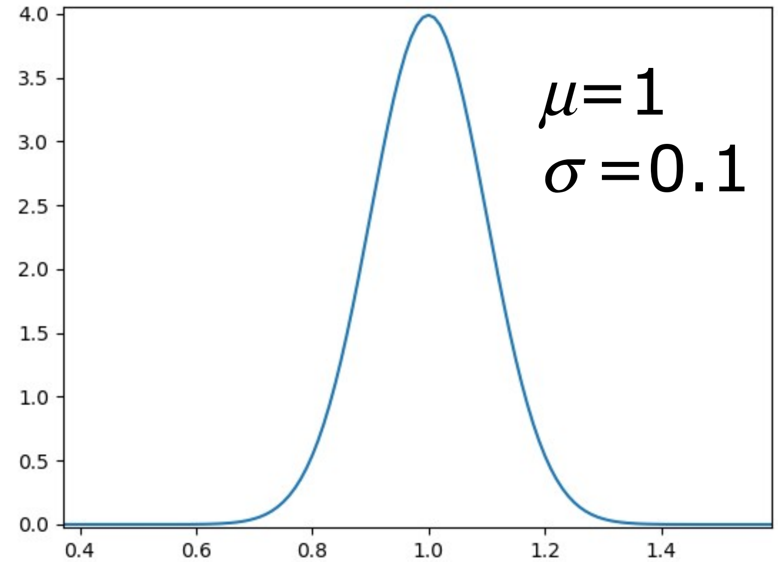
```
x := Quantity(2.84, 0.03);
            x := Quantity(2.84, 0.03)
y := Quantity(6.73, 0.12e-1);
            y := Quantity(6.73, 0.012)
u := combine(x+y, errors);
            u := Quantity(9.57, 0.03231098884)
ApplyRule(u, round3g[1]);
            Quantity(9.57, 0.03)
```

Multiplikation

```
x := Quantity(2.84, 0.03);
            x := Quantity(2.84, 0.03)
y := Quantity(6.73, 0.12e-1);
            y := Quantity(6.73, 0.012)
u := combine(x*y, errors);
            u := Quantity(19.1132, 0.2047560900)
ApplyRule(u, round3g[1]);
            Quantity(19.11, 0.20)
```

Normalfordeling





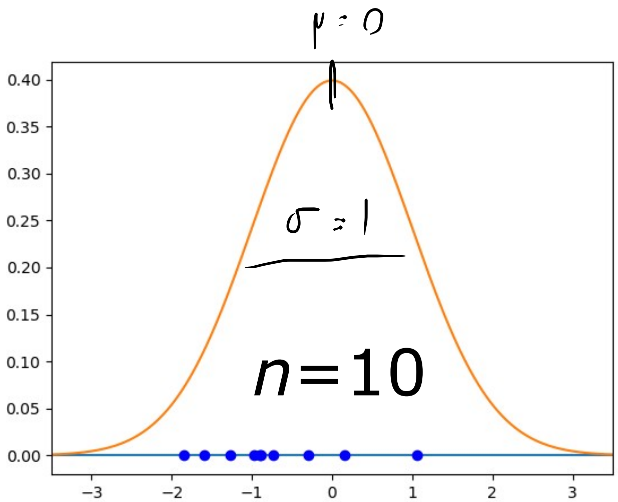
$$y=f(x)=\frac{e^{\frac{-(x-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

$$P(x_1<y<x_2)=\int_{x_1}^{x_2} f(x)\,dx$$

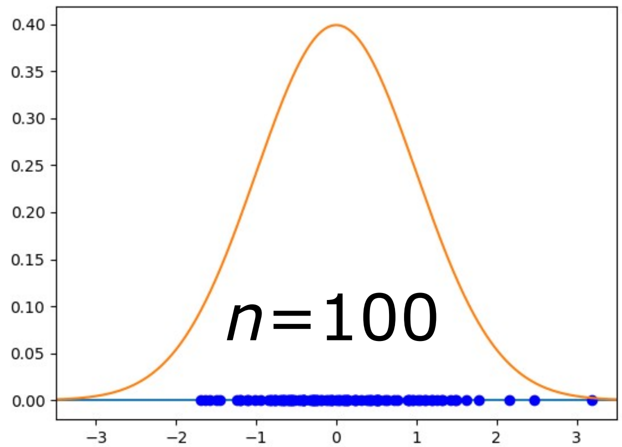
$$\int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx=\sigma^2$$

Normalfordeling

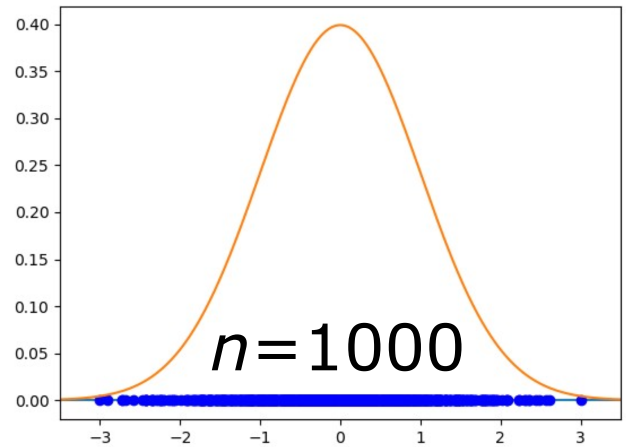
$$\bar{x} = \frac{1}{n}\sum_i x_i \qquad \sigma = \sqrt{\frac{1}{n-1}\sum_i (x_i - \bar{x})^2}$$



$\bar{x} \approx -0.73 \qquad \sigma \approx 0.85$



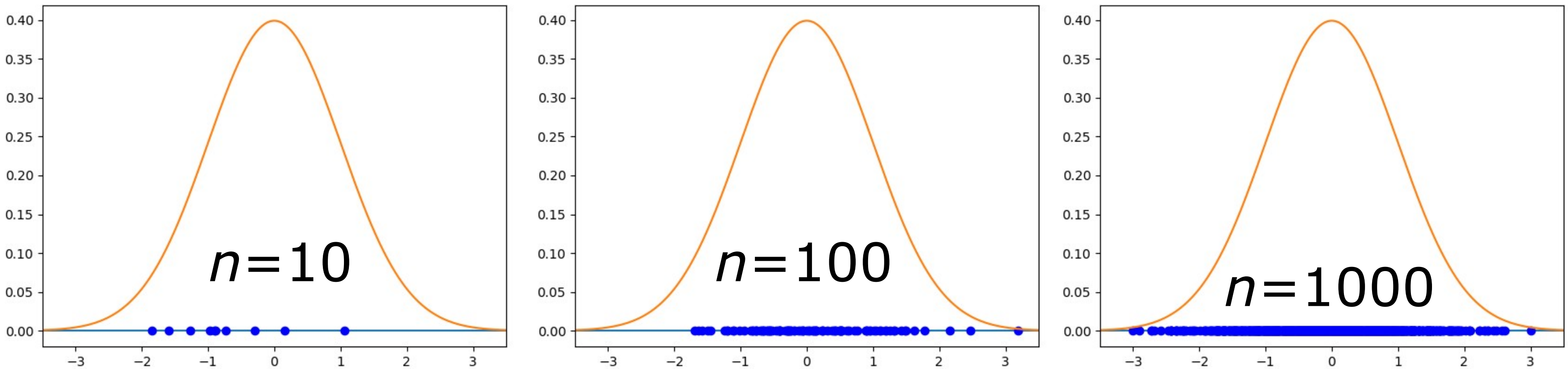
$\bar{x} \approx 0.066 \qquad \sigma \approx 0.93$



$\bar{x} \approx -0.026 \qquad \sigma \approx 0.97$

Usikkerhed på $\bar{x}$: $\delta \bar{x} \sim \frac{\sigma}{\sqrt{n}}$ (generelt estimat)

Normalfordeling

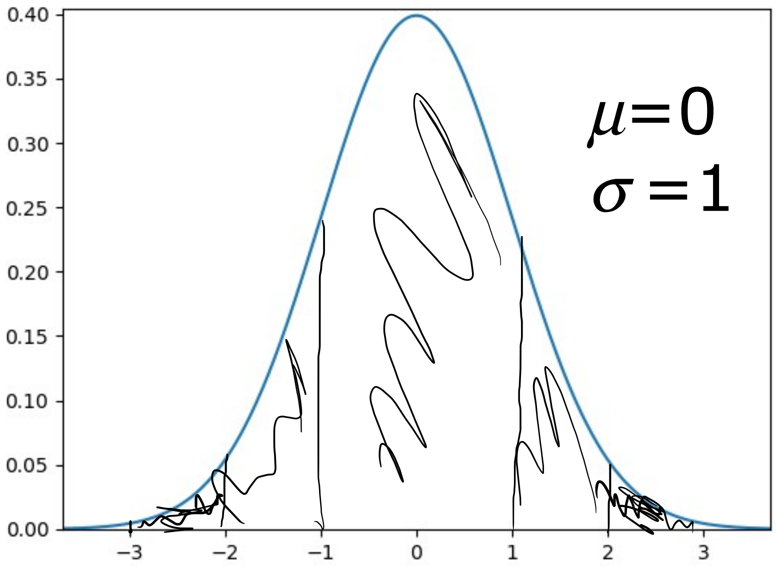


$\bar{x} \approx -0.73 \qquad \sigma \approx 0.85$

$\bar{x} \approx 0.066 \qquad \sigma \approx 0.93$

$\bar{x} \approx -0.026 \qquad \sigma \approx 0.97$

Normalfordeling



$$P\left(|y-\mu|<\sigma\right)\approx 0.68269$$

$$P\left(|y-\mu|<2\sigma\right)\approx 0.95450$$

$$P\left(|y-\mu|<3\sigma\right)\approx 0.99730$$

Test af hypotese: Sammenlign målt værdi *y* med forventet værdi y_0:

Fint	OK	Hmm...	Nix!
$\lvert y-y_0\rvert<\sigma$	$\lvert y-y_0\rvert<2\sigma$	$\lvert y-y_0\rvert<3\sigma$	$\lvert y-y_0\rvert>3\sigma$

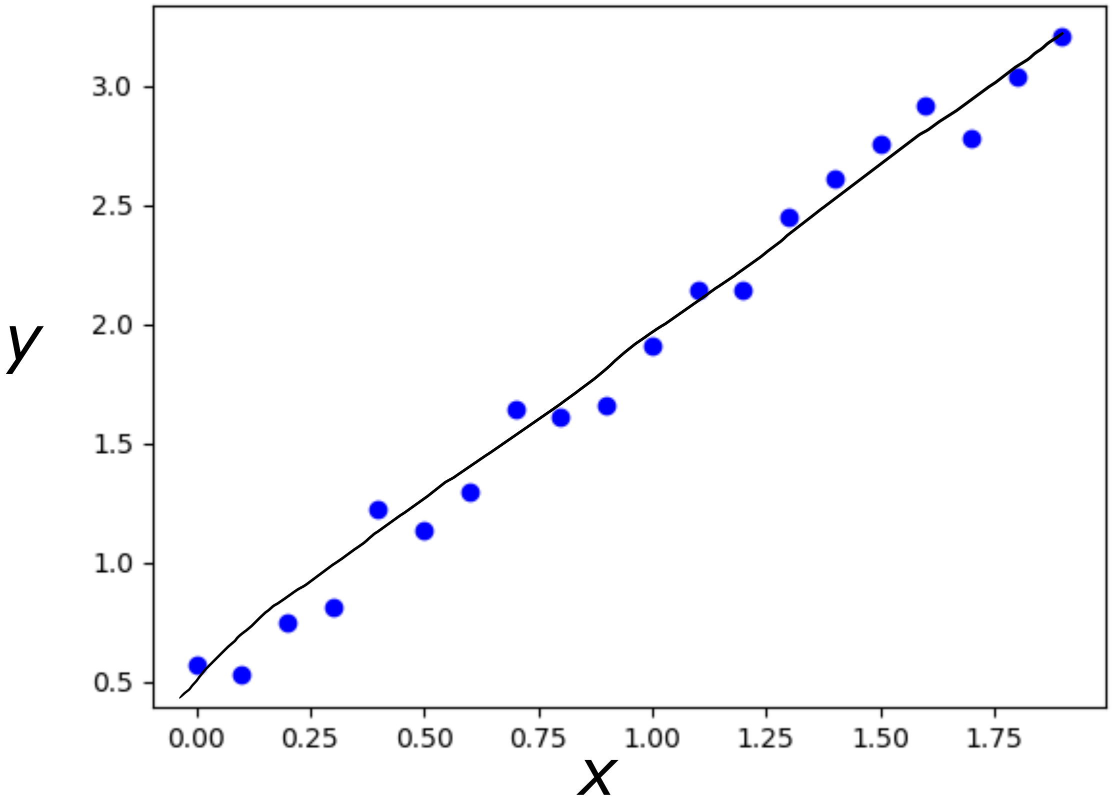
Vægtet middel

Forskellige måleserier for samme størrelse, f.eks. forskellige laboratorier, samplestørrelse, etc.

$$x_1 \pm \delta x_1, x_2 \pm \delta x_2, \ldots, x_n \pm \delta x_n$$

$$\bar{x} = \frac{\sum_i x_i w_i}{\sum_i w_i} \qquad w_i = \frac{1}{\delta x_i^2} \qquad \delta x = \frac{1}{\sqrt{\sum_i w_i}}$$

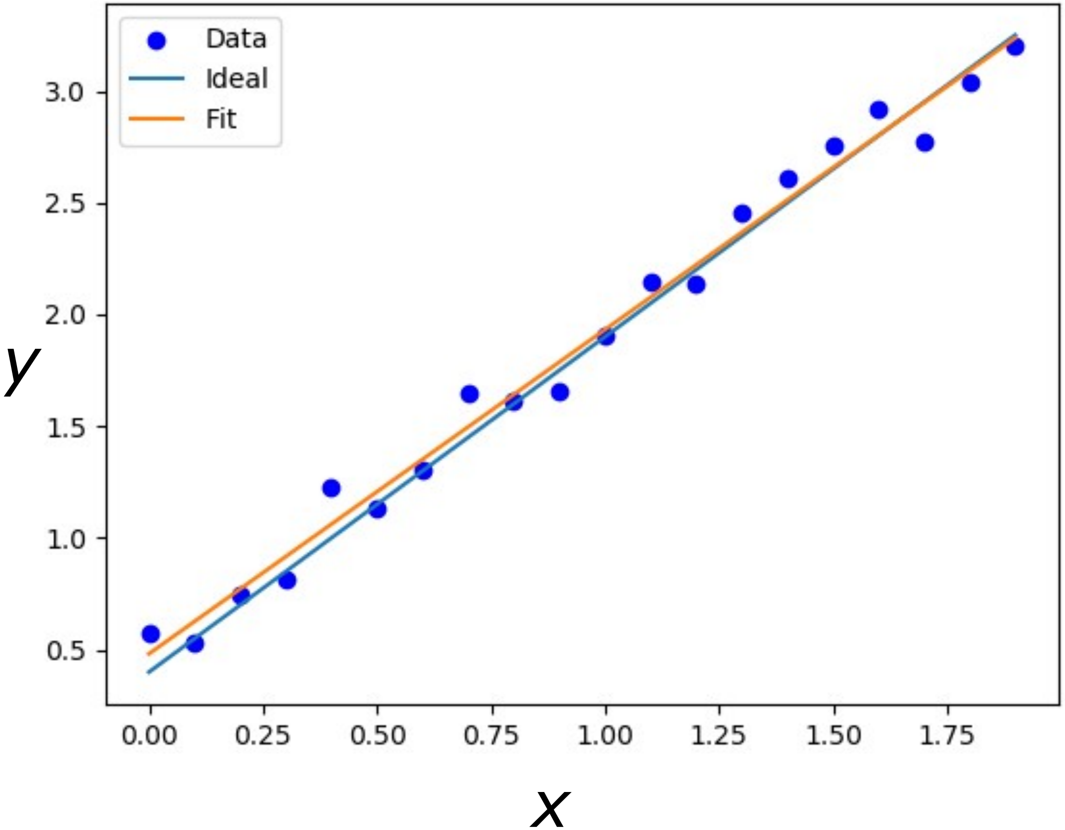
Lineær regression



Fit målte data til ret linje. Minimér *kvadratsummen:*

$$\sqrt{\frac{1}{n-2}\sum_i \left(y_i - y_{fit}\left(x_i\right)\right)^2}$$

Lineær regression



Ideal linje: $y=0.4+x\cdot 1.5$

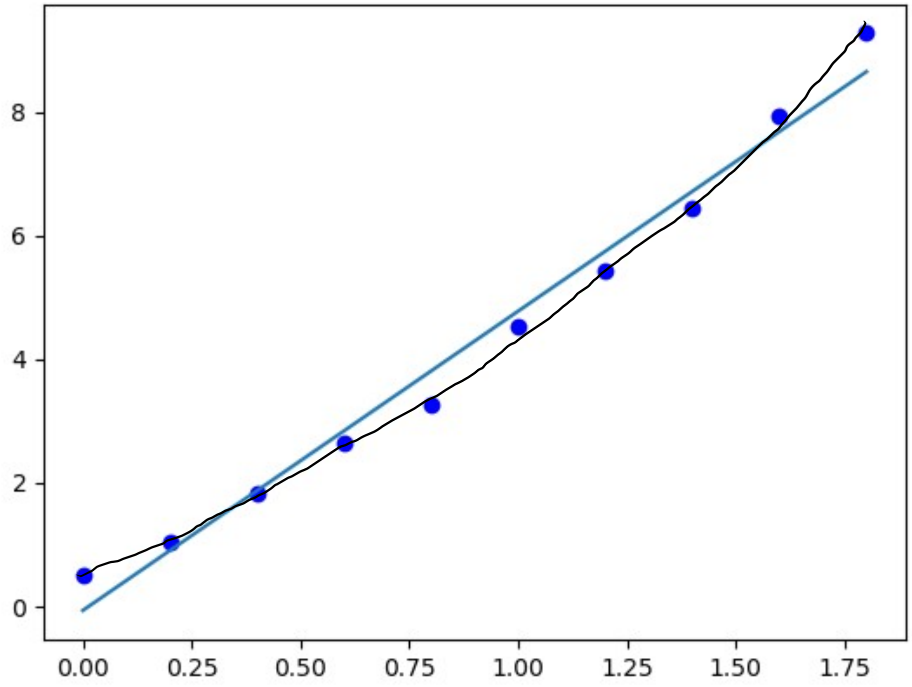
Data: Ideal linje + normalfordeling med $\mu=0$, $\sigma=0.1$.

Fit: $y=0.48..+x\cdot 1.45..$

$$\sqrt{\frac{1}{n-2}\sum_i \left(y_i - y_{fit}(x_i)\right)^2} \approx 0.102..$$

Højereordens regression

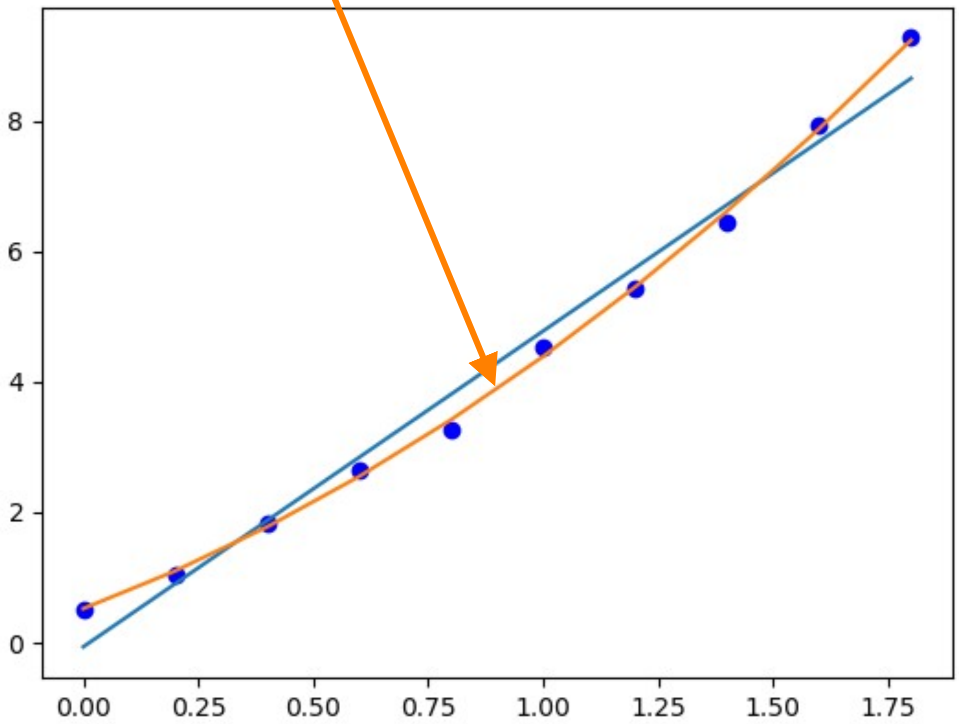
Lineær regression, *y*=4.84·*x*-0.0616; OK?



$$\sqrt{\frac{1}{n-2}\sum_i \left(y_i - y_{fit}(x_i)\right)^2} \approx 0.412..$$

Højereordens regression

Fit til 2.grads polynomium: y=1.22·x^2+2.64·x+0.52



$$\sqrt{\frac{1}{n-3}\sum_i \left(y_i - y_{fit}\left(x_i\right)\right)^2} \approx 0.112..$$

Regression i Maple

```
with(Statistics);
x := Vector([30., 40., 50., 60., 70., 80., 90., 100., 110., 120., 130., 140.]);
y := Vector([19., 19., 22., 25., 21., 25., 26., 23., 27., 29., 30., 29.]);
LinearFit(B*t+A, x, y, t);
LinearFit(B*t+A, x, y, t, summarize = true);
Summary:
----------------
Model: 16.410256+.96153846e-1*t
----------------
Coefficients:
Estimate   Std. Error  t-value   P(>|t|)
Parameter 1    16.4103    1.2998     12.6251   0.0000
Parameter 2    0.0962     0.0142     6.7866    0.0000
----------------
R-squared: 0.8216, Adjusted R-squared: 0.8038
LinearFit(B*t+A, x, y, t, output = parametervector);
LinearFit(B*t+A, x, y, t, output = standarderrors);
LinearFit(B*t+A, x, y, t, output = residualstandarddeviation);
```

```
with(plots):
with(Statistics):
x := Vector([1, 2, 3, 4, 5, 6]);
y := Vector([3.4, 5.2, 7.0, 9.0, 11.2, 13.4]);
N := 6;
plot(x, y, style = point);
LinearFit(B*t+A, x, y, t);
f := unapply(%, t);
display(plot(x, y, style = point), plot(f(t), t = x(1) .. x(N), color = red));
res := LinearFit(B*t+A, x, y, t, output = residuals);
plot(x, res, style = point);
LinearFit(C*t^2+B*t+A, x, y, t);
f := unapply(%, t);
display(plot(x, y, style = point), plot(f(t), t = x(1) .. x(N), color = red));
res := LinearFit(C*t^2+B*t+A, x, y, t, output = residuals);
plot(x, res, style = point);
```

Opsummering

Tilfældig usikkerhed: $\delta x_t = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$

Tilfældig +
systematisk usikkerhed: $\delta x = \sqrt{\left(\delta x_t\right)^2 + \left(\delta x_s\right)^2}$

Middelværdi: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ $\delta \bar{x} = \frac{\delta x}{\sqrt{n}}$ (tilfældig usikkerhed)

Opsummering

Ophobningsloven: $y = f\left(x_1, x_2, .., x_N\right)$

$$\delta y = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 \delta x_1^2 + .. + \left(\frac{\partial f}{\partial x_N}\right)^2 \delta x_N^2}$$

δx_i uafhængige

$$\delta y = \left|\frac{\partial f}{\partial x_1} \delta x_1\right| + .. + \left|\frac{\partial f}{\partial x_N} \delta x_N\right|$$

δx_i afhængige

Opsummering

Uafhængige usikkerheder: $y = \sum_i x_i \Rightarrow \delta y = \sqrt{\sum_i \delta x_i^2}$

$$y = \frac{x_1 x_2 .. x_N}{z_1 z_2 .. z_M} \Rightarrow \frac{\delta y}{|y|} = \sqrt{\sum_{i=1}^{N} \left(\frac{\delta x_i}{x_i}\right)^2 + \sum_{j=1}^{M} \left(\frac{\delta z_j}{z_j}\right)^2}$$

Afhængige usikkerheder: $y = \sum_i x_i \Rightarrow \delta y = \sum_i \delta x_i$

$$y = \frac{x_1 x_2 .. x_N}{z_1 z_2 .. z_M} \Rightarrow \frac{\delta y}{|y|} = \sum_{i=1}^{N} \frac{\delta x_i}{|x_i|} + \sum_{j=1}^{M} \frac{\delta z_j}{|z_j|}$$

Kinematik i 2D og 3D, B & W kap. 3

DTU Fotonik
Institut for Fotonik

Fra ~~sidste~~ forrige gang

Hastighed og acceleration fra stedfunktion *x(t)*

$$v_x = \frac{dx}{dt} \qquad a_x = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Farten er $|v(t)|$

Ligninger for lineær bevægelse med konstant acceleration - f.eks. frit fald med $a_y = -g$

$$\boxed{\begin{aligned} v(t) &= v_0 + at \\ x(t) &= x_0 + v_0 t + \frac{1}{2}at^2 \\ x(t) &= x_0 + \frac{1}{2}\left(v(t) + v_0\right)t \\ v^2(x) &= v_0^2 + 2a\left(x - x_0\right) \end{aligned}}$$

Denne uges læringsmål

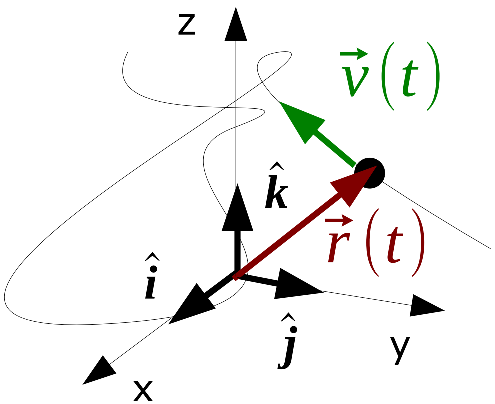
-Generalisér begreberne *stedfunktion, hastighed, acceleration*, til flere dimensioner

-Beskrive kastebevægelse

-Forstå hastighed og acceleration i jævn og ujævn cirkelbevægelse

-Forstå relativ bevægelse i flere dimensioner

Hastighed og acceleration i 3D



$$\vec{r}(t) = x(t)\,\hat{\boldsymbol{i}} + y(t)\,\hat{\boldsymbol{j}} + z(t)\,\hat{\boldsymbol{k}}$$

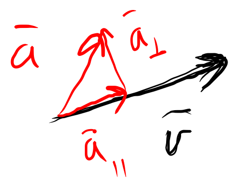
$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\,\hat{\boldsymbol{i}} + \frac{dy}{dt}\,\hat{\boldsymbol{j}} + \frac{dz}{dt}\,\hat{\boldsymbol{k}} \equiv v_x(t)\,\hat{\boldsymbol{i}} + v_y(t)\,\hat{\boldsymbol{j}} + v_z(t)\,\hat{\boldsymbol{k}}$$

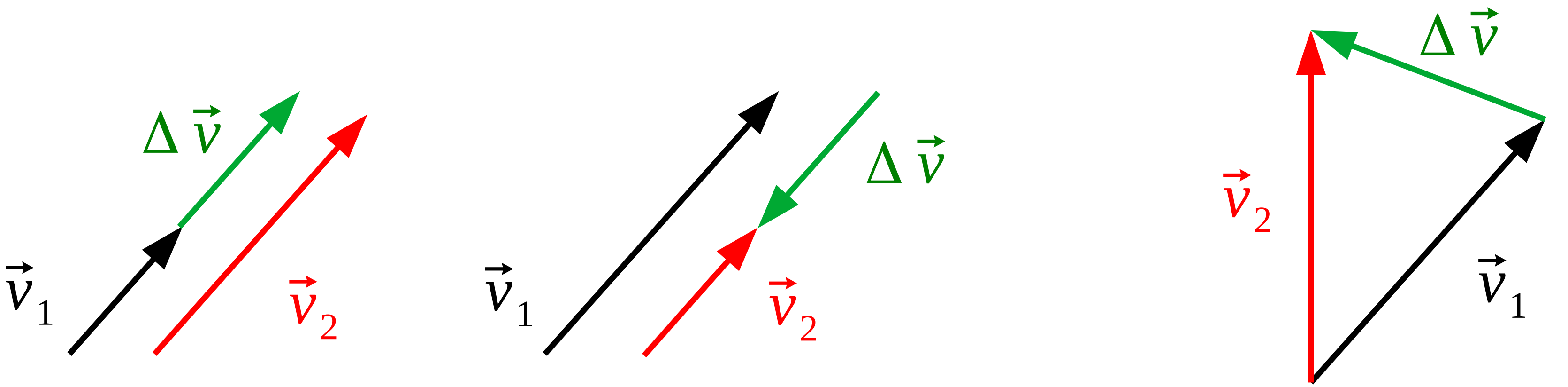
$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2x}{dt^2}\,\hat{\boldsymbol{i}} + \frac{d^2y}{dt^2}\,\hat{\boldsymbol{j}} + \frac{d^2z}{dt^2}\,\hat{\boldsymbol{k}} \equiv a_x(t)\,\hat{\boldsymbol{i}} + a_y(t)\,\hat{\boldsymbol{j}} + a_z(t)\,\hat{\boldsymbol{k}}$$

Hastighed og acceleration i 3D

$$\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

I 2D og 3D kan vi have acceleration uden at *farten* $|\vec{v}|$ ændres

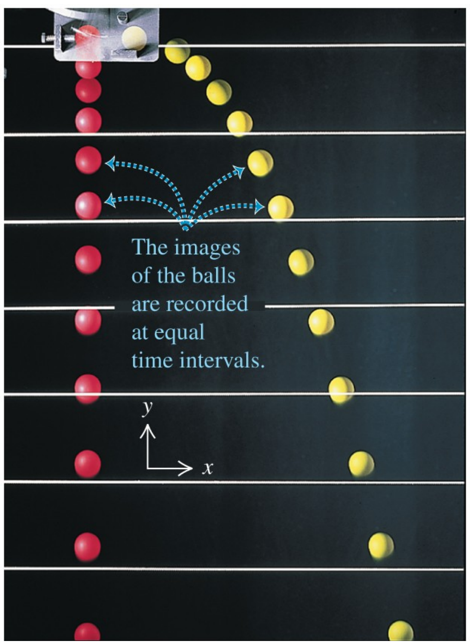


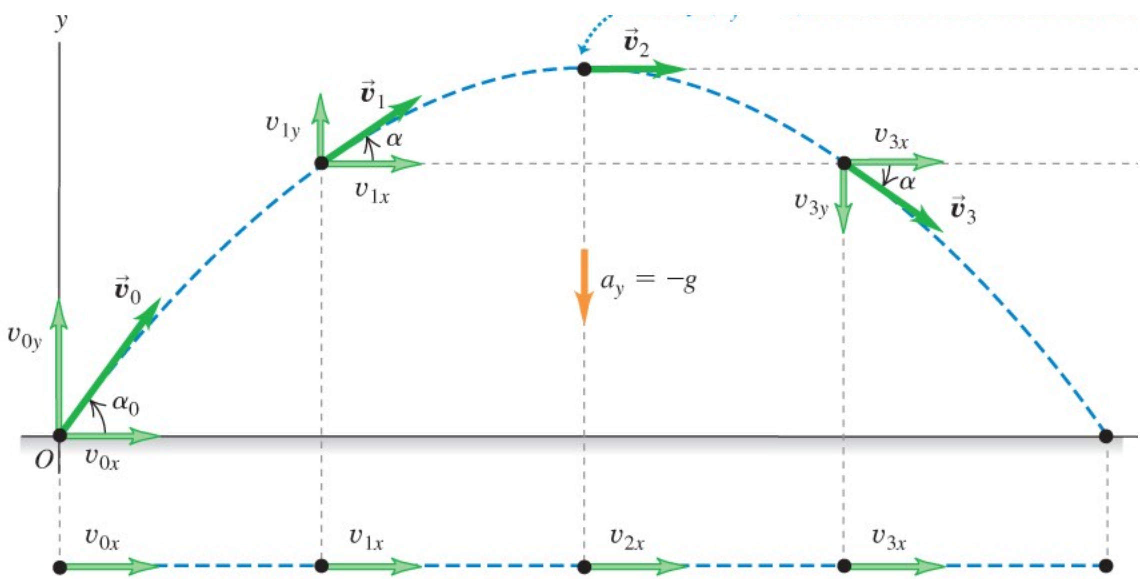


Kastebevægelse ('projectile motion')

Frit fald i 2D (3D) med begyndelseshastighed $\vec{v}_0$

$$a_x(t) = 0 \qquad a_y(t) = -g$$



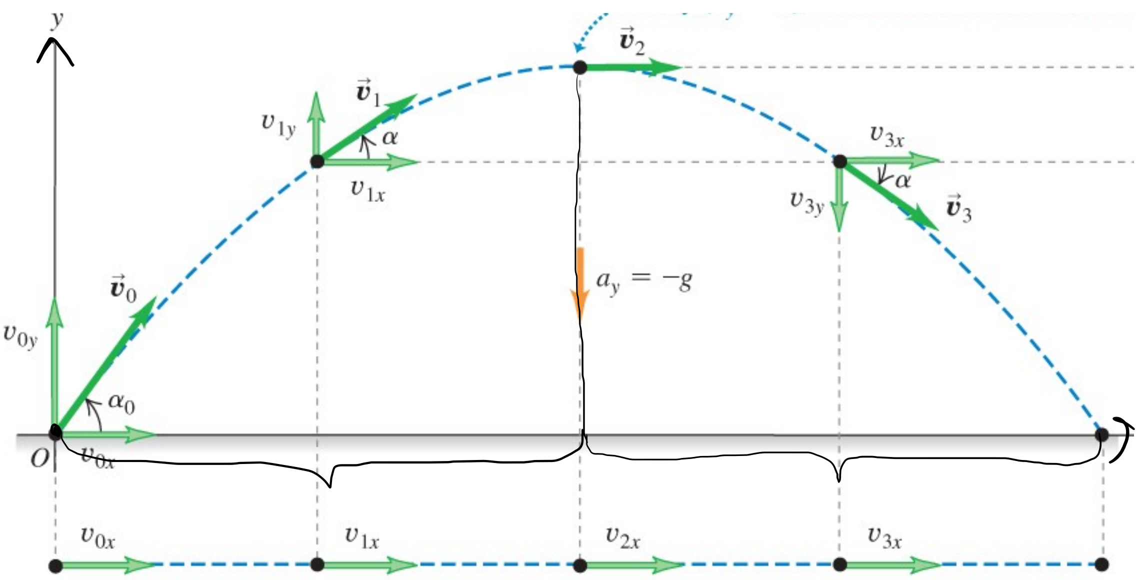


Kastebevægelse ('projectile motion')

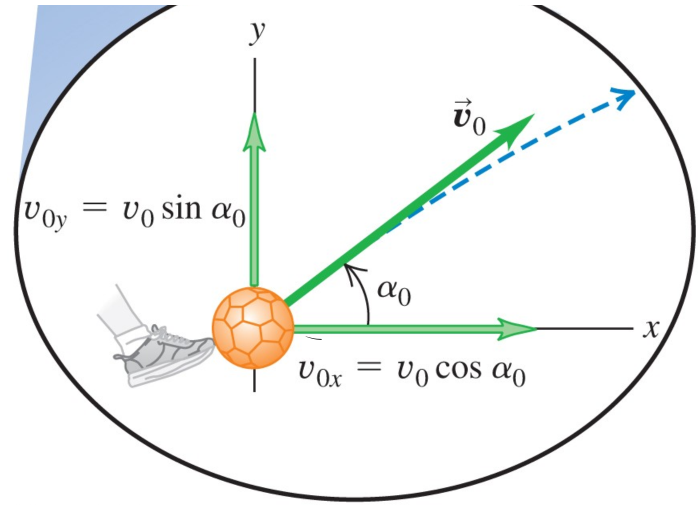
Frit fald i 2D (3D) med begyndelseshastighed $\vec{v}_0$

$$x(t) = x_0 + v_{0x}\,t + \cancel{\tfrac{1}{2} a_x t^2}$$

$$y(t) = y_0 + v_{0y}\,t + \tfrac{1}{2} a_y t^2 = y_0 + v_{0y}\,t - \tfrac{1}{2} g t^2$$



Kasteparabel



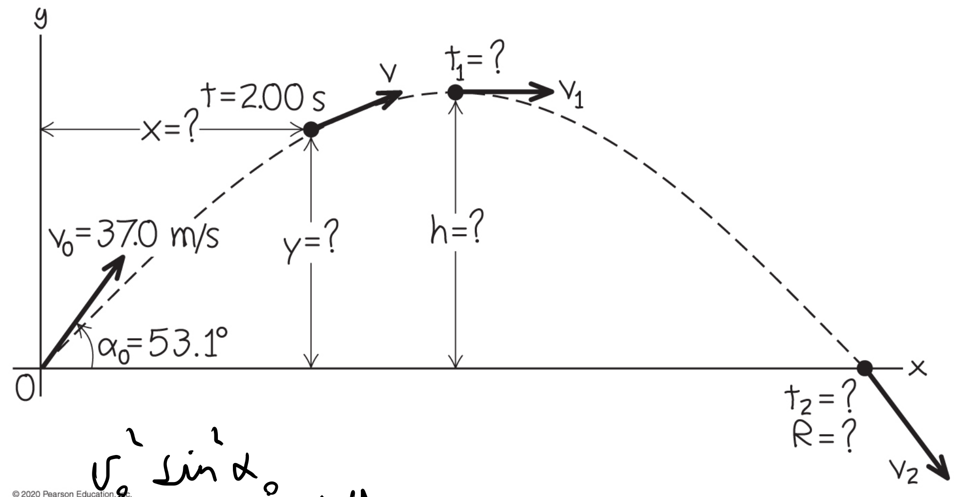
$$x(t) = x_0 + v_{0x} t \Rightarrow t = \frac{x - x_0}{v_{0x}}$$

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2 =$$

$$y_0 + v_{0y} \frac{x - x_0}{v_{0x}} - \frac{1}{2} g \frac{(x - x_0)^2}{v_{0x}^2} \overset{x_0 = 0}{=} y_0 + \frac{v_{0y}}{v_{0x}} x - \frac{1}{2} \frac{g}{v_{0x}^2} x^2 =$$

$$y_0 + \frac{\cancel{v_0} \sin \alpha_0}{\cancel{v_0} \cos \alpha_0} x - \frac{1}{2} \frac{g}{v_0^2 \cos^2 \alpha_0} x^2 = y_0 + \operatorname{tg} \alpha_0 \, x - \frac{g}{2 v_0^2 \cos^2 \alpha_0} x^2$$

Kasteparabel



$$v_y^2 = v_{0y}^2 + 2a(y - y_0) =$$

$$v_{0y}^2 - 2g(h - y_0) = 0$$

$$\Downarrow$$

$$v_{0y}^2 = 2g(h - y_0) \Rightarrow h = \frac{v_{0y}^2}{2g} + y_0 = \frac{v_0^2 \sin^2\alpha_0}{2g} + y_0$$

$$R = v_{0x} t_2 = 2 v_{0x} t_1 \qquad \boxed{v_y = v_{0y} + a_y t = v_{0y} - gt \Rightarrow v_{0y} - g t_1 = 0 \Rightarrow t_1 = \frac{v_{0y}}{g}}$$

$$\Downarrow$$

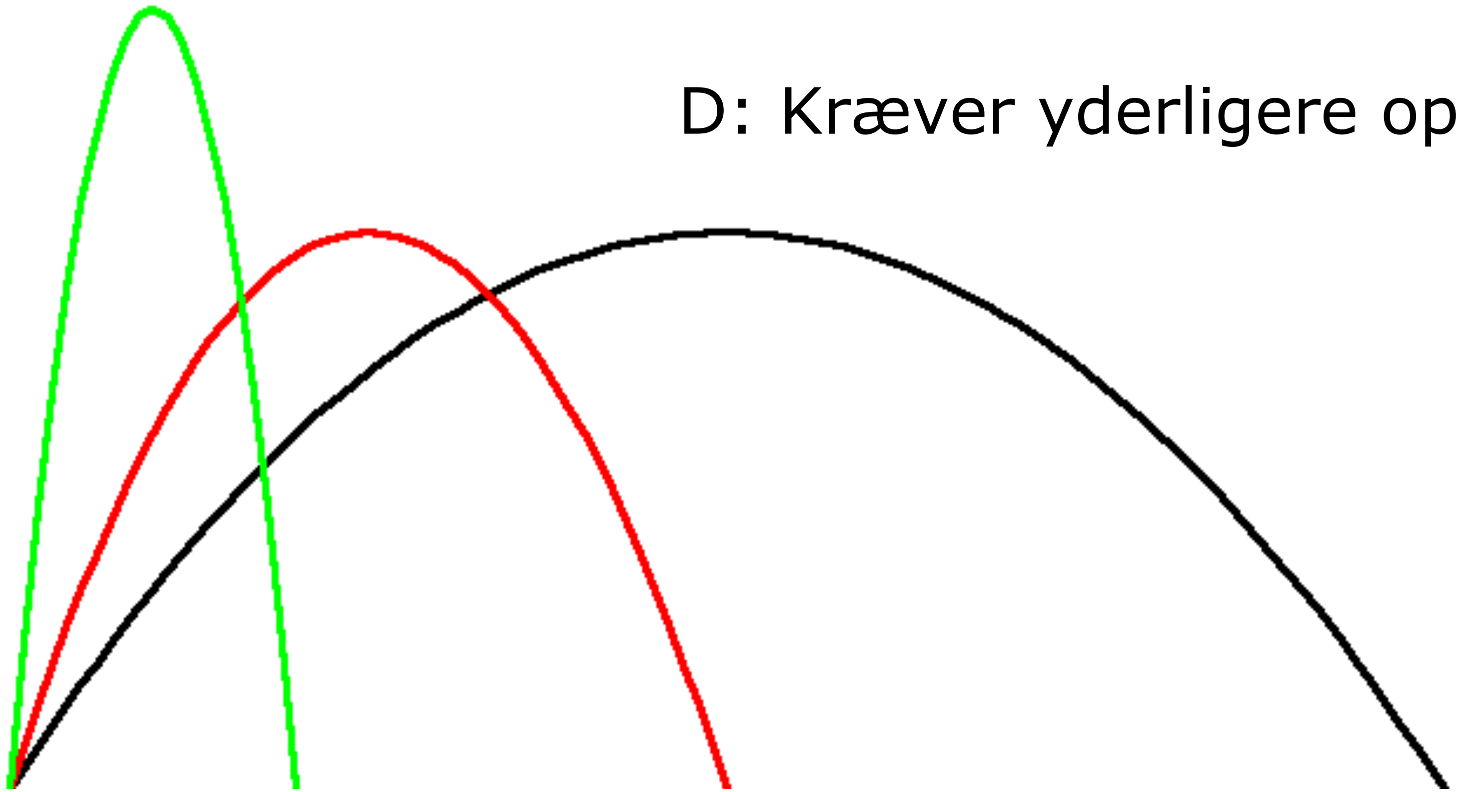
$$R = \frac{2 v_{0x} v_{0y}}{g} = 2\,\frac{v_0^2 \cos\alpha_0 \sin\alpha_0}{g} = \frac{v_0^2 \sin 2\alpha_0}{g}$$

Quiz: Flyvetid

Hvilken bane har længst flyvetid?

A: Sort B: Rød C: Grøn

D: Kræver yderligere oplysninger



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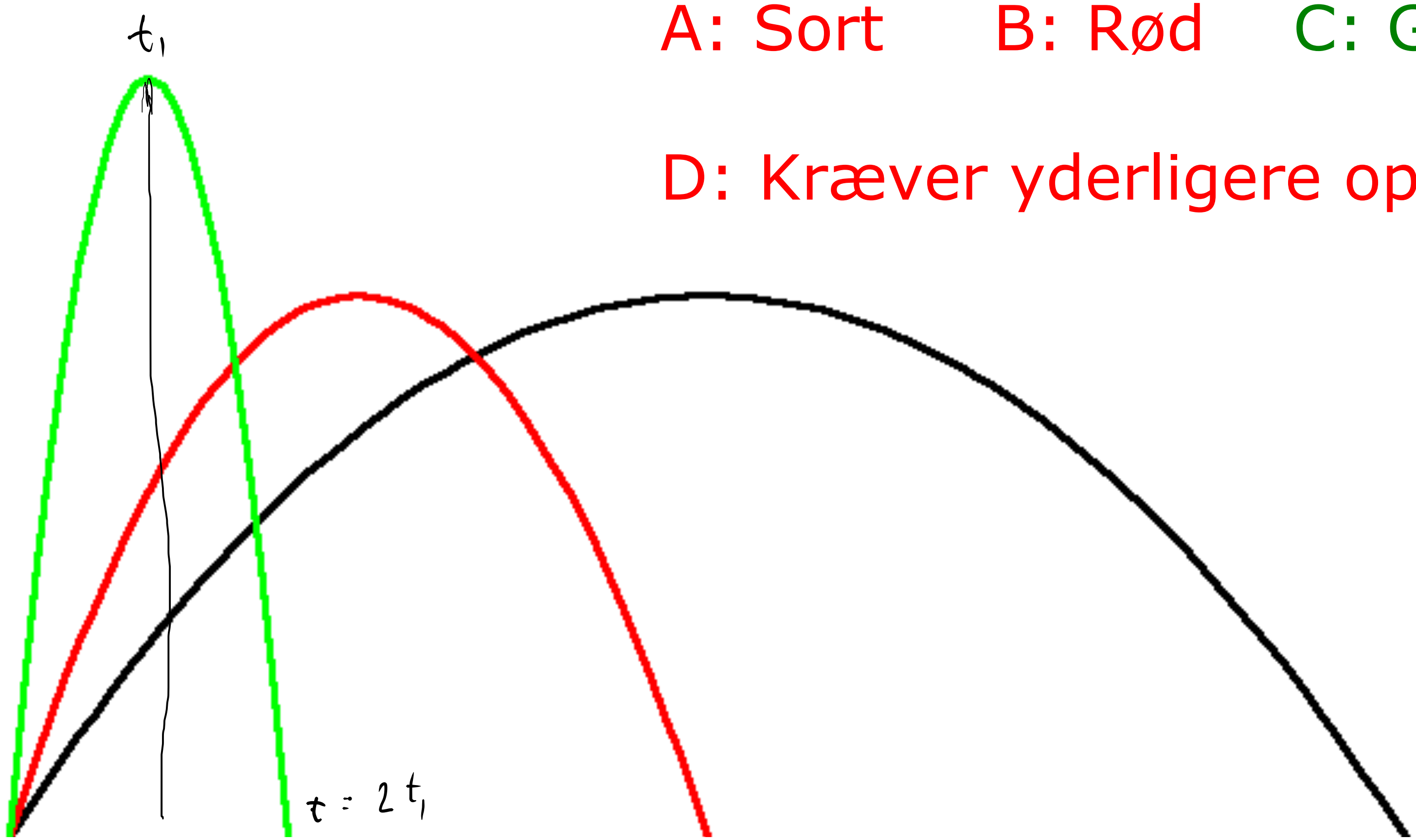
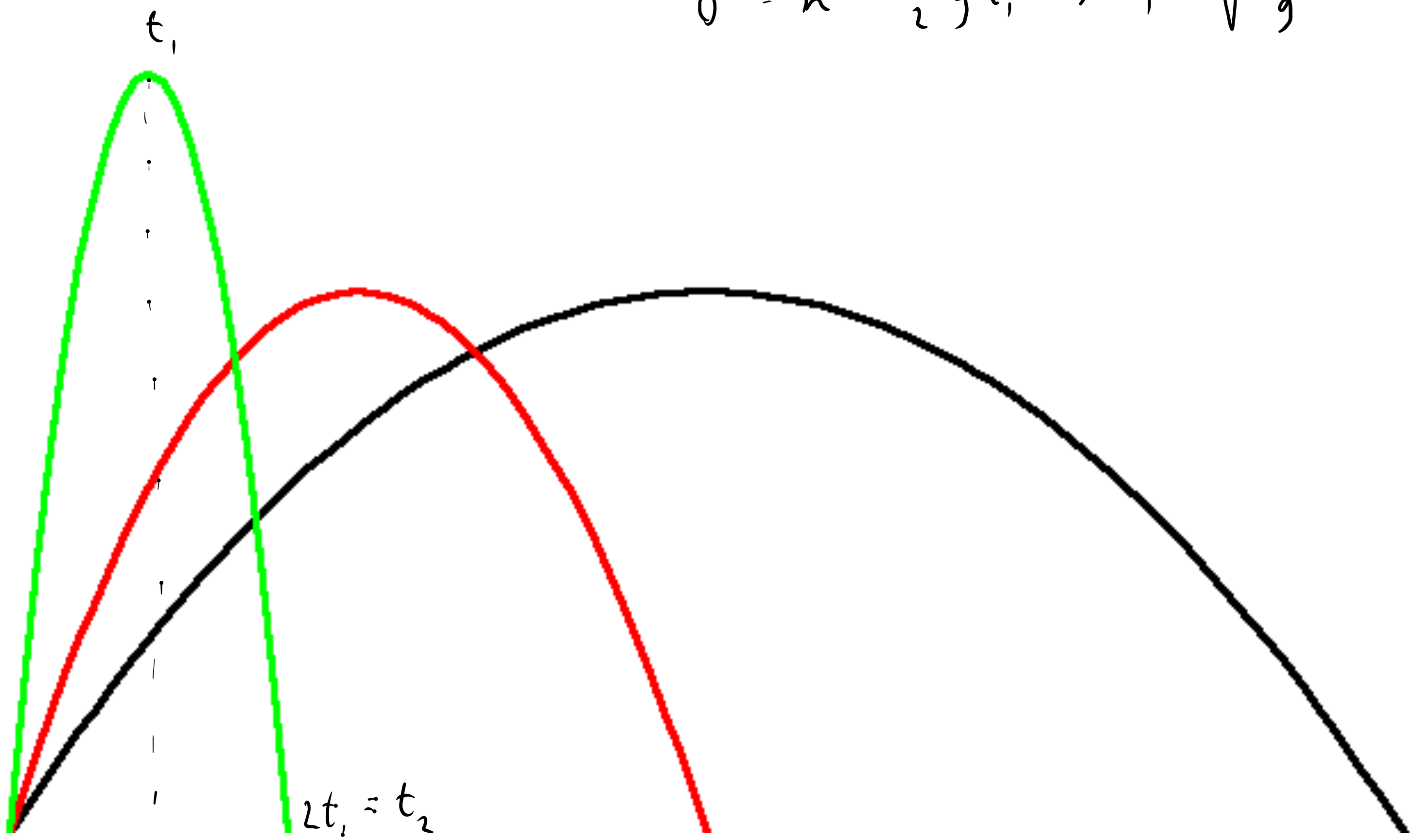


Flyvetid

$$y(2t_1) = y(t_1) + v_y(t_1)(t_2 - t_1) - \frac{1}{2} g (t_2 - t_1)^2$$

$$\Downarrow$$

$$0 = h - \frac{1}{2} g t_1^2 \Rightarrow t_1 = \sqrt{\frac{2h}{g}} \Rightarrow t_2 = 2\sqrt{\frac{2h}{g}}$$



Eksempel: Bueskydning

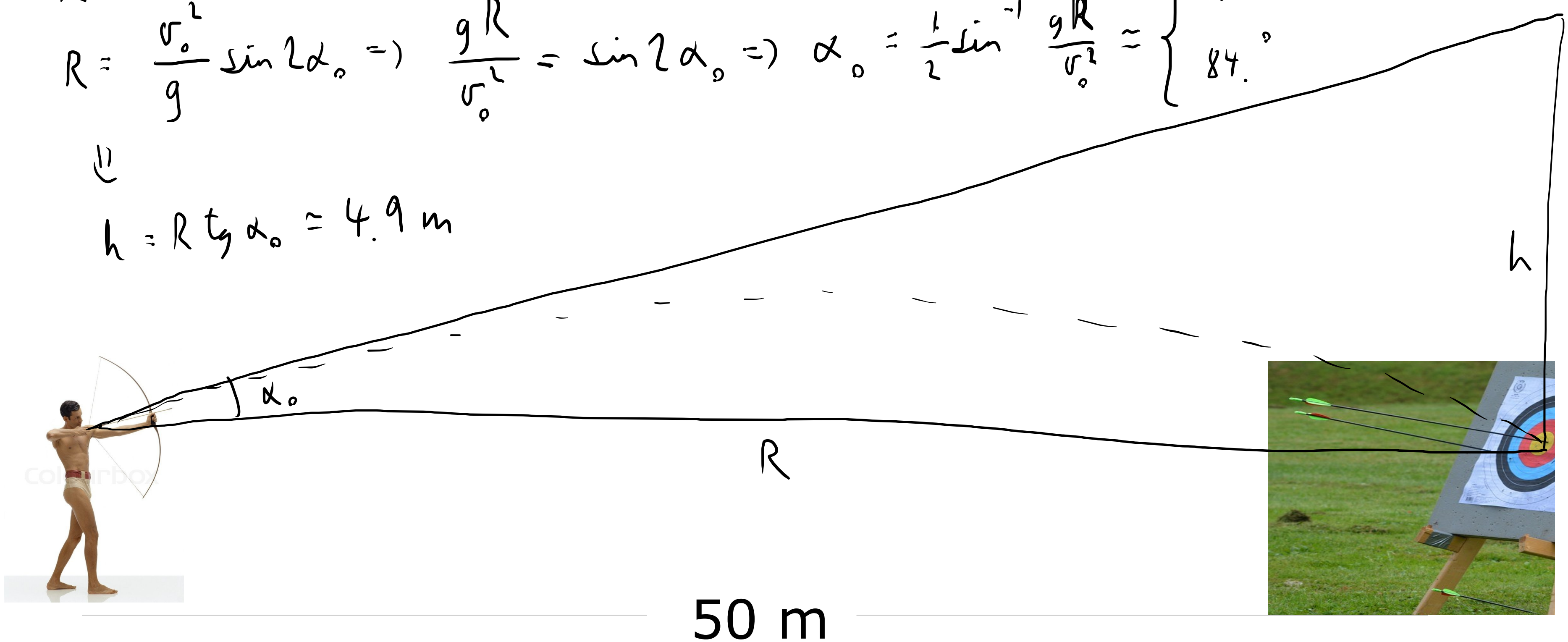
Hvor højt skal skytten sigte? v_0=50 m/s

$$\frac{h}{R} = \mathrm{tg}\,\alpha_0 \Rightarrow h = R\,\mathrm{tg}\,\alpha_0$$

$$R = \frac{v_0^2}{g}\sin 2\alpha_0 \Rightarrow \frac{gR}{v_0^2} = \sin 2\alpha_0 \Rightarrow \alpha_0 = \frac{1}{2}\sin^{-1}\frac{gR}{v_0^2} \simeq \begin{cases} 5.7^\circ \\ 84.^\circ \end{cases}$$

$$\Downarrow$$

$$h = R\,\mathrm{tg}\,\alpha_0 \simeq 4.9\ \mathrm{m}$$

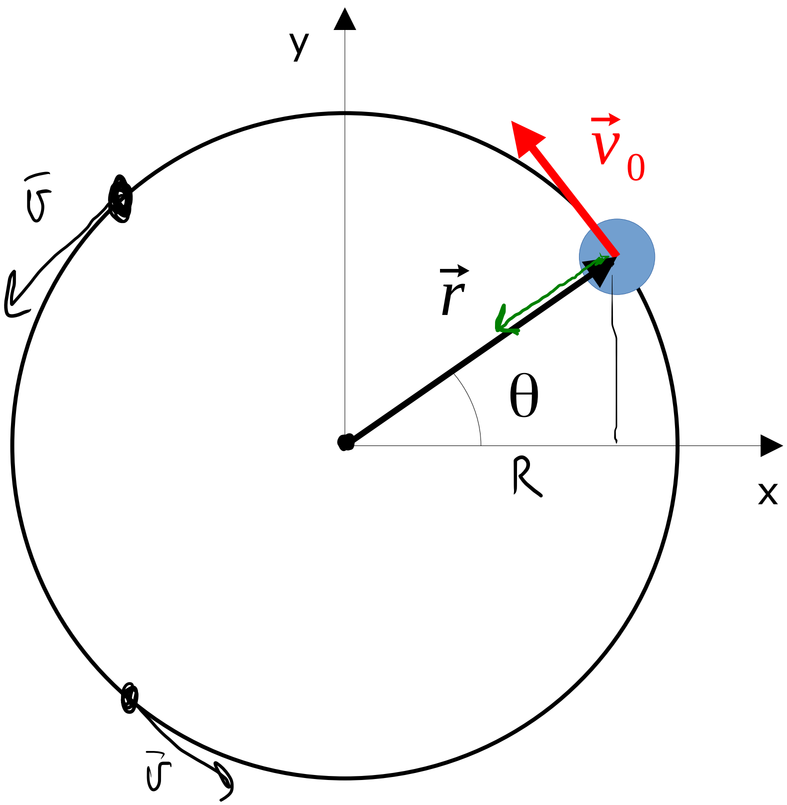


Eksempel: Bueskydning

Hvor højt skal skytten sigte? v_0=50 m/s

50 m

Jævn cirkelbevægelse



$$v_0 = |\vec{v}_0| \text{ konstant} \quad |\vec{r}| = R \text{ konstant}$$

$$x(t) = R\cos\theta(t) = R\cos\omega t$$

$$y(t) = R\sin\theta(t) = R\sin\omega t$$

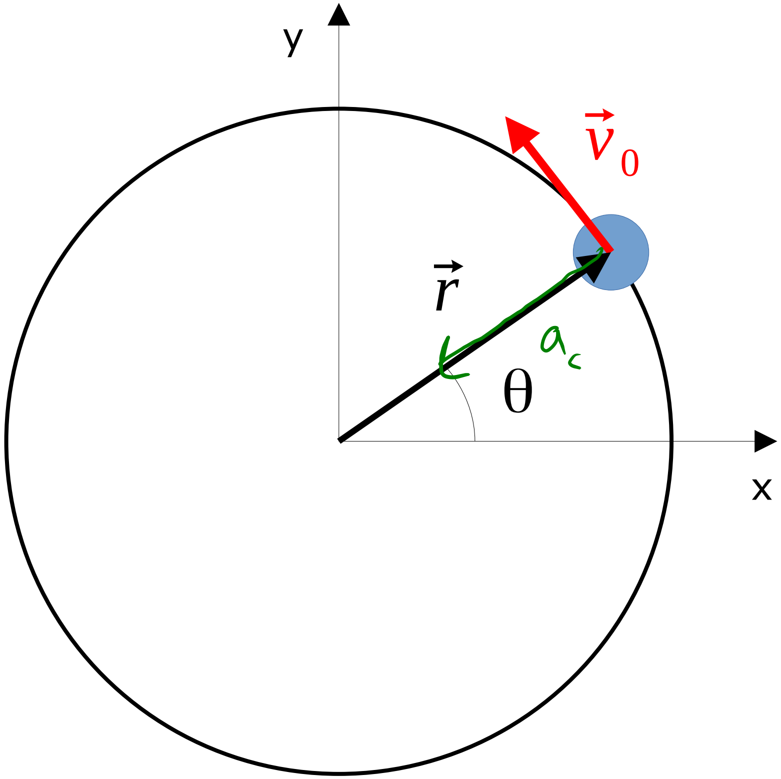
$$v_x = \frac{dx}{dt} = -R\omega\sin\omega t; \quad v_y = R\omega\cos\omega t$$

$$a_x = \frac{dv_x}{dt} = -R\omega^2\cos\omega t = -\omega^2 x$$

$$a_y = \frac{dv_y}{dt} = -R\omega^2\sin\omega t = -\omega^2 y$$

$$\bar{a} = -\omega^2\bar{r}$$

Jævn cirkelbevægelse



$$v_0 = |\vec{v}_0| \text{ konstant} \qquad |\vec{r}| = R \text{ konstant}$$

$$\bar{a} = -\omega^2 \bar{r} \Rightarrow |\bar{a}| = \omega^2 |\bar{r}| = \omega^2 R$$

$$v_x = -R\omega \sin \omega t \qquad v_y = R\omega \cos \omega t$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(R\omega)^2(\cos^2 \omega t + \sin^2 \omega t)}$$

$$= R\omega = v_0 \Rightarrow \omega = \frac{v_0}{R}$$

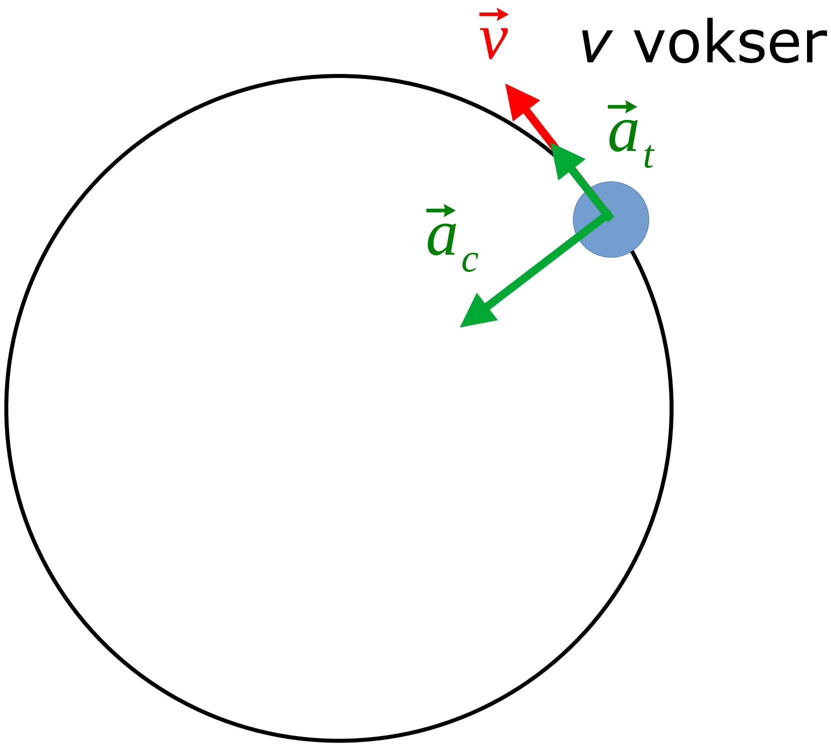
$$\Downarrow$$

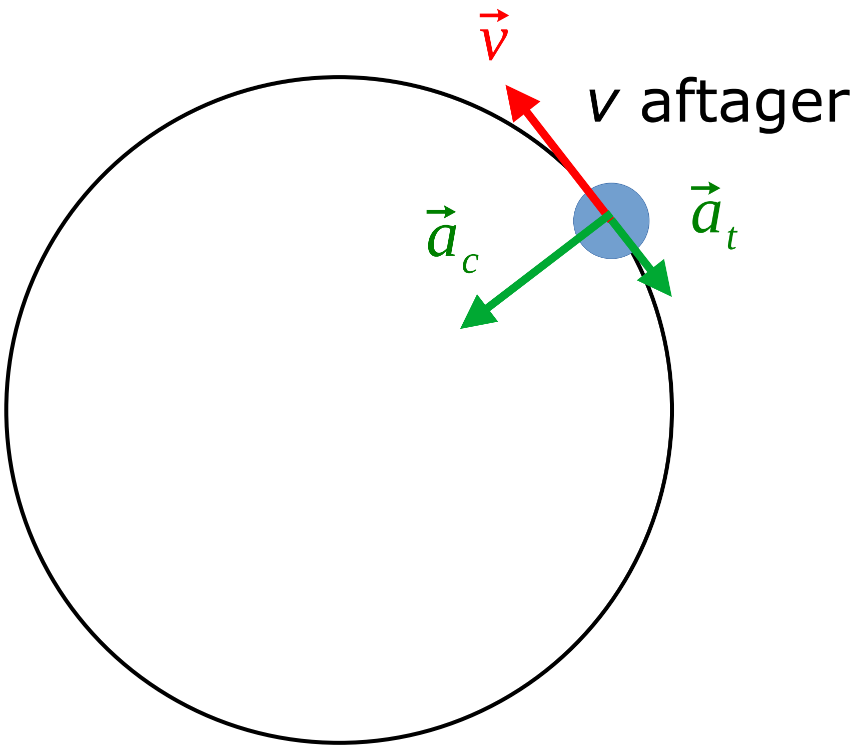
$$a_c = \left(\frac{v_0}{R}\right)^2 R = \frac{v_0^2}{R}$$

(U)jævn cirkelbevægelse

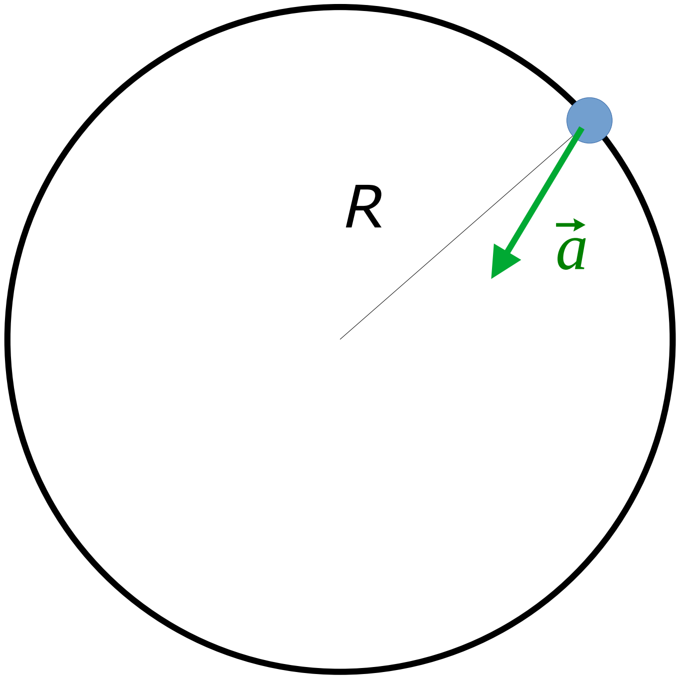
$$|\vec{r}| = R \text{ konstant}$$

$$\boxed{a_c = \frac{v^2}{R} \qquad |a_t| = |\vec{a}_t| = \left|\frac{dv}{dt}\right| \qquad a_t \equiv \frac{dv}{dt}}$$





Quiz - cirkelbevægelse



Hvad kan vi sige om farten i cirkelbevægelsen?

A: Farten aftager

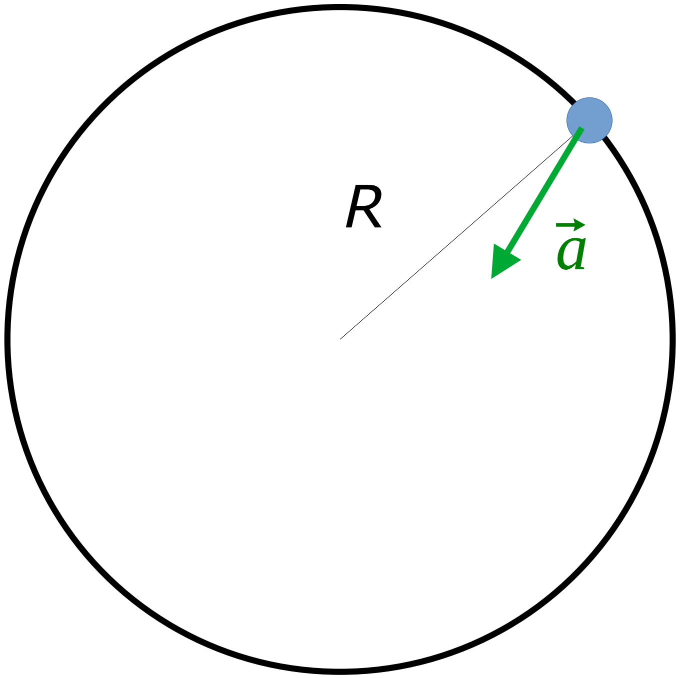
B: Farten vokser

C: Farten er konstant

D: Farten vokser eller aftager

E: Vi kan ikke sige noget.

Quiz - cirkelbevægelse



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A: Farten aftager

B: Farten vokser

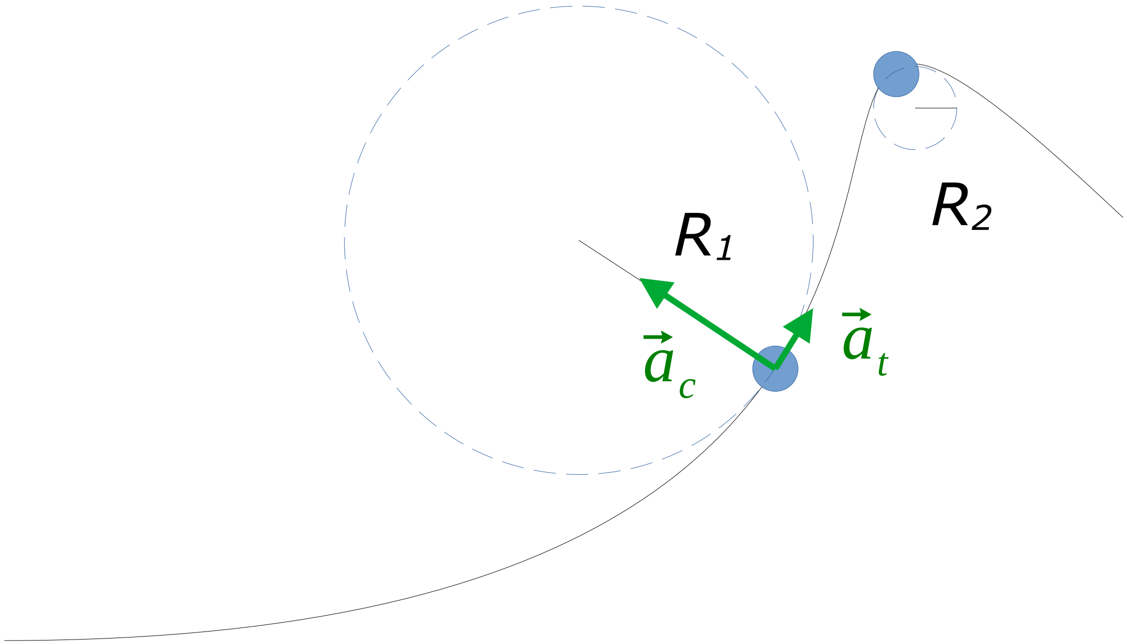
C: Farten er konstant

D: Farten vokser eller aftager

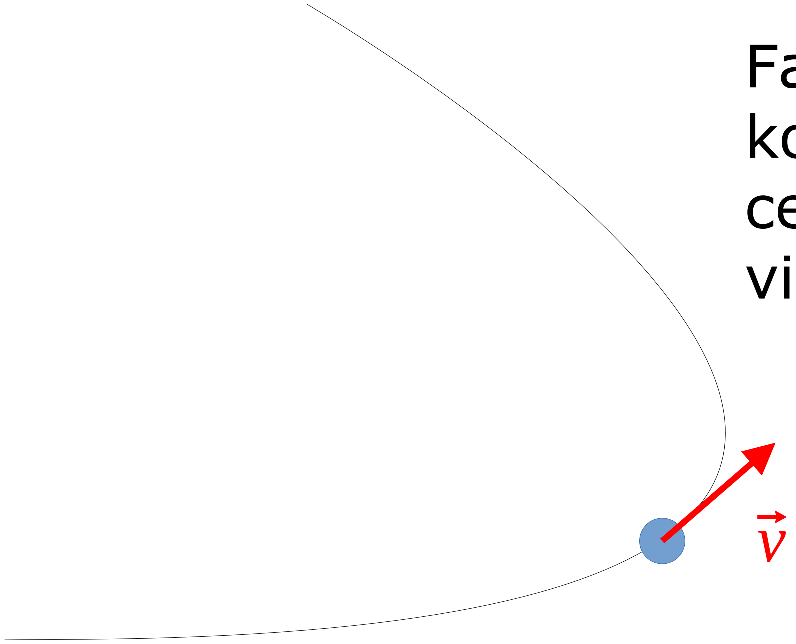
E: Vi kan ikke sige noget

Kurvebevægelse

$$a_c = \frac{v^2(t)}{R(t)} \qquad a_t = \frac{dv}{dt}$$



Quiz - kurvebevægelse



Farten i kurvebevægelsen er konstant. Hvad gælder om centripetalaccelerationen i det viste punkt?

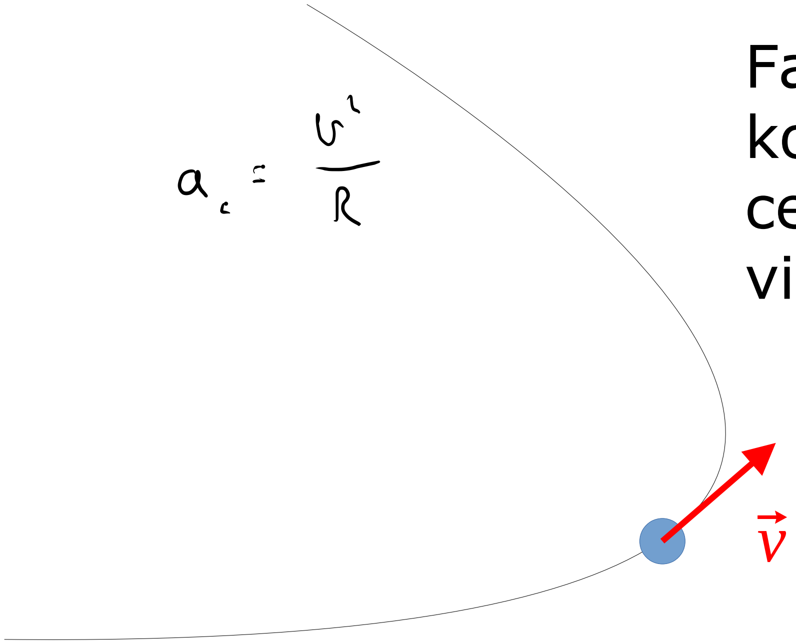
A: a_c vokser

B: a_c aftager

C: a_c er konstant

D: Kan ikke afgøres

Quiz - kurvebevægelse



Farten i kurvebevægelsen er konstant. Hvad gælder om centripetalaccelerationen i det viste punkt?

A: a_c vokser

B: a_c aftager

C: a_c er konstant

D: Kan ikke afgøres

Eksempel - cykelhjul

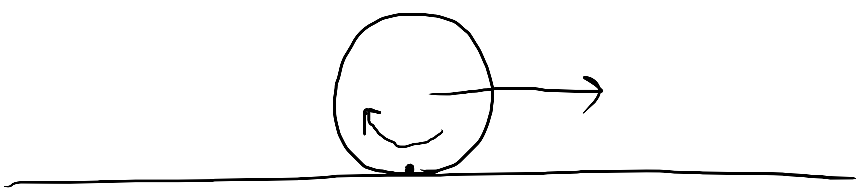
$v = 5.0\,\mathrm{m/s}$

Find a_c for et punkt på dækket

$$a_c = \frac{v^2}{R} \qquad R \simeq 0.35\,\mathrm{m}$$

$$\Downarrow$$

$$a_c \simeq 71.\ \mathrm{m/s^2}$$



Relativ bevægelse - Galileitransformationen

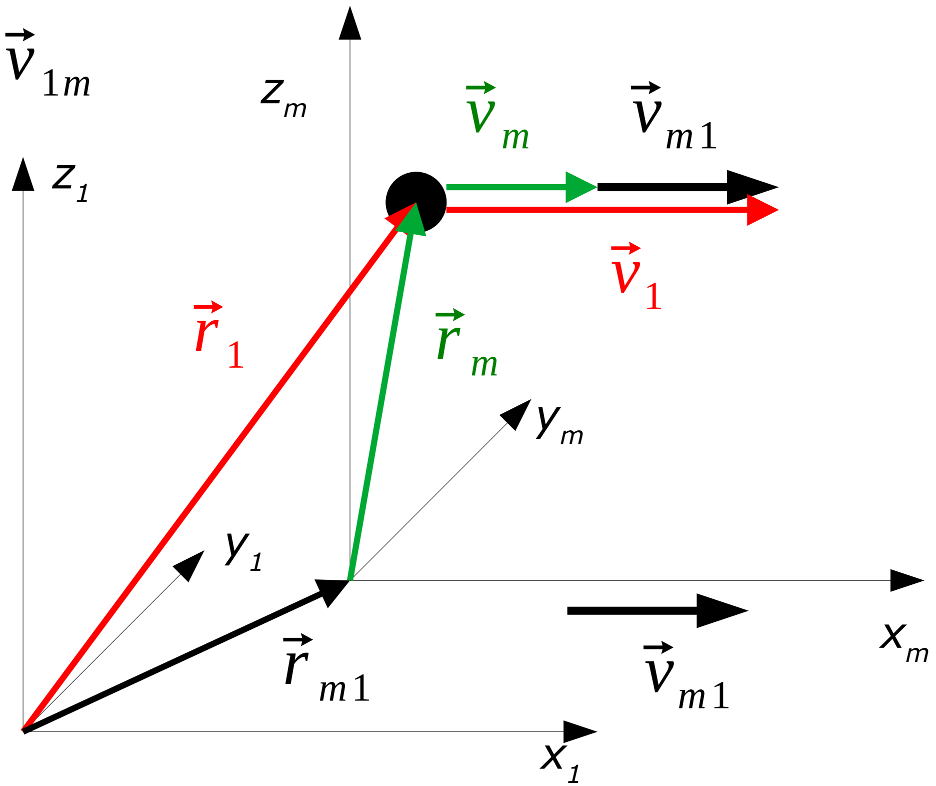
$$\vec{r}_1 = \vec{r}_{m1} + \vec{r}_m \Rightarrow \vec{r}_m = \vec{r}_1 - \vec{r}_{m1} = \vec{r}_1 + \vec{r}_{1m} \qquad \vec{r}_{1m} = -\vec{r}_{m1}$$

$$\Downarrow$$

$$\vec{v}_1 = \vec{v}_{m1} + \vec{v}_m \Rightarrow \vec{v}_m = \vec{v}_1 - \vec{v}_{m1} = \vec{v}_1 + \vec{v}_{1m}$$

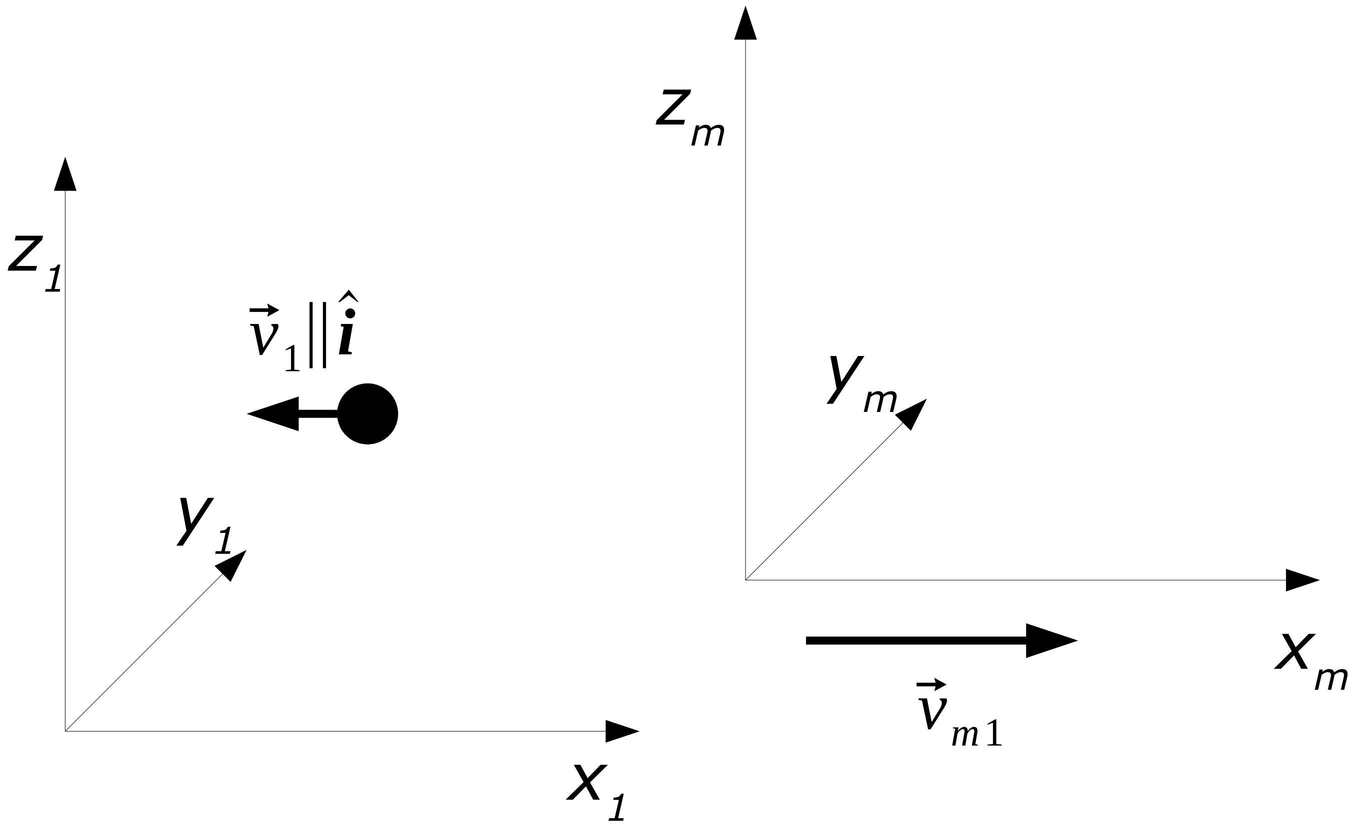
$$\text{Konstant } \vec{v}_{m1} \Rightarrow \vec{a}_1 = \vec{a}_m$$

Fysikkens love er ens i to koordinatsystemer med konstant relativ hastighed
(A. Einstein)



Quiz – relativ bevægelse

Hvad gælder om partiklens *fart* i de to referencesystemer?



A: $v_1 > v_m$

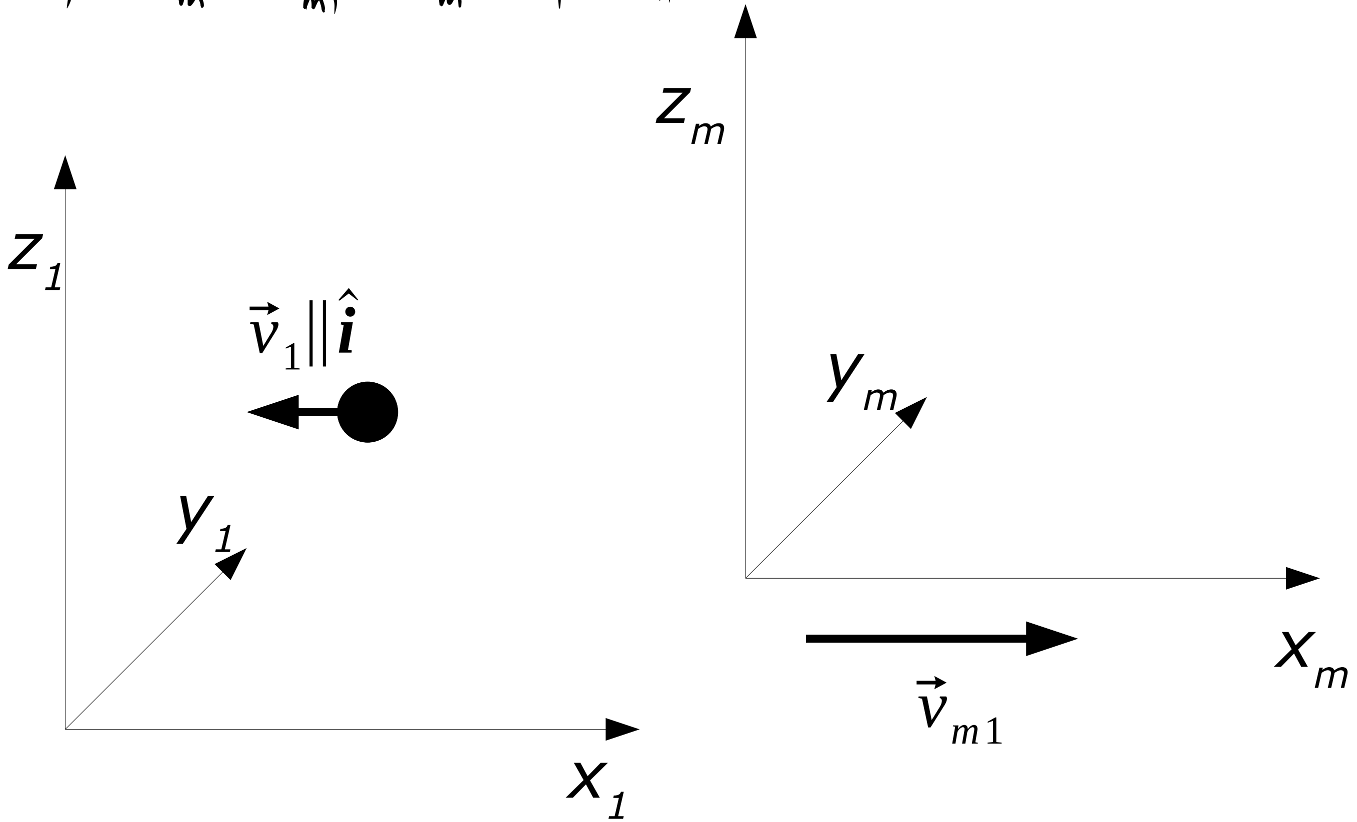
B: $v_1 < v_m$

C: $v_1 = v_m$

Quiz – relativ bevægelse

Hvad gælder om partiklens *fart* i de to referencesystemer?

$\vec{v}_1 = \vec{v}_m + \vec{v}_{m1} \Rightarrow \vec{v}_m = \vec{v}_1 - \vec{v}_{m1}$



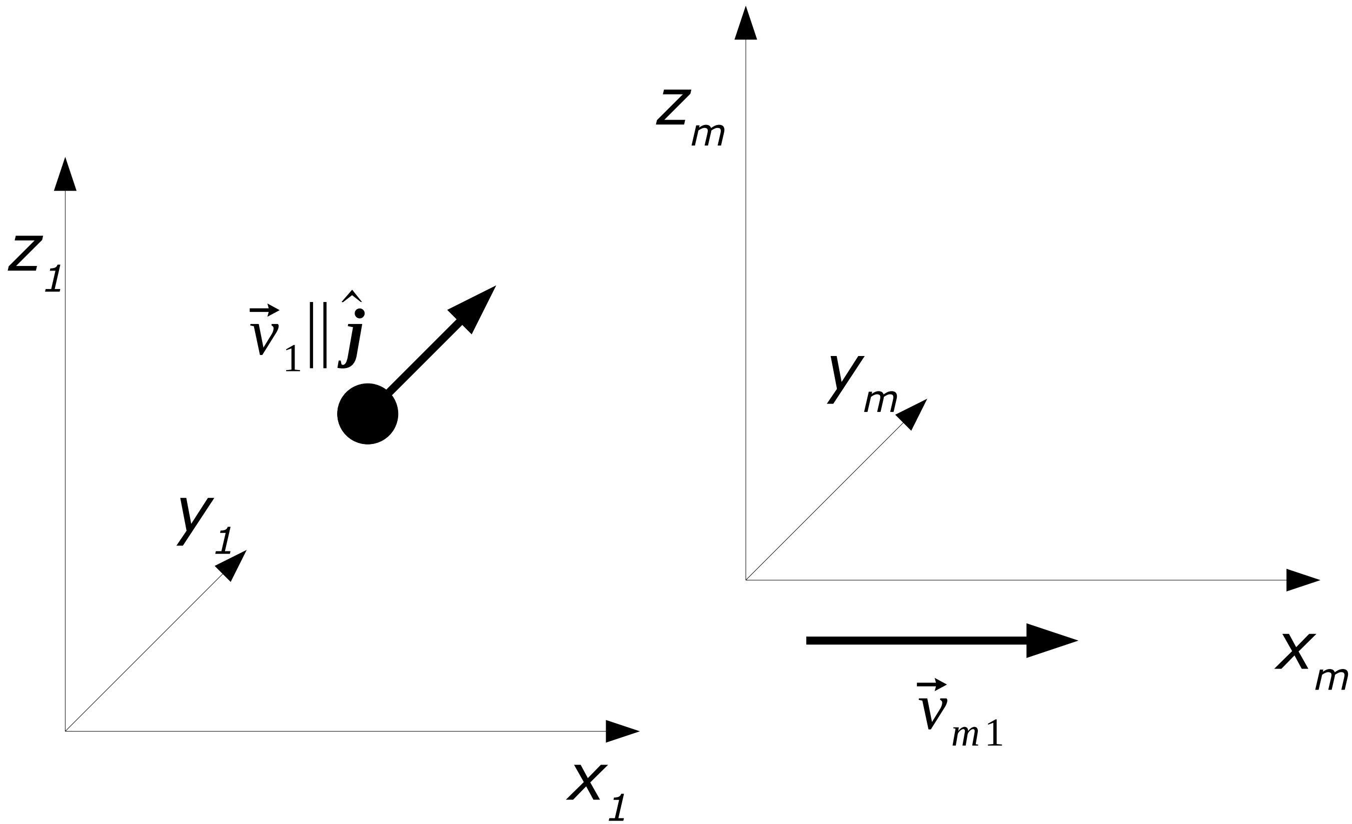
A: $v_1 > v_m$

B: $v_1 < v_m$

C: $v_1 = v_m$

Quiz – relativ bevægelse

Hvad gælder om partiklens *fart* i de to referencesystemer?



A: $v_1 > v_m$

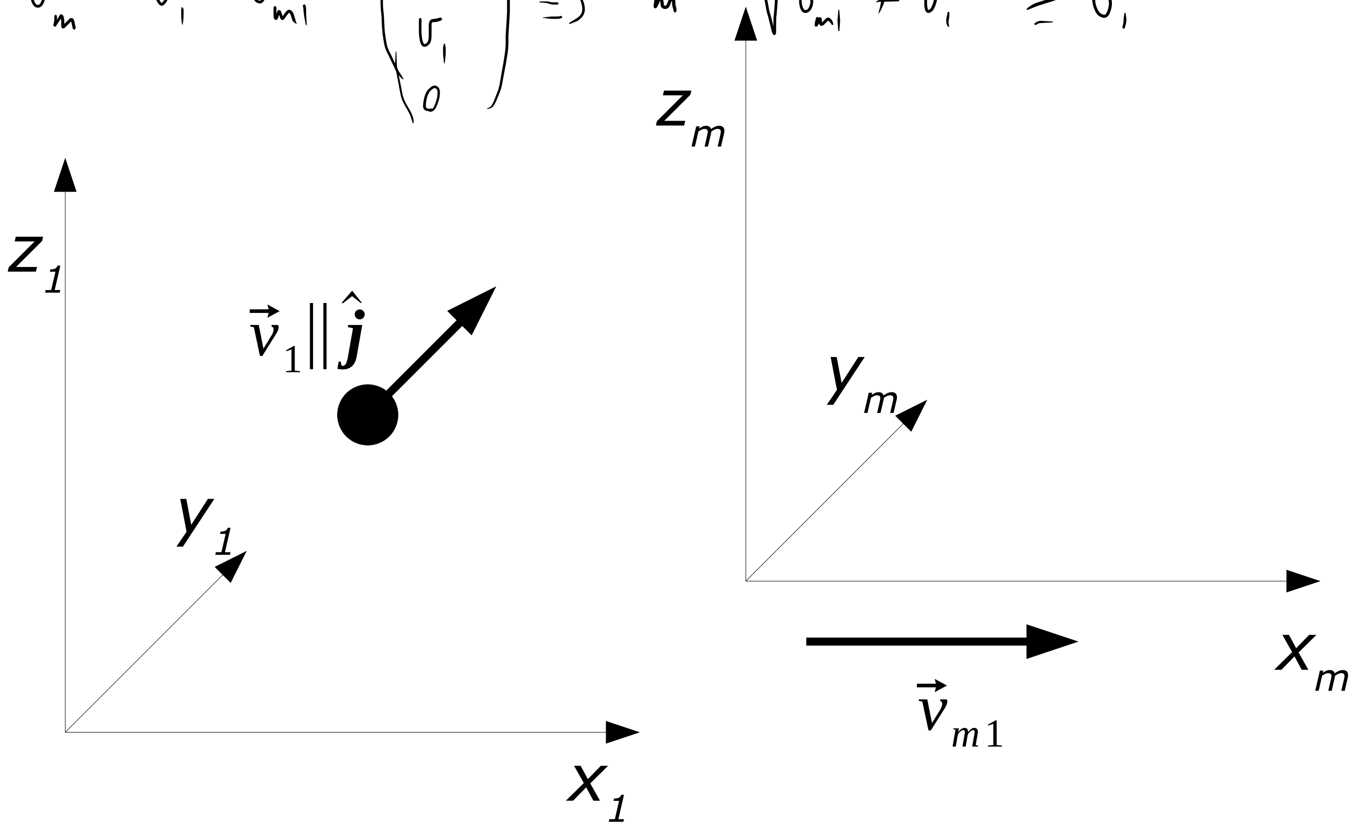
B: $v_1 < v_m$

C: $v_1 = v_m$

Quiz – relativ bevægelse

Hvad gælder om partiklens *fart* i de to referencesystemer?

$$\vec{v}_m = \vec{v}_1 - \vec{v}_{m1} = \begin{pmatrix} -v_{m1} \\ v_1 \\ 0 \end{pmatrix} \Rightarrow v_m = \sqrt{v_{m1}^2 + v_1^2} \geq v_1$$



A: $v_1 > v_m$

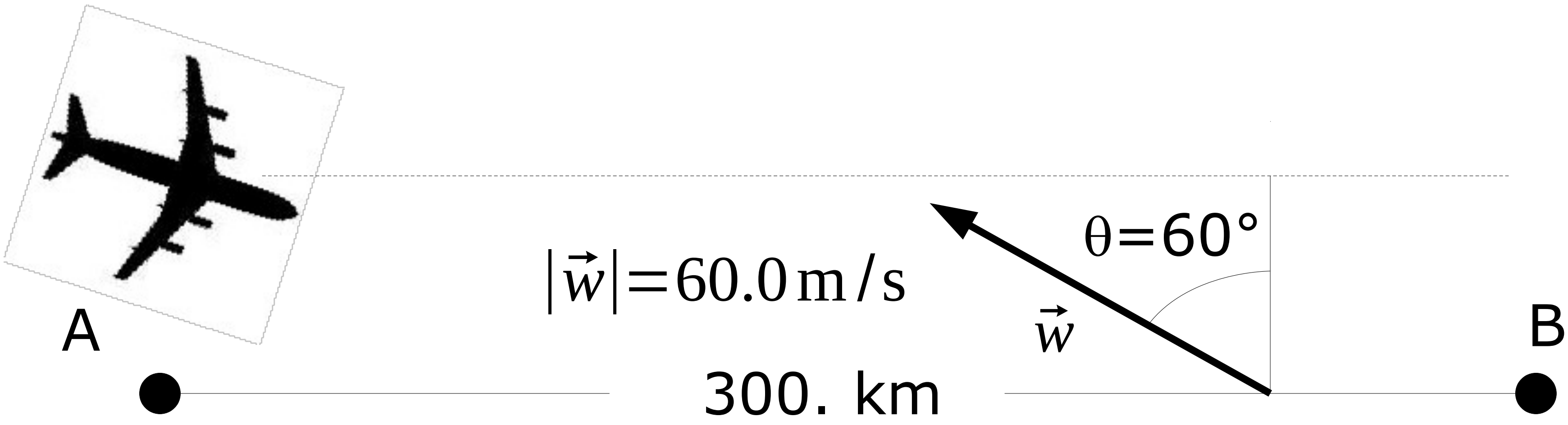
B: $v_1 < v_m$

C: $v_1 = v_m$

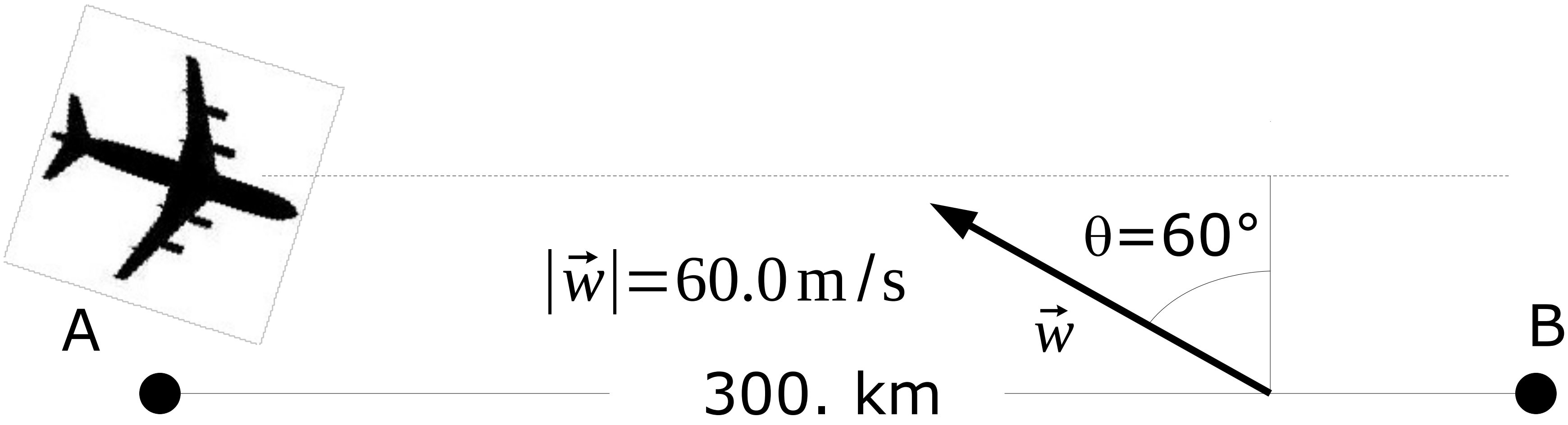
Eksempel: Fly i skrå sidevind

Flyet flyver 250. m/s i forhold til vinden.

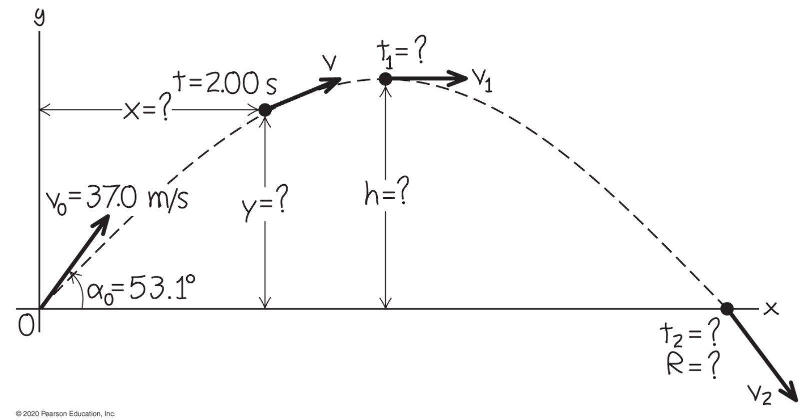
Hvor hurtigt kommer flyet fra A til B?



Eksempel: Fly i skrå sidevind



Opsummering



Kasteparabel:

$$y(x) = y_0 + \frac{v_{0y}}{v_{0x}} x - \frac{g}{2 v_{0x}^2} x^2 = y_0 + \tan(\alpha_0) x - \frac{g}{2 v_0^2 \cos^2(\alpha_0)} x^2$$

$$h = y_0 + \frac{v_{0y}^2}{2g} = y_0 + \frac{v_0^2 \sin^2(\alpha_0)}{2g}$$

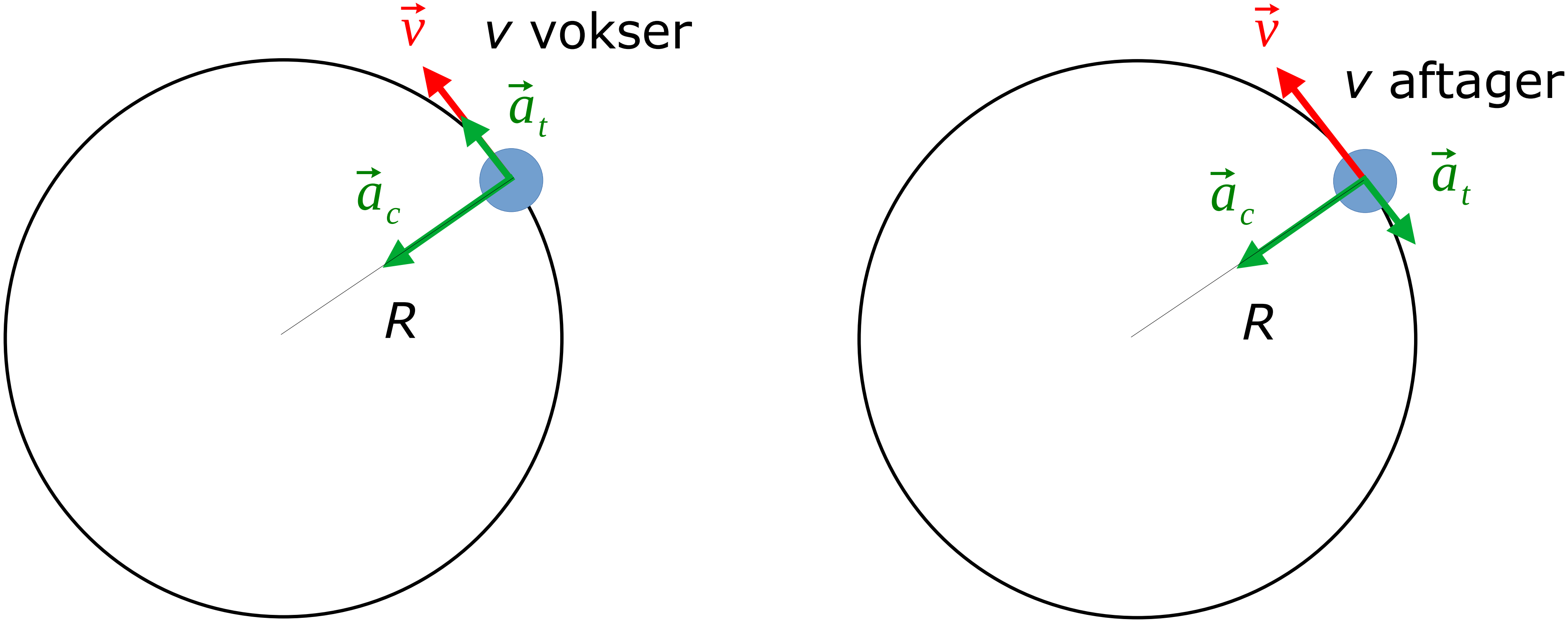
$$R = 2 \frac{v_{0x} v_{0y}}{g} = \frac{v_0^2}{g} \sin 2\alpha_0$$

Kasteparablen er *symmetrisk* – lige lang op- og nedtur.

Opsummering

a_c centripetalacceleration
a_t tangentialacceleration

$$a_c = \frac{v^2}{R} \qquad a_t = |\vec{a}_t| = \frac{dv}{dt}$$



Opsummering

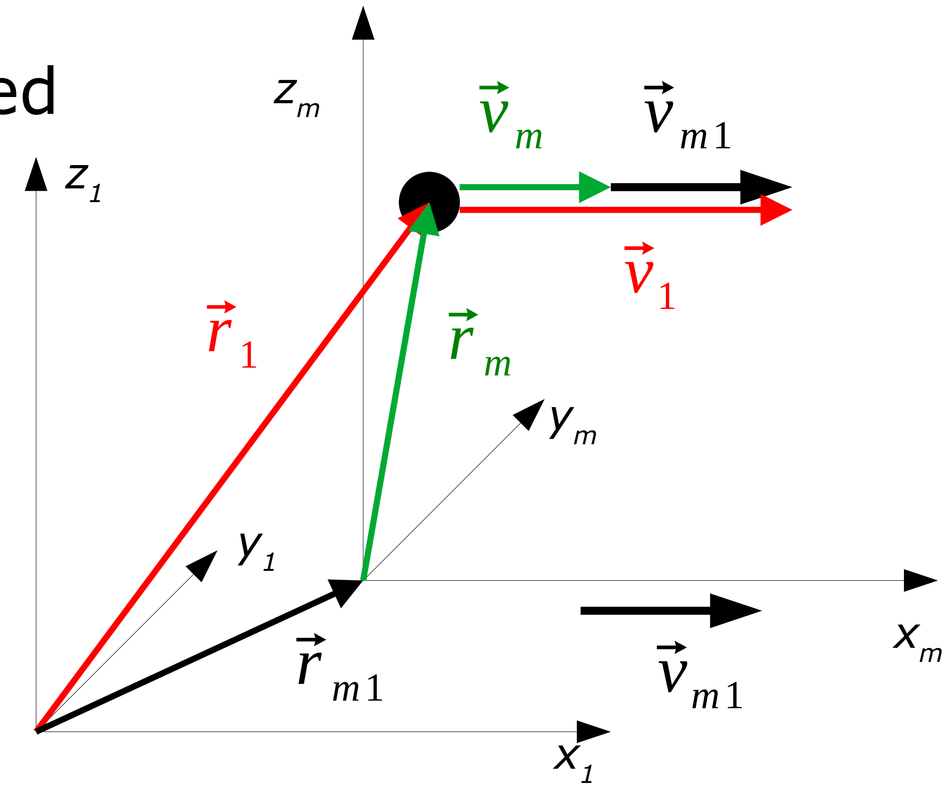
Relativ bevægelse - transformation mellem koordinatsystemer med konstant indbyrdes hastighed

$$\vec{r}_1 = \vec{r}_{m1} + \vec{r}_m \Rightarrow \vec{r}_m = \vec{r}_1 - \vec{r}_{m1}$$

$$\Downarrow$$

$$\vec{v}_1 = \vec{v}_{m1} + \vec{v}_m \Rightarrow \vec{v}_m = \vec{v}_1 - \vec{v}_{m1}$$

Konstant $\vec{v}_{m1} \Rightarrow \vec{a}_1 = \vec{a}_m$



Kræfter og Newtons love, Y & F kap. 4

DTU Fotonik
Institut for Fotonik

Fra sidste gang

$$\vec{r}(t) = x(t)\,\hat{\boldsymbol{i}} + y(t)\,\hat{\boldsymbol{j}} + z(t)\,\hat{\boldsymbol{k}}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\boldsymbol{i}} + \frac{dy}{dt}\hat{\boldsymbol{j}} + \frac{dz}{dt}\hat{\boldsymbol{k}}$$

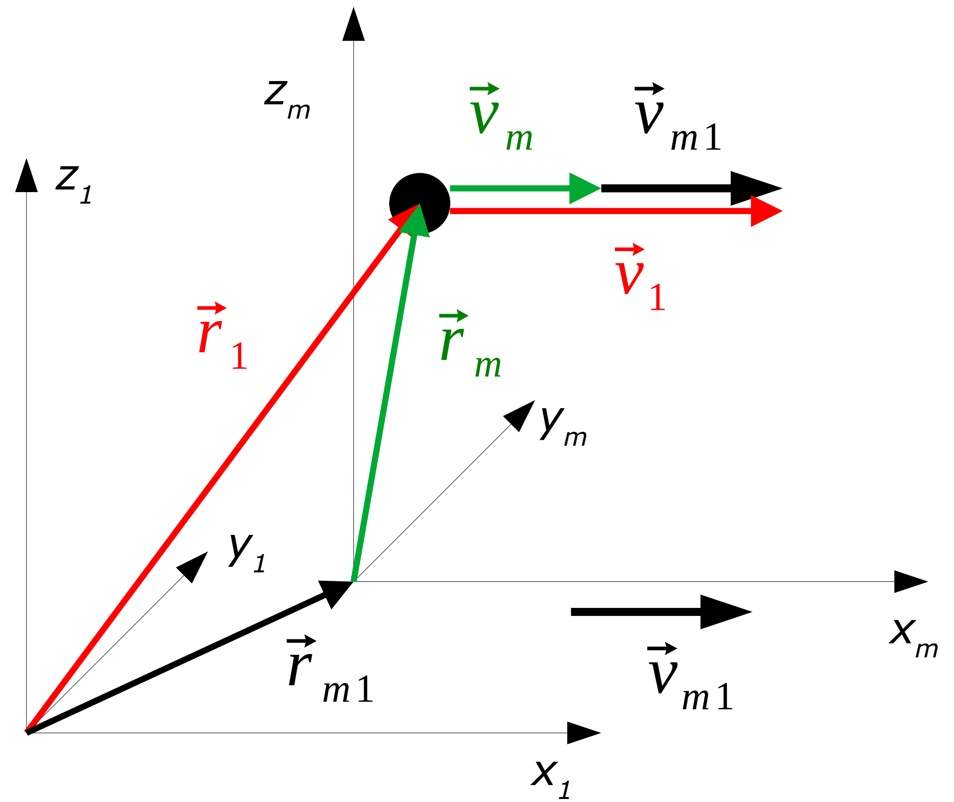
$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2x}{dt^2}\hat{\boldsymbol{i}} + \frac{d^2y}{dt^2}\hat{\boldsymbol{j}} + \frac{d^2z}{dt^2}\hat{\boldsymbol{k}}$$

Relativ bevægelse:

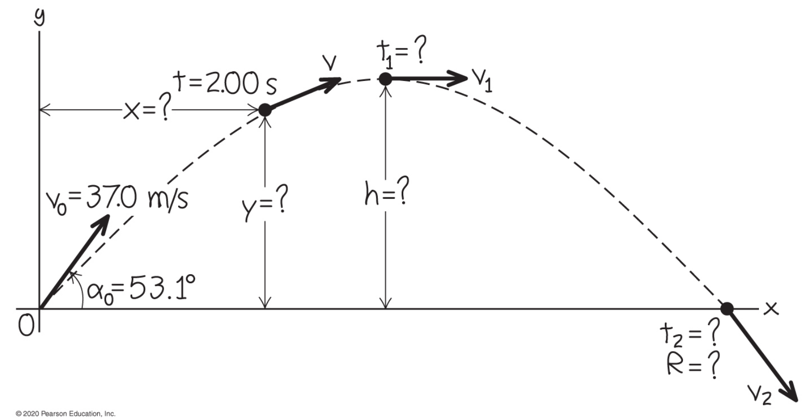
$$\vec{r}_1 = \vec{r}_{m1} + \vec{r}_m \Rightarrow \vec{r}_m = \vec{r}_1 - \vec{r}_{m1}$$

$$\vec{v}_1 = \vec{v}_{m1} + \vec{v}_m \Rightarrow \vec{v}_m = \vec{v}_1 - \vec{v}_{m1}$$

Konstant $\vec{v}_{m1} \Rightarrow \vec{a}_1 = \vec{a}_m$



Fra sidste gang



Kasteparabel:

$$x(t) = x_0 + v_{0x} t \qquad y(t) = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

$$y(x) = y_0 + \frac{v_{0y}}{v_{0x}} x - \frac{g}{2 v_{0x}^2} x^2 = y_0 + \tan(\alpha_0) x - \frac{g}{2 v_0^2 \cos^2(\alpha_0)} x^2$$

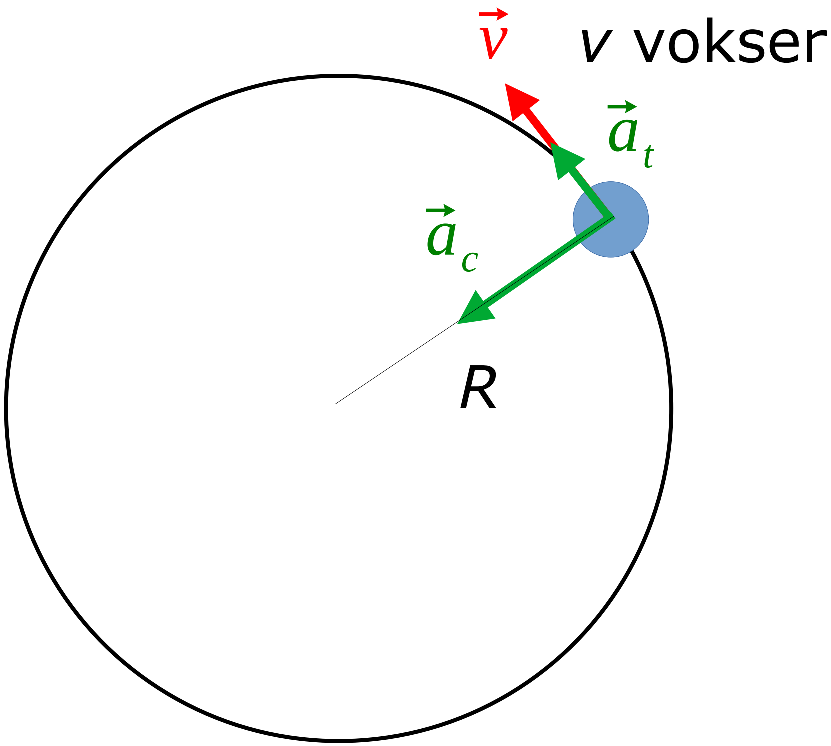
$$h = y_0 + \frac{v_{0y}^2}{2g} = y_0 + \frac{v_0^2 \sin^2(\alpha_0)}{2g} \qquad R = 2\frac{v_{0x} v_{0y}}{g} = \frac{v_0^2}{g} \sin 2\alpha_0$$

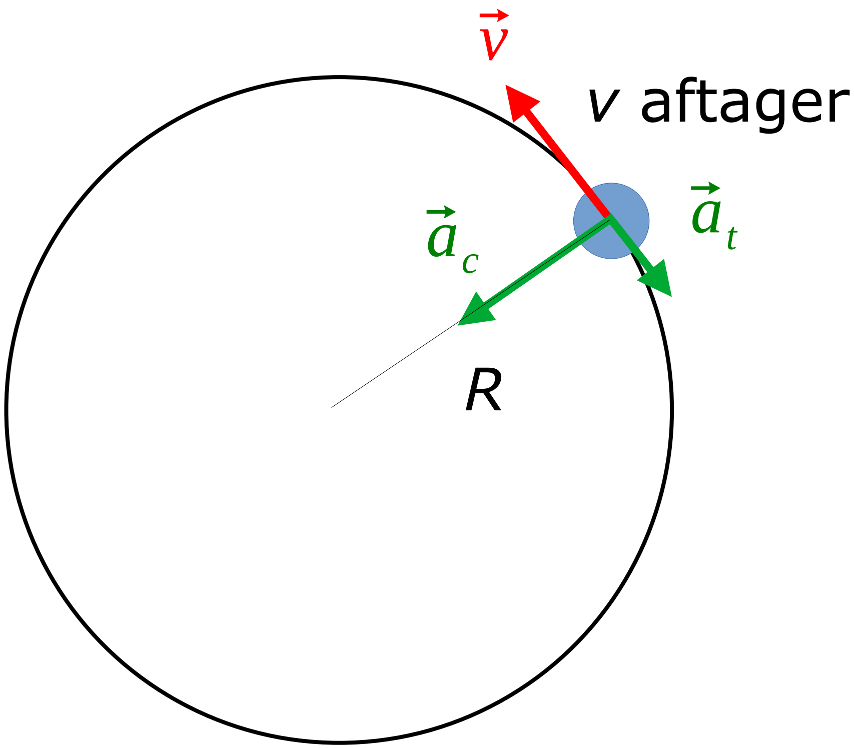
Kasteparablen er *symmetrisk* – lige lang op- og nedtur.

Fra sidste gang

a_c centripetalacceleration
a_t tangentialacceleration

$$\boxed{a_c = \frac{v^2}{R} \qquad |a_t| = |\vec{a}_t| = \left|\frac{dv}{dt}\right| \qquad a_t \equiv \frac{dv}{dt}}$$





Denne uges læringsmål

-Forstå kraftbegrebet og Newtons love

-Forstå *normalkraft, tyngdekraft, snorekraft*

-Benytte kraftdiagrammer

-Løse konkrete mekanikproblemer

Kraftbegrebet intuitivt

Kræfter udøves for at flytte på ting

Kræfter har *retning*, dvs. de beskrives med *vektorer*

Kræfter på samme legeme kan lægges sammen

Newtons love

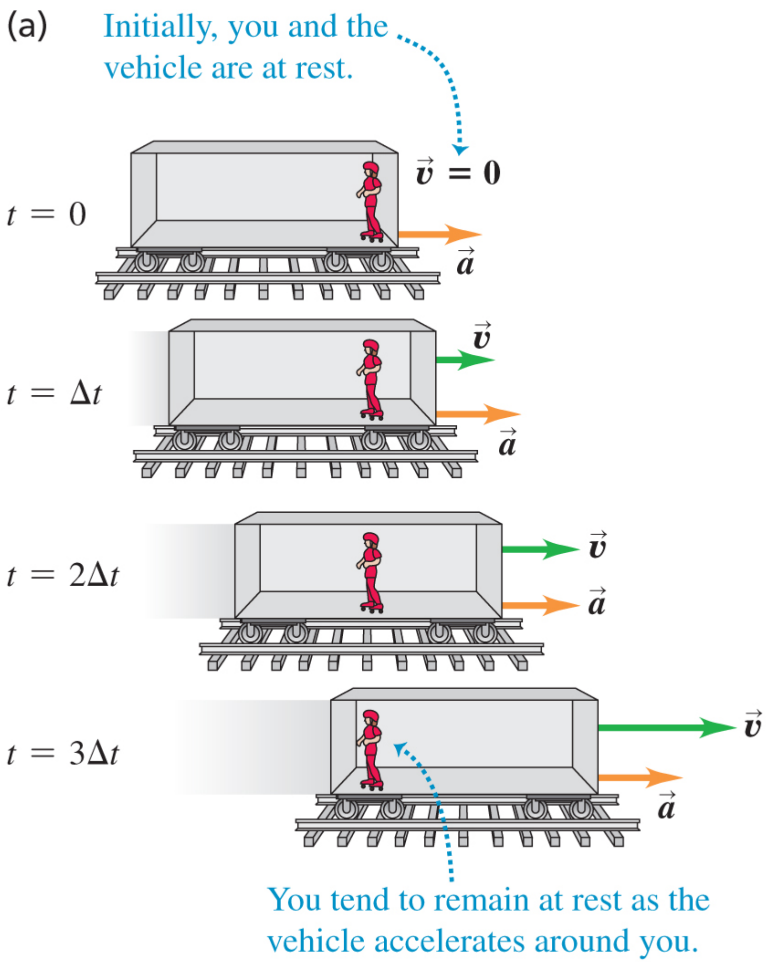
Newton I: Hvis den *resulterende* (samlede) kraft på et legeme er nul vil legemet ligge stille eller bevæge sig med konstant hastighed

Newton II: Et legemes acceleration kan findes ud fra den resulterende kraft på legemet ved formlen

$$\boxed{\vec{F}_{res} = m\vec{a}}$$

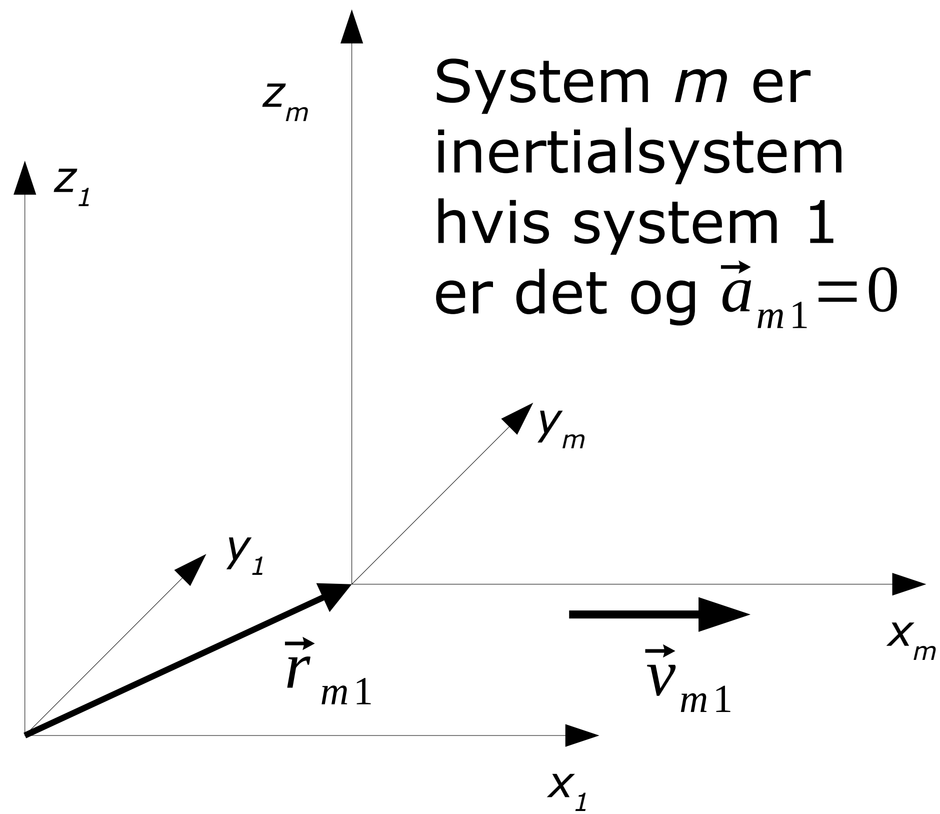
Newton III: To legemer påvirker hinanden med kræfter der er lige store og modsat rettede

Newton I - Inertialsystemer



Inertialsystem=koordinatsystem hvori Newtons love gælder.

System *m* er inertialsystem hvis system 1 er det og $\vec{a}_{m1} = 0$



Quiz - inertialsystemer

Hvis landevejen er et inertialsystem, hvilke af følgende er da også inertialsystemer?

A) Bil på lige vej, bremser

B) Bil på lige vej, konstant fart

C) Bil i kurve, konstant fart

D) Bil på jævn stigning, konstant fart

E) Bil som flyver af vejen, uden kontakt med underlaget

Quiz - inertialsystemer

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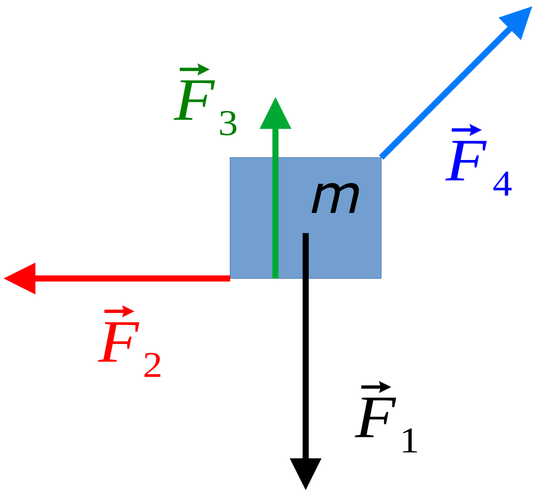
D) Bil på jævn stigning, konstant fart

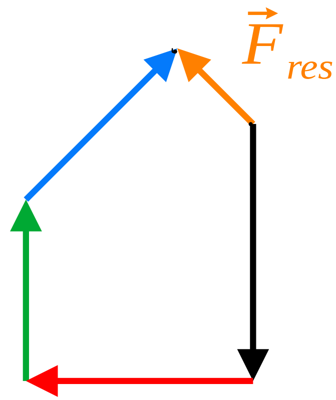
E) Bil som flyver af vejen, uden kontakt med underlaget

Newton II – resulterende kraft, kraftdiagram

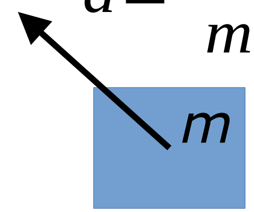
$$\bar{F}_{res} = m\bar{a} \Rightarrow \bar{a} = \frac{\bar{F}_{res}}{m}$$

$$\vec{F}_{res} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$





$$\vec{a} = \frac{\vec{F}_{res}}{m}$$

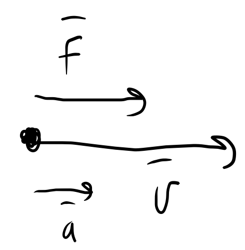


Newton II – kraft og acceleration

Et legeme bevæger sig mod højre under påvirkning af en enkelt kraft der peger mod højre. Størrelsen af kraften aftager som funktion af tiden. Hvad gælder om farten af legemet?

⇨ Legemets fart øges	3	(30 %)
Legemets fart aftager	7	(70 %)
Legemets fart er konstant	0	(0 %)

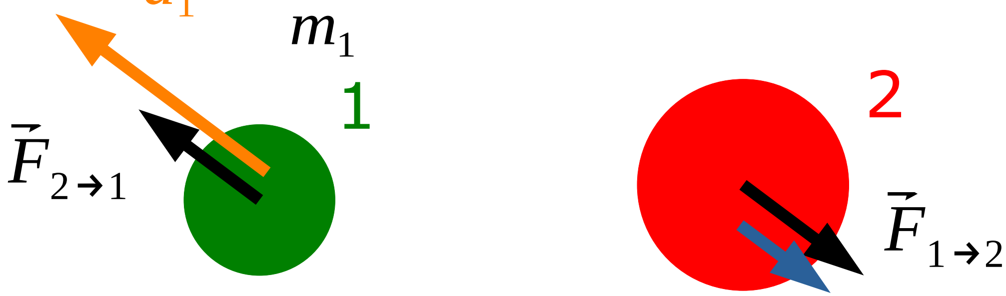
$\bar{a} = \frac{\bar{F}}{m}$



Newton III - aktion/reaktion

Hvis der ikke er *ydre* kræfter:

$$\vec{a}_1 = \frac{\vec{F}_{2\to 1}}{m_1}$$



$$\vec{a}_2 = \frac{\vec{F}_{1\to 2}}{m_2} = -\frac{\vec{F}_{2\to 1}}{m_2}$$

$$\boxed{\vec{a}_1 = -\frac{m_2}{m_1}\vec{a}_2}$$

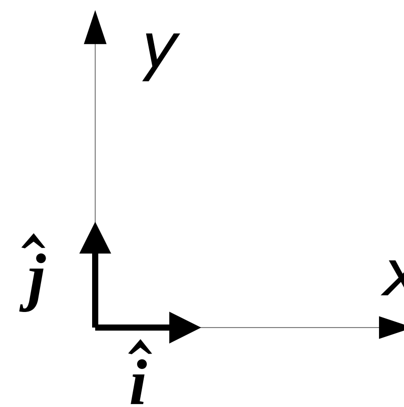
Tyngdekraft

Nær jordoverfladen er tyngdekraften uafhængig af højden (ca.) og proportional med massen.

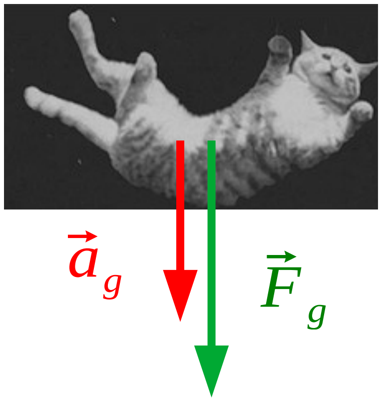
$$\vec{F}_g = -mg\,\hat{\boldsymbol{j}}$$

Indsæt i Newton II:

$$\vec{F}_{res} = -\cancel{m}g\,\hat{\boldsymbol{j}} = \cancel{m}\vec{a} \Rightarrow \vec{a} = -g\,\hat{\boldsymbol{j}}$$



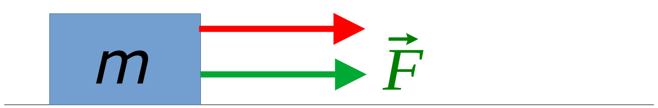
Masse vs. vægt



$\vec{a}_g$ $\vec{F}_g$

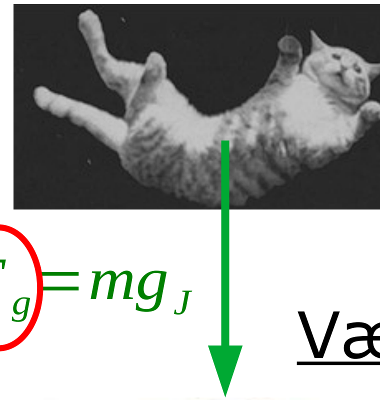
$$\vec{F}_{res} = m\vec{a}$$

Masse



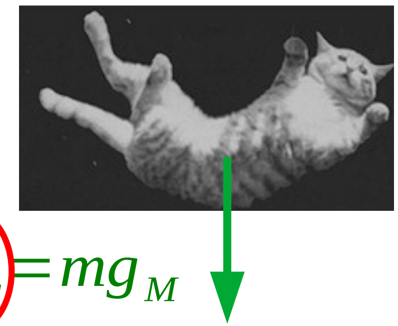
$\vec{a}$

m $\vec{F}$



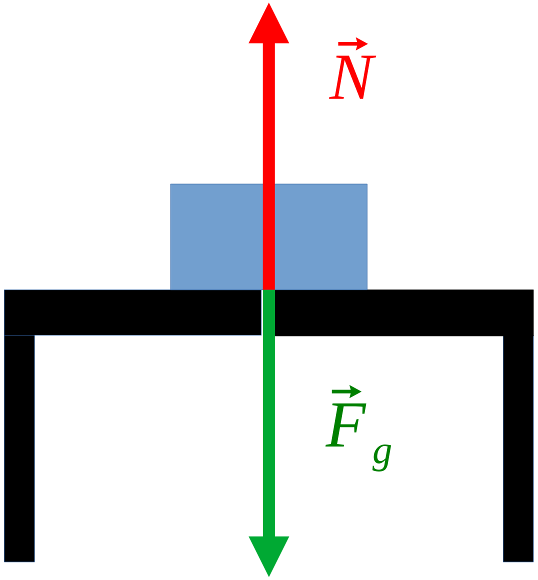
$$F_g = mg_J$$

Vægt



$$F_g = mg_M$$

Normalkraft



Normalkræfter forhindrer ting i at falde igennem faste overflader.

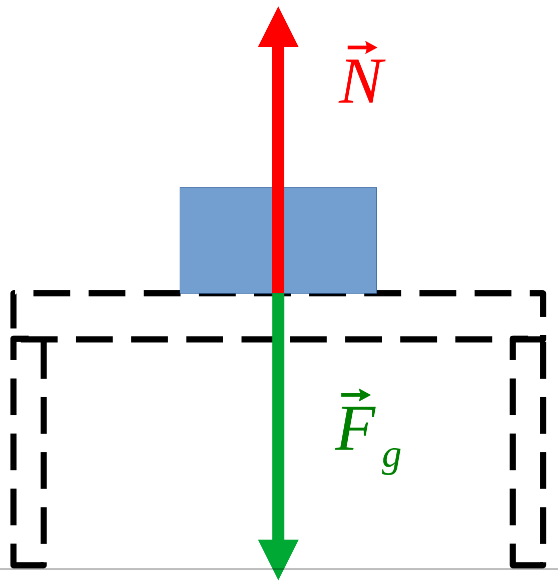
Normalkraften er altid vinkelret på fladen.

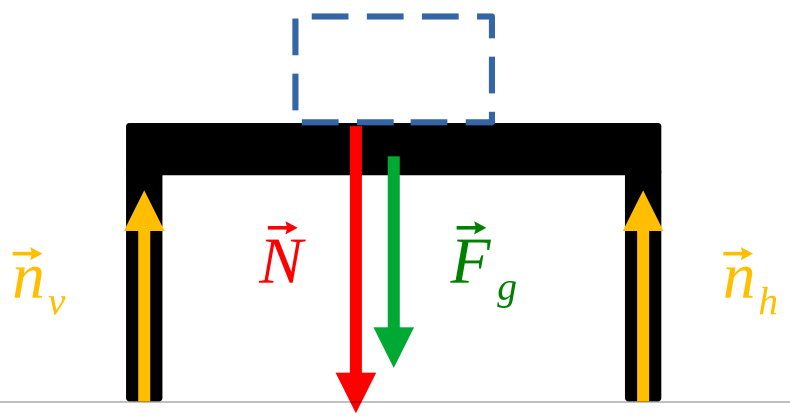
Normalkraften har netop den størrelse som er nødvendig for at genstanden ikke falder igennem.

Normalkræfter kan skubbe, ikke trække (medmindre overfladen er klistret, men så kalder vi det noget andet).

Kraftdiagram ("free body diagram")

Diagram der viser alle kræfter *på et enkelt legeme*





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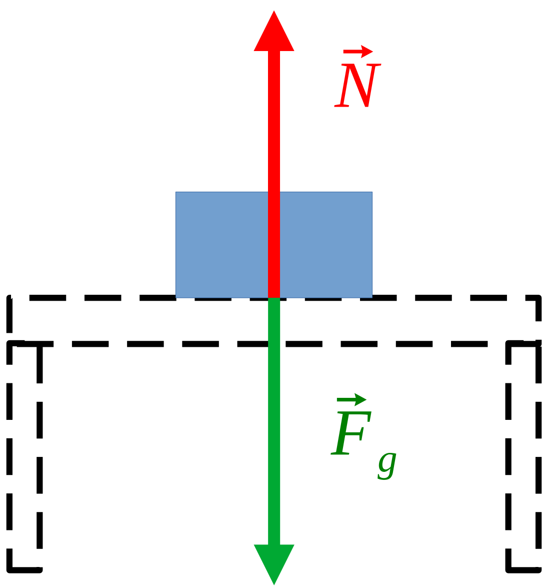
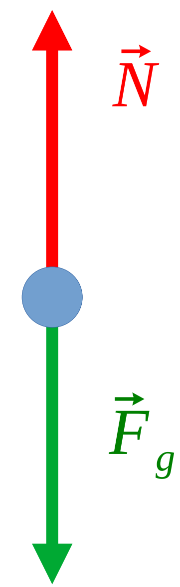


~Punktpartikel
(vi ser på udstrakte legemer senere)

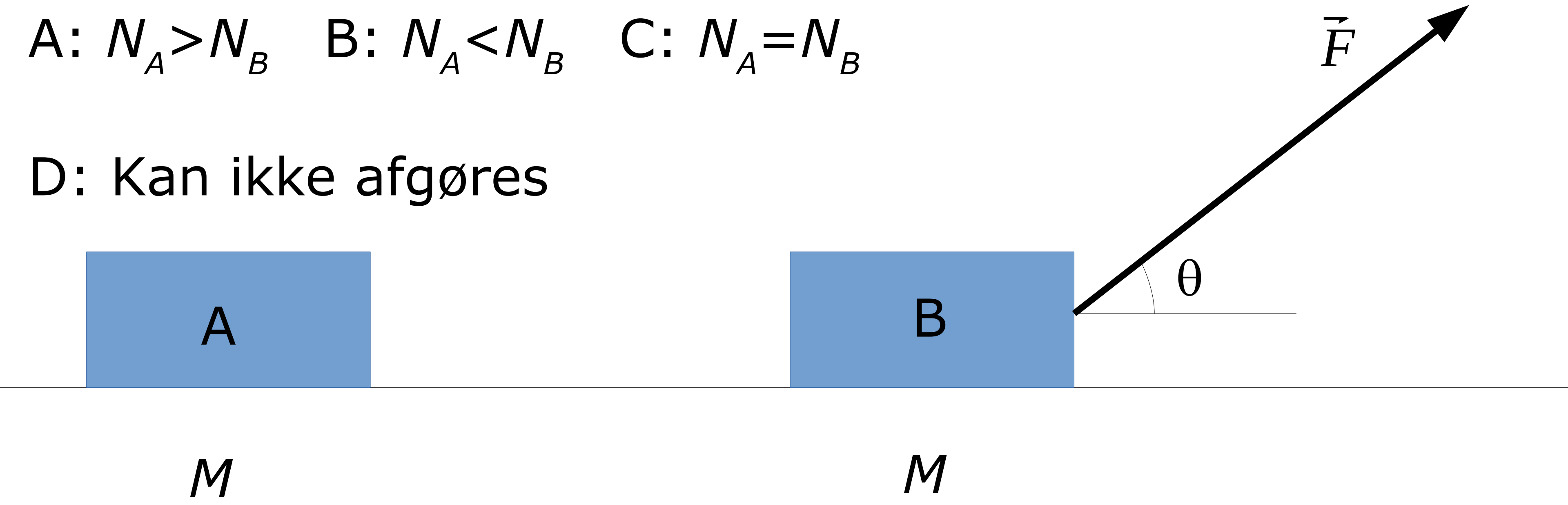


Quiz - normalkraft

Klods A ligger stille, klods B trækkes hen ad bordet med kraften $\vec{F}$. Begge klodser har massen M. Hvad gælder om normalkræfterne på klodserne N_A , N_B?

A: $N_A > N_B$ B: $N_A < N_B$ C: $N_A = N_B$

D: Kan ikke afgøres



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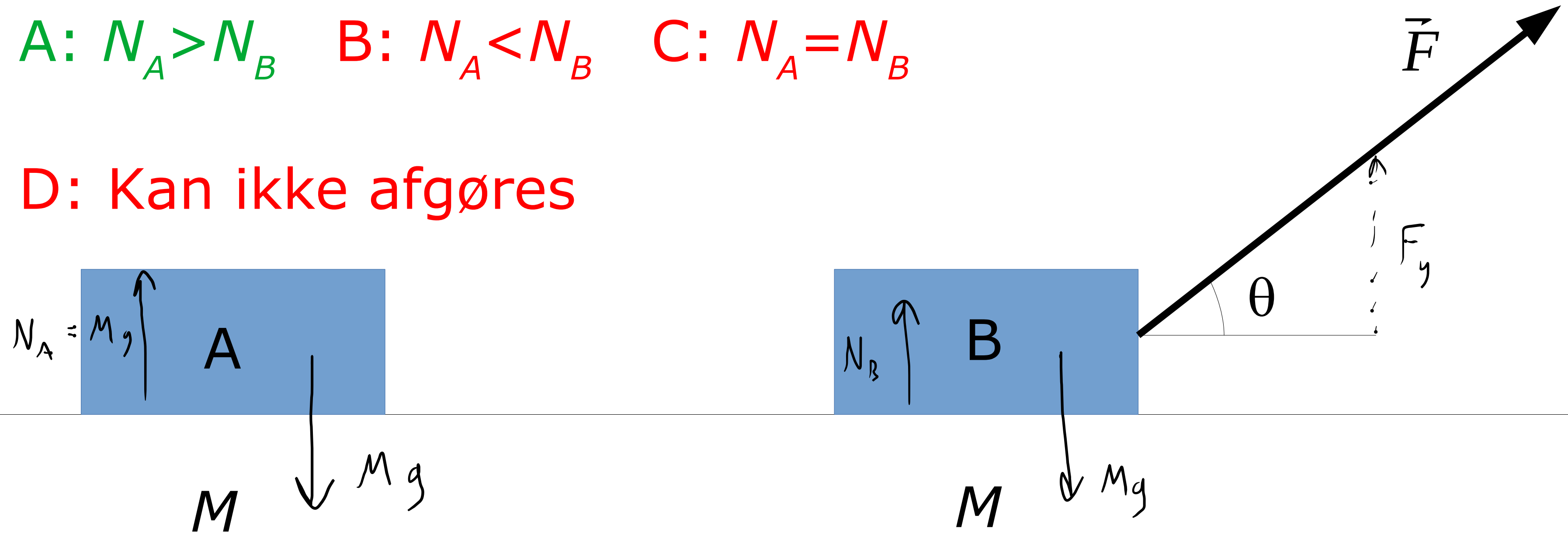
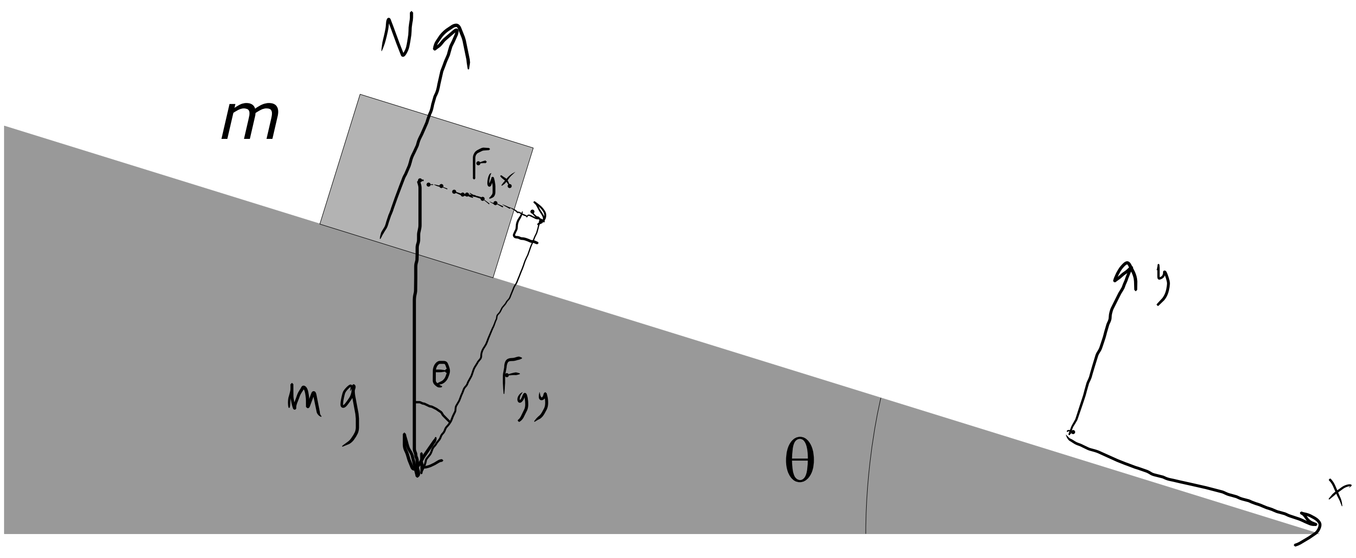


Eksempel: Klods på skråplan I

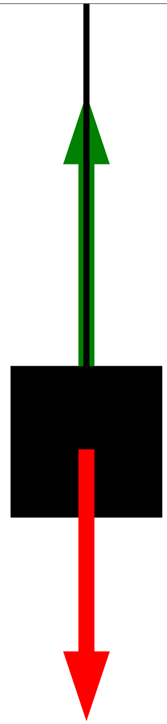
Hvad bliver klodsens acceleration?

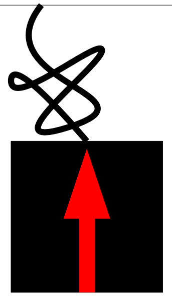
$$N\text{II}_x: F_{res,x} = F_{gx} = \cancel{m} g\sin\theta = \cancel{m} a_x \Rightarrow a_x = g\sin\theta$$

$$N\text{II}_y: N + F_{gy} = N - mg\cos\theta = m a_y = 0 \Rightarrow N = mg\cos\theta$$



Snorekraft





Snorekraften *formidler* kraften fra loftet der holder loddet fast

Snorekraften er altid rettet langs snoren

Snorekraften kan *trække*, men ikke skubbe

Snorspænding

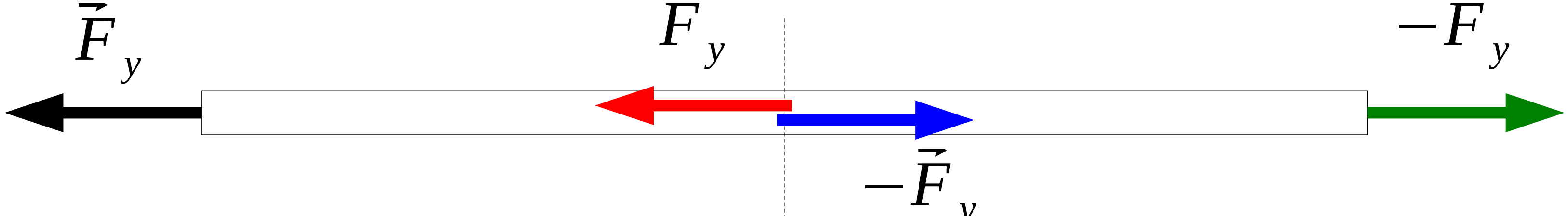
Ydre kraft $\vec{F}_y$

Snorkraft på kugle $\vec{F}_s$

Masseløs snor:

Reaktionskraft på snor fra kugle $\vec{F}_r = -\vec{F}_s$

$$\vec{F}_{res} = \vec{F}_y + \vec{F}_r = m\vec{a} = 0 \Rightarrow \vec{F}_r = -\vec{F}_y \quad \Rightarrow \vec{F}_y = \vec{F}_s$$



Snorspænding, T=F$_y$ (Størrelsen af $\vec{F}_y$) konstant gennem *masseløs* snor. Hvis ingen andre kræfter langs snoren.

Quiz - snorekraft

To personer med samlet masse *m* trækkes op mod helikopteren med konstant fart via en masseløs snor. Snorspændingen er..

A: $T>mg$ B: $T=mg$ C: $T<mg$

D: $T=0$

Quiz - snorekraft

To personer med samlet masse *m* trækkes op mod helikopteren med <u>konstant fart</u> via en masseløs snor.
Snorspændingen er..

A: $T>mg$ B: $T=mg$ C: $T<mg$

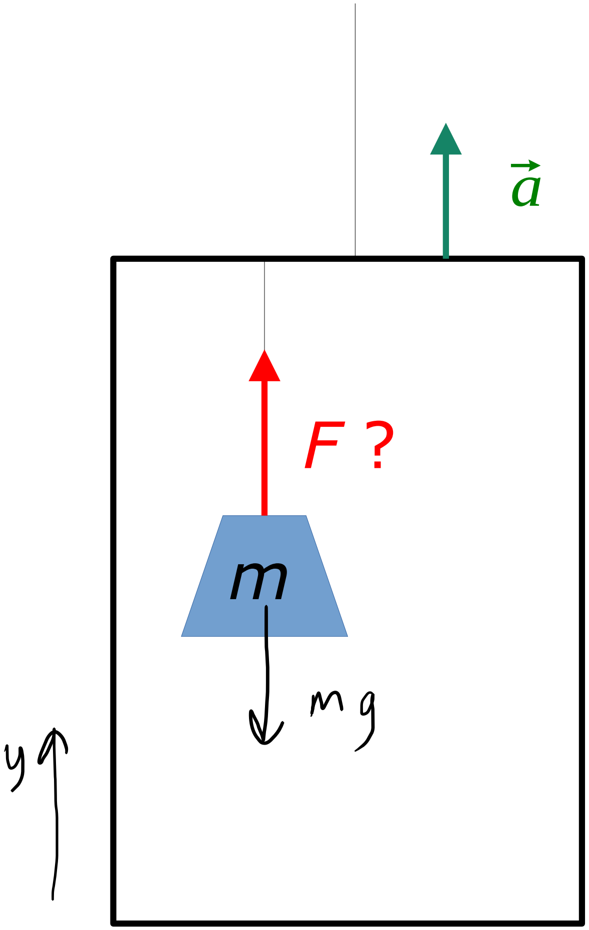
D: $T=0$

Eksempel: Elevator

$$F_{net,y} = F - mg = ma_y$$

$$\Downarrow$$

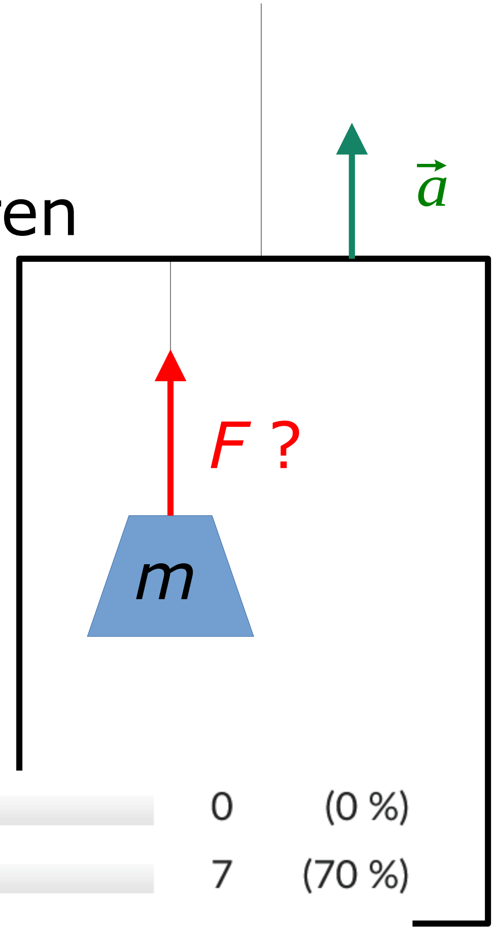
$$F = m(g + a_y)$$



Eksempel: Elevator

Et 10 kg lod hænger i en kraftmåler, der hænger fra loftet i en elevator. Elevatoren bevæger sig og kraftmåleren viser 127 N. Hvad kan man sige om bevægelsesretningen af elevatoren?

$mg \simeq 98.1\ N$



Elevatoren bevæger sig ned ad.		0	(0 %)
Elevatoren bevæger sig op ad.		7	(70 %)
⇨ Man kan ikke afgøre hvilken vej elevatoren bevæger sig.		3	(30 %)

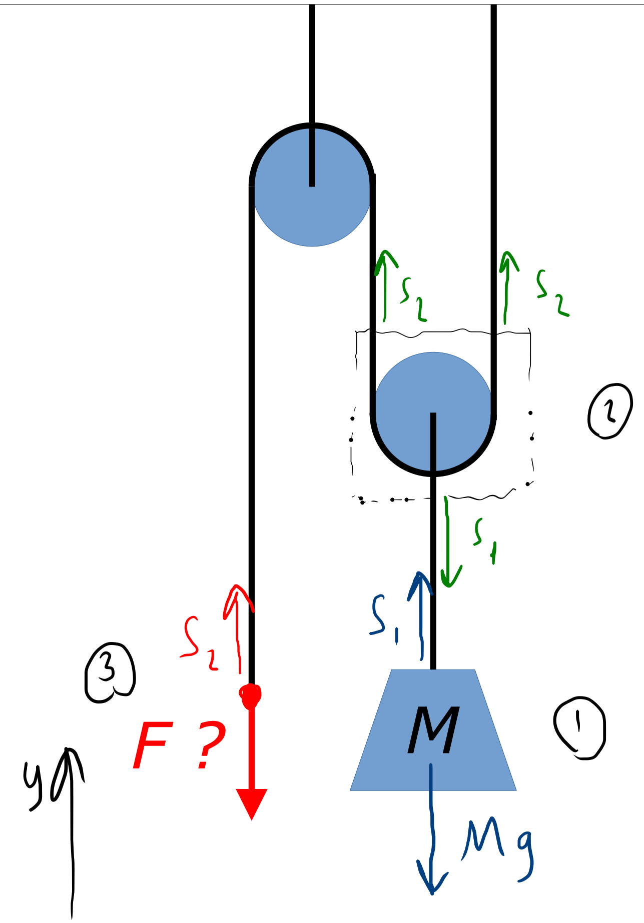
Eksempel – lodder og trisser

Snore og trisser er masseløse.

$$NII_{1}: \quad S_1 - Mg = 0 \Rightarrow S_1 = Mg$$

$$NII_{2}: \quad 2S_2 - S_1 = 0 \Rightarrow S_2 = \frac{1}{2} S_1 = \frac{1}{2} Mg$$

$$NII_{3}: \quad S_2 - F = 0 \Rightarrow F = S_2 = \frac{1}{2} Mg$$



Eksempel: Klods på skråplan II

$$NII_x: \; F_{res,x} = mg\sin\theta - S = ma_x$$

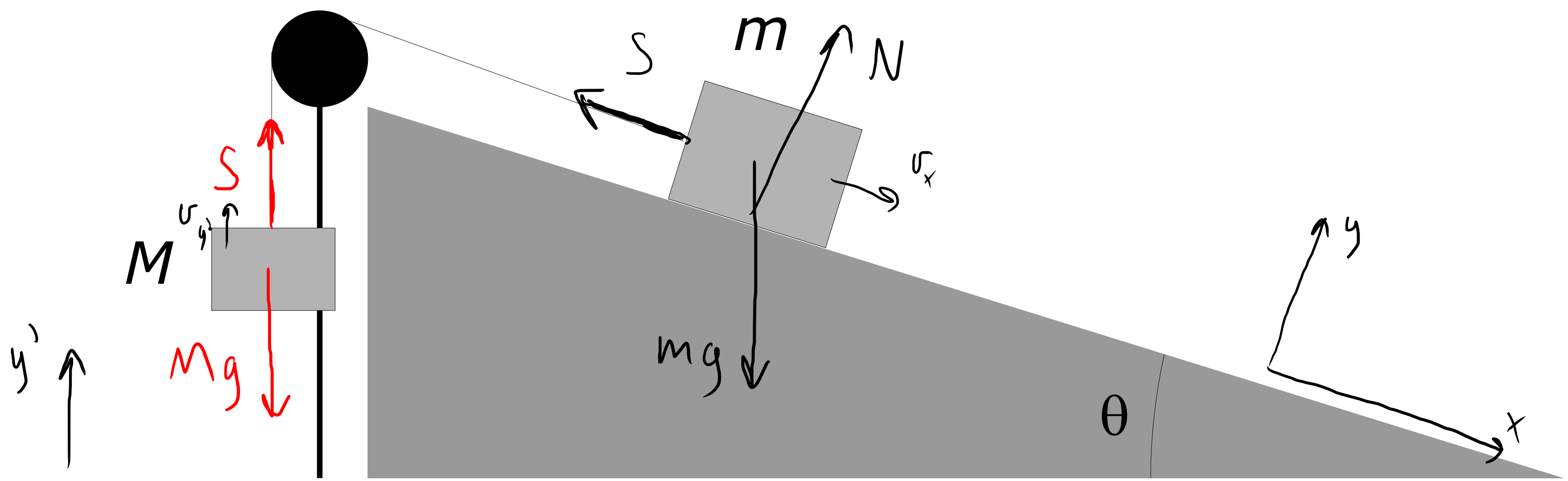
$$NII_y: \; N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$$

$$NII_{y'}: \; S - Mg = Ma_{y'}$$

$$\Downarrow \text{GB}$$

$$S - Mg = Ma_x$$

$$v_x = v_{y'} \Rightarrow \boxed{a_x = a_{y'}}$$ Geometrisk bånd (GB)



Eksempel: Klods på skråplan II

$$mg\sin\theta - S = ma_x \implies mg\sin\theta - M(g + a_x) = ma_x$$

(1)

$$S - Mg = Ma_x$$

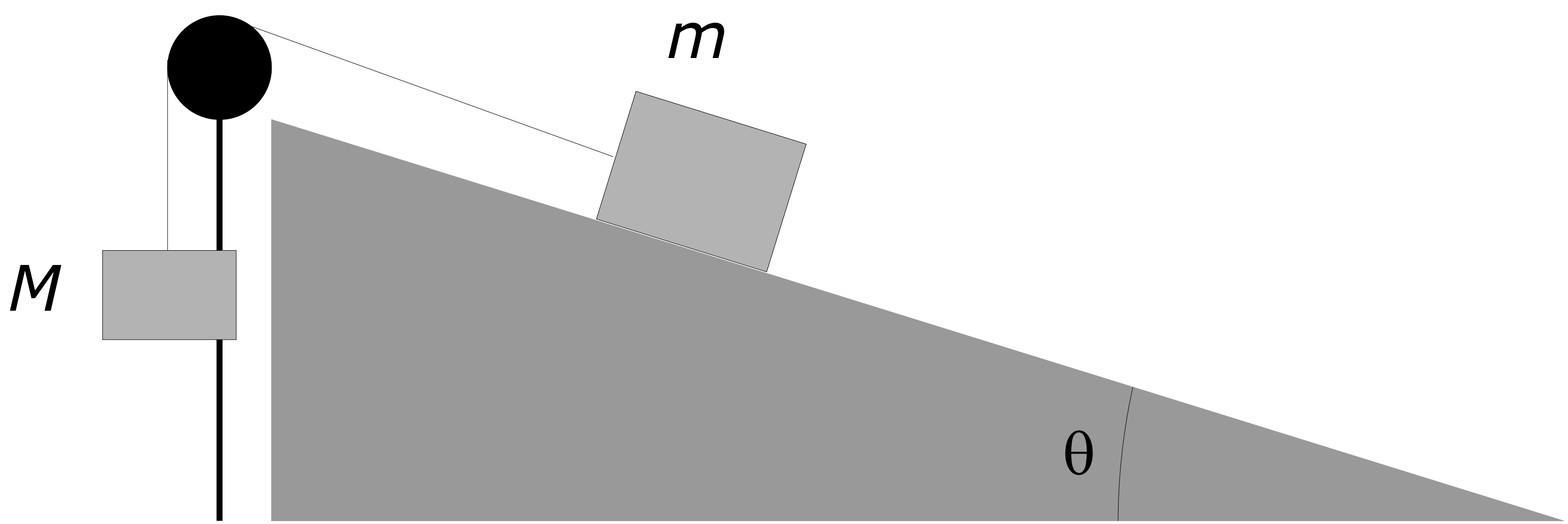
$$\Downarrow$$

$$mg\sin\theta - Mg = (m+M)a_x$$

$$\Downarrow$$

(1) $S = M(g + a_x)$

$$g\,\frac{m\sin\theta - M}{m+M} = a_x$$



Opsummering

Newton I: $\vec{F}_{res} = 0 \Rightarrow \vec{v}\ \text{konstant}$

Newton II: $\vec{F}_{res} = m\vec{a}$

Newton III: To legemer påvirker hinanden med lige store og modsat rettede kræfter

Opsummering

Tyngdekraft trækker nedad proportionalt med massen

Snorekraft er trækkraft i snorens retning, hvis størrelse overalt er *snorspændingen T* (for masseløs snor)

Normalkraft N er vinkelret på fast overflade, og forhindrer genstande i at falde igennem denne.

Mere om Newtons love, Y & F kap. 5

DTU Fotonik
Institut for Fotonik

Fra sidste gang

Newton I: $\vec{F}_{res} = 0 \Rightarrow \vec{v}\ \text{konstant}$

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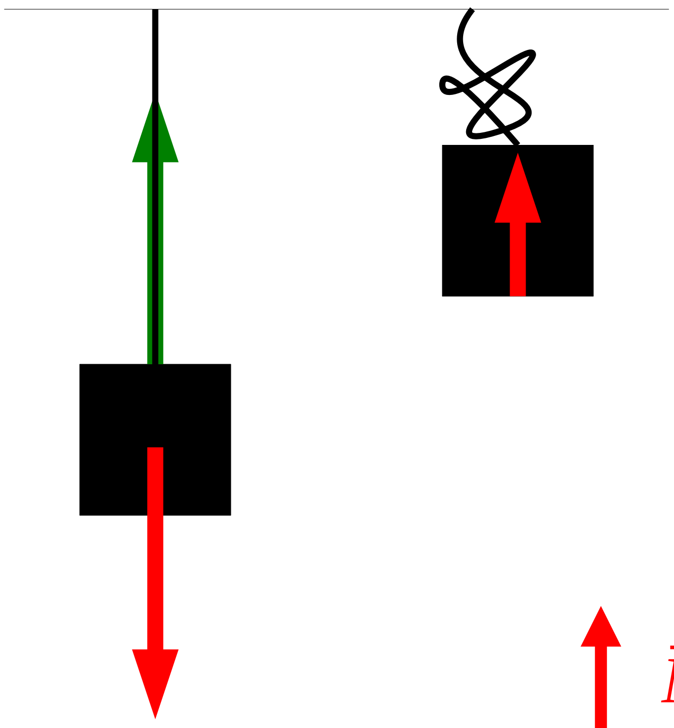
Fra sidste gang

Snorekraft er trækkraft i snorens retning, hvis størrelse overalt er *snorspændingen T* (for masseløs snor)



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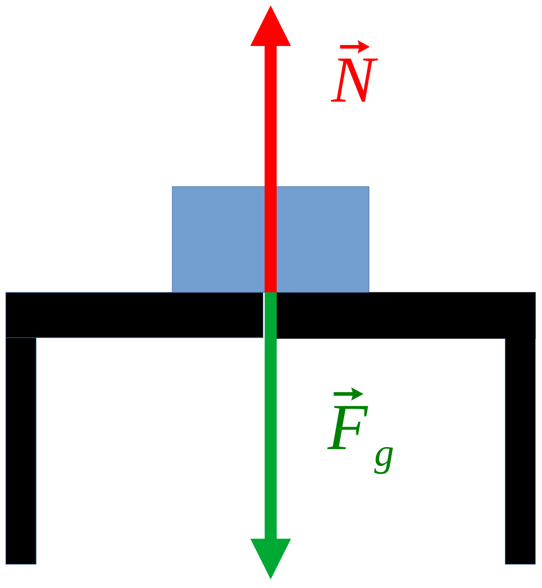
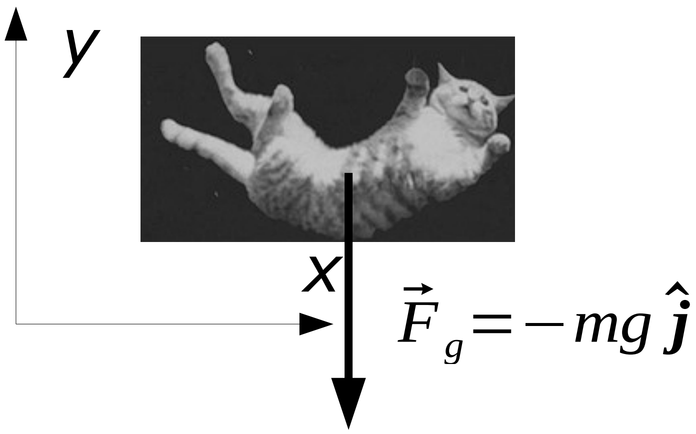
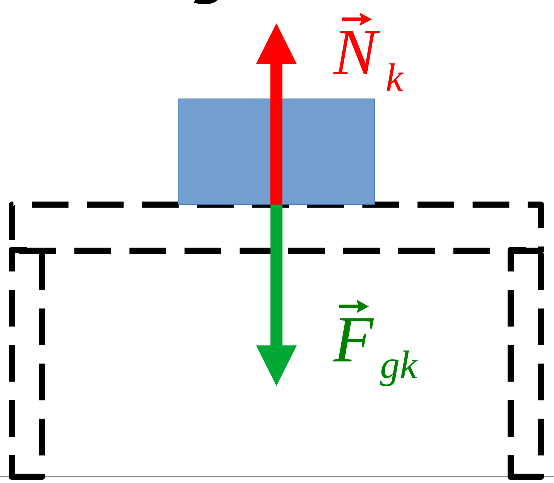


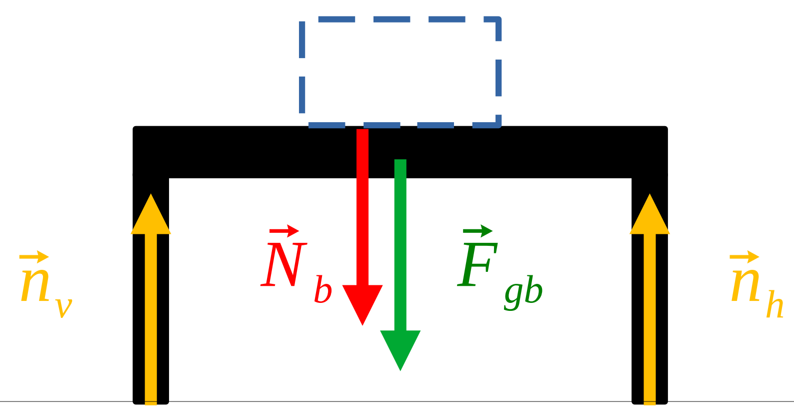
Fra sidste gang

Tyngdekraft er altid rettet nedad, og med konstant størrelse *mg* tæt ved jordoverfladen.



Kraftdiagram viser alle kræfter *på et enkelt legeme*





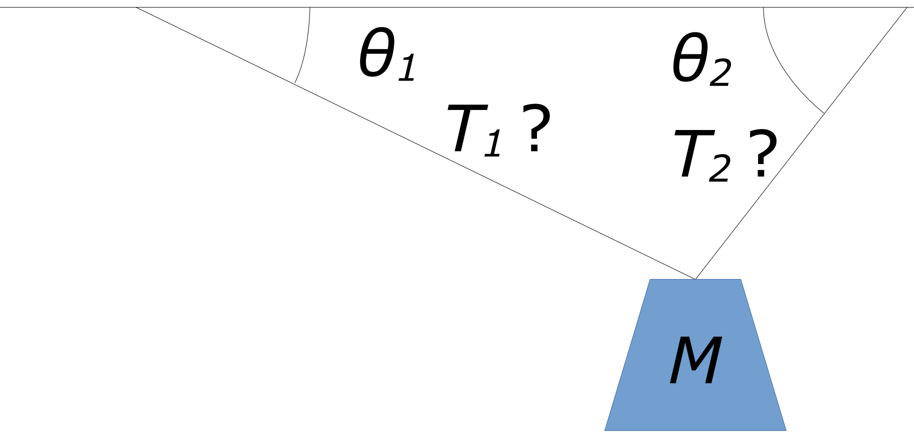
Denne uges læringsmål

-Benytte *standardmetoden* til løsning af ligevægtsproblemer og dynamiske problemer i mekanik

-Forstå *statisk* og *kinetisk kontaktfriktion*

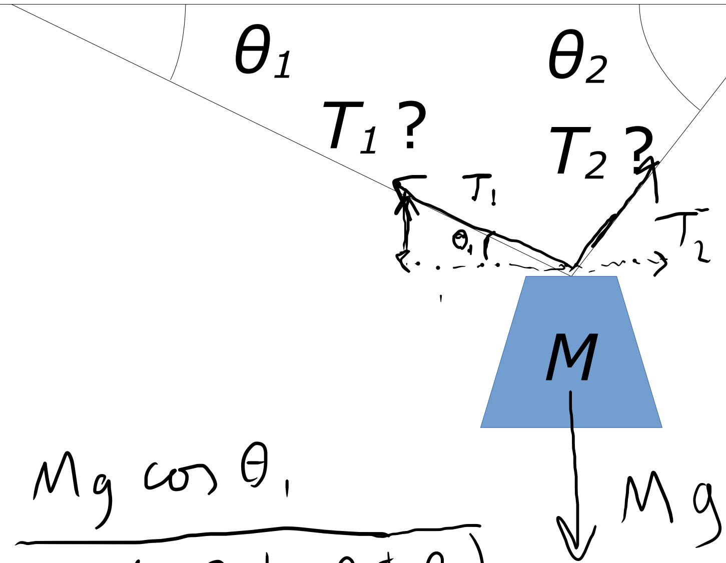
-Beskrive kræfter og acceleration i cirkelbevægelse

Eksempel: Ligevægtsproblem (statik)



Hvad bliver snorspændingerne i de to snore, T_1 og T_2 ?

Eksempel: Ligevægtsproblem (statik)



$$① \; NI_x: -T_1 \cos\theta_1 + T_2 \cos\theta_2 = 0$$

$$② \; NI_y: T_1 \sin\theta_1 + T_2 \sin\theta_2 - Mg = 0$$

$$\Downarrow$$

$$① \; T_2 \cos\theta_2 = T_1 \cos\theta_1 \Rightarrow T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2}$$

$$\Downarrow ②$$

$$T_1 \sin\theta_1 + T_1 \sin\theta_2 \frac{\cos\theta_1}{\cos\theta_2} = Mg$$

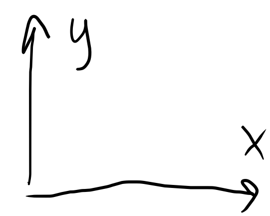
$$\Downarrow$$

$$T_1 (\sin\theta_1 + tg\,\theta_2 \cos\theta_1) = Mg$$

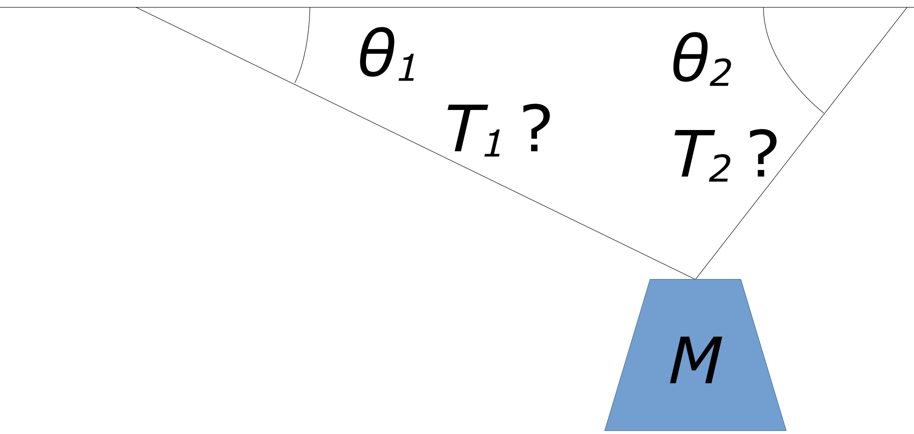
$$\Downarrow$$

$$T_1 = \frac{Mg}{\sin\theta_1 + tg\,\theta_2 \cos\theta_1}$$

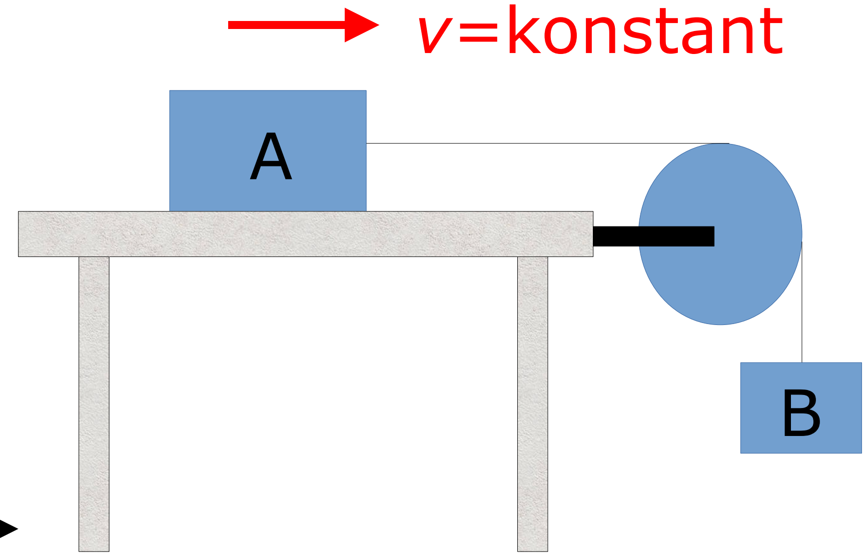
$$T_2 = \frac{Mg \cos\theta_1}{\cos\theta_2 (\sin\theta_1 + \cos\theta_1 \, tg\,\theta_2)}$$



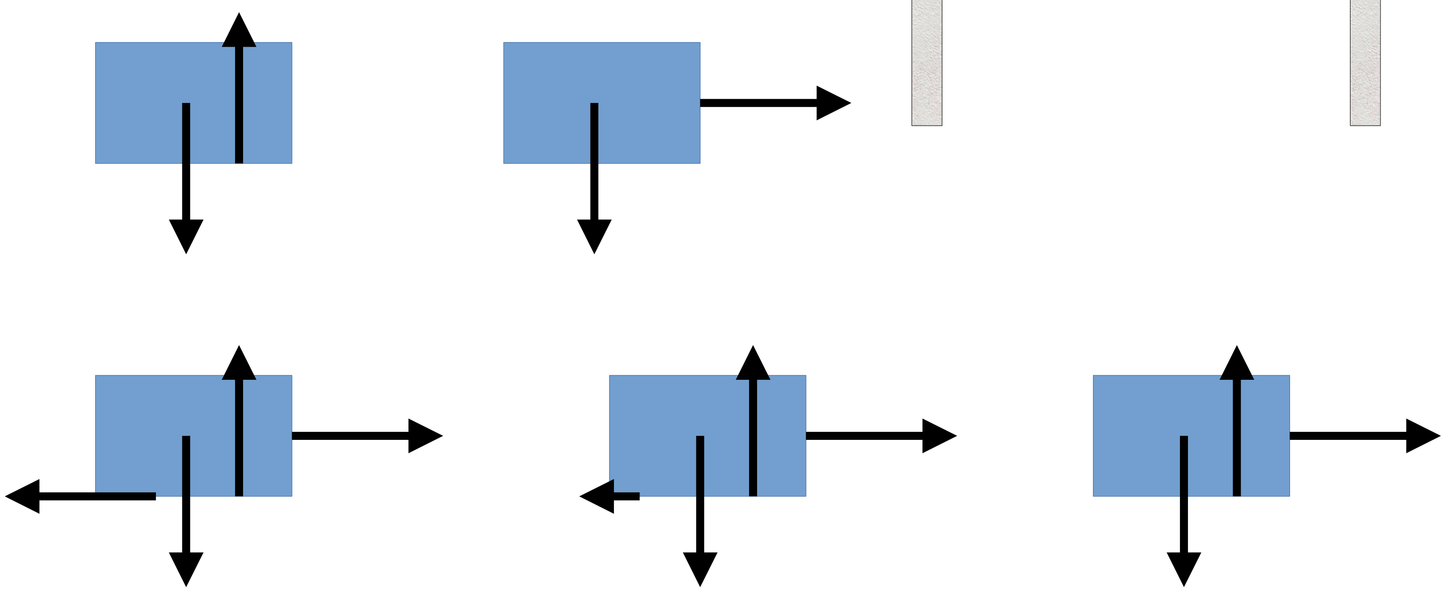
Eksempel: Ligevægtsproblem (statik)



Quiz: Kraftdiagram

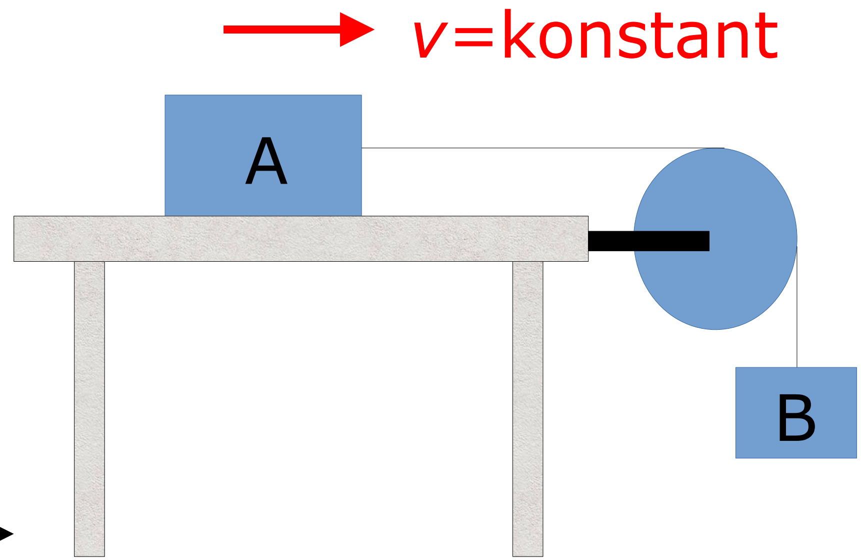


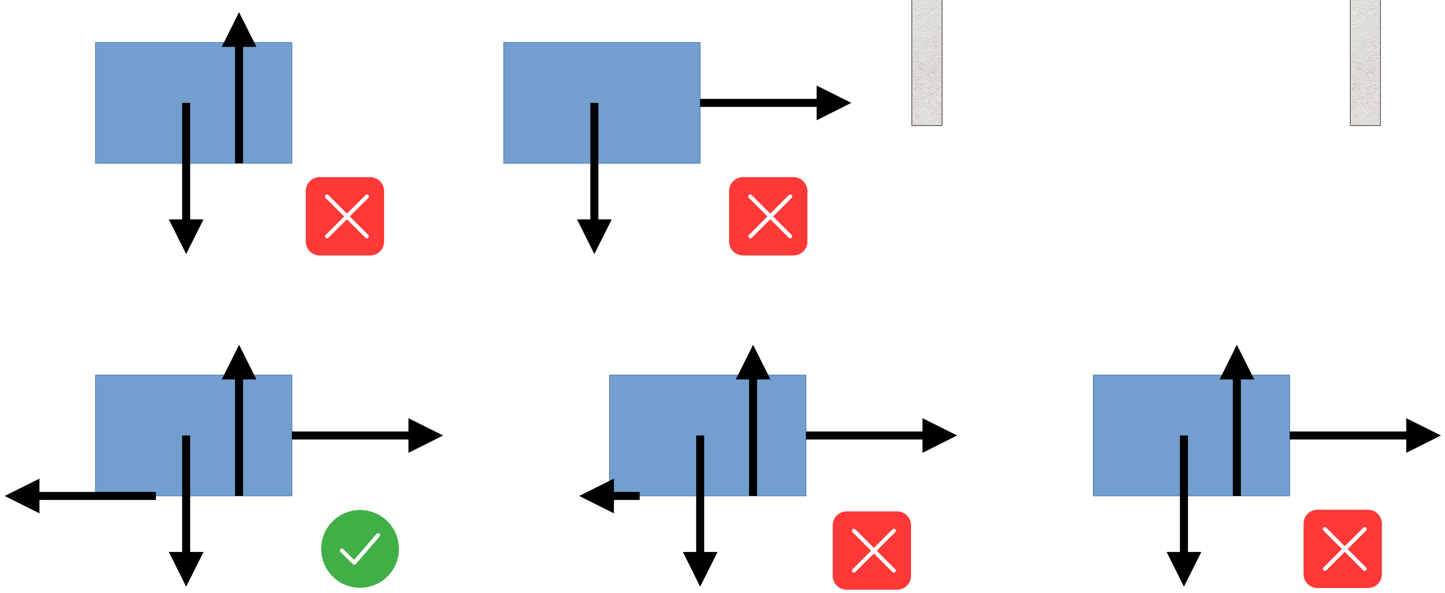
Hvilket kraftdiagram er korrekt for klods A?



Quiz: Kraftdiagram

Hvilket kraftdiagram er korrekt for klods A?





Eksempel: Dynamisk problem

'Atwood maskine': Hvad bliver loddernes acceleration?

$$\mathrm{NII}_{My}: \; S - Mg = M a_M \quad ① \qquad \mathrm{NII}_{my}: \; S - mg = m a_m \quad ②$$

$$\boxed{a_M = -a_m \quad \mathrm{GB}}$$

$$\overset{②}{\Rightarrow} \; S - mg = -m a_M \Rightarrow S = m(g - a_M)$$

$$\Downarrow ①$$

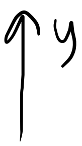
$$m(g - a_M) - Mg = M a_M$$

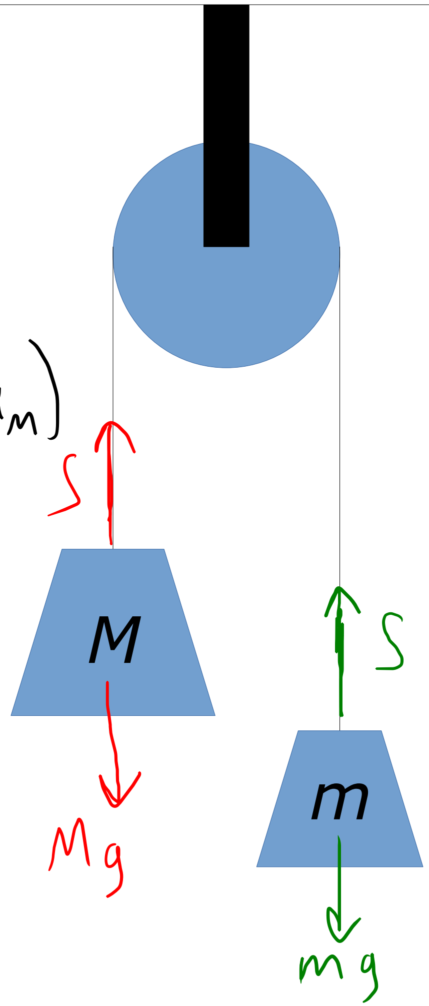
$$\Downarrow$$

$$mg - Mg = (m+M) a_M$$

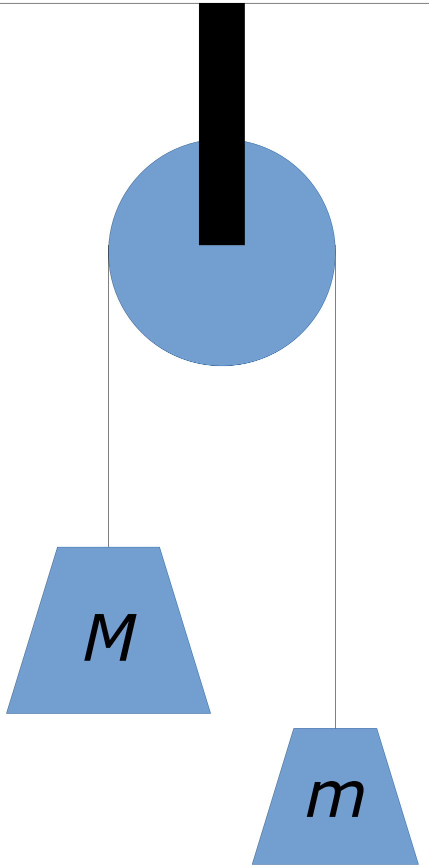
$$\Downarrow$$

$$a_M = g\,\frac{m - M}{m + M} = \frac{\frac{m}{M} - 1}{\frac{m}{M} + 1}\, g$$





Eksempel: Dynamisk problem



”Standardmetoden”

-Vælg et koordinatsystem

-Tegn et eller flere kraftdiagrammer

-Bestem resulterende kræfter på relevante legemer,
opstil Newton II for disse.

-Udnyt eventuelle 'geometriske bånd' – f.eks. hvis
to legemers bevægelse er bundet til hinanden via en snor.

-Løs ligningerne, bestem de ubekendte størrelser.

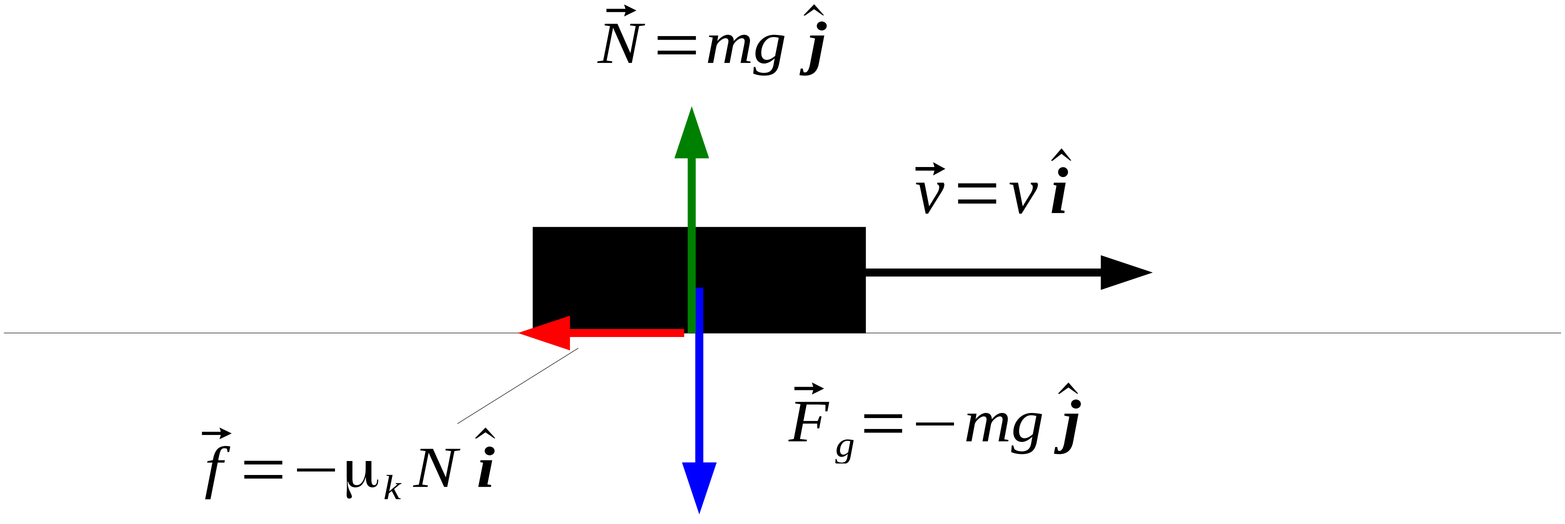
-Benyt evt. kinematiske ligninger til at finde
hastigheder, forskydninger etc..

Friktionskræfter - kontaktfriktion

Kinetisk friktion f virker mod bevægelsesretningen

Uafhængig af fartens størrelse, blot der er bevægelse

Proportional med *normalkraften N*

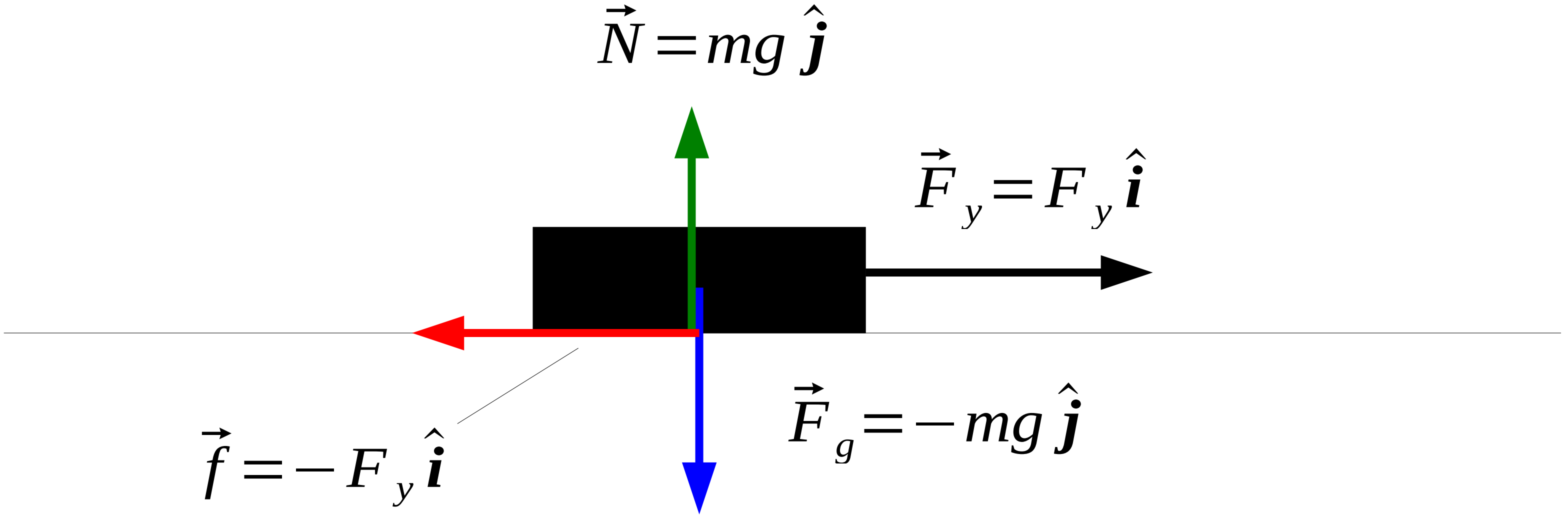


Friktionskræfter - kontaktfriktion

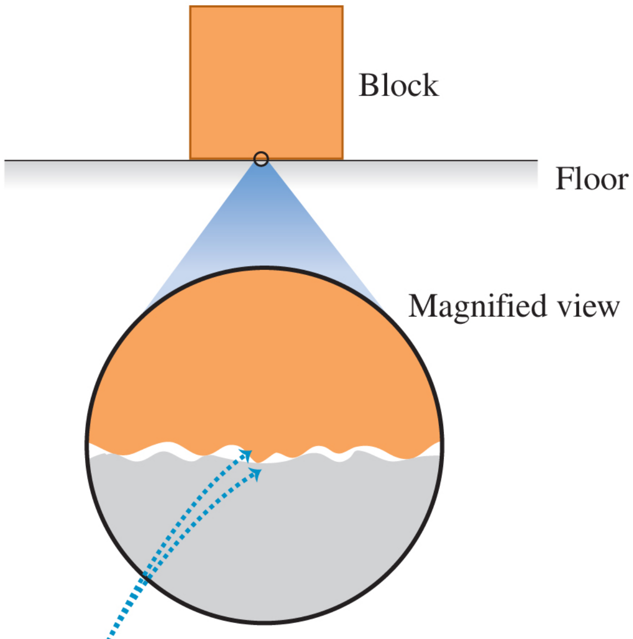
Statisk friktion virker mod en ydre kraft ved stilstand

Kraften er præcis stor nok til at modvirke bevægelse

Kraftens størrelse kan ikke overstige $\mu_s N$



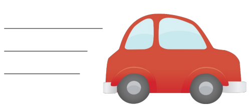
Friktionskræfter - kontaktfriktion



The friction and normal forces result from interactions between molecules in the block and in the floor where the two rough surfaces touch.

TABLE 5.1 Approximate Coefficients of Friction

Materials	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Brass on steel	0.51	0.44
Zinc on cast iron	0.85	0.21
Copper on cast iron	1.05	0.29
Glass on glass	0.94	0.40
Copper on glass	0.68	0.53
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

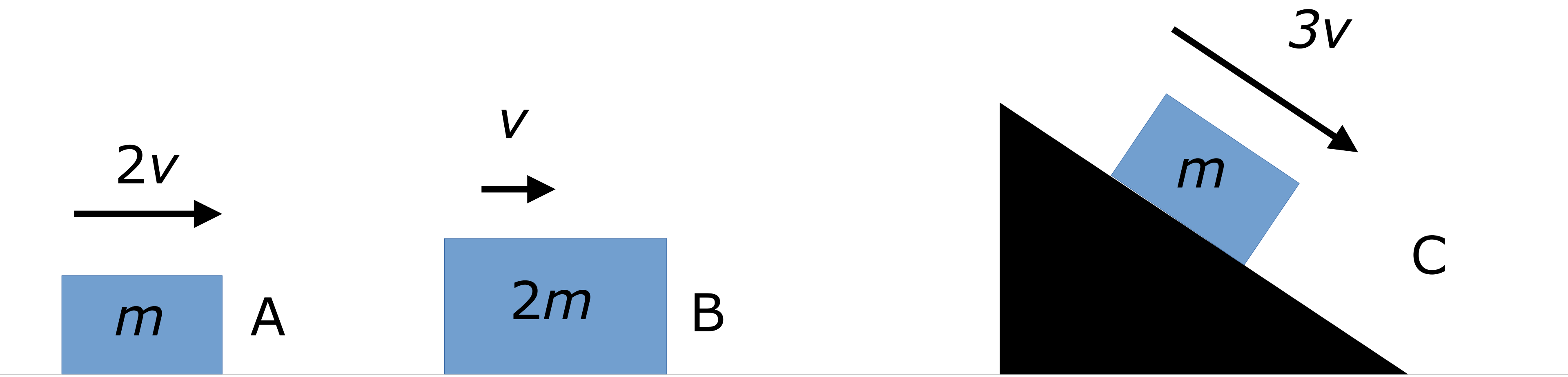


Quiz - friktionskraft

Hvad kan vi sige om friktionskraften på klodserne?

i) $f_C > f_A > f_B$ ii) $f_B > f_A = f_C$ iii) $f_B > f_A > f_C$

iv) $f_C > f_A = f_B$ v) $f_A = f_B = f_C$ vi) Vi kan ikke sige noget



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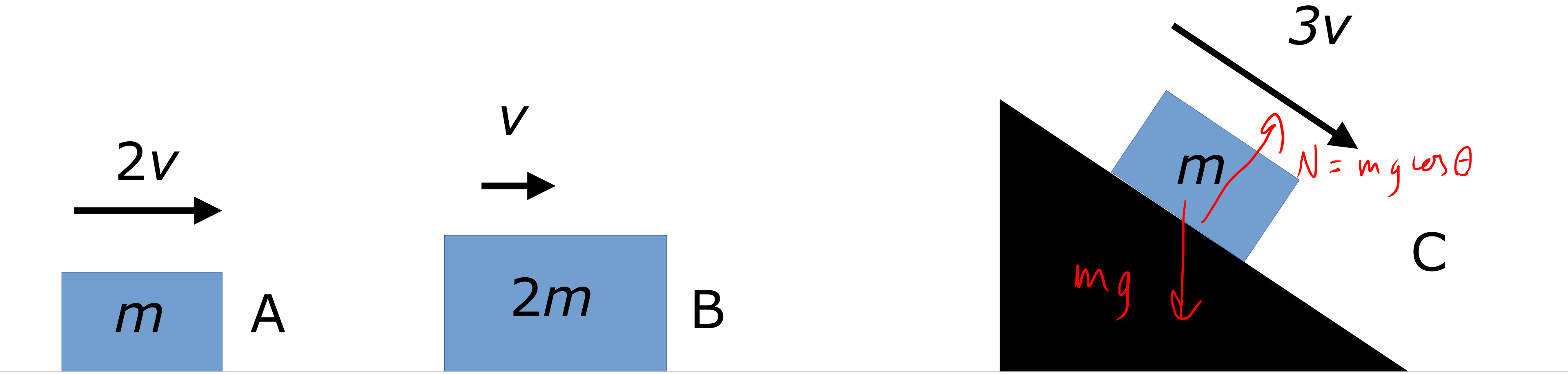

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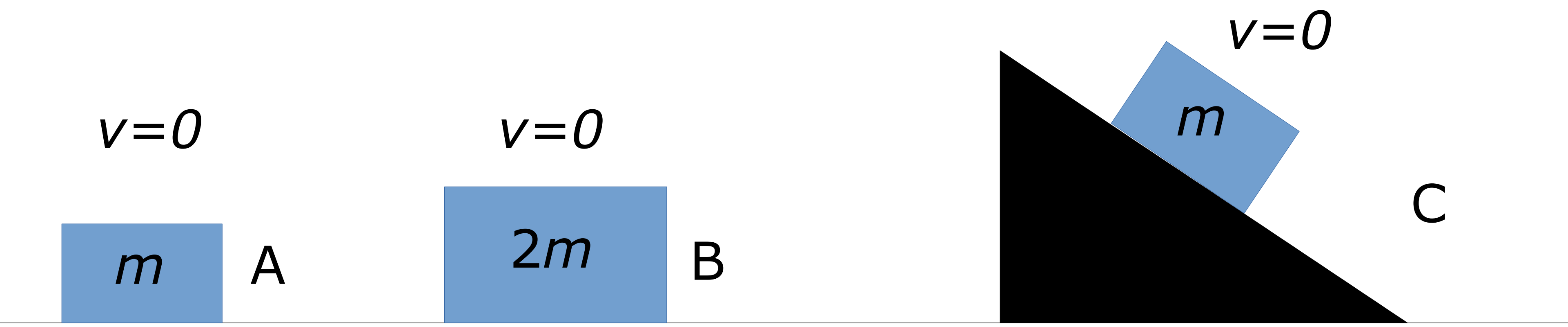

Quiz - friktionskraft

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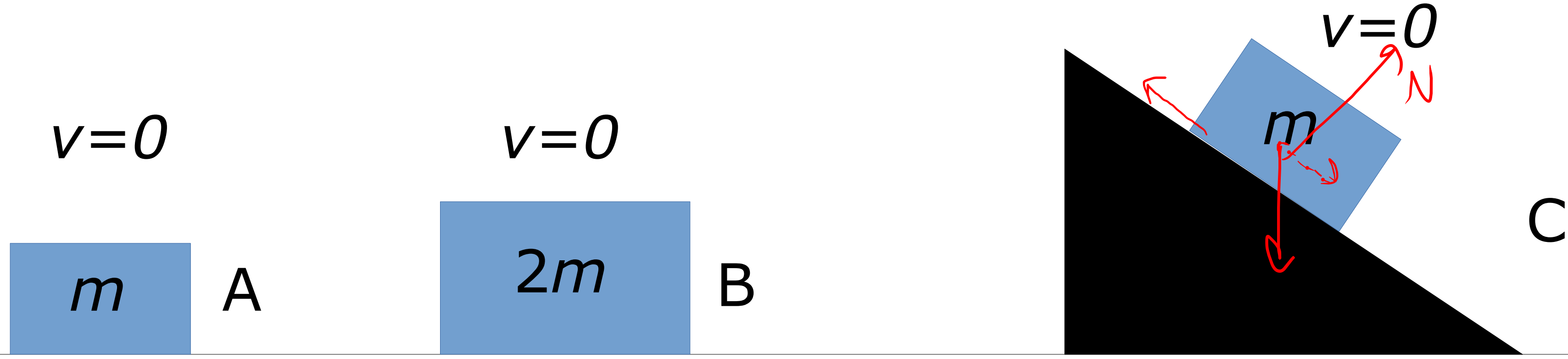


Eksempel: Klods på skråplan III

Hvad er minimumsvinklen hvor klodsen kan glide?

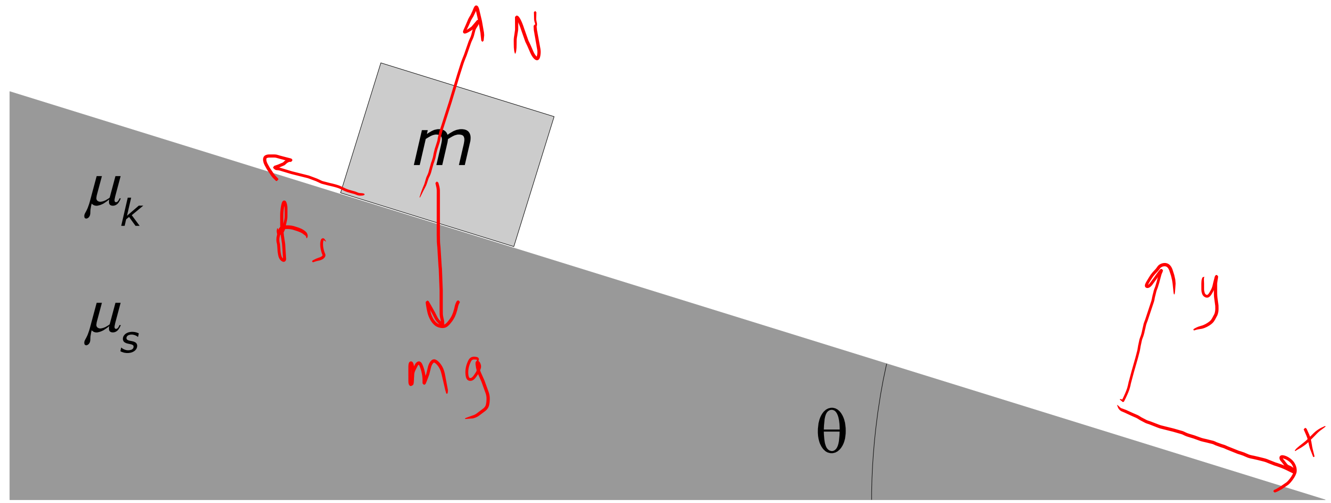
$$N\mathrm{II}_x: \; mg\sin\theta - f_s = ma_x = 0 \Rightarrow f_s = mg\sin\theta \leq \mu_s N = \mu_s mg\cos\theta$$

$$N\mathrm{II}_y: \; N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$$

$$\cancel{mg}\sin\theta \leq \mu_s \cancel{mg}\cos\theta$$

$$\Downarrow$$

$$\tan\theta \leq \mu_s$$



Eksempel: Klods på skråplan III

$$NII_x: \; mg\sin\theta - f_k = mg\sin - \mu_k N = mg\sin\theta - \mu_k mg\cos\theta = m a_x$$

$$mg\sin\theta - \mu_k mg\cos\theta \geq 0 \Rightarrow mg(\sin\theta - \mu_k\cos\theta) \geq 0 \Rightarrow \sin\theta \geq \mu_k \cos\theta$$

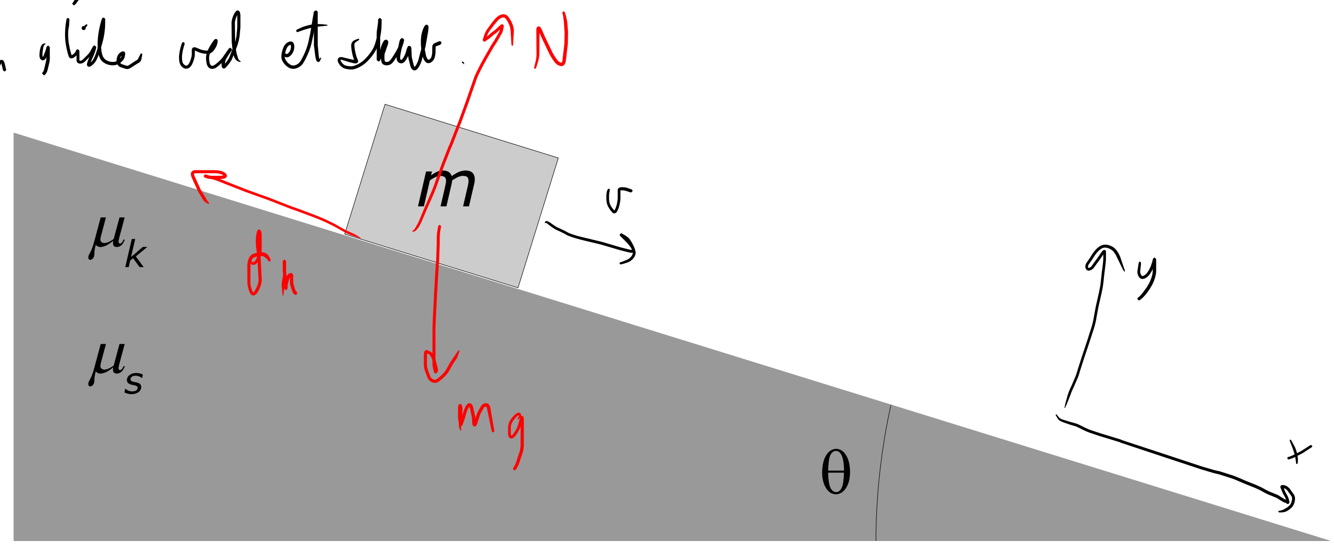
$$\Downarrow$$

$$tg\,\theta \geq \mu_k$$

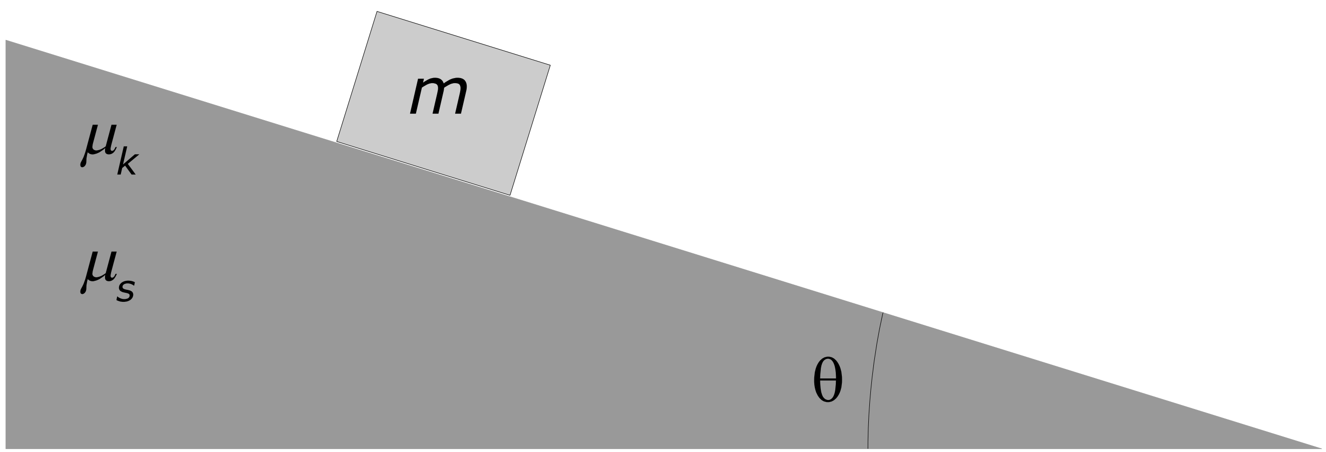
$tg\,\theta \geq \mu_s$ Klodsen glide

$\mu_k \leq tg\,\theta \leq \mu_s$ Klodsen glide ved et skub.

$tg\,\theta \leq \mu_k$ Klodsen glide ikke



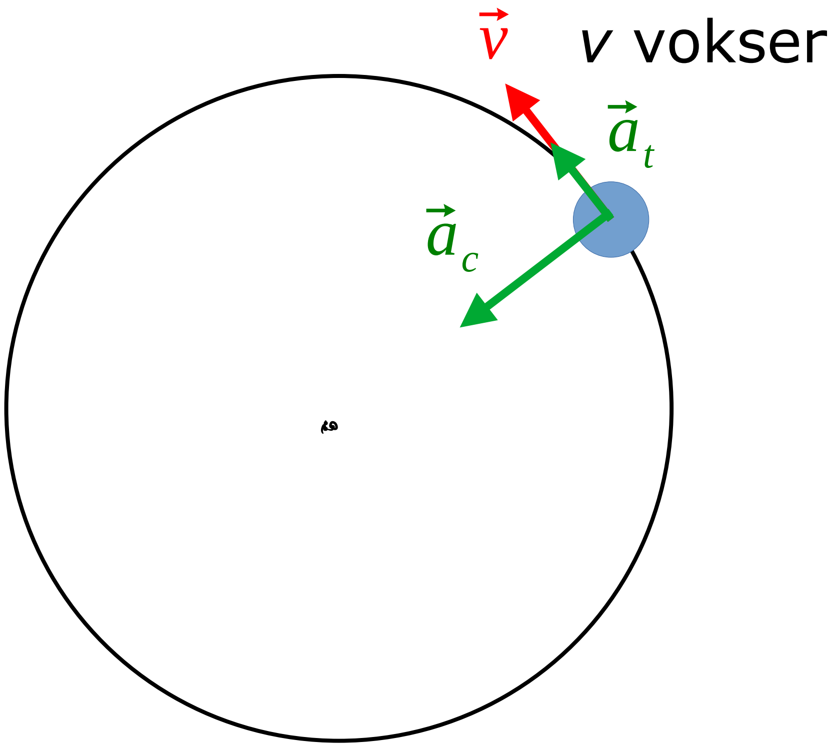
Eksempel: Klods på skråplan III

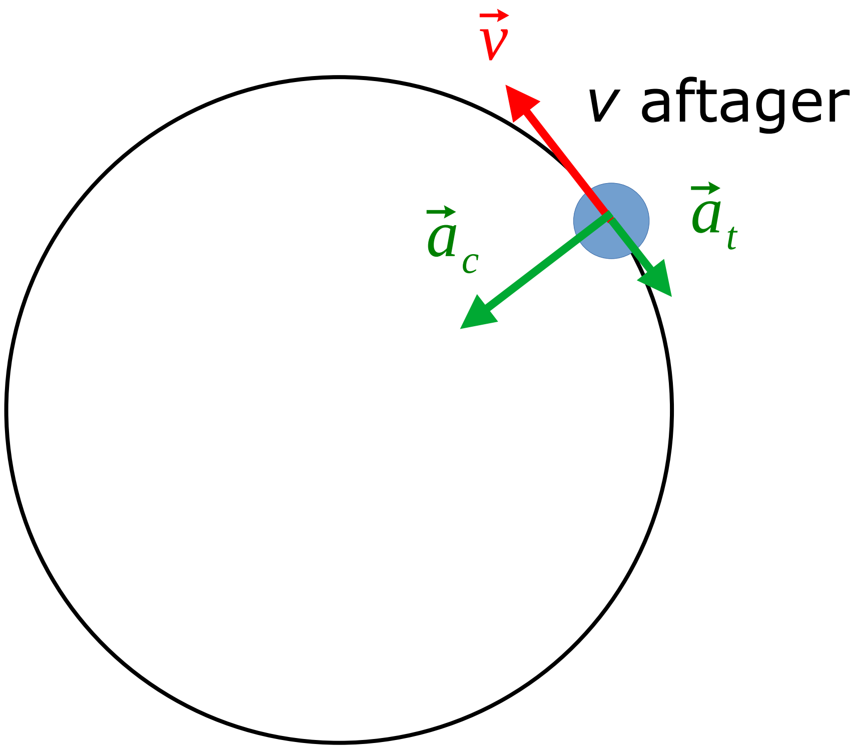


(U)jævn cirkelbevægelse - repetition

$$|\vec{r}| = R \text{ konstant}$$

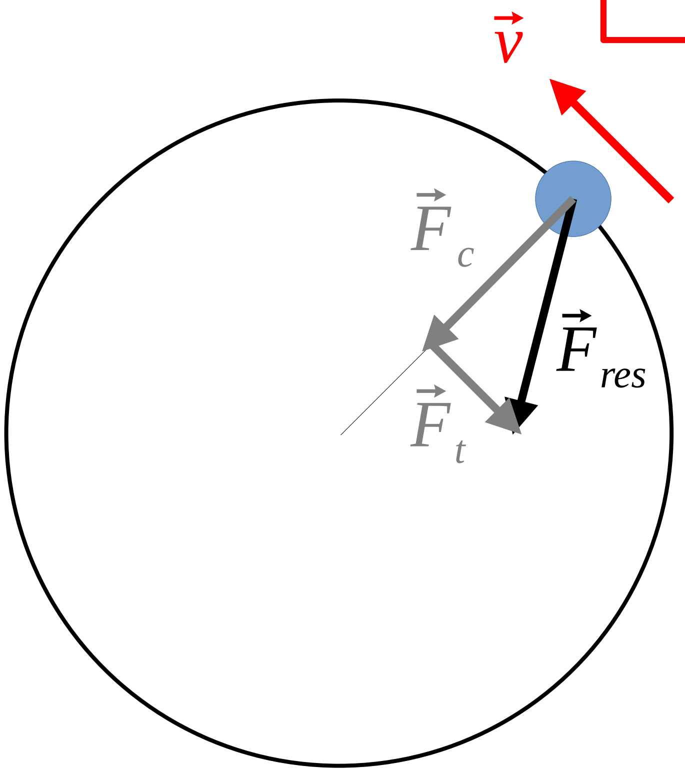
$$\boxed{a_c = \frac{v^2}{R} \qquad |a_t| = |\vec{a}_t| = \left|\frac{dv}{dt}\right| \qquad a_t \equiv \frac{dv}{dt}}$$





Kræfter i cirkelbevægelse

$$a_c = \frac{v^2}{R} \qquad |a_t| = |\vec{a}_t| = \left|\frac{dv}{dt}\right| \qquad a_t \equiv \frac{dv}{dt}$$

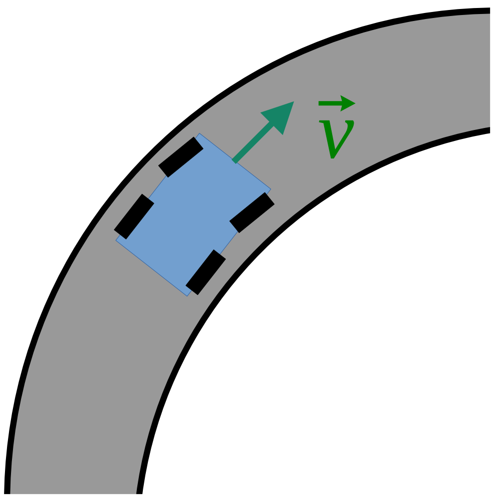


Newton II: $\vec{F}_{res} = m\vec{a}$

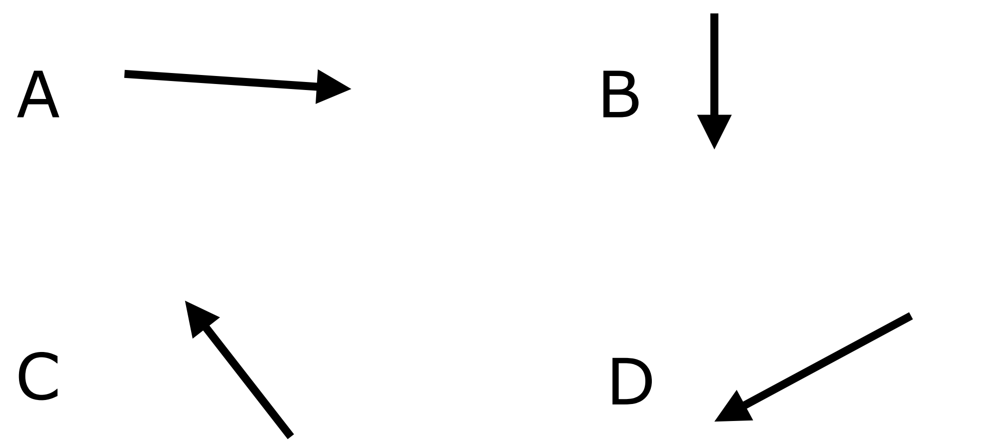
$$F_c = m a_c = m\frac{v^2}{R}$$ Centripetalkraft

$$F_t = m a_t = m\frac{dv}{dt}$$ Tangentialkraft

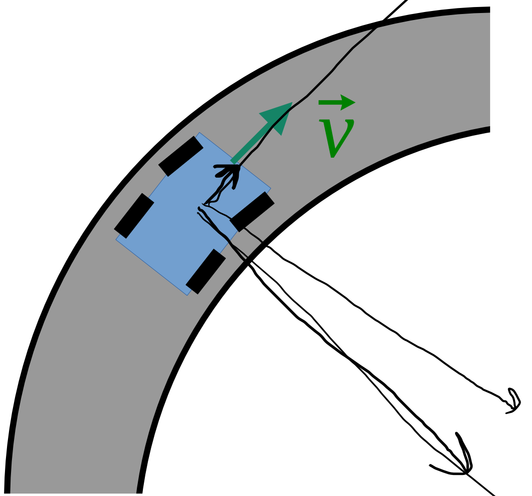
Quiz – bil i sving



Bilen øger sin fart gennem svinget, som har konstant krumningsradius. Hvilken af nedenstående vektorer kunne beskrive friktionskraften på dækkene?



Quiz – bil i sving



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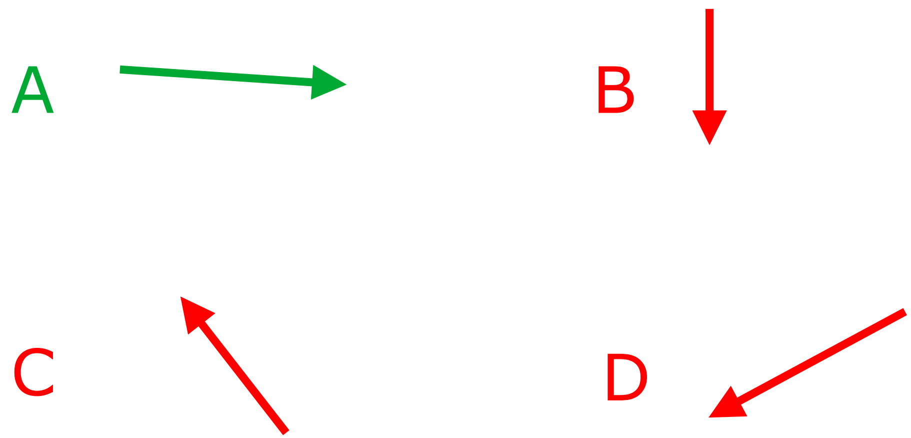
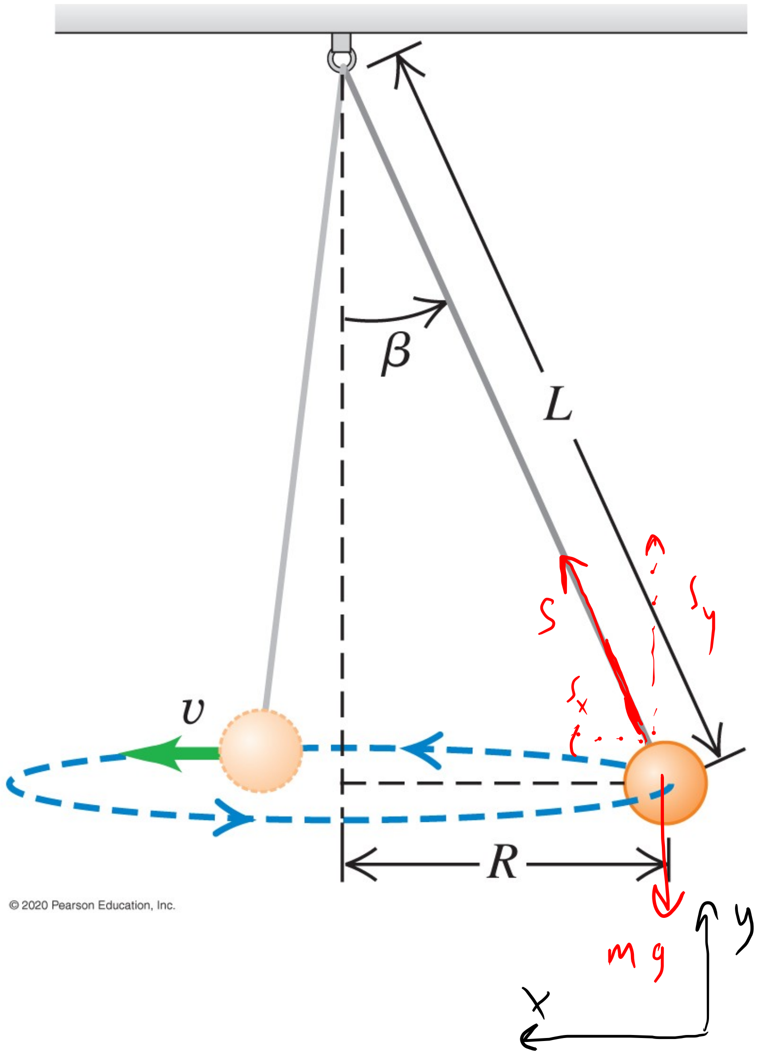


Eksempel: Konisk pendul

Bestem omløbstiden

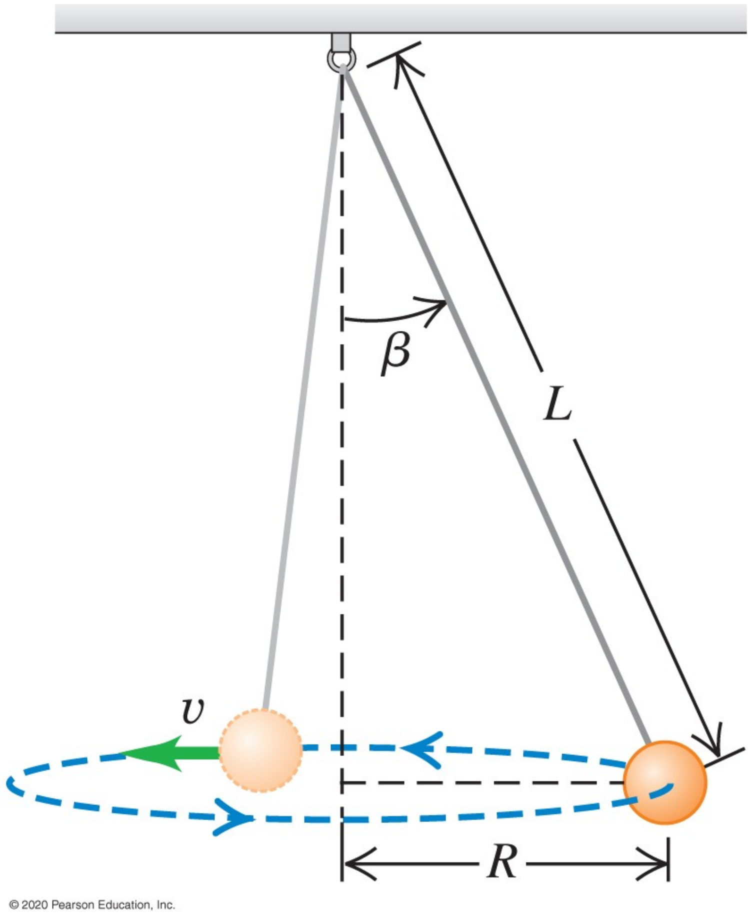
$$N\text{II}_y: \; S_y - mg = 0$$

$$N\text{II}_x: \; S_x = m a_x = m \frac{v^2}{R}$$



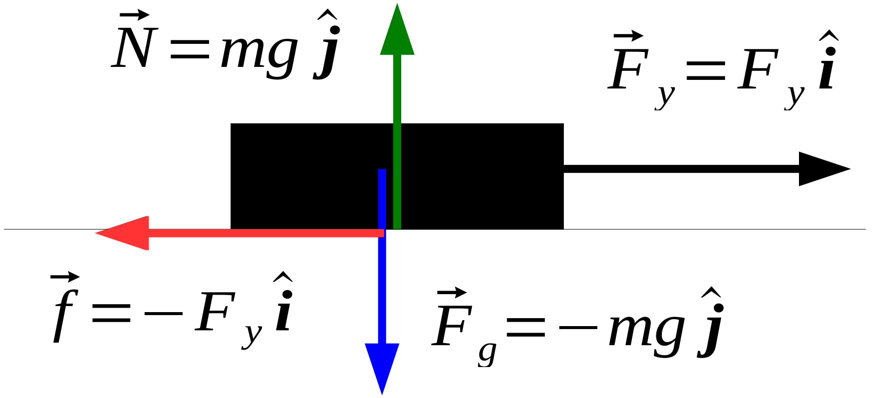
Eksempel: Konisk pendul

Bestem omløbstiden

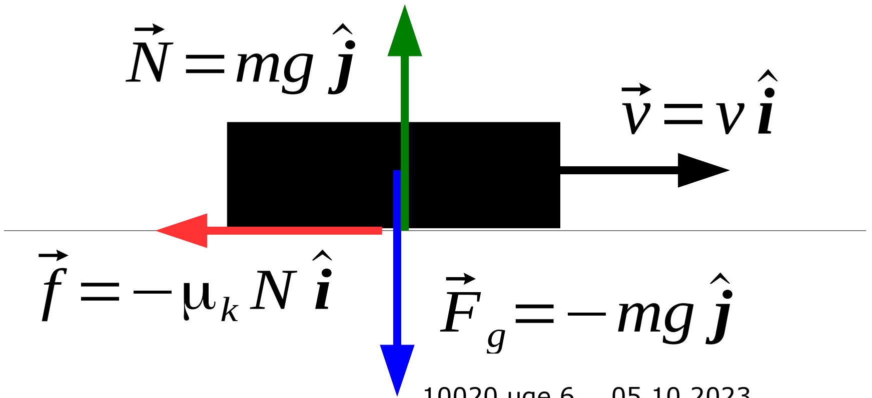


Opsummering

Statisk friktion er $\leq \mu_s N$ og fastholder et objekt i hvile på en flade.



Kinetisk friktion virker mod bevægelsesretningen, og har størrelsen $\mu_k N$



Opsummering - ”Standardmetoden”

-Vælg et koordinatsystem

-Tegn et eller flere kraftdiagrammer

-Bestem resulterende kræfter på relevante legemer, opstil Newton II for disse.

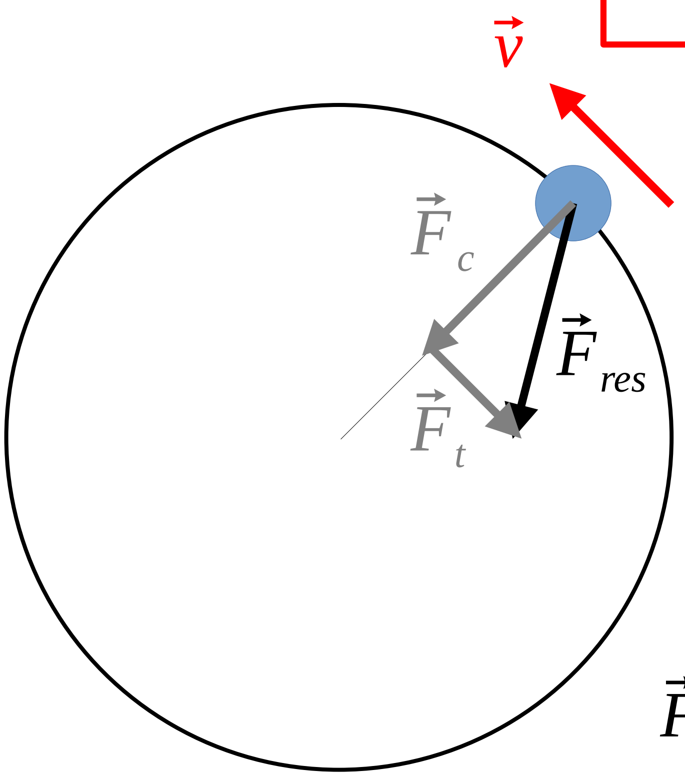
-Udnyt eventuelle 'geometriske bånd' – f.eks. hvis to legemers bevægelse er bundet til hinanden via en snor.

-Løs ligningerne, bestem de ubekendte størrelser.

-Benyt evt. kinematiske ligninger til at finde hastigheder, forskydninger etc..

Opsummering: Kræfter i cirkelbevægelse

$$a_c = \frac{v^2}{R} \qquad |a_t| = |\vec{a}_t| = \left|\frac{dv}{dt}\right| \qquad a_t \equiv \frac{dv}{dt}$$



$$F_c = m\,a_c = m\frac{v^2}{R}$$ Centripetalkraft

$$F_t = m\,a_t = m\frac{dv}{dt}$$ Tangentialkraft

$$\vec{F}_{res} = \vec{F}_c + \vec{F}_t$$

$\vec{F}_c$ og $\vec{F}_t$ er *ikke* selvstændige kræfter!

Mere om Newtons love II, Y & F kap. 5

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Fra sidste gang - ”Standardmetoden”

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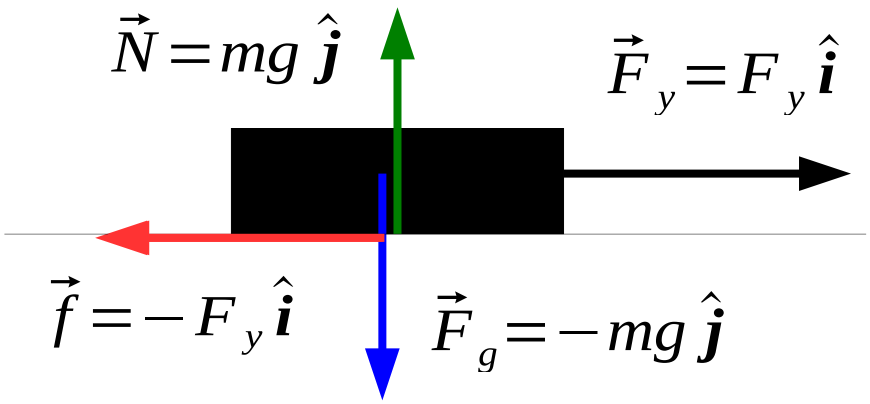
-Løs ligningerne, bestem de ubekendte størrelser.

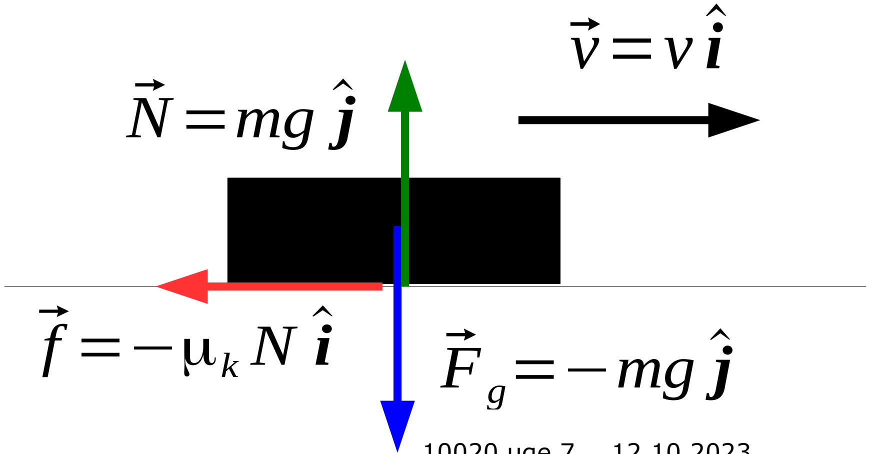
-Benyt evt. kinematiske ligninger til at finde hastigheder, forskydninger etc..

Fra sidste gang

Statisk friktion er $\leq \mu_s N$ og fastholder et objekt i hvile på en flade.



Kinetisk friktion virker mod bevægelsesretningen, og har størrelsen $\mu_k N$



Opsummering: Kræfter i cirkelbevægelse

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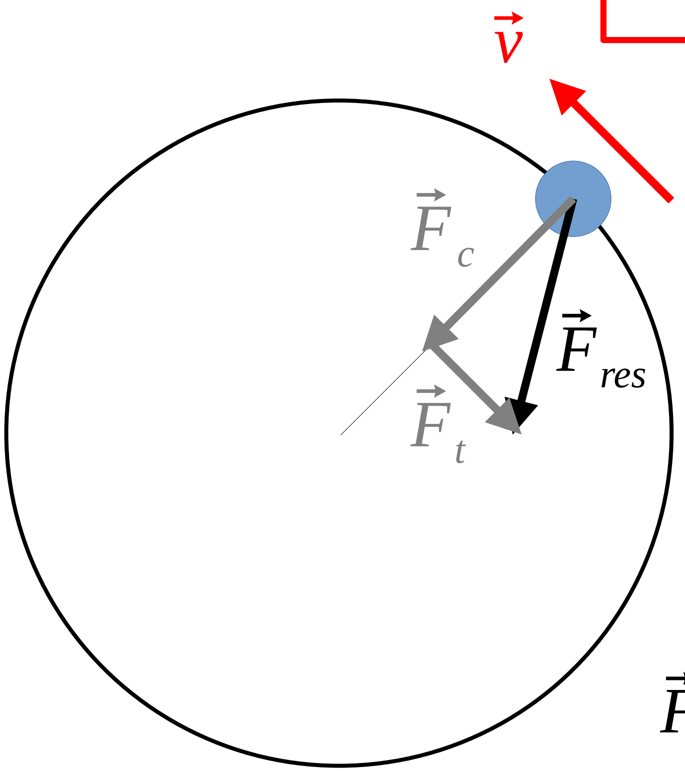


$$F_c = m a_c = m\frac{v^2}{R}$$ Centripetalkraft

$$F_t = m a_t = m\frac{dv}{dt}$$ Tangentialkraft

$$\vec{F}_{res} = \vec{F}_c + \vec{F}_t$$

$\vec{F}_c$ og $\vec{F}_t$ er *ikke* selvstændige kræfter!

Denne uges læringsmål

-Eksempler på cirkelbevægelse

-Forstå *væskefriktion*

-Forstå friktion i *rullebevægelse*

Quiz: Lodret cirkelbevægelse

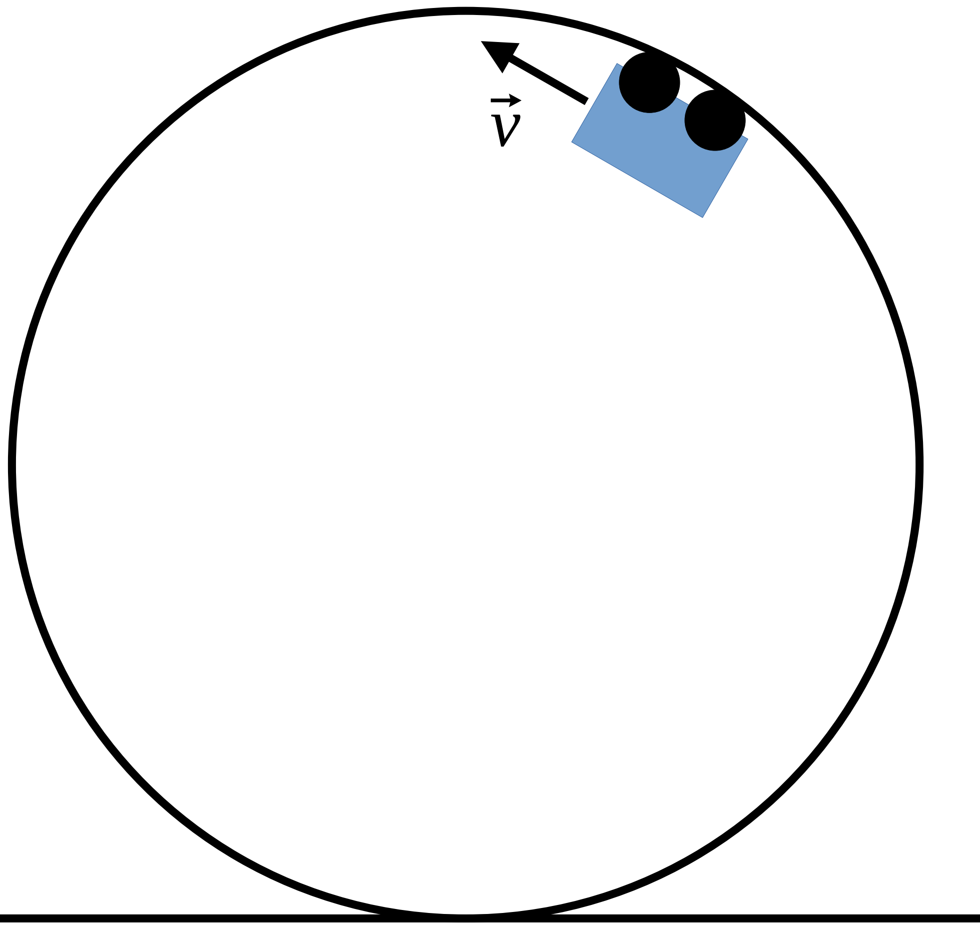
Hvilke kræfter påvirker vognen?

A) Tyngdekraft

B) Tyngdekraft, normalkraft

C) Tyngdekraft, centripetalkraft

D) Tyngdekraft, normalkraft, centripetalkraft



Quiz: Lodret cirkelbevægelse

Hvilke kræfter påvirker vognen?

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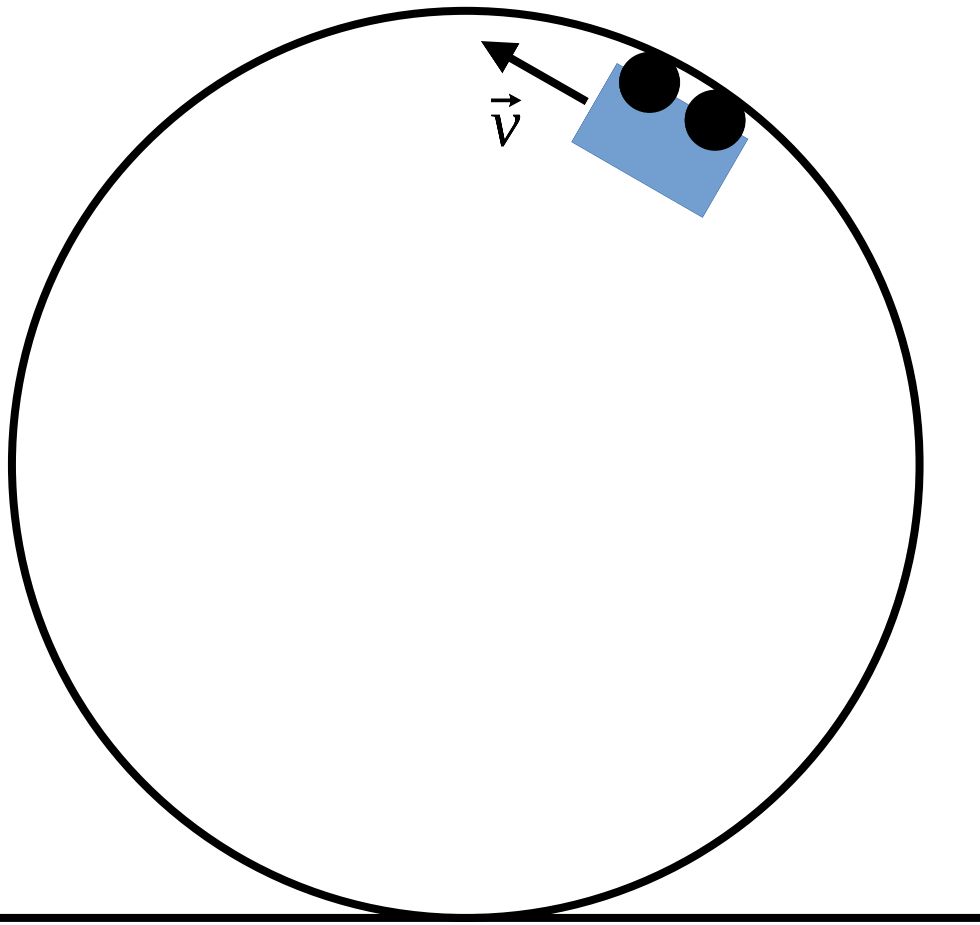
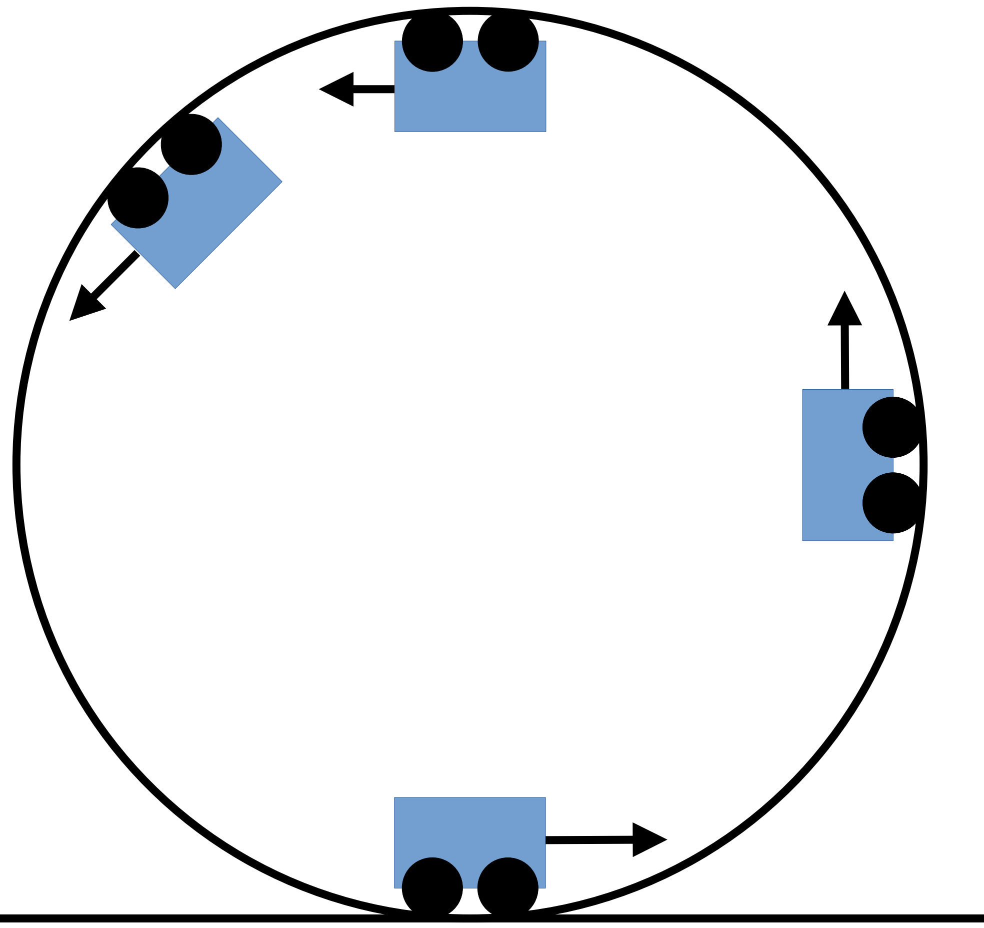


Quiz: Lodret cirkelbevægelse

I hvilke(n) position(er) er tangentialaccelerationen nul?



Quiz: Lodret cirkelbevægelse

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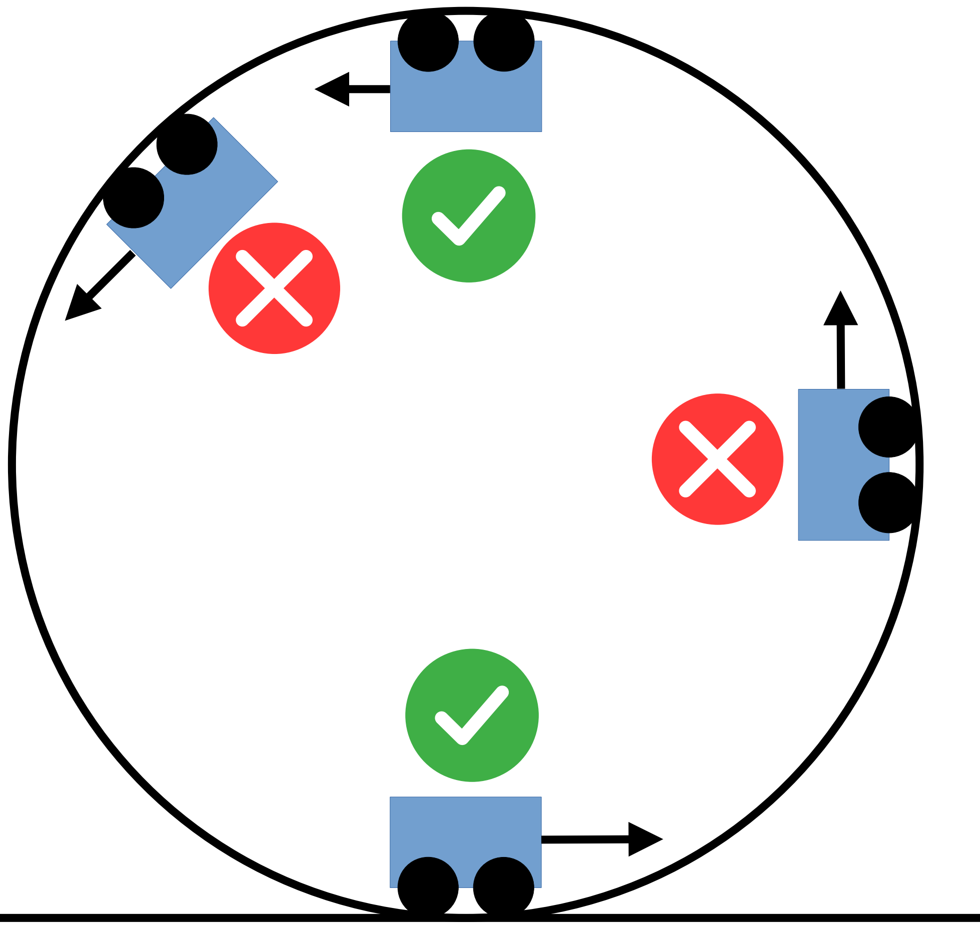
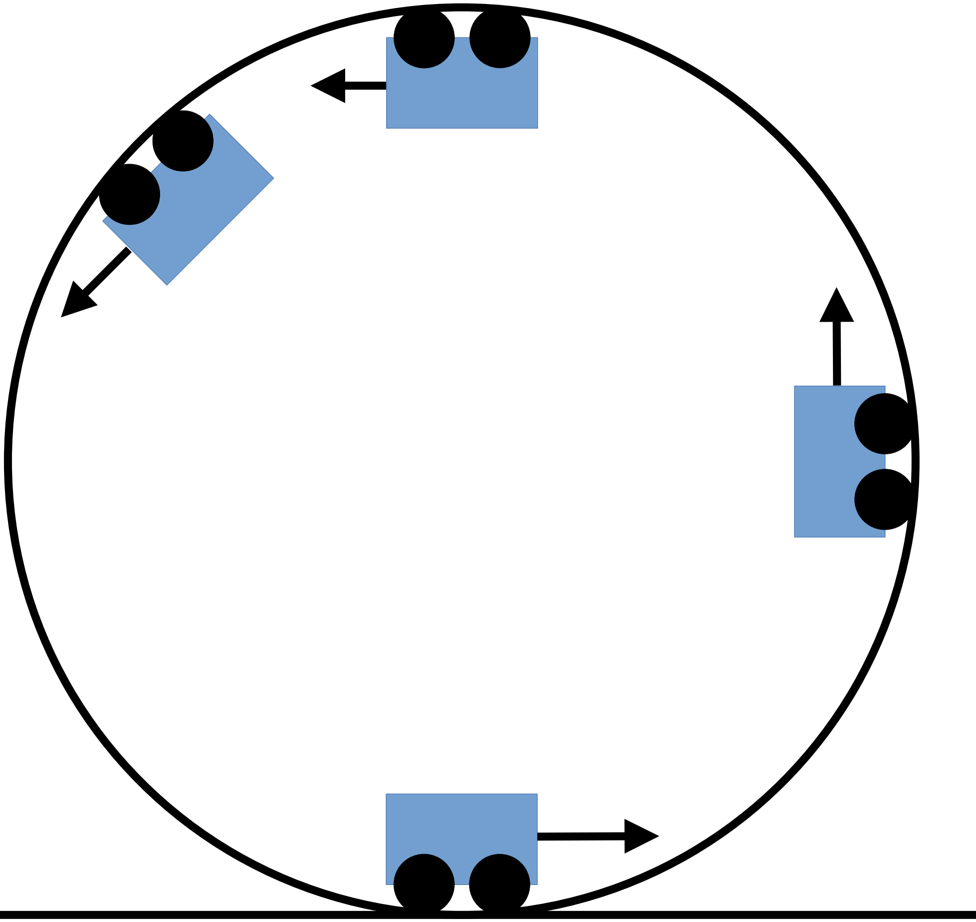


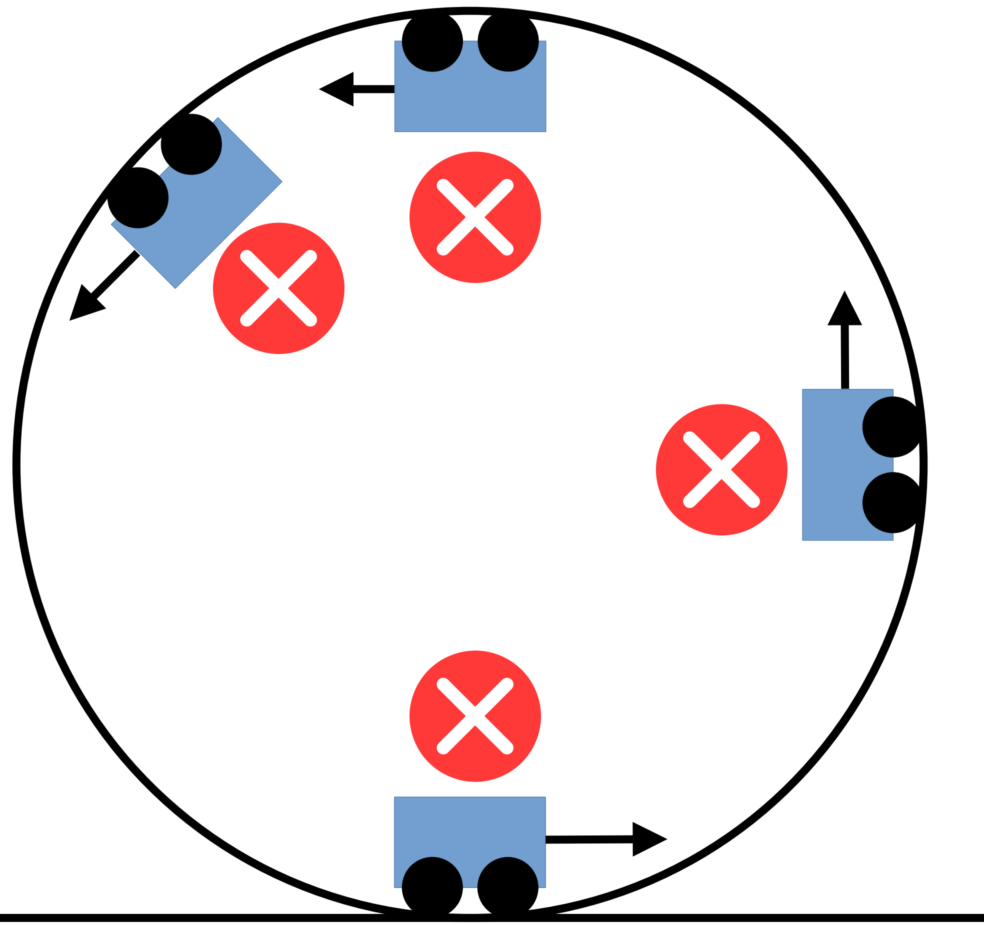
Quiz: Lodret cirkelbevægelse

I hvilke(n) position(er) er centripetalaccelerationen nul?



Quiz: Lodret cirkelbevægelse

I hvilke(n) position(er) er centripetalaccelerationen nul?



Eksempel: Lodret cirkelbevægelse

Bestem normalkraft og tangential-acceleration.

$$NII_c: n_1 + n_2 + mg\sin\theta = m a_c = m\frac{v^2}{R}$$

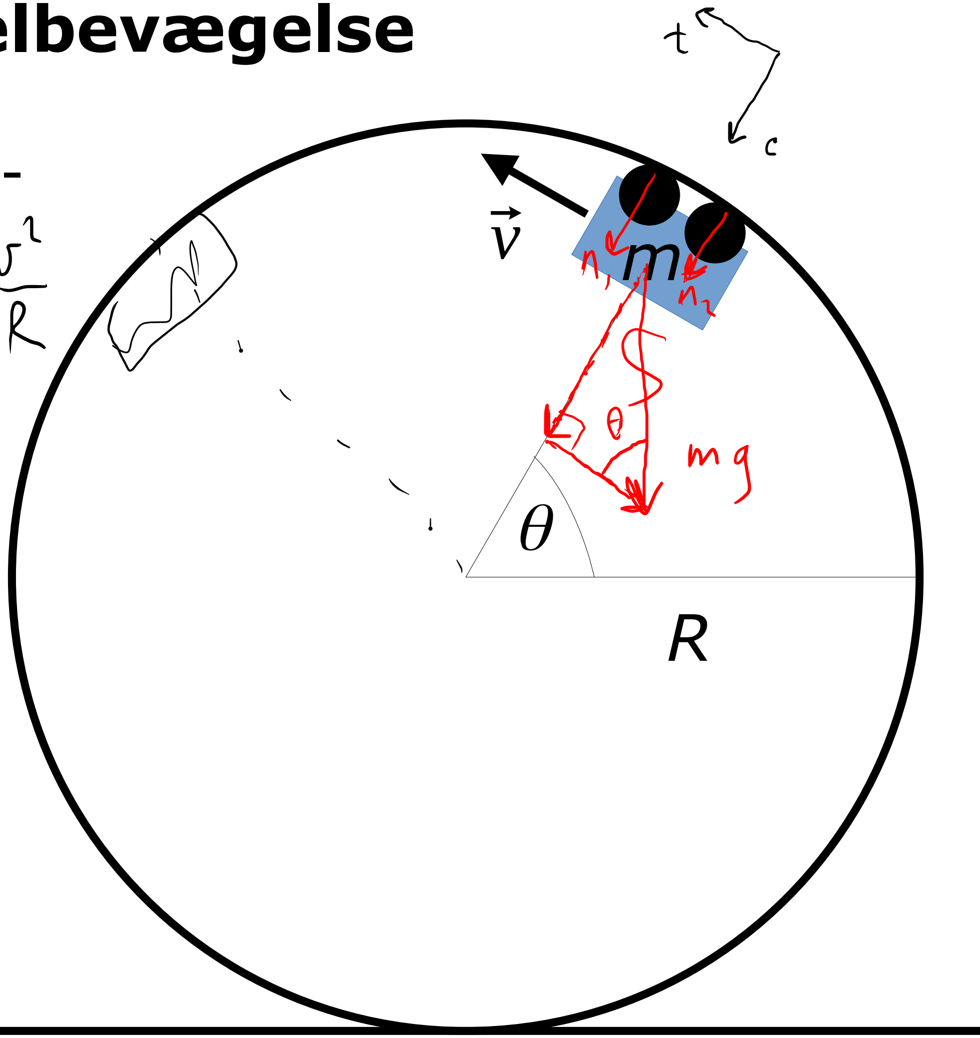
$$\Downarrow$$

$$n_1 + n_2 = m\frac{v^2}{R} - mg\sin\theta$$

$$NII_t: -mg\cos\theta = m a_t$$

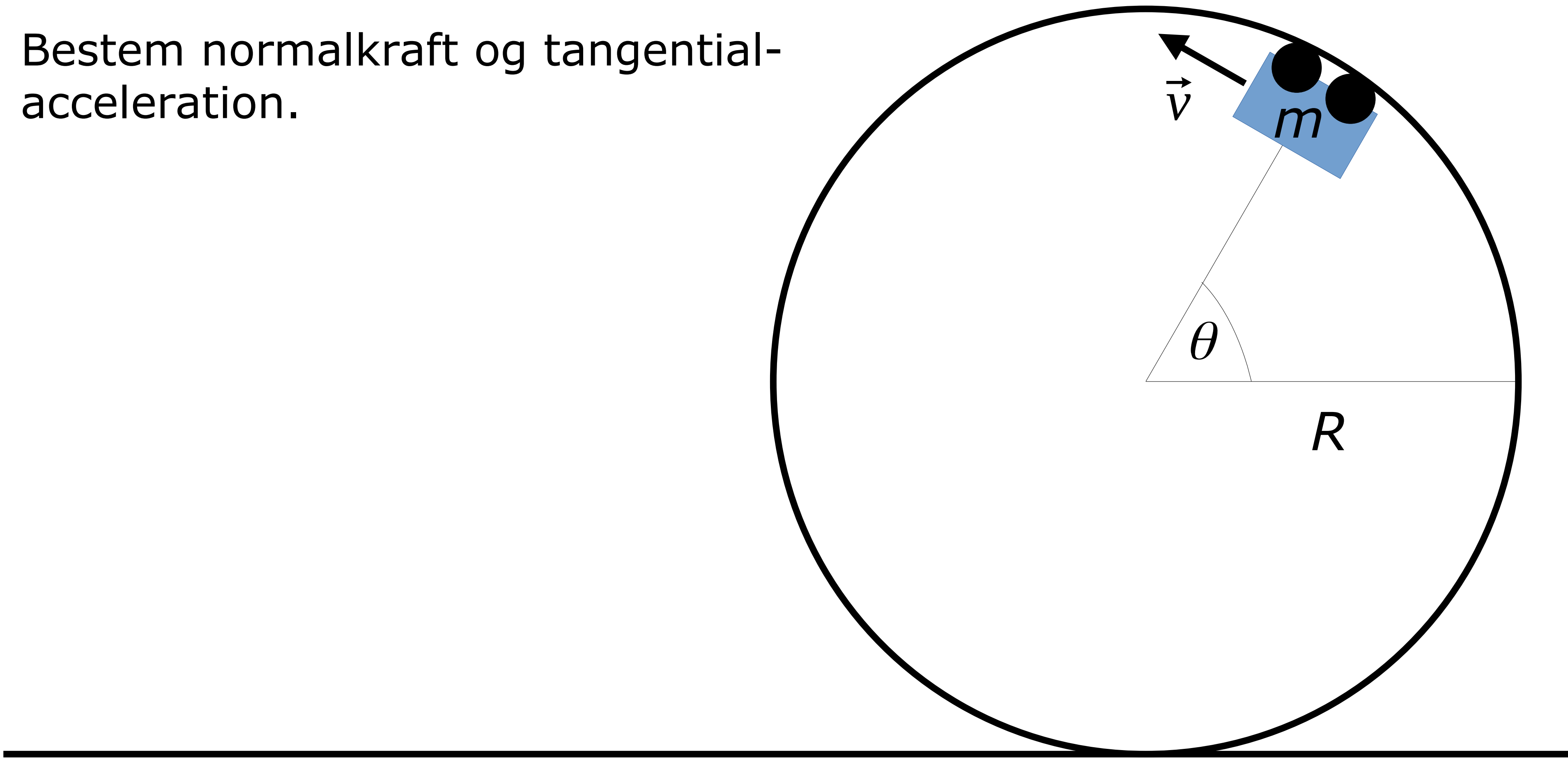
$$\Downarrow$$

$$a_t = -g\cos\theta$$



Eksempel: Lodret cirkelbevægelse

Bestem normalkraft og tangential-acceleration.



Eksempel: Fly som drejer

Hvad er drejeradius?

$$NII_y: \quad L\cos\beta - mg = 0$$

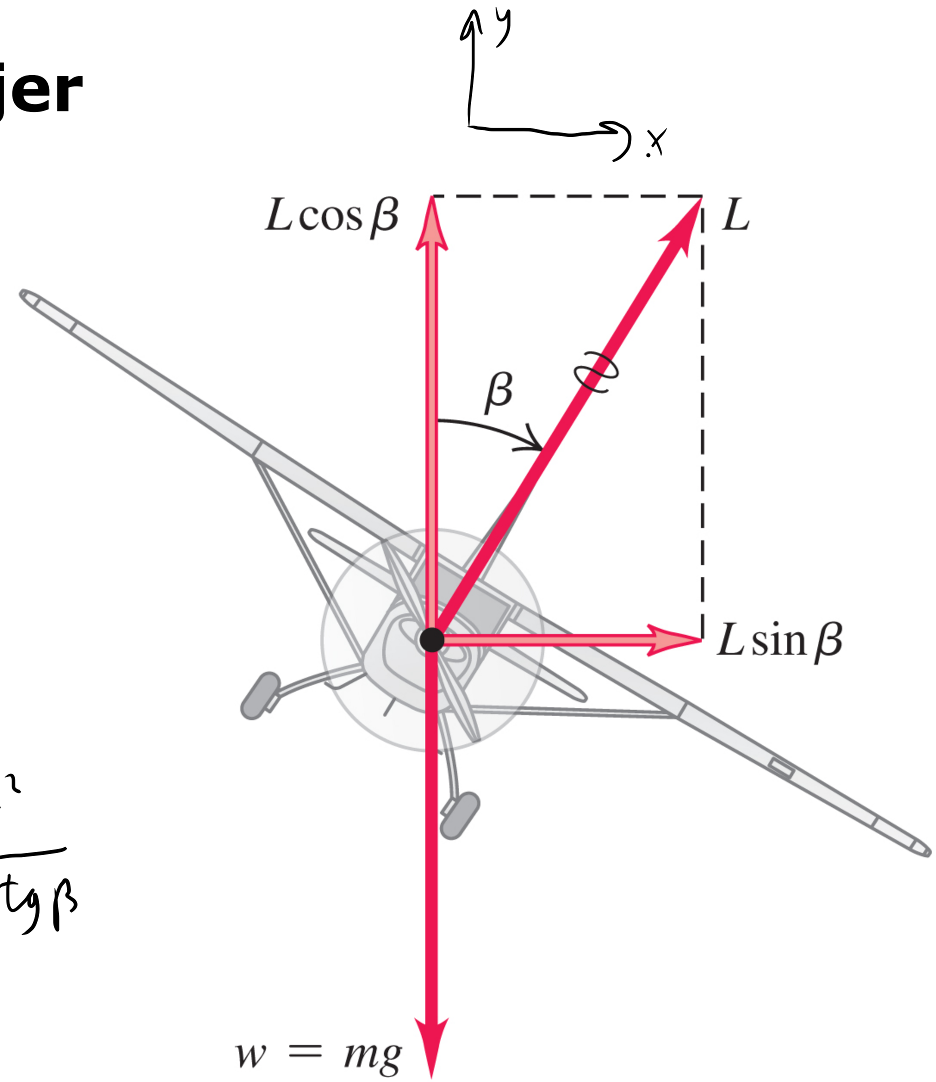
$$\Downarrow$$

$$L = \frac{mg}{\cos\beta}$$

$$NII_x: \quad L\sin\beta = m a_c = m\frac{v^2}{R}$$

$$\Downarrow$$

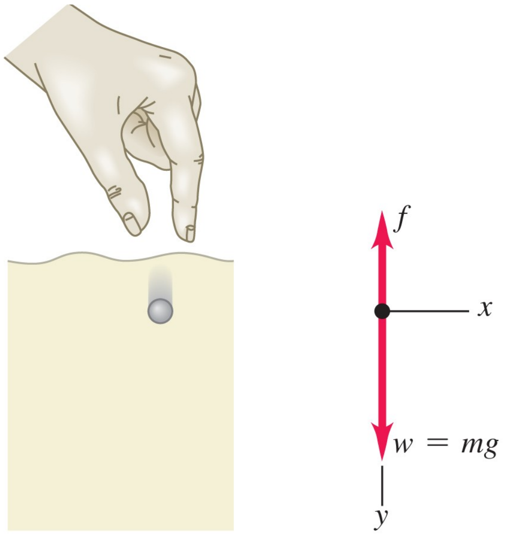
$$R = \frac{mv^2}{L\sin\beta} = \frac{\cancel{m}v^2}{\frac{\cancel{m}g\sin\beta}{\cos\beta}} = \frac{v^2}{g\,\mathrm{tg}\,\beta}$$



Væskefriktion

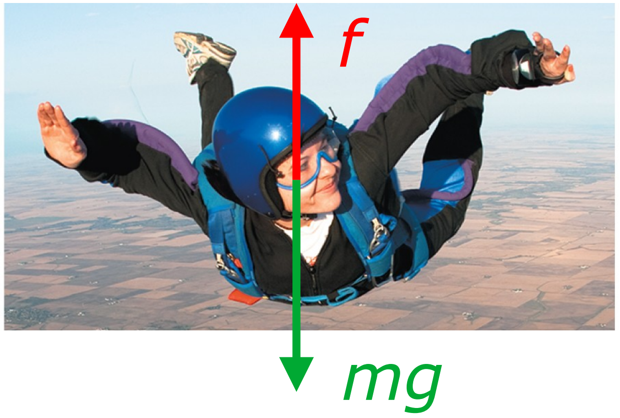
Modstand mod bevægelse i 'fluider' (væske/luft):

Tyk væske/lav fart: $\vec{f} \approx -k\vec{v}$



Tynd væske (luft)/høj fart:

$$\vec{f} \approx -D|v|\vec{v} \Rightarrow f = Dv^2$$

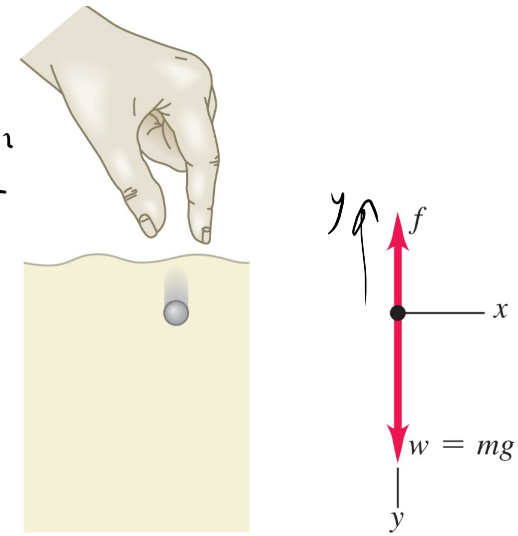


Terminalfart

$$\vec f \approx -k\vec v \qquad \frac{N}{m/s} \sim \frac{kg\, m/s^2}{m/s} \sim \frac{kg}{s}$$

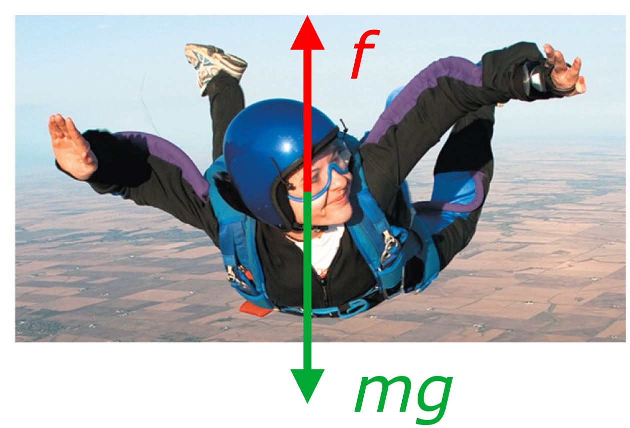
$$\frac{m/s}{s}$$

$$F_{res} = f - mg = kv_t - mg = 0 \Rightarrow v_t = \frac{mg}{k} \sim \frac{\cancel{kg}\, m/s^2}{\cancel{kg}/s}$$



$$\vec f \approx -D|v|\vec v \Rightarrow f = Dv^2$$

$$Dv_t^2 - mg = 0 \Rightarrow v_t \sqrt{\frac{mg}{D}}$$



Eksempel: Fart som funktion af tid

$$\text{NII}_y: \cancel{m}g - kv = m a_y = \cancel{m}\frac{dv}{dt} \Rightarrow \frac{dv}{dt} = g - \frac{kv}{m}$$

$$\Downarrow$$

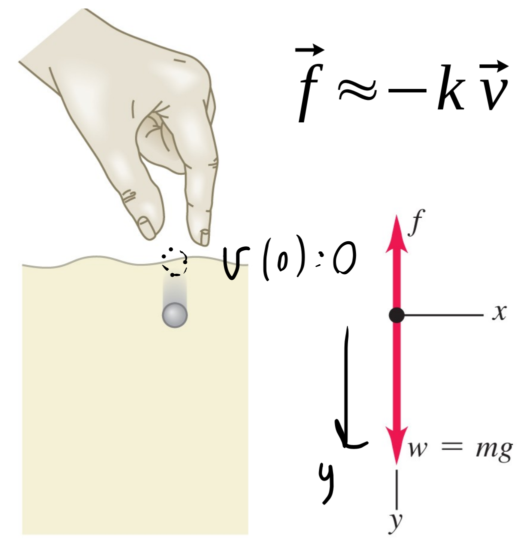
$$\frac{dv}{dt} = -\frac{k}{m}\left(v - \frac{m}{k}g\right) = -\frac{k}{m}\overbrace{(v - v_t)}^{u} = -\frac{k}{m}u$$

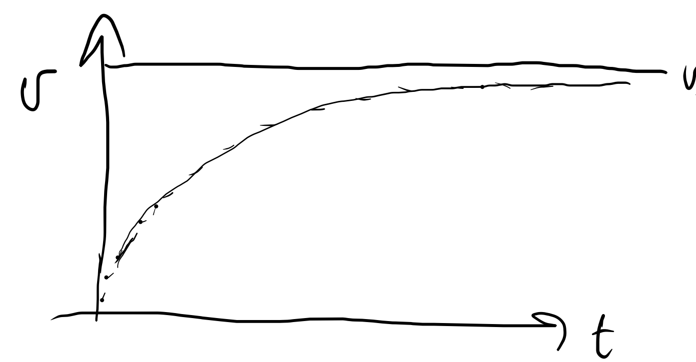
$$\frac{du}{dt} = \frac{dv}{dt} = -\frac{k}{m}u \Rightarrow u(t) = \underbrace{u(0)}_{-v_t}e^{-\frac{k}{m}t}$$

$$\Downarrow$$

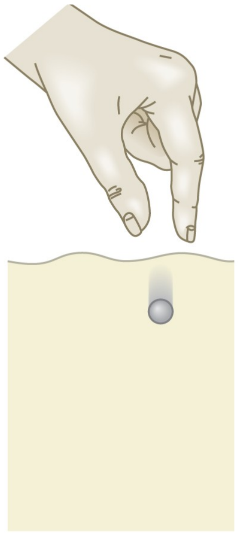
$$v(t) - v_t = -v_t e^{-\frac{k}{m}t} \Rightarrow v(t) = v_t\left(1 - e^{-\frac{k}{m}t}\right)$$

$$\vec{f} \approx -k\vec{v}$$

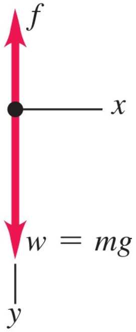




Eksempel: Fart som funktion af tid



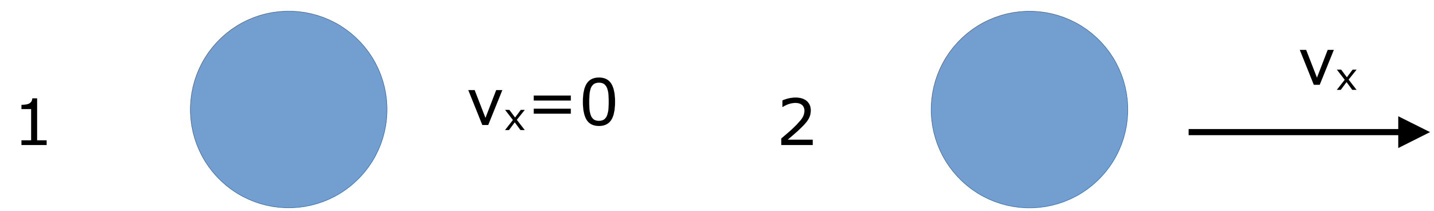
$$\vec{f} \approx -k\,\vec{v}$$



Quiz – faldtid med luftmodstand

$\vec{f} = -D|\vec{v}|\vec{v}$

De to kugler er netop smidt ud fra et fly.
Hvad kan vi sige om faldtiden for de to kugler?



A) #1 kommer hurtigst ned B) #2 kommer hurtigst ned

C) De kommer lige hurtigt ned D) Vi kan ikke sige noget

Quiz – faldtid med luftmodstand

De to kugler er netop smidt ud fra et fly.
Hvad kan vi sige om faldtiden for de to kugler?

A) #1 kommer hurtigst ned B) #2 kommer hurtigst ned

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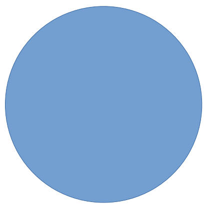
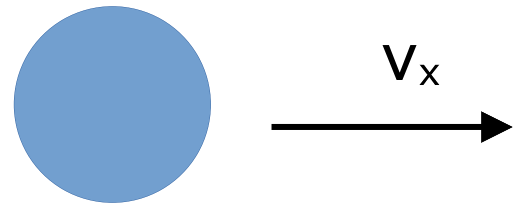
Quiz – faldtid med luftmodstand

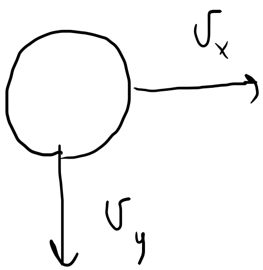
1  $v_x=0$

2 

$$f_x = -D v\, v_x = -D\sqrt{v_x^2+v_y^2}\; v_x$$

$$f_y = -D\sqrt{v_x^2+v_y^2}\; v_y$$

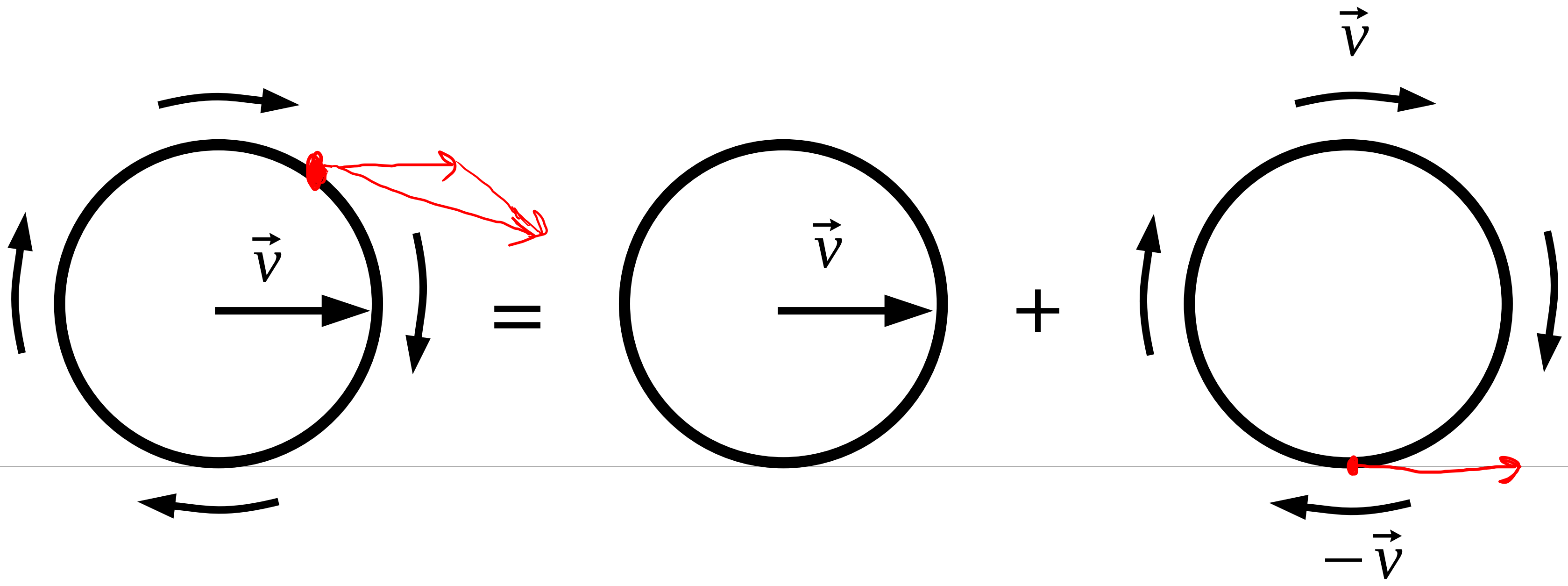
$$\bar{f} = -D|\bar{v}|\,\bar{v}$$



Rullebevægelse

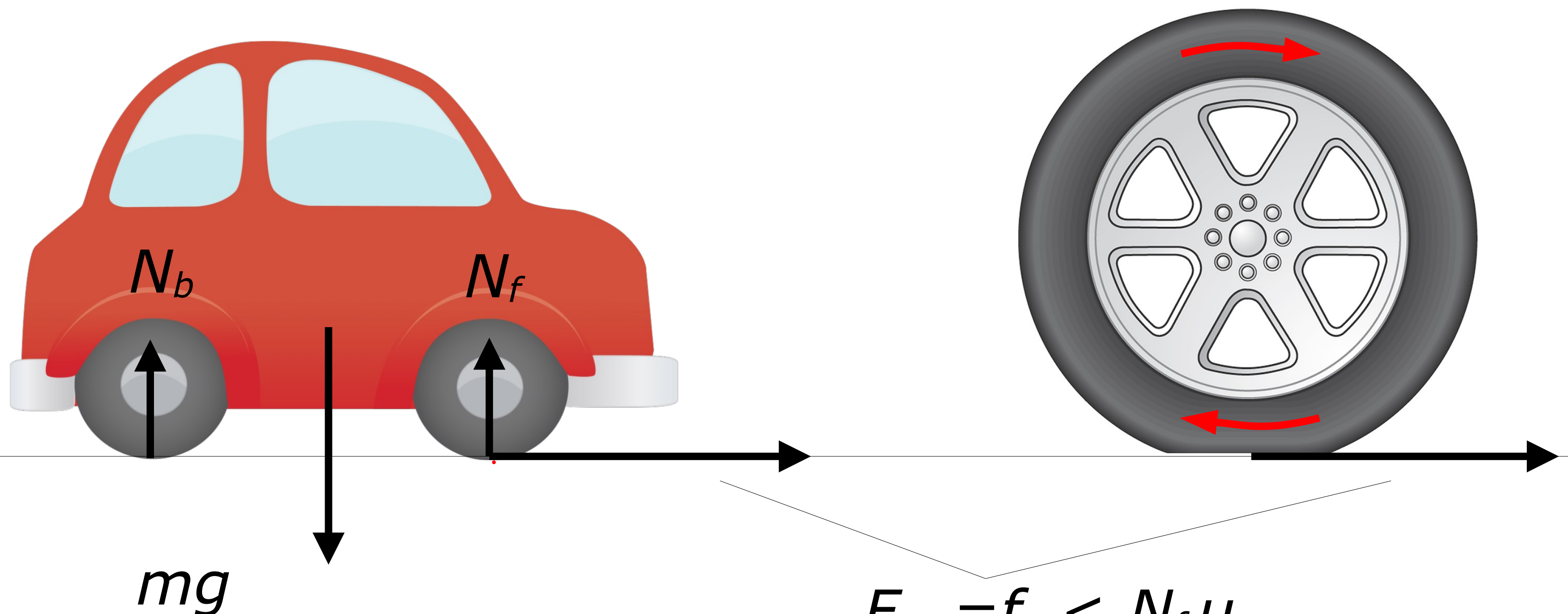
Hjulets kontaktpunkt står stille i forhold til vejen

Statisk friktion mellem hjul og vej



Rullebevægelse - kraftoverføring

Fremdriften på bilen kommer ved *statisk friktion*

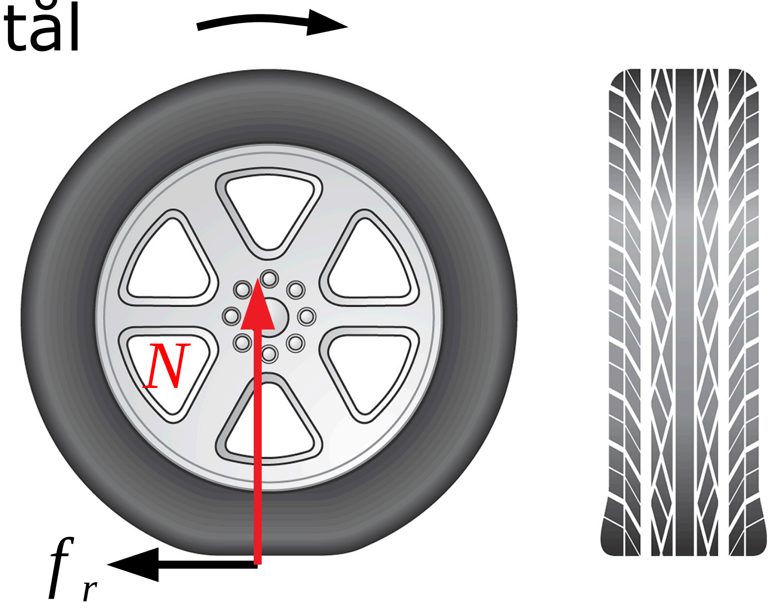


$$F_m = f_s < N_f \mu_s$$

Rullemodstand

Modstand mod hjulets fremadrettede bevægelse som følge af hjulets deformation: $f_r = N\mu_r$

μ_r~0.01-0.02 for gummi mod asfalt,
~0.001-0.002 for stål mod stål



Eksempel: Ligevægtsproblem II

Hvor stejl en bakke kan vi køre op ad?

$$NI_x: \; f_f + f_b - Mg\sin\theta = 0 \Rightarrow f_f + f_b = Mg\sin\theta$$

$$NI_y: \; n_f + n_b - Mg\cos\theta = 0 \Rightarrow n_f + n_b = Mg\cos\theta$$

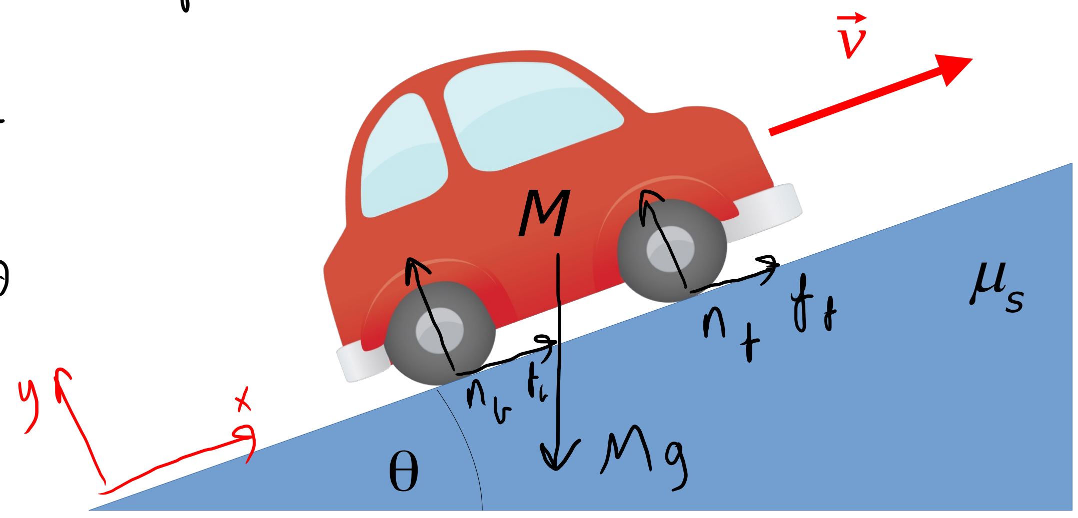
$$f_f \leq \mu_s n_f \; ; \; f_b \leq \mu_s n_b$$

$$\Downarrow$$

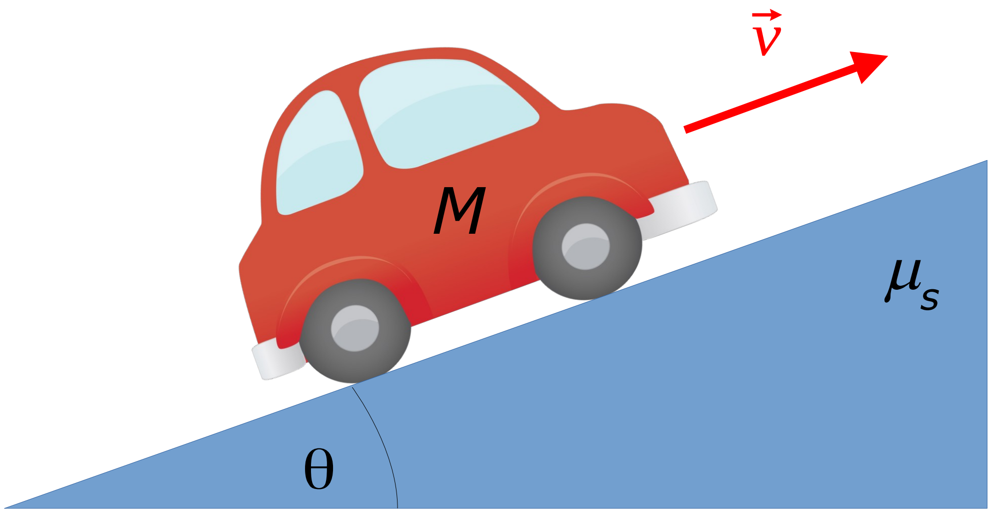
$$f_f + f_b \leq \mu_s (n_f + n_b) = \mu_s Mg\cos\theta$$

$$\Downarrow$$

$$\cancel{Mg}\sin\theta \leq \mu_s \cancel{Mg}\cos\theta \Rightarrow \tan\theta \leq \mu_s$$

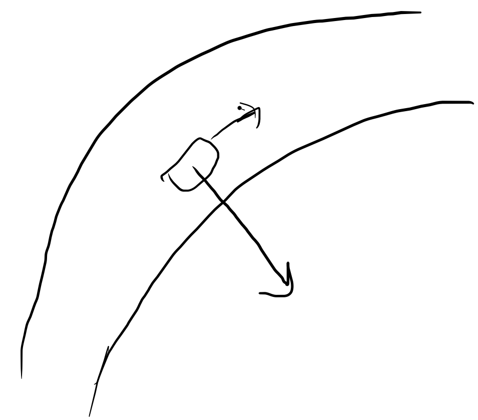


Eksempel: Ligevægtsproblem II



Eksempel: Bil i (fladt) sving

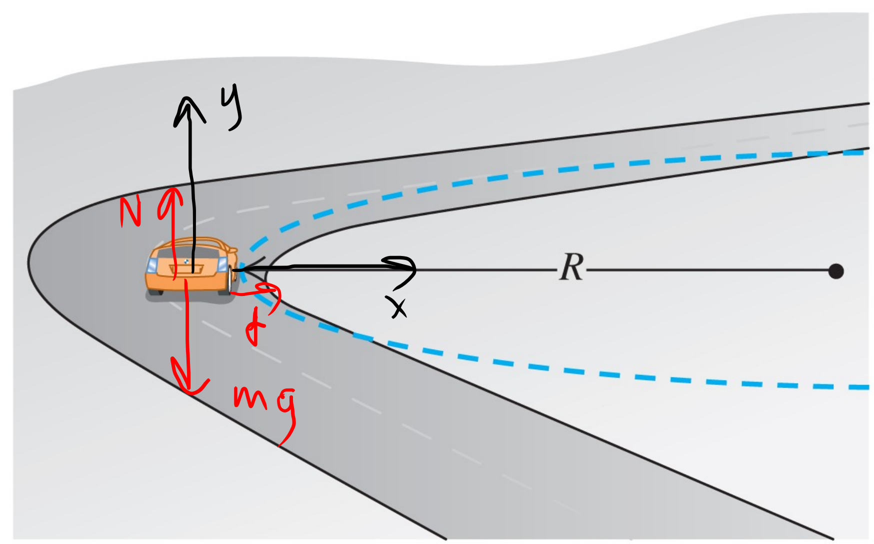
Hvor hurtigt kan svinget køres?



$$N\,\text{II}_x: \quad f = m\frac{v^2}{R} \le \mu_s N$$

$$N\,\text{II}_y: \quad N - mg = 0 \Rightarrow N = mg$$

$$\cancel{m}\frac{v^2}{R} \le \mu_s \cancel{m} g \Rightarrow v = \sqrt{\mu_s g R}$$

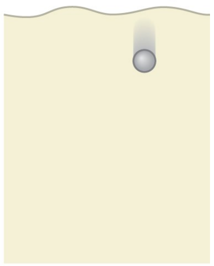


Opsummering: Væskefriktion

Modstand mod bevægelse i 'fluider' (væske/luft):

Terminalfart:

Tyk væske/lav fart: $\vec{f} \approx -k\,\vec{v}$



$$v_t = \frac{mg}{k}$$

Tynd væske (luft)/høj fart:

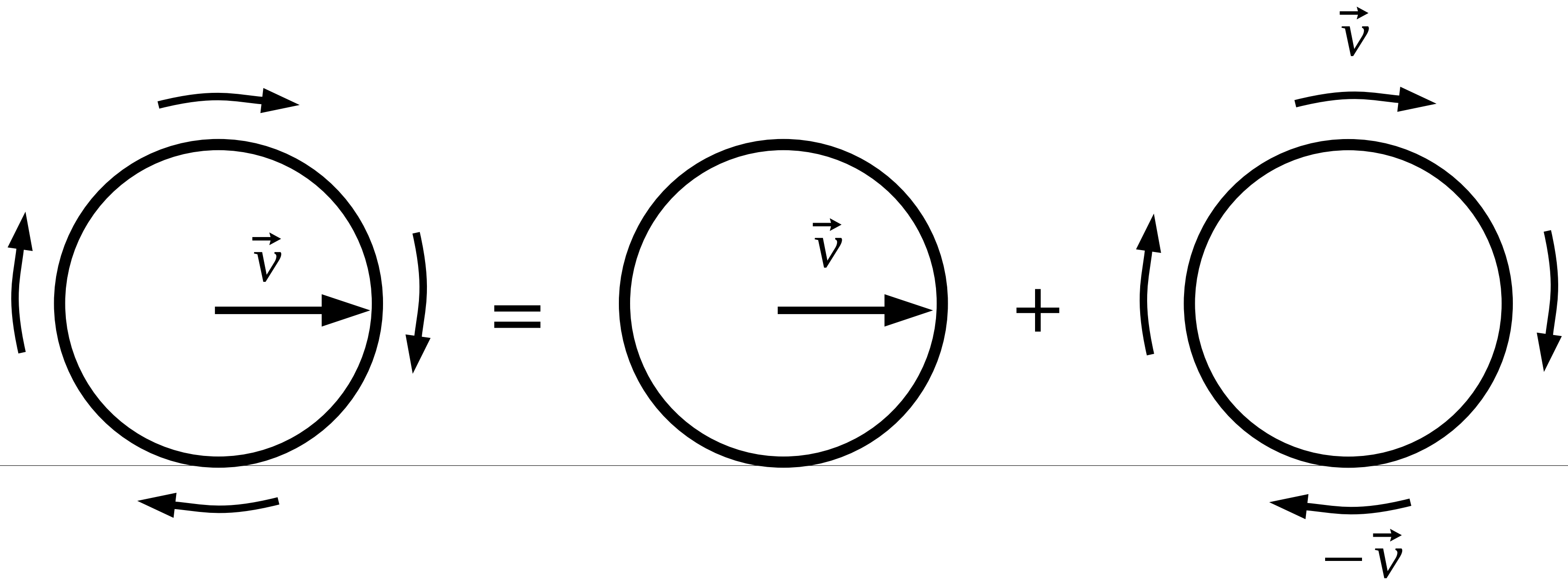
$$\vec{f} \approx -D\,|v|\,\vec{v} \Rightarrow f = Dv^2$$

$$v_t = \sqrt{\frac{mg}{D}}$$

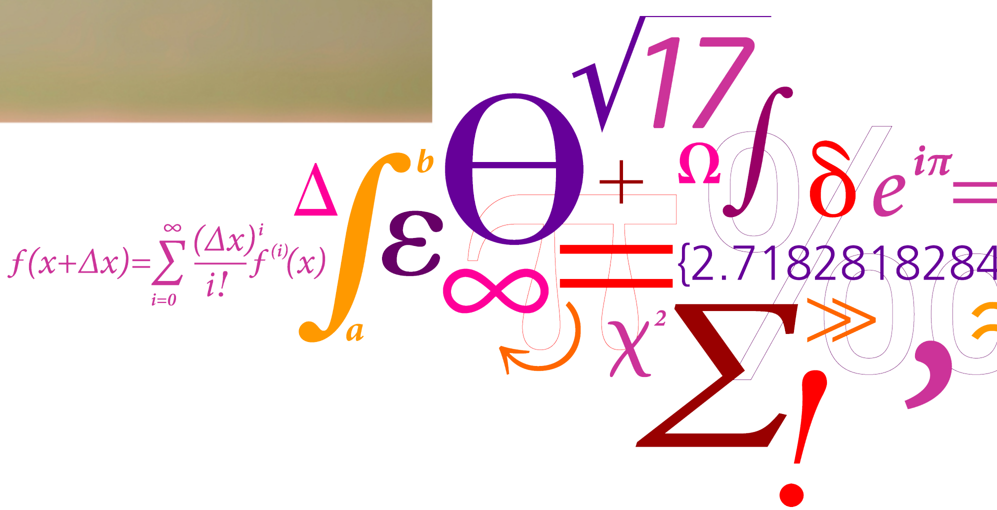
Opsummering

Rullebevægelse: Hjulets kontaktpunkt står stille i forhold til vejen
Statisk friktion mellem hjul og vej

Dertil *rullemodstand* fra deformation af hjulet:
Effektiv friktionskraft $f = \mu_r N$ (N=normalkraft)



Arbejde og kinetisk energi, Y & F kap. 6



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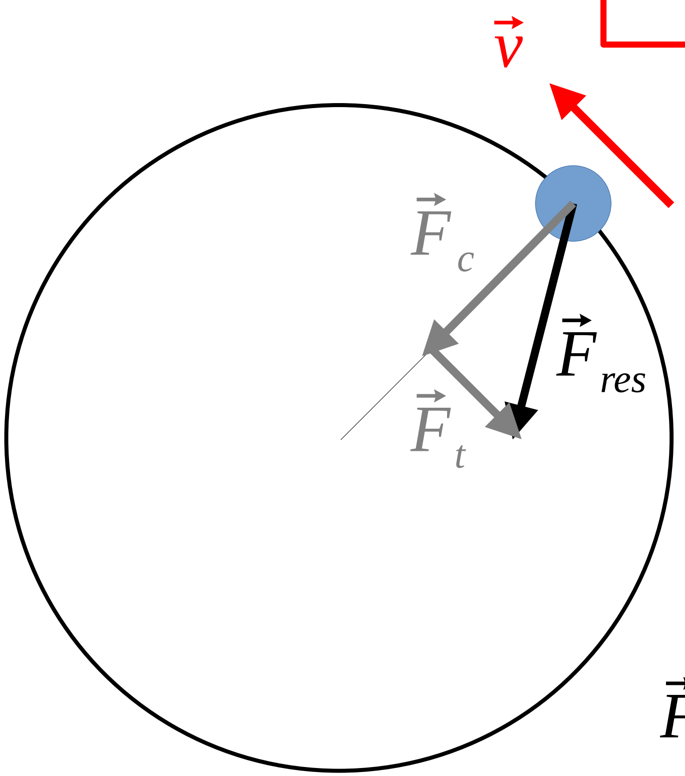
Fra sidste gang: Kræfter i cirkelbevægelse

$$a_c = \frac{v^2}{R} \qquad |a_t| = |\vec{a}_t| = \left|\frac{dv}{dt}\right| \qquad a_t \equiv \frac{dv}{dt}$$



$$F_c = m\,a_c = m\frac{v^2}{R}$$ Centripetalkraft

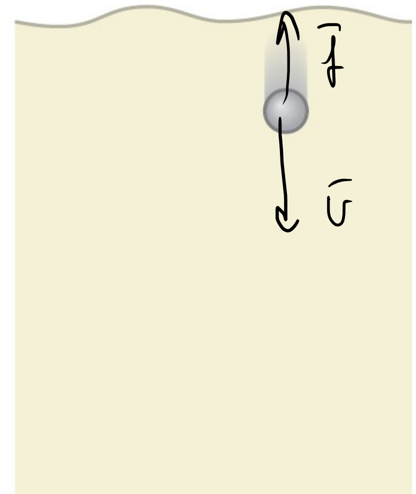
$$F_t = m\,a_t = m\frac{dv}{dt}$$ Tangentialkraft

$$\vec{F}_{res} = \vec{F}_c + \vec{F}_t$$

$\vec{F}_c$ og $\vec{F}_t$ er *ikke* selvstændige kræfter!

Fra sidste gang: Friktion

Væskefriktion:



Tyk væske/lav fart: $\vec{f} \approx -k\,\vec{v}$

Tynd væske (luft)/høj fart:

$$\vec{f} \approx -D\left|v\right|\vec{v} \Rightarrow f = Dv^2$$

Denne uges læringsmål

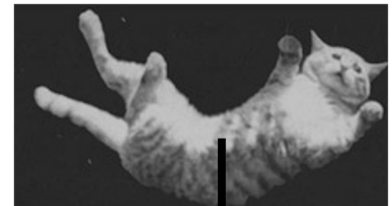
-Forstå sammenhængen mellem arbejde og kinetisk energi

-Benytte energibetragtninger ved bevægelse i tyngdefeltet og fjederbevægelse

-Forstå effektbegrebet

1D-bevægelse, konstant kraft

v_0



y_0

y

$$\vec{F} = -mg\,\hat{\boldsymbol{j}}$$

$$v^2(y) = v_0^2 + 2a(y - y_0)$$

$$F_{res} = m a \Rightarrow a = \frac{F_{res}}{m}$$

$$\Downarrow$$

$$v^2(y) = v_0^2 + 2\frac{F_{res}}{m}(y - y_0)$$

$$\Downarrow$$

$$\frac{1}{2} m v^2(y) = \frac{1}{2} m v_0^2 + F_{res}(y - y_0)$$

$$\Downarrow$$

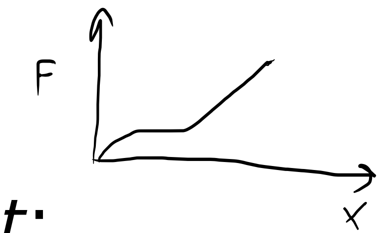
$$\Delta K = \frac{1}{2} m\left(v^2(y) - v_0^2\right) = F_{res}(y - y_0)$$

$y,\ v(y)$

Definér kinetisk energi $K = \frac{1}{2} m v^2$

Arbejdssætningen, 1D

$$F_{res} = ma = m\frac{dv}{dt}$$



Generelt – kraften kan variere med *x*,*t*:

$$\int_{x_0}^{x_1} F_{res}(x)\,dx = \int_{x(t_0)}^{x(t_1)} F_{res}(x(t))\,dx = \int_{t_0}^{t_1} F_{res}(x(t))\frac{dx}{dt}\,dt = \int_{t_0}^{t_1} F_{res}(x(t))\,v\,dt =$$

$$m\int_{t_0}^{t_1} \frac{dv}{dt}\,v\,dt = m\int_{v_0}^{v_1} v\,dv = \frac{1}{2}m\left[v^2\right]_{v_0}^{v_1} = \frac{1}{2}m\left(v_1^2 - v_0\right) = K_1 - K_0 = \Delta K$$

$$\int_{t_0}^{t_1} f(u(t))\frac{du}{dt}\,dt = \int_{u_0}^{u_1} f(u)\,du$$

Arbejdssætningen, 1D

Generelt – kraften kan variere med *x*,*t*:

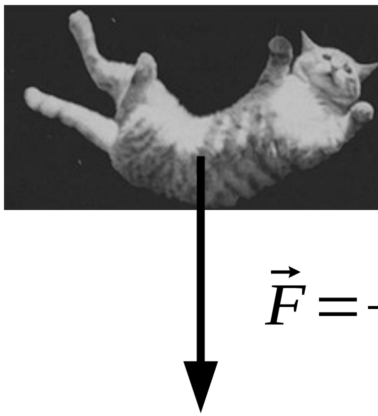
$$\int_{x_0}^{x_1} F_{res}\,dx = \frac{1}{2} m\left(v_1^2 - v_0^2\right)$$

Definér *arbejdet* $W = \int F_{res}\,dx$ - enhed Nm=kg m²/s² =J(oule)

$\frac{1}{2} m\left(v_1^2 - v_0^2\right)$ er ændringen i kinetisk energi, $K = \frac{1}{2} m v^2$

Arbejdssætningen (for kinetisk energi): $W = \Delta K$

Arbejdssætningen, 1D



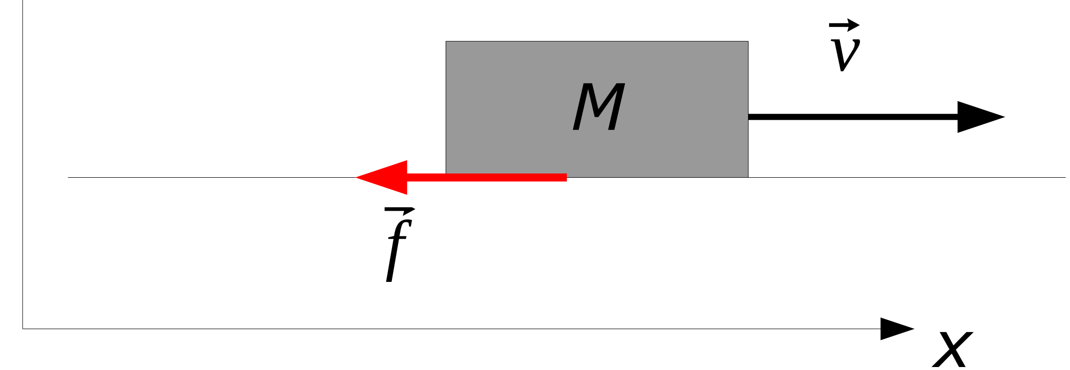
$W = F_g (y - y_0)$

$$\vec{F} = -mg\,\hat{\boldsymbol{j}}$$

$\vec{F}$ og $\vec{v}$ ensrettede - kinetisk energi vokser.

$$W = \int_{x_0}^{x_1} F_{res}\, dx = \Delta K$$

$\vec{f}$ og $\vec{v}$ modsatrettede - kinetisk energi falder.



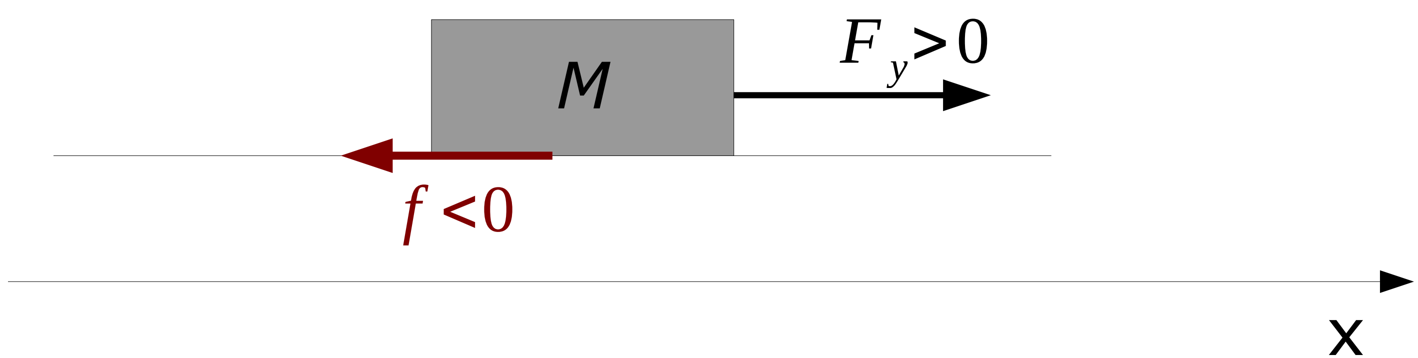
Arbejdssætningen, 1D

Arbejde fra ydre kraft F_y

$$W_{tot} = \int F_{res}\,dx = \int \left(F_y + f\right) dx = \overbrace{\int F_y\,dx}^{W_y} + \underbrace{\int f\,dx}_{W_f} = W_y + W_f$$

Totalt arbejde

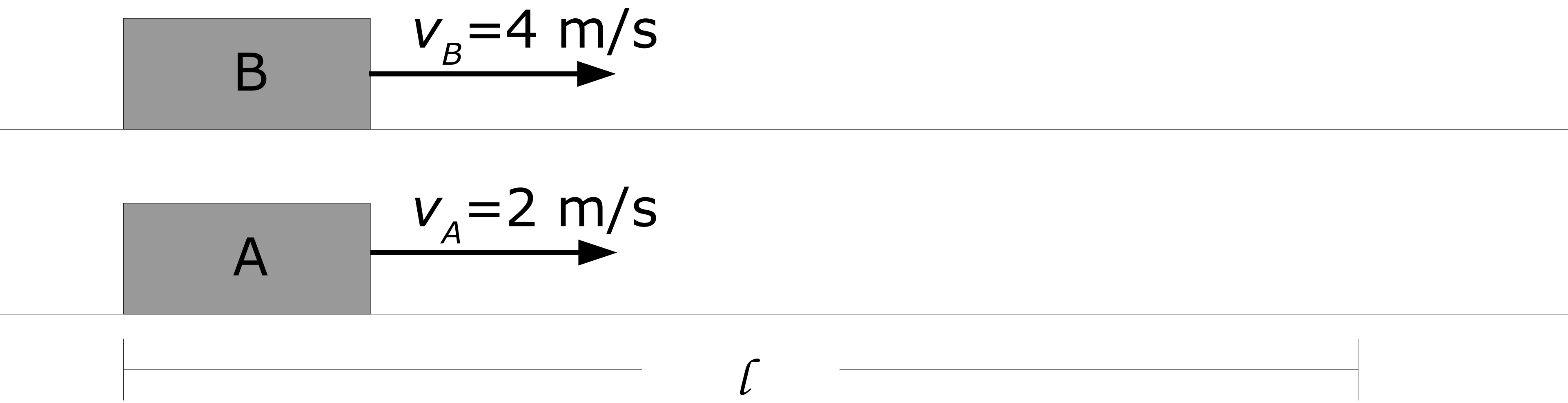
Arbejde fra friktionskraft



Quiz – arbejde mod friktion

Klodserne trækkes med konstant fart mod kinetisk friktion. Klodserne har samme masse og μ_k. Hvordan forholder trækkraftens arbejde på A og B sig til hinanden?

1: $W_A > W_B$ 2: $W_A = W_B$ 3: $W_A < W_B$



Quiz – arbejde mod friktion

Klodserne trækkes med konstant fart mod kinetisk friktion. Klodserne har samme masse og μ_k. Hvordan forholder trækkraftens arbejde på A og B sig til hinanden?

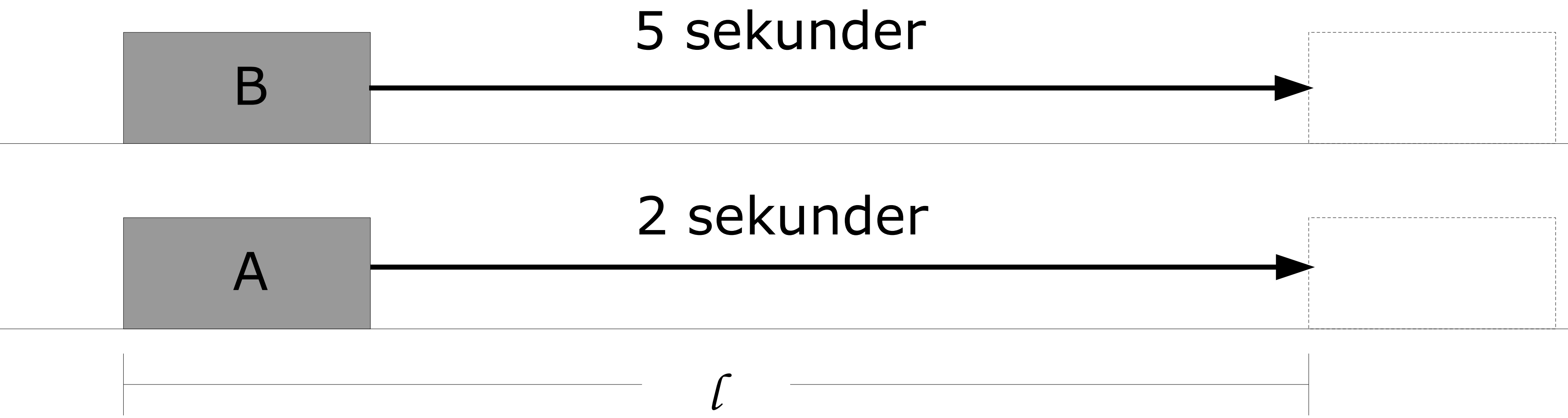
1: $W_A > W_B$ 2: $W_A = W_B$ 3: $W_A < W_B$

$\Delta K = 0 = W = W_y + W_f \Rightarrow W_y = -W_f$

Quiz – arbejde mod friktion

Klodserne trækkes fra stilstand med konstant kraft mod kinetisk friktion. Klodserne har samme masse. Hvordan forholder trækkraftens arbejde på A og B sig til hinanden?

1: $W_A > W_B$ 2: $W_A = W_B$ 3: $W_A < W_B$



Quiz – arbejde mod friktion

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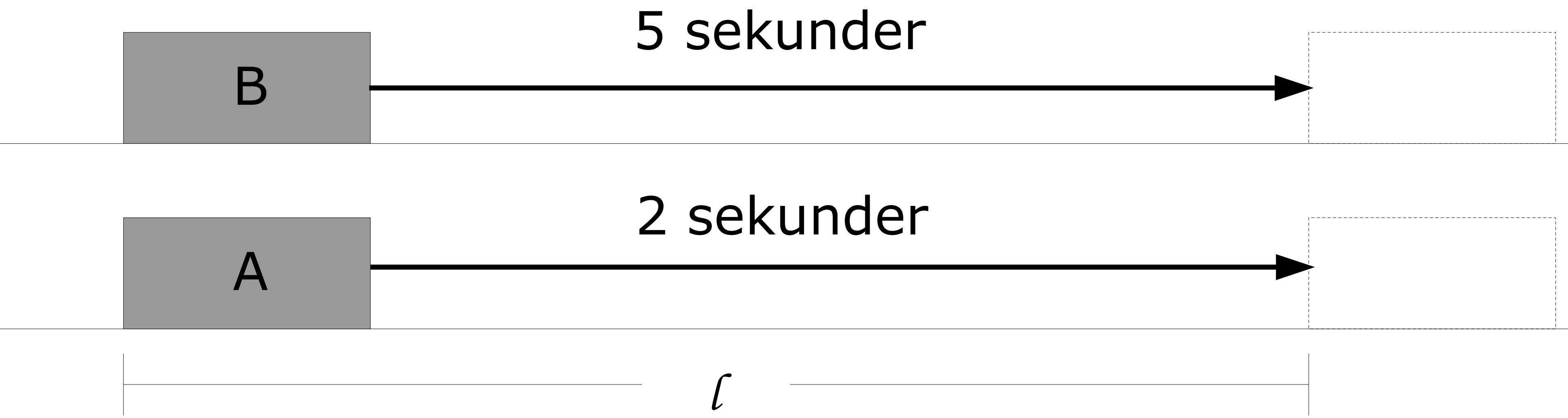


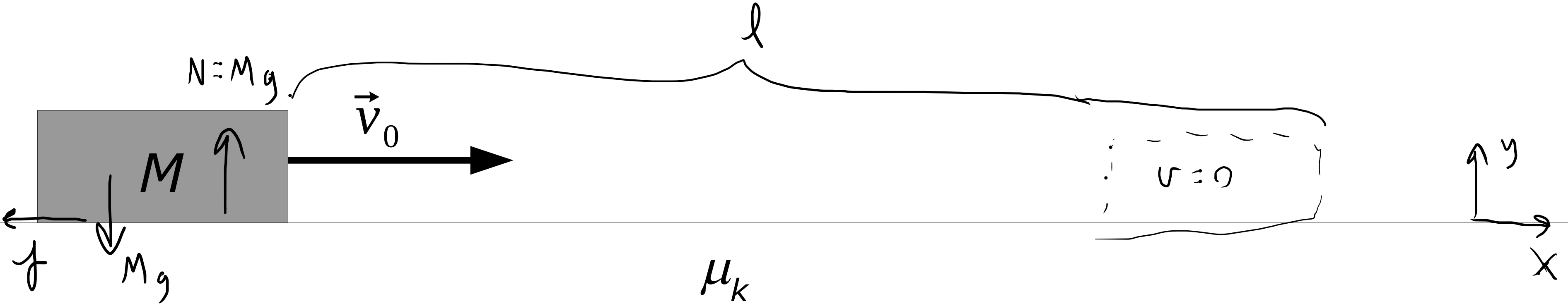
Eksempel – arbejde mod friktion

Hvor langt glider kassen? $\Delta K = W$

$$\Delta K = \frac{1}{2} m\left(0^2 - v_0^2\right) = -\frac{1}{2}\cancel{M} v_0^2 = W = fl = -\mu_k \cancel{M} g l$$

$$\Downarrow$$

$$l = \frac{v_0^2}{\mu_k g}$$



Eksempel: Klods på skråplan

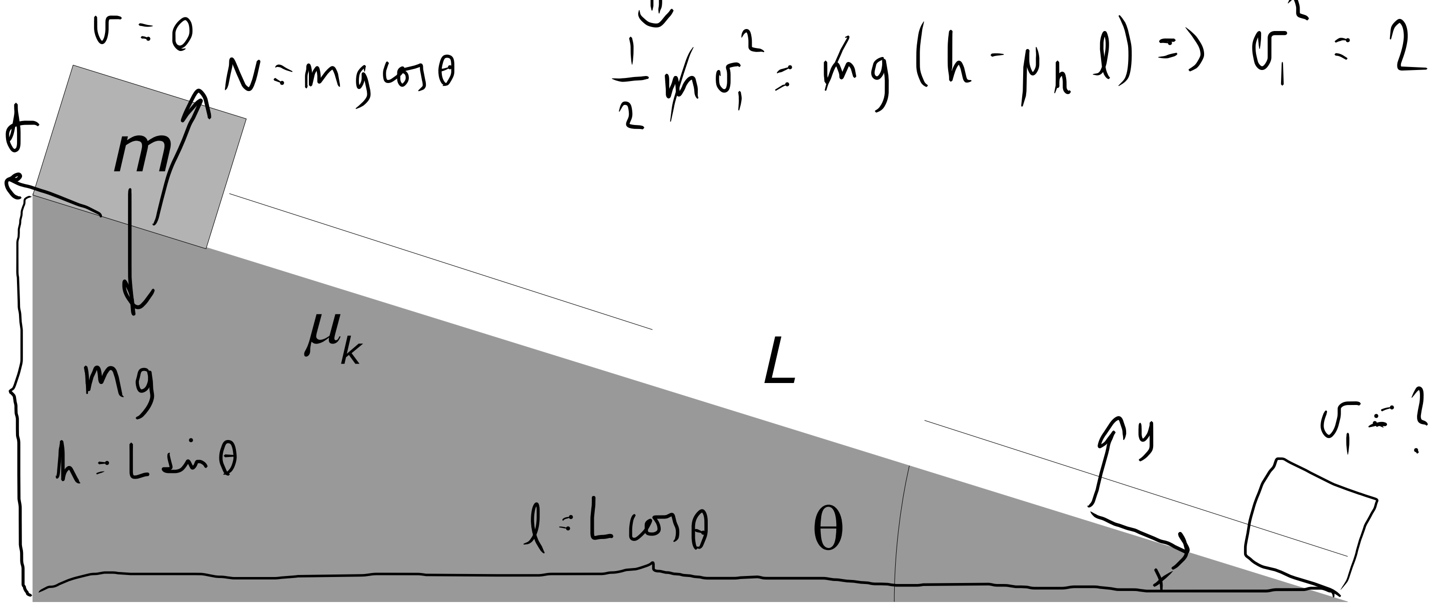
Hvad er klodsens fart ved bunden af skråplanet?

$$f = -\mu_k N = -\mu_k m g \cos\theta \qquad W = \Delta K = \frac{1}{2} m \left(v_1^2 - 0^2\right) = \frac{1}{2} m v_1^2$$

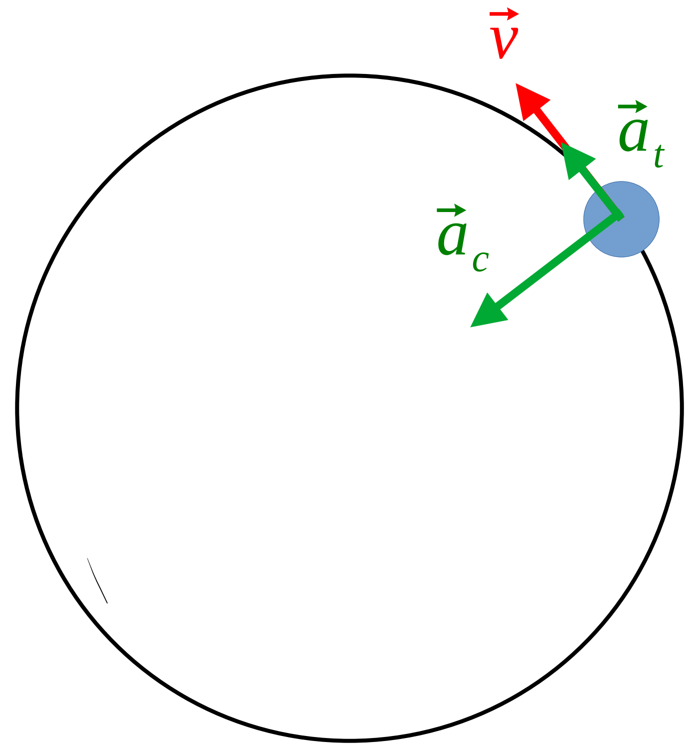
$$W = F_{res} L = (mg\sin\theta - \mu_k mg\cos\theta) L = mgh - mg\mu_k l$$

$$\Downarrow$$

$$\frac{1}{2} \cancel{m} v_1^2 = \cancel{m} g (h - \mu_k l) \Rightarrow v_1^2 = 2g(h - \mu_k l)$$



Arbejde i flere dimensioner

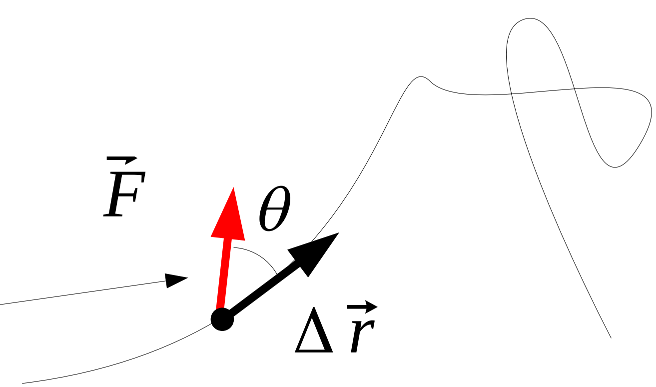


$$a_c = \frac{v^2}{R} \qquad |a_t| = |\vec{a}_t| = \left|\frac{dv}{dt}\right| \qquad a_t \equiv \frac{dv}{dt}$$

Kun acceleration (kraft) parallelt med hastigheden ændrer *v* og dermed kinetisk energi $K = \frac{1}{2} m v^2$

I 2D/3D: $\boxed{W = \int \vec{F} \cdot d\vec{r} = \Delta K}$

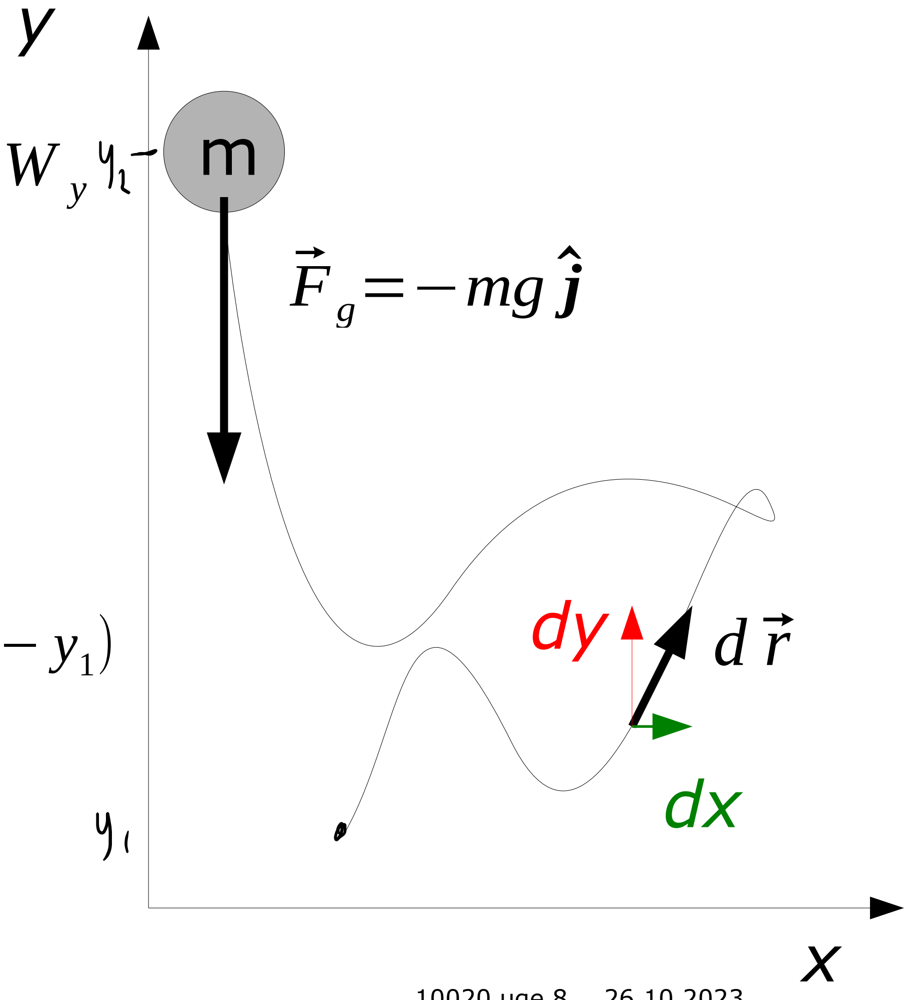
$$\Delta W = F \Delta r \cos\theta$$



Generel bevægelse i tyngdefelt

Kræfter er additive:

$$\Delta K = \int \vec{F}_{tot} \cdot d\vec{r} = \int (\vec{F}_g + \vec{F}_y) \cdot d\vec{r} = W_g + W_y$$



Arbejde *fra tyngdekraften* på partiklen:

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_g \cdot d\vec{r} = -mg \int_{\vec{r}_1}^{\vec{r}_2} \hat{\boldsymbol{j}} \cdot d\vec{r} = -\int_{y_1}^{y_2} mg\,dy = -mg(y_2 - y_1)$$

Eksempel: Rutsjebaneloop

Hvad bliver farten i toppen af loopet?

$$W = \Delta K \qquad \Delta K = \frac{1}{2} m \left(v_1^2 - v_0^2 \right)$$

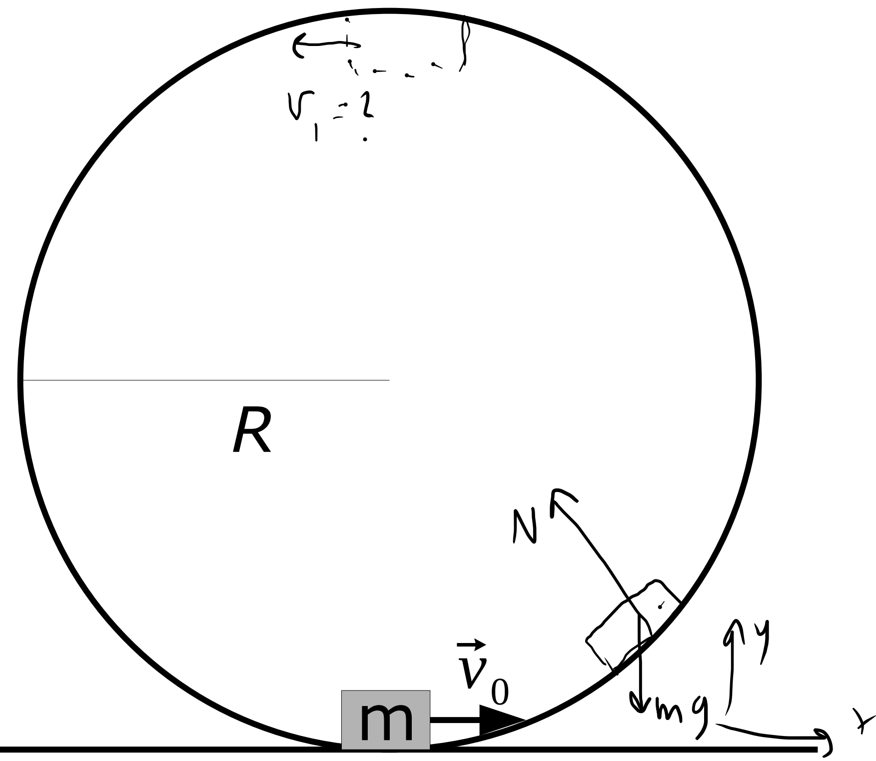
$$W = W_g = -mg\left(y_1 - y_0 \right) = -mg2R$$

$$\Downarrow$$

$$\frac{1}{2} \cancel{m} \left(v_1^2 - v_0^2 \right) = -2\cancel{m}gR$$

$$\Downarrow$$

$$v_1^2 - v_0^2 = -4gR \Rightarrow v_1 = \sqrt{v_0^2 - 4gR}$$



Quiz - vægtløftning

Hvor meget arbejde udfører vægtløftersmølf på vægten fra han starter løftet til vægten er sat på jorden igen?

A: 0 J

B: 2000 J

C: 4000 J

D: Kræver yderligere oplysninger

Quiz - vægtløftning

Hvor meget arbejde udfører vægtløftersmølf på vægten fra han starter løftet til vægten er sat på jorden igen?

$$\Delta K = 0 = W = W_s + W_g = W_s$$

A: 0 J

B: 2000 J

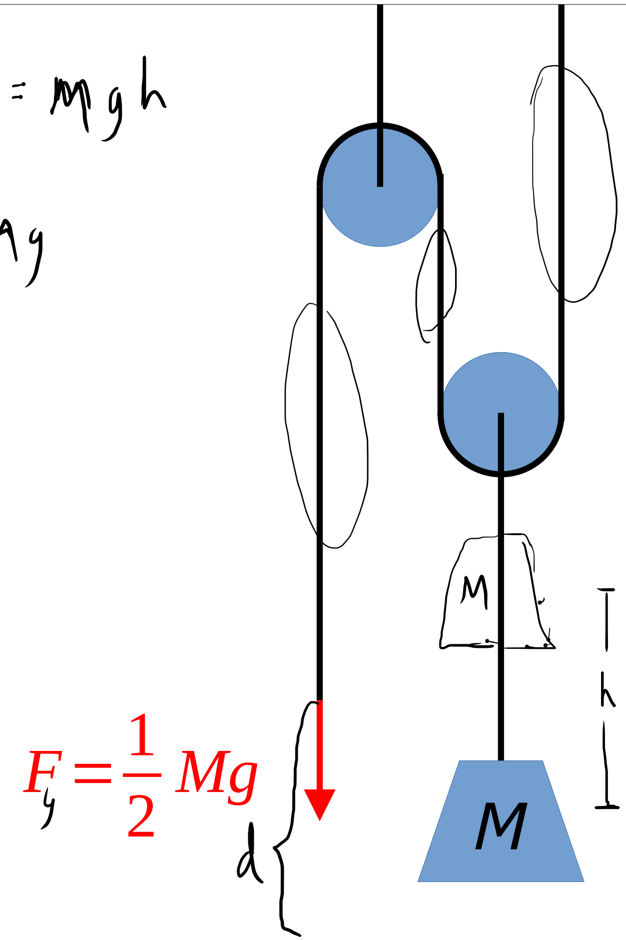
C: 4000 J

D: Kræver yderligere oplysninger

Eksempel – lodder og trisser

$$\Delta K = 0 = W_y + W_g = W_y - mgh \Rightarrow W_y = mgh$$

$$W_y = F_y d = F_y 2\cancel{h} = Mg\cancel{h} \Rightarrow F_y = \frac{1}{2} Mg$$

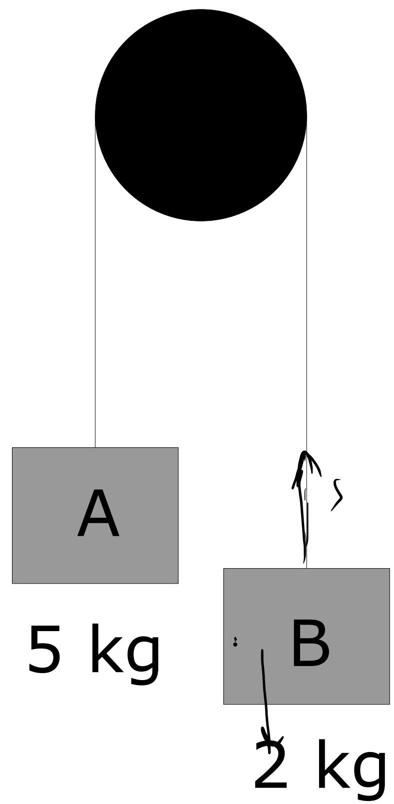


Quiz

Sandt/falsk?

1: Snorekraften udfører positivt arbejde på B

2: Snorekraften udfører positivt arbejde på A



3: Tyngdekraften udfører positivt arbejde på B

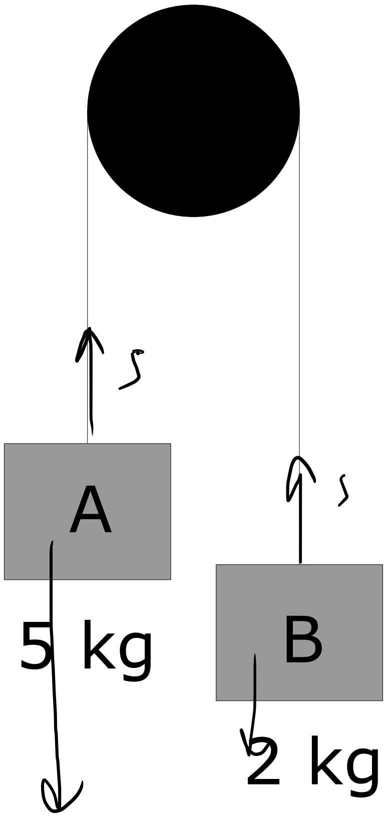
4: Tyngdekraften udfører samlet set positivt arbejde på A+B

Quiz

Sandt/falsk?

1: Snorekraften udfører positivt arbejde på B

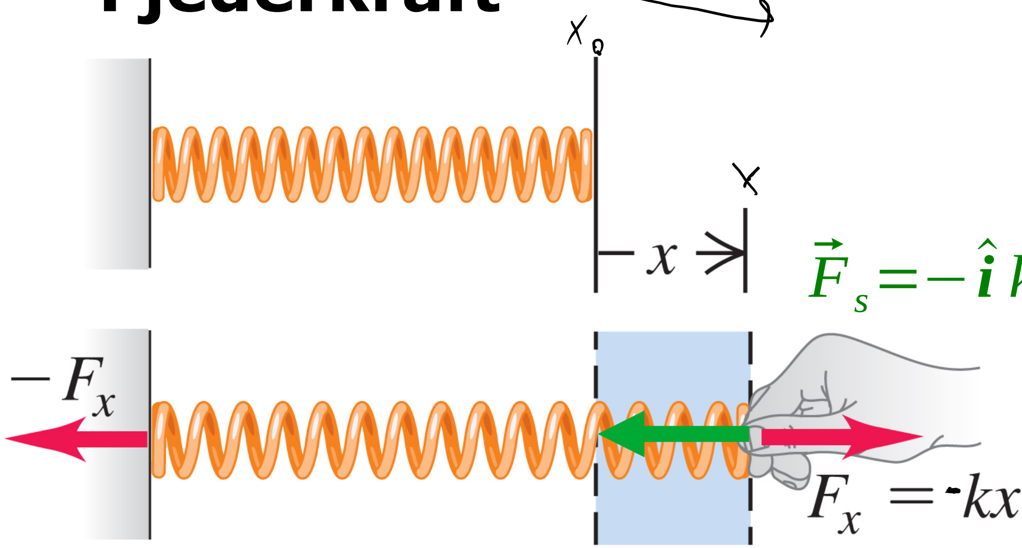
2: Snorekraften udfører positivt arbejde på A



3: Tyngdekraften udfører positivt arbejde på B

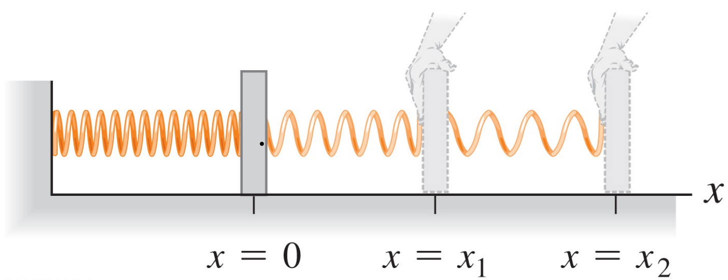
4: Tyngdekraften udfører samlet set positivt arbejde på A+B

Fjederkraft



$$\vec{F}_s = -\hat{\mathbf{i}}\, k\,(x - x_0)$$

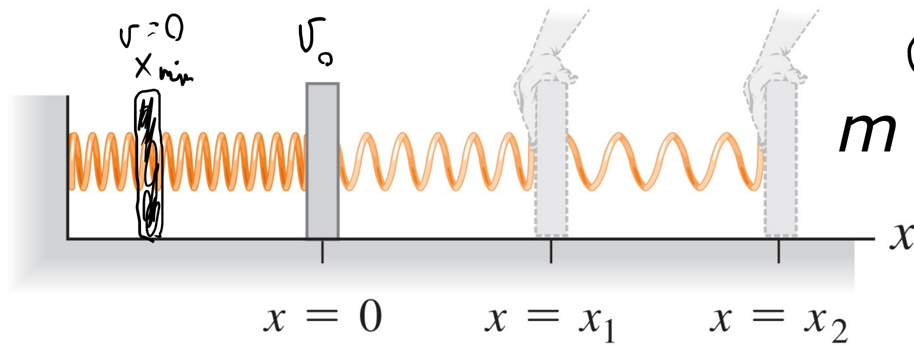
$x_0 = 0$



$$\int_0^{x_1} F_s\,dx = -k\int_0^{x_1} x\,dx = -\frac{1}{2}k\,x_1^2$$

$$\int_{x_1}^{x_2} F_s\,dx = -k\int_{x_1}^{x_2} x\,dx = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

Eksempel – vandret fjederbevægelse



(1) Klodsen slippes i x_2. Hvad er farten i $x=0$? Hvor langt mod venstre kommer den? (2)

$$\boxed{W = \Delta K}$$

$$\Delta K = \frac{1}{2} m \left(v_0^2 - 0^2\right) = \cancel{\frac{1}{2}} m v_0^2 = W = -\frac{1}{2} k \left(0^2 - x_2^2\right) =$$

(1)

$$\cancel{\frac{1}{2}} k x_2^2 \Rightarrow v_0^2 = \frac{k}{m} x_2^2 \Rightarrow v_0 = \sqrt{\frac{k}{m}}\, x_2$$

---

(2)

$$\Delta K = \frac{1}{2} m \left(0^2 - 0^2\right) = 0 = W = -\frac{1}{2} k \left(x_{min}^2 - x_2^2\right) = 0 \Rightarrow x_{min}^2 = x_2^2 \Rightarrow x_{min} = -x_2$$

Effekt – arbejde pr. tid

Effekt: $P = \frac{dW}{dt}$ $\qquad W(t) = \int_{\vec{r}(0)}^{\vec{r}(t)} \vec{F} \cdot d\vec{r} = \int_0^t \vec{F} \cdot \frac{d\vec{r}}{dt_1} dt_1 = \int_0^t \vec{F} \cdot \vec{v}\, dt_1$

$$\Downarrow$$

$$\boxed{P = \frac{d}{dt} \int_0^t \vec{F} \cdot \vec{v}\, dt_1 = \vec{F} \cdot \vec{v}}$$

-enhed J/s=W(att)

$$F_{luft} = \frac{1}{2} c_d A \rho v^2$$

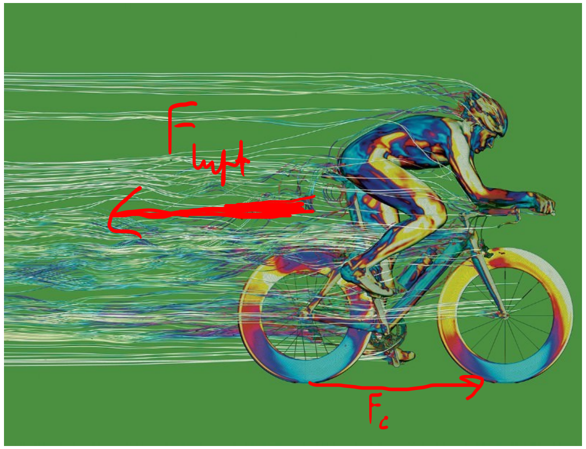
Fart

Luftens massetæthed

Formfaktor

Frontareal

$$P = \vec{F}_c \cdot \vec{v} = \frac{1}{2} c_d A \rho v^2 \cdot v = \frac{1}{2} c_d A \rho v^3$$



Quiz - luftmodstand

Antag at elbilen kun påvirkes af luftmodstand. Når farten *fordobles* ændres motoreffekt/rækkevidde med faktor:

Rækkevidde:	Motoreffekt:
1/2	2
1/4	4
1/8	8

Quiz - luftmodstand

Antag at elbilen kun påvirkes af luftmodstand. Når farten *fordobles* ændres motoreffekt/rækkevidde med faktor:

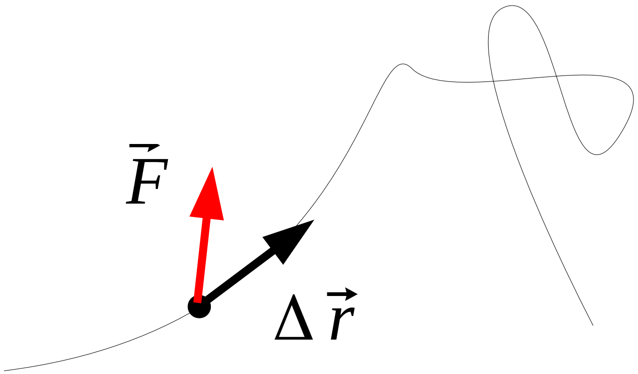
Rækkevidde:	Motoreffekt:
1/2	2
1/4	4
1/8	8

$W = F_{luft} \cdot L$

Opsummering

$W=\int \vec{F}\cdot d\vec{r}$ er *arbejdet* på en partikel der udfører en bevægelse under påvirkning af resulterende kraft $\vec{F}$

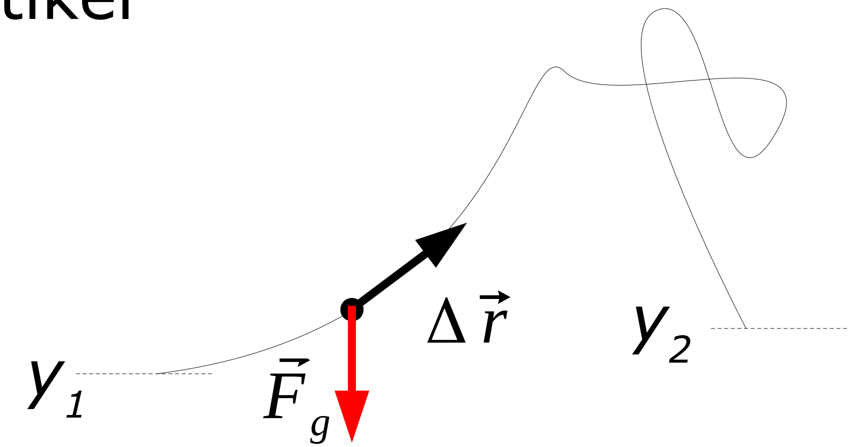
$K=\frac{1}{2}mv^2$ er *kinetisk energi*



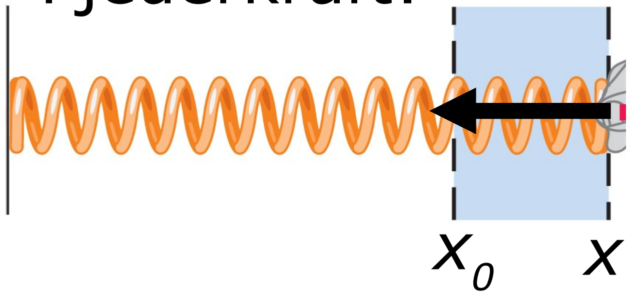
$\boxed{W=\Delta K}$ *Arbejdssætningen* for kinetisk energi - arbejdet giver ændringen i kinetisk energi fra start til slut i bevægelsen

Opsummering

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_g \cdot \vec{dr} = -mg\left(y_2 - y_1\right)$$ - tyngdekraftens arbejde på partikel



Fjederkraft:



$$\vec{F} = -K\left(x - x_0\right)\hat{\boldsymbol{i}}$$

$$W = \frac{1}{2}K\left(x - x_0\right)^2$$

Opsummering

$$F_{drag} = \frac{1}{2} c_d A \rho v^2$$

Formfaktor

Frontareal

Luftens massetæthed

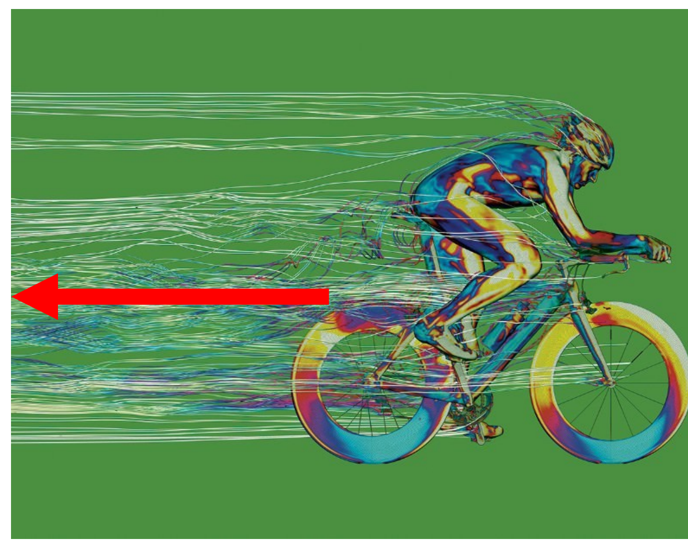
Effekt:

$$P = \frac{d}{dt} \int_0^t \vec{F} \cdot \vec{v}\, dt_1 = \vec{F} \cdot \vec{v}$$

$$P = \frac{dW}{dt} = \frac{1}{2} c_d A \rho v^3$$

Effekt krævet for at overvinde luftmodstand ved konstant fart

$\vec{F}_{drag}$



Impuls og stødprocesser, Y & F kap. 8

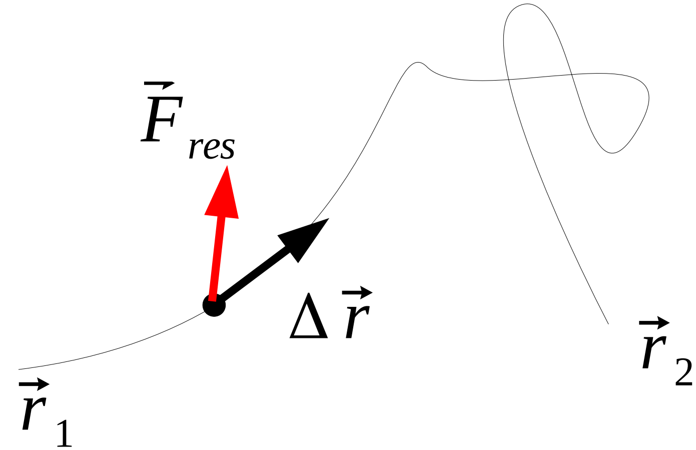
DTU Fotonik
Institut for Fotonik

Fra sidste gang

$$K=\frac{1}{2}mv^2$$ *Kinetisk energi*

$$W=\int \vec{F}_{res}\cdot d\vec{r}$$ *Arbejde*

$W = \Delta K$



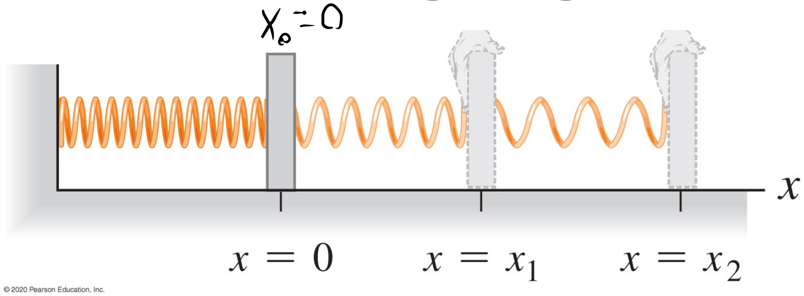
$$\vec{F}=-\nabla U \Leftrightarrow W_F=-(U(\vec{r}_2)-U(\vec{r}_1))=-\Delta U$$

U potentialfunktion
for *konservativ kraft*

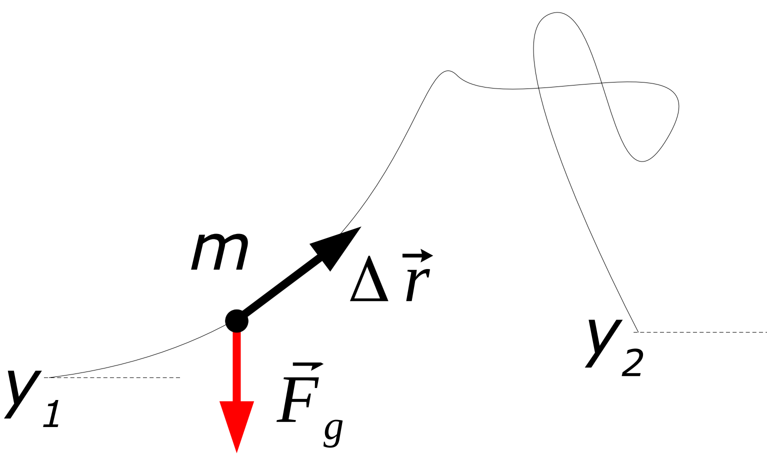
$$\boxed{W_{other}=\Delta K+\Delta U}$$

Arbejdssætningen - *W_{other}* er arbejdet fra ikke-konservative bidrag til $\vec{F}_{res}$

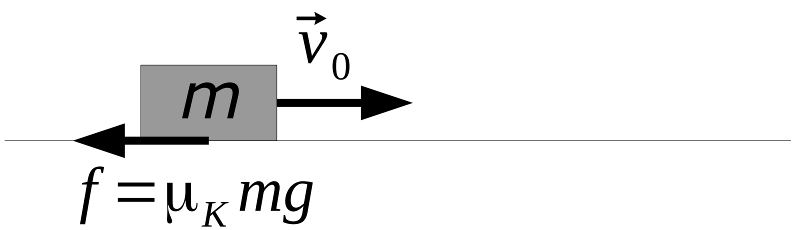
Fra sidste gang



$$U(x) = \frac{1}{2}k(x - x_0)^2$$



$$U(y) = mgy$$



Friktionskraft ikke konservativ, ingen potentialfunktion

Denne uges læringsmål

-Forstå begrebet *impuls*

-Forstå betingelserne for *impulsbevarelse*

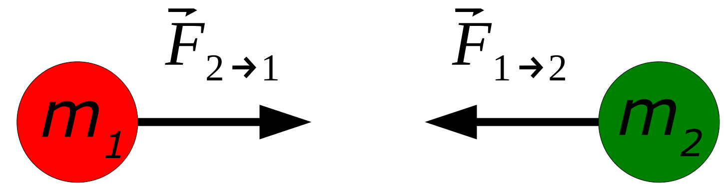
-Regne på *elastiske* og *inelastiske* stødprocesser

-Forstå begrebet massemidtpunkt, og *massemidtpunktssætningen*

-Forstå raketbevægelse

Impulsbevarelse

To partikler "alene i verden":



N2: $\vec{F}_{2\to1} = m_1\vec{a}_1 = m_1\dfrac{d\vec{v}_1}{dt}$ $\qquad\vec{F}_{1\to2} = m_2\vec{a}_2 = m_2\dfrac{d\vec{v}_2}{dt}$

N3: $\vec{F}_{2\to1} = -\vec{F}_{1\to2}$

$$\Downarrow$$

$$m_1\frac{d\vec{v}_1}{dt} = -m_2\frac{d\vec{v}_2}{dt} \Rightarrow m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = 0 \Rightarrow \frac{d}{dt}\left(\overbrace{m_1\vec{v}_1}^{\bar{p}_1} + \overbrace{m_2\vec{v}_2}^{\bar{p}_2}\right) = 0$$

Definér *totalimpuls* $\vec{P}_{tot} \equiv \vec{p}_1 + \vec{p}_2 \equiv m_1\vec{v}_1 + m_2\vec{v}_2 \Rightarrow \dfrac{d\vec{P}_{tot}}{dt} = 0$

Impuls og impulsændring

Én partikel: $\vec{p} \equiv m\vec{v}$ impuls (eng. *momentum*, gl. dansk *bevægelsesmængde*)

Newton 2: $\vec{F} = m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt} \Rightarrow \int_{t_1}^{t_2} \vec{F}\,dt = m(\vec{v}_2 - \vec{v}_1) = \Delta\vec{p}$

$$\boxed{\int_{t_1}^{t_2} \vec{F}\,dt = \Delta\vec{p}}$$

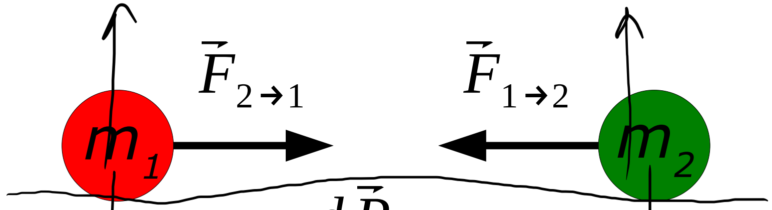
Impulsændring, da. 'kraftens impuls', eng. *impulse*

$$\vec{F}_{ave} = \frac{\Delta\vec{p}}{t_2 - t_1}$$

'Gennemsnitlig' kraft i tidsinterval

Impulsbevarelse

To partikler "alene i verden":



$$\frac{d\vec{P}_{tot}}{dt}=0$$

$$\vec{P}_{tot}=\vec{p}_1+\vec{p}_2=m\,\vec{v}_1+m\,\vec{v}_2$$

$\Delta t \simeq 0.01\,s$

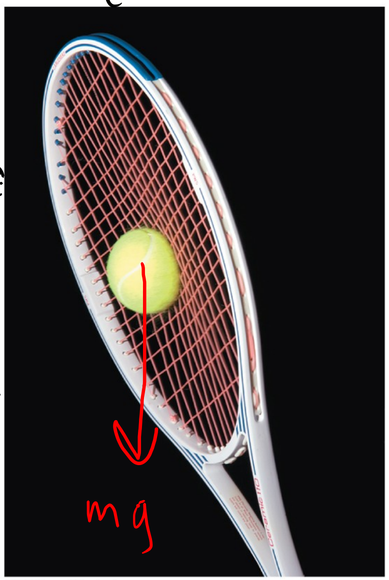
$m \simeq 0.06\,kg$ $\quad mg = 0.06\,kg\cdot 9.8\,\frac{m}{s^2} \simeq 0.6\,N$

$\Delta p = m(v_u - v_i) = 0.06\,kg\cdot 50\,\frac{m}{s} = 3\cdot Ns$ $\quad F_{ave} = \frac{3\,Ns}{0.01\,s} \simeq 300\,N$

$v_i = -20\,\frac{m}{s}$

$v_u = 30\,\frac{m}{s}$

x

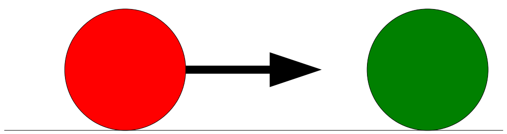


Almindeligvis er der ydre kræfter på partiklerne.

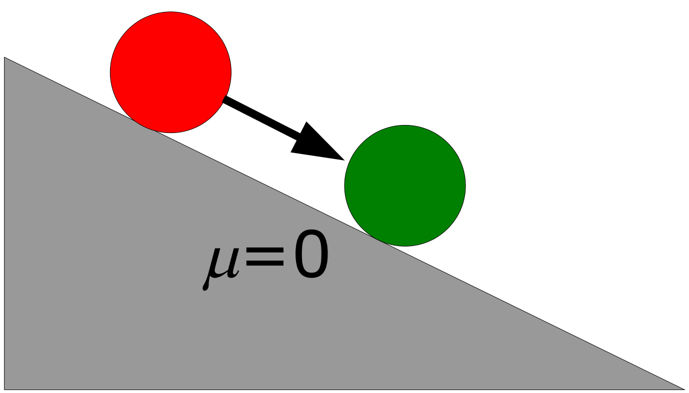
I kollisioner ('stød') er kræfterne mellem de involverede partikler typisk meget større end ydre kræfter

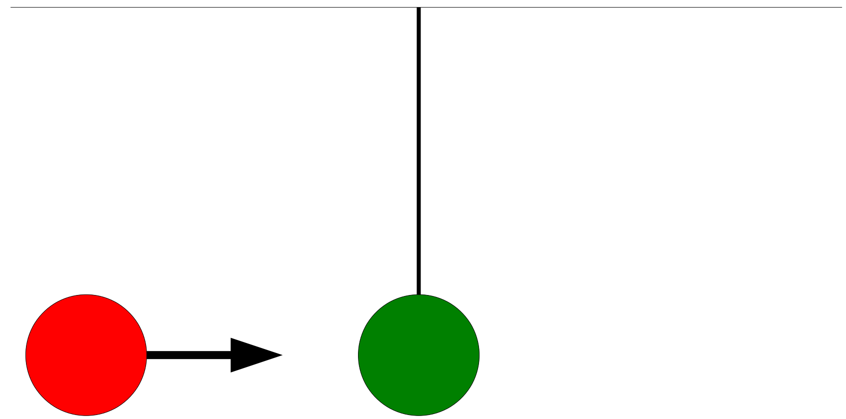
Quiz: Impulsbevarelse

I hvilke situationer kan vi regne med impulsbevarelse i stødøjeblikket?



$\mu=0$

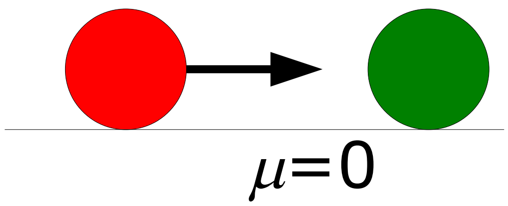


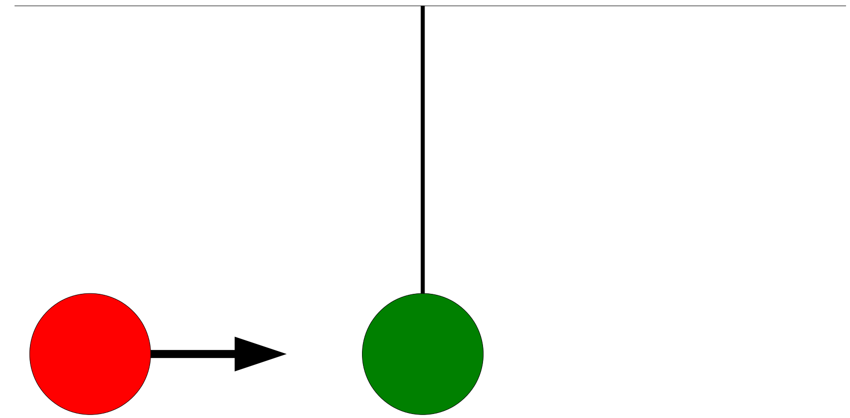


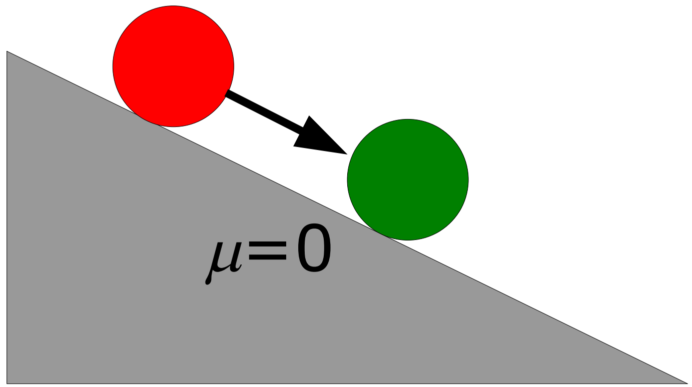
$\mu=0$

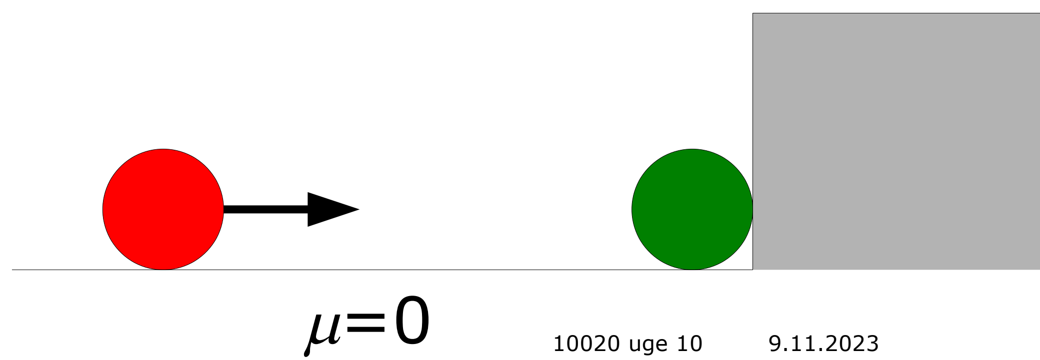
Quiz: Impulsbevarelse

I hvilke situationer kan vi regne med impulsbevarelse i bevægelsen efter stødet?



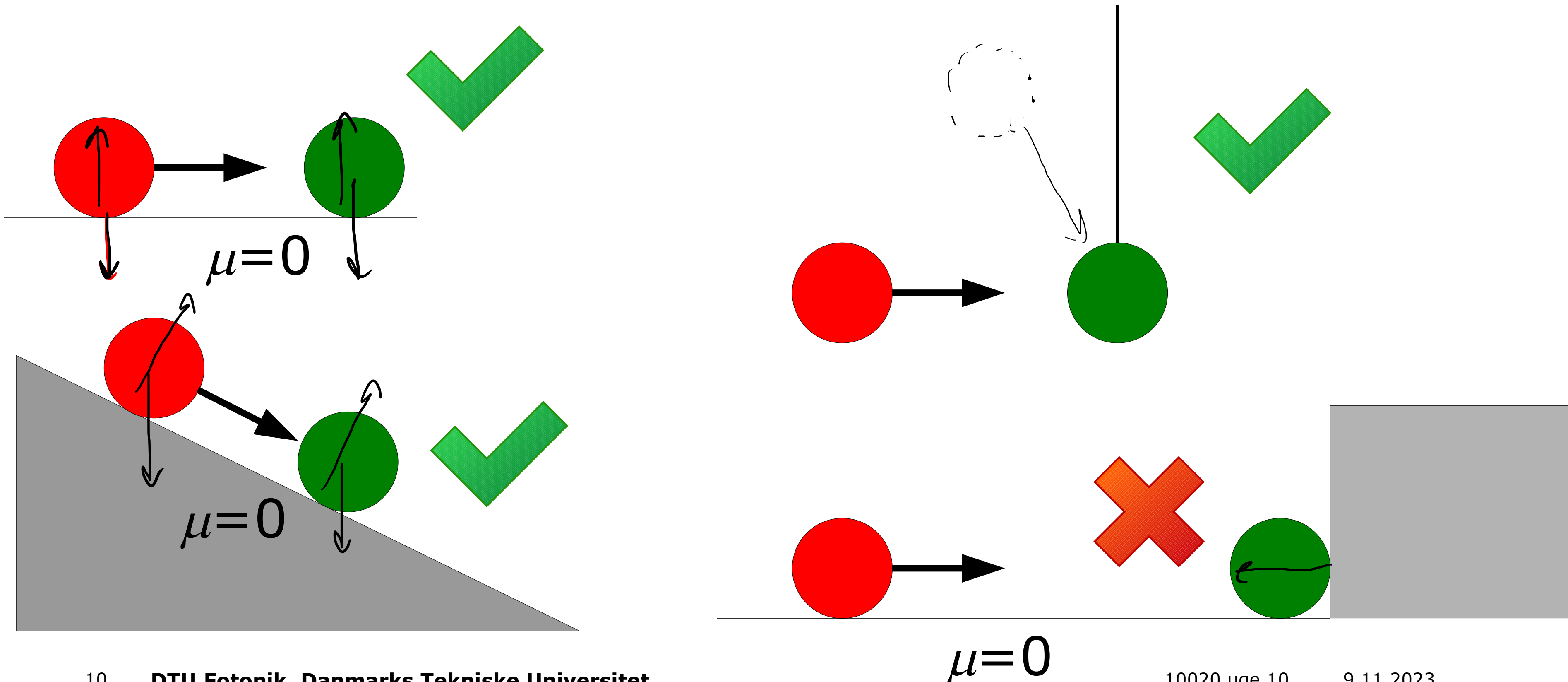






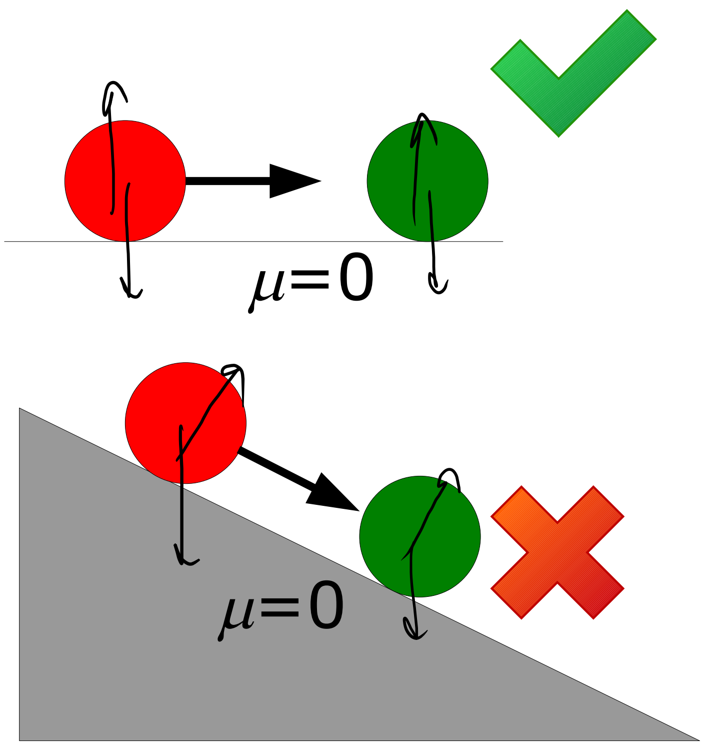
Quiz: Impulsbevarelse

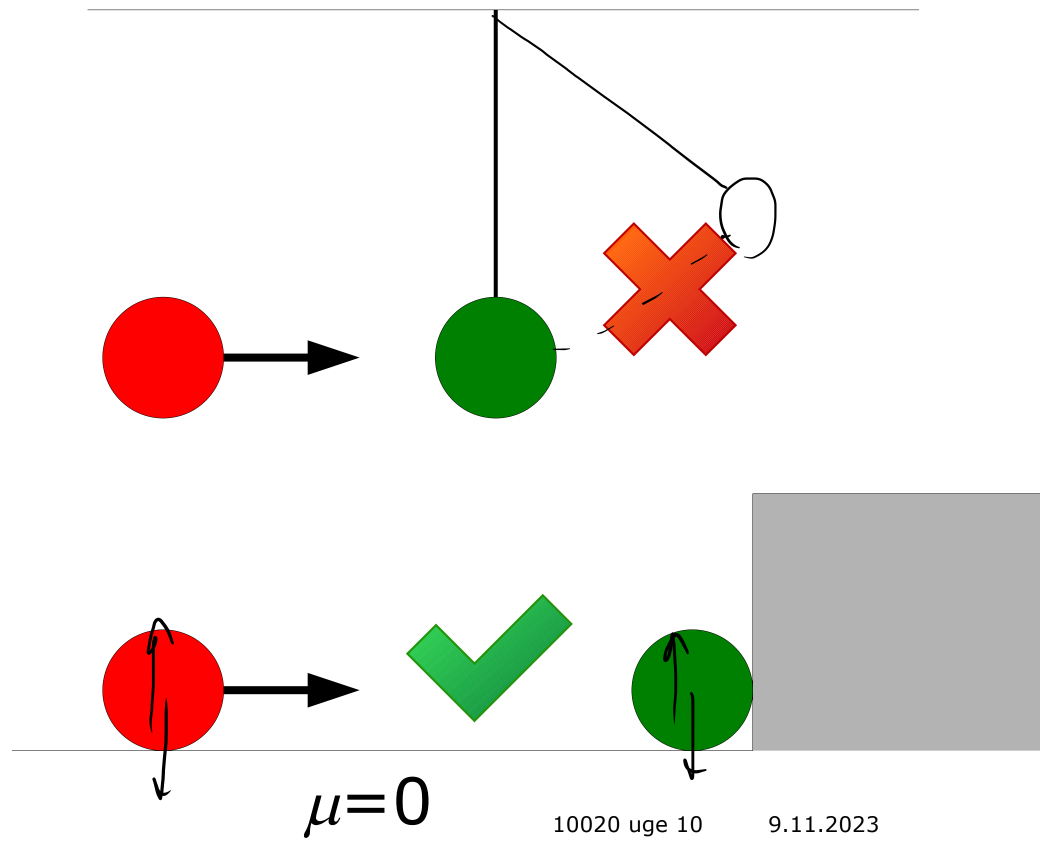
I hvilke situationer kan vi regne med impulsbevarelse i stødøjeblikket?



Quiz: Impulsbevarelse

I hvilke situationer kan vi regne med impulsbevarelse i bevægelsen efter stødet?



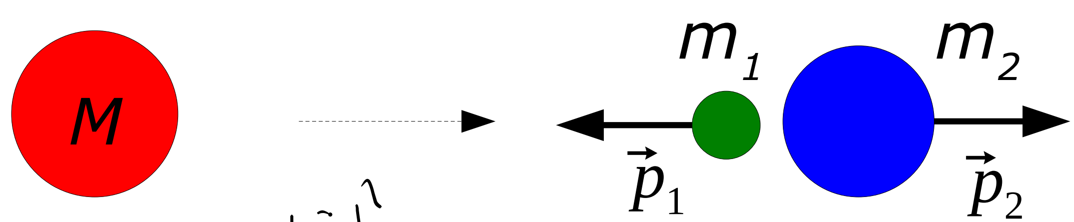


Impuls og kinetisk energi

$$\bar{v} = \frac{\bar{p}}{m}$$

$$E_{kin} = \frac{1}{2} m v^2 = \frac{1}{2} m |\vec{v}|^2 \qquad \vec{p} = m\vec{v} \Rightarrow E_{kin} = \frac{1}{2} m \left|\frac{\vec{p}}{m}\right|^2 = \frac{|\vec{p}|^2}{2m}$$

Eksempel: Radioaktivt henfald – hvilken datterkerne får størst energi?



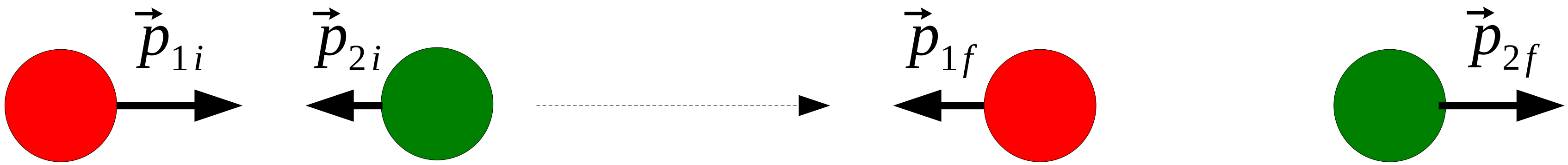
$$M \cdot 0 = 0 = \bar{p}_1 + \bar{p}_2 \Rightarrow \bar{p}_2 = -\bar{p}_1$$

$$E_1 = \frac{|\bar{p}_1|^2}{2m_1}$$

$$E_2 = \frac{|\bar{p}_2|^2}{2m_2} = \frac{|\bar{p}_1|^2}{2m_2} = E_1 \frac{m_1}{m_2}$$

Elastisk stød

Et stød kaldes *elastisk* hvis den samlede kinetiske energi er uændret.



$\vec{p}_{1i}+\vec{p}_{2i}=\vec{p}_{1f}+\vec{p}_{2f}$ altid

$$\frac{|\vec{p}_{1i}|^2}{2m_1}+\frac{|\vec{p}_{2i}|^2}{2m_2}=\frac{|\vec{p}_{1f}|^2}{2m_1}+\frac{|\vec{p}_{2f}|^2}{2m_2}$$ elastisk

1 dimension: $p_{1ix}+p_{2ix}=p_{1fx}+p_{2fx}$

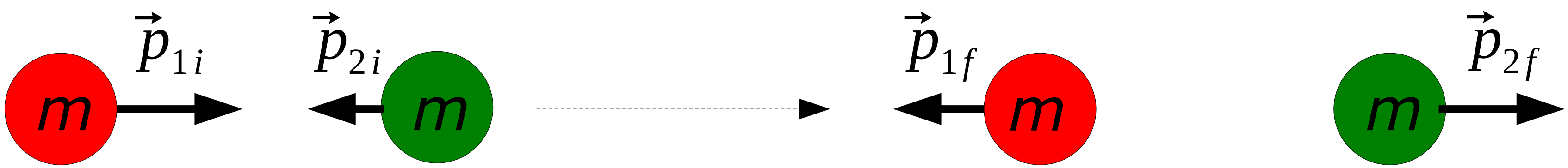
$$\frac{p_{1ix}^2}{2m_1}+\frac{p_{2ix}^2}{2m_2}=\frac{p_{1fx}^2}{2m_1}+\frac{p_{2fx}^2}{2m_2}$$

$$\Downarrow$$

$$p_{1fx}=\frac{m_1-m_2}{m_1+m_2}p_{1ix}+\frac{2m_1}{m_1+m_2}p_{2ix} \qquad p_{2fx}=\frac{2m_2}{m_1+m_2}p_{1ix}+\frac{m_2-m_1}{m_1+m_2}p_{2ix}$$

Elastisk stød – 1D billard

$$p_{1fx} = \frac{m_1 - m_2}{m_1 + m_2} p_{1ix} + \frac{2m_1}{m_1 + m_2} p_{2ix} \qquad p_{2fx} = \frac{2m_2}{m_1 + m_2} p_{1ix} + \frac{m_2 - m_1}{m_1 + m_2} p_{2ix}$$



$$p_{1fx} = \frac{2m}{2m} p_{2ix} = p_{2ix}$$

$$p_{2fx} = p_{1ix}$$

$$p_{1fx} = \frac{m_1 - m_2}{m_1 + m_2} p_{1ix} + \frac{2m_1}{m_1 + m_2} p_{2ix}$$

$$p_{2fx} = \frac{2m_2}{m_1 + m_2} p_{1ix} + \frac{m_2 - m_1}{m_1 + m_2} p_{2ix}$$

Elastisk stød

$$\cancel{m} v_{1f} = \frac{m - M}{m + M} \cancel{m} v_1 + \frac{2\cancel{m}}{m + M} M v_2$$

$$\Downarrow$$

$$v_{1f} = \frac{m - M}{m + M} v_1 + \frac{2M}{m + M} v_2 \simeq -v_1 + 2v_2$$

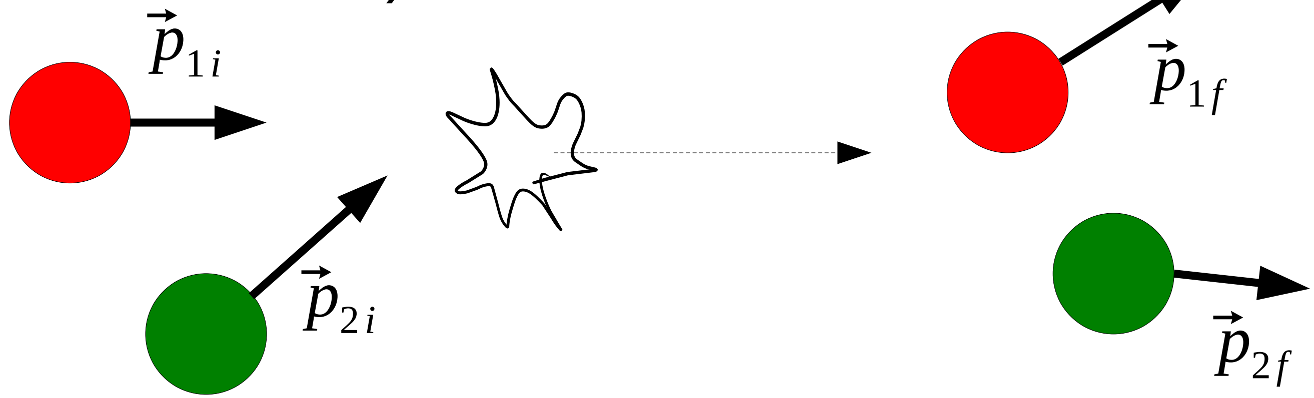
$$v_{2f} = \frac{M - m}{M + m} v_2 + \frac{2m}{m + M} v_1 \simeq$$

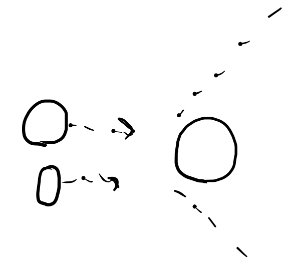
v_{1f}

@ t~ ...

v_2

Elastisk stød – 2 dimensioner





$$p_{1ix}+p_{2ix}=p_{1fx}+p_{2fx} \qquad p_{1iy}+p_{2iy}=p_{1fy}+p_{2fy}$$

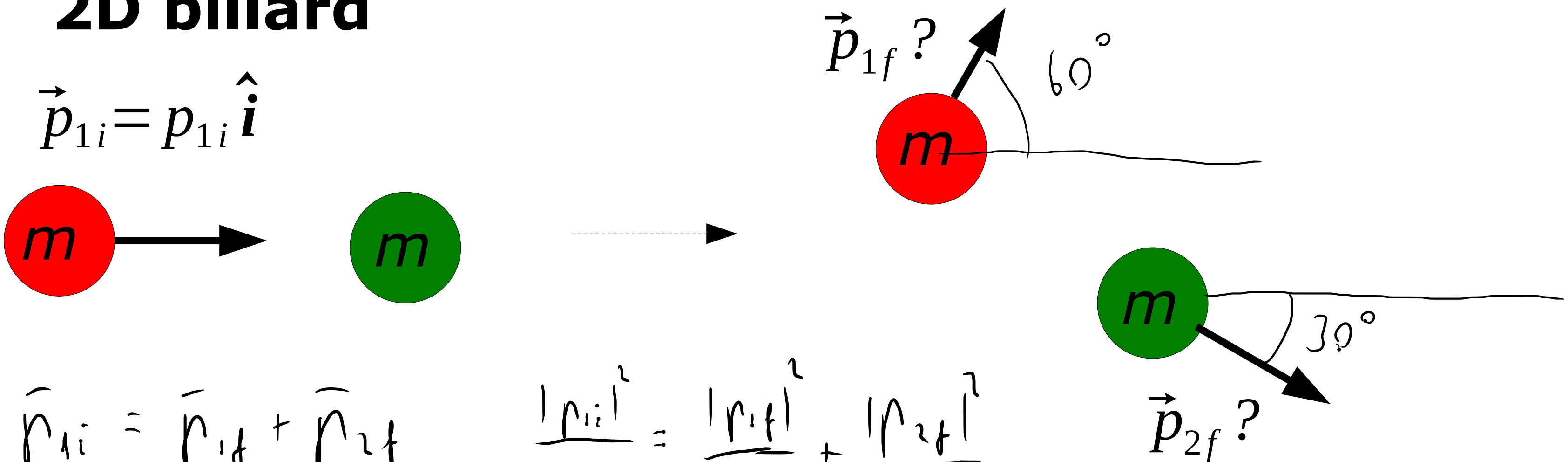
$$\frac{p_{1ix}^2+p_{1iy}^2}{2m_1}+\frac{p_{2ix}^2+p_{2iy}^2}{2m_2}=\frac{p_{1fx}^2+p_{1fy}^2}{2m_1}+\frac{p_{2fx}^2+p_{2fy}^2}{2m_2}$$

3 ligninger, 4 ubekendte!

$|\vec{v}_{2f}-\vec{v}_{1f}|=|\vec{v}_{2i}-\vec{v}_{1i}|$ kan vises for elastisk stød

2D billard

$\vec{p}_{1i} = p_{1i}\,\hat{\boldsymbol{i}}$



$$\bar{p}_{1i} = \bar{p}_{1f} + \bar{p}_{2f} \qquad \frac{|p_{1i}|^2}{\cancel{2m}} = \frac{|p_{1f}|^2}{\cancel{2m}} + \frac{|p_{2f}|^2}{\cancel{2m}}$$

$$|\bar{p}_{1i}|^2 = \bar{p}_{1i}\cdot\bar{p}_{1i} = (\bar{p}_{1f}+\bar{p}_{2f})\cdot(\bar{p}_{1f}+\bar{p}_{2f}) = \cancel{|\bar{p}_{1f}|^2 + |p_{2f}|^2} + 2\,\bar{p}_{1f}\cdot p_{2f} = \cancel{|\bar{p}_{1f}|^2 + |\bar{p}_{2f}|^2}$$

$$\Downarrow$$

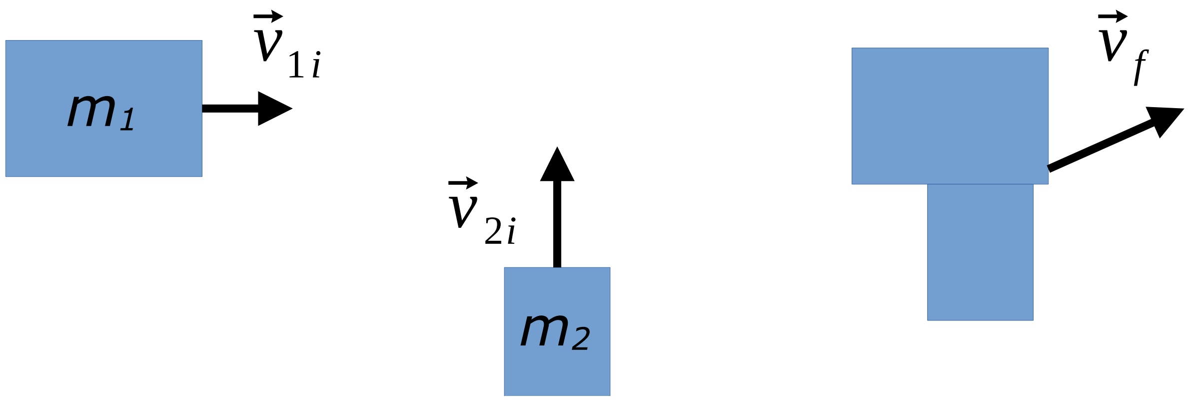
$$2\,\bar{p}_{1f}\cdot p_{2f} = 0 \Rightarrow \begin{cases} \bar{p}_{1f} = 0 \\ \bar{p}_{2f} = 0 \\ \bar{p}_{1f} \perp p_{2f} \end{cases}$$

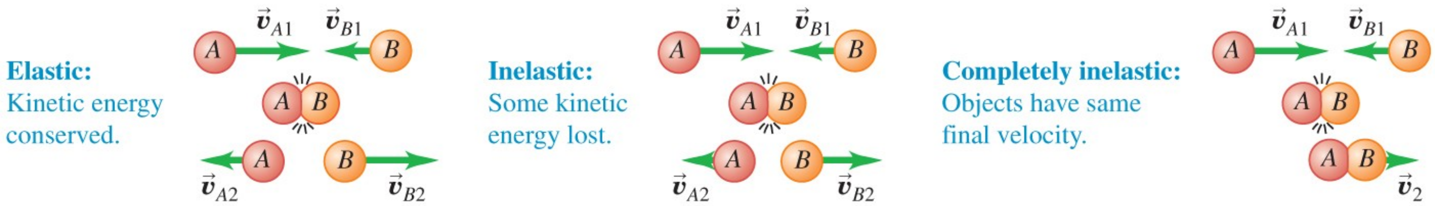
$\boxed{\text{Kun for } \bar{p}_{2i} = 0 \text{ og } m_1 = m_2 = m}$

Fuldstændig uelastisk stød

$\vec{v}_{1f} = \vec{v}_{2f}$ uanset objekternes masser.

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} = (m_1 + m_2)\vec{v}_f \Rightarrow \boxed{\vec{v}_f = \frac{m_1}{m_1 + m_2}\vec{v}_{1i} + \frac{m_2}{m_1 + m_2}\vec{v}_{2i}}$$





Eksempel: Ballistisk pendul

$$\mathrm{I}: \quad m v_r = (m+M) v_1 \Rightarrow v_1 = \frac{m}{m+M} v_r$$

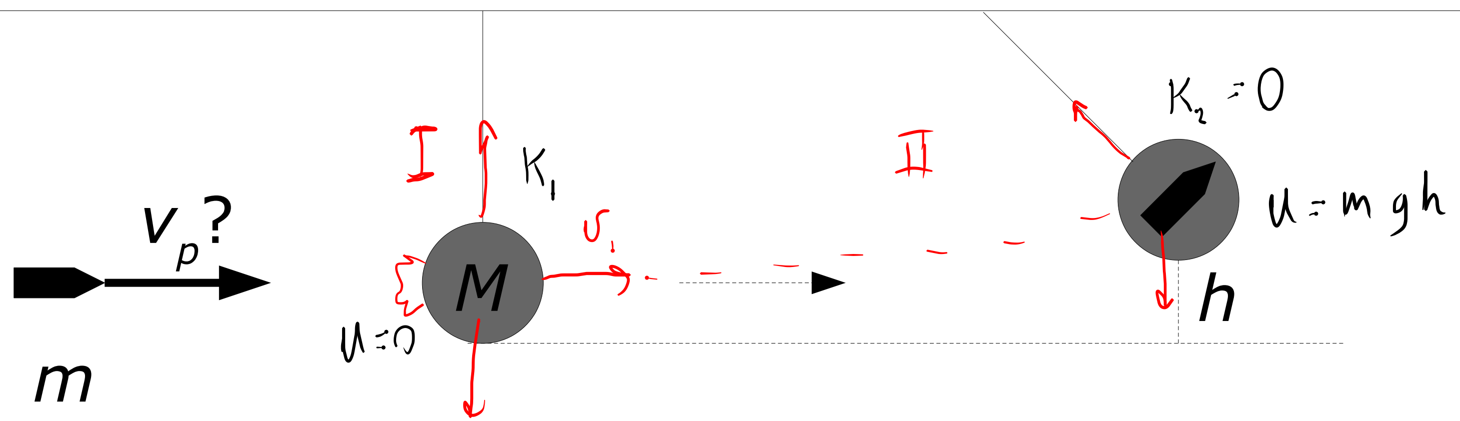
$$\mathrm{II}: \quad W = W_g \Rightarrow \Delta K + \Delta U = 0 \Rightarrow 0 - \frac{1}{2}(m+M) v_1^2 + ((m+M)gh - 0)) = 0$$

$$\Downarrow$$

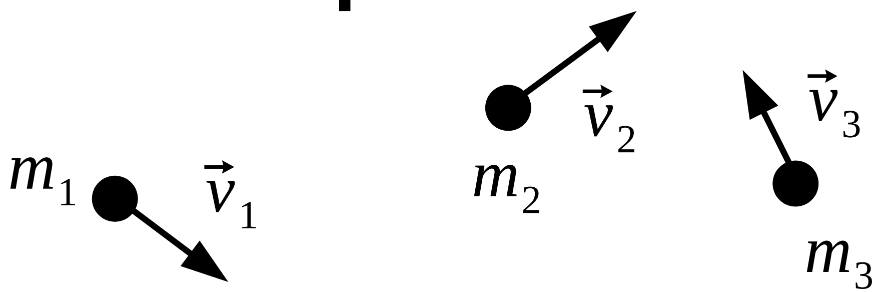
$$\cancel{(m+M)} g h = \frac{1}{2} \cancel{(m+M)} v_1^2 \Rightarrow v_1^2 = 2gh$$

$$\Downarrow$$

$$\left(\frac{m}{m+M}\right)^2 v_r^2 = 2gh \Rightarrow v_r = \frac{m+M}{m}\sqrt{2gh}$$



Massemidtpunkt



Én partikel:

$$\vec{p}_i = m_i \vec{v}_i = m_i \frac{d\vec{r}_i}{dt}$$

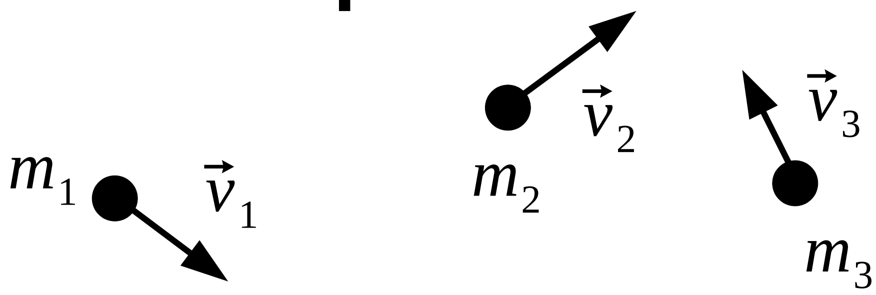
Samling af partikler: $\vec{P} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i = \sum_i m_i \frac{d\vec{r}_i}{dt} = \frac{d}{dt} \sum_i m_i \vec{r}_i$

Definér *massemidtpunkt* $\boxed{\vec{R} = \frac{1}{M} \sum_i m_i \vec{r}_i \qquad M = \sum_i m_i}$

$$\boxed{\vec{P} = \frac{d}{dt} \sum_i m_i \vec{r}_i = M \frac{d\vec{R}}{dt} \equiv M\vec{V}}$$

Totalimpuls kan beregnes som om al massen er samlet i $\vec{R}$ med hastighed $\vec{V}$

Massemidtpunkt



Én partikel:

$$\frac{d\vec{p}_i}{dt} = \vec{F}_{res,i} \quad \text{(Newton 2)}$$

$$\vec{F}_{res,i} = \vec{F}_{ext,i} + \sum_{j \neq i} \vec{F}_{j \to i}$$

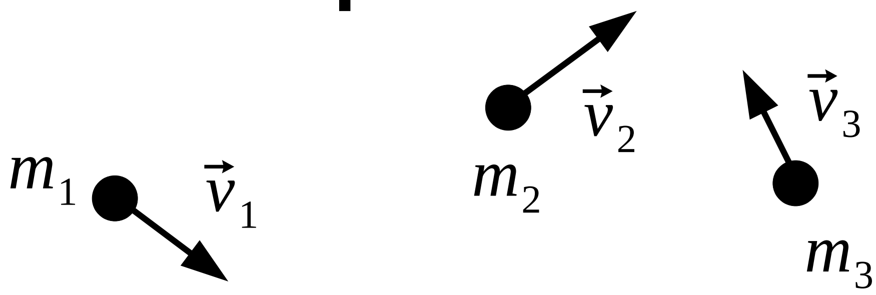
Samlet *ydre* kraft på *i*

Kræfter mellem partikler

Samling af partikler: $$\frac{d\vec{P}}{dt} = \sum_i \frac{d\vec{p}_i}{dt} = \sum_i \vec{F}_{ext,i} + \sum_i \sum_{j \neq i} \vec{F}_{j \to i}$$

Newton 3: $$\vec{F}_{i \to j} = -\vec{F}_{j \to i} \Rightarrow \sum_i \sum_{j \neq i} \vec{F}_{j \to i} = 0 \Rightarrow \boxed{\frac{d\vec{P}}{dt} = \sum_i \vec{F}_{ext,i}}$$

Massemidtpunkt



Én partikel:

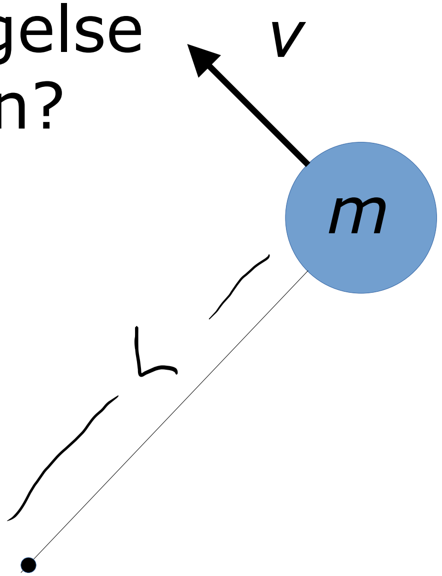
$$\frac{d\vec{p}_i}{dt} = \vec{F}_{res,i} \quad \text{(Newton 2)}$$

$$\boxed{\frac{d\vec{P}}{dt} = M\frac{d\vec{V}}{dt} = M\frac{d^2\vec{R}}{dt^2} = \sum_i \vec{F}_{ext,i} \qquad \textit{Massemidtpunktssætningen}}$$

Massemidtpunktets bevægelse er bestemt af de ydre kræfter alene via Newtons 2. lov.

Quiz – bold i snor, cirkelbevægelse

Massiv bold med diameter d, masse m. Snorens længde er L. Jævn cirkelbevægelse med farten v. Hvad er centripetalkraften?



A) $F_c = \frac{mv^2}{L}$

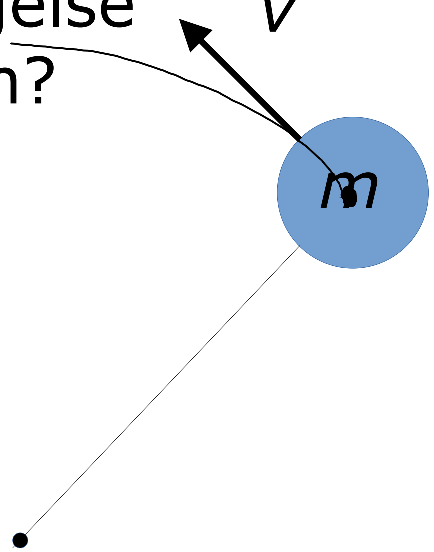
B) $F_c = \frac{mv^2}{L + d/2}$

C) $F_c = \frac{mv^2}{L + d}$

D) Kræver yderligere oplysninger

Quiz – bold i snor, cirkelbevægelse

Massiv bold med diameter *d*, masse *m*. Snorens længde er *L*. Jævn cirkelbevægelse med farten *v*. Hvad er centripetalkraften?



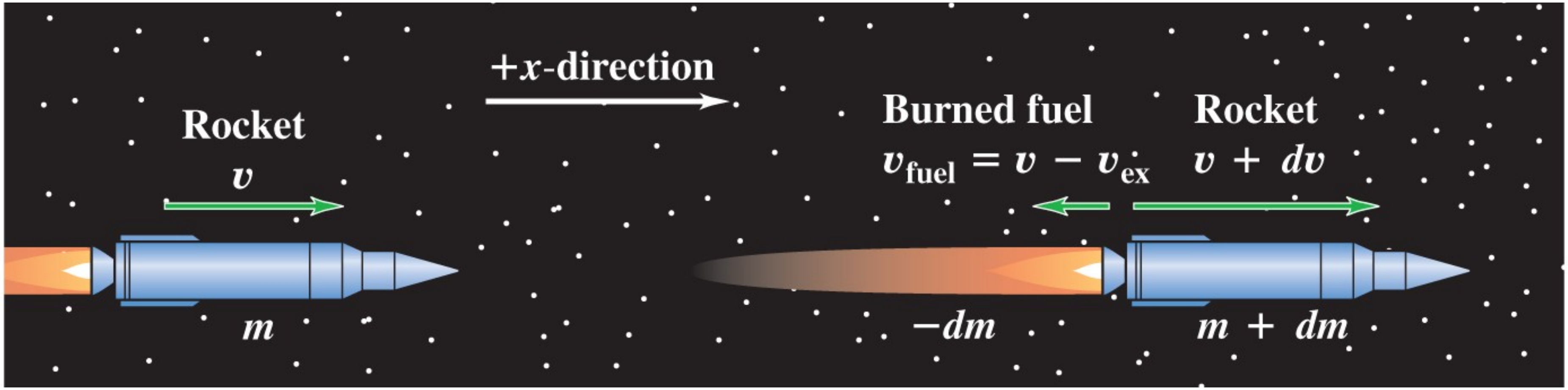
A) $F_c = \frac{mv^2}{L}$

B) $F_c = \frac{mv^2}{L + d/2}$

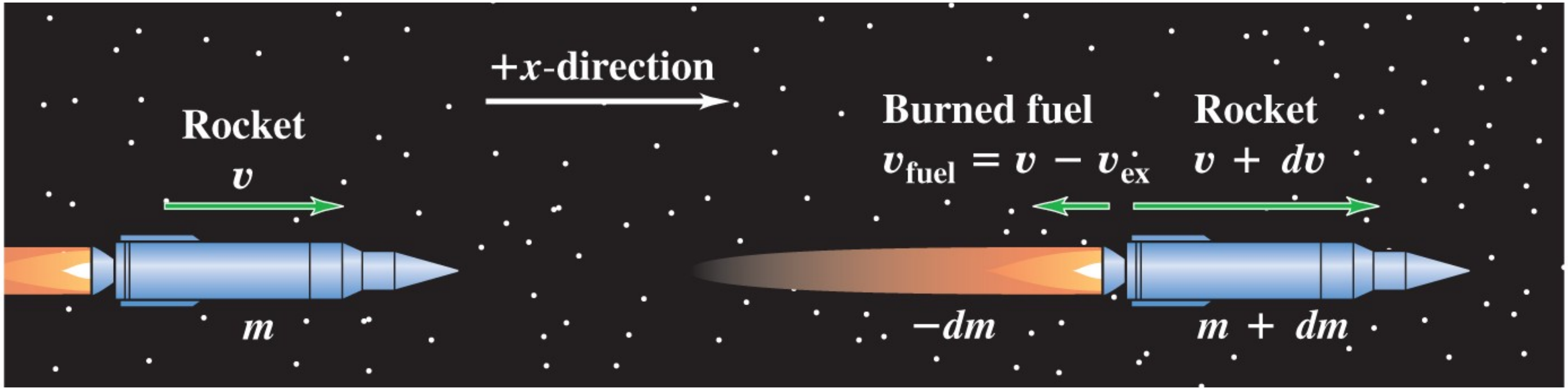
C) $F_c = \frac{mv^2}{L + d}$

D) Kræver yderligere oplysninger

Raketbevægelse



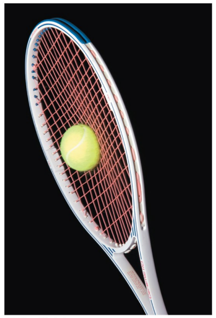
Raketbevægelse



Opsummering

Totalimpulsen $\vec{P}_{tot} \equiv \vec{p}_1 + \vec{p}_2 \equiv m_1\vec{v}_1 + m_2\vec{v}_2$ for to partikler er bevaret hvis påvirkningen fra ydre kræfter er negligibel

Impulsbevarelse kan typisk anvendes i *stødprocesser* hvor kræfterne mellem to legemer momentant er meget store



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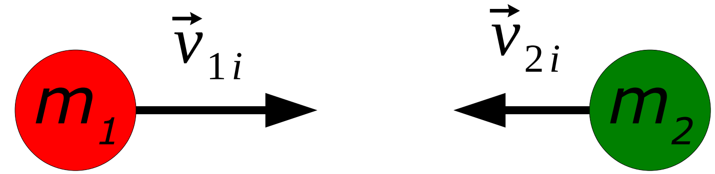
I et *elastisk stød* er kinetisk energi bevaret i selve stødet

I et *fuldstændig uelastisk stød* har de to legemer samme hastighed efter stødet, og tabet af energi er maksimalt.

Opsummering: Formler

Elastisk stød, 1 dimension:



$$p_{1fx} = \frac{m_1 - m_2}{m_1 + m_2} p_{1ix} + \frac{2m_1}{m_1 + m_2} p_{2ix}$$

$$v_{1fx} = \frac{m_1 - m_2}{m_1 + m_2} v_{1ix} + \frac{2m_2}{m_1 + m_2} v_{2ix}$$

$$p_{2fx} = \frac{2m_2}{m_1 + m_2} p_{1ix} + \frac{m_2 - m_1}{m_1 + m_2} p_{2ix}$$

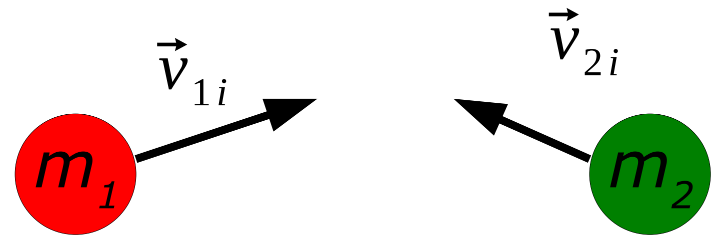
$$v_{2fx} = \frac{2m_1}{m_1 + m_2} v_{1ix} + \frac{m_2 - m_1}{m_1 + m_2} v_{2ix}$$

Relativ hastighed: $v_{2fx} - v_{1fx} = -\left(v_{2ix} - v_{1ix}\right)$

I 2D/3D: $\left|\vec{v}_{2f} - \vec{v}_{1f}\right| = \left|\vec{v}_{2i} - \vec{v}_{1i}\right|$

Opsummering: Formler

Fuldstændig uelastisk stød:



$$\vec{v}_f = \frac{m_1}{m_1+m_2}\vec{v}_{1i} + \frac{m_2}{m_1+m_2}\vec{v}_{2i}$$

$$K_i - K_f = \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\left|\vec{v}_{1i} - \vec{v}_{2i}\right|^2$$

Massemidtpunkt

$$\vec{R} = \frac{1}{M}\sum_i m_i \vec{r}_i \qquad M = \sum_i m_i$$

Massemidtpunktssætningen

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{V}}{dt} = M\frac{d^2\vec{R}}{dt^2} = \sum_i \vec{F}_{ext,i}$$

Mekanik – systematisk opgaveløsning

DTU Fotonik
Institut for Fotonik

APUK model for problemløsning

Analyse – hvilken fysik beskriver problemet?

Planlægning – hvilke ligninger skal løses?

Udførelse – beregninger gennemføres.

Kontrol – ser det rigtigt ud?

Temadag om systematisk opgaveløsning

Metoder til løsning af mekanikopgaver:

-'Standardmetode' – Newtons love direkte

-Impuls/energibetragtninger – arbejdssætningen, bevarelseslove.

Temadag om systematisk opgaveløsning

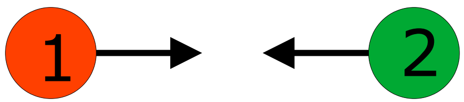
-'Standardmetoden':

-Tegning, koordinatsystem, kraftdiagram

-Newtons love: Acceleration fra kræfter

N2: $\vec{F}_{res} = m\vec{a}$

N3: $\vec{F}_{2\rightarrow 1} = -\vec{F}_{1\rightarrow 2}$



-'Geometriske bånd' (GB)

-Løsning af ligninger og kontrol

Temadag om systematisk opgaveløsning

Dette opgavesæt: Hver opgave har 3 (4) dele:
[A] + [B] + [C] (+ [D])

[A] Tegning, koordinatsystem, kraftdiagram

[B] Opstil relevante ligninger

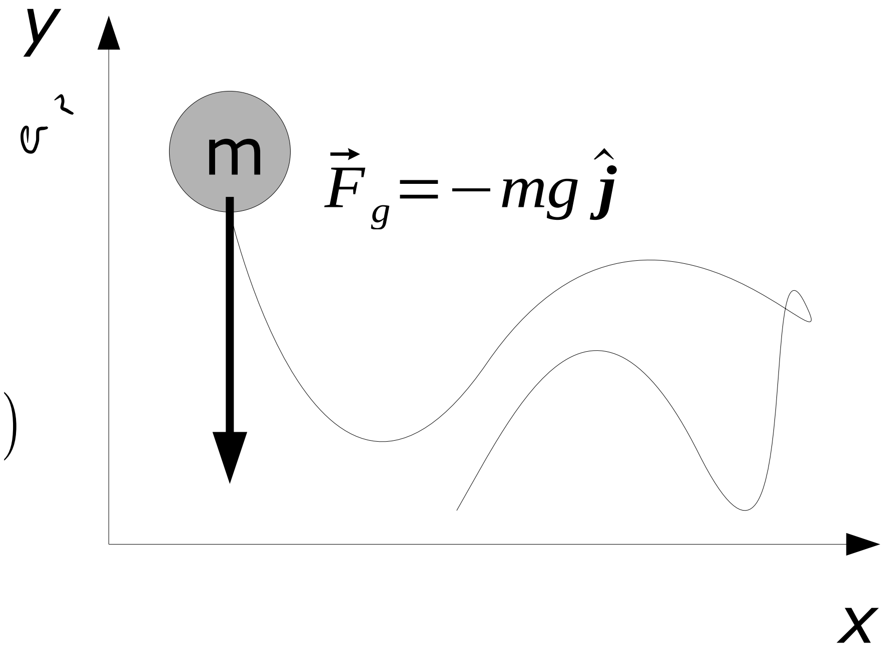
[C] Løsning af de opstillede ligninger, kontrol

[D] Alternative teknikker, energibetragtninger

'Alternativet' – energi/impuls, bevarelseslove

$\boxed{\text{Arbejdssætningen: } \Delta K = W = \int \vec{F}_{res} \cdot d\vec{r}}$

$K = \frac{1}{2} m v^2$



$\vec{F}_g = -mg\,\hat{\boldsymbol{j}}$

Arbejde fra tyngdekraften:

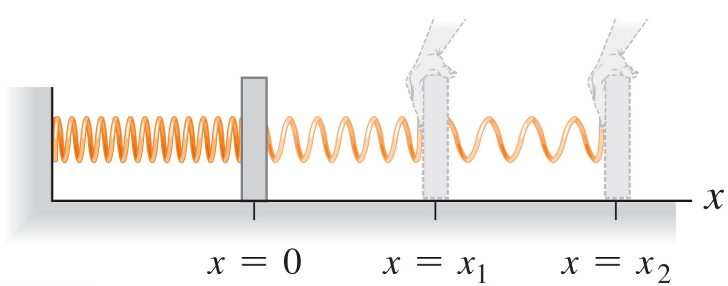
$$W_g = -mg(y_2 - y_1) = -(U(y_2) - U(y_1))$$

$$U(y) = mgy$$

Arbejde fra fjederkraft:

$$\int_{x_1}^{x_2} F_s\, dx = -\frac{1}{2}k(x_2^2 - x_1^2) = -(U(x_2) - U(x_1))$$

$$U(x) = \frac{1}{2}k\,x^2$$

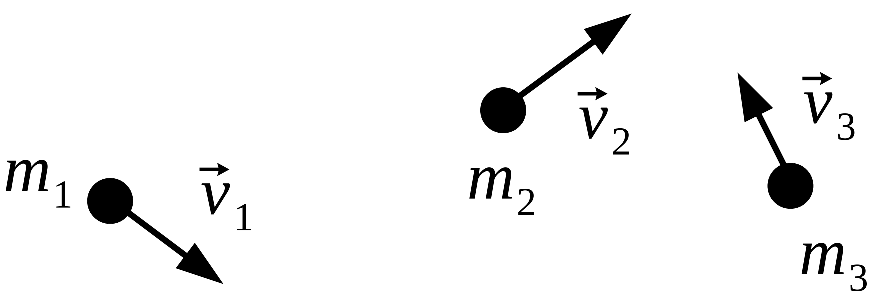


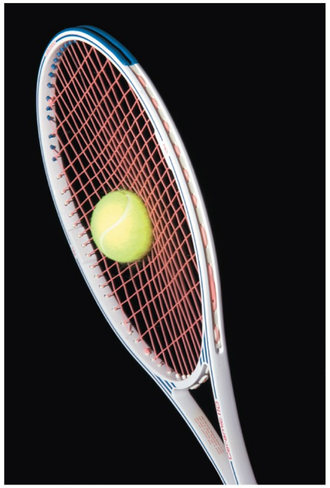
'Alternativet' – energi/impuls, bevarelseslove

Mekanisk energibevarelse hvis kun konservative kræfter

$$W=-\left(U\left(y_2\right)-U\left(y_1\right)\right)=-\Delta U=\Delta K \Rightarrow \Delta U+\Delta K=0 \Rightarrow U+K=\text{konst}.$$

Impulsbevarelse: $\vec{P}_{tot}=\sum_i \vec{p}_i=\sum_i m_i \vec{v}_i$ Bevaret hvis der ikke er ydre kræfter



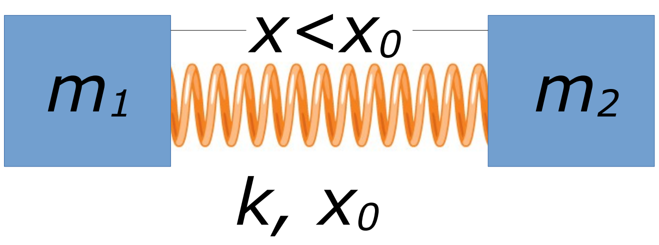


Quiz

Hvordan findes accelerationer når klodser slippes i hvile?
Masseløs fjeder, glat underlag

A) Standardmetoden B) Energi/impulsbetragtninger

C) Begge er mulige D) Begge er nødvendige

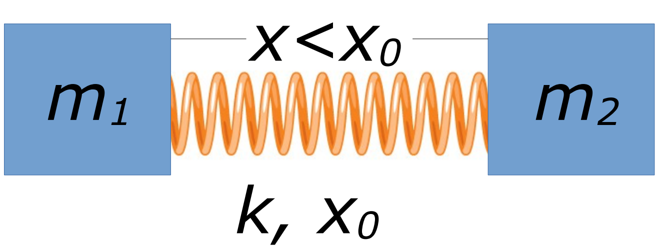


Quiz

Hvordan findes sluthastigheder når klodser slippes i hvile? Masseløs fjeder, glat underlag.

A) Standardmetoden B) Energi/impulsbetragtninger

C) Begge er mulige D) Begge er nødvendige

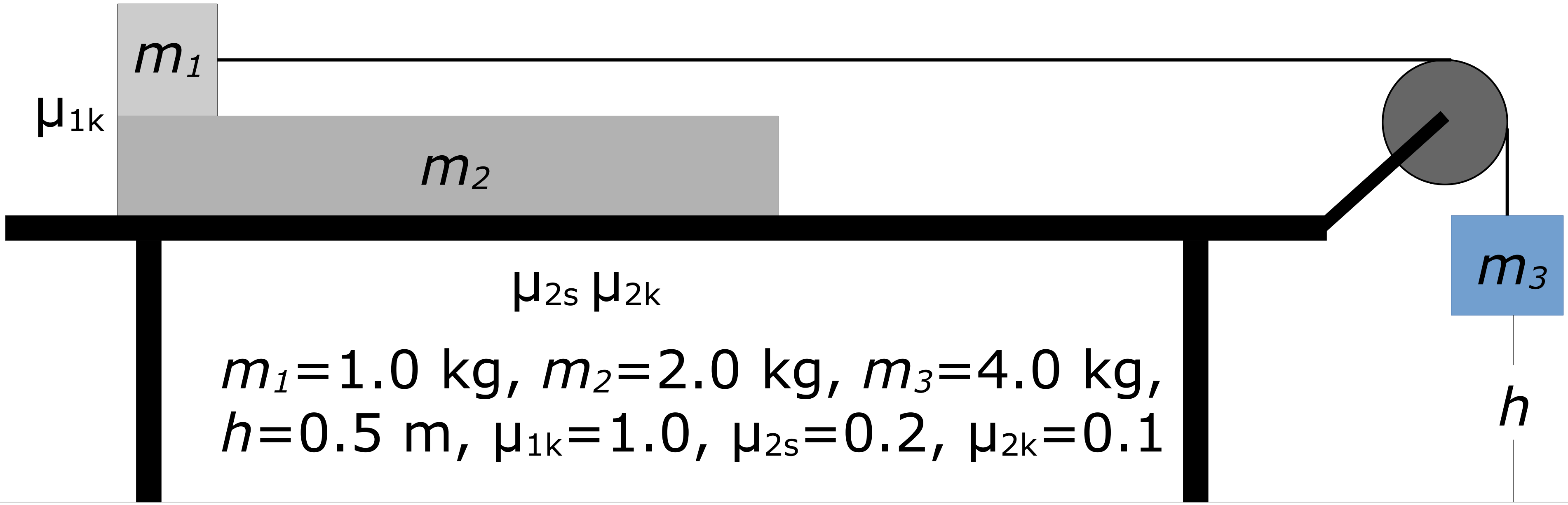


Quiz

Hvordan findes farten af m_1 og m_2 når m_3 rammer gulvet?

A) Standardmetoden B) Energi/impulsbetragtninger

C) Begge er mulige D) Begge er nødvendige



m_1=1.0 kg, m_2=2.0 kg, m_3=4.0 kg,
h=0.5 m, μ_{1k}=1.0, μ_{2s}=0.2, μ_{2k}=0.1

Quiz

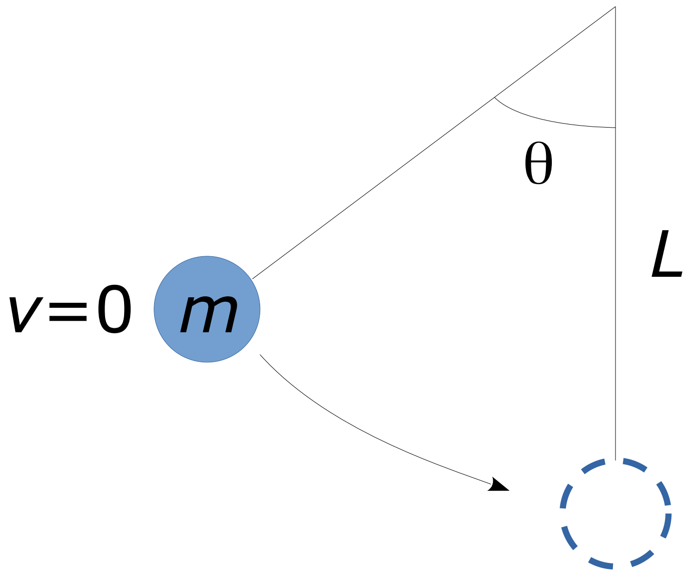
Hvordan findes snorkraften i bunden af pendulsvinget?

A) Standardmetoden

B) Energi/impulsbetragtninger

C) Begge er mulige

D) Begge er nødvendige

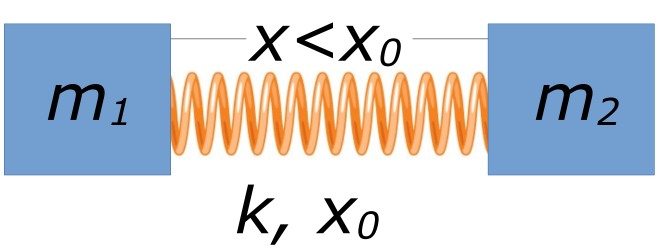


Quiz

Hvordan findes accelerationer når klodser slippes i hvile?
Masseløs fjeder, glat underlag

A) Standardmetoden B) Energi/impulsbetragtninger

C) Begge er mulige D) Begge er nødvendige

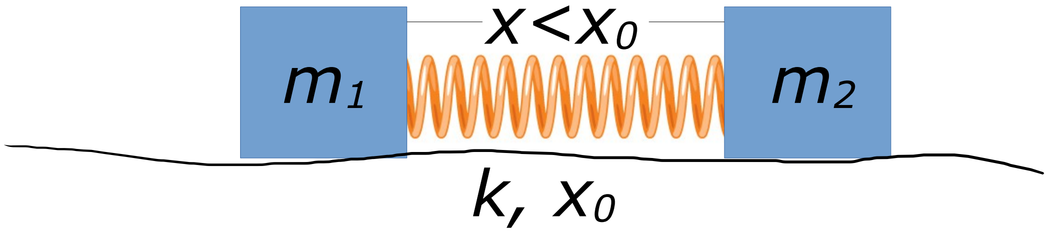


Quiz

Hvordan findes sluthastigheder når klodser slippes i hvile?
Masseløs fjeder, glat underlag.

A) Standardmetoden B) Energi/impulsbetragtninger

C) Begge er mulige D) Begge er nødvendige

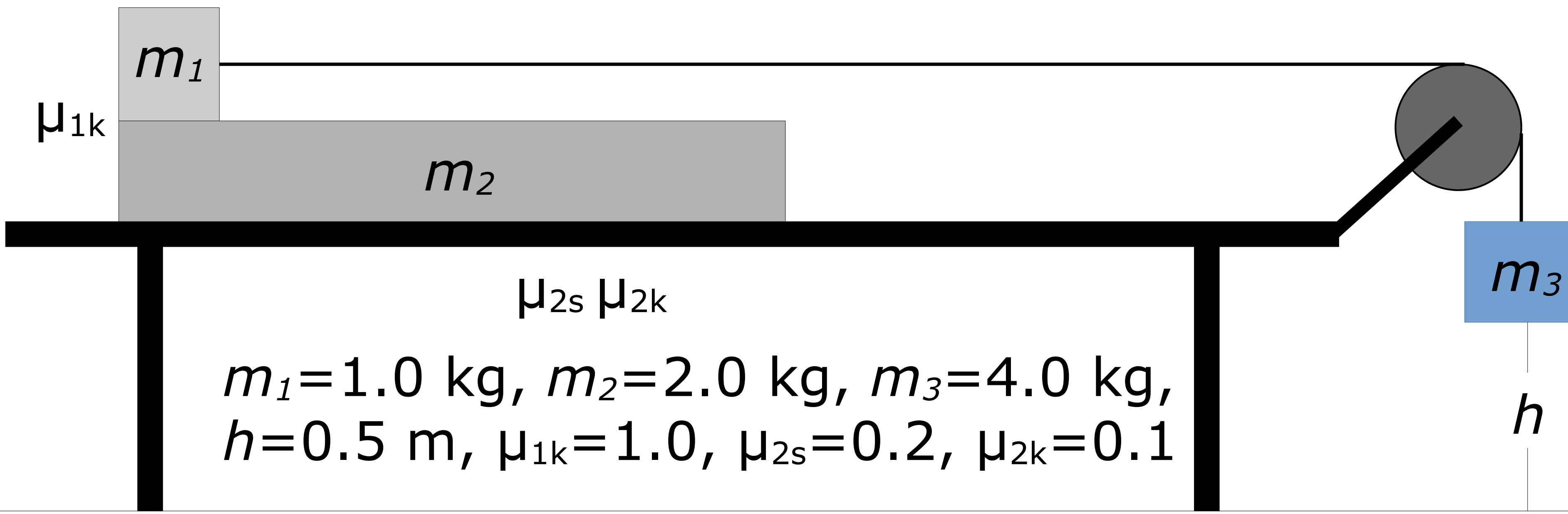


Quiz

Hvordan findes farten af m_1 og m_2 når m_3 rammer gulvet?

A) Standardmetoden B) Energi/impulsbetragtninger

C) Begge er mulige D) Begge er nødvendige



m_1=1.0 kg, m_2=2.0 kg, m_3=4.0 kg,
h=0.5 m, μ_{1k}=1.0, μ_{2s}=0.2, μ_{2k}=0.1

Quiz

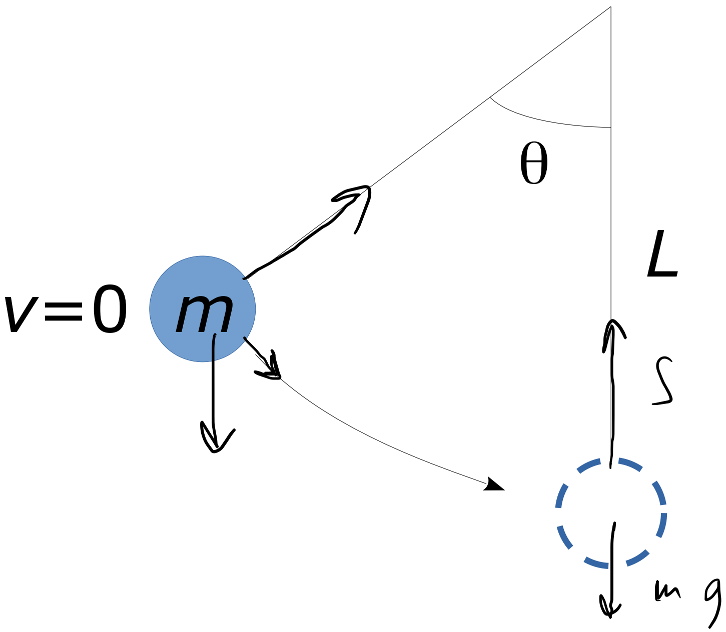
Hvordan findes snorkraften i bunden af pendulsvinget?

A) Standardmetoden

B) Energi/impulsbetragtninger

C) Begge er mulige

D) Begge er nødvendige

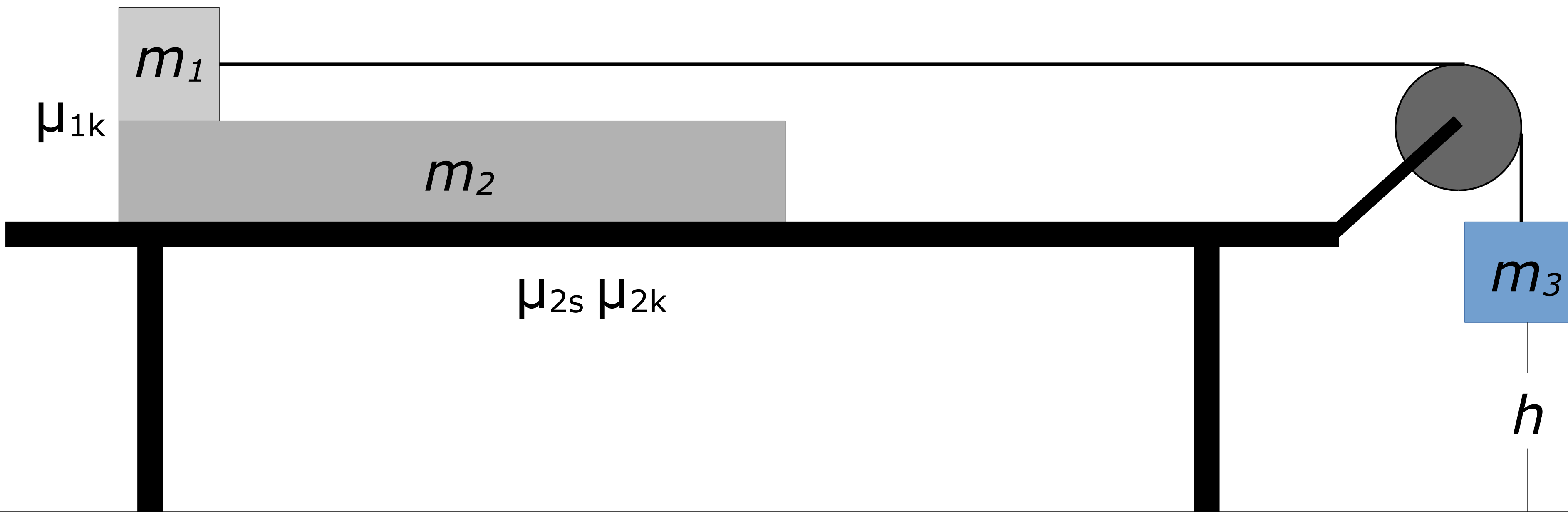


Eksempel 1

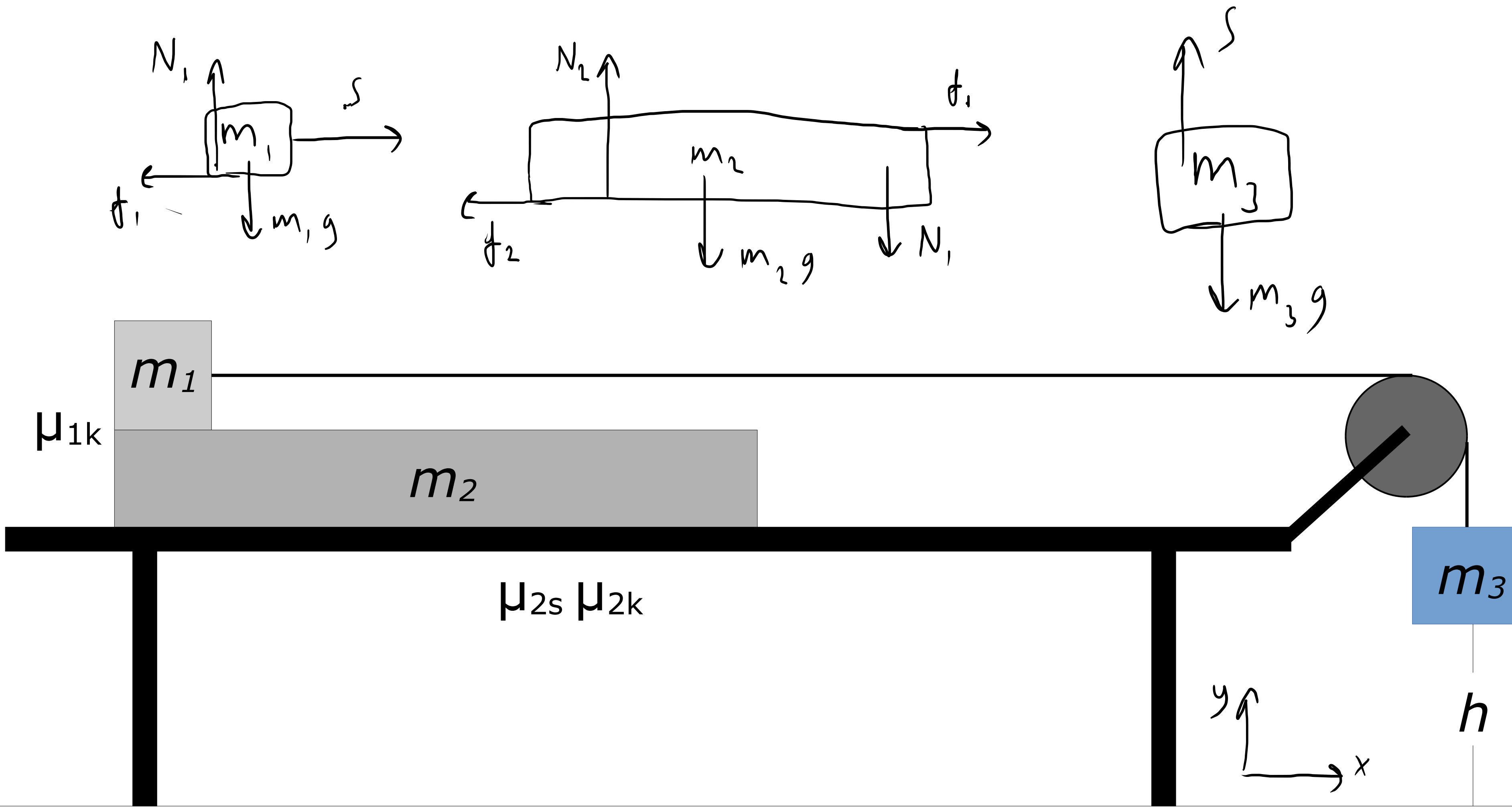
Klodserne starter i hvile. Hvor hurtigt bevæger klods 1 og 2 sig når klods 3 rammer gulvet? Klods 1 glider på klods 2.

m_1=1.0 kg, m_2=2.0 kg, m_3=4.0 kg, h=0.5 m

μ_{1k}=1.0, μ_{2s}=0.2, μ_{2k}=0.1



Eksempel 1 [A]



N_1
S
m_1
f_1
$m_1 g$
N_2
f_1
m_2
f_2
$m_2 g$
N_1
S
m_3
$m_3 g$
m_1
μ_{1k}
m_2
μ_{2s} μ_{2k}
m_3
h
y
x

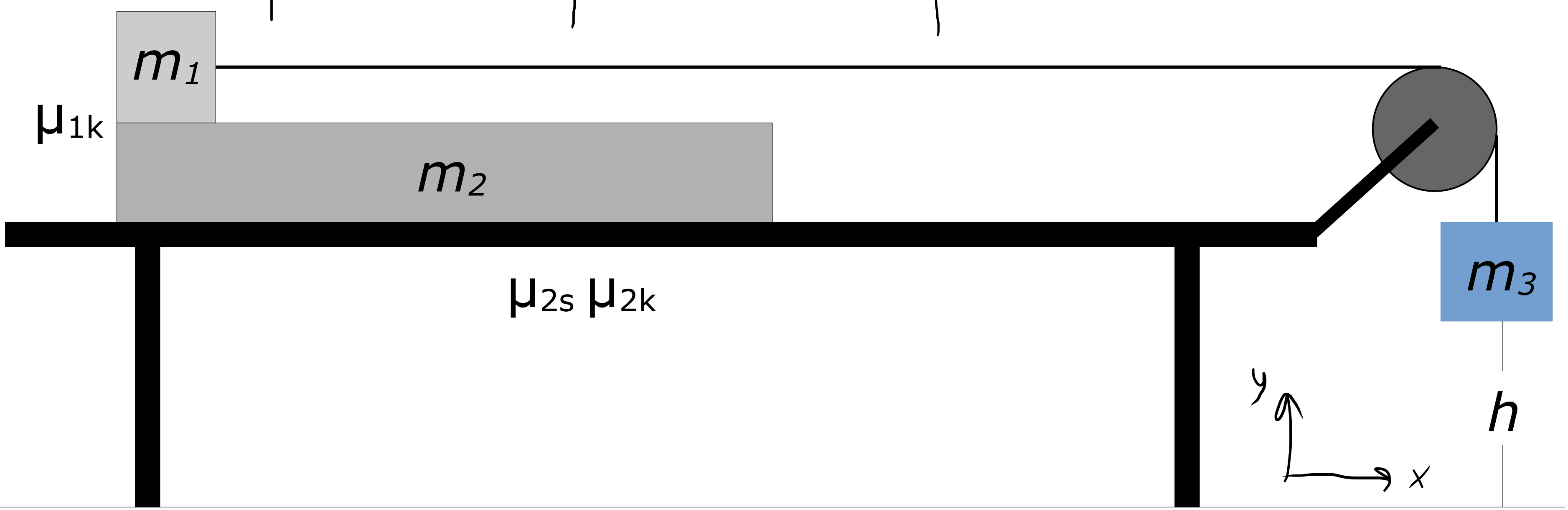
Eksempel 1 [B]

$GB: \; a_{3y} = -a_{1x}$

$f_1 = N_1 \mu_{1k}$

$f_2 \begin{cases} \leq \mu_{2s} N_2 \; ; \; a_{2x} = 0 \\ = \mu_{2k} N_2 \; ; \; a_{2x} > 0 \end{cases}$

	1	2	3
$N II_x$	$S - f_1 = m_1 a_{1x}$	$f_1 - f_2 = m_2 a_{2x}$	—
$N II_y$	$N_1 - m_1 g = 0$	$N_2 - N_1 - m_2 g = 0$	$S - m_3 g = m_3 a_{3y}$



Eksempel 1

2 $N\mathrm{II}_x: f_1 - f_2 = m_2 a_{2x} = 0 ?$

$\mu_{1k} N_1 = f_1 = f_2 \leq \mu_{2s} N_2 \Rightarrow \mu_{1k} N_1 \leq \mu_{2s} N_2$

$N\mathrm{II}_{y1}: N_1 - m_1 g = 0 \Rightarrow N_1 = m_1 g$

$N\mathrm{II}_{y2}: N_2 - N_1 - m_2 g = 0 \Rightarrow N_2 = (m_1 + m_2) g$

$\mu_{1k} m_1 \cancel{g} \leq \mu_{2s} (m_1 + m_2) \cancel{g}$

$\Downarrow$

$1.0\,\mathrm{kg} \leq 0.2 \cdot 3.0\,\mathrm{kg} \Rightarrow 1.0\,\mathrm{kg} \leq 0.6\,\mathrm{kg}$

%

$g = 9.8\ \mathrm{m/s^2}$

$f_1 - f_2 = \mu_{1k} m_1 g - \mu_{2k} N_2 =$

$\mu_{1k} m_1 g - \mu_{2k} (m_1 + m_2) g \simeq$

$9.8\,\mathrm{N} - 0.3\,\mathrm{kg} \cdot g =$

$9.8\,\mathrm{N} - 2.94\,\mathrm{N}$

$f_1 - f_2 = m_2 a_{2x} \Rightarrow$

$a_{2x} = \dfrac{6.86\,\mathrm{N}}{2.0\,\mathrm{kg}} \simeq 3.43\ \mathrm{m/s^2}$

Eksempel 1

$1+3$

$$N\text{II}_{x,1}: \; S - \mu_{1k} m_1 g = m_1 a_{1x}$$

$$N\text{II}_{y,3}: \; S - m_3 g = m_3 a_{3y} = -m_3 a_{1x}$$

$$\Downarrow$$

$$S = m_1(\mu_{1k} g + a_{1x}) \Rightarrow m_1(\mu_{1k} g + a_{1x}) - m_3 g = -m_3 a_{1x}$$

$$\Downarrow$$

$$(m_1 + m_3) a_{1x} = (m_3 - m_1 \mu_{1k}) g \Rightarrow a_{1x} = \frac{m_3 - m_1 \mu_{1k}}{m_1 + m_3} g \simeq \frac{3.0}{5.0} g \simeq$$

$$5.88 \, m/s^2$$

Eksempel 1

$$h = \frac{1}{2} a_{1x} t^2 \Rightarrow t = \sqrt{\frac{2h}{a_{1x}}} \simeq 0.412\,s$$

$$\Downarrow$$

$$v_{1x} = a_{1x} t \simeq 2.4\ m/s$$

$$v_{2x} = a_{2x} t \simeq 1.4\ m/s$$

Eksempel 2

Hvad bliver klodsernes sluthastigheder? Masseløs fjeder, glat underlag.

$$\Delta K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\boxed{\begin{aligned} & m_1 v_1 + m_2 v_2 = 0 \\ & \Downarrow \\ & v_2 = -\frac{m_1}{m_2} v_1 \end{aligned}} \longrightarrow$$

$$\Delta K = -\Delta U = -\left(0 - \frac{1}{2} k (x - x_0)^2\right)$$

$$\Downarrow$$

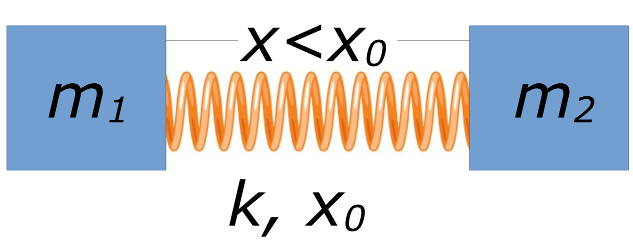
$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} k (x - x_0)^2$$

$$\Downarrow$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2} v_1\right)^2 = \cancel{\frac{1}{2}} v_1^2 \left(m_1 + \frac{m_1^2}{m_2}\right) = \cancel{\frac{1}{2}} k (x - x_0)^2$$

$$\Downarrow$$

$$v_1^2 = \frac{k (x - x_0)^2}{m_1 + \frac{m_1^2}{m_2}}$$



Eksempel 3

Find snorkraften i bunden af pendulsvinget.

$$N\mathrm{II}_c: \quad S - mg = m a_c = m\frac{v^2}{L}$$

$$\Delta K = \frac{1}{2}\cancel{m} v^2 = -(0 - mgh) = \cancel{m}gh$$

$$\Downarrow$$

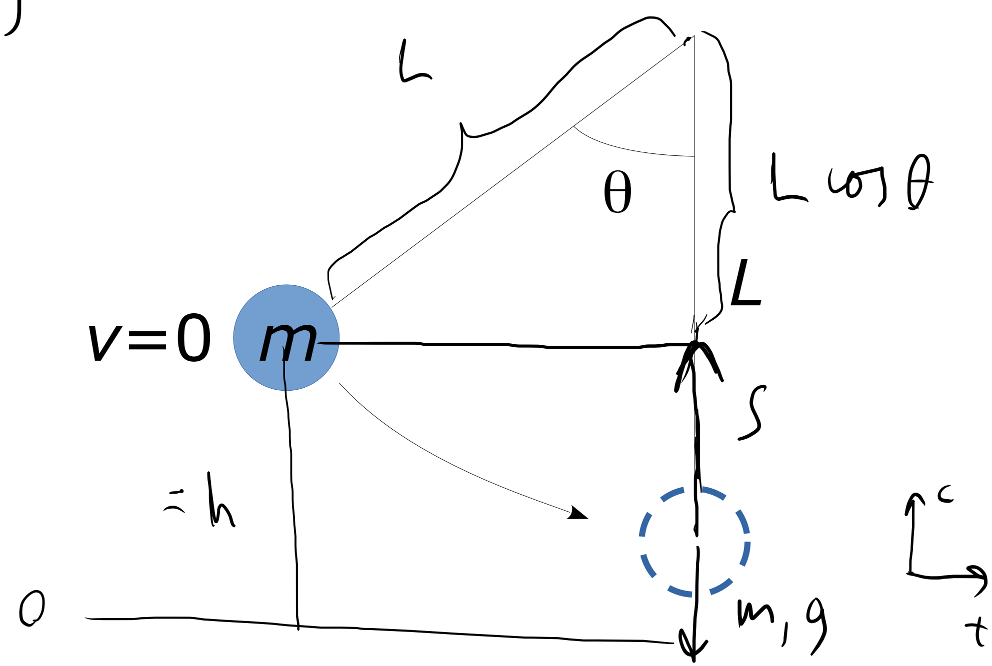
$$v^2 = 2gh = 2gL(1-\cos\theta)$$

$$\Downarrow$$

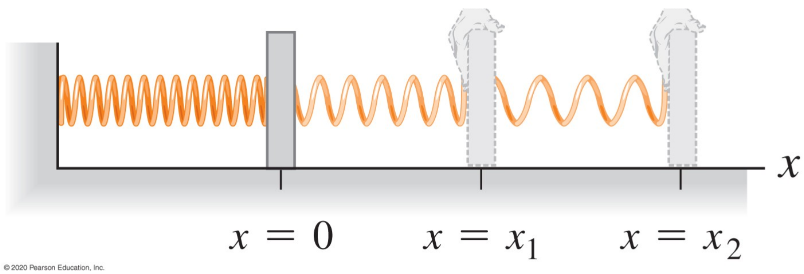
$$S - mg = \frac{m\,2g\cancel{L}(1-\cos\theta)}{\cancel{L}}$$

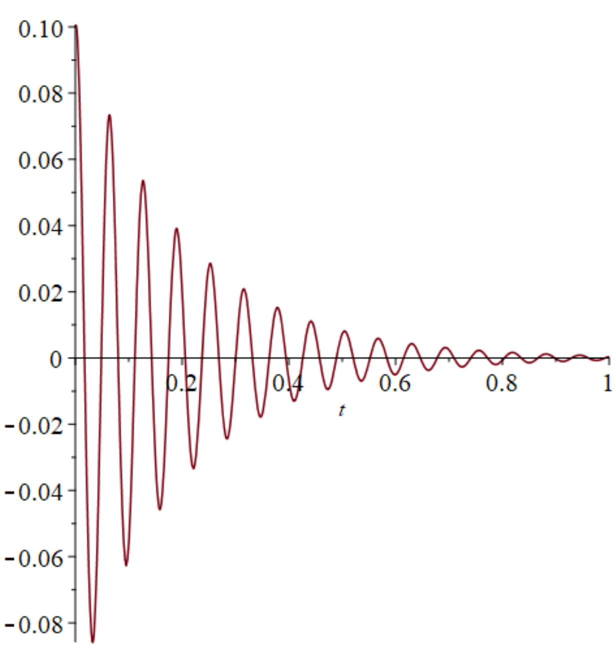
$$\Downarrow$$

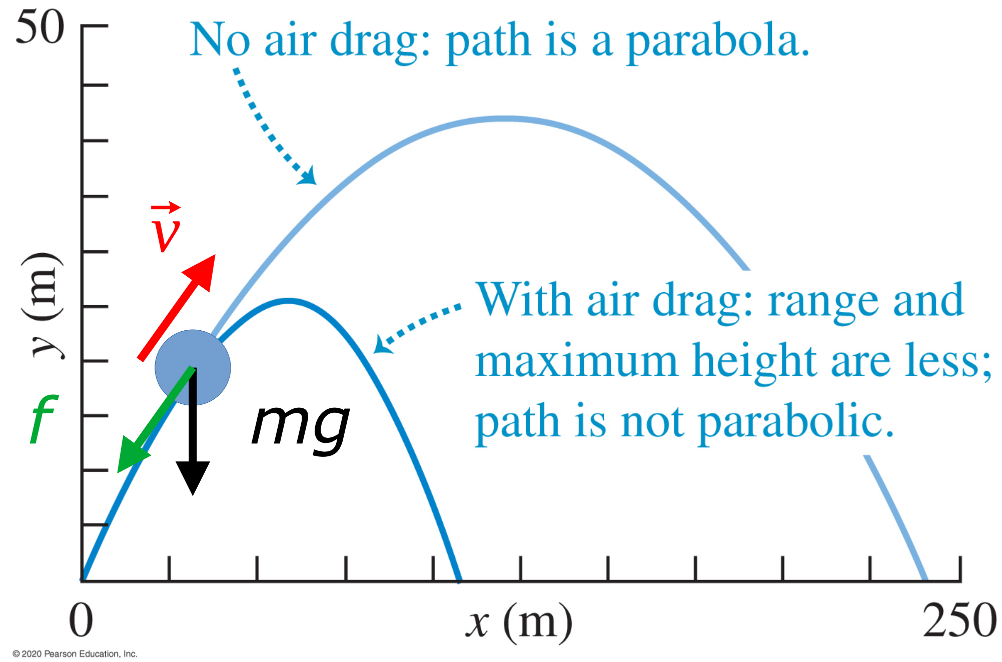
$$S = mg\big(1 + 2(1-\cos\theta)\big)$$

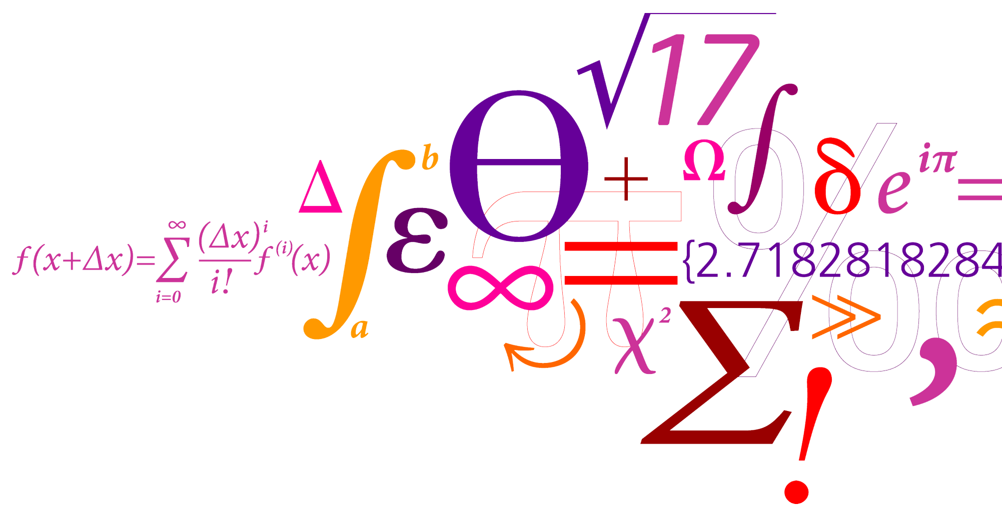


Temadag om differentialligningsmodeller









DTU Fotonik
Institut for Fotonik

Temadag

-Numerisk opgaveløsning i Maple (der er også nogle udledninger)

-Optimering, visualisering, diskussion

-Denne forelæsning:

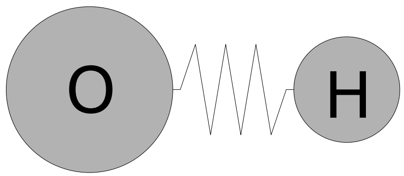
Numerisk løsning af differentialligninger

Dimensionsanalyse

Maple syntakseksempler

Newton 2 som differentialligning

Simulering af 'ukendt' problem



$$ma = m\frac{d^2x}{dt^2} = -kx(t) \qquad v(t) = \frac{dx}{dt}$$

m=1.67·10^{-27} kg, k=7.8·10^{2} N/m,
x(0)=10^{-11} m, v(0)=0

Simulér bevægelsen som funktion af t

$> \ ode := m \cdot diff(x(t), t, t) = -k \cdot x(t);$

$$ode := m\left(\frac{d^2}{dt^2}\,x(t)\right) = -k\,x(t)$$

$> \ init := x(0) = 1.e-11, D(x)(0) = 0.;$

$$init := x(0) = 1.\ 10^{-11}, D(x)(0) =$$

$> \ sol := dsolve(\{ode, init\}, numeric, output = listprocedure, parameters = [m, k]);$

$$sol := \left[t = \mathbf{proc}(t)\ \ldots\ \mathbf{end\ proc}, x(t) = \mathbf{proc}(t)\ \ldots\ \mathbf{end\ proc}, \right.$$

$> \ sol(parameters = [1.67e-27, 7.8e2])$

$$\left[t(parameters = [1.67\ 10^{-27}, 780.]) = [m = 1.67\ 10^{-27}, k = 780.], x(t)(parameter\right.$$

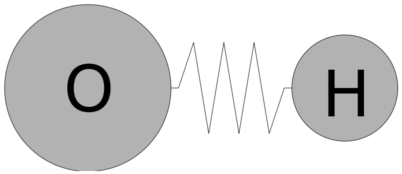
$$= 780.], \left(\frac{d}{dt}\,x(t)\right)(parameters = [1.67\ 10^{-27}, 780.]) = [m = 1.67\ 10^{-27}, k =$$

$> \ xout := eval(x(t), sol)$

$$xout := \mathbf{proc}(t)\ \ldots\ \mathbf{end\ proc}$$

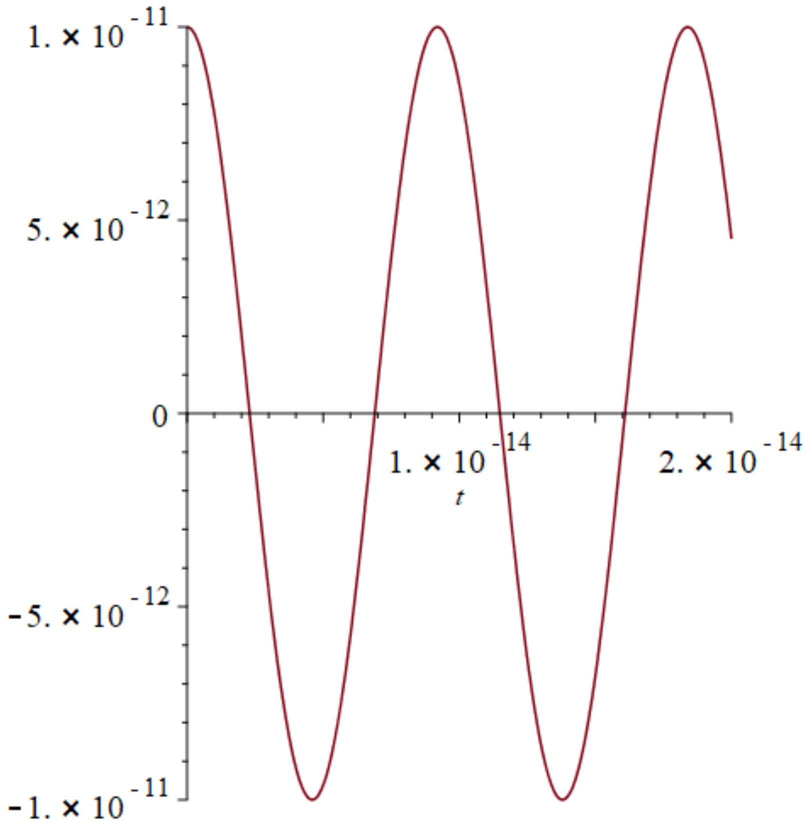
$> \ plot([xout(t)], t = 0..2e-14)$

Simulering af 'ukendt' problem

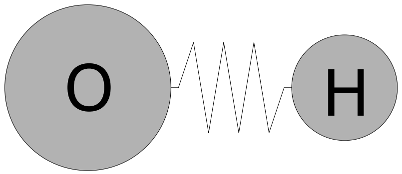


m=1.67·10^{-27} kg, k=7.8·10^{2} N/m,
x(0)=10^{-11} m

$$\sqrt{\frac{m}{k}} \approx 1.5 \cdot 10^{-15} s$$



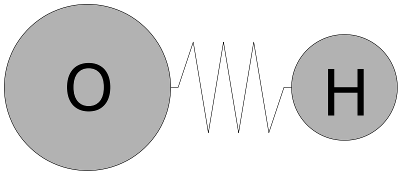
Dæmpet svingning



$$ma = m\frac{d^2x}{dt^2} = -kx(t) - b\frac{dx}{dt}$$

m=1.67·10^{-27} kg, k=7.8·10^{2} N/m,
b=5·10^{-14} kg/s, x(0)=10^{-11} m

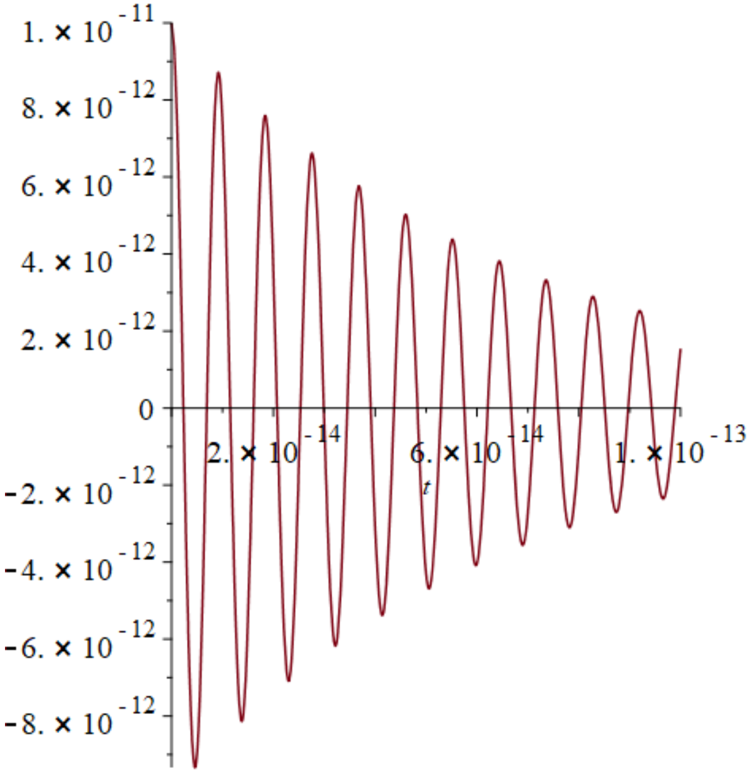
Dæmpet svingning



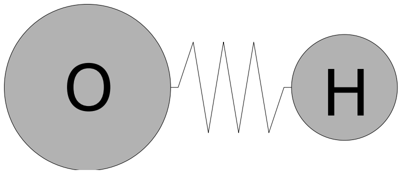
m=1.67·10^{-27} kg, k=7.8·10^{2} N/m, b=5·10^{-14} kg/s, $x(0)$=10^{-11} m

$$> \; ode := m \cdot diff(x(t), t, t) = -k \cdot x(t) - b \cdot diff(x(t), t);$$

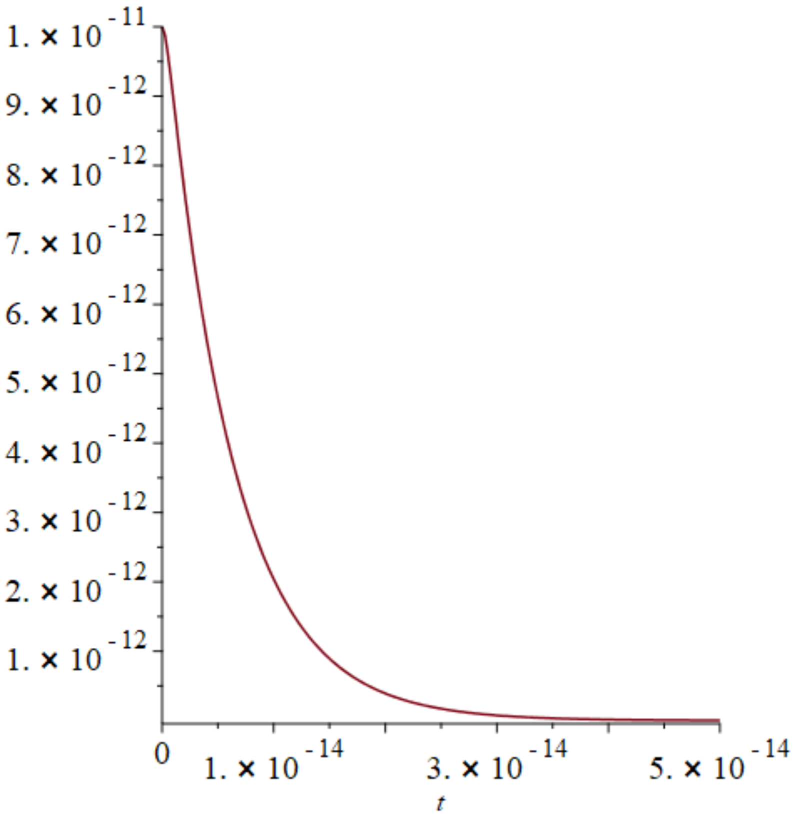
$$ode := m \left(\frac{d^2}{dt^2} x(t) \right) = -k\, x(t) - b \left(\frac{d}{dt} x(t) \right)$$



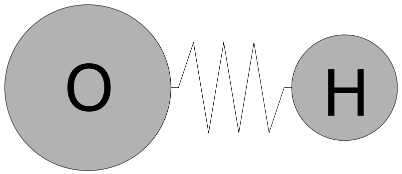
Dæmpet svingning



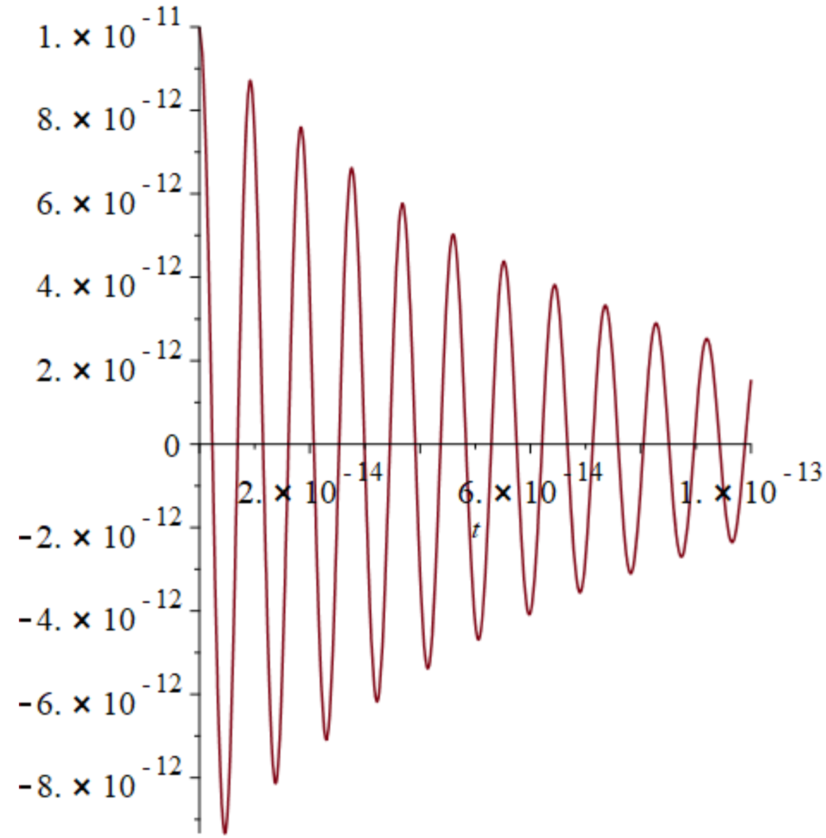
m=1.67·10^{-27} kg, k=7.8·10^{2} N/m,
b=5·10^{-12} kg/s, x(0)=10^{-11} m



Periode vs. *b?*



$$ma = m\frac{d^2x}{dt^2} = -kx(t) - b\frac{dx}{dt}$$



Benyt fsolve til at finde første nulpunkt, det skulle give ca. den kvarte periode.

Periode vs. *b* - algoritme

$> \; bvalues := Vector(20);$

$> \; firstzero := Vector(20);$

$> \; \mathbf{for}\ i\ \mathbf{from}\ 1\ \mathbf{by}\ 1\ \mathbf{to}\ 20\ \mathbf{do}\ sol(parameters = [1.67e-27, 7.8e2, i \cdot 5.e-14]);$

$xout := eval(x(t), sol); bvalues[i] := i \cdot 50.;$

$firstzero[i] := fsolve(xout(t), t = 1.e-15) \cdot 1.e15; \mathbf{end\ do}$

$> \; with(Statistics):$

$> \; LinearFit([1, t, t^2], bvalues, firstzero, t, summarize = true);$

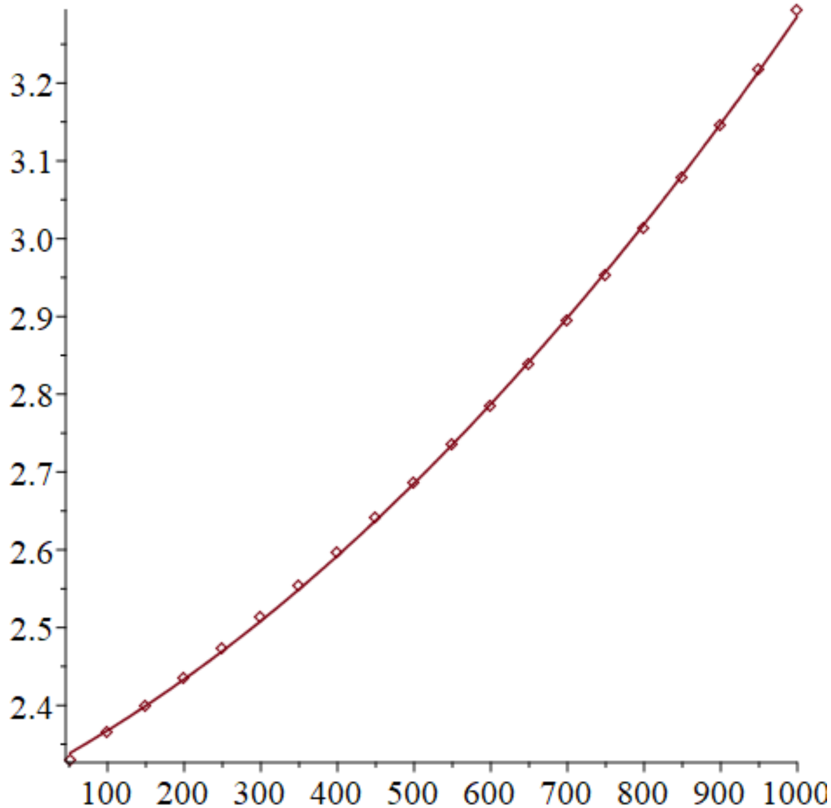
Periode vs. *b* - resultater

```
Summary:
----------------
Model: 2.3109783+.52295242e-3*t+.45192985e-6*t^2
----------------
Coefficients:
              Estimate  Std. Error  t-value   P(>|t|)
Parameter 1    2.3110    0.0034      689.3386   0.0000
Parameter 2    0.0005    0.0000      35.5631    0.0000
Parameter 3    0.0000    0.0000      33.2218    0.0000
----------------
R-squared: 0.9998, Adjusted R-squared: 0.9998
```

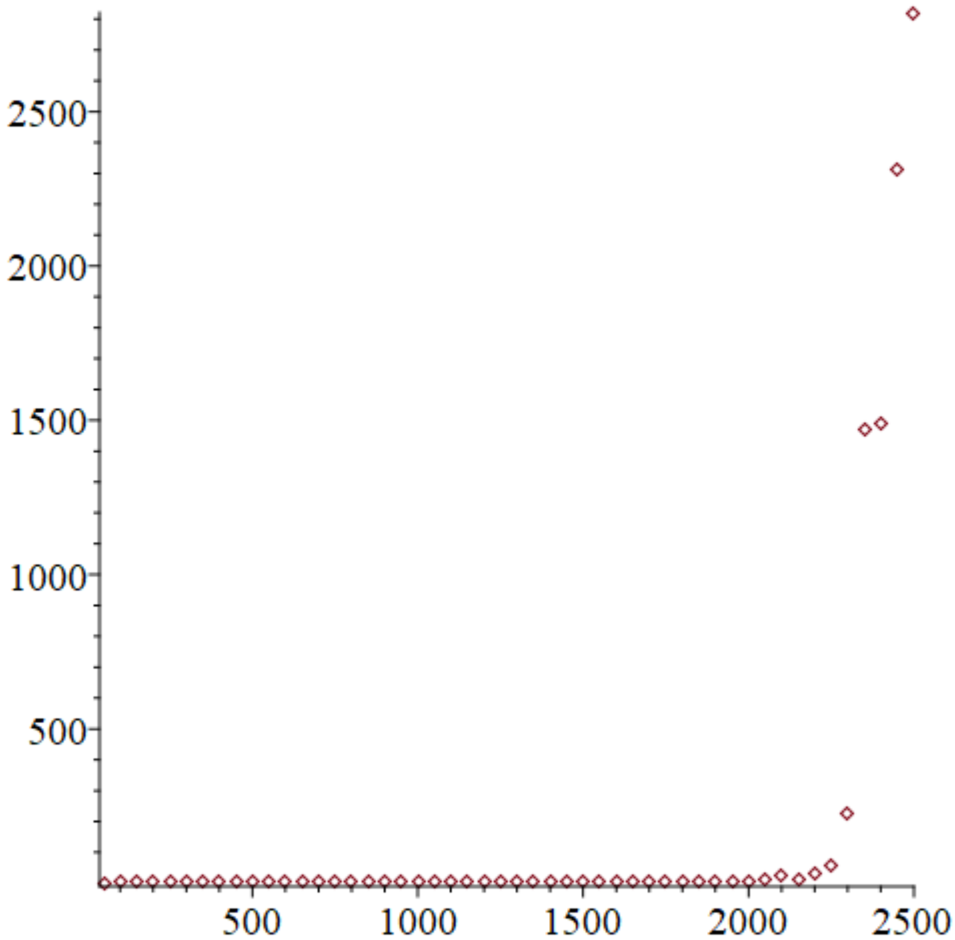
$$2.31097833736316 + 0.000522952418946002\, t + 4.51929851401230\; 10^{-7}\, t^2$$

Periode vs. *b* - resultater

$$> \ display(plot(bvalues, firstzero, style = point), plot(unapply(\%, t), 50..1000));$$



Periode vs. *b* - resultater



Kapitel 10-2

Rotation og dynamik

Rotation – nu med dynamiske effekter - 2

Angulært moment

Angular momentum of a particle relative to origin O of an inertial frame of reference

Position vector of particle relative to O

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} \qquad (10.24)$$

Linear momentum of particle = mass times velocity

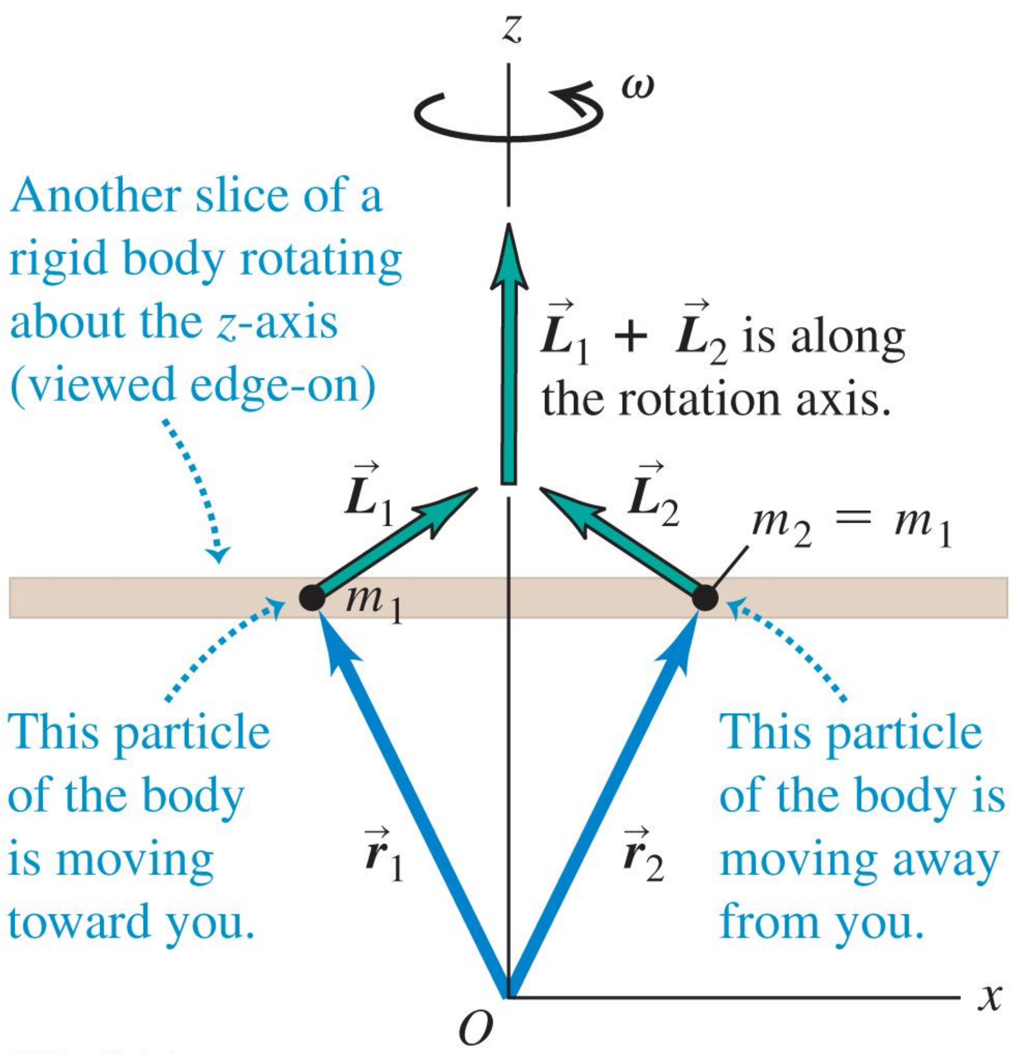
Angular momentum of a rigid body rotating around a symmetry axis

Moment of inertia of rigid body about symmetry axis

$$\vec{L} = I\vec{\omega} \qquad (10.28)$$

Angular velocity vector of rigid body

Symmetry on rotation axis



Opgave 10.38

10.38 •• A woman with mass 55 kg is standing on the rim of a large horizontal disk that is rotating at 0.47 rev/s about an axis through its center. The disk has mass 119 kg and radius 3.5 m. Calculate the magnitude of the total angular momentum of the woman–disk system. (Assume that you can treat the woman as a point.)

Impulsmomentsætningen (IMS)

For a system of particles:

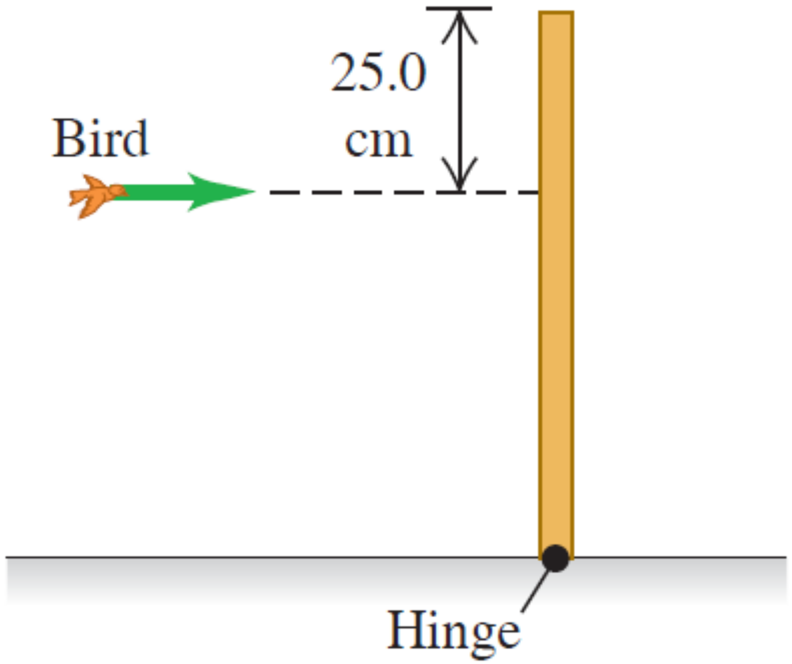
Sum of external torques on the system $\sum \vec{\tau} = \frac{d\vec{L}}{dt}$ Rate of change of total angular momentum $\vec{L}$ of system

Inc.

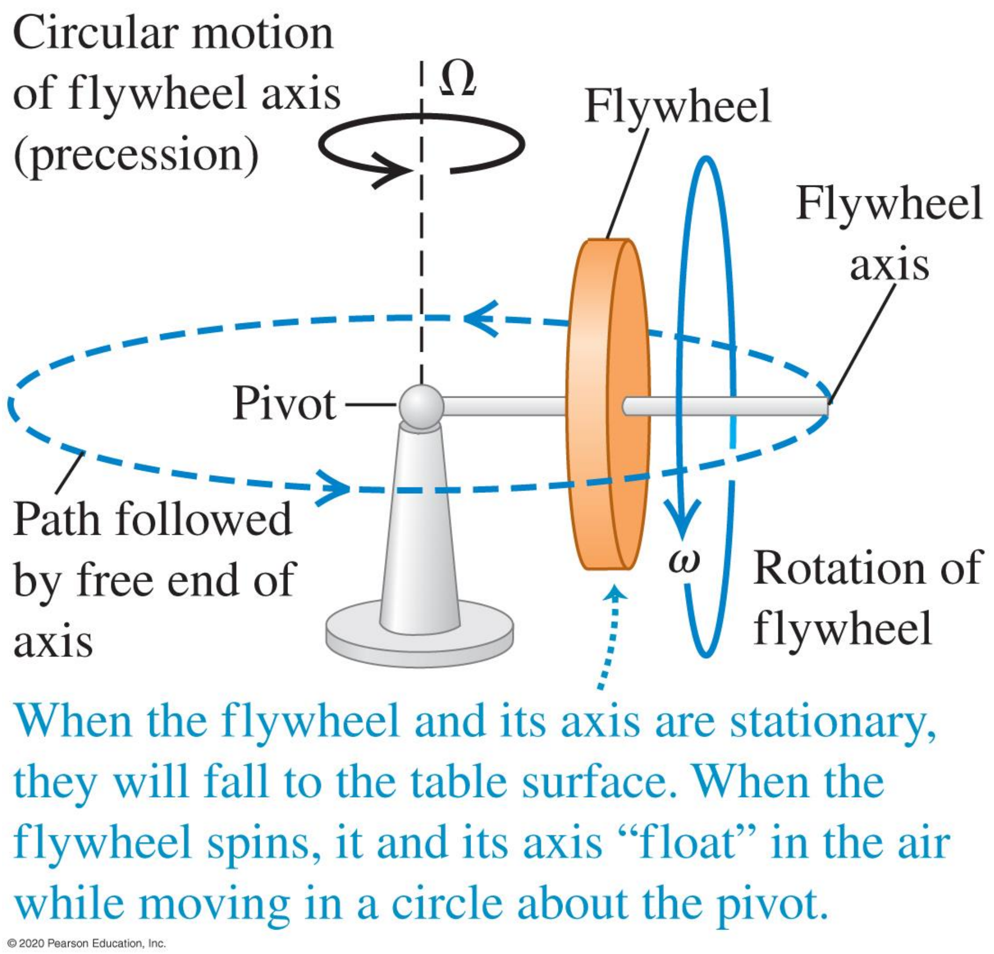
Opgave 10.85

10.85 ••• A 500.0 g bird is flying horizontally at 2.25 m/s, not paying much attention, when it suddenly flies into a stationary vertical bar, hitting it 25.0 cm below the top (**Fig. P10.85**). The bar is uniform, 0.750 m long, has a mass of 1.50 kg, and is hinged at its base. The collision stuns the bird so that it just drops to the ground afterward (but soon recovers to fly happily away). What is the angular velocity of the bar (a) just after it is hit by the bird and (b) just as it reaches the ground?

Figure **P10.85**



Gyroscope



Circular motion
of flywheel axis
(precession)
Ω
Flywheel
Flywheel
axis
Pivot
Path followed
by free end of
axis
ω
Rotation of
flywheel
When the flywheel and its axis are stationary,
they will fall to the table surface. When the
flywheel spins, it and its axis “float” in the air
while moving in a circle about the pivot.
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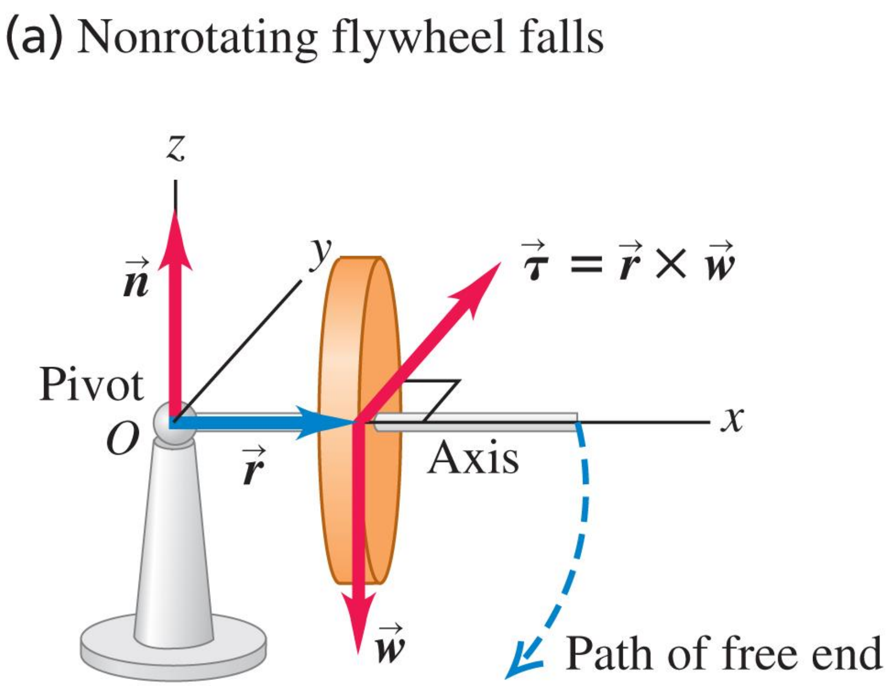
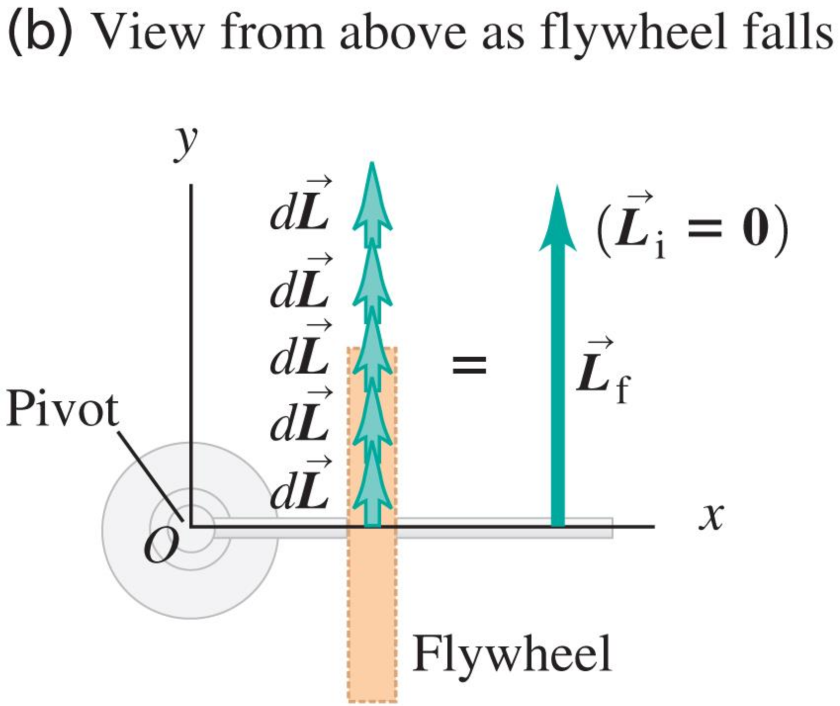
Gyroscope – no rotation



When the flywheel is not rotating, its weight creates a torque around the pivot, causing it to fall along a circular path until its axis rests on the table surface.

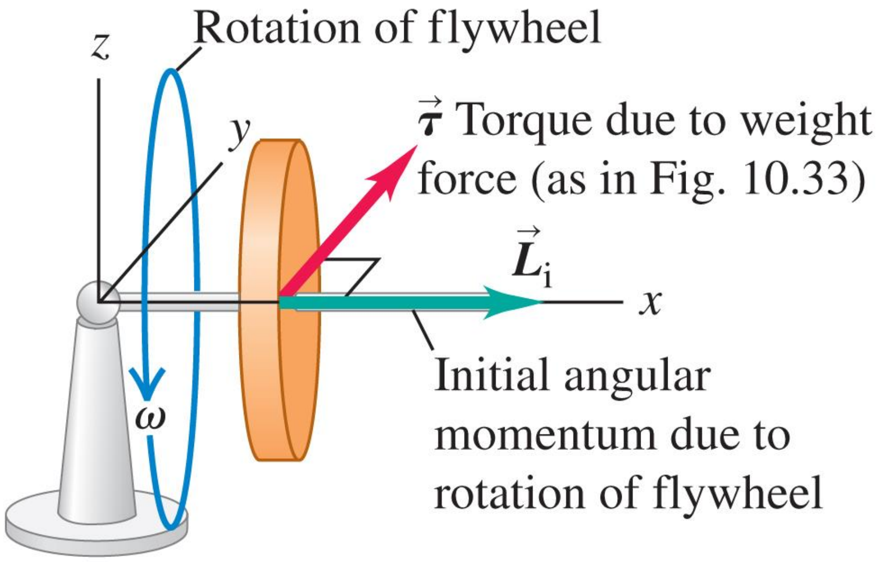


In falling, the flywheel rotates about the pivot and thus acquires an angular momentum $\vec{L}$. The *direction* of $\vec{L}$ stays constant.

Roterende gyroskop

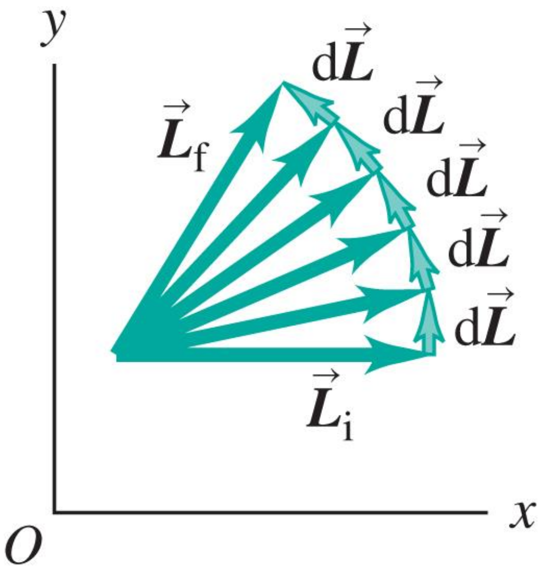
(a) Rotating flywheel

When the flywheel is rotating, the system starts with an angular momentum $\vec{L}_i$ parallel to the flywheel's axis of rotation.



(b) View from above

Now the effect of the torque is to cause the angular momentum to precess around the pivot. The gyroscope circles around its pivot without falling.



A video illustration

https://www.youtube.com/watch?v=V6XSsNAWg00

Kapitel 11

Ligevægt og elasticitet

Ligevægt

First condition for equilibrium: For the center of mass of an object at rest to remain at rest ...

$$\sum \vec{F} = 0 \quad (11.1)$$

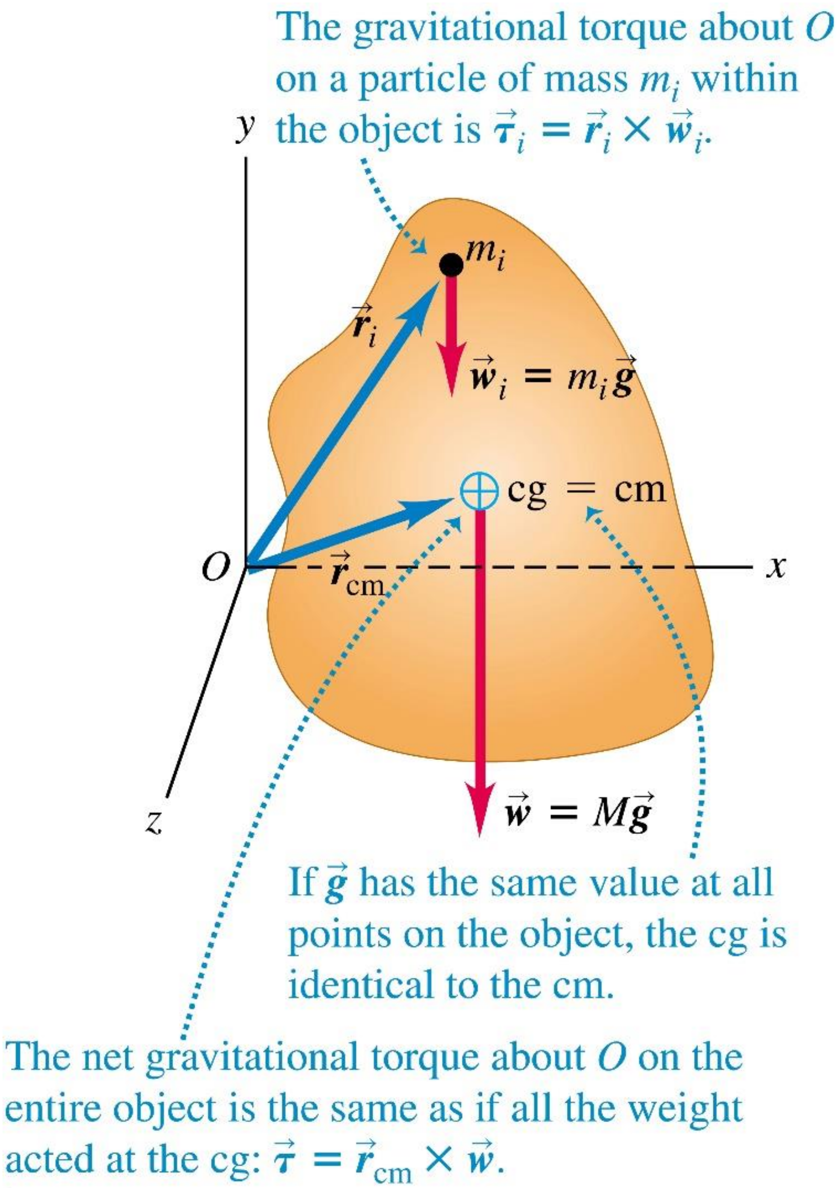
... the *net external force* on the object must be *zero*.

Second condition for equilibrium: For a nonrotating object to remain nonrotating ...

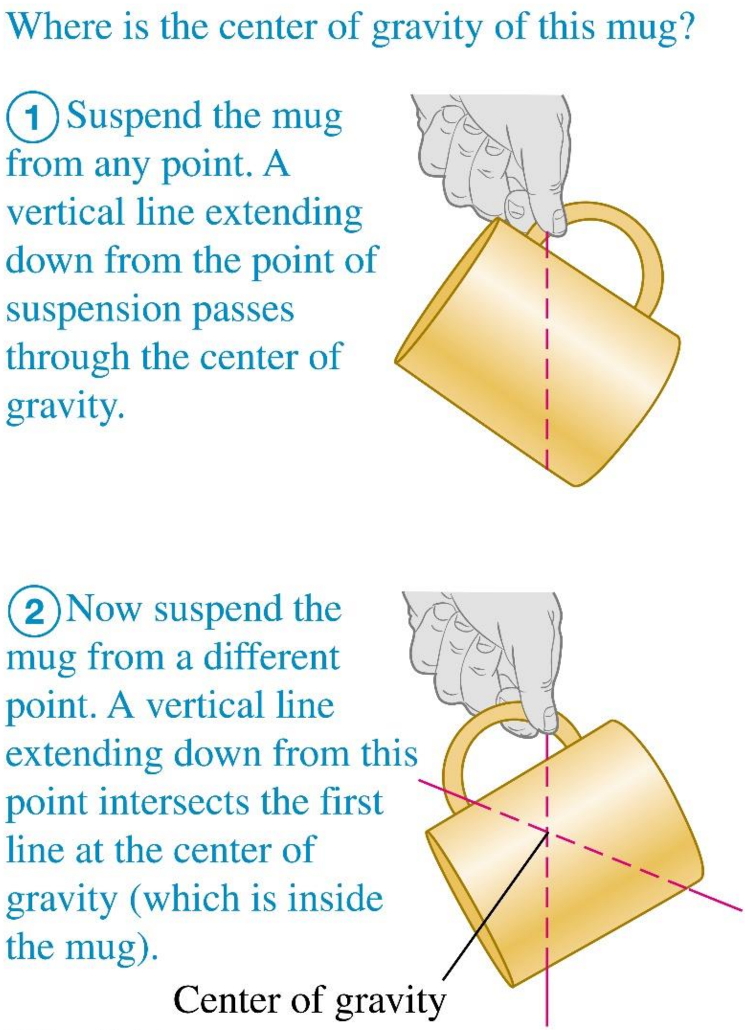
$$\sum \vec{\tau} = 0 \quad (11.2)$$

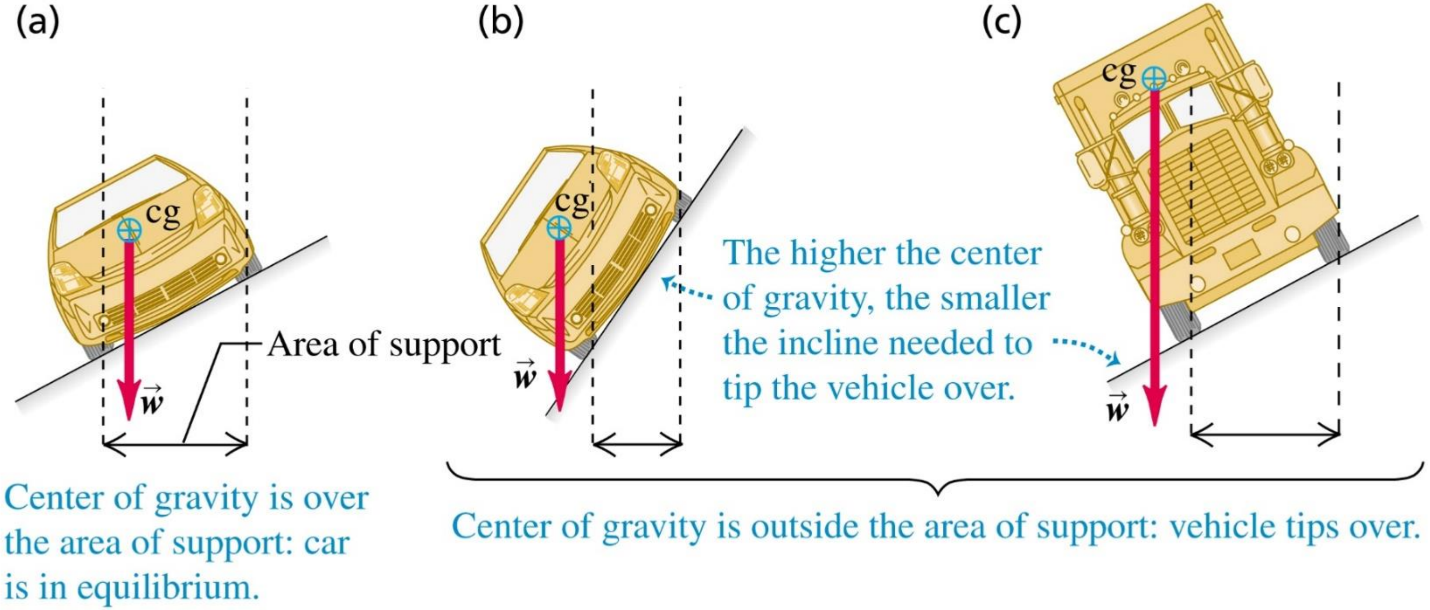
...the *net external torque around any point* on the object must be *zero*.

Tyngdepunkt



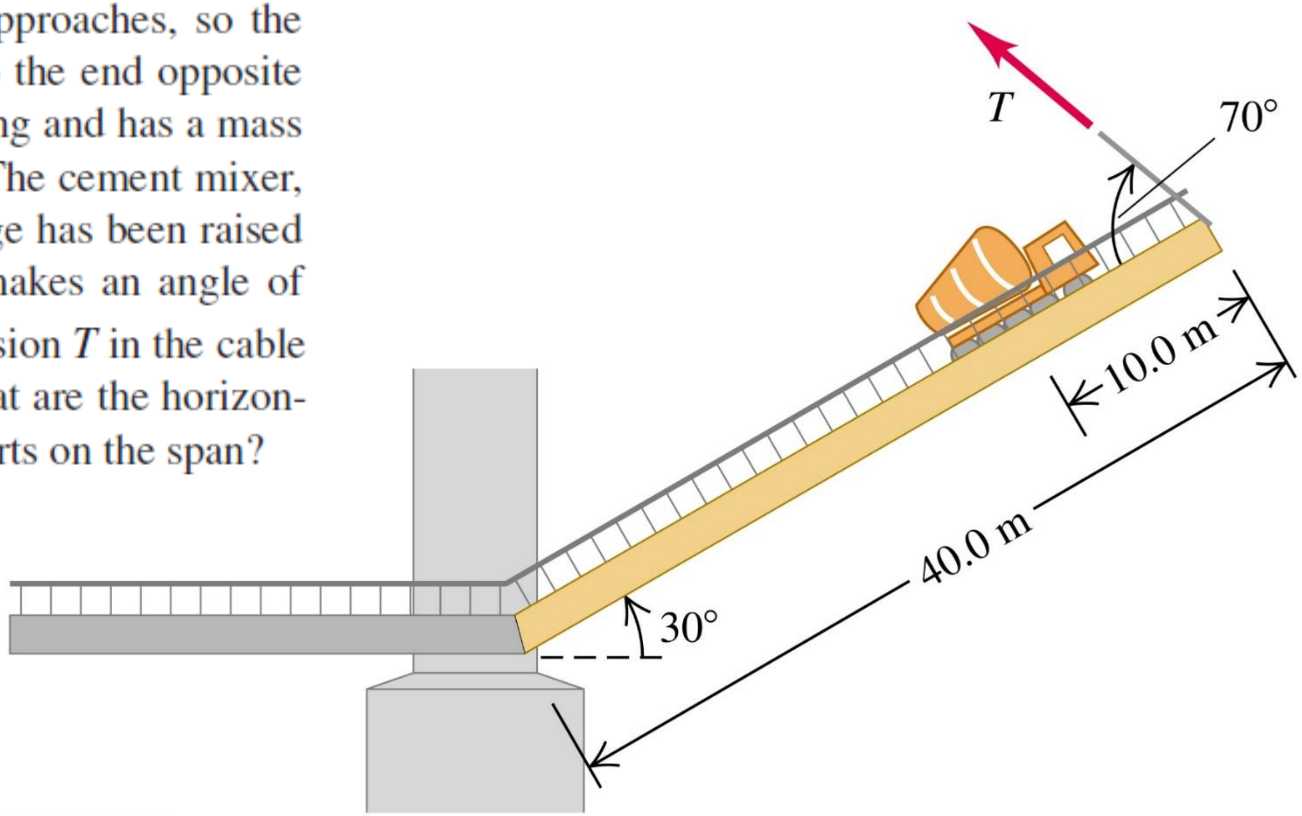
Tyngdepunkt





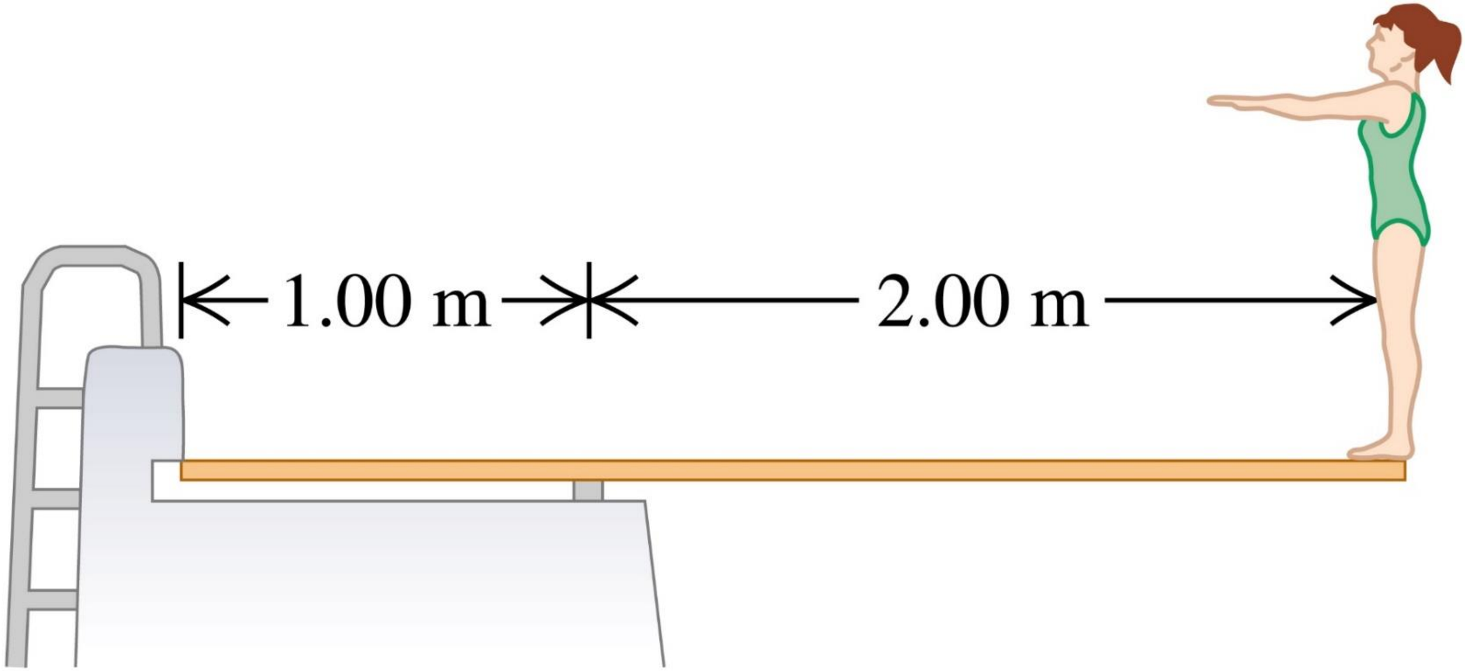
Opg 11.56

11.56 • **A Truck on a Drawbridge.** A loaded cement mixer drives onto an old drawbridge, where it stalls with its center of gravity three-quarters of the way across the span. The truck driver radios for help, sets the handbrake, and waits. Meanwhile, a boat approaches, so the drawbridge is raised by means of a cable attached to the end opposite the hinge (**Fig. P11.56**). The drawbridge is 40.0 m long and has a mass of 18,000 kg; its center of gravity is at its midpoint. The cement mixer, with driver, has mass 30,000 kg. When the drawbridge has been raised to an angle of 30° above the horizontal, the cable makes an angle of 70° with the surface of the bridge. (a) What is the tension T in the cable when the drawbridge is held in this position? (b) What are the horizontal and vertical components of the force the hinge exerts on the span?

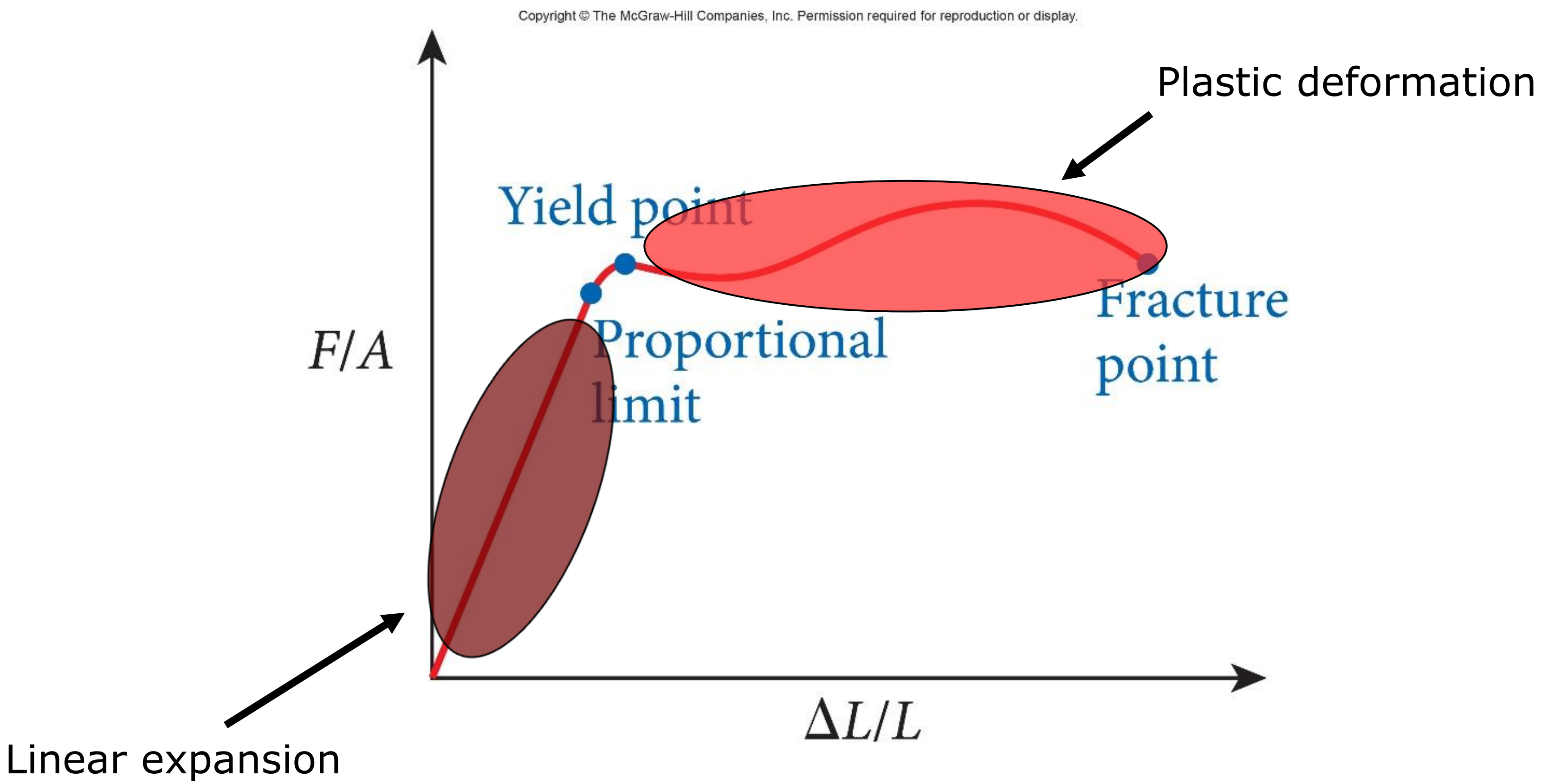


Opg 11.13

11.13 • A diving board 3.00 m long is supported at a point 1.00 m from the end, and a diver weighing 500 N stands at the free end (**Fig. E11.13**). The diving board is of uniform cross section and weighs 280 N. Find (a) the force at the support point and (b) the force at the left-hand end.



Response to external force



Tension

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

Y is **Youngs modulus**

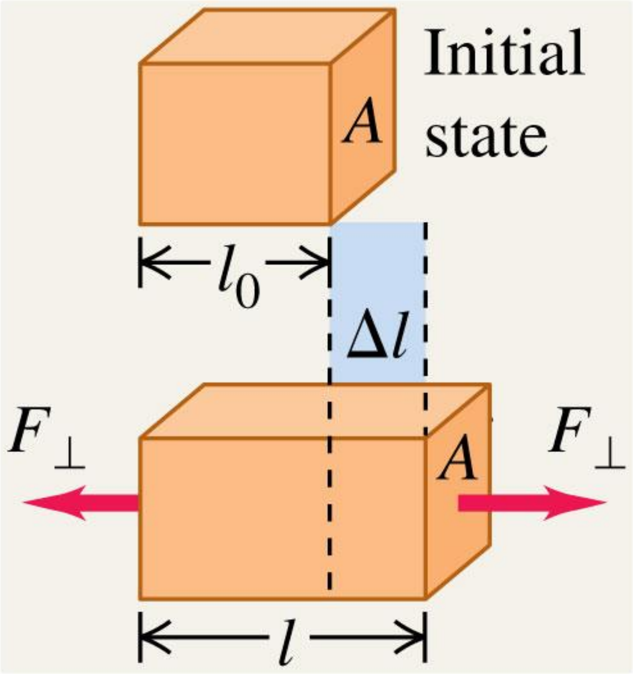
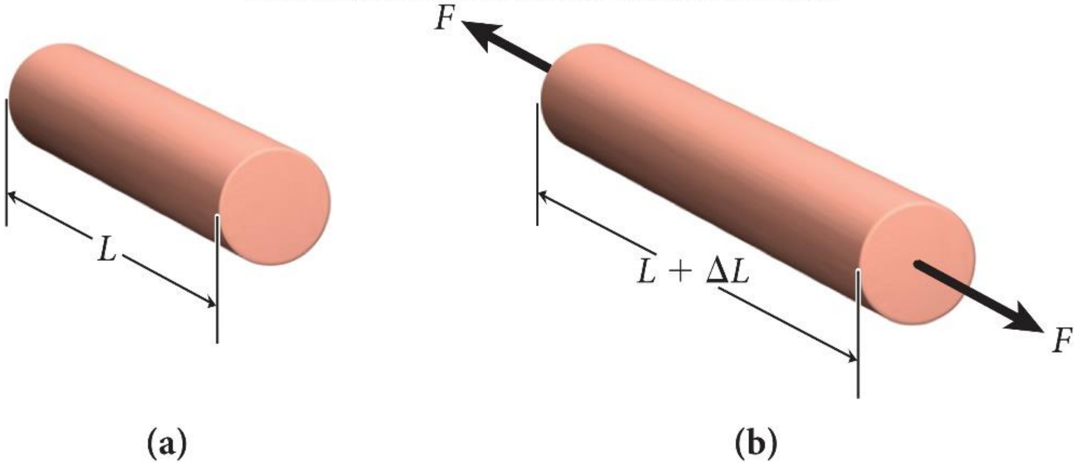


Table 13.1 Some Typical Values of Young's Modulus

Material	Young's Modulus (10^9 N/m^2)
Aluminum	70
Bone	10–20
Concrete	20–30 (compression)
Diamond	1000–1200
Glass	70
Polystyrene	3
Rubber	0.01–0.1
Steel	200
Titanium	100–120
Tungsten	400
Wood	10–15



Bulk compression

$$\frac{F}{A} = B\frac{\Delta V}{V}$$

B is **bulk modulus**

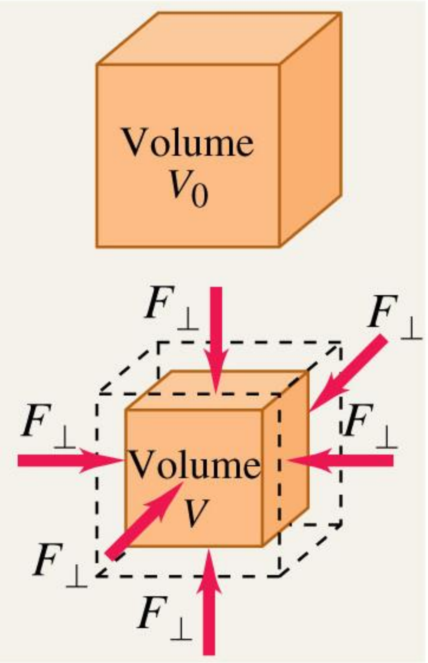
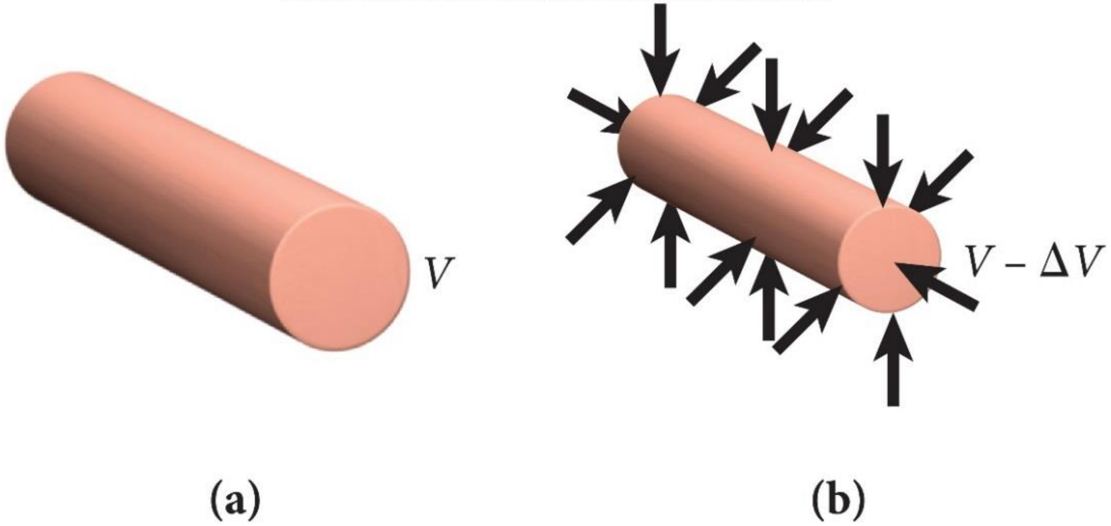


Table 13.2 Some Typical Values of the Bulk Modulus

Material	Bulk Modulus (10^9 N/m^2)
Air	0.000142
Aluminum	76
Basalt rock	50–80
Gasoline	1.5
Granite rock	10–50
Mercury	28.5
Steel	160
Water	2.2



Shear

$$\frac{F}{A} = G\frac{\Delta x}{L}$$

G is **shear modulus**

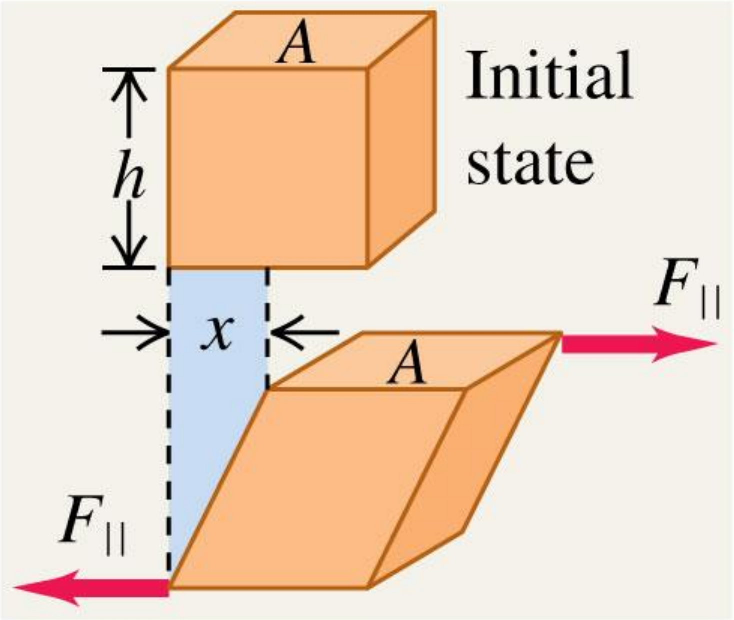
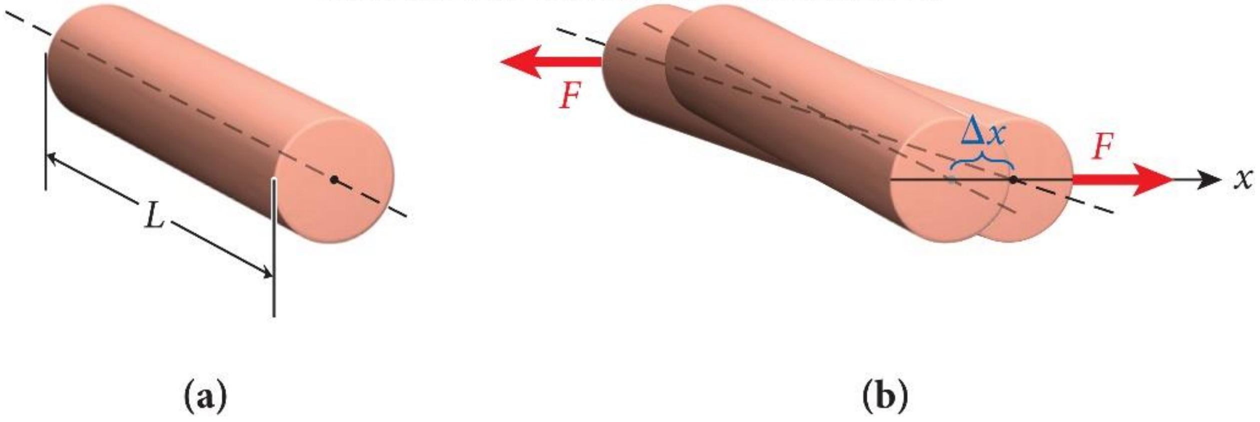


Table 13.3	Some Typical Values of the Shear Modulus
Material	**Shear Modulus (10^9 N/m^2)**
Aluminum	25
Copper	45
Glass	26
Polyethylene	0.12
Rubber	0.0003
Titanium	41
Steel	70–90

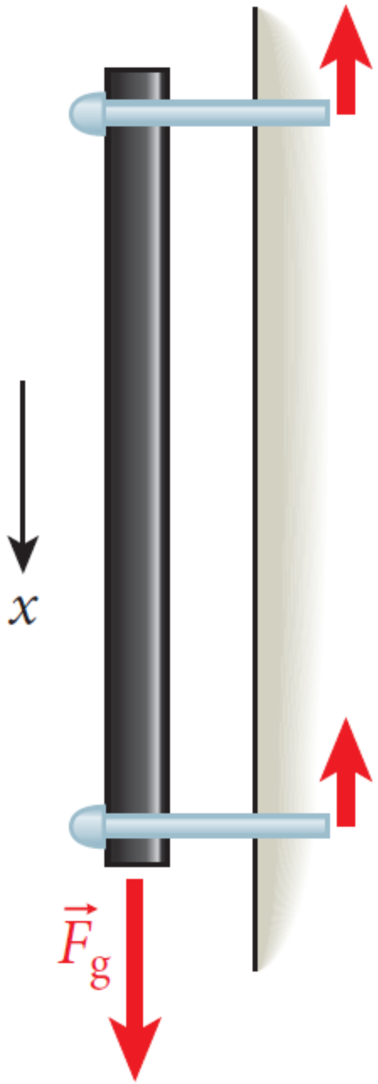


Problem

You've just bought a new flat-panel TV (Figure 13.10) and want to mount it on the wall with four bolts, each with a diameter each of 0.50 cm. You cannot mount the TV flush against the wall but have to leave a 10.0-cm gap between wall and TV for air circulation.

PROBLEM 1

If the mass of your new TV is 42.8 kg, what is the shear stress on the bolts?



Opg 11.30

11.30 •• **Stress on a Mountaineer's Rope.** A nylon rope used by mountaineers elongates 1.10 m under the weight of a 65.0 kg climber. If the rope is 45.0 m in length and 7.0 mm in diameter, what is Young's modulus for nylon?

DTU

Seniorforsker Søren B. Korsholm, PPFE, DTU Fysik

Kapitel 10 Rotationsdynamik

Program for i dag

13:00	Introduktion
13:10	Kapitel 10 – Rotationsdynamik (10.1-10.4)
13:50	Pause
14:10	Kapitel 10 – Rotationsdynamik (10.1-10.4)
14:30	Pause og Gruppereregning
17:00	Tak for i dag

Undervisere og hjælpelærere

Undervisere

- Cathrine Frandsen – F24

- Søren B Korsholm – F24

Kursusansvarlig

- Carsten Knudsen

Hjælpelærere

- Ida Grum-Schwensen Andersen

- Adam Jabiri

- Lucas Borg Clausen
- Joachim Christian Christensen

Søren Bang Korsholm

Seniorforsker i plasmafysik og fusionsenergi (sv.t. lektor) siden 2007

Læste Teknisk Fysik ~ FysNan på DTU 1992-1998
PhD på Risø/DTU Fysik 1998-2002
PostDoc MIT/Risø 2002-2006

Primært ansvarsområde: Ansvarlig og projektleder for udviklingen af Collective Thomson Scattering diagnostikken til ITER og til DEMO

Underviser (10402, 10020) og vejleder/medvejleder på MSc, BSc, fagprojekter mm

Formidling om fusionsenergi overfor offentligheden, pressen og gymnasieelever mv

Kursusplan for foråret

01-02-2024	Kap. 9	alec
08-02-2024	Kap. 10	sbko
15-02-2024	Kap. 10+11	sbko
22-02-2024	Kap. 13	sbko
29-02-2024	Temadag	sbko
07-03-2024	Kap. 14	sbko
14-03-2024	Kap. 21	fraca
21-03-2024	Kap. 22	fraca
28-03-2024	Påske	
04-04-2024	Kap. 23	fraca
11-04-2024	Kap. 27	fraca
18-04-2024	Kap. 27+28	fraca
25-04-2024	Kap. 28	fraca
02-05-2024	Opsamling	fraca

Praktiske oplysninger

DTU Learn har al den information I skal have – ellers skriv til os

Forelæsninger torsdage kl 13-15 i B306 aud. 34

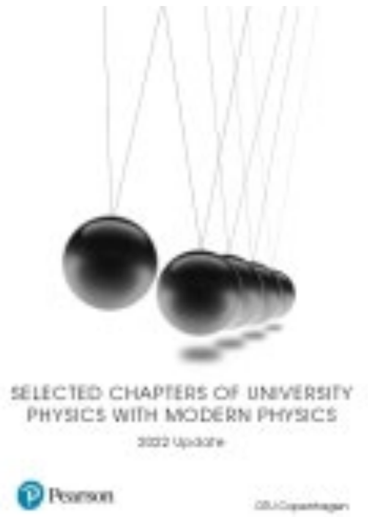
Grupperegning torsdage kl 15-17 er i holdområderne 96, 97, 98, 99, 108a og 108b i Bygning 306.

Opgaverne er på DTU Learn og løsningerne kommer sidst på ugen

Eksamen: Skriftlig eksamen i eksamensperioden F24.

Lærebog: Selected chapters of University Physics with Modern Physics (2022 update) af Young and Freedman

Inkluderer e-bog ved køb via Polyteknisk Boghandel



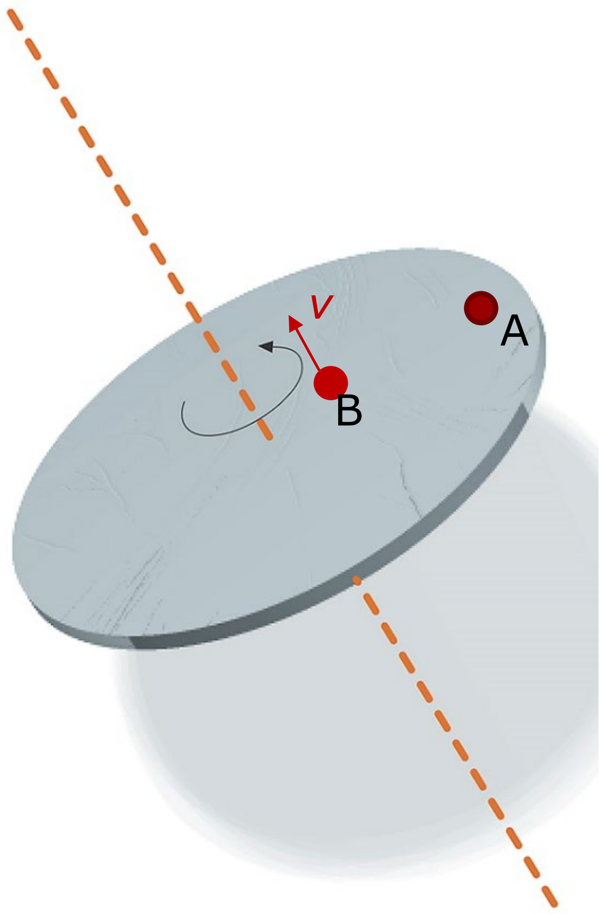
Jeres indsats

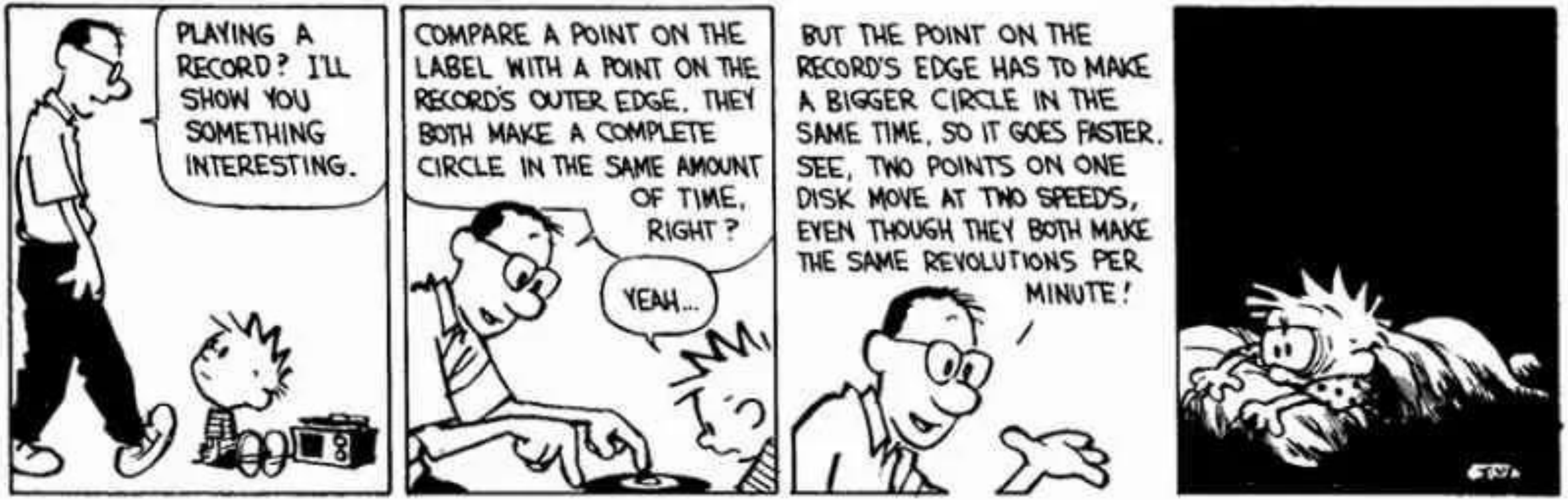
- Læs bogen før forelæsningen
- Tag en evt. quiz senest dagen før forelæsningen
- Deltag i forelæsningerne
- Regn opgaverne
  - At læse løsningerne gør dig god til at læse løsninger, ikke nødvendigvis til at regne opgaverne
  - Gruppearbejde er godt for meget, men du skal også lære at regne for dig selv
  - De fleste opgaver har facit, som man kan bruge under regningen
  - De fleste opgaver har også løsninger, som bliver frigivet efter grupperegningen – evt. senere på ugen

Fra sidste gang - Quiz

To punkter, A og B, er på en disk, der roterer om en akse. Punkt A er tre gange så langt væk fra rotationsaksen som punkt B. Hvis farten for punkt B er v, hvad er så farten for punkt A?

A. v
B. $3v$
C. $v/3$
D. $9v$




PLAYING A RECORD? I'LL SHOW YOU SOMETHING INTERESTING.
COMPARE A POINT ON THE LABEL WITH A POINT ON THE RECORD'S OUTER EDGE. THEY BOTH MAKE A COMPLETE CIRCLE IN THE SAME AMOUNT OF TIME, RIGHT?
YEAH...
BUT THE POINT ON THE RECORD'S EDGE HAS TO MAKE A BIGGER CIRCLE IN THE SAME TIME, SO IT GOES FASTER. SEE, TWO POINTS ON ONE DISK MOVE AT TWO SPEEDS, EVEN THOUGH THEY BOTH MAKE THE SAME REVOLUTIONS PER MINUTE!

Kinematik for cirkelbevægelse med konstant vinkelacceleration

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Analogi:

Translation:

$$x(t)$$

$$v(t)$$

$$a(t)$$

Konstant acceleration:

$$v_x = v_{0x} + a_x t$$

$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x (x - x_0)$$

Rotation:

$$\theta(t)$$

$$\omega(t)$$

$$\alpha(t)$$

Konstant vinkelacceleration:

$$\omega = \omega_0 + \alpha t$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Rotation af stift legeme om fast akse

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$v = r\omega, \quad a = r\alpha$$

	Translation:	Rotation:
Rotation: Kinematik	$x(t)$	$\theta(t)$
	$v(t)$	$\omega(t)$
	$a(t)$	$\alpha(t)$
	m	I
	$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$
	$F = ma$	$\tau = I\alpha$ (I dag!)

Roatationsdynamik – Kapitel 10 (10.1-10.4)

Kraftmoment - torque

Hvilken ville du vælge?

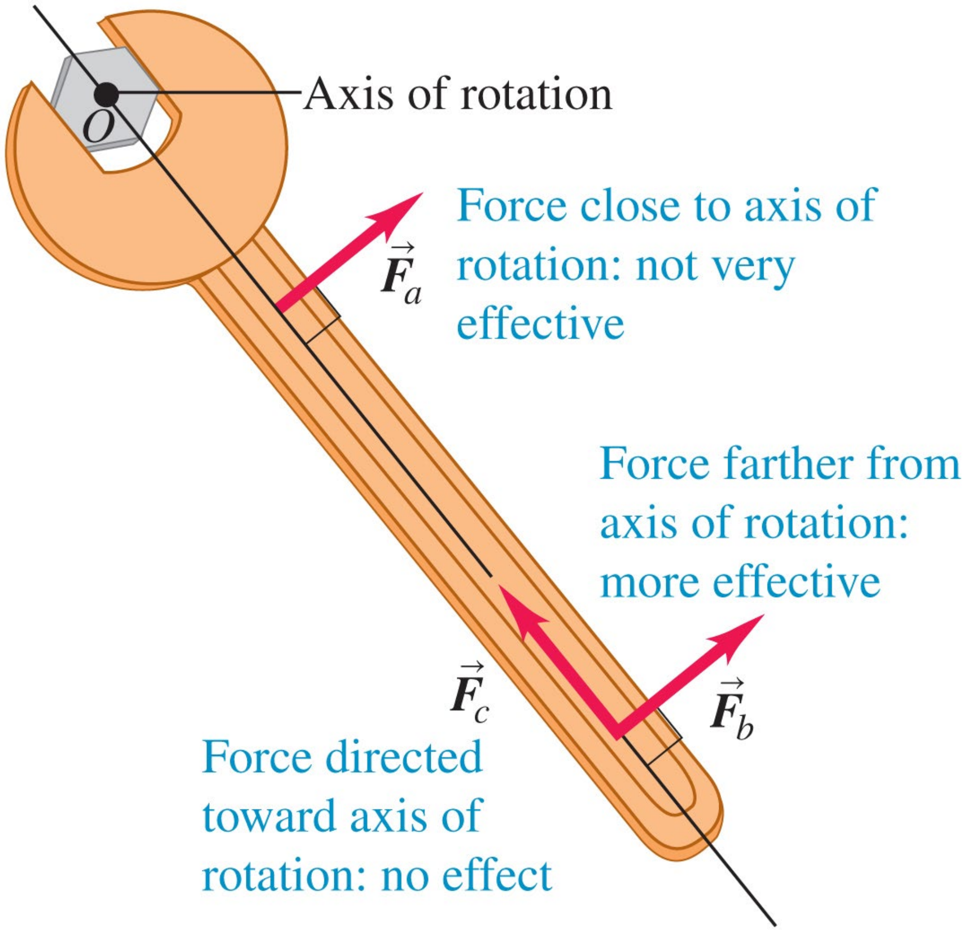
$$\tau = Fl$$

Kraftmoment om en rotationsakse

Hold øje med retningen af kraften i forhold til rotationen

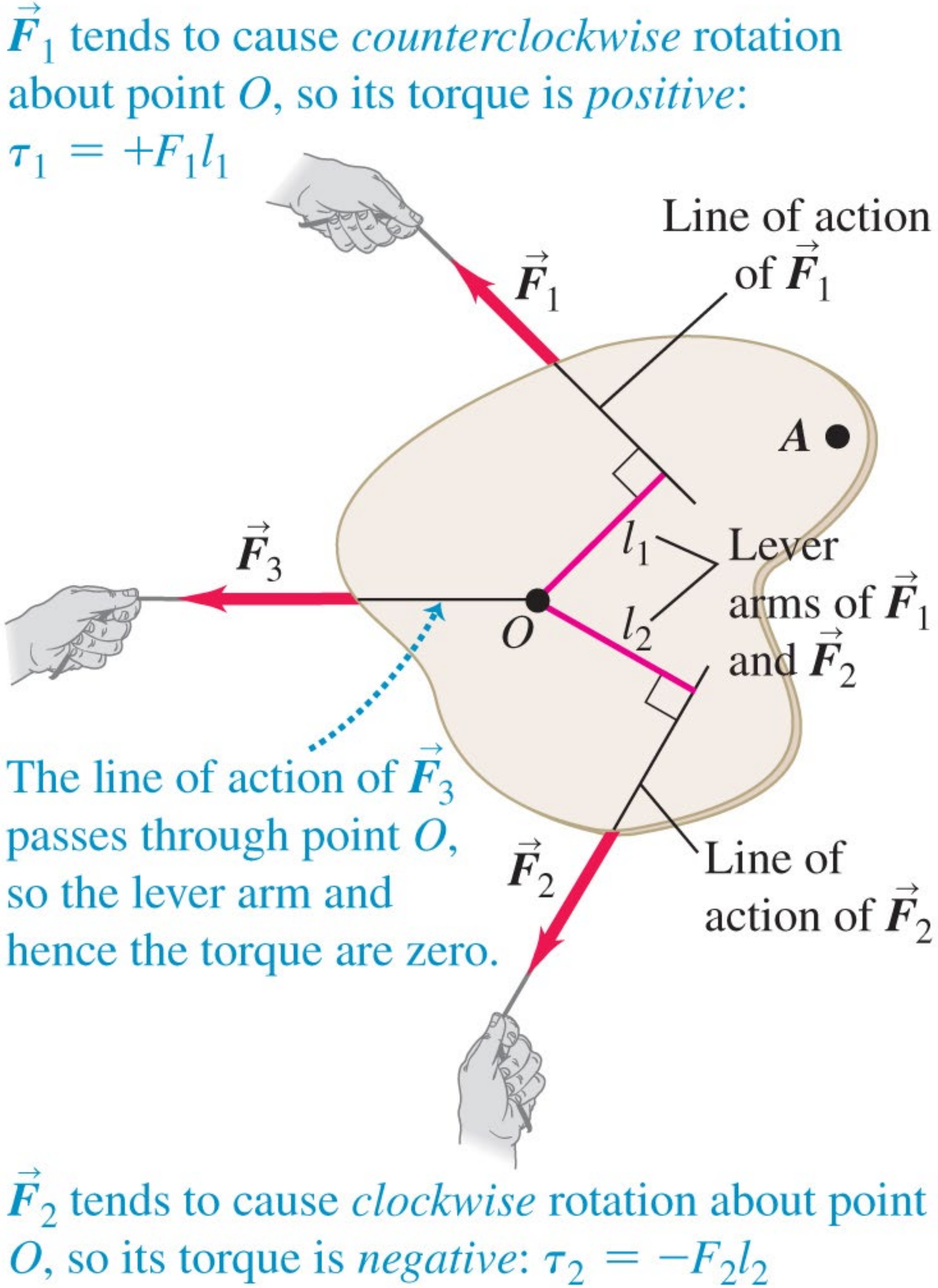
Og med afstanden fra rotationsaksen til kraftens angrebspunkt.

$$\tau = Fl$$

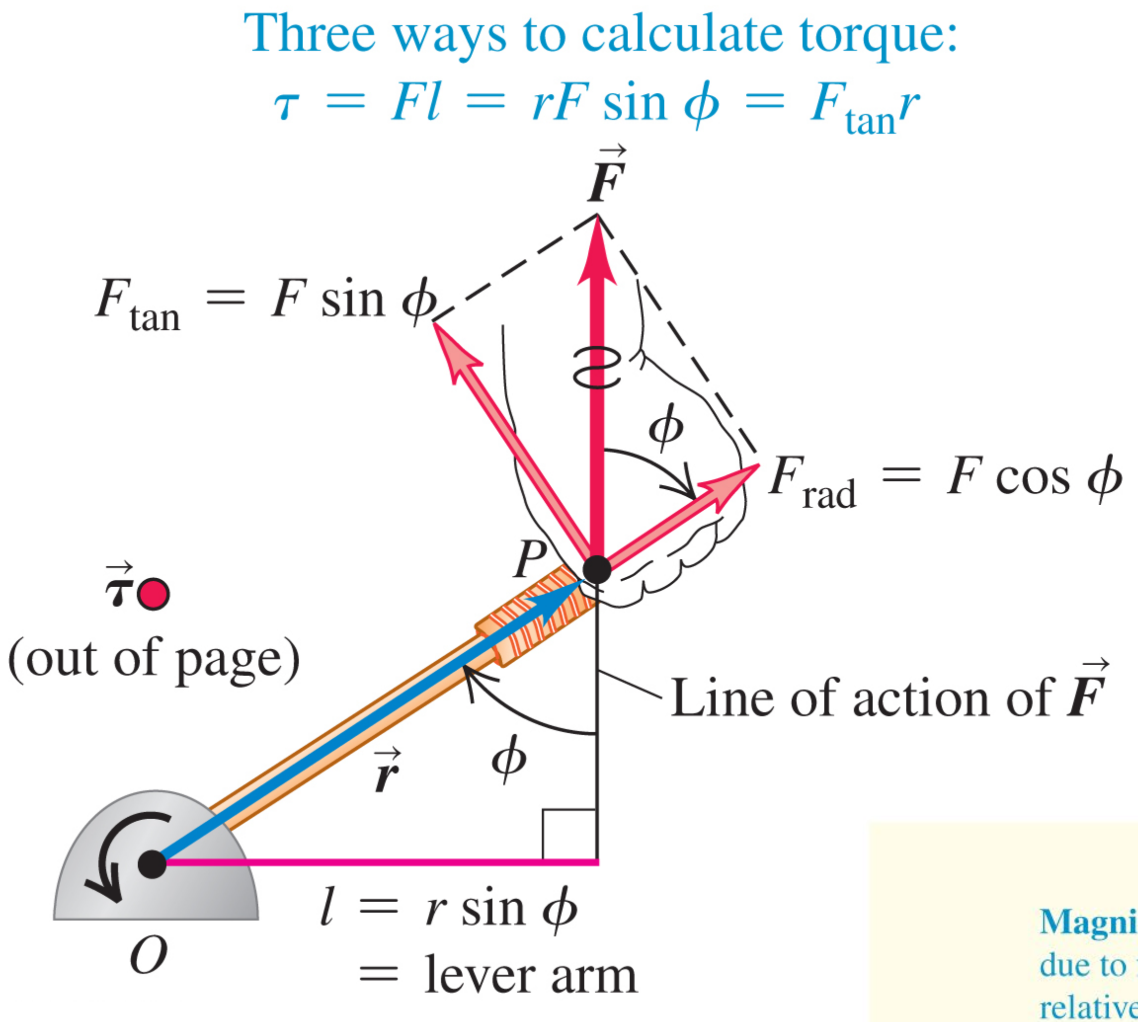


Kraftmoment

- Normalt regner vi positiv kraftmoment for påvirkning af rotationen i retning mod uret



Forskellige måder at regne kraftmomentet



Magnitude of $\vec{r}$ (vector from O to where $\vec{F}$ acts)

Lever arm of $\vec{F}$

Magnitude of torque due to force $\vec{F}$ relative to point O

$$\tau = Fl = rF\sin\phi = F_{tan}r \qquad (10.2)$$

Magnitude of $\vec{F}$

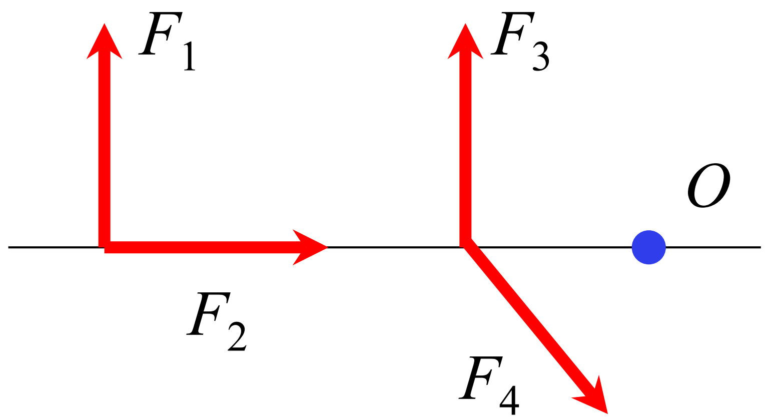
Angle between $\vec{r}$ and $\vec{F}$

Tangential component of $\vec{F}$

Quiz

De fire kræfter på figuren har alle samme størrelse: $F_1 = F_2 = F_3 = F_4$.

Hvilken kraft giver det største kraftmoment med hensyn til punktet O?



A. F_1

B. F_2

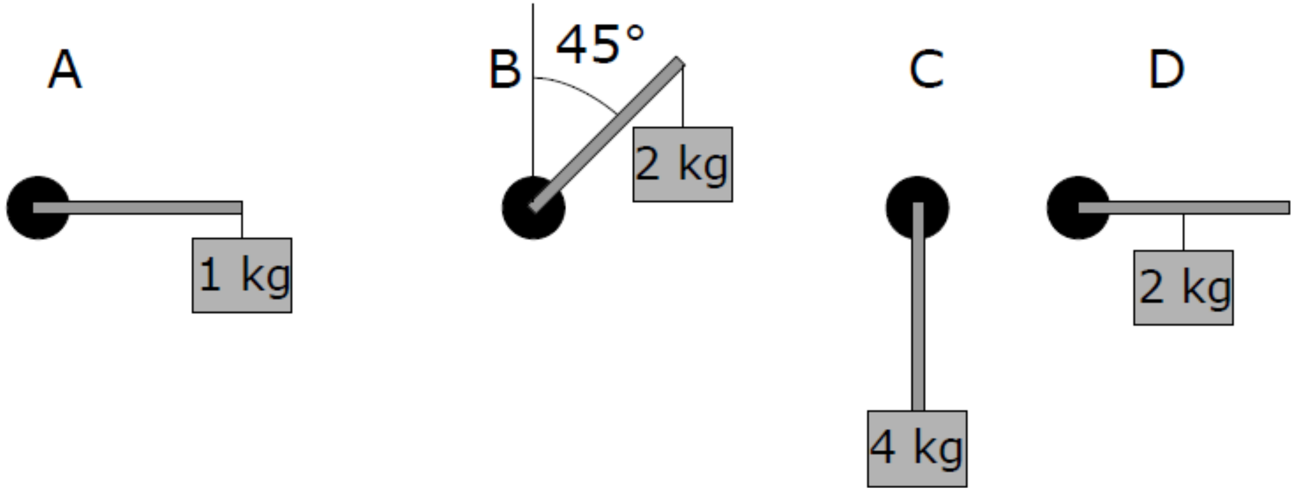
C. F_3

D. F_4

E. Ikke tilstrækkelig information

Quiz

I hvilken situation er kraftmomentet fra loddet størst?



Quiz – faldende stang

Hvad er kraftmomentet på stangen fra tyngdekraften?

$$\tau = mgL \qquad \tau = mgL\cos\theta \qquad \tau = mgL\sin\theta$$

$$\tau = mg\frac{L}{2} \qquad \tau = mg\frac{L}{2}\cos\theta \qquad \tau = mg\frac{L}{2}\sin\theta$$

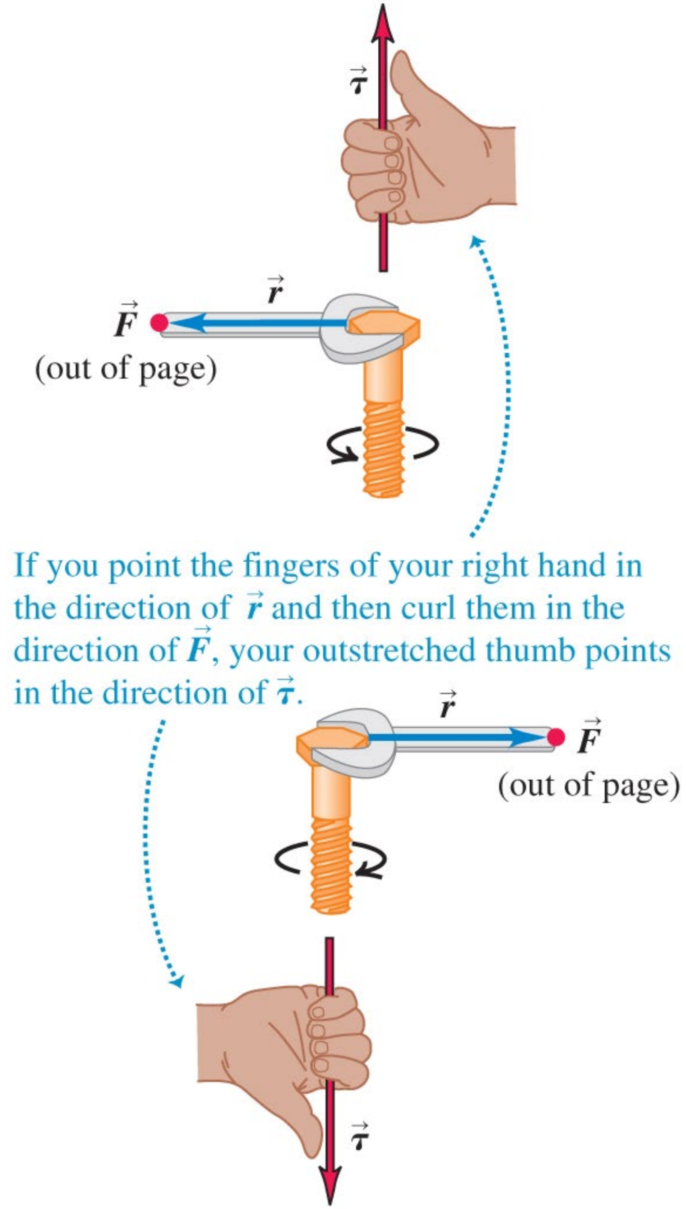
Kraftmomentet er en vektor

Torque vector due to force $\vec{F}$ relative to point O

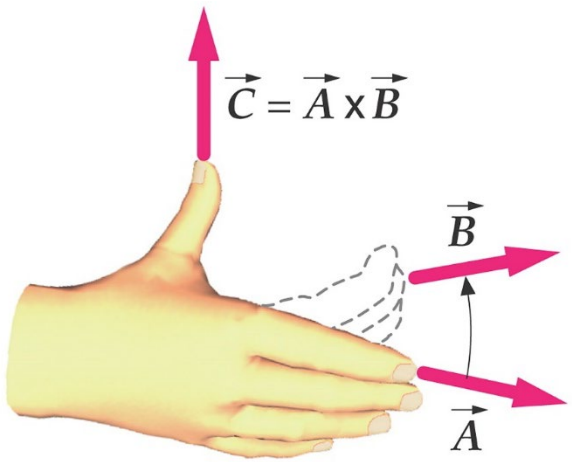
$$\vec{\tau} = \vec{r} \times \vec{F} \qquad (10.3)$$

Vector from O to where $\vec{F}$ acts

Force $\vec{F}$

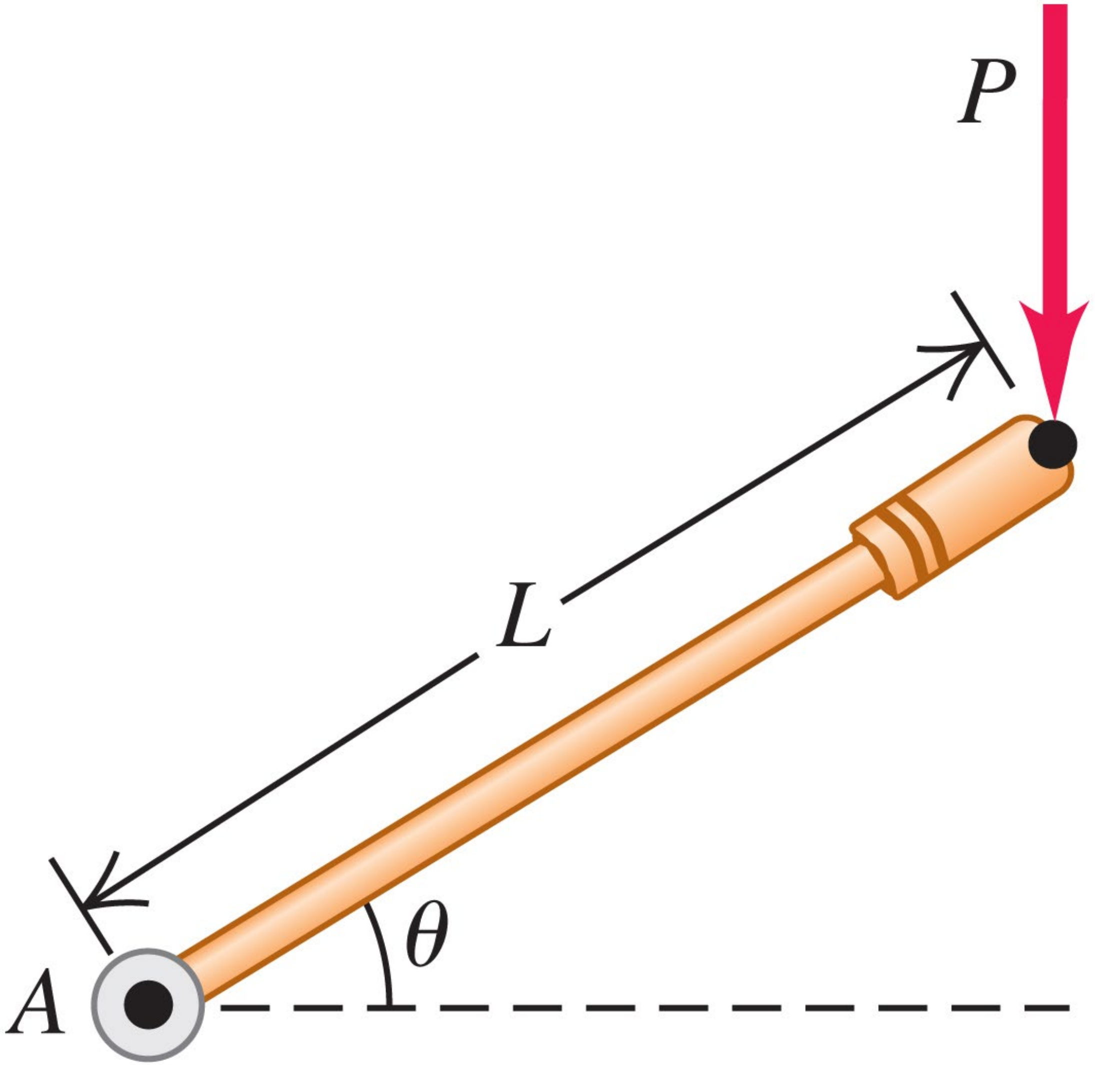


HUSK



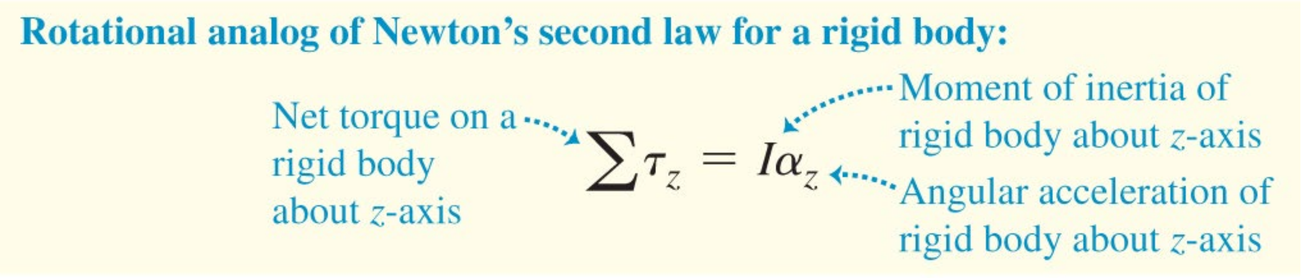
Eksempel

- Hvad er kraftmomentet fra P omkring A?

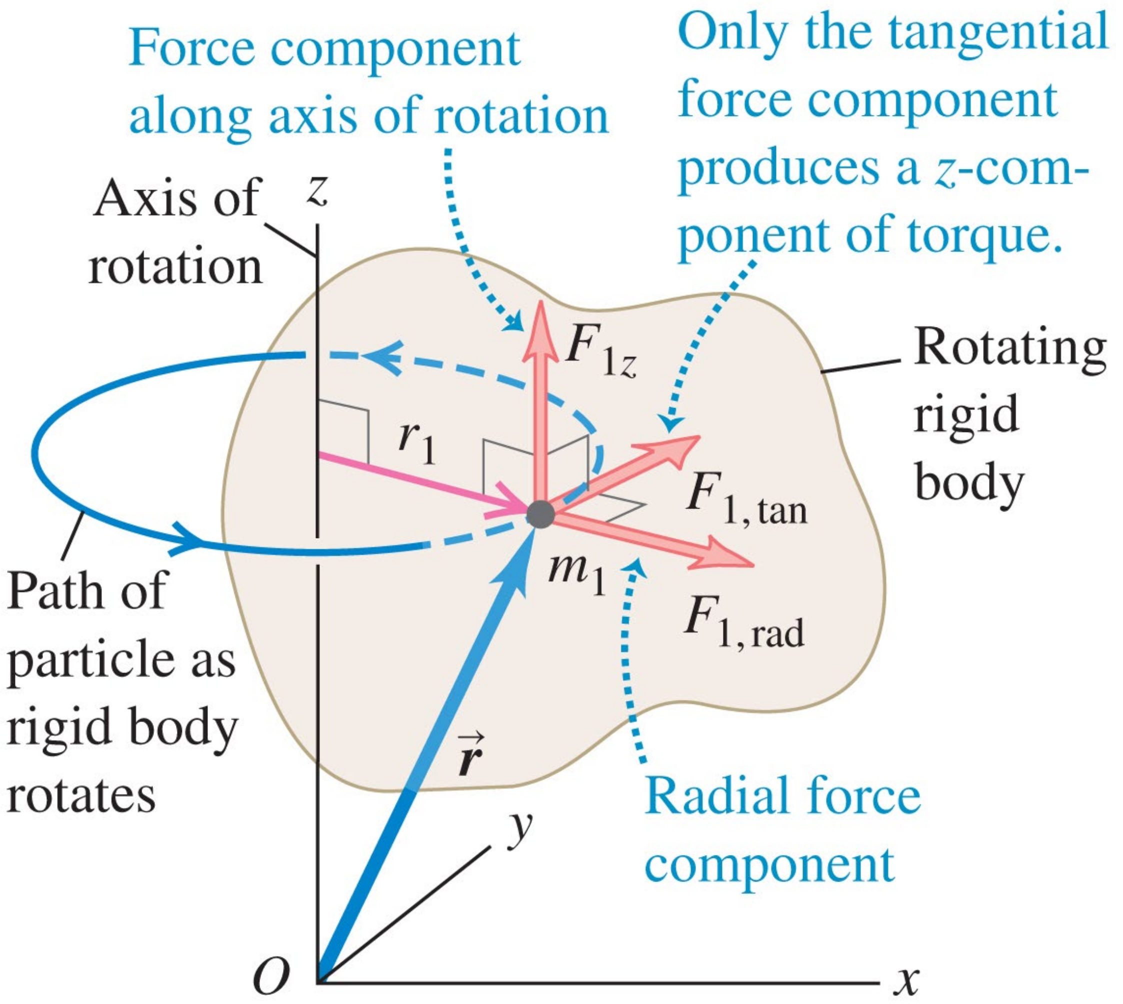


Kraftmoment og vinkelacceleration

Rotation af et stift legeme om en fast akse (z):



(svarer til N2: $\sum F = ma$ for 1D-translation)



PROBLEM-SOLVING STRATEGY 10.1 Rotational Dynamics for Rigid Bodies

Our strategy for solving problems in rotational dynamics is very similar to Problem-Solving Strategy 5.2 for solving problems involving Newton's second law.

IDENTIFY *the relevant concepts:* Equation (10.7), $\sum \tau_z = I\alpha_z$, is useful whenever torques act on a rigid body. Sometimes you can use an energy approach instead, as we did in Section 9.4. However, if the target variable is a force, a torque, an acceleration, an angular acceleration, or an elapsed time, using $\sum \tau_z = I\alpha_z$ is almost always best.

SET UP *the problem* using the following steps:

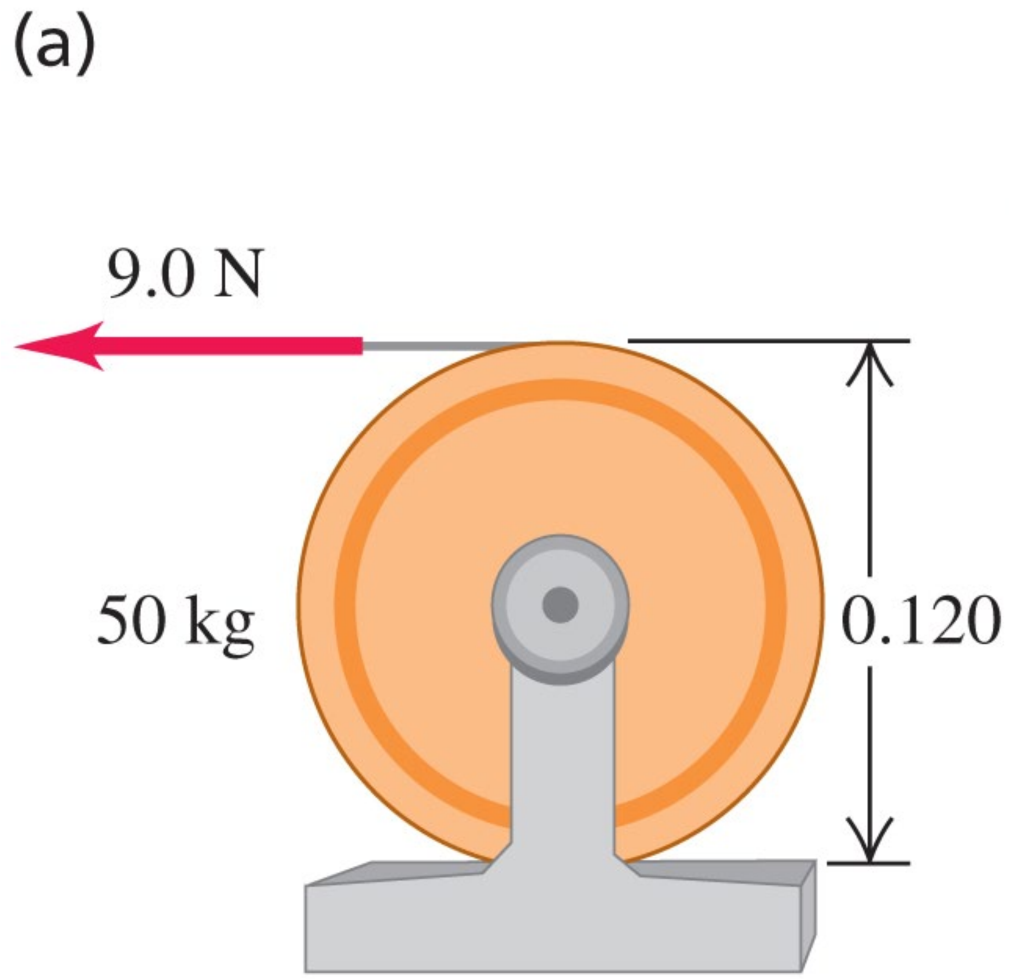
1. Sketch the situation and identify the body or bodies to be analyzed. Indicate the rotation axis.
2. For each body, draw a free-body diagram that shows the body's *shape,* including all dimensions and angles. Label pertinent quantities with algebraic symbols.
3. Choose coordinate axes for each body and indicate a positive sense of rotation (clockwise or counterclockwise) for each rotating body. If you know the sense of α_z, pick that as the positive sense of rotation.

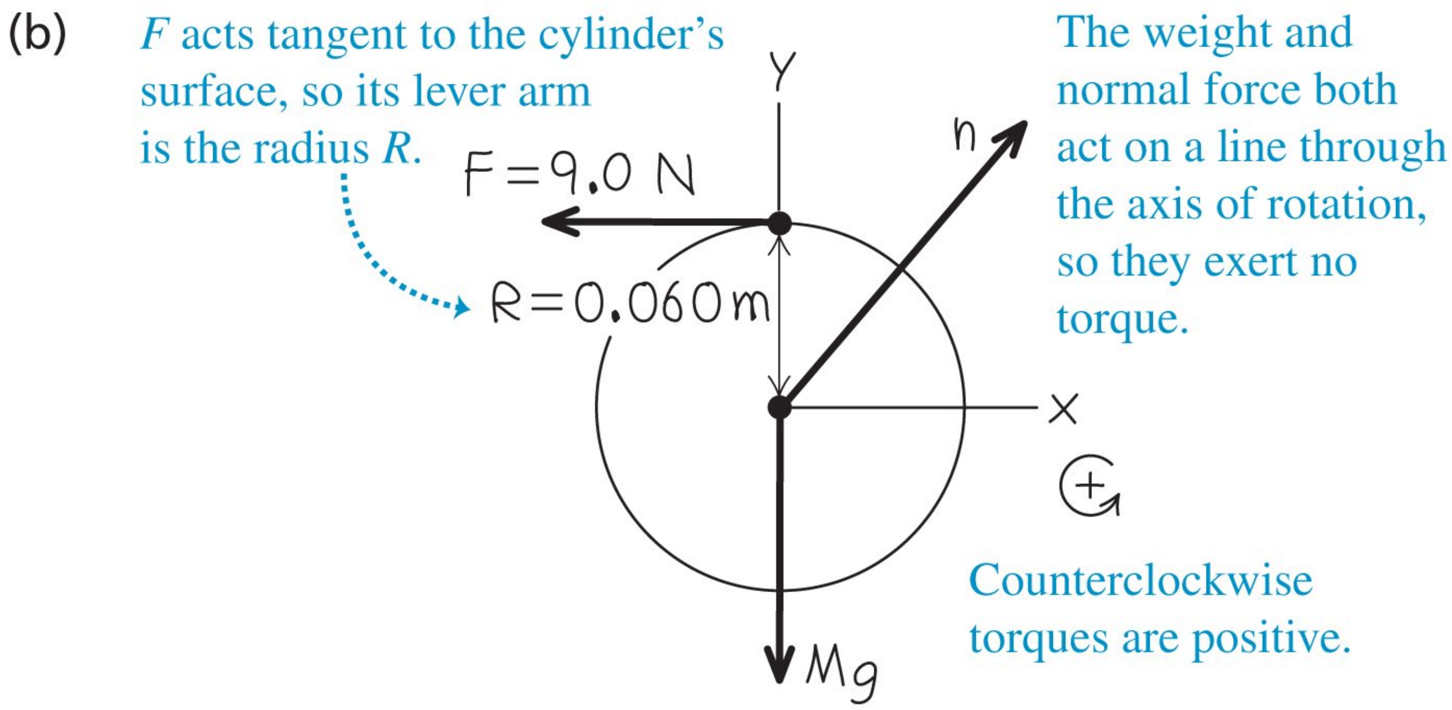
EXECUTE *the solution:*

1. For each body, decide whether it undergoes translational motion, rotational motion, or both. Then apply $\sum \vec{F} = m\vec{a}$ (as in Section 5.2), $\sum \tau_z = I\alpha_z$, or both to the body.
2. Express in algebraic form any *geometrical* relationships between the motions of two or more bodies. An example is a string that unwinds, without slipping, from a pulley or a wheel that rolls without slipping (discussed in Section 10.3). These relationships usually appear as relationships between linear and/or angular accelerations.
3. Ensure that you have as many independent equations as there are unknowns. Solve the equations to find the target variables.

EVALUATE *your answer:* Check that the algebraic signs of your results make sense. As an example, if you are unrolling thread from a spool, your answers should not tell you that the spool is turning in the direction that rolls the thread back onto the spool! Check that any algebraic results are correct for special cases or for extreme values of quantities.

Eksempel 10.2





Rotation og translation

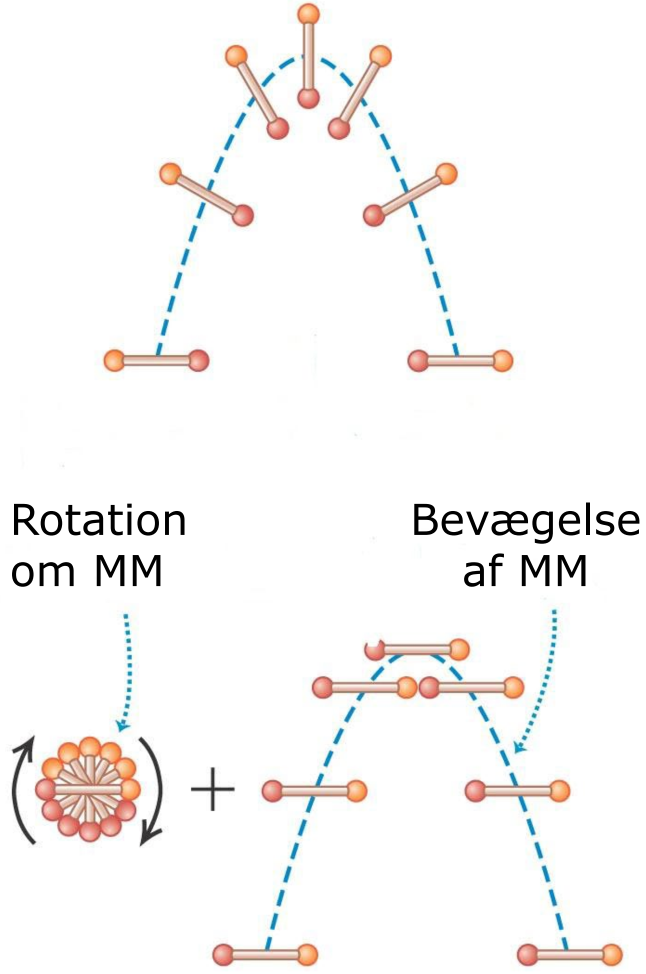
Bevægelsen opdeles i
1. bevægelsen af massemidtpunkt
2. rotationen om massemidtpunkt

Rotation af MM

$$I_{\mathrm{cm}}\alpha = \sum \tau_{\mathrm{cm}}$$

Bevægelse af MM

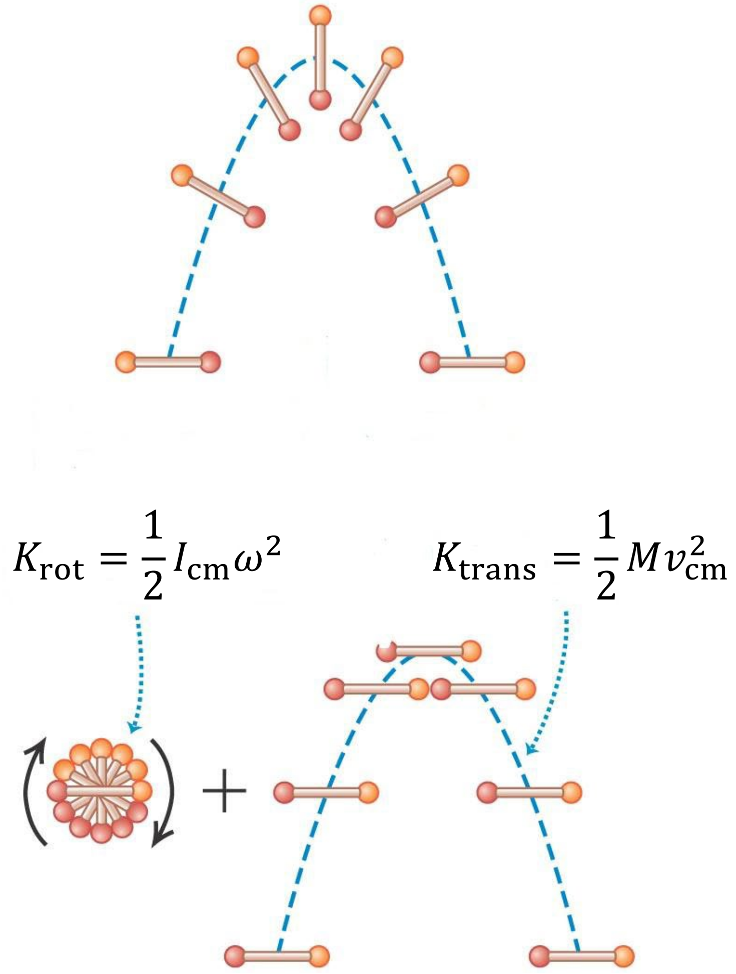
$$M\vec{a}_{\mathrm{cm}} = \sum \vec{F}_{\mathrm{ydre}}$$



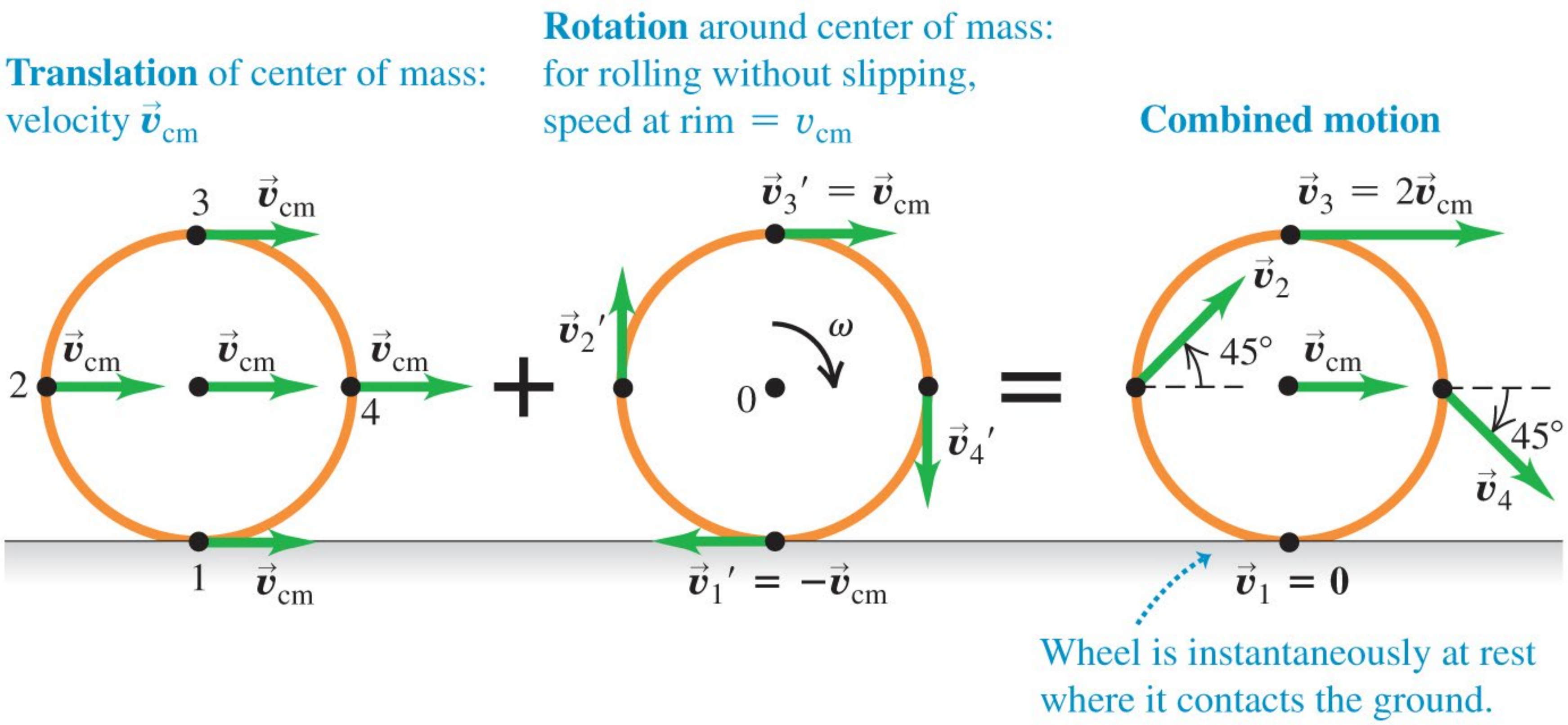
Kinetisk energi af bevægende og roterende legeme

Vi kan naturligt opdele i rotations og translations kinetisk energi

$$\boxed{K = K_{\mathrm{rot}} + K_{\mathrm{trans}} = \frac{1}{2} I_{\mathrm{cm}}\omega^2 + \frac{1}{2} M v_{\mathrm{cm}}^2}$$



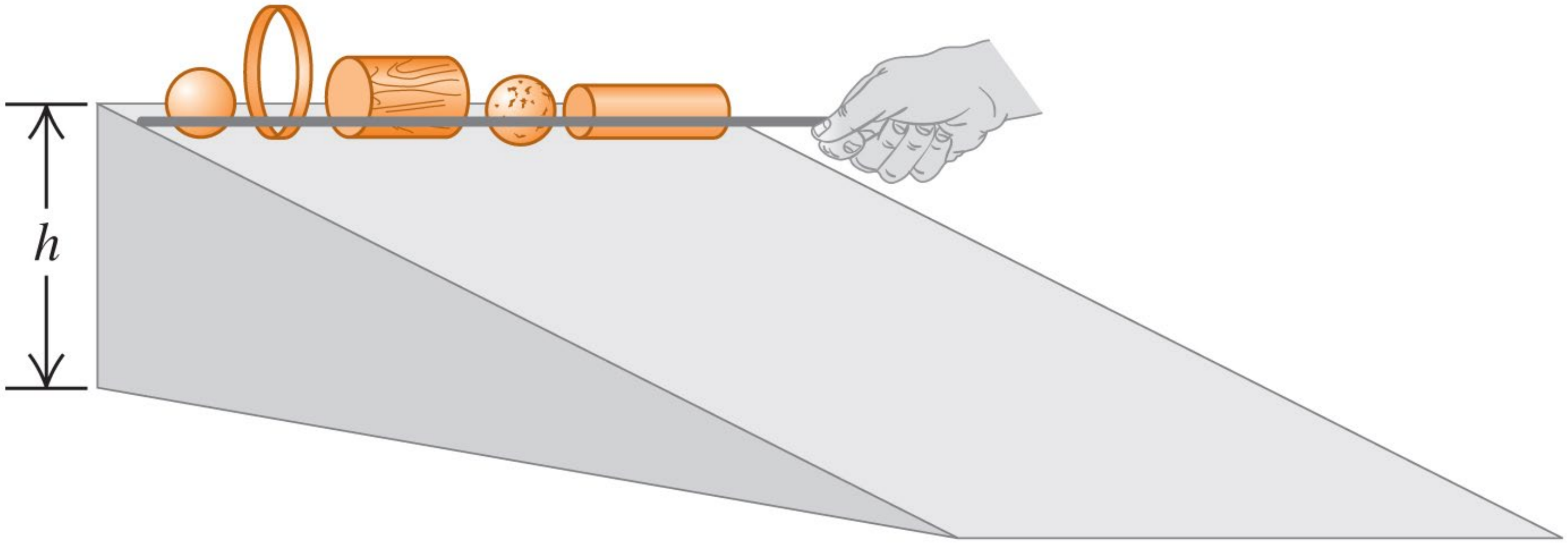
Ren rulning – rulning uden glidning



Ren rulning: Benyt rullebetingelsen (geometrisk bånd):

$$x_{cm} = R\theta, \quad v_{cm} = R\omega, \quad a_{cm} = R\alpha$$

Husk energibevarelse – og komponenterne



$$E_{tot} = K + U = K_{rot} + K_{trans} + U$$

Dynamik for planbevægelse med translation og rotation

$$\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}} \qquad (10.12)$$

$$\sum \tau_z = I_{\text{cm}}\alpha_z \qquad (10.13)$$

Dynamik for planbevægelse med translation og rotation

Newton II for CM
x-translation :

$$\sum F_x = Ma_x$$

Newton II for CM
y-translation :

$$\sum F_y = Ma_y$$

Rotation omkring CM
z-akse (IMS):

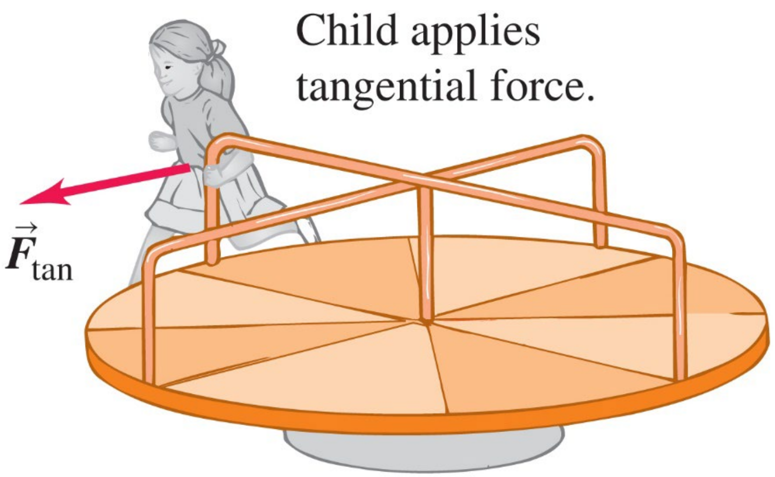
$$\sum \tau_z = I_{cm}\alpha_z$$

Standardmetoden: Udvidet løsningsstrategi

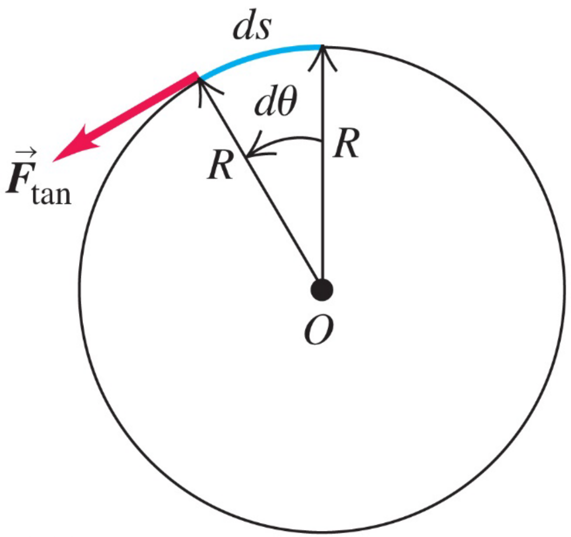
- Tegning, koordinatsystem, omløbsretning
- Kraftdiagram for hvert enkelt legeme, betegnelser
- Newton II-translation for legeme(r) (”massemidtpunktssætningen” (MMS))
- Newton II-rotation for legeme(r) (”Impulsmomentsætningen” (IMS))
- Geometrisk(e) bånd
- Løs de fremkomne ligninger

Arbejde ved rotation

(a)



(b) Overhead view of merry-go-round



Work done by a torque τ_z

$$W = \int_{\theta_1}^{\theta_2} \tau_z \, d\theta \qquad (10.20)$$

Upper limit = final angular position

Integral of the torque with respect to angle

Lower limit = initial angular position

Work done by a constant torque τ_z

$$W = \underbrace{\tau_z}_{\text{Torque}}\underbrace{(\theta_2 - \theta_1)}_{\text{Final minus initial angular position}} = \tau_z \underbrace{\Delta\theta}_{\text{angular displacement}} \qquad (10.21)$$

Final minus initial angular position = angular displacement

Effekt fra et kraftmoment

$$P = \tau_z \omega_z \qquad (10.23)$$

Power due to a torque acting on a rigid body: P

Torque with respect to rigid body's rotation axis: τ_z

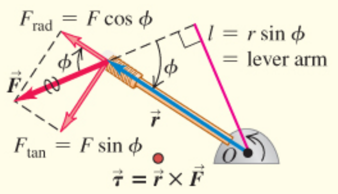
Angular velocity of rigid body about axis: ω_z

Kort opsummering af 10.1-10.4

Torque: When a force $\vec{F}$ acts on an object, the torque of that force with respect to a point O has a magnitude given by the product of the force magnitude F and the lever arm l. More generally, torque is a vector $\vec{\tau}$ equal to the vector product of $\vec{r}$ (the position vector of the point at which the force acts) and $\vec{F}$. (See Example 10.1.)

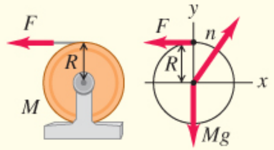
$$\tau = Fl = rF\sin\phi = F_{\tan}r \qquad (10.2)$$

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad (10.3)$$



Rotational dynamics: The rotational analog of Newton's second law says that the net torque acting on an object equals the product of the object's moment of inertia and its angular acceleration. (See Examples 10.2 and 10.3.)

$$\sum\tau_z = I\alpha_z \qquad (10.7)$$



Combined translation and rotation: If a rigid body is both moving through space and rotating, its motion can be regarded as translational motion of the center of mass plus rotational motion about an axis through the center of mass. Thus the kinetic energy is a sum of translational and rotational kinetic energies. For dynamics, Newton's second law describes the motion of the center of mass, and the rotational equivalent of Newton's second law describes rotation about the center of mass. In the case of rolling without slipping, there is a special relationship between the motion of the center of mass and the rotational motion. (See Examples 10.4–10.7.)

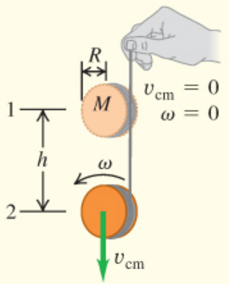
$$K = \tfrac{1}{2}Mv_{cm}^2 + \tfrac{1}{2}I_{cm}\omega^2 \qquad (10.8)$$

$$\sum\vec{F}_{ext} = M\vec{a}_{cm} \qquad (10.12)$$

$$\sum\tau_z = I_{cm}\alpha_z \qquad (10.13)$$

$$v_{cm} = R\omega \qquad (10.11)$$

(rolling without slipping)



Work done by a torque: A torque that acts on a rigid body as it rotates does work on that body. The work can be expressed as an integral of the torque. The work–energy theorem says that the total rotational work done on a rigid body is equal to the change in rotational kinetic energy. The power, or rate at which the torque does work, is the product of the torque and the angular velocity (See Example 10.8.)

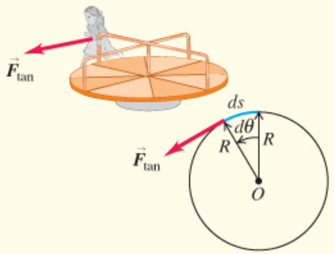
$$W = \int_{\theta_1}^{\theta_2} \tau_z \, d\theta \qquad (10.20)$$

$$W = \tau_z(\theta_2 - \theta_1) = \tau_z\Delta\theta \qquad (10.21)$$

(constant torque only)

$$W_{tot} = \tfrac{1}{2}I\omega_2^2 - \tfrac{1}{2}I\omega_1^2 \qquad (10.22)$$

$$P = \tau_z\omega_z \qquad (10.23)$$



DTU

Kapitel 13
Gravitation

Kursusplan for foråret

01-02-2024	Kap. 9	alec
08-02-2024	Kap. 10	sbko
15-02-2024	Kap. 10+11	sbko
22-02-2024	Kap. 13	sbko
29-02-2024	Temadag	sbko
07-03-2024	Kap. 14	sbko
14-03-2024	Kap. 21	fraca
21-03-2024	Kap. 22	fraca
28-03-2024	Påske	
04-04-2024	Kap. 23	fraca
11-04-2024	Kap. 27	fraca
18-04-2024	Kap. 27+28	fraca
25-04-2024	Kap. 28	fraca
02-05-2024	Opsamling	fraca

Emne for Temadag:
Rotation

Indhold i kapitel 13

LEARNING OUTCOMES

In this chapter, you'll learn...

13.1 How to calculate the gravitational forces that any two objects exert on each other.

13.2 How to relate the weight of an object to the general expression for gravitational force.

13.3 How to use and interpret the generalized expression for gravitational potential energy.

13.4 How to calculate the speed, orbital period, and total mechanical energy of a satellite in a circular orbit.

13.5 How to apply and interpret the three laws that describe the motion of planets.

13.6 Why the gravitational force exerted by a spherically symmetric planet is the same as if all of the planet's mass were concentrated at its center.

13.7 How the earth's rotation affects the apparent weight of an object at different latitudes.

13.8 What black holes are, how to calculate their properties, and how astronomers discover them.

- Opgaverne til regneøvelserne er i lærebogen:
  - 3, 7, 16, 19, 28, 32, 41

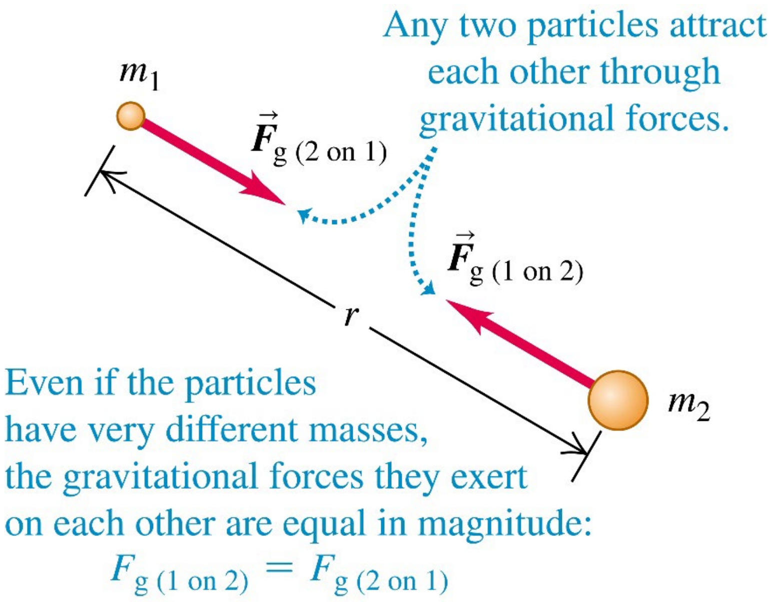
Tyngdekraften

Kraften mellem to objekter med massen m_1 og m_2, og afstanden mellem massemidtpunkterne r er

Newtons gravitationslov $$F_\mathrm{G} = G\frac{m_1 m_2}{r^2}$$

hvor vi eksperimentelt har fundet $G = 6.67 \cdot 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$

Bemærk at begge objekter påvirkes af hinanden



Nogle eksempler

$$F_g = G\frac{m_1 m_2}{r^2}$$

Tyngdekraften imellem Jorden og Solen: $m_1 = 6*10^{24}$ kg; $m_2 = 2*10^{30}$ kg ; $r = 1.5*10^{11}$ m => $\boldsymbol{F_g = 3.6*10^{22}}$ **N**

Tyngdekraften imellem Jorden og Månen: $m_1 = 6*10^{24}$ kg; $m_2 = 7*10^{22}$ kg ; $r = 3.84*10^{8}$ m => $\boldsymbol{F_g = 2.0*10^{20}}$ **N**

Tiltrækning imellem to mennesker: $m_1 = 80$ kg; $m_2 = 70$ kg ; $r = 1$ m => $\boldsymbol{F_g = 3.7*10^{-7}}$ **N**

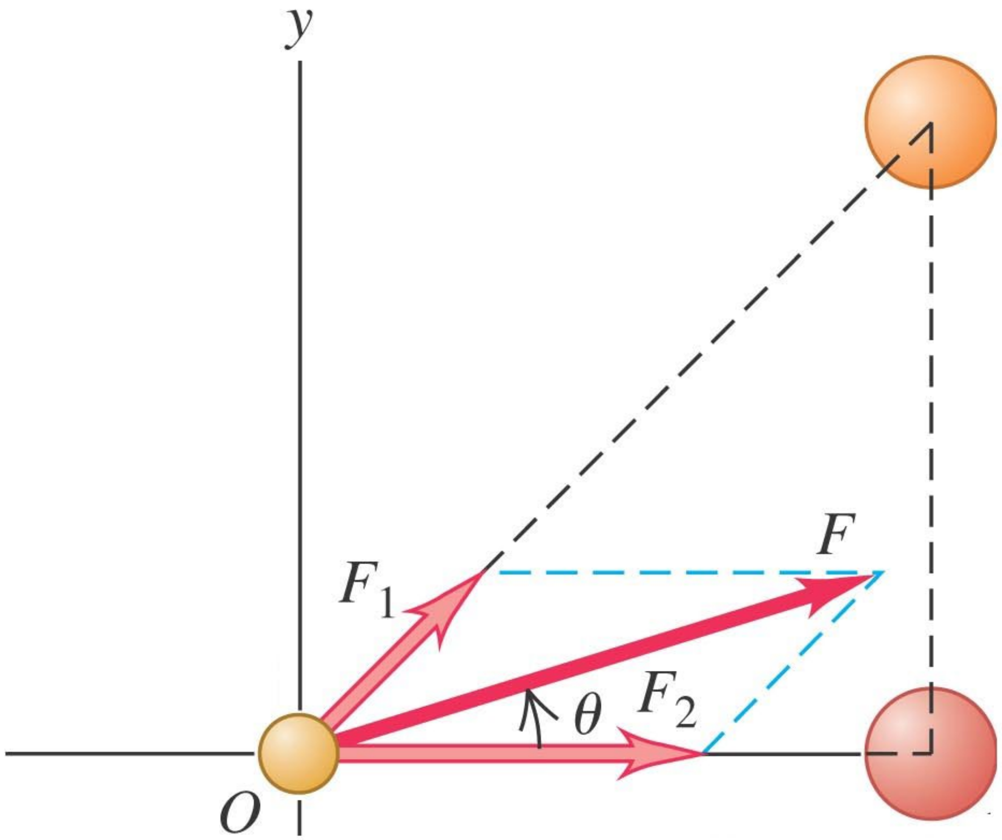
Tyngdekraften imellem to protoner i en atomkerne: $m_1 = m_2 = 1.7*10^{-27}$ kg ; $r = 2*10^{-15}$ m => $\boldsymbol{F_g = 4.8*10^{-35}}$ **N**

Superposition af gravitationelle kræfter

Vi lærte i kapitel 4, at superpositions

Princippet gælder, dvs. at hvis flere kræfter virker er den resulterende kraft vektorsummen af kræfterne

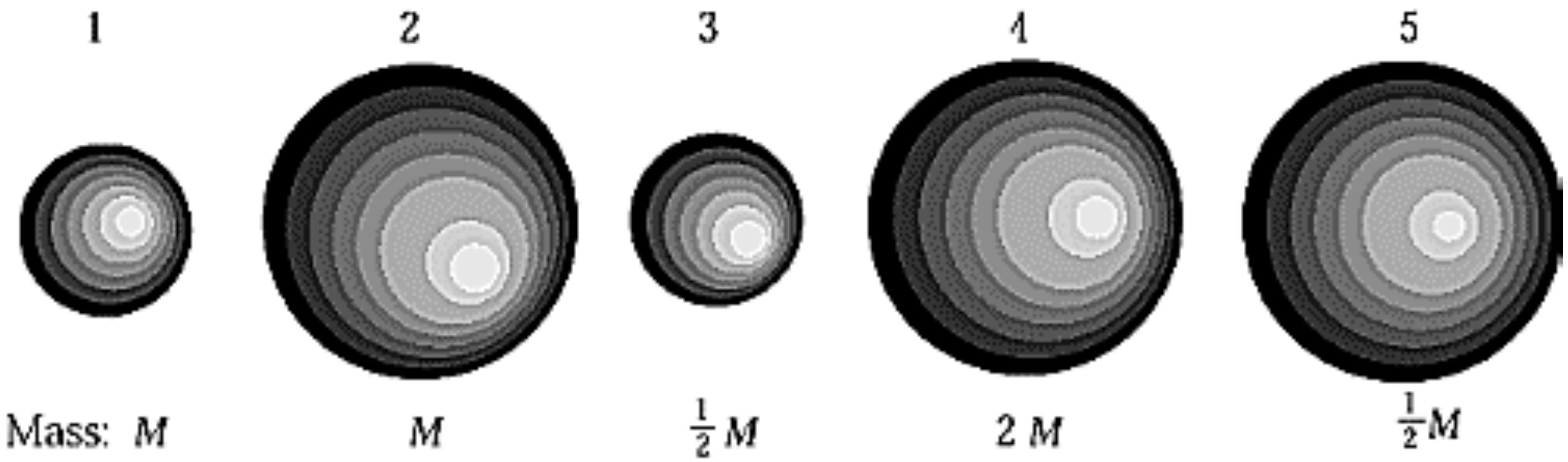
Tyngdekraften på et legeme forårsaget af N andre legemer er altså blot vektorsummen af de enkelte kræfter



F er den resulterende gravitationskraft på planeten *O* fra de to større planeter

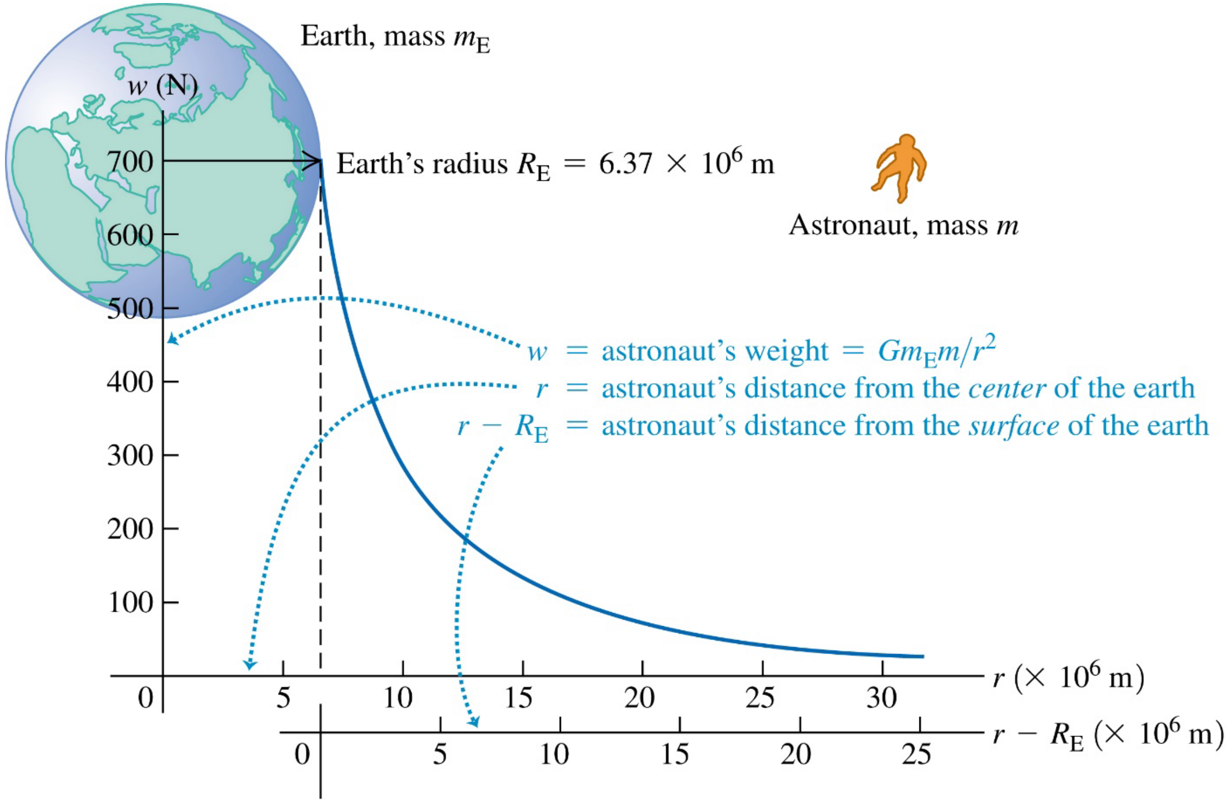
Quiz

Fem homogene planeter har forskellige masser og radiusser, som angivet på figuren. Planeterne 2, 4 og 5 har præcist dobbelt så stor radius som planeterne 1 og 3. På hvilken planet ville du veje mindst?



Vægt

- Vægten er summen af alle dele af universets tyngdekraft på et objekt. På overfladen af Jorden mærker vi i praksis kun tyngdekraften fra Jorden



$$w = F_g = \underbrace{G\frac{M_J}{R_J^2}}_{g} m$$

vægt

masse

Hvad vejer Jorden?

$$M_J = \underbrace{g\frac{R_J^2}{G}}$$

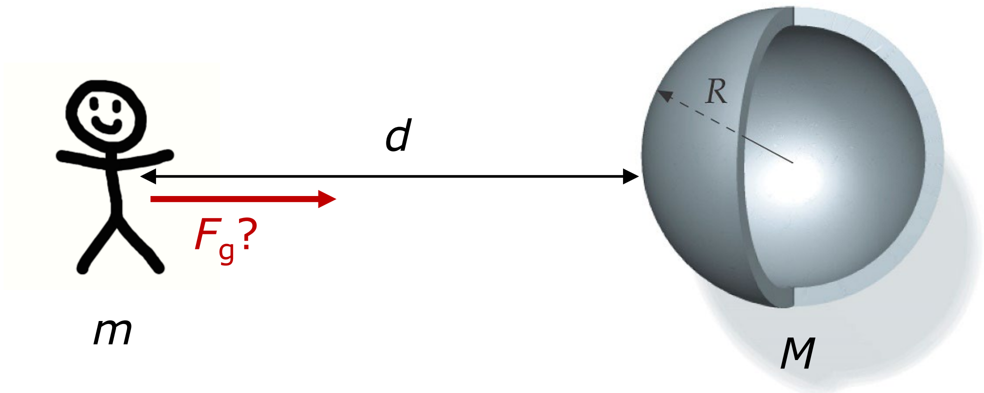
Quiz

- Forestil dig en planet der har den halve masse af Jorden og den halve jordradius. Hvor stor er tyngdeaccelerationen på planeten i forhold til Jordens tyngdeacceleration *g*?

A. Det dobbelte
B. Det samme
C. Det halve
D. En fjerdedel
E. Ingen af disse

$$g = \frac{Gm_J}{R_J^2}$$

Quiz: Gravitationskraft uden for en kugleskal



- Forestil dig en særlig planet, som er udformet som en kugleskal med massen *M* og radius *R*. Du er en afstand *d* fra denne planet. Hvad er gravitationskraften på dig fra planeten?

A. $F_g = \frac{GMm}{R^2}$

B. $F_g = \frac{GMm}{d^2}$

C. $F_g = \frac{GMm}{(d+R)^2}$

Gravitationskraften nær jordoverfladen

Størrelsen af kraften, som virker på et legeme med masse m i frit fald nær Jordens overflade er mg

$$mg = G\frac{M_J m}{R_J^2} \Rightarrow g = \frac{G\,M_J}{R_J^2}$$

Som funktion af højde, h, over Jorden:

$$g = \frac{G\,M_J}{\left(R_J + h\right)^2}$$

TABLE 13.1

Free-Fall Acceleration g at Various Altitudes Above the Earth's Surface

Altitude h (km)	g (m/s²)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13
∞	0

QUIZ!

Hvis du vejer 1000 N på Jorden, hvor meget vejer du så på Mars?

$$\frac{M_{♂}}{M_{\oplus}} \approx 0.1$$

$$\frac{R_{♂}}{R_{\oplus}} \approx 0.5$$

Tilsyneladende vægt i et accelereret koordinatsystem

Grundet centrifugalkraften (en fiktiv kraft), vil din vægt føles mindre. Din tilsyneladende vægt er

$$w_{\text{tilsyneladende}} = w_{\text{ægte}} - ma$$

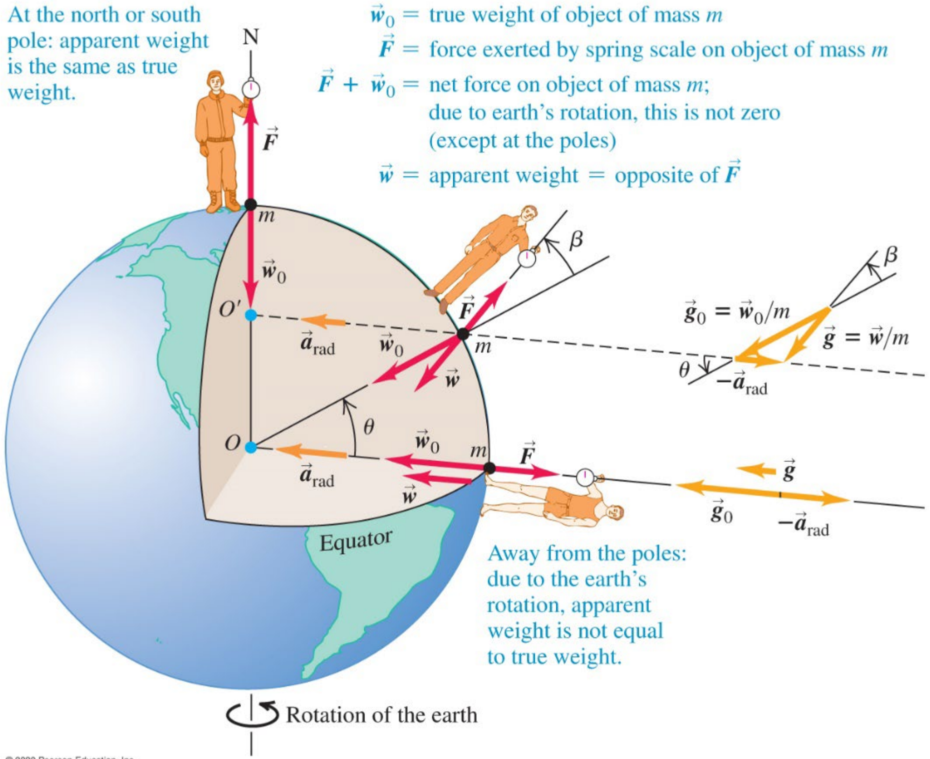
hvor a her er centripetalaccelerationen

$$a = \frac{v^2}{r} = \omega^2 r$$

Så du vejer (tilsyneladende) lidt mindre ved ækvator end her i Danmark

TABLE 13.1 Variations of g with Latitude and Elevation

Station	North Latitude	Elevation (m)	g (m/s^2)
Canal Zone	09°	0	9.78243
Jamaica	18°	0	9.78591
Bermuda	32°	0	9.79806
Denver, CO	40°	1638	9.79609
Pittsburgh, PA	40.5°	235	9.80118
Cambridge, MA	42°	0	9.80398
Greenland	70°	0	9.82534

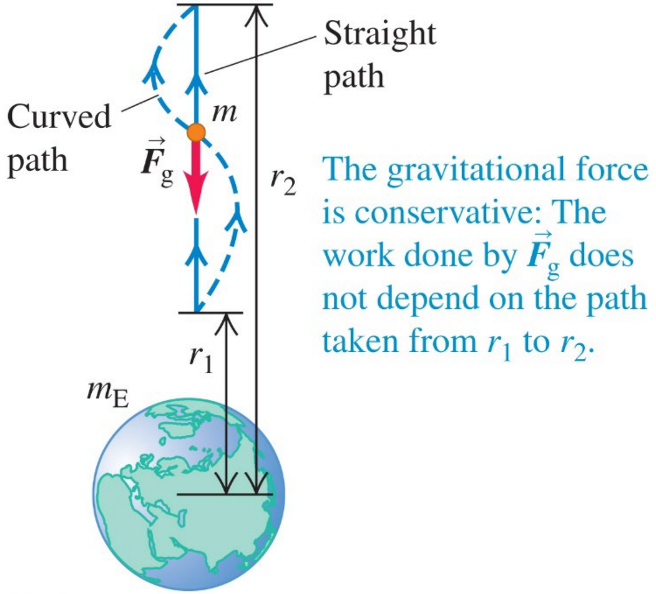


Gravitationel potentiel energi

Ligesom tyngdekraften, er den potentielle energi vi kender $U = mgh$ en tilnærmelse, som virker nær Jordens overflade

Betragt arbejdet fra tyngdekraften når vi flytter m_A fra r_1 til r_2 afstand fra m_B

$$W_G = \int_{r_1}^{r_2} \vec{F}_g \cdot d\vec{r} = -\int_{r_1}^{r_2} G\frac{m_A m_B}{r^2}dr = -Gm_A m_B\left[-\frac{1}{r}\right]_{r_1}^{r_2} = G\frac{m_A m_B}{r_2} - G\frac{m_A m_B}{r_1}$$

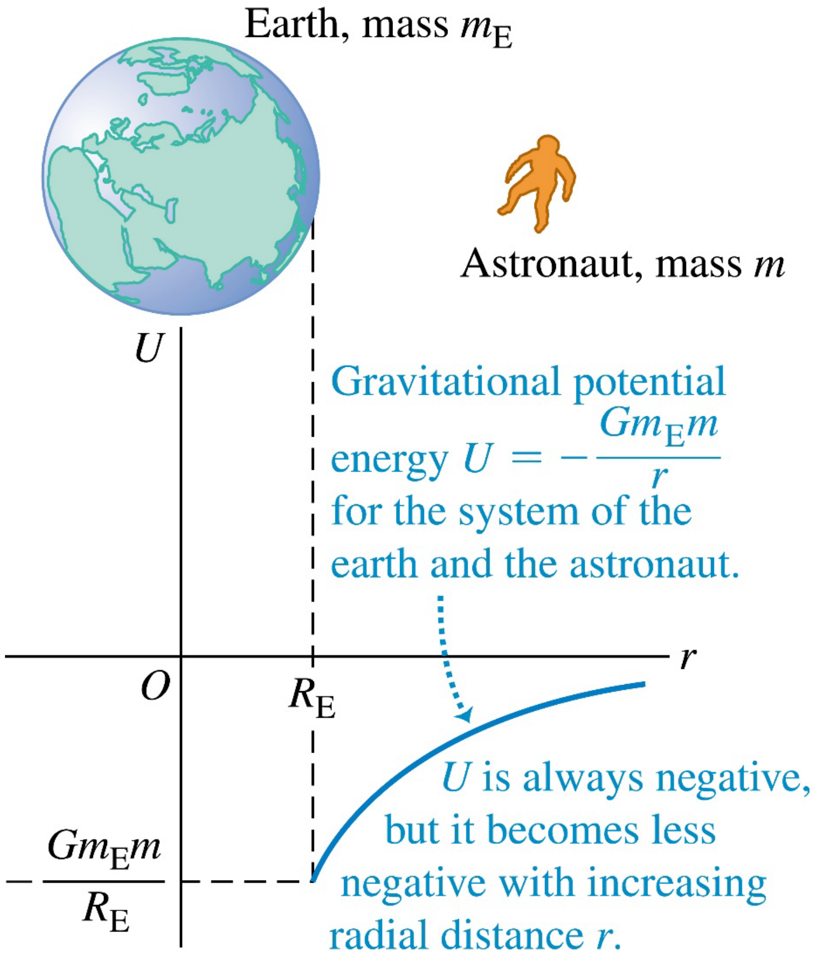


Samme definition som fra (7.3) $W_G = -\Delta U_G$,

og sættes det gravitationelle nulpunkt uendeligt langt væk fra Jorden fås

$$U = -G\frac{M_J m}{r}$$

Gravitationel potentiel energi



Gravitationel potentiel energi – hvad med mgh?

Ligesom tyngdekraften, er den potentielle energi vi kender, $U = mgh$, en tilnærmelse, som virker nær Jordens overflade

$$\Delta U_G = -W_g = G\frac{M_J m}{r_1} - G\frac{M_J m}{r_2} = mM_J G\frac{r_2 - r_1}{r_1 r_2}$$

Skriv nu $r_1 = R_J + y_1$ og $r_2 = R_J + y_2$, hvor $y_1, y_2 \ll R_J$

$$\Delta U_G = mM_J G\frac{y_2 - y_1}{(R_J + y_1)(R_J + y_2)} = m\frac{M_J G}{R_J^2}\frac{y_2 - y_1}{(1 + y_1/R_J)(1 + y_2/R_J)} \approx mg\underbrace{(y_2 - y_1)}_{h}$$

Opsummering

Kraften mellem to objekter med massen m_1 og m_2 er

$$F_{\mathrm{G}} = G\frac{m_1 m_2}{r^2}$$

Den gravitationelle energi af to punkter adskilt af afstanden r (med nulpunkt uendeligt langt væk)

$$U = -G\frac{Mm}{r}$$

Fra Jorden til Månen

EXAMPLE 13.5 "From the earth to the moon"

In Jules Verne's 1865 story with this title, three men went to the moon in a shell fired from a giant cannon sunk in the earth in Florida. (a) Find the minimum muzzle speed needed to shoot a shell straight up to a height above the earth equal to the earth's radius R_E. (b) Find the minimum muzzle speed that would allow a shell to escape from the earth completely (the *escape speed*). Neglect air resistance, the earth's rotation, and the gravitational pull of the moon. The earth's radius and mass are $R_E = 6.37 \times 10^6$ m and $m_E = 5.97 \times 10^{24}$ kg.

Regn: Hvad er undvigelseshastigheden?

- Hvis kanonløbet er 50 meter langt, og accelerationen er konstant. Hvad skal **a** være for at nå undvigelseshastigheden?

Sorte huller

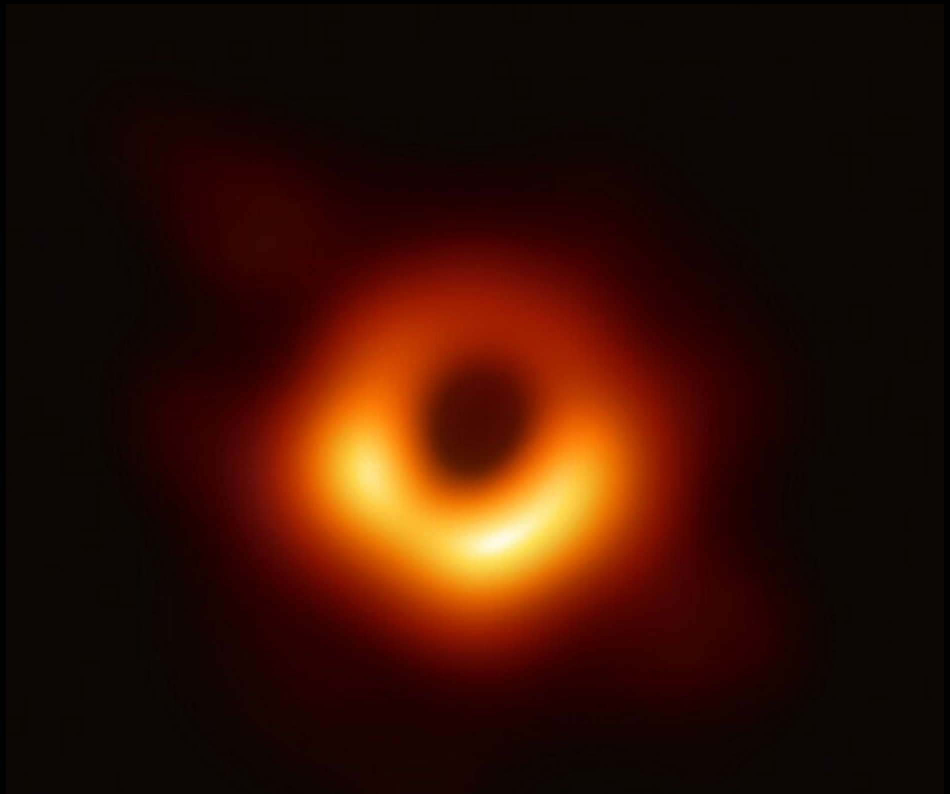
Undvigelseshastigheden er generelt givet ved

$$v = \sqrt{\frac{2GM}{R}}$$

For et sort hul kan lys ikke slippe væk fra massen indenfor en radius (lysets fart i vakuum $v = c$):

$$R_S = \frac{2GM}{c^2}$$

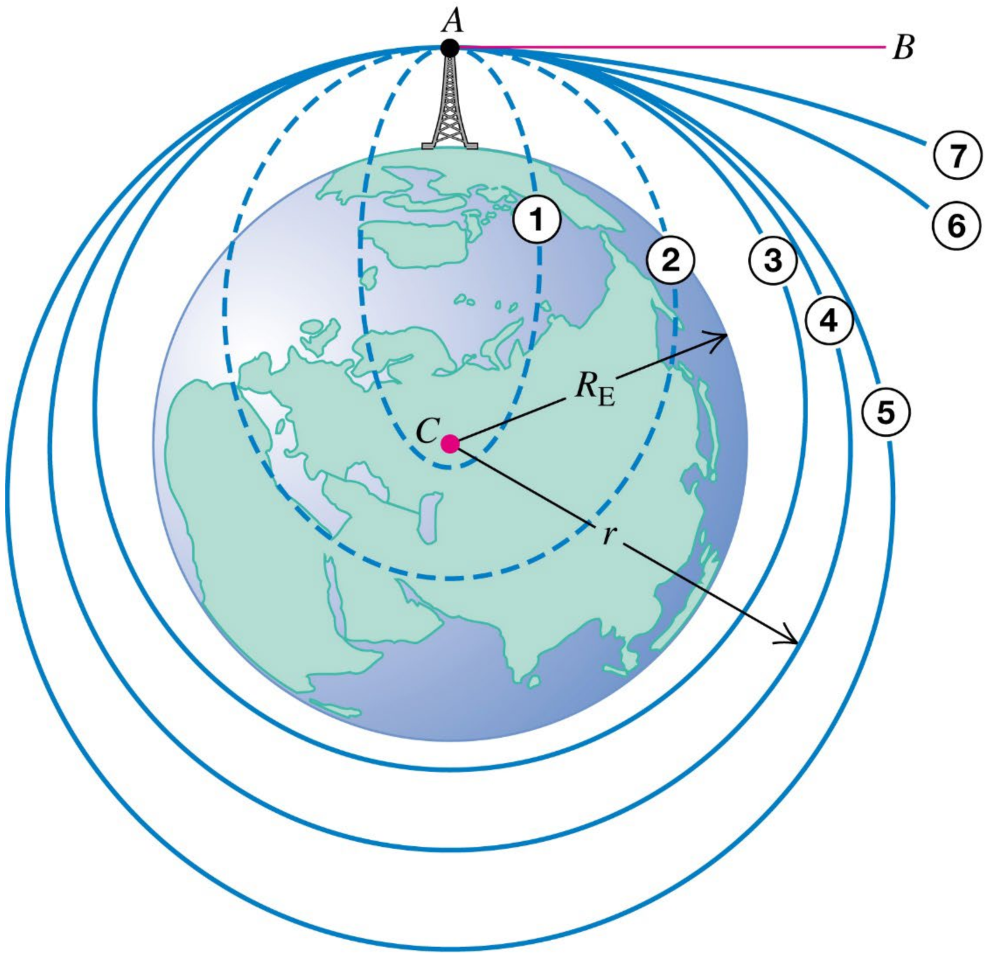
Denne radius kaldes Schwarzschild radius og overfladen for det sorte huls begivenhedshorisont. Indenfor denne slipper intet – ikke engang lys – væk.



Using the Event Horizon Telescope, scientists obtained an image of the black hole at the center of the galaxy M87

2019

Kredsløb (om Jorden)



A projectile is launched from *A* toward *B*. Trajectories 1 through 7 show the effect of increasing initial speed.

Cirkulært kredsløb

I cirkulært kredsløb gælder, at tyngdekraften er lig centripetalkraften

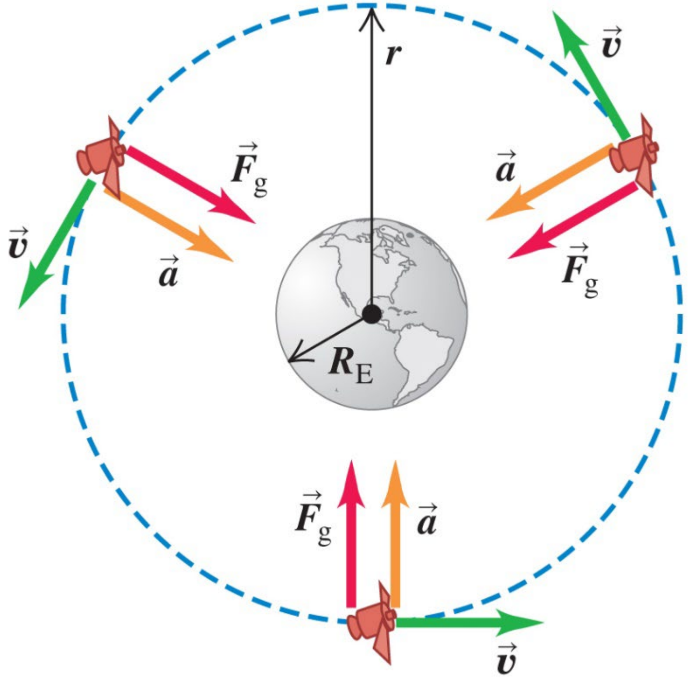
$$G\frac{M_J}{r^2}m = \frac{mv^2}{r}$$

Hvilket giver en fart for cirkulært kredsløb i en radius fra Jordens centrum r

$$v = \sqrt{G\frac{M_J}{r}}$$

Og omløbstid

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\left(GM_J\right)^{1/2}}$$

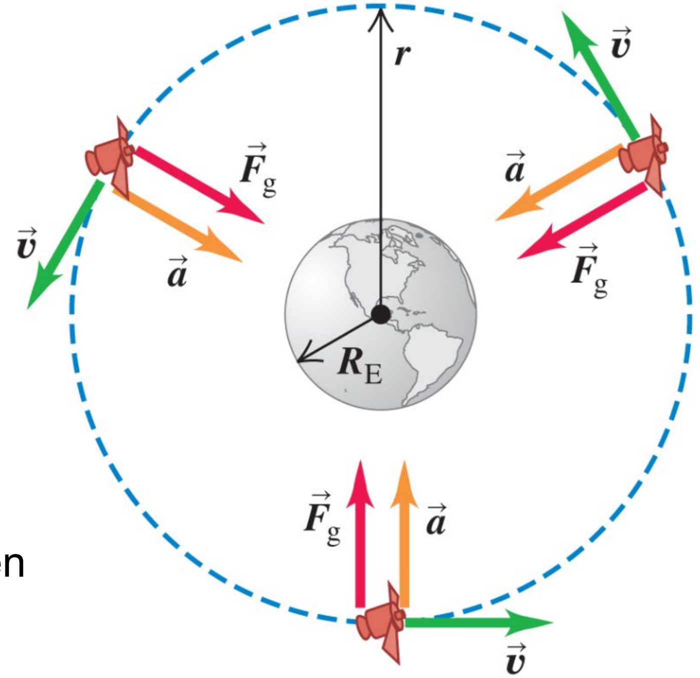


The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

Astronauter i en rumstation

$$v = \sqrt{G\frac{M_J}{r}}$$

Uafhængigt af massen



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

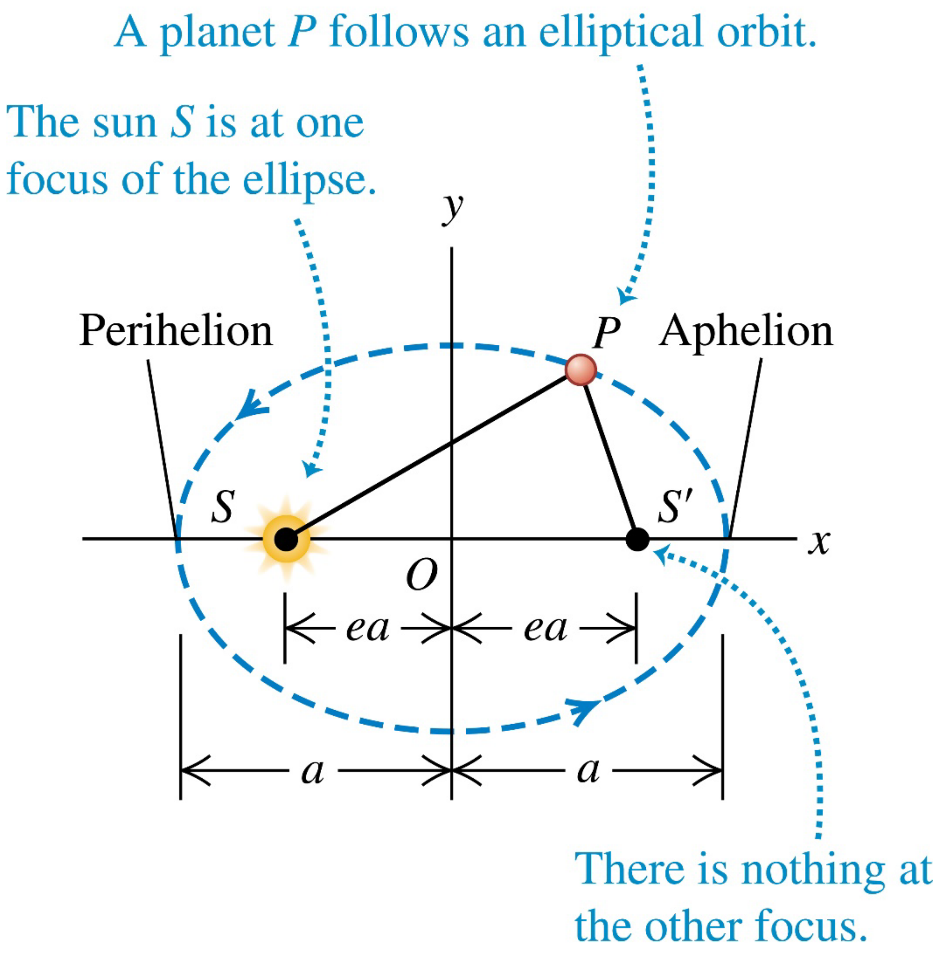
Keplers 3 love

Keplers 3 love:

1. **Alle planeter bevæger sig i elliptiske baner, med Solen som det ene fokuspunkt**

2. En linje fra Solen til planeten spænder over det samme areal over samme tid

3. Planeternes omløbsperiode er proportional med storakselængden i 3/2 potens



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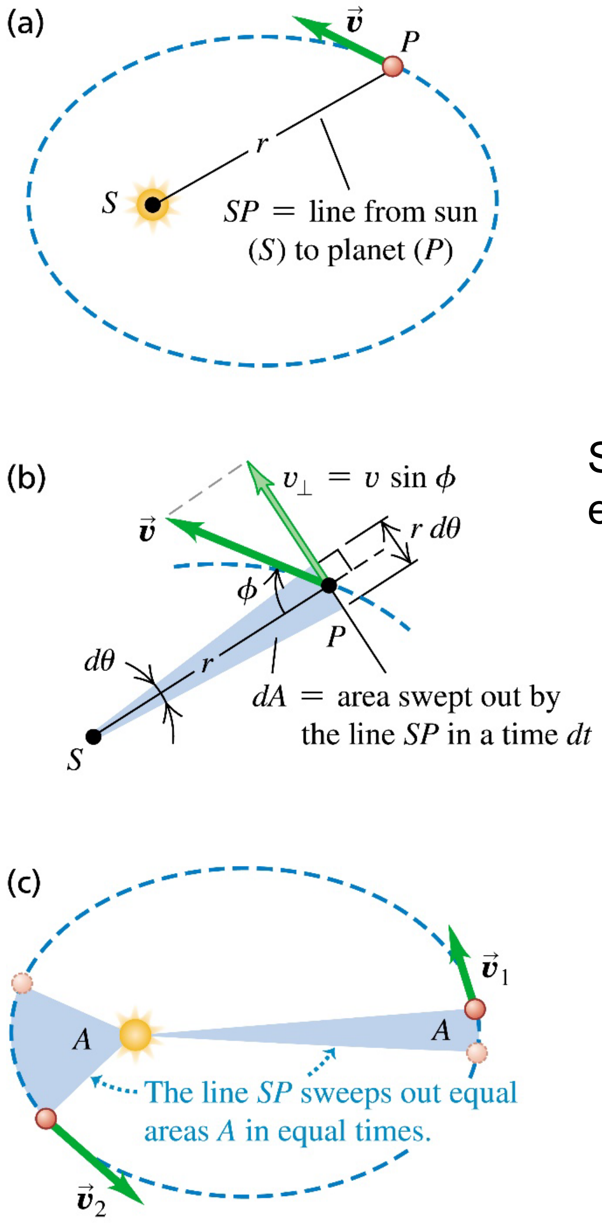


Sektorhastigheden $\frac{dA}{dt}$ er konstant:

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

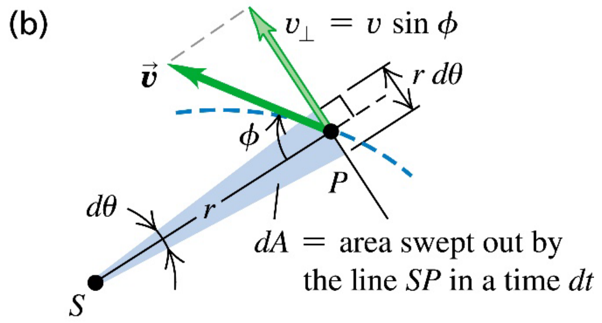
Keplers 3 love

Keplers 3 love:

1. Alle planeter bevæger sig i elliptiske baner, med Solen som det ene fokuspunkt

2. **En linje fra Solen til planeten spænder over det samme areal over samme tid**

3. Planeternes omløbsperiode er proportional med storakselængden i 3/2 potens



$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}rv_\perp = \frac{1}{2}rv\sin\theta$$
$$= \frac{1}{2m}|\vec{r} \times m\vec{v}| = \frac{L}{2m}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = 0$$

Keplers 3 love

Keplers 3 love:

1. Alle planeter bevæger sig i elliptiske baner, med Solen som det ene fokuspunkt

2. En linje fra Solen til planeten spænder over det samme areal over samme tid

3. **Planeternes omløbsperiode er proportional med storakselængden i 3/2 potens**

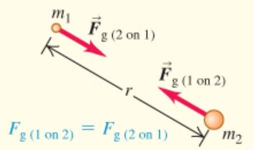
$$T = \frac{2\pi a^{3/2}}{(GM_{Sol})^{1/2}}$$

Kort opsummering af Kapitel 13

CHAPTER 13 SUMMARY

Newton's law of gravitation: *Any* two particles with masses m_1 and m_2, a distance r apart, attract each other with forces inversely proportional to r^2. These forces form an action–reaction pair and obey Newton's third law. When two or more objects exert gravitational forces on a particular object, the total gravitational force on that individual object is the vector sum of the forces exerted by the other objects. The gravitational interaction between spherical mass distributions, such as planets or stars, is the same as if all the mass of each distribution were concentrated at the center. (See Examples 13.1–13.3 and 13.10.)

$$F_g = \frac{Gm_1m_2}{r^2} \quad (13.1)$$



Gravitational force, weight, and gravitational potential energy: The weight w of an object is the total gravitational force exerted on it by all other objects in the universe. Near the surface of the earth (mass m_E and radius R_E), the weight is essentially equal to the gravitational force of the earth alone. The gravitational potential energy U of two masses m and m_E separated by a distance r is inversely proportional to r. The potential energy is never positive; it is zero only when the two objects are infinitely far apart. (See Examples 13.4 and 13.5.)

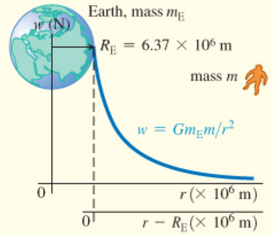
$$w = F_g = \frac{Gm_Em}{R_E^2} \quad (13.3)$$

(weight at earth's surface)

$$g = \frac{Gm_E}{R_E^2} \quad (13.4)$$

(acceleration due to gravity at earth's surface)

$$U = -\frac{Gm_Em}{r} \quad (13.9)$$



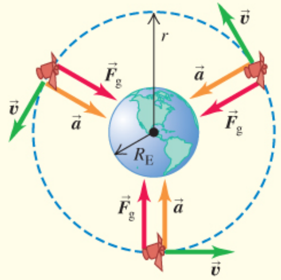
Orbits: When a satellite moves in a circular orbit, the centripetal acceleration is provided by the gravitational attraction of the earth. Kepler's three laws describe the more general case: an elliptical orbit of a planet around the sun or a satellite around a planet. (See Examples 13.6–13.9.)

$$v = \sqrt{\frac{Gm_E}{r}} \quad (13.10)$$

(speed in circular orbit)

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$

(period in circular orbit) (13.12)



Black holes: If a nonrotating spherical mass distribution with total mass M has a radius less than its Schwarzschild radius R_S, it is called a black hole. The gravitational interaction prevents anything, including light, from escaping from within a sphere with radius R_S. (See Example 13.11.)

$$R_S = \frac{2GM}{c^2} \quad (13.30)$$

(Schwarzschild radius)

If all of the object is inside its Schwarzschild radius $R_S = 2GM/c^2$, the object is a black hole.

Så skal der regnes

- Opgaverne 3, 7, 16, 19, 28, 32, 41 i lærebogen (pp. 449-452)

PROBLEM-SOLVING STRATEGY 1.1 Solving Physics Problems

IDENTIFY *the relevant concepts:*

- Use the physical conditions stated in the problem to help you decide which physics concepts are relevant.
- Identify the **target variables** of the problem—that is, the quantities whose values you're trying to find, such as the speed at which a projectile hits the ground, the intensity of a sound made by a siren, or the size of an image made by a lens.
- Identify the known quantities, as stated or implied in the problem. This step is essential whether the problem asks for an algebraic expression or a numerical answer.

SET UP *the problem*:

- Given the concepts, known quantities, and target variables that you found in the IDENTIFY step, choose the equations that you'll use to solve the problem and decide how you'll use them. Study the worked examples in this book for tips on how to select the proper equations. If this seems challenging, don't worry—you'll get better with practice!
- Make sure that the variables you have identified correlate exactly with those in the equations.
- If appropriate, draw a sketch of the situation described in the problem. (Graph paper and a ruler will help you make clear, useful sketches.)

EXECUTE *the solution*:

- Here's where you'll "do the math" with the equations that you selected in the SET UP step to solve for the target variables that you found in the IDENTIFY step. Study the worked examples to see what's involved in this step.

EVALUATE *your answer:*

- Check your answer from the SOLVE step to see if it's reasonable. (If you're calculating how high a thrown baseball goes, an answer of 1.0 mm is unreasonably small and an answer of 100 km is unreasonably large.) If your answer includes an algebraic expression, confirm that it correctly represents what would happen if the variables in it had very large or very small values.
- For future reference, make note of any answer that represents a quantity of particular significance. Ask yourself how you might answer a more general or more difficult version of the problem you have just solved.

DTU

10022 Fysik 1 | forelæsning 1

Stive legemers rotation

Velkommen tilbage til Fysik 1!

- Lokaler

  Forelæsninger er i B306 Aud 34. Torsdag 13.00 – 15.00

  Grupperegning er i holdområderne 96, 97, 98, 99, 108a og 108b i Bygning 306.
  Torsdag 15.00 – 17.00

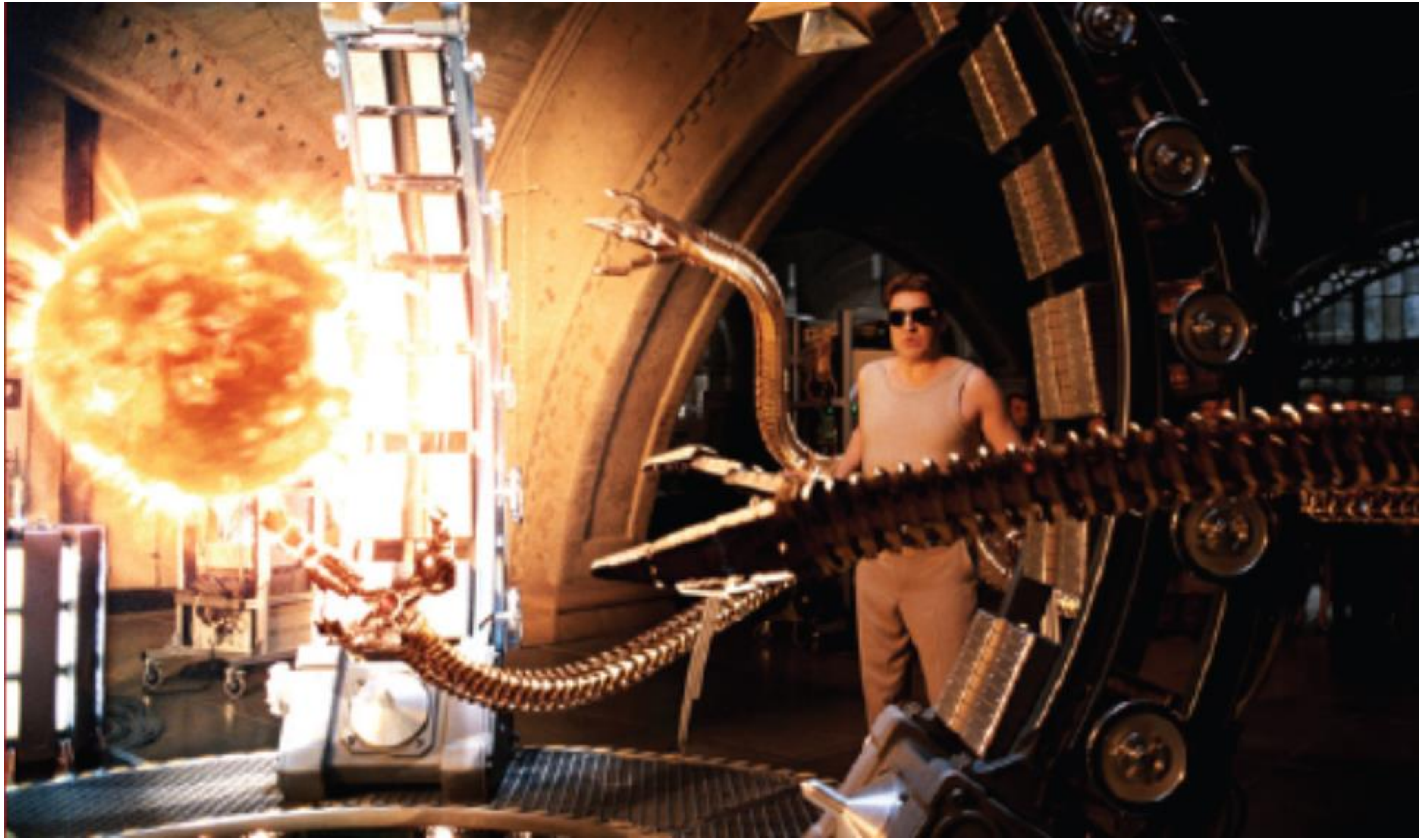
- Nye forelæsere

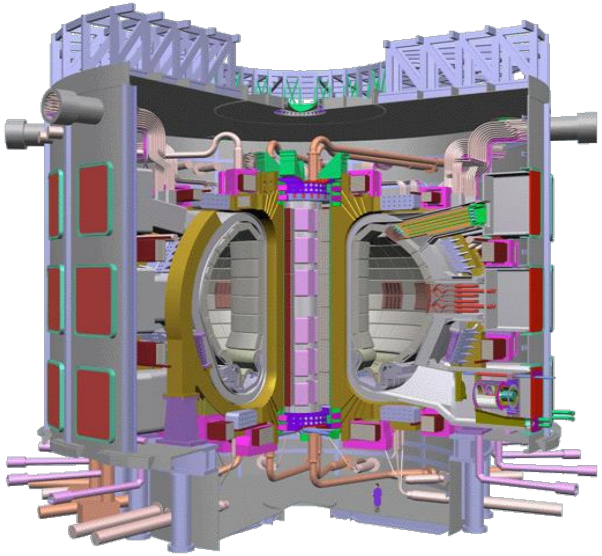
Søren + Cathrine

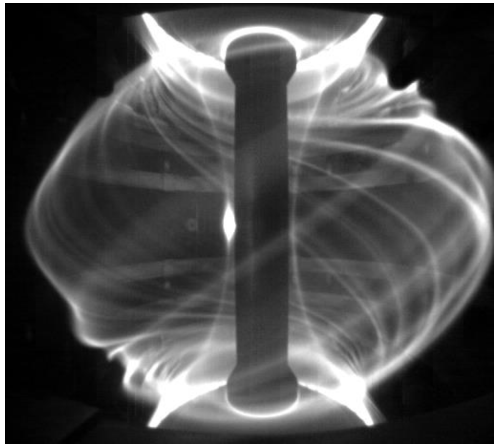
- Nye emner!

Fusionsenergi (mit (og Sørens) forskningsfelt)

= sej fysik der kan være med til at løse energiforsyningskrisen







Fysik 1

Content

Motion in 1+2+3 dimensions. Newton's laws. Work and kinetic energy. Potential energy and energy conservation. Momentum and collisions. Rotation of rigid bodies. Dynamics of rotational motion. Equilibrium and elasticity. Gravitation. Periodic motion. Fluid mechanics. Electric charge and electric field. Magnetism.

klassisk mekanik

01-02-2024	Kap. 9	alec
08-02-2024	Kap. 10	sbko
15-02-2024	Kap. 10+11	sbko
22-02-2024	Kap. 13	sbko
29-02-2024	Temadag	sbko
07-03-2024	Kap. 14	sbko

14-03-2024	Kap. 21	fraca
21-03-2024	Kap. 22	fraca
28-03-2024	Påske	
04-04-2024	Kap. 23	fraca
11-04-2024	Kap. 27	fraca
18-04-2024	Kap. 27+28	fraca
25-04-2024	Kap. 28	fraca
02-05-2024	Opsamling	fraca

elektromagnetisme

QUIZ! Når tårnet vælter, hvornår begynder det at gå i stykker?

I dag: Kap. 9 – Stive legemers rotation

Rotationel kinematik

LEARNING OUTCOMES

In this chapter, you'll learn...

9.1 How to describe the rotation of a rigid body in terms of angular coordinate, angular velocity, and angular acceleration.

9.2 How to analyze rigid-body rotation when the angular acceleration is constant.

9.3 How to relate the rotation of a rigid body to the linear velocity and linear acceleration of a point on the body.

Inertimoment

9.4 The meaning of a body's moment of inertia about a rotation axis, and how it relates to rotational kinetic energy.

9.5 How to relate the values of a body's moment of inertia for two different but parallel rotation axes.

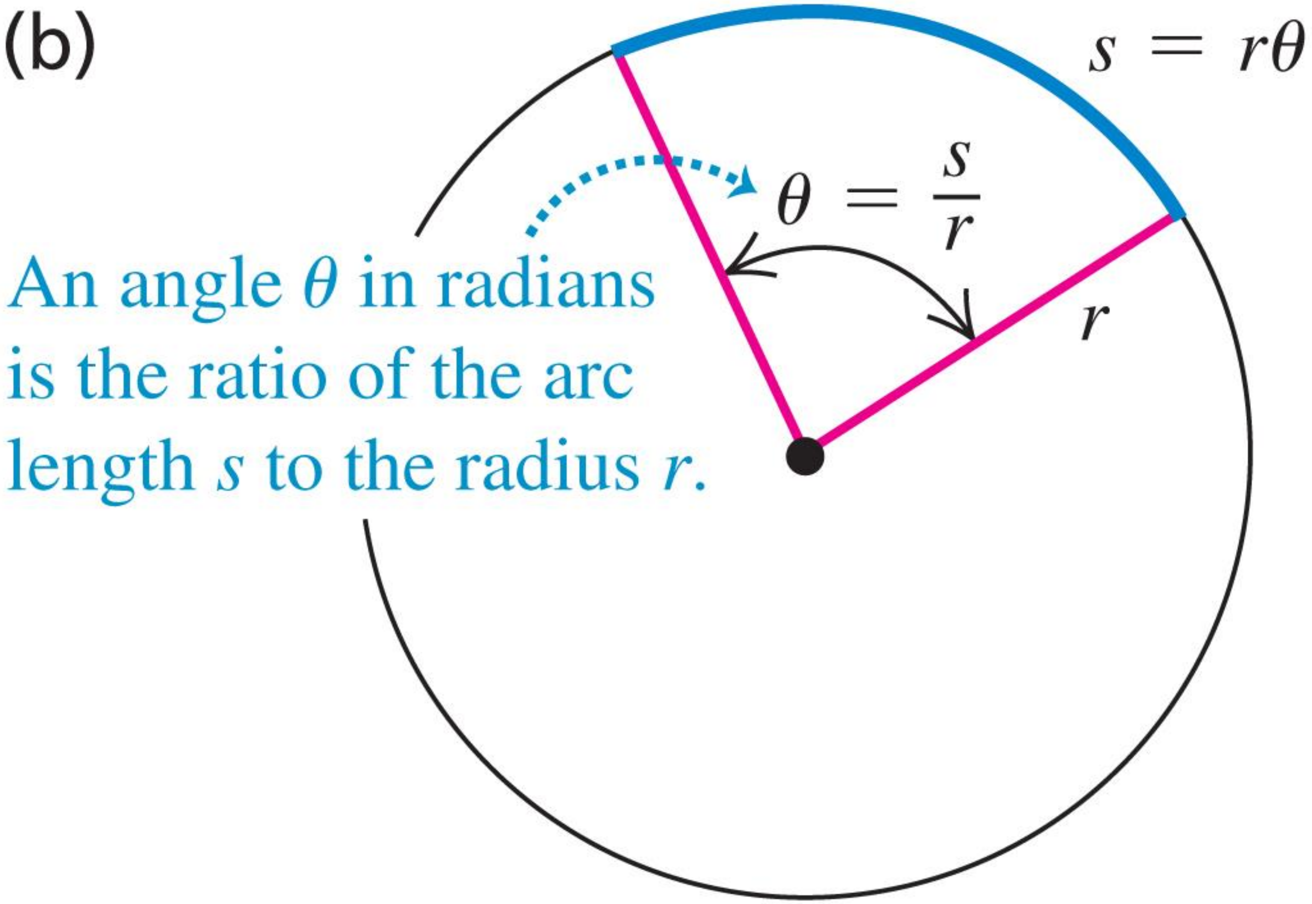
9.6 How to calculate the moment of inertia of bodies with various shapes.

Rotationel kinematik

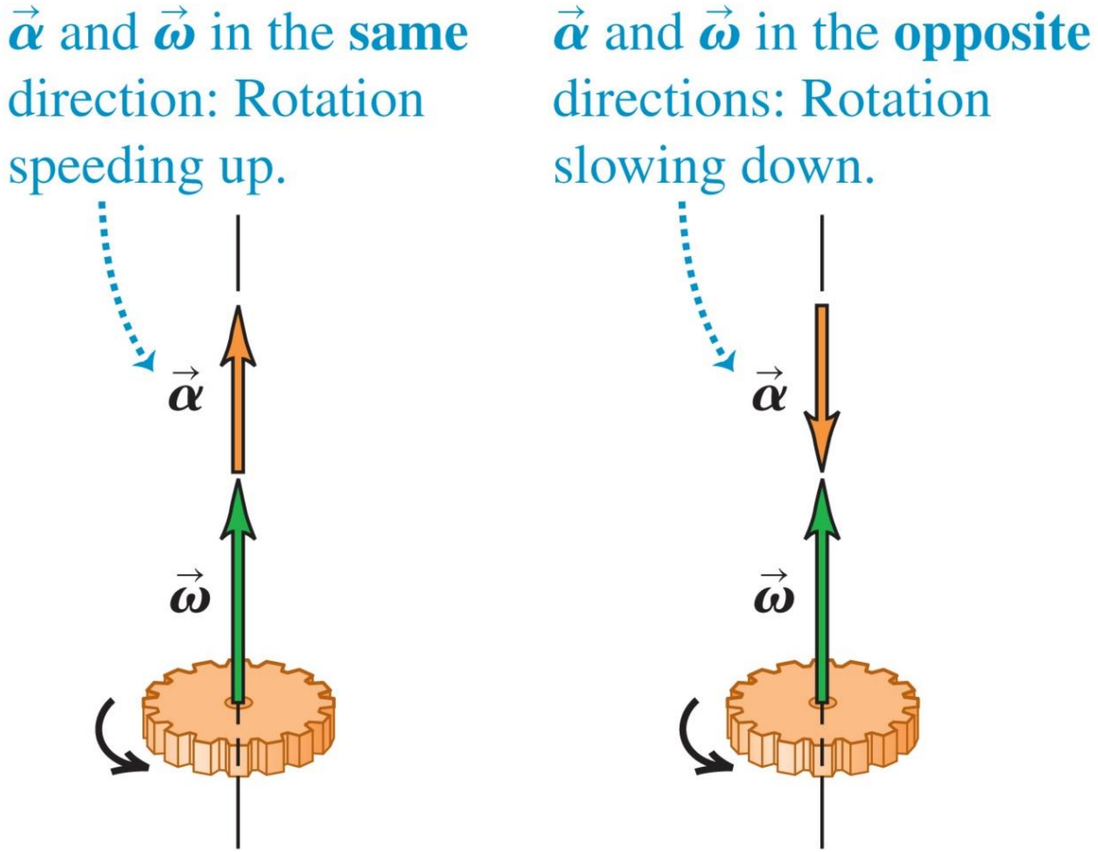
Rotationelle variable

Linear kinematik (Efterår 2023)		Rotationel kinematik (Forår 2024)	
Positionskoordinat	$x\ (y, z, \dots)$	θ	Vinkelkoordinat
Hastighed	$v_x = \frac{dx}{dt}$	$\omega_z = \frac{d\theta}{dt}$	Vinkelhastighed
Acceleration	$a_x = \frac{dv_x}{dt}$	$\alpha_z = \frac{d\omega_z}{dt}$	Vinkelacceleration

Rotationelle variable



Rotationelle variable



<u>Rotationel</u> kinematik (Forår 2024)

θ	Vinkelkoordinat
$\omega_z = \dfrac{d\theta}{dt}$	Vinkelhastighed
$\alpha_z = \dfrac{d\omega_z}{dt}$	Vinkelacceleration

Rotation med konstant vinkelacceleration

TABLE 9.1 Comparison of Linear and Angular Motions with $\boxed{\text{Constant Acceleration}}$

Straight-Line Motion with Constant Linear Acceleration		Fixed-Axis Rotation with Constant Angular Acceleration
$a_x = \text{constant}$		
$v_x = v_{0x} + a_x t$	(2.8)	
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	(2.12)	
$v_x^2 = v_{0x}{}^2 + 2a_x(x - x_0)$	(2.13)	
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$	(2.14)	

QUIZ!

Opsummering: Rotationel kinematik

Rotational kinematics: When a rigid body rotates about a stationary axis (usually called the z-axis), the body's position is described by an angular coordinate θ. The angular velocity ω_z is the time derivative of θ, and the angular acceleration α_z is the time derivative of ω_z or the second derivative of θ. (See Examples 9.1 and 9.2.) If the angular acceleration is constant, then θ, ω_z, and α_z are related by simple kinematic equations analogous to those for straight-line motion with constant linear acceleration. (See Example 9.3.)

$$\omega_z = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad (9.3)$$

$$\alpha_z = \lim_{\Delta t \to 0} \frac{\Delta\omega_z}{\Delta t} = \frac{d\omega_z}{dt} \quad (9.5)$$

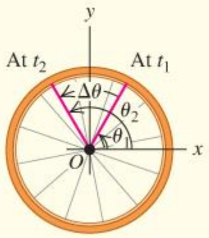
Constant α_z only:

$$\theta = \theta_0 + \omega_{0z}t + \tfrac{1}{2}\alpha_z t^2 \quad (9.11)$$

$$\theta - \theta_0 = \tfrac{1}{2}(\omega_{0z} + \omega_z)t \quad (9.10)$$

$$\omega_z = \omega_{0z} + \alpha_z t \quad (9.7)$$

$$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0) \quad (9.12)$$



I dag: Kap. 9 – Stive legemers rotation

LEARNING OUTCOMES

In this chapter, you'll learn...

Rotationel kinematik

- **9.1** How to describe the rotation of a rigid body in terms of angular coordinate, angular velocity, and angular acceleration.
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- **9.3** How to relate the rotation of a rigid body to the linear velocity and linear acceleration of a point on the body.

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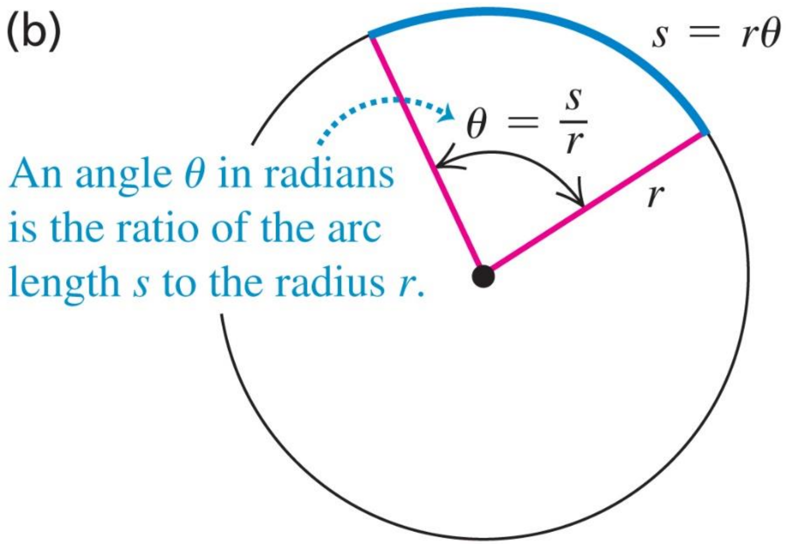
- **9.4** The meaning of a body's moment of inertia about a rotation axis, and how it relates to rotational kinetic energy.
- **9.5** How to relate the values of a body's moment of inertia for two different but parallel rotation axes.
- **9.6** How to calculate the moment of inertia of bodies with various shapes.

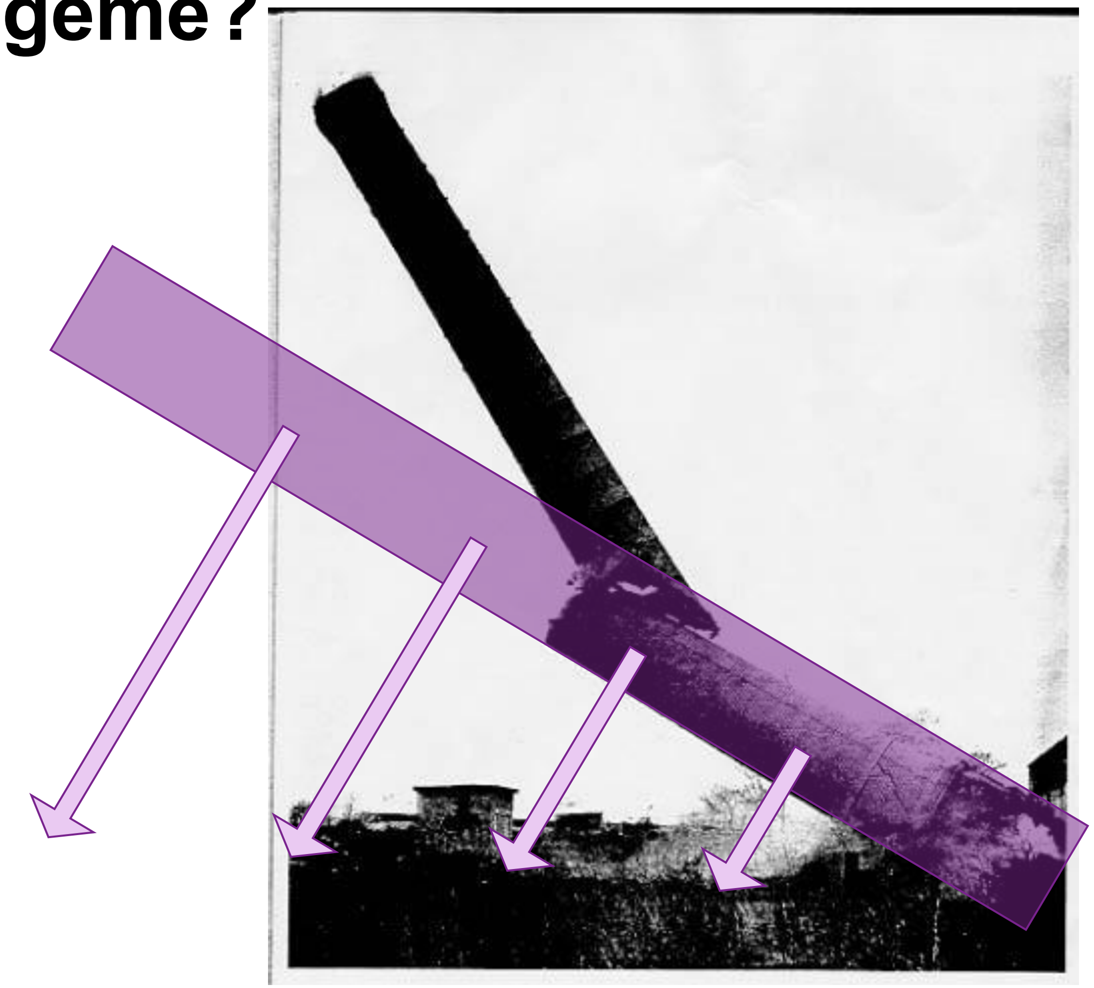
Rotationelle variable



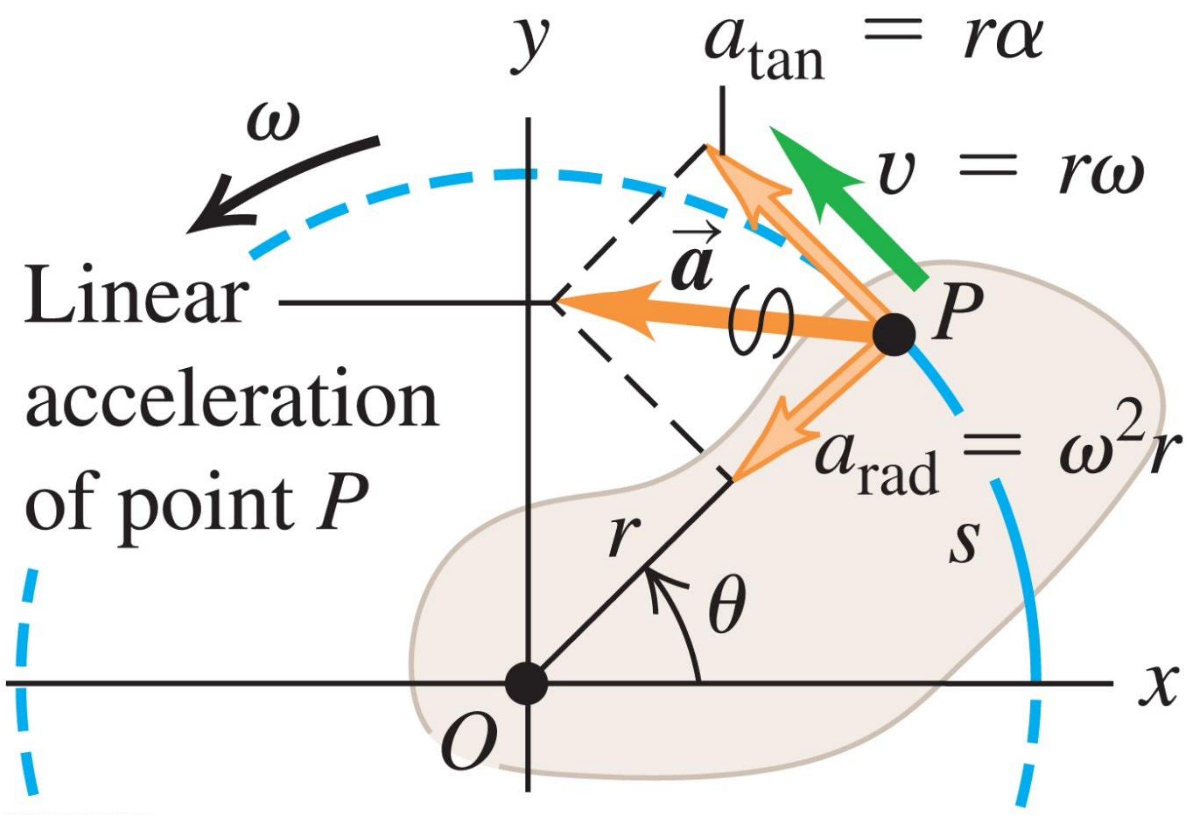
$$\left|\frac{\mathrm{d}s}{\mathrm{d}t}\right| = \left|r\frac{\mathrm{d}\theta}{\mathrm{d}t}\right| = r\left|\frac{\mathrm{d}\theta}{\mathrm{d}t}\right| \quad \rightarrow \quad v = r\omega$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = r\frac{\mathrm{d}\omega}{\mathrm{d}t} \quad \rightarrow \quad a_{\tan} = r\alpha$$

QUIZ! Hvordan varierer den tangentielle acceleration langs tårnet, når dette roterer som et stift legeme?



Sammenhæng mellem lineær og rotationel kinematik



QUIZ! Hvad gælder for et roterende stift legeme?

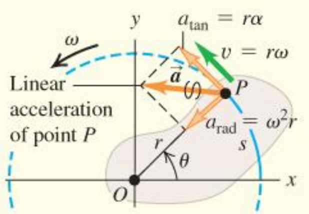
Opsummering: Sammenhæng mellem lineær og rotationel kinematik

Relating linear and angular kinematics: The angular speed ω of a rigid body is the magnitude of the body's angular velocity. The rate of change of ω is $\alpha = d\omega/dt$. For a particle in the body a distance r from the rotation axis, the speed v and the components of the acceleration $\vec{a}$ are related to ω and α. (See Examples 9.4 and 9.5.)

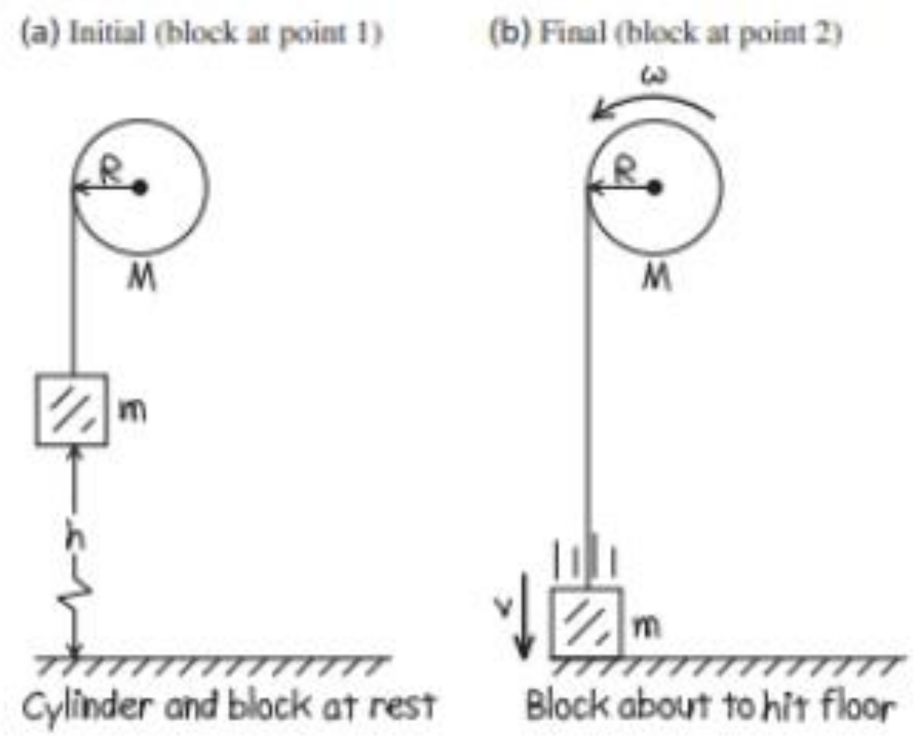
$$v = r\omega \qquad (9.13)$$

$$a_{\text{tan}} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha \qquad (9.14)$$

$$a_{\text{rad}} = \frac{v^2}{r} = \omega^2 r \qquad (9.15)$$



Anvendelse



Hvad er v når klodsen rammer gulvet?

I dag: Kap. 9 – Stive legemers rotation

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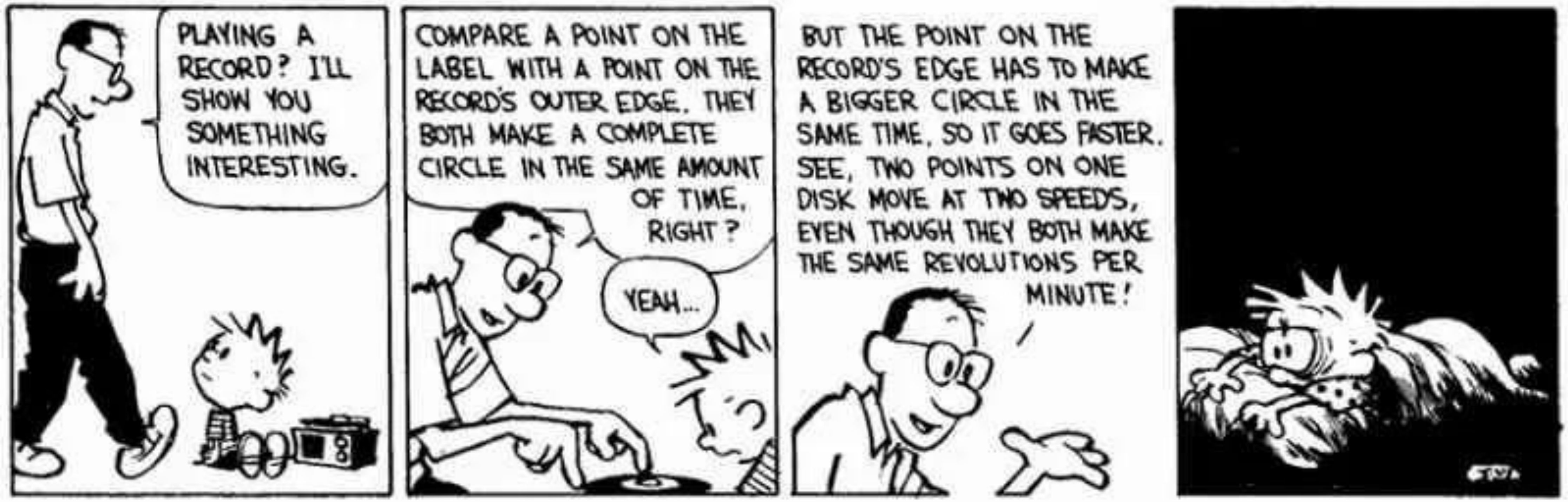
PAUSE!



PLAYING A RECORD? I'LL SHOW YOU SOMETHING INTERESTING.
COMPARE A POINT ON THE LABEL WITH A POINT ON THE RECORD'S OUTER EDGE. THEY BOTH MAKE A COMPLETE CIRCLE IN THE SAME AMOUNT OF TIME, RIGHT?
YEAH...
BUT THE POINT ON THE RECORD'S EDGE HAS TO MAKE A BIGGER CIRCLE IN THE SAME TIME, SO IT GOES FASTER. SEE, TWO POINTS ON ONE DISK MOVE AT TWO SPEEDS, EVEN THOUGH THEY BOTH MAKE THE SAME REVOLUTIONS PER MINUTE!

Inertimoment

I dag: Kap. 9 – Stive legemers rotation

Rotationel kinematik

Inertimoment

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Rotationel kinetisk energi

Alle små dele af et roterende legeme, med massen m_i og hastigheden v_i for den i'ende del, har den kinetiske energi $K_i = \frac{1}{2} m_i v_i^2$.

Den totale kinetiske energi er da

$$K = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (r_i \omega)^2 = \frac{1}{2} \sum_i m_i r_i^2 \, \omega^2 = \frac{1}{2} \left(\sum_i m_i r_i^2 \right) \omega^2 = \frac{1}{2} I \omega^2$$

hvor I kaldes inertimomentet

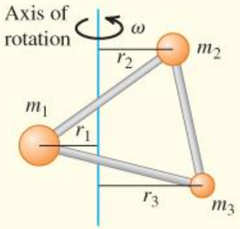
QUIZ

I dag: Kap. 9 – Stive legemers rotation

Moment of inertia and rotational kinetic energy: The moment of inertia I of a body about a given axis is a measure of its rotational inertia: The greater the value of I, the more difficult it is to change the state of the body rotation. The moment of inertia can be expressed as a sum over the particles m_i that make up the body, each of which is at its own perpendicular distance r_i from the axis. The rotational kinetic energy of a rigid body rotating about a fixed axis depends on the angular speed ω and the moment of inertia I for that rotation axis. (See Examples 9.6–9.8.)

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots = \sum_i m_i r_i^2 \quad (9.16)$$

$$K = \tfrac{1}{2} I \omega^2 \quad (9.17)$$



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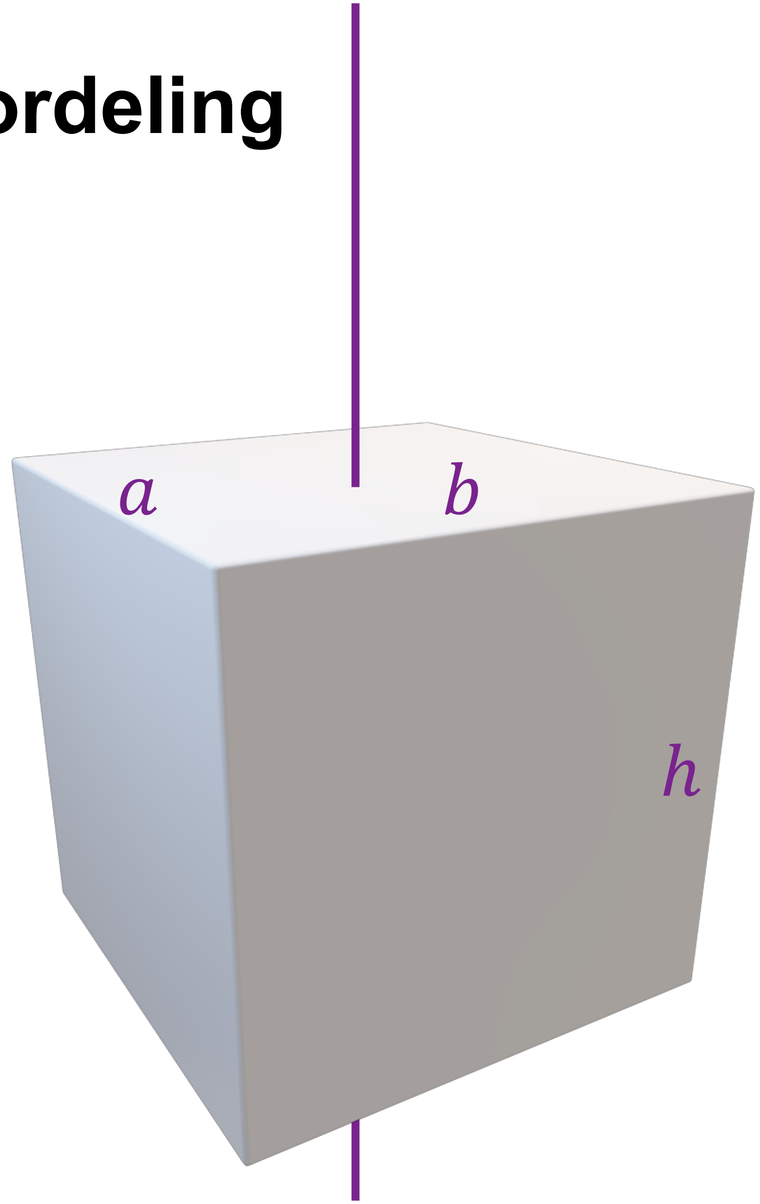
Inertimoment for kontinuert massefordeling

For en kontinuert massefordeling, kan inertimomentet defineres som et integral over alle masseelementerne og deres vinkelrette afstand til rotationsaksen

$$I = \sum_i m_i r_i^2 \quad \rightarrow \quad I = \int r^2 \, \mathrm{d}m$$

For eksempel er inertimomentet for en klods med massen M og dimensionerne a, b, h omkring en akse gennem massemidtpunktet og langs klodsens højde h.



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For eksempel er inertimomentet for en klods med massen M og dimensionerne a, b, h omkring en akse gennem massemidtpunktet og langs klodsens højde h.

Massetæthed $\rho = \frac{\mathrm{d}m}{\mathrm{d}V} = \frac{M}{V}$ og volumen $\mathrm{d}V = \mathrm{d}x\mathrm{d}y\mathrm{d}z$ og $V = abh$. Hvis vi lægger koordinatsystemet så rotationsaksen går igennem $(x, y) = (0,0)$ fås $r^2 = x^2 + y^2$

$$I = \int r^2\,\mathrm{d}m = \int r^2\,\rho\mathrm{d}V = \int_0^h \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} (x^2 + y^2)\rho\mathrm{d}x\mathrm{d}y\mathrm{d}z = \frac{M}{12}(a^2 + b^2)$$

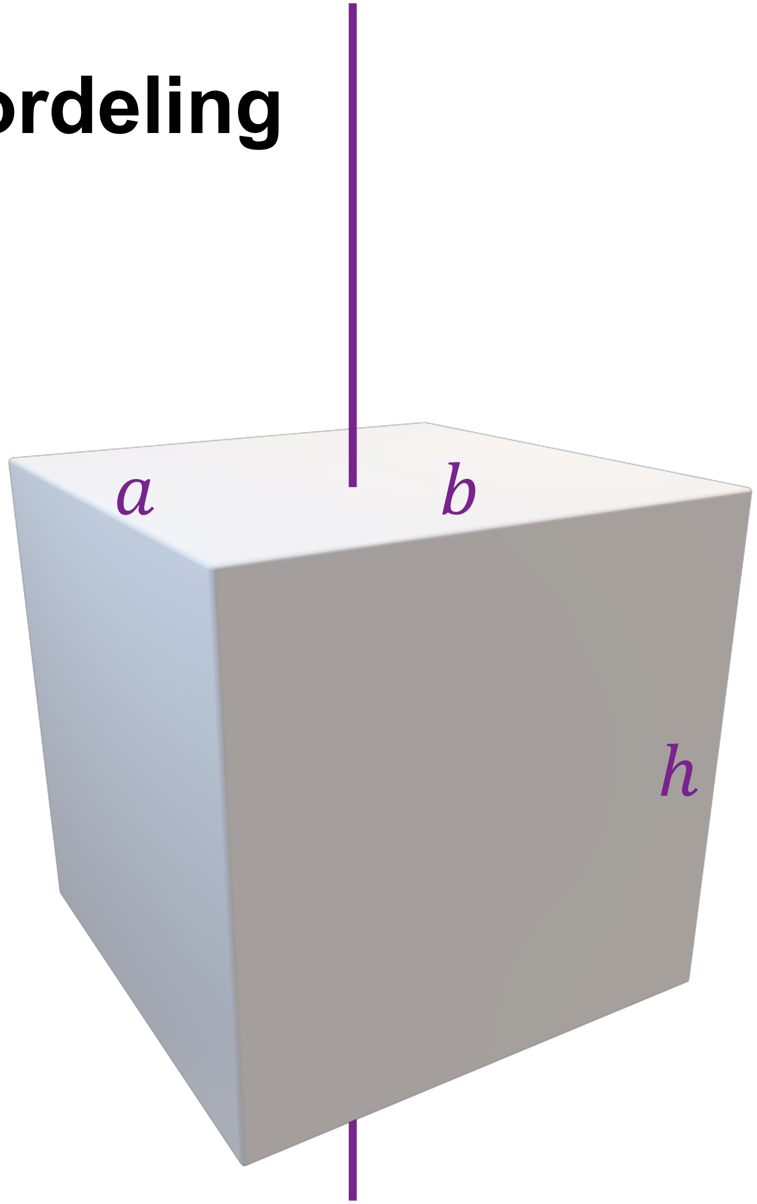
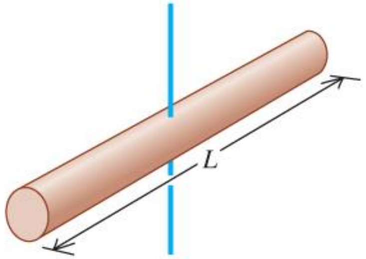


TABLE 9.2 Moments of Inertia of Various Bodies

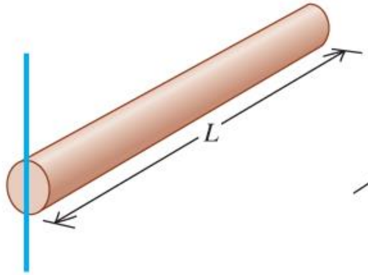
(a) Slender rod, axis through center

$$I = \frac{1}{12}ML^2$$



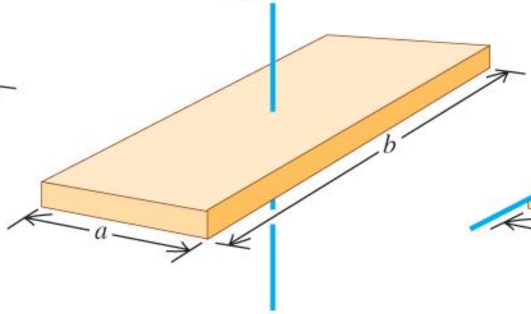
(b) Slender rod, axis through one end

$$I = \frac{1}{3}ML^2$$



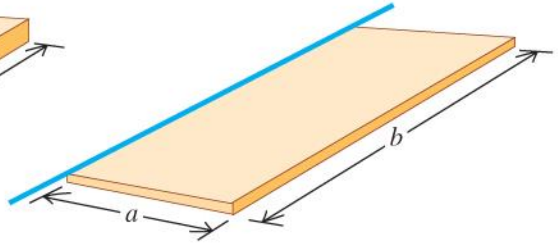
(c) Rectangular plate, axis through center

$$I = \frac{1}{12}M\left(a^2 + b^2\right)$$



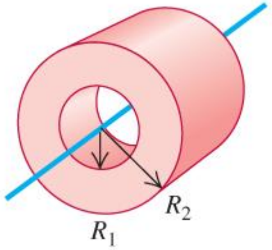
(d) Thin rectangular plate, axis along edge

$$I = \frac{1}{3}Ma^2$$



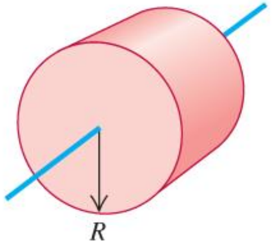
(e) Hollow cylinder

$$I = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$$



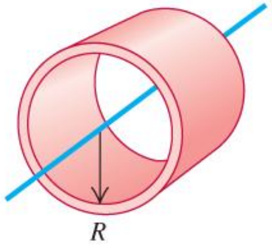
(f) Solid cylinder

$$I = \frac{1}{2}MR^2$$



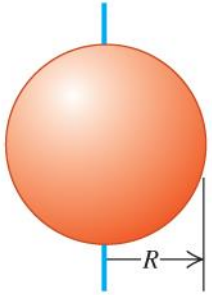
(g) Thin-walled hollow cylinder

$$I = MR^2$$



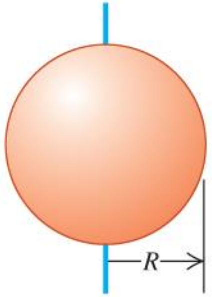
(h) Solid sphere

$$I = \frac{2}{5}MR^2$$

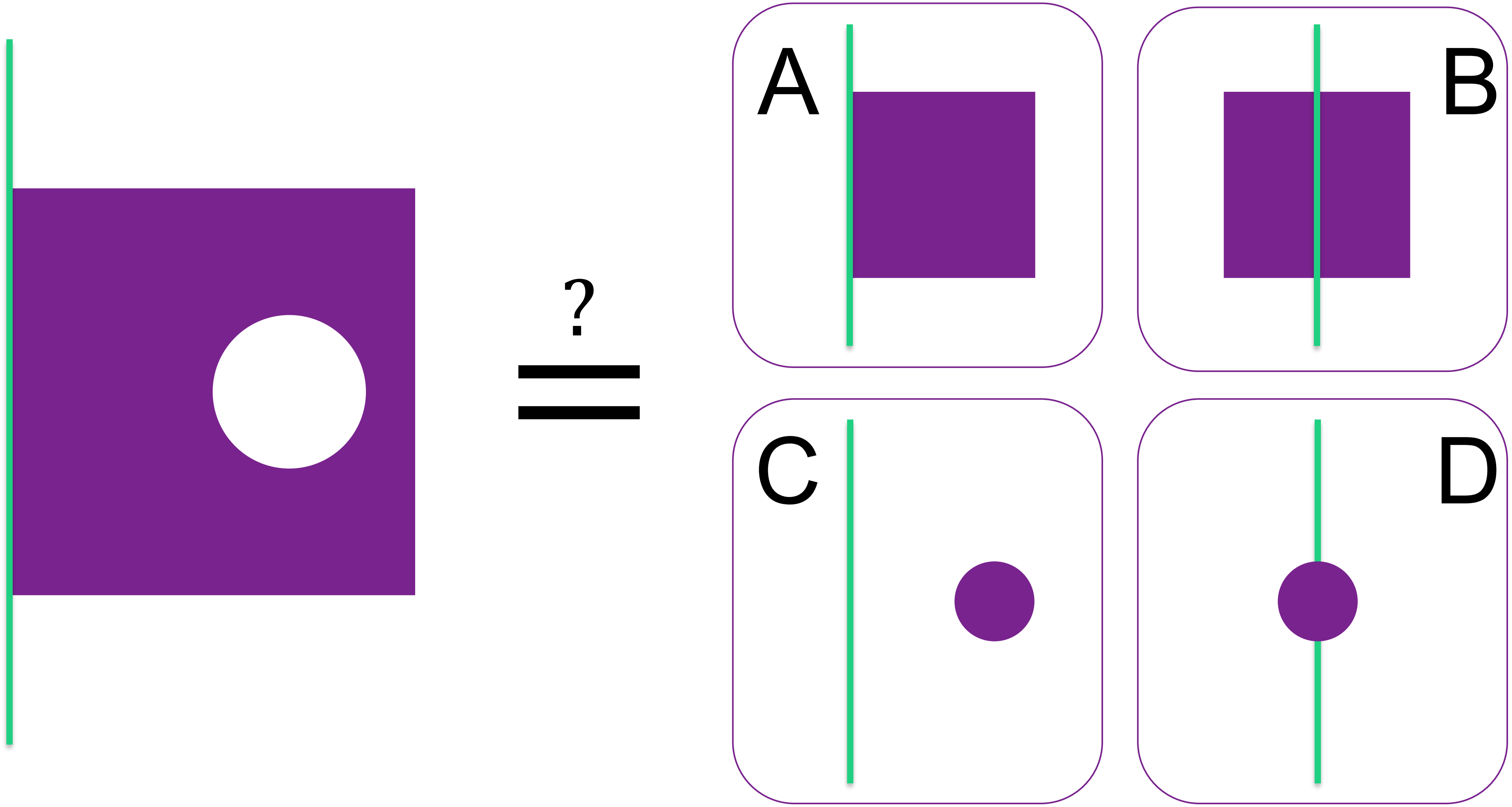


(i) Thin-walled hollow sphere

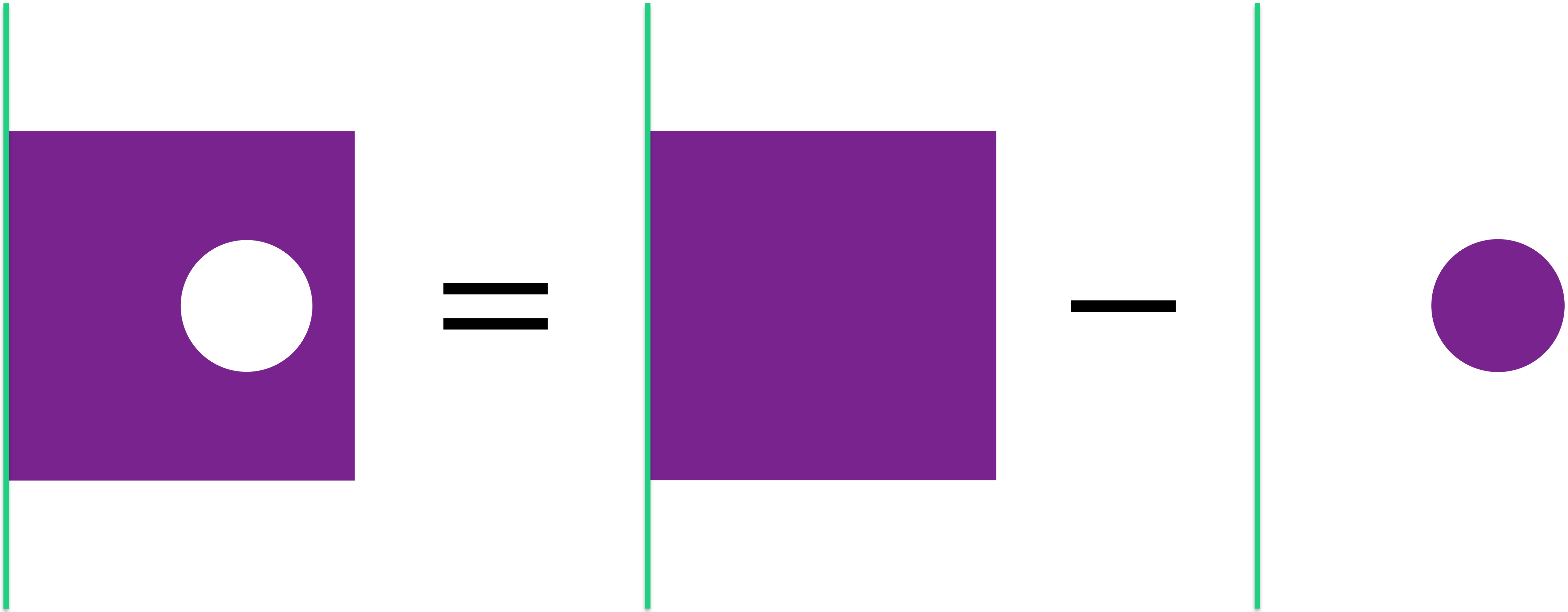
$$I = \frac{2}{3}MR^2$$



QUIZ! Sammensatte legemer



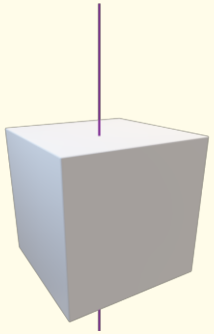
QUIZ! Sammensatte legemer



I dag: Kap. 9 – Stive legemers rotation

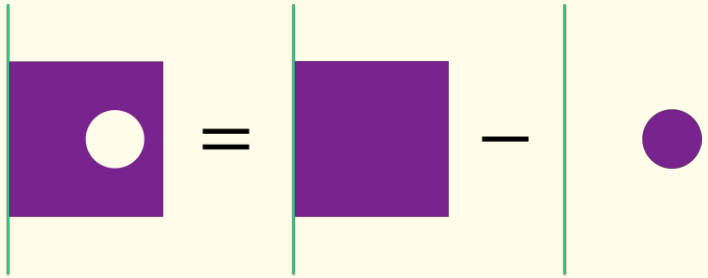
Inertimoment for kontinuert massefordeling

$$I = \sum_i m_i r_i^2 \quad \rightarrow \quad I = \int r^2 \, \mathrm{d}m$$

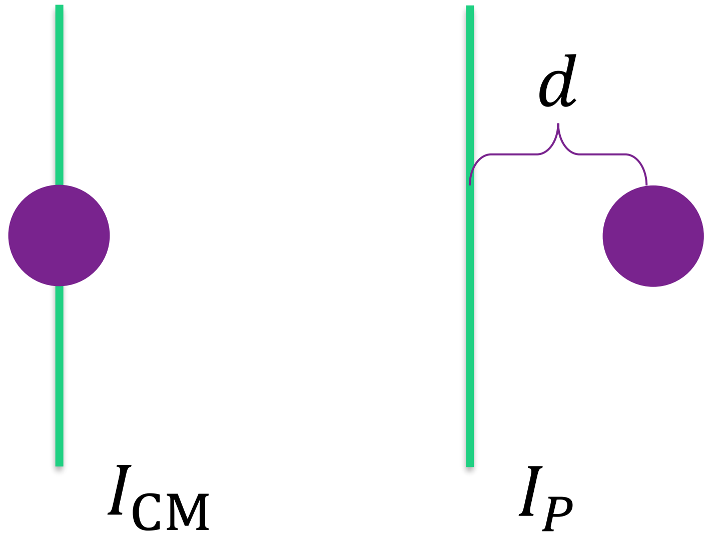


Inertimomenter af sammensatte legemer (eller legemer med huller) kan findes ved at tillægge eller fratrække de enkelte deles inertimomenter med samme rotationsakse

$$I = \sum_j I_j$$



Parallelakseteoremet



$$r_i = r_{i,\mathrm{CM}} + d$$

$$I_P = \sum_i m_i r_i^2 = \sum_i m_i \left(r_{i,\mathrm{CM}} + d\right)^2 = \sum_i m_i {r_{i,\mathrm{CM}}}^2 + \sum_i m_i d^2 + 2\underbrace{\sum_i m_i r_{i,\mathrm{CM}} d}_{0} = I_{\mathrm{CM}} + Md^2$$

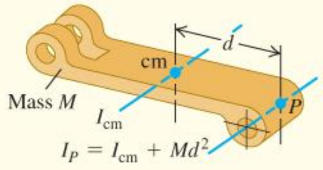
$$\boxed{I_P = I_{\mathrm{CM}} + Md^2}$$

QUIZ

I dag: Kap. 9 – Stive legemers rotation

Calculating the moment of inertia: The parallel-axis theorem relates the moments of inertia of a rigid body of mass M about two parallel axes: an axis through the center of mass (moment of inertia I_{cm}) and a parallel axis a distance d from the first axis (moment of inertia I_P). (See Example 9.9.) If the body has a continuous mass distribution, the moment of inertia can be calculated by integration. (See Examples 9.10 and 9.11.)

$$I_P = I_{cm} + Md^2 \qquad (9.19)$$



I dag: Kap. 9 – Stive legemers rotation

LEARNING OUTCOMES

In this chapter, you'll learn...

Rotationel kinematik

- **9.1** How to describe the rotation of a rigid body in terms of angular coordinate, angular velocity, and angular acceleration.
- **9.2** How to analyze rigid-body rotation when the angular acceleration is constant.
- **9.3** How to relate the rotation of a rigid body to the linear velocity and linear acceleration of a point on the body.

Inertimoment

- **9.4** The meaning of a body's moment of inertia about a rotation axis, and how it relates to rotational kinetic energy.
- **9.5** How to relate the values of a body's moment of inertia for two different but parallel rotation axes.
- **9.6** How to calculate the moment of inertia of bodies with various shapes.

I dag: Kap. 9 – Stive legemers rotation

Hvad er lige nu mest uklart?

Problemløsning

PROBLEM-SOLVING STRATEGY 1.1 Solving Physics Problems

IDENTIFY *the relevant concepts:*

- Use the physical conditions stated in the problem to help you decide which physics concepts are relevant.
- Identify the **target variables** of the problem—that is, the quantities whose values you're trying to find, such as the speed at which a projectile hits the ground, the intensity of a sound made by a siren, or the size of an image made by a lens.
- Identify the known quantities, as stated or implied in the problem. This step is essential whether the problem asks for an algebraic expression or a numerical answer.

SET UP *the problem:*

- Given the concepts, known quantities, and target variables that you found in the IDENTIFY step, choose the equations that you'll use to solve the problem and decide how you'll use them. Study the worked examples in this book for tips on how to select the proper equations. If this seems challenging, don't worry—you'll get better with practice!
- Make sure that the variables you have identified correlate exactly with those in the equations.
- If appropriate, draw a sketch of the situation described in the problem. (Graph paper and a ruler will help you make clear, useful sketches.)

EXECUTE *the solution:*

- Here's where you'll "do the math" with the equations that you selected in the SET UP step to solve for the target variables that you found in the IDENTIFY step. Study the worked examples to see what's involved in this step.

EVALUATE *your answer:*

- Check your answer from the SOLVE step to see if it's reasonable. (If you're calculating how high a thrown baseball goes, an answer of 1.0 mm is unreasonably small and an answer of 100 km is unreasonably large.) If your answer includes an algebraic expression, confirm that it correctly represents what would happen if the variables in it had very large or very small values.
- For future reference, make note of any answer that represents a quantity of particular significance. Ask yourself how you might answer a more general or more difficult version of the problem you have just solved.

EXAMPLE 9.5 Designing a propeller

WITH VARIATION PROBLEMS

You are designing an airplane propeller that is to turn at 2400 rpm (**Fig. 9.13a**). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the propeller tips through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the propeller tips moved faster, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

IDENTIFY and SET UP We consider a particle at the tip of the propeller; our target variables are the particle's distance from the axis and its acceleration. The speed of this particle through the air, which cannot exceed 270 m/s, is due to both the propeller's rotation and the forward motion of the airplane. Figure 9.13b shows that the particle's velocity $\vec{v}_{tip}$ is the vector sum of its tangential velocity due to the propeller's rotation of magnitude $v_{tan} = \omega r$, given by Eq. (9.13), and the forward velocity of the airplane of magnitude $v_{plane} = 75.0$ m/s. The propeller rotates in a plane perpendicular to the direction of flight, so $\vec{v}_{tan}$ and $\vec{v}_{plane}$ are perpendicular to each other, and we can use the Pythagorean theorem to obtain an expression for v_{tip} from v_{tan} and v_{plane}. We'll then set $v_{tip} = 270$ m/s and solve for the radius r. The angular speed of the propeller is constant, so the acceleration of the propeller tip has only a radial component; we'll find it by using Eq. (9.15).

EXECUTE We first convert ω to rad/s (see Fig. 9.11):

$$\omega = 2400 \text{ rpm} = \left(2400 \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 251 \text{ rad/s}$$

(a) From Fig. 9.13b and Eq. (9.13),

$$v_{tip}^2 = v_{plane}^2 + v_{tan}^2 = v_{plane}^2 + r^2\omega^2 \quad \text{so}$$

$$r^2 = \frac{v_{tip}^2 - v_{plane}^2}{\omega^2} \quad \text{and} \quad r = \frac{\sqrt{v_{tip}^2 - v_{plane}^2}}{\omega}$$

If $v_{tip} = 270$ m/s, the maximum propeller radius is

$$r = \frac{\sqrt{(270 \text{ m/s})^2 - (75.0 \text{ m/s})^2}}{251 \text{ rad/s}} = 1.03 \text{ m}$$

(b) The centripetal acceleration of the particle is, from Eq. (9.15),

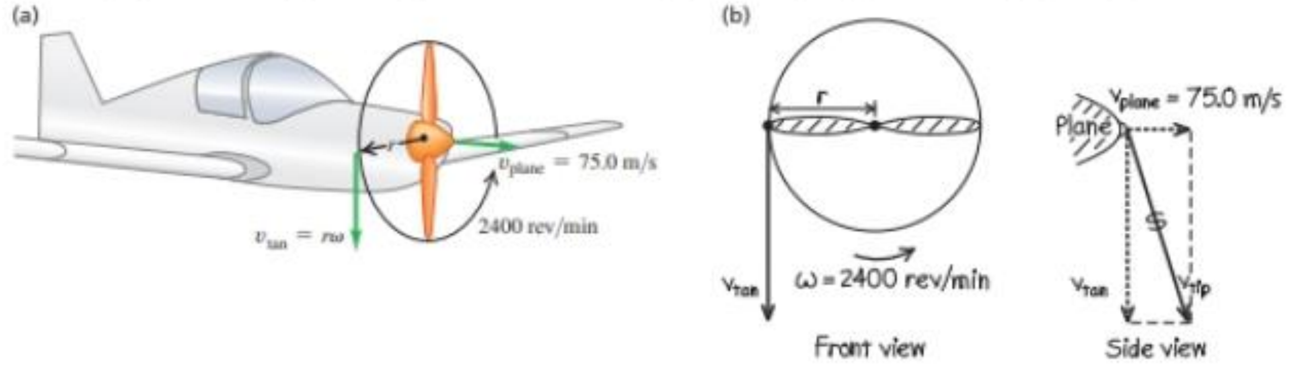
$$a_{rad} = \omega^2 r = (251 \text{ rad/s})^2(1.03 \text{ m})$$
$$= 6.5 \times 10^4 \text{ m/s}^2$$

The tangential acceleration a_{tan} is zero because ω is constant.

EVALUATE From $\sum \vec{F} = m\vec{a}$, the propeller must exert a force of 6.5×10^4 N on each kilogram of material at its tip! This is why propellers are made out of tough material such as aluminum alloy.

KEYCONCEPT If a rotating rigid body is also moving as a whole through space, use vector addition to find the velocity of a point on the rigid body.

Figure 9.13 (a) A propeller-driven airplane in flight. (b) Our sketch showing the velocity components for the propeller tip.



EXAMPLE 9.8 An unwinding cable II

WITH VARIATION PROBLEMS

We wrap a light, nonstretching cable around a solid cylinder with mass M and radius R. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass m and release the block from rest at a distance h above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

IDENTIFY As in Example 9.7, the cable doesn't slip and so friction does no work. We assume that the cable is massless, so that the forces it exerts on the cylinder and the block have equal magnitudes. At its upper end the force and displacement are in the same direction, and at its lower end they are in opposite directions, so the cable does no *net* work and $W_{\text{other}} = 0$. Only gravity does work, and mechanical energy is conserved.

SET UP **Figure 9.17a** shows the situation before the block begins to fall (point 1). The initial kinetic energy is $K_1 = 0$. We take the gravitational potential energy to be zero when the block is at floor level (point 2), so $U_1 = mgh$ and $U_2 = 0$. (We ignore the gravitational potential energy for the rotating cylinder, since its height doesn't change.) Just before the block hits the floor (Fig. 9.17b), the block has kinetic energy due to its translational motion and the cylinder has kinetic energy due to its rotation. The total kinetic energy is the sum of these:

$$K_2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

The moment of inertia of the cylinder is $I = \tfrac{1}{2}MR^2$. Also, $v = R\omega$ since the speed of the falling block must be equal to the tangential speed at the outer surface of the cylinder.

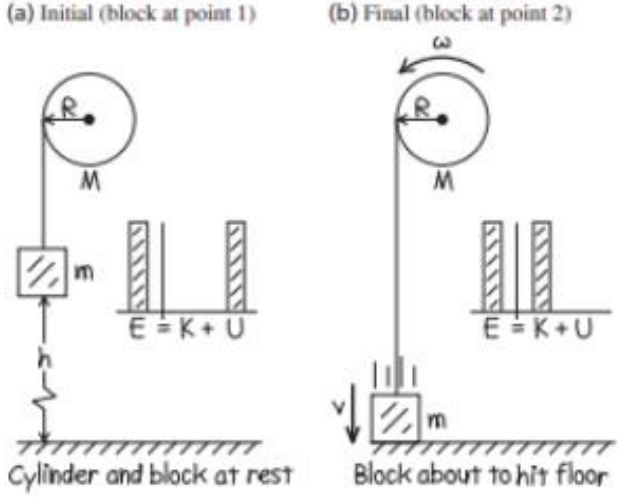
EXECUTE We use our expressions for K_1, U_1, K_2, and U_2 and the relationship $\omega = v/R$ in Eq. (7.4), $K_1 + U_1 = K_2 + U_2$, and solve for v:

$$0 + mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 + 0 = \tfrac{1}{2}\left(m + \tfrac{1}{2}M\right)v^2$$

$$v = \sqrt{\frac{2gh}{1 + M/2m}}$$

The final angular speed of the cylinder is $\omega = v/R$.

Figure 9.17 Our sketches for this problem.



EVALUATE When M is much larger than m, v is very small; when M is much smaller than m, v is nearly equal to $\sqrt{2gh}$, the speed of a body that falls freely from height h. Both of these results are as we would expect.

KEYCONCEPT When a block is attached to a string that wraps around a cylinder or pulley of radius R, the speed v of the block is related to the angular speed ω of the cylinder or pulley by $v = R\omega$. You can use this to find the combined kinetic energy of the two objects.