1 Put Pricing

$$S_0 = \$90,\ c = \$5.2,\ K = S_0,\ r = 0.9\%,\ T = 2/12$$
$$D_0 = c_1 * e^{-r*t_1} = 1.1 * e^{-0.009*1/12} \approx \$1.099175$$

The price is determined using the put-call parity, for discrete income assets:

$$c + e^{-rT}K = p + S_0 - D_0 \implies p = c + e^{-rT}K - (S_0 - D_0)$$
$$p = 5.2 + 90 * e^{-0.009*2/12} - (90 - 1.099175) \approx \$6.164277$$

2 GBP Arbitrage

$$S_0 = \$1.3050,\ r = 0.8\%,\ q = 1.2\%,\ K = 1.3,\ T = 1,\ c = \$0.048,\ p = \$0.03$$

The put-call parity for assets with continous yield:

$$c + e^{-rT}K = p + S_0e^{-qT}$$

$$0.048 + 1.3 * e^{-0.008} = \$1.337641 > 0.03 + 1.305 * e^{-0.012} = \$1.319434$$

As the value of the portfolio with the call option is higher, an arbitrage strategy is to sell the call and buy the put and the asset (GBP).

- **At time T=0** - Sell call option, buy put and asset. This is done by borrowing $p + S_0 - c = \$1.287$.
- **At time T=1 year**: Value of loan: $1287 * e^{rT} \approx \$1.297337$
  - If $S_T < 1.3$: Sell GBP and get $\$1.3 * e^{qT}$ and pay off loan.
    Profit: $1.3 * e^{qT} - 1.297337 \approx \$0.018357$
  - If $S_T > 1.3$: Sell GBP and get $\$S_T * e^{qT}$ and pay off loan.
    Profit: $S_T * e^{qT} - 1.297337 \geq \$0.018357$

A profit of at least \$0.018357/unit is guaranteed, and this is thus an arbitrage opportunity.

3 Call Bear Spread

$$S_0 = \$703,\ K_2 = \$700,\ K_1 = \$680,\ p_2 = \$38,\ p_1 = \$52$$

A bear spread using calls is constructed by buying the call with the higher strike price, and selling the one with the lower. This results in a "credit" of $p_1 - p_2 = 52 - 28 = \$14$.

The P&L for this setup is as follows: $(S_T - K_2)^+ - (S_T - K_1)^+ + 14$.

There are three cases to consider:

$$P\&L = \begin{cases} S_T - 700 - S_T + 680 + 14 = -6, & if\ S_T > \$700 \\ -S_T - 680 + 14 = 694 - S_T, & if\ \$680 < S_T < \$700 \\ 0 - 0 + 14 = 14, & if\ S_T < \$680 \end{cases}$$

From this, it is easy to identify the maximum profit as $\$14/option$, the maximum loss as $-\$6/option$ and the break even point as $S_T = \$694$

4 Binomial I

$$S_0 = \$20,\ u = 1.2,\ d = 0.8,\ r = 1\%,\ Payoff = max(0, 400 - S^2)$$

$$f(S_\delta) : f_u = f(uS_0) = max(0, 400 - (1.2 * 20)^2) = 0.0$$
$$f_d = f(dS_0) = max(0, 400 - (0.8 * 20)^2) = \$144$$

To replicate the option, we construct a portfolio with Δ number of shares, and borrow Ψ dollars. Δ and Ψ are given by:

$$\Delta = \frac{f_u - f_d}{S_0(u-d)} = \frac{0 - 144}{20 * (1.2 - 0.8)} = -18$$
$$\Psi = \frac{df_u - uf_d}{e^{r\delta}(u-d)} = \frac{0 - 1.2 * 144}{e^{0.01*6/12} * (1.2 - 0.8)} \approx -\$429.845391$$

The value of the derivative (price of the power put) at time zero for no arbitrage should be:

$$f_0 = S_0\Delta - \Psi = 20 * -18 - (-429.845391) \approx \$69.845391$$

5 Binomial II

$$u = 1.1,\ d = 0.8,\ T = \delta = 3/12,\ r = 1\%,\ K = \$95$$
$$f_u = max(K - uS_0, 0) = 0,\ f_d = max(K - u * dS_0, 0) = 15$$

For the risk neutral pricing, we first need the risk neutral probability p^*.

$$p^* = \frac{e^{r\delta} - d}{u - d} = \frac{e^{0.01*3/12} - 0.8}{1.1 - 0.8} \approx 0.675010$$

The risk neutral price is then given as the risk neutral expectation of the payoff discounted at the risk free rate:

$$f_0 = e^{-r\delta}\mathbb{E}[f(S_\delta)] = e^{-r\delta}\left(p^* f_u + (1 - p^*) f_d\right) = e^{-0.01*3/12}\left((1 - 0.675010) * 15\right) = \$4.862672$$

A delta hedge is constructed by purchasing Δ shares according to:

$$\Delta = \frac{f_u - f_d}{S_0(u-d)} = 100 * \frac{0 - 15}{100 * (1.1 - 0.8)} = -50$$

As such, you should (short) sell 50 shares to construct a delta hedge.