1 Forward Hedge

$$P_0 = \$5,\ K = \$5.15,\ S_T = \$4.85$$

$$Payoff_{long} = S_T - K = \$4.85 - \$5.15 = -\$0.30 \text{ per gallon}$$

The reason for doing so is eliminating uncertainty in costs. When the cost structure for a given term is certain, the company can focus their energy on conducting their core business instead. Furthermore, the risk of unforeseen events increasing the fuel price drastically is circumvented by employing the hedge.

2 Put Hedge

$$S_0 = \$200,\ N_{stocks} = 1000$$
$$K = \$198,\ P_0 = \$5,\ N_{put} = 1000$$

$$P\&L_{put} = (K - S_T)^+ - P_0, \quad P\&L_{stock} = (S_T - S_0)$$

$$P\&L_{hedge} = \begin{cases} 1000 * (K - S_0) - 1000 * P_0 = -2000 - 5000 = -\$7000, & \text{if } S_T < 198 \\ 1000 * (S_T - S_0) - 1000 * P_0, & \text{else} \end{cases}$$

I.e, the put option is only exercised when the share price is below the strike price and when it is exercised it limits the otherwise unlimited potential loss. The second expression gives the break-even point as:

$$1000 * (S_T - 200) - 1000 * 5 = 0 \implies S_T = \frac{5000}{1000} + 200 = \$205$$

Thus, the hedge limits the loss to $-\$7000$. A loss is obtained for share prices below \$205

3 TV loan

$$P = 0.9 * 4000 = 3600, i = 12\%/12 = 1\%, N = 12$$

The first payment is obtained with the formula:

$$A = \frac{P * i}{1 - (1+i)^{-N}} = \frac{3600 * 0.01}{1 - (1.01)^{-12}} = 319.86$$

The composition of the payments was calculated with excel using the **PPMT**-formula and resulted in:

	1	2	3	4	5	6	7	8	9	10	11	12
Principal	283.86	286.69	289.56	292.46	295.38	298.34	301.32	304.33	307.37	310.45	313.55	316.69
Interest	36.00	33.16	30.29	27.40	24.47	21.52	18.54	15.52	12.48	9.41	6.30	3.17

4 Bonds I

$$y = 0.03, r_c = 0.02, F = \$100$$

Each coupon equals $\frac{r_c * 100}{2} = \$1$. The price P is calculated as:

$$P = c_{0.5} * e^{-0.5y} + c_1 * e^{-y} + (F + c_{1.5}) * e^{-1.5y} = e^{-0.5*0.03} + e^{-0.03} + 101 * e^{-1.5*0.03} = \$98.511303$$

The zero rates are calculated as:

$$\begin{aligned}
r_{0.5} &: \$99 = \$100 * e^{-0.5*r_{0.5}} \implies r_{0.5} = \frac{\log(99/100)}{-0.5} \approx 2.010067\% \\
r_1 &: \$97.5 = \$100 * e^{r_1} \implies r_1 = \log(97.5/100) \approx 2.531781\% \\
r_{1.5} &: \$98.511303 = e^{-0.5*r_{0.5}} + e^{-r_1} + 101 * e^{-1.5*r_{1.5}} \\
&\implies r_{1.5} = \frac{\log(96.546303/101)}{-1.5} = 3.006520\%
\end{aligned}$$

For coupon rate = 2.4%, each coupon is equal to $\frac{0.024*100}{2} = \$1.2$. The price ($P^*$) is given by:

$$P^* = 1.2 * e^{-0.5*r_{0.5}} + 1.2 * e^{-r_1} + 101.2 * e^{-1.5*r_{1.5}} \approx \$114.370856$$

5 Bonds II

$$r_c = 3\%, \ P = \$100, \ c = 0.03 * \frac{100}{2} = \$1.5$$

Yield is calculated with the following formula:

$$\begin{aligned}
P &= c * e^{-0.5y} + c * e^{-y} + c * e^{-1.5y} + (F + c) * e^{-2y} \\
100 &= 1.5 * e^{-0.5y} + 1.5 * e^{-y} + 1.5e^{-1.5y} + 101.5e^{-2y}
\end{aligned}$$

The last expression can be solved using the Solver in Excel, and the result is: $y \approx 2.977722\%$.

Next, the duration is calculated using the formula:

$$\begin{aligned}
D &= \sum_{i=1}^{n} t_i \left(\frac{c_i e^{-y t_i}}{P} \right) \\
&= 0.5 * \frac{1.5e^{-0.5y}}{100} + 1 * \frac{1.5e^{-y}}{100} + 1.5 * \frac{1.5e^{-1.5y}}{100} + 2 * \frac{101.5e^{-2y}}{100} \approx 1.956100
\end{aligned}$$

The convexity is given by:

$$
\begin{aligned}
C &= \sum_{i=1}^{n} t_i^2 \left(\frac{c_i e^{-y t_i}}{P} \right) \\
&= 0.5^2 * \frac{1.5e^{-0.5y}}{100} + 1 * \frac{1.5e^{-y}}{100} + 1.5^2 * \frac{1.5e^{-1.5y}}{100} + 2^2 * \frac{101.5e^{-2y}}{100} \approx 3.875798
\end{aligned}
$$

To approximate the price change of the bond for an increase in the yield by 100 basis points one uses the Duration and Convexity in the following way, using a Taylor-approximation:

$$
\Delta P = \frac{\partial P}{\partial y} \Delta y + 0.5 \frac{\partial^2 P}{\partial y^2} \Delta y^2 = -DP\Delta y + 0.5CP\Delta y^2 = -0.01DP + 0.00005CP \approx -1.936721
$$

The new price is therefore: $P_{new} = P_{old} + \Delta P = 100 - 1.936721 = \$98.063279$