

## 1 Forward Price

$$S_0 = \$180, r = 1.6\%, c = \$0.51, T = 6$$

The forward price is given by,  $F_0 = (S_0 - D_0)e^{rT}$

$$D_0 = c * e^{-r*1/12} + c * e^{-r*4/12} \approx 1.0166$$

$$F_0 = (180 - 1.0166)e^{0.016*6/12} \approx 180.4210022$$

After 2 month:  $S_t = \$160, t = 2, K = F_0$

$$D_t = c * e^{-r*2/12} = 0.508642$$

$$F_t = e^{r(T-t)}(S_t - D_t) = e^{0.016*4/12}(160 - 0.508642) = 160.344251$$

The value at time t, is given by,

$$V_t = e^{-r(T-t)}(F_t - K) = e^{-0.016*4/12}(160.344251 - 180.4210022) = -\$19.969960$$

## 2 GBP Forward I

$$S_0 = 1.3, T = 6, r = 1.6\%, q = 1.0\%$$

For zero-value the forward price should be:  $F_0 = S_0 * e^{(r-q)T} = 1.3 * e^{(0.016-0.01)/2} \approx \$1.30391$ . As the quoted price is lower than this, an arbitrage opportunity exists for a long forward contract.

At time = 0:

- Long forward to buy 1 M GBP in 6 months
- Loan  $e^{-qT}$  Million GBP and sell to receive  $S_0 * e^{-qT}$  Million USD.
- Deposit the dollars for 6 months with interest rate  $r$ .

At time = T:

- Withdraw  $S_0 * e^{(r-q)T}$  Million USD.
- Buy 1 Million GBP for forward price  $\$1.30151/GBP$  and pay back loan

At this time, the profit equals the amount withdrawn from the account subtracted by the cost to pay back the loan:

$$S_0 * e^{(r-q)T} MUSD - 1.30151 MUSD \approx \$2395.855854$$

### 3 LIBOR FRA

$$r_1 = 1.6\%, r_2 = 2\%, t_1 = 6, t_2 = 18, L = \$1,000,000$$

The interest rate to ensure a zero-cost FRA is given by:

$$r_K = \frac{r_2 t_2 - r_1 t_1}{t_2 - t_1} = \frac{0.02 * 18/12 - 0.016 * 6/12}{(18 - 6)/12} = 2.2\%$$

After 6 months:  $r_M = 2.4\%$ ,  $r = 2.0\%$

The value for the borrower at time  $t_2$  is calculated as:  $V(t_2) = L(e^{r_M(t_2-t_1)} - e^{r_K(t_2-t_1)})$ . To get the value of the FRA for the borrower at the time of settlement ( $t_1$ ), this has to be discounted by the risk free interest rate:

$$\begin{aligned} V(t_1) &= e^{-r(t_2-t_1)} V(t_2) = L e^{-r(t_2-t_1)} (e^{r_M(t_2-t_1)} - e^{r_K(t_2-t_1)}) = \\ &= 1000000 * e^{-0.02} (e^{0.024} - e^{0.022}) \approx \$2006.009343 \end{aligned}$$

### 4 GBP Forward II

$$S_0 = \$1.3/GBP, r_b = 1.7\%, r_d = 1.5\%, q = 1\%$$

There are two arbitrage opportunities: If the price is low enough the arbitrageur can long the forward and if the price is too high the arbitrageur can short the forward.

- Long arbitrage profit:  $S_0 e^{(r_d - q)T} - F_0 = 1.3e^{0.005} - F_0$
- Short arbitrage profit:  $F_0 - S_0 e^{(r_b - q)T} = F_0 - 1.3e^{0.007}$

To eliminate both opportunities, the following must hold:

$$\begin{cases} F_0 \geq 1.3e^{0.005} \\ F_0 \leq 1.3e^{0.007} \end{cases}$$

This gives the answer as:  $1.306516 \leq F_0 \leq 1.309132$

### 5 Put Arbitrage

$$K_1 = \$315, P_1 = \$5.60, K_2 = \$317.5, P_2 = \$5.40$$

An arbitrage strategy for this situation would be to short the put option with the lower strike price, and buy the put option with the higher strike price.

At time 0: Short 315-puts and receive \$5.60/*option*. Buy equally many 317.5-puts for \$5.40/*option*. The payoff for a short put:  $-(K - S)^+$ , and for a long put:  $(K - S)^+$ . For this setup the combined P&L thus amounts to:

$$(K_2 - S)^+ - (K_1 - S)^+ + (P_1 - P_2) = (317.5 - S)^+ - (315 - S)^+ + 0.2$$

Considering the stock price  $S$  when exercising the option, there are three possibilities.

1.  $S > \$317.5$ . This means that none of the options are exercised and the P&L becomes:  
 $P\&L = \$0.2$
2.  $\$315 < S < \$317.5$ . In this situation, only the 317.5-put is exercised:  
 $P\&L = 317.5 - S + 0.2 > \$0.2$
3.  $S < \$315$ . Finally, this case means that both options are exercised:  
 $P\&L = 317.5 - S - 315 + S + 0.2 = \$2.7$

Thus, no matter the share price  $S$  the strategy implies a profit of at least \$0.2/*option* and is thus an example of arbitrage.