

- Credits are given to **solutions with detailed derivations/calculations** only.
- (4 points) Consider the monthly returns of two risky assets. The return of the first asset has a mean of 2% and standard deviation of 3%. The return of the second asset has a mean of 1.5% and standard deviation of 2%. The correlation coefficient of the two returns is 0.3. How can the minimum variance portfolio (MVP) be constructed? What are the mean and standard deviation of the return of the MVP? Consider a portfolio with 50% invested in asset 1 and 50% invested in asset 2. Is such a portfolio efficient?
 - (4 points) The monthly prices of MCD, LLY, JPM, AMZN and SP500 are given in the attached Excel file. Estimate the expected returns and the covariance matrix of the returns of the above assets and fill out the following table. Some values have been given to you for double checking.

Mean	Asset	MCD	LLY	JPM	AMZN	SP500
	MCD				0.001085	
	LLY		0.003056			
	JPM	0.001315				
	AMZN					0.001964
0.5608%	SP500			0.002189		

- (4 points) Consider portfolios constructed using MCD, LLY, JPM and AMZN. Assume no short selling. Using the parameters obtained in question #2, construct the minimum variance portfolio. If the desired monthly expected rate of return is 2%, how should the portfolio be constructed? Is this portfolio efficient? Using SP500 as a proxy for the market portfolio, estimate the beta's of the four stocks.

4. (4 points) Suppose the risk free rate of return is 0.1% for a 1-month period. Use SP500 as a proxy for the market portfolio. Using the parameters obtained in question #2, compute the market price of risk. Give the expression for the capital market line. For an efficient portfolio on the capital market line with standard deviation 5%, what's the expected rate of return? What's the beta of this portfolio?

5. (4 points) A company needs to hedge the purchase of 1 million units of asset A in two months using 3-month futures on a similar asset B. The futures contract size is 4000 units of the asset B. The following are historical data on the changes of the spot price of asset A and the changes of the futures price of asset B in two-month periods. What's the optimal hedge ratio? How many futures contracts are needed for this hedge?

spot price change	0.50	0.61	-0.22	-0.35	0.79	0.04	0.15	0.70	-0.51	-0.41
futures price change	0.56	0.63	-0.12	-0.44	0.60	-0.06	0.01	0.80	-0.56	-0.46