Forward Hedge

$$P_0 = \$5,\ K = \$5.15,\ S_T = \$4.85$$

$$Payoff_{long} = S_T - K = \$4.85 - \$5.15 = -\$0.30 \text{ per gallon}$$

The reason for doing so is eliminating uncertainty in costs.

Put Hedge

$$S_0 = \$200,\ N_{stocks} = 1000$$
$$K = \$198,\ P_0 = \$5,\ N_{put} = 1000$$

$$P\&L_{put} = (K - S_T)^+ - P_0, \quad P\&L_{stock} = (S_T - S_0)$$

$$P\&L_{hedge} = \begin{cases} 1000 * (K - S_0) - 1000 * P_0 = -2000 - 5000 = -\$7000, \text{ if } S_T \leq 198 \\ 1000 * (S_T - S_0) - 1000 * P_0, \text{ else} \end{cases}$$

I.e, the put option is only exercised when the share price is below the strike price. The second expression gives the break-even point as:

$$1000 * (S_T - 200) - 1000 * 5 = 0 \implies S_T = \frac{5000}{1000} + 200 = \$205$$

Thus, the hedge limits the loss to $-\$7000$. A loss is obtained for share prices below \$205

TV loan

$$P = 0.9 * 4000 = 3600, i = 12\%/12 = 1\%, N = 12$$

The first payment is obtained with the formula:

$$A = \frac{P * i}{1 - (1+i)^{-N}} = \frac{3600 * 0.01}{1 - (1.01)^{-12}} = 319.86$$

The composition of the payments was calculated with excel and resulted in:

	1	2	3	4	5	6	7	8	9	10	11	12
Principal	283.86	286.69	289.56	292.46	295.38	298.34	301.32	304.33	307.37	310.45	313.55	316.69
Interest	36.00	33.16	30.29	27.40	24.47	21.52	18.54	15.52	12.48	9.41	6.30	3.17

Bonds I

$$y = 0.03, r_c = 0.02, F = \$100$$

Each coupon equals $\frac{r_c * 100}{2} = \$1$. The price P is calculated as:

$$P = c_{0.5} * e^{-0.5y} + c_1 * e^{-y} + (F + c_{1.5}) * e^{-1.5y} = e^{-0.5*0.03} + e^{-0.03} + 101 * e^{-1.5*0.03} = \$98.511303$$

The zero rates are calculated as:

$$\begin{aligned}
r_{0.5} &: \$99 = \$100 * e^{-0.5*r_{0.5}} \implies r_{0.5} = \frac{\log(99/100)}{-0.5} \approx 2.010067\% \\
r_1 &: \$97.5 = \$100 * e^{r_1} \implies r_1 = \log(97.5/100) \approx 2.531781\% \\
r_{1.5} &: \$98.511303 = e^{-0.5*r_{0.5}} + e^{-r_1} + 101 * e^{-1.5*r_{1.5}} \\
&\implies r_{1.5} = \frac{\log(96.546303/101)}{-1.5} = 3.006520\%
\end{aligned}$$

For coupon rate $= 2.4\%$, each coupon is equal to $\frac{0.024*100}{2} = \$1.2$. The price (P^*) is given by:

$$P^* = 1.2 * e^{-0.5*r_{0.5}} + 1.2 * e^{-r_1} + 101.2 * e^{-1.5*r_{1.5}} \approx \$114.370856$$

Bonds II

$$r_c = 3\%,\ P = \$100,\ c = 0.03 * \frac{100}{2} = \$1.5$$

Yield is calculated with the following formula:

$$\begin{aligned}
P &= c * e^{-0.5y} + c * e^{-y} + c * e^{-1.5y} + (F + c) * e^{-2y} \\
100 &= 1.5 * e^{-0.5y} + 1.5 * e^{-y} + 1.5e^{-1.5y} + 101.5e^{-2y}
\end{aligned}$$

The last expression can be solved using the Solver in Excel, and the result is: $y \approx 2.977722\%$.

Next, the duration is calculated using the formula:

$$\begin{aligned}
D &= \sum_{i=1}^{n} t_i \left(fracc_i e^{-yt_i} P \right) \\
&= 0.5 * \frac{1.5e^{-0.5y}}{100} + 1 * \frac{1.5e^{-y}}{100} + 1.5 * \frac{1.5e^{-1.5y}}{100} + 2 * \frac{101.5e^{-2y}}{100} \approx 1.956100
\end{aligned}$$

The convexity is given by:

$$
\begin{aligned}
C &= \sum_{i=1}^{n} t_i^2 \left(\frac{c_i e^{-yt_i}}{P} \right) \\
&= 0.5^2 * \frac{1.5e^{-0.5y}}{100} + 1 * \frac{1.5e^{-y}}{100} + 1.5^2 * \frac{1.5e^{-1.5y}}{100} + 2^2 * \frac{101.5e^{-2y}}{100} \approx 3.875798
\end{aligned}
$$

To approximate the price change of the bond for an increase in the yield by 100 basis points one uses the Duration and Convexity in the following way:

$$
\Delta P = \frac{\partial P}{\partial y} \Delta y + 0.5 \frac{\partial^2 P}{\partial y^2} \Delta y^2 = -DP\Delta y + 0.5CP\Delta y^2 = -0.01DP + 0.00005CP \approx -1.936721
$$

The new price is therefore: $P_{new} = P_{old} + \Delta P = 100 - 1.936721 = \$98.063279$