1 Forward Price

$$S_0 = \$180, r = 1.6\%, c = \$0.51, T = 6$$

The forward price is given by, $F_0 = (S_0 - D_0)e^{rT}$

$$D_0 = c * e^{-r*1/12} + c * e^{-r*4/12} \approx 1.0166$$

$$F_0 = (180 - 1.0166)e^{0.016*6/12} \approx 180.4210022$$

After 2 month: $S_t = \$160, t = 2, K = F_0$

$$D_t = c * e^{-r*2/12} = 0.508642$$

$$F_t = e^{r(T-t)}(S_t - D_t) = e^{0.016*4/12}(160 - 0.508642) = 160.344251$$

The value at time t, is given by,

$$V_t = e^{-r(T-t)}(F_t - K) = e^{-0.016*4/12}(160.344251 - 180.4210022) = -\$19.969960$$

2 GBP Forward I

$$S_0 = 1.3, T = 6, r = 1.6\%, q = 1.0\%$$

For zero-value the forward price should be: $F_0 = S_0 * e^{(r-q)T} = 1.3 * e^{(0.016-0.01)/2} \approx \$1.30391$. As the quoted price is lower than this, an arbitrage opportunity exists for a long forward contract.

At time = 0:

- Long forward to buy 1 M GBP in 6 months
- Loan e^{-qT} Million GBP and sell to receive $S_0 * e^{-qT}$ Million USD.
- Deposit the dollars for 6 months with interest rate r.

At time = T:

- Withdraw $S_0 * e^{(r-q)T}$ Million USD.
- Buy 1 Million GBP for forward price $\$1.30151/GBP$ and pay back loan

At this time, the profit equals the amount withdrawn from the account subtracted by the cost to pay back the loan:

$$S_0 * e^{(r-q)T} MUSD - 1.30151 MUSD \approx \$2395.855854$$

3 LIBOR FRA

$$r_1 = 1.6\%,\ r_2 = 2\%,\ t_1 = 6,\ t_2 = 18,\ L = \$1,000000$$

The interest rate to ensure a zero-cost FRA is given by:

$$r_K = \frac{r_2 t_2 - r_1 t_1}{t_2 - t_1} = \frac{0.02 * 18/12 - 0.016 * 6/12}{(18-6)/12} = 2.2\%$$

In 6 months: $r_M = 2.4\%,\ r = 2.0\%$

The value for the borrower at time t_2 is calculated as: $V(t_2) = L\left(e^{r_M(t_2-t_1)} - e^{r_K(t_2-t_1)}\right)$. To get the value of the FRA for the borrower at the time of settlement (t_1), this has to be discounted by the risk free interest rate:

$$V(t_1) = e^{-r(t_2-t_1)}V(t_2) = Le^{-r(t_2-t_1)}\left(e^{r_M(t_2-t_1)} - e^{r_K(t_2-t_1)}\right) =$$

$$= 1000000 * e^{-0.02}\left(e^{0.024} - e^{0.022}\right) \approx \$2006.009343$$

4 GBP Forward II

$$S_0 = \$1.3/GPB,\ r_b = 1.7\%,\ r_d = 1.5\%,\ q = 1\%$$

There are two arbitrage opportunities: If the price is low enough the arbitrager can long the forward and if the price is too high the arbitrager can short the forward.

- Long arbitrage profit: $S_0 e^{r_d - q}T - F_0 = 1.3e^{0.005} - F_0$
- Short arbitrage profit: $F_0 - S_0 e^{r_b - q}T = F_0 - 1.3e^{0.007}$

To eliminate both opportunities, the following must hold:

$$\begin{cases} F_0 \geq 1.3e^{0.005} \\ F_0 \leq 1.3e^{0.007} \end{cases}$$

This gives the answer as: $1.306516 \leq F_0 \leq 1.309132$

5 Put Arbitrage

$$K_1 = \$315,\ P_1 = \$5.60\ K_2 = \$317.5,\ P_2 = \$5.40$$

An arbitrage strategy for this situation would be to short the put option with the lower strike price, and buy the put option with the higher strike price.

At time 0: Short 315-puts and receive \$5.60/*option*. Buy equally many 317.5-puts for \$5.40/*option*. The payoff for a short put: $-(K-S)^+$, and for a long put: $(K-S)^+$. For this setup the combined P&L thus amounts to:

$$(K_2 - S)^+ - (K_1 - S)^+ + (P_1 - P_2) = (317.5 - S)^+ - (315 - S)^+ + 0.2$$

Considering the stock price S when exercising the option, there are three possibilities.

1. $S > \$317.5$. This means that none of the options are exercised and the P&L becomes:
   $P\&L = \$0.2$

2. $\$315 < S < \$317.5$. In this situation, only the 317.5-put is exercised:
   $P\&L = 317.5 - S + 0.2 > \$0.2$

3. $S < \$315$. Finally, this case means that both options are exercised:
   $P\$L = 317.5 - S - 315 + S + 0.2 = \$2.7$

Thus, no matter the share price S the strategy implies a profit of at least \$0.2/*option*