Name:

Let (a_n) and (b_n) be sequences of real numbers and suppose that

$$\lim_{n\to\infty} a_n = l \text{ and } \lim_{n\to\infty} b_n = m.$$

Set

$$c_n = a_n + 6b_n.$$

(a) Does the sequence (c_n) converge? If so what is $\lim_{n\to\infty} c_n$?

The sequence (c_n) converges to $l + 6m$

(b) Use the definition of a limit to prove carefully that your answer in part (a) is correct (do not use the limit rules).

Let $\epsilon > 0$. Since $\lim_{n\to\infty} a_n = l$ there exists an N_1 so that if $n \geq N_1$ then $|a_n - l| < \epsilon/2$. Since $\lim_{n\to\infty} b_n = m$ there exists an N_2 so that if $n \geq N_2$ then $|b_n - m| < \epsilon/12$. Set $N = \max\{N_1, N_2\}$. Then if $n \geq N$ we have

$$\begin{aligned}
|a_n + 6b_n - (l + 6m)| &\leq |a_n - l| + |6b_n - 6m| \\
&= |a_n - l| + 6|b_n - m| \\
&< \epsilon/2 + 6(\epsilon/12) \\
&= \epsilon/2 + \epsilon/2 = \epsilon.
\end{aligned}$$