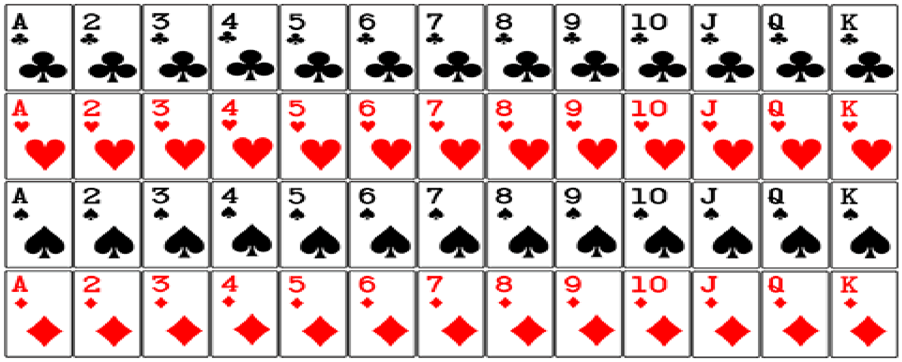
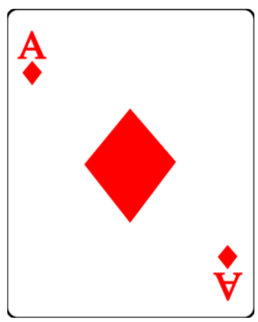
Subject: Statistics & Probability
Course code: STA-301

Define 52 play cards: standard deck **playing card** games. A "standard" deck of playing cards consists of 52 Cards in each of the 4 suits of Spades, Hearts, Diamonds, and Clubs.



We know 4 type of 52 play card:

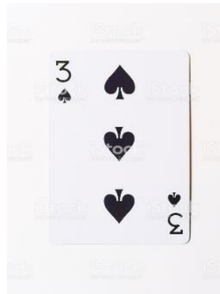
1. **Diamond:** Diamonds ♦ or ♦ (four-color deck) is one of the four suits of playing cards in the standard French deck.



2. **Hearts:** The object of the game is to either avoid taking tricks containing these cards, or to "shoot the moon" by capturing all of them. It is red heart symbol.

3. **Reversible:** Reversible is one of the four suits of playing cards in the standard French deck.

4. **Spade:** Spades is one of the four suits of playing cards in the standard French deck. It is a black heart.



Problem 1: A card is drawn from a pack of 52 cards find the probability that it is:
1. A red card?
2. A spade card?
3. An Ace?
4. Not a spade?
5. A king or queen?

Solution: When a card is drawn from a pack of 52 card, the total number of equality likely, naturally exclusive and exhaustive outcomes are 52.
That is here n = 52
We know,

$\text{Total play card} = 52$
$\text{Total red card} = 26$
$\text{Total spade card} = 13$
$\text{Total Reversible card} = 13$
$\text{Total Ace card} = 4$

1. Let A be the event of drawing a red card , there are 26 red card and 26 black card in a pack and any one of the red cards can be drawn in 26 ways that is m = 26. Then the probability of a red card is $= 26/52 = 1/2\ (Ans.)$
2. Lt B the event of drawing a spade. There are 13 spade in a pack of 52 cards that is m = 13
∴ Then the probability of a spade card $P(B) = {}^{13}/_{52} = {}^{1}/_{4}\ (Ans.)$
3. Let C be the event of drawing an ace. There are four aces in all, one of each suit. That is m = 4 then the probability of Ace $P(C) = \frac{4}{52} = \frac{1}{13}(Ans:)$
4. Let D be the event that the card is not a spades in 52 cards. Only 13 cards are spade and the remaining 39 cards are not spade ∴ m= 39
Probability of the card not spade is $P(D) = 39/52 = 3/4\ (Ans.)$
Or,

$$\mathrm{P}(D) = 1 - \mathrm{P(A)}$$
$$P(D) = 1 - \frac{1}{4} = \frac{3}{4}\ (Ans.)$$

5. Let E be the event that the card is a king or a queen out of 52 carda. There are 4 king and queens.

That means m = 8.
Hence A king or queen: $P(K + Q) = P(K) + P(Q)$

$$= \frac{4}{52} + \frac{4}{52}$$
$$= \frac{8}{52}$$

$$= \frac{3}{13} (Ans:)$$

Problem: Two fair coin are tossed simultaneously. Let A be the event head on first coin, and B be the event tail on the second coin. Show that A and B are Independent.

Solve: We construct a sample space for the above experiment.

S = {HH, HT, TH, TT}

Let, A denote event "head on the first coin" and B denote the event tail on the second coin.

A= {HH, HT} B= {TT, HT}

$$A \cap B = \{ HT \}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$$

We know that,

$$P(A \cap B) = P(A).P(B)$$

$$\frac{1}{4} = \frac{1}{2}.\frac{1}{2}$$

$$\frac{1}{4} = \frac{1}{4}$$

There, the event A and B are independent.

Problem: Three horses A, B, and C are in a race. A is twice as likely of win B and B is twice as likely of win B and B is twice as likely of win as C. what are their respective probabilities of winning that is $P(A), P(B)\ and\ P(C)$

<u>Solution</u>: Let,

$$P(C) = M\ then\ P(B) = 2.P(C)$$

$$= 2M$$

$$\therefore P(A) = 2.P(B) = 2.2M = 4M$$

Now, Let Probability of $P(A, B, C) = 1$

$$\therefore P(A) + P(B) + P(C) = 1$$

$$=> 4M + 2M + M = 1$$

$$=> 7M = 1$$

$$\therefore M = \frac{1}{7}$$

$$\therefore P(C) = \frac{1}{7}$$

$$\therefore P(B) = 2.P(C) = 2.\frac{1}{7} = \frac{2}{7}$$

$$\therefore P(A) = 2.P(B) = 2.\frac{2}{7} = \frac{4}{7}$$

$\therefore P(A) = \frac{4}{7}, P(B) = \frac{2}{7}, P(C) = \frac{1}{2}$ (Ans.)

<u>Additive Law</u>: If A and B are two event in event space then $P(A \cup B)$ = P(A) +P(B) –$P(A \cap B)$

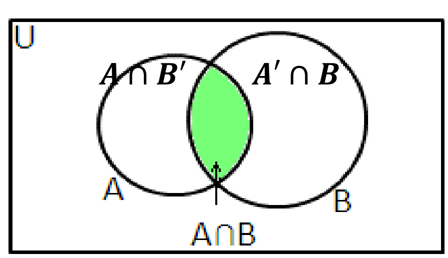
Proof: It is obvious from the Venn diagram

$$A = (A \cap B') \cup (A \cap B)$$

Therefore,

$$P(A) = P[(A \cap B') \cup (A \cap B)]$$

Since$A \cap B$ and $A \cap B'$ are mutually exclusive, then



$P(A) = P(A \cap B') + P(A \cap B)$ …………(1)

Similarly, $B = (A \cap B) \cup (A' \cap B)$ and

$P(B) = P(A \cap B) + P(A' \cap B)$…………… (2)

Since$A \cap B$ and $A' \cap B$ are mutually exclusive

Now, $A \cup B = (A \cap B') \cup (A \cap B) \cup (A' \cap B)$

Therefore, $P(A \cup B) = (A \cap B') + (A \cap B) + (A' \cap B)$ ……….(3)

Since $A \cap B'$,$A \cap B$ and $A' \cap B$ are mutually exclusive.

Combining (1) and (2) we have

$$P(A) + P(B) = P(A \cap B') + P(A \cap B) + P(A \cap B) + P(A' \cap B)$$
$$= P(A \cap B') + P(A \cap B) + P(A' \cap B) + P(A \cap B)$$
$$= (A \cup B) \cup (A \cap B)$$ [Using (3) we have

$$=> P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This theorem is known as additive law.

Probability

(A)Define the following terms with an example:

(i) Random Experiment (ii) Sample Space (iii) Mutually Exclusive Event (iv) Events (v) Out Come (vi) Experiment (vii) Random Experiment

Probability: A numerical measures of uncertainty of an event of an experiment is called probability.

If an experiment can result in ‘n’ exhaustive, mutually exclusive and equally likely outcomes and ‘m’ of these outcomes are favorable to an event A, then the probability of event A is defined as, $(A) = \frac{m}{n}$.

In tossing a die, $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$

Independent Event: Two Events E and F are said to be independent if and only if

$$P(E \cap F) = P(E).P(F)$$

Event: One or more outcomes constitute an event. An event is a subset of a sample space and usually denoted by A, B, X, Y, Z etc. S = {HH, HT, TH, TT}

Example: Let E is an event that appears head is tossing two coins. E = {HH}.

A = both coin shown similar face = {HH, HT}

B = At least one coin shown head is {HH, HT, TH}

Sample Space: The set of all possible outcomes of an experiment is called sample space. It is denoted by ‘S’.

Example:

(1) If the experiment consists of tossing two coins that

S = {HH, HT, TH, TT}

(2) If the experiment consists of tossing or throwing a die that

S= {1, 2, 3, 4, 5, 6}

Experiment: An experiment is an act that can be repeated under given condition.

Example: Tossing of a coin is a trial and getting head and tail are out comes.

Random experiment: Experiments are called random experiments if the outcomes depend on chance and cannot be predicted with certainty.

A collection of all possible outcomes of a random experiment is called a sample space.

Example: (1) Tossing so a coin (2) Throwing of a deicide.

Out comes: The results of an experiment are known as outcomes.

Example: Tossing of coin that S = {H, T}

<u>Mutually exclusive event:</u> If A and B be two events in event space, then they are said to be mutually exclusive if $A \cap B = \emptyset$. That is, two events are said to be mutually exclusive if they have no common points.

<u>Complementary Event:</u> Let A be an event in a sample space S. The probability of A under the condition that the event B has already been happened is called the condition of probability of A given B and if is written as

$$P\left({}^{A}/_{B}\right) = \frac{P(A\cap B)}{P(B)}, \text{ where } P(B) > 0 \text{ or } P(B) \neq 0$$

And $$P\left({}^{B}/_{A}\right) = \frac{P(A\cap B)}{P(A)}, \text{ where } P(A) > 0$$

Q. What is additive law of probability? Prove the additive law of probability.

<u>Additive law of Probability</u>: Let A and B be two not mutually exclusive events. Then the probability of happening two events A and B is P (A) and P (B) and happening two events A and B at a time is $P(A \cap B)$. The probability of happening any event from the two event is

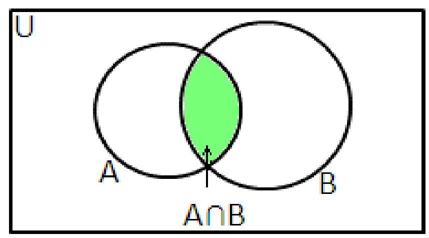
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This theorem is called additive law.

<u>Proof</u>: Let A and B are two not mutually exclusive events include in an experiment. The number of sample space S is n(S). The number of elements of events A, B and $A \cap B$ are n (A), n (B), and $n(A \cap B)$

Here, $$P\ (A) = \frac{n(A)}{n(S)}, P(B) = \frac{P(B)}{n(S)}, P(A \cap B) = \frac{n(A\cap B)}{n(S)}$$

From Venn diagram, we have,



$$P(A \cup B) = \frac{n(A\cup B)}{n(S)}$$

$$P(A \cup B) = \frac{n(A)+n(B)-n(A\cap B)}{n(S)}$$

$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A\cap B)}{n(S)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \text{ (Proved)}$$

<u>Example</u>: Two coins are tossed, A is the event getting two heads and B is the event second coin shows head. Evaluate $P(A \cup B)$

<u>Solution</u>: The Sample space for this experiment

S = {HH, HT, TH, TT}

And the events A, B and $A \cap B$ are

A = {HH}, B = {HH, TH}

$\therefore A \cap B = \{HH\}$ And the associated probabilities are

$$P(A) = {}^{1}/_{4},\ P(B) = \frac{2}{4} = \frac{1}{2} \text{ and } P(A \cap B) = \frac{1}{4}$$

Hence, $P(AUB) = P(A) + P(B) - P(A \cap B)$ (Ans.)

<u>Example</u>: Two coins are tossed together. Let A be the event that the first coin shows head and B be the event that at least one coin shows head. Evaluate $P(A \cup B)$.

<u>Solution</u>: The sample space for this experiment

S = {HH, HT, TH, TT}

Let A be the event that first coin shows head, then A = {HH, HT} $\therefore P(A) = \frac{2}{4} = \frac{1}{2}$

And Let B be the event that at least one coin shows head, then

B = {HH, HT, TH} $\therefore P(B) = \frac{3}{4}$

And $A \cap B = \{HH, HT\}$ $\therefore P(A \cap B) = \frac{2}{4} = \frac{1}{2}$

Hence,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$= \frac{1}{2} + \frac{3}{4} - \frac{1}{2}$

$= \frac{3}{4}$(Ans.)

Example: A hamburger chain found that 75% of all customers use mustered, 80% use ketch up and 65% use both. What is the probability that a customer will use at least one of these?

Solution: Let A be the event "customer uses muster" and B be the event "customer uses ketch up".

Thus we have P (A) $= \frac{75}{100} = 0.75$

$P\ (B)\ = \frac{80}{100} =\ 0.80$

$P(A \cap B) = \frac{65}{100} = 0.65$

Hence,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.75 + 0.80 - 0.65$$
$$= 0.90$$

Neither muster nor ketch up customer will use

$$P(A \cup B)' = 1 - P(A \cap B)$$
$$= 1 - 0.90$$
$$=\ 0.10$$
$= 10\%$(Ans.)

Example: A box containing 5 red and 6 blue balls. If three balls out of the box. What is the probability that two are and one is blue?

Solution: Total ball in the box containing 11 and out 3 balls from the box, then $n(S) =\ 11c_3 = 165$

The Number of events $n(E) = 5c_2 * 6c_1 = 60$

The Probability of the event $P(E) = \frac{n(E)}{n(S)} = \frac{60}{165} = \frac{12}{33}$ (ans.)

Example: Three fair coin are tossed simultaneously, write down the sample spade and hence calculate the probability of (i) at least head one will shows (ii) at most two head will shows
(iii) Tails on the last two coins. Evaluate $P(A \cup B)$. Solution: The sample space for the above experiment

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Let, A be the event that of least one head will shows

A = {HHH, HHT, HTH, HTT, THH, THT, TTH}, n (A) =7

$$\therefore P(A) = \frac{7}{8}$$

Let B be the event that at most two head will shows

B = {HHH, HHT, HTH, THH}, n (B) = 4

$\therefore P\ (B)\ = \frac{4}{8} = \frac{1}{2}$

Their intersection $A \cap B$ = {HHH, HHT, HTH, THH}, n (A B) = 4

$\therefore P(A \cap B) = \frac{4}{8} = \frac{1}{2}$

Hence, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{7}{8} + \frac{1}{2} - \frac{1}{2} = \frac{7}{8}$

Let, 'c' be the event that tails on the last two coins

C = {HTT, TTT}, n (c) =2

$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{2}{8} = \frac{1}{4}$(Ans.)

<u>Complementary event</u>: Let A be any event defined on a sample space S than the complementary of A, denoted by A', is the event consisting of all the sample points in S but not in A.

Q. Define conditional probability. Show that $P(A \cup B) = P(A/B)\,.\,P(B) = P(B/A)\,.\,P(A)$

<u>Solution</u>: Discuses the multiplication rule of conditional probability

<u>Conditional Probability</u>: Let A and B are far events, the probability of a under the condition that event B has already been happened is called the condition of probability of A given B if is written as $P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B/A) = \frac{P(A \cap B)}{P(A)}$,

<u>Multiplication rule</u>: If A and B are two events in event space of a given probability space (S, A, P [.]) And for given event A and B satisfying P (A) > 0 and P (B) > 0 then

$$P(A \cap B) = P(A)\,.\,P(B/A) = P(B)\,.\,P(A/B)$$

Proof: From the definition of condition of probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Then, $P(A \cap B) = P(B)\,.\,P(A/B)$………….. (1)

Similarly, $P(B/A) = \frac{P(A \cap B)}{P(A)}$

$\therefore P(A \cap B) = P(A)\,.\,P(B/A)$………………. (2)

Combining two equation (1) and (2) we have

$$P(A \cap B) = P(A)\,.\,P(B/A) = P(B)\,.\,P(A/B)$$

Hence the theorem as proves.

Give example of equally likely outcomes, exhaustive cases and mutually exclusive outcomes. Favorable cases.

<u>Mutually exclusive outcomes:</u> Outcomes are said to be mutually exclusive if the happening of any one of them precludes the happening of all others.

<u>Example</u>: In tossing a Coin, the outcomes head and tail are mutually exclusive.

<u>Equally Likely outcomes:</u> Outcomes or cases of a trail are said to be equally likely if have no reason to except any one rather than other.

<u>Example</u>: In tossing a fair coin, the outcomes head and tail are equally likely.

<u>Favorable outcomes:</u> The outcomes of an experiment are said to be favorable to an event if they entail the happening of the event.

<u>Example:</u> If throwing a die, the favorable outcomes of the even numbers on the faces of the die will be 2, 4, and 6.

<u>Exhaustive outcomes:</u> Outcomes of an experiment are said to be exhaustive if they include all possible outcomes.

<u>Example:</u> In throwing a die, exhaustive numbers of outcomes are 6. In tossing of a coin, exhaustive outcome are 3.

<u>Example</u>: Jony feels that the probability that he will pass mathematics is $2/3$ and statistics is $5/6$. If the probability that he will pass both course is $3/5$, what is the probability that he will pass at least one of the course?

Solution: Let M be the event that he will pass in mathematics

And S be the event that he will pass in statistics

$$P(S) = 5/6 \text{ And } P(M \cap B) = 3/5$$

The event $M \cup S$ means that at least one of M or S occurs.

Hence,

$$P(M \cup S) = P(M) + P(S) - P(M \cap S)$$

$$= {}^{2}/_{3} + {}^{5}/_{6} - {}^{3}/_{5}$$
$$= \frac{9}{10}\text{(Ans.)}$$

Question: In a bolt factory machine A produces 55% of the output and B produces the rest. On a average 10 items in 100 produced by machine are defective and 3 items in 500 produced by B are defective. In a day's run, the two machines produce 25000 items. An items is drawn of random from a day's output and is found to be defective. What is probability that defective item was produced by machine A? What is the probability the defective item was produced by B?

<u>SOLUTION:</u> $B1 =$ item produced by machine A,

$B2 =$ item produced by machine B,

$A =$ a defective item

We have, $P(B_1) = 55\% = 0.55$

$P(B_2) = 45\% = 0.45$

$$\therefore P({}^{A}/_{B1}) = \frac{P(A \cap B_1)}{P(B_1)} = \frac{10}{1000} = \frac{1}{100} = 0.01$$

And $P\,({}^{A}/_{B2}) = \frac{P(A \cap B_2)}{P(B_2)} = \frac{3}{500} = 0.006$

From Bayes theorem

$$P(B_k/A) = \frac{P(B_k).P(A/B_k)}{\sum_{i=1}^{n} P(B_i)\,P(A/B_i)}$$

Total Probability

$$\sum P(B_i)\,\mathrm{P}(\mathrm{A}/B_i) = P(A) = P(B_1)\,P\left(\frac{\mathrm{A}}{B_1}\right) + P(B_2)\,P\left(\frac{\mathrm{A}}{B_2}\right)$$
$$= .55 * 0.01 + .45 * 0.006 = 0.0055 + 0.0027$$
$$= 0.0082$$

$$P\,(B1/A) = \frac{P(B_1)\,P(A/B_1)}{\sum P(B_i)\,P(A/B_i)} = \frac{.55*0.01}{0.0082} = 0.67 = \frac{55}{82}$$
$$P(B_2/A) = \frac{P(B_2)\,P(A/B_2)}{\sum P(B_i)\,P(A/B_i)} = \frac{.45*0.006}{0.0082} = 0.33 = \frac{27}{82}$$

<u>Probability function of Poisson distribution:</u>

A discrete random variable X is said to have a Poisson distribution if is probability function is given by $P(x:\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}$ Where x=0, 1, 2---- ∞

Where e=2.71828 and λ is the parameter of the distribution which is the mean number of success.

<u>Derivation of Poisson distribution from binomial distribution:</u>

Poisson distribution com be derived from the binomial distribution under the following condition.

(i) P, the probability of success in a Bernoulli trial is very small that is p $\to 0$.

(ii) n, the number of trials is very large, That is n $\to \infty$

(iii) np= λ is finite constant, That is the average number of success is finite.

$$\therefore np = \lambda$$

$\triangleright\ P = \lambda / n$

$\triangleright\ q = 1 - \lambda / n$

$\triangleright\ q = 1 - p$

The probability function of binomial variant x with parameter n and p is

$$P(x;n,p) = \binom{n}{x} p^x q^{n-x}$$

$$= \frac{x!}{X!(n-x)!}\left(\frac{\lambda}{n}\right)^x \left(1 - \lambda/n\right)^{n-x}$$

$$= \frac{n!}{X!(n-x)!} \cdot \frac{\lambda^n}{n^x} \cdot \left(1 - \lambda/n\right)^n \cdot \left(1 - \lambda/n\right)^{-x}$$

$$= \frac{\lambda^x}{x!}\left(1 - \frac{\lambda}{n}\right)^{-x} . \left(1 - \frac{\lambda}{n}\right)^n . \frac{n!}{n^x (n-x)!}$$

For fixed x $\lim\limits_{n\to\infty}\left(1 - \frac{\lambda}{n}\right)^{-x} = 1$

$$And\ \lim_{n\to\infty} \frac{n!}{n^x\,(n-x)!} = \lim_{n\to\infty} \frac{n(n-1)(n-2).....[n-(x-1)](n-x)!}{n^x(n-x)!}$$

$$= \lim_{n\to\infty} = \frac{n^x\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right).......\left(1-\frac{x-1}{n}\right)}{n^x}$$

$$= \lim_{n\to\infty} n^x \left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right) \dots \dots . \left(1 - \frac{x-1}{n}\right)$$

Again, $\lim\limits_{n\to\infty}\left(1 - \frac{\lambda}{n}\right)^n = e^{\frac{-\lambda}{n}.n} = e^{-\lambda}$

Hence, $P(x;\, n,\, p) = \frac{e^{-\lambda}\,\lambda^x}{x!} = P(x;\lambda)$

Which is the probability function of poison distribution with parameter λ

Q: <u>Mean And Variance of poison distribution:</u>

<u>Mean and Variance are equal in Poisson distribution:</u>

<u>Proof</u>: The probability function of poison variate x with parameter λ is

$P(x, \lambda)\frac{e^{-\lambda}\ \lambda^x}{x!}$; x = 0, 1, 2………∞

<u>Mean</u>: $〚M'〛\_1 = E[x]$

$$= \sum x\, P(x,\lambda)$$

$$= \sum x\, . \frac{e^{-\lambda}\ \lambda^x}{x!} = \sum x\ \frac{e^{-\lambda}\ \lambda^x}{x(x-1)!}$$

$$= e^{-\lambda} \sum \frac{\lambda^x}{(x-1)!}$$

$M'_1 = e^{-\lambda} . \lambda \sum \frac{\lambda^{x-1}}{(x-1)!}$ is

$$= \lambda e^{-\lambda} e^{\lambda}$$

$$M'_1 = \lambda = M_1$$

Variance: $M\_2 = E[x^2] - [E(x)]^2$ ---------------------- (1)

Now $E[x]^2 = E[x(x-1)+x]$

$$= E[x(x-1)] + E[x]$$

$$= \sum x(x-1)\, P(x,\lambda) + \lambda$$

$$= \sum x(x-1)\, \frac{e^{-\lambda}\ \lambda^x}{x!} + \lambda$$

$$= e^{-\lambda} \sum x(x-1)\ . \frac{\lambda^x}{(x-1)(x-2)!} + \lambda$$

$$= e^{-\lambda}.\ \lambda^2 \sum \frac{\lambda^{x-2}}{(x-2)} + \lambda$$

$$= e^{-\lambda}.\ \lambda^2 . e^{\lambda} + \lambda$$

$$= \lambda^2 + \lambda$$

From (1) we have

Variance, $M_2 = \lambda^2 + \lambda - \lambda^2$

$$= \lambda$$

Hence mean and variance of Poisson distribution are equal.

Bernoulli trial:

A random experiment whose outcomes have been classified into two categories, called 'success' and Failure represented by letter s and f respectively is called a Bernoulli trial.

If p is the probability of success and q= 1-p is the probability failure of Bernoulli trial, Then the probability of x is

$$f(x = \{p\}, if\ \ x = 01$$

$$\{1-p\}; if\ \ x = 0$$

This probability function is called Bernoulli Probability function or the Bernoulli Probability distribution.

Bernoulli distribution:

A discrete random variable x is said to have a Bernoulli distribution if its probability function is given by

$$p(x,p) = p^x (1-p)^{1-x}\ For\ x = 0, 1$$

Where p is the parameter of the distribution satisfying

$0 \le p \le 1$. And $p + q = 1$

Binomial Distribution: A discrete random variable X is said to have a binomial distribution if its probability function is defined by

$$\text{P(x; n, p)} = \binom{n}{x} p^x q^{n-x} \text{ for } x = 0,1,2, \dots \dots ..n$$

Where the two parameters n and p satisfy 0≤P≤1 and n is a positive integer and q=1-P

If P is the probability of success in a Bernoulli trail. P remain the same from trail, then probability of success in n trails is

$$\text{P[X=x]} = \text{P(x; n, p)} = \binom{n}{x} p^x (1-P)^{n-x}$$

$$= \binom{n}{x} p^x q^{n-x}$$

1st raw moment,

Mean: $M'_1 = \text{E[x]} = \sum x \binom{n}{x} p^x q^{n-x}$

$= \sum x \frac{n!}{x!(n-x)!} p^x q^{n-x}$

$= \sum x \frac{n(n-1)!}{x(x-1)!(n-x)!} p^x q^{n-x}$

$= np \sum \frac{(n-1)!}{x(x-1)!(n-x)!} p^{x-1} q^{n-x}$

$= np \sum \binom{n-1}{x-1} p^{x-1} q^{n-x}$

$=$ np $(p+q)^{n-1}$

$=$ np$(1)^{n-1}$ [Since p+q=1]

= np

2nd raw moment,

Variance = E $[x^2]$ = $\sum x^2$p(x; np)

Here, E $[x^2]$ = E[x(x-1) +x]

= E[x(x-1)] +E[x]

=$\sum$x(x-1) p(x; n, p) +np

$\therefore M'_2 = \sum$ x (x-1)p(x;n,p)+np

$= \sum$ x (x-1) $\binom{n}{x} p^x q^{n-x}$+np

$= \sum$ x (x-1) $\frac{n!}{x!(n-x)!} p^x q^{n-x}$ + np

$=\sum$ x (x-1) $\frac{n(n-1)(n-2)!}{x(x-1)(x-2)!(n-x)!} p^2 . p^{x-2} q^{n-x}$ + np

$=n(n-1)p^2 \sum \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} q^{n-x}$ + np

$= n(n-1)p^2 \, (p+q)^{n-2}$+np

$= n(n-1)p^2 \, (1)^{n-2}$+np

$= n(n-1)p^2$ + np

∴The first moment about origin is the mean of the distribution that is mean= M_1=$M'_1 = np$

Variance is the second central moment of the distribution

Var[x] = M_2=$M'_2 - (M'_1)^2$

=n (n-1) p^2 +np-$(np)^2$

= $n^2 \, p^2 - np^2$ +np$-n^2 \, p^2$

= np(1-p)

= npq

∴The mean of the binomial distribution is always greater than it's variance.

Problem 1: A card is drawn from a pack of 52 cards find the probability that it is:

1. A red card?

2. A spade card?

3. An Ace?

4. Not a spade?

5. A king or queen?

Solution: When a card is drawn from a pack of 52 card, the total number of equality likely, naturally exclusive and exhaustive outcomes are 52.

That is here n = 52

We know,

Total play card = 52

Total red card = 26

Total spade card = 13

Total Reversible card = 13

Total Ace card = 4

1. Let A be the event of drawing a red card , there are 26 red card and 26 black card in a pack and any one of the red cards can be drawn in 26 ways that is m = 26. Then the probability of a red card is $= 26/52 = 1/2\ (Ans.)$

2. Lt B the event of drawing a spade. There are 13 spade in a pack of 52 cards that is m = 13

∴ Then the probability of a spade card $P(B) = {}^{13}/_{52} = {}^{1}/_{4}\ (Ans.)$

3. Let C be the event of drawing an ace. There are four aces in all, one of each suit. That is m = 4 then the probability of Ace $P(C) = \frac{4}{52} = \frac{1}{13}(Ans:)$

4. Let D be the event that the card is not a spades in 52 cards. Only 13 cards are spade and the remaining 39 cards are not spade ∴ m= 39

Probability of the card not spade is $P(D) = 39/52 = 3/4\ (Ans.)$

Or,

$$\mathrm{P}(D) = 1 - \mathrm{P(A)}$$

$$P(D) = 1 - \frac{1}{4} = \frac{3}{4}\ (Ans.)$$

6. Let E be the event that the card is a king or a queen out of 52 card. There are 4 king and queens.

That means m = 8.

Hence A king or queen: $P(K + Q) = P(K) + P(Q)$

$$= \frac{4}{52} + \frac{4}{52}$$

$$= \frac{8}{52}$$

$$= \frac{3}{13}(Ans:)$$

Problem: Three horses A, B, and C are in a race. A is twice as likely of win B and B is twice as likely of win B and B is twice as likely of win as C. what are their respective probabilities of winning that is $P(A), P(B)\ and\ P(C)$

Solution: Let,

$$P(C) = M\ then\ P(B) = 2.P(C)$$

$$= 2M$$

$$\therefore P(A) = 2.P(B) = 2.2M = 4M$$

Now, Let Probability of $P(A, B, C) = 1$

$$\therefore P(A) + P(B) + P(C) = 1$$

$$=> 4M + 2M + M = 1$$

$$=> 7M = 1$$

$$\therefore M = \frac{1}{7}$$

$$\therefore P(C) = \frac{1}{7}$$

$$\therefore P(B) = 2.P(C) = 2.\frac{1}{7} = \frac{2}{7}$$

$$\therefore P(A) = 2.P(B) = 2.\frac{2}{7} = \frac{4}{7}$$

$\therefore P(A) = \frac{4}{7}, P(B) = \frac{2}{7}, P(C) = \frac{1}{2}$ (Ans.)

Problem: Two coins are tossed, A is the event getting two heads and B is the event second coin shows head. Evaluate $P(A \cup B)$

<u>Solution</u>: The Sample space for this experiment

S = {HH, HT, TH, TT}

And the events A, B and $A \cap B$ are

A = {HH}, B = {HH, TH}

$\therefore A \cap B = \{HH\}$ And the associated probabilities are

$$P(A) = {}^{1}/_{4},\ P(B) = \frac{2}{4} = \frac{1}{2}\ and\ P(A \cap B) = \frac{1}{4}$$

Hence, $P(AUB) = P(A) + P(B) - P(A \cap B)$ (Ans.)

<u>Example</u>: Two coins are tossed together. Let A be the event that the first coin shows head and B be the event that at least one coin shows head. Evaluate $P(A \cup B)$.

<u>Solution</u>: The sample space for this experiment

S = {HH, HT, TH, TT}

Let A be the event that first coin shows head, then A = {HH, HT} $\therefore P(A) = \frac{2}{4} = \frac{1}{2}$

And Let B be the event that at least one coin shows head, then

B = {HH, HT, TH} $\therefore P(B) = \frac{3}{4}$

And $A \cap B = \{HH, HT\}$ $\therefore P(A \cap B) = \frac{2}{4} = \frac{1}{2}$

Hence,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{3}{4} - \frac{1}{2}$$

$= \frac{3}{4}$(Ans.)

Problem: Two fair coin are tossed Simultaneously , Let A be the event head on first coin and B be the Event tail on the Second coin , Show that A and B are Independent .

Solution : We Construct a sample space for the above Experiment : S={ HH, HT, TH, TT }

Let , A Denote event "Head on the first coin " & B Denote the Event "Tail in the Second Coin"

A={ HH,HT } B={ HT, TT }

$A \cap B = \{ HT \}$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$$

We know, For Independent

P$(A \cap B) = P(A) \,.\, P(B)$

L.H.S:

P$(A \cap B)$=$\frac{1}{4}$

R.H.S :

$P(A) \,.\, P(B)$

$\frac{1}{2} * \frac{1}{2} = \frac{1}{4}$

Hence, L.H.S = R.H.S

A and B are Independent .

Problem : Verify the Following Functions are probability function .

$$f(x) = \frac{2x-1}{8} \quad x = 0,1,2,3$$

Solution : Necessary Condition to be a Probability Function:

I. $f(x) \geq 0$
II. $\sum_x^n f(x) = 1$
III. $P(X = x) = f(x)$

Check The First Condition :

$$f(x) = \frac{2x-1}{8}$$

$$f(0) = \frac{-1}{8} \quad f(1) = \frac{1}{8} \quad f(2) = \frac{3}{8} \quad f(3) = \frac{5}{8}$$

Not Satisfied .

Check The Second Condition :

$$\sum_{x}^{n} f(x) = 1$$

$$f(0) + f(1) + f(2) + f(3)$$

$$\frac{-1}{8} + \frac{1}{8} + \frac{3}{8} + \frac{5}{8}$$

$$\frac{3+5}{8} = \frac{8}{8} = 1$$

Second Condition Satisfied.

Check The Thired Condition :

$$P(X = 0) = f(0) = \frac{-1}{8}$$

Third Condition Satisfied.

Hence, The Function f(x) is not a probability function.

Problem : Verify the Following Functions are probability function . (H.W)

$$f(x) = \frac{x+1}{16} \quad x = 0,1,2,3$$

Problem : The probability function of a discrete Random Variable X is

$$f(x) = \begin{cases} \alpha \left(\frac{3}{4}\right)^{x} & ; x = 0,1,2,3, \ldots \ldots . \propto \\ 0 & ; \quad Other \end{cases}$$

Evaluate : $\propto$ $and\ find\ P(x \leq 3)$

Solution : Since, $f(x)$ is Probability Function :

$$\sum_{x=0}^{\propto} f(x) = 1$$

$$f(0) = \propto \left(\frac{3}{4}\right)^{0} = \propto$$

$$f(1) = \propto \left(\frac{3}{4}\right)^1 = \propto \left(\frac{3}{4}\right)$$

$$f(2) = \propto \left(\frac{3}{4}\right)^2 = \propto \left(\frac{3}{4}\right)^2$$

$$f(3) = \propto \left(\frac{3}{4}\right)^3 = \propto \left(\frac{3}{4}\right)^3 \text{ and so on}$$

$$\sum_{x=0}^{\propto} f(x) = f(0) + f(1) + f(2) + f(3) + \cdots \ldots \ldots \ldots .$$

$$= \propto + \propto \left(\frac{3}{4}\right) + \propto \left(\frac{3}{4}\right)^2 + \propto \left(\frac{3}{4}\right)^3 + \cdots \ldots \ldots \ldots \ldots ..$$

$$= \propto \left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2\right]$$

We Know , $S_{\propto} = \frac{a}{1-r} = \propto . \frac{1}{1-\frac{3}{4}} = 4 \propto$

Since :

$\sum_{x=0}^{3} f(x) = 1$, $4 \propto = 1$, $\propto = \frac{1}{4}$ Ans

$$P(x \le 3) = \frac{1}{4}\left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3\right]$$

$$= \frac{1}{4}\left[1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64}\right]$$

Problem : A Random Variable X has the following from

$$f(x) = \begin{cases} kx , 0 < x < 4 \\ 0 , otherwise \end{cases}$$

I. Determaine k for which f(x) is density function .
II. Calculate P(1<x<2)
III. P(x>2)
IV. P(x<3)

Solution :

Condition of Density Function :

- $f(x) \ge 0$
- $\int_{-\infty}^{\infty} f(x)dx = 1$
- $P(a < x < b) = \int_a^b f(x)dx$

(a) For X to be density Function , We Have

$\int_{-\infty}^{\infty} f(x)dx = 1$

Thus: $\int_0^4 kx\ dx = 1$

$$k\int_0^4 x\ dx = 1$$

$$k\left[\frac{x}{2}\right]^4 = 1$$

$$k\left[\frac{16}{2} - 0\right] = 1$$

$$8k = 1$$

$$k = \frac{1}{8}$$

$$(b). \qquad P(1 < x < 2) = \int_1^2 kx\ dx$$

$$k\int_1^2 x\ dx$$

$$= k\left[\frac{x}{2}\right]^2$$

$$\frac{1}{8}\left[\frac{4}{2} - \frac{1}{2}\right]$$

$$\frac{1}{8}.\frac{4-1}{2}$$

$$\frac{1}{8}.\frac{3}{2} = \frac{3}{16}$$

2. $p(x > 2) = \int_2^4 \frac{1}{8}x\ dx$

$$= \frac{1}{8}\int_2^4 x\ dx$$

$$= \frac{1}{8}\left[\frac{x^2}{2}\right]^4$$

$$\frac{1}{8}\left[\frac{16}{2} - \frac{4}{2}\right]$$

$$= \frac{3}{4}$$

4. $p(x < 3) = \int_0^3 \frac{1}{8}x\ dx$

$$= \frac{1}{8}\int_0^3 x\ dx$$

$$= \frac{1}{8}\left[\frac{x^2}{2}\right]^3$$

$$= \frac{1}{8}(\frac{9}{2} - \frac{0}{2})$$

$$= \frac{9}{16} \ Ans\ .$$

Problem: A Continuous Random Variable has the following density function .

$$f(x) = \begin{cases} \frac{2}{27}(1+x); 2 < x < 5 \\ 0 \quad ; \quad otherwise \end{cases}$$

a. Verify that it Satisfies the condition $\int_{-\infty}^{\infty} f(x)dx = 1$

b. Find $P(x < 4)$&$P(x > 3)$

c. Find $P(3 < x < 4)$

Solution :

Condition of Density Function :

- $f(x) \geq 0$
- $\int_{-\infty}^{\infty} f(x)dx = 1$
- $P(a < x < b) = \int_{a}^{b} f(x)dx$

1, $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\int_{2}^{5} \frac{2}{27}(1+x)dx = 1$$

$$\frac{2}{27}\int_{2}^{5}(1+x)dx = 1$$

$$\frac{2}{27}\left[x + \frac{x^2}{2}\right]^5 = 1$$

$\frac{2}{27}[5 + \frac{25}{2} - (2 + 4/2)]$=1

$$\frac{2}{27}\left[5 + \frac{25}{2} - 2 - 2\right] = 1$$

$$\frac{2}{27}\left[5 + \frac{25}{2} - 4\right] = 1$$

$$\frac{2}{27}\left[\frac{10 + 25 - 8}{2}\right] = 1$$

$$\frac{2}{27} \cdot \frac{27}{2} = 1$$

Hence: $\int_{-\infty}^{\infty} f(x)dx = 1\ (\ Verified\ )$

2, $p(x < 4) = \int_{2}^{4} \frac{2}{27}(1+x)dx$

$$\frac{2}{27}\int_2^4 (1+x)dx$$

$$\frac{2}{27}\left[x+\frac{x^2}{2}\right]^4$$

$$\frac{2}{27}[\,4+\frac{16}{2}-(2+\frac{4}{2})\,]$$

$$\frac{2}{27}(\,4+8-2-2\,)$$

$$\frac{2}{27}\,.8=\frac{16}{27}\ Ans\,.$$

And , $P(x>3)=\int_3^5 \frac{2}{27}(1+x)dx$

$$\frac{2}{27}\int_3^5 (1+x)dx$$

$$\frac{2}{27}\left[x+\frac{x^2}{2}\right]^5$$

$$\frac{2}{27}\left[\,5+\frac{25}{2}-3-\frac{9}{2}\right]$$

$$\frac{2}{27}\left[\frac{10+25-6-9}{2}\right]$$

$$\frac{2}{27}*10=\frac{20}{27}\ Ans$$

3, $P(3<x<4)=\int_3^4 f(x)dx=\ \int_3^4 \frac{2}{27}\ (1+x)dx$

$$\frac{2}{27}\int_3^4 (1+x)dx$$

$$\frac{2}{27}\left[x+\frac{x^2}{2}\right]^4$$

$$\frac{2}{27}\left(4+\frac{16}{2}-3-\frac{9}{2}\right)$$

$$\frac{2}{27}\left(4+8-3-\frac{9}{2}\right)$$

$$\frac{2}{27}(\,1+8-\frac{9}{2})$$

$$\frac{2}{27}\left(9-\frac{9}{2}\right)$$

$$\frac{2}{27}(\frac{18-9}{2})$$

$$\frac{2}{27}\,.\frac{9}{2}=\ \frac{1}{3}\ Ans\,.$$

Page 1 : Problem : Let the Random Variable Variable X have the Following Probability Distribution .

x	-1	0	1
F(x)	0.2	0.3	0.5

Compute: $E(x\text{-}1)^2$ $E(x^2)$ and $E(3x+1)$

Solution :We Know: $E(X) = \sum_{x=-1}^{1}(\mathrm{x}-1)^2\ \mathrm{f(x)}$

$$(x-1)^2[\,f(-1)+f(0)+f(1)]$$

$$(x-1)^2 f(-1)+(x-1)^2 f(0)+(x-1)^2 f(1)$$

$$(-1-1)^2 f(-1)+(0-1)^2 f(0)+(1-1)^2 f(1)$$

$$-2^2 * 0.2 \pm 1^2 * 0.3 + 0^2 * 0.5$$

$$(4 * 0.2)+(1 * 0.3) = 1.1\ Ans$$

2, $E(x)^2 = \sum_{x=-1}^{1}\mathrm{x}^2\ \mathrm{f(x)} = -1^2 f(-1)+0^2 f(0)+1 f(1)$

$$= (1 * 0.2)+(0 * 0.3)+(1 * 0.5) = 0.7\ Ans\ .$$

3, $E(3x+1) = \sum_{x=-1}^{1}(3\mathrm{x}+1)\mathrm{f(x)}$

$$= \{3(-1)+1\} f(-1)+\{3 * 0+1\} f(0)+(3 * 1+1) f(1)$$

$$= (-3+1) * 0.2 + 1 * 0.3 + 4 * 0.5$$

$$(-2 * 0.2)+(1 * 0.3)+(4 * 0.5)$$

$$-0.4+0.3+2 = 1.9\ Ans\ .$$

Problem: Let the Random Variable X have the following Probability Distribution.

x	-1	0	1
F(x)	0.2	0.3	0.5

Compute: $E(2x), E(x+2)$

Solution : we know : $E(X) = \sum x f(x)$

Here, $E(2x) = 2E(x) = 2\sum_{x=-1}^{1} x\ f(x)$

$$= 2\{(-1 * 0.2)+(0 * 0.3)+(1 * 0.5)\}$$

$$= 2(-0.2+0+0.5) = \frac{3}{5} = 0.6\ Ans\ .$$

$$E(x+2) = \sum_{x=-1}^{1}(x+2) f(x)$$

$$= (-1+2) f(1)+(0+2) f(0)+(1+2) f(1)$$

$$= (1 * 0.2)+(2 * 0.3)+(3 * 0.5)$$

$$= 0.2 + 0.6 + 1.5 = 2.3\ Ans\ .$$

Problem : Let x denotes the number of spots showing on the face of a well balanced dice after it is rolled once . If $y = x^2 + 2x$

Evaluate E(x), E(y) and hence the Variance of x and y .

Solution : The Random variable x and y together with the Probability Distribution are Shown in the follow table :

Values of x	1	2	3	4	5	6
Values of y	3	8	15	24	35	48
F(x)=f(y)	1/6	1/6	1/6	1/6	1/6	1/6

$$E(X) = \sum_{x=1}^{6} xf(x) = \frac{1}{6}[1 + 2 + 3 + 4 + 5 + 6] = 3.5$$

$$E(Y) = \sum_{y=1}^{6} yf(y) = \frac{1}{6}[3 + 8 + 15 + 24 + 35 + 48] = 22.167$$

We Know :

$\sigma^2 = E(x^2) - \{E(x)\}^2$……………………..(1)

$\sigma^2 = E(y^2) - \{E(y)\}^2$ ……………………...(2)

$$E(x^2) = \sum_{x=1}^{6} x^2 f(x) = \frac{1}{6}[\ 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2\ ] = 15.167$$

$$E(y^2) = \sum_{y=1}^{6} y^2 f(y) = \frac{1}{6}[\ 3^2 + 8^2 + 15^2 + 24^2 + 35^2 + 48^2\ ] = 733.8333$$

1 no Eqation :

$$\sigma^2 = 15.167 - 3.5^2 = 2.917\ Ans\ .$$

2 no Eqation :

$$\sigma^2 = 733.833 - 22.167^2 = 242.454\ Ans\ .$$

Regression Equation , When Y depends on X is :

$Y-\bar{Y}=b_{yx}(x-x)$

.x=X-X

.y=y- $\bar{Y}$

Where: $b_{xy}=\frac{\sum xy}{\sum x^2}$

Regression Equation , When X depends on Y is :

X- =b_{yx}(x-x)

.x=X-X

.y=Y- Ȳ

Where: $b_{xy}=\frac{\sum xy}{\sum y^2}$

Problem: In the following table are recorded data Sharing the test Scores made by Salesman on an intelligence test and their Weekly Sales :

Salesman	1	2	3	4	5	6	7	8	9	10
Test Scores	40	70	50	60	80	50	90	40	60	60
Sales(R'S)	2.5	6.0	4.0	5.0	4.0	2.5	5.5	3.0	4.5	3.0

Calculate the regression eq^n of sales on test scores and estimate . The probable weekly sales if a salesman makes a score of 100 .

Solution: Let Sates be denoted by Y and test scores X , we have to fit a regression eq^n of Y an X , that is Y- Ȳ=b_{yx}=(X-X)

salesman	Test scores(X)	x=X-X	X^2	Sales Y	Y= Y- Ȳ	Y^2	xy
1	40	-20	400	2.5	-1.5	2.25	30
2	70	10	100	6.0	2	4	20
3	50	-10	100	4.0	0	0	0
4	60	0	0	5.0	1	1	0
5	80	20	400	4.0	0	0	0
6	50	-10	100	2.50	-1.5	2.25	15
7	90	30	900	5.50	1.5	2.25	45
8	40	-20	400	3.0	-1	1	20
9	60	0	0	4.50	0.5	0.25	0
10	60	0	0	3.00	-1	1	0

N=10	$\sum x = 600,$ $X = \frac{\sum x}{N} = \frac{600}{10} = 60$	$\sum x = 0$	$\sum x^2 = 2400$	$\sum y = 40, \bar{Y} = \frac{\sum y}{N} = 4.0$	$\sum y = 0$	$\sum y^2 = 14$	$\sum xy = 130$

Problem : Suppose that X is a Continuous random variable with probability density function .

$$f(x) = \begin{cases} 2x \,; 0 < x < 1 \\ 0 \quad ; otherwise \end{cases}$$

Find the expected value of X .

Sulution: Let X be a random variable with probability density function .

$$f(x) = \begin{cases} 2x \,; 0 < x < 1 \\ 0 \quad ; otherwise \end{cases}$$

Find the mean of 1, X 2, X^2 3,$\sqrt{X}$ 4, 1-X and $X^2 + 1 - X + \sqrt{X}$

1, By definition mean of X is : $E(x) = \int_a^b xf(x)dx$

$$E(X) = \int_0^1 x\,.2x\;dx = 2\int_0^1 x^2 = 2\left[\frac{x^3}{3}\right]^1 = 2\,.\frac{1}{3}$$

$$E(X) = \frac{2}{3}\;Ans.$$

2, $E[\,X^2\,] = \int_0^1 x^2.\,2x\;dx = 2\int_0^1 x^3 dx = 2.\left[\frac{x^4}{4}\right]^1 = \frac{2.1}{4}$

$$E(X^2) = \frac{2}{4} = \frac{1}{2}\;Ans\;.$$

3, Mean of $\sqrt{X}$ is :

$E\left[\sqrt{X}\right] = \int_0^1 \sqrt{x}\;2x\;dx = 2\int_0^1 x^{\frac{1}{2}}.\,x\;dx = 2\int_0^1 x^{\frac{1}{2}+1}\;dx$

$$2\int_0^1 x^{\frac{1+2}{2}}\,dx = 2\int_0^1 x^{\frac{2}{3}}dx = 2\left[\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right]^1$$

$2\left[\frac{x^{\frac{3+2}{2}}}{\frac{3+2}{2}}\right]^1 = 2\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]^1$

$$2\left[\frac{1^{\frac{5}{2}}}{\frac{5}{2}}\right]^1$$

$$2\left[\frac{1}{\frac{5}{2}}\right] = 2 * \frac{2}{5} = \frac{4}{5}\ Ans\ .$$

4, Mean of (1-x) is :

$$E(1-x) = 1 - E(X) = 1 - \frac{2}{3} = \frac{1}{3}$$

And, Mean of $X^2 + 1 - X + \sqrt{X}$ is :

$$E\left(X^2 + 1 - X + \sqrt{X}\right) = E(X^2) + 1 - E(X) + E(\sqrt{X})$$

$$= \frac{1}{2} + 1 - \frac{2}{3} + \frac{4}{5} - \frac{49}{30} = 1.63$$

Problem: Find the marginal density of x and y from the following join probability density function and verify that the marginal distribution are also probability distribution .

$$f(x,y) = \begin{cases} \frac{1}{8}(6-x-y); 0 < x < 2; 2 < y < 4 \\ 0 \qquad\qquad ; \qquad\qquad otherwise \end{cases}$$

Formula:

Marginal density of X is :

$$g(x) = \int_{-\infty}^{\infty} f(x,y)dy$$

Marginal density of Y is :

$$h(y) = \int_{-\infty}^{\infty} f(x,y)dx$$

Solution: By the definition of marginal densities of X :

$$g(x) = \int_{2}^{4} \frac{1}{8}(6-x-y)dy$$

$$\frac{1}{8}\int_{2}^{4} (6-x-y)dy$$

$$\frac{1}{8}\left[6y - xy - \frac{y^2}{2}\right]^4$$

$$\frac{1}{8}\{[24-4x-8] - [12-2x-2]\}$$

$$\frac{1}{8}\{16-4x-10+2x\}$$

$$\frac{1}{8}(6-2x)$$

$$\frac{6-2x}{8} = \frac{2(3-x)}{8} = \frac{3-x}{4}$$

$$\frac{1}{4}(3-x) for\ 0 < x < 2\ Ans\ .$$

By The definitation marginal densities of y .

$$h(y) = \int_0^2 \frac{1}{8}(6-x-y)dx$$

$$= \frac{1}{8}\int_0^2 (6-x-y)\ dx$$

$$= \frac{1}{8}\left[6x - \frac{x^2}{2} - yx\right]^2$$

$$= \frac{1}{8}\{(12-2-2y)-0\}$$

$$= \frac{1}{8}(10-2y)$$

$$= \frac{2(5-y)}{8} = \frac{5-y}{4} = \frac{1}{4}(5-y) for\ 2 < y < 4$$

Now we verify that g(x) and h(y) are probability distribution , it is clear that in the give range of the variables x and y . $g(x) \geq 0\ and\ h(y) \geq 0$

The other Condition to be fulfilled is that

$$\int_{-\infty}^{\infty} g(x)dx = 1\ and\ \int_{-\infty}^{\infty} h(y)dy = 1$$

Now $\int_0^2 g(x)dx = \frac{1}{4}\int_0^2 (3-x)dx = \frac{1}{4}\left[3x - \frac{x^2}{2}\right]^2 = 1$

Similarly, $\int_2^4 h(y)dy = \frac{1}{4}\int_2^4 (5-y)dy = \frac{1}{4}\left[5y - \frac{y}{2}\right]^4 = 1$