Discrete Mathematics DSBA

Classwork 1. Logic in Life and Mathematics

1. Split the following statements into connectives and logical atoms reasonably; then rewrite them using symbols:

a) Roses are red but violets are blue.

b) If at least one rose is not red, then it is not the case that violets are blue.

c) Neither roses are red nor John likes them.

d) If John likes roses and violets, then either Mary likes violets or John likes Mary.

e) The fact that roses are blue implies that Mary likes John only if John likes violets.

2. Let n be some fixed natural number. Which of the following statements are true assuming some knowledge of arithmetic? How does this depend on n?

a) if n is divisible by 4, then n is divisible by 2 or by 3;

b) if n is divisible by 2 and by 3, then n is divisible by 4;

c) if n is divisible by 4 but not by 2, then n equals 8;

d) if n is either odd or even, then n is greater than 3.

3. Which of the following statements (put in symbols) are tautologies? Construct truth tables or better try to avoid using them.

a) $((A \to B) \to A) \to A$;

b) $(A \to B) \to ((A \to \neg B) \to \neg A)$;

c) $\neg(A \to C) \vee ((B \wedge \neg C) \to (A \to (B \vee C)))$;

d) $(D \to (A \wedge C)) \to (\neg B \to ((C \wedge D) \to (A \vee B \vee \neg C)))$.

4. Prove the following logical equivalences:

a) $A \to B \equiv \neg A \vee B$;

b) $\neg(A \to B) \equiv A \wedge \neg B$;

c) $A \to B \equiv \neg B \to \neg A$

d) $A \to (B \to C) \equiv (A \wedge B) \to C$;

e) $\neg(A \vee B) \equiv \neg A \wedge \neg B$;

f) $\neg(A \wedge B) \equiv \neg A \vee \neg B$.

5. Put the following arguments in symbols and check their validity (i.e. whether you get a tautology):

a) If he doesn't tell her, she'll never find out. If she doesn't ask, he won't tell. She did find out. So she must have asked.

b) Unless taxes are raised, there will be a deficit. If there is a deficit, state services will be curtailed. Therefore, if taxes are raised, state services will not be curtailed.

6. Revisit the the statements from Problem 1. Translate them into logical symbolism now using predicates and quantifiers. Consider quantifier domains carefully.

7. Translate the following statements into logical symbolism with a reasonable choice of logical atoms:

a) Peter is either older than Paul or is wiser than any other man.

b) Each odd natural number is a sum of four square numbers at least one of which is odd.

c) The following system of inequalities has a solution in real numbers for some positive value of the parameter α:

$$\begin{cases} x^2 + \alpha y \geqslant 2 \\ \alpha x^5 + 2x - 3y = 0. \end{cases}$$

8. Consider the schematic map of a city's downtown.

	1	2	3	4	5
1	B S		H	S	
2	S			R	
3		R	S		
4		S B	H		B
5	H R		R		

Avenues running from the north to the south are crossed by streets which run from the west to the east. These are marked with their numbers; the letters at crossings denote local businesses. The statement $B(i, j)$ means that there is a Bank at the crossing of the i-th Ave and the j-th Street. Likewise for Hotels, Restaurants, and Supermarkets. Check if the following statements are true and translate them into/from logical symbolism:

a) There is a Hotel on each street where a Bank is.

b) There is an avenue with a Bank crossing a street that has neither Hotel nor Restaurant.

c) Every street has a Supermarket or crosses an avenue with a Bank.

d) There exists a Restaurant with no Bank on the same avenue.

e) $\exists n \forall m \, (S(n, m) \to \neg \exists k \, R(k, m))$.

f) $\forall n \exists m \, B(n, m) \lor \exists n \exists m \forall k \, (\neg H(n, m) \land (S(n, k) \lor S(k, m)))$.

g) $\exists m \forall n \forall k \, ((B(n, m) \land B(k, m)) \to n = k)$.

h) $\forall m \forall u \forall v \, (\exists x \exists y (B(u, m) \land R(v, m) \land H(x, m) \land S(y, m)) \to (\neg B(m, m) \land B(m, m)))$.

9. Put the following arguments in symbols and check their validity:

a) No animals are immortal. All unicorns are animals. Therefore some unicorns are not immortal.

b) There is a white unicorn and there is a tame unicorn. Therefore some white unicorns are tame.

c) Some students are studious. No student is unqualified. Therefore some unqualified students are not studious.

d) Nothing effective is easy. Something easy is popular. Therefore something popular is not effective.

e) Any fool could do that. I cannot do that. Therefore I am not a fool.

f) If anyone can solve this problem, some mathematician can solve it. Paul is a mathematician and cannot solve the problem. Therefore the problem cannot be solved.

Discrete Mathematics **DSBA**

Homework 1.

Please notice that each your answer should be supported by an argument.

1. Check whether the following statements are tautologies:

a) $((((A \to B) \to A) \to A) \to A) \to A$;

b) $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$;

c) $(A \to (C \land D)) \to (((A \to B) \land (E \to \neg D)) \to ((C \to B) \lor (D \land B \land \neg E)))$.

2. Prove the following logical equivalences:

a) $(A \land B) \lor C \equiv (A \lor C) \land (B \lor C)$;

b) $(A \land B) \lor A \equiv A \equiv A \land (A \lor B)$;

c) $\neg A \to B \equiv \neg B \to A$.

3. Alice has chosen a natural number x. She makes the following statements, of which exactly one is true. Which statement is true?

a) 12 is divisible by x;

b) $x = 4$ or $x = 11$;

c) if x is even, then $x = 6$;

d) $4 \leq x \leq 6$;

e) 22 is divisible by x but $x < 22$;

f) $x = 7$ or $x = 12$.

4. Consider the schematic map of another city's downtown.

	1	2	3	4	5
1	H			S	
2		H R		R	
3	S	B S	S	S	S H
4	B				B
5	B		R		S R

Translate the following statements into/from logical symbolism and check if they are true:

a) There is a street with at least two different Bank offices.

b) Only Restaurants and Supermarkets can share a crossing with a Hotel.

c) Every avenue with a Supermarket has a Restaurant as well.

d) If you mistake streets for avenues and vice versa, the map is still accurate.

e) $\exists m \forall n\, S(n, m) \to \forall n \exists m\, R(n, m)$.

f) $\exists i \exists j \exists n \exists m\, (B(i, j) \land S(i, m) \land S(n, j) \land B(n, m))$.

5. Put the following arguments in symbols and check their validity:

a) Only birds have feathers. No mammal is a bird. Therefore each mammal is featherless.

b) Everyone loves himself. Therefore someone is loved by somebody.

c) Any mathematician can solve this problem if anyone can. Paul is a mathematician and cannot solve the problem. Therefore the problem cannot be solved.

d) Anyone who can solve this problem is a mathematician. Paul cannot solve this problem. Therefore Paul is not a mathematician.

e) Anyone who can solve this problem is a mathematician. No mathematician can solve this problem. Therefore the problem cannot be solved.

6*. Suppose you are given with the following question when taking a multiple choice test :

What percentage of answers to this question are correct?

a) 50%;

b) 25%;

c) 0%;

d) 50%.

What would be your answer? Why?