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INVESTIGATING THE FUSION OF MASCON AND NEURAL NETWORK GRAVITY MODELS

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This paper addresses the asteroid gravity modeling problem, which poses significant challenges due to the irregular shapes and varying density distributions of these celestial bodies. Conventional global gravity approximations (e.g. spherical harmonics) struggle to accurately capture these complexities. This study proposes a novel approach by fusing mascon models with neural networks models. The key idea involves using a pre-trained mascon model as the foundation of the joint model. Subsequently, a neural network is fused to the previous model and trained to enhance gravity accuracy, particularly in regions where the mascon model exhibits limitations. Numerical results are then evaluated by comparing this new approach with past neural gravity modeling efforts, before being deployed within within the open-source Basilisk simulation framework thus being available for the entire astrodynamics community.

Asteroids, small rocky bodies orbiting the Sun, are known for their irregular shapes and non-uniform internal density distributions. These characteristics pose a significant challenge when attempting to accurately resolve a global gravity model within their vicinity. This issue is relevant across various mission phases, particularly when the spacecraft must operate at low-altitudes where non-Keplerian gravity perturbations are most significant. It is widely acknowledged that in these gravity-dominated regimes, careful attention must be paid to the gravity field as the complex dynamics can place the spacecraft on escape trajectories or collisions paths with the body.[1] Hence, precise gravity modeling is required to ensure safe mission operations. Furthermore, gravity models intended for on-board deployment to enable spacecraft autonomy must remain computationally lightweight and memory efficient.

Traditionally, gravity has been modeled using the spherical harmonics series expansion,[2] which efficiently encodes spatial variations in gravity field, particularly over long-wavelengths. However, spherical harmonics suffers from a significant drawback: divergence within the body's circumscribing sphere which becomes most apparent for elongated or irregularly-shaped asteroids. A widely adopted alternative is the polyhedral model,[3] which avoids divergence in this regime and better captures the dynamics induced by these irregular geometries. Despite this, this model still relies on a constant density assumption which may not be valid, and it necessitates looping over all polyhedron edges and faces which can lead to computational burdens for high-fidelity models.

Discretized approaches, such as mascon (mass concentrations) models, offer a potential remedy. Initially developed to model large mass anomalies of the Moon,[4] mascon models aggregate the gravitational contributions of distributed, discretized mass volumes — often represented as point

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masses. These mass distributions can be prescribed, such as arranging layers of point masses at the centroids of discretized elements,[5] or optimized directly to best match an available dataset.[6–10] Notably, solving the non-linear optimization of point mass placement significantly impacts accuracy,[7–9] particularly with a low number of masses (< 500). Moreover, even when carefully attention is paid to the optimization, mascon models may introduce large errors near the surface. To minimize the impact of these errors, this study turns to machine learning to offer a potential remedy.

Machine learning has revolutionized multiple fields within science and engineering within the last decade. As it relates to gravity modeling, multiple efforts have emerged to leverage machine learning. At a high-level these efforts seek to regress a mapping between position and acceleration using tools Extreme Learning Machines,[11] traditional neural networks,[12] physics informed neural networks,[13] and even neural density fields.[14]

Of these models, one of the matured approaches is the use of physics-informed neural networks (PINNs)[15] which provide a powerful framework to model solutions of complex differential equations from data alone. Martin and Schaub[13,16,17] have leveraged PINNs to solve the gravity modeling in small-body environments, most recently having introduced their third generation PINN gravity model (PINN-GM-III).[13] This latest model introduces a variety of modifications to maximize modeling acccuracy and reliability. On particular modification of note is the PINN-GM-III's ability to seamlessly fuse the neural gravity model with an analytic, point-mass model. This fusion ensures that the hybrid model maintains reliability even beyond the bounds of the training set by ensuring a smooth transition to a reliable gravity model when the network grows uncertain.

The present study aims to further develop the concept of fusing neural gravity representations with analytic models, but instead focuses attention on the use of mascon gravity models. In comparison to the single point-mass model utilized in Ref.13, the optimized mascon model offers a precise initial approximation of the asteroid's gravity field. This diminishes the neural network's burden of learning well-established behaviors through analytic models, allowing it to focus on deciphering challenging complex patterns within the field. For instance, mascon models maintain accuracy within the Brillouin sphere until very close to the surface. Nonetheless, standalone lightweight mascons (< 500 point masses) struggle to grasp intricate patterns, even with abundant data. Hence, it proves beneficial to enhance the pre-trained mascon with a neural network that introduces additional basis functions capable of unlocking the remaining intricate patterns (refer to Fig. 1 for preliminary results). The overall setup mirrors that of the PINN-GM-III model in Ref.13, wherein the total potential is expressed as $U = U_{\text{mascon}} + w_{nn}U_{\text{nn}}$. Here, the term w_{nn} denotes a weight function depending on the orbital radius, smoothly deactivating the neural network's contribution beyond dataset bounds.

The mascon-NN model is currently integrated into the Basilisk[i] astrodynamics simulation framework.[18] With the previous potential addition, the mascon contribution can be separately incorporated from the neural network, eliminating the necessity for redundant gradient computation. However, the integration of the neural network component presents certain challenges which will be investigated here. Moreover, to cater to the broad astrodynamics community, the neural network has been converted into an Open Neural Network Exchange (ONNX)[ii] format, which is the most versatile option for neural networks deployment. Nevertheless, ONNX does not support automatic differentiation during inference, complicating the integration of PINNs from Ref. 13. Currently, we

[i] https://hanspeterschaub.info/basilisk/
[ii] https://onnxruntime.ai

resort to finite differentiation of the potential network to bypass this challenge, but remain optimistic that alternative solutions can be found.

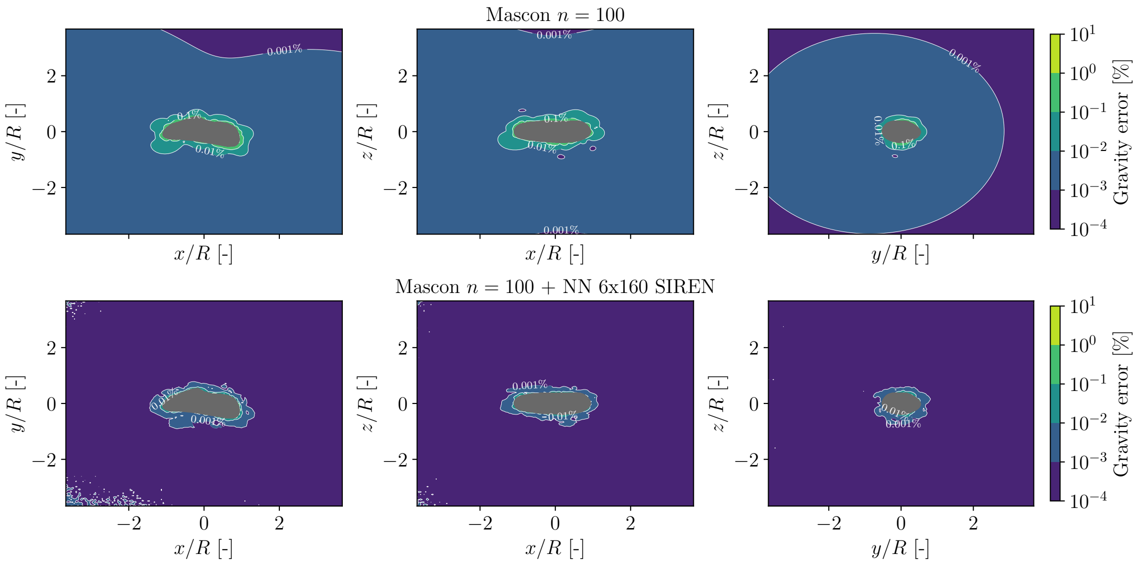

Figure 1. Top: Eros gravity errors with mascon model; bottom: Eros gravity errors with mascon-NN model.

Future work to be completed before the conference may seek to address the following questions: 1) when compared to a single point-mass model, what is the actual contribution of the pre-trained mascon model in the fused model accuracy and adjustment, and 2) given the difficulty of performing automatic differentiation online via ONNX models, what is the trade off (accuracy/computational speed/deployment complexity) of the physics-based neural network with respect to directly learning the gravity acceleration field? The answers to these questions aim to shed light on how analytic and numerical models can work together to provide a competitive gravity modeling option for the broader astrodynamics community.

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