

# QUIZ2 ADEI CURS 20-21Q1

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## Contents

Network KPI	1
Quiz 2 - Questions	1
Question 1	1
Question 2	3
Question 3	4
Question 4	5
Question 5	6
Question 6	6
Question 7	7
Question 8	9
Question 9	12
Question 10	13

## Network KPI

```
#### Loading the R package that will be used (previously installed)
# Clear plots
if(!is.null(dev.list())) dev.off()

## null device
##      1

rm(list = ls()) # Remove all objects
```

## Quiz 2 - Questions

### Question 1

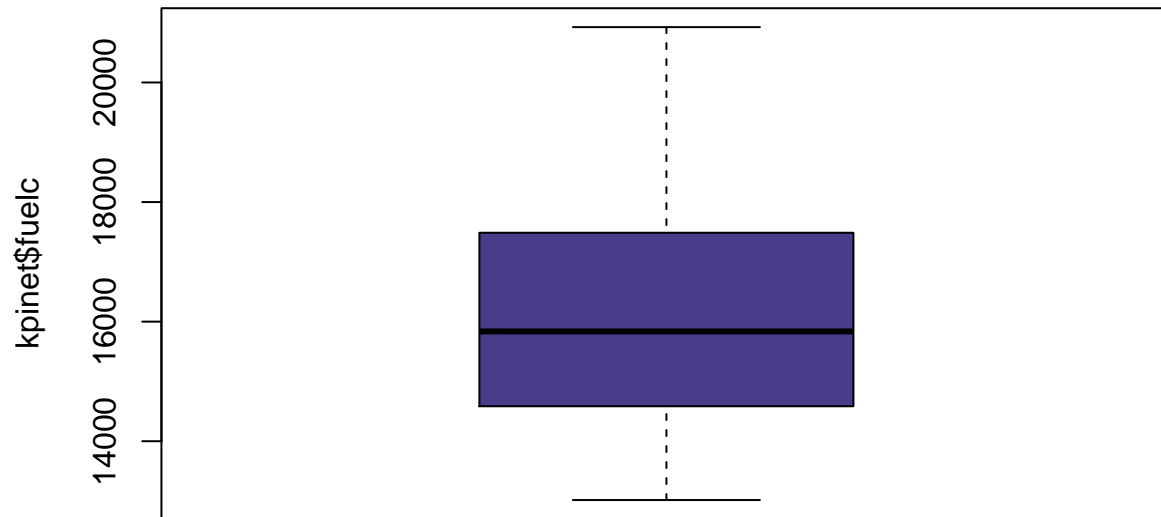
1. Variable *fuelc* contains the total fuel consumption in liters of the selected configuration. Summarize numerically and graphically the response variable. Make an interpretation of the results. Do you think that *fuelc* may be considered normally distributed?

- És evident que les dades no es distribueixen normalment, ja que veiem que, primerament, no són simètriques.
- El 50% de les mostres es troben entre els 14589 i els 17475 litres, ambdós quantils Q1 i Q3.
- La mitjana i la mediana no s'acosten, per tant declarem que hi ha asimetia.
- No hi ha valors atípics, ja que els valors mínims i màxims no són valors extrems.
- La prova de Shapiro rebutja la nul·la hipòtesi de normalitat.

```
summary(kpinet$fuelc)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    13016   14589   15837   16193   17475   20926
```

```
Boxplot(kpinet$fuelc,col="darkslateblue")
```



```
hist(kpinet$fuelc,freq=F,10,col="darkslateblue")
mm<-mean(kpinet$fuelc);dd<-sd(kpinet$fuelc);mm;dd
```

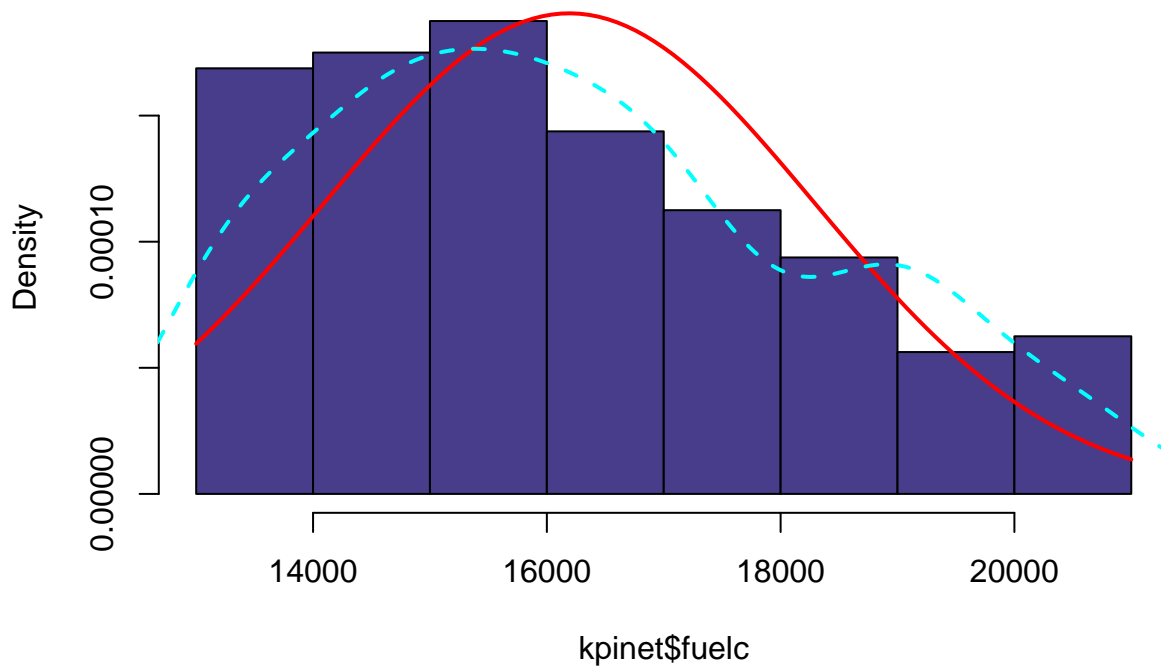
```
## [1] 16193.08
```

```
## [1] 2093.585
```

```
curve(dnorm(x,mm,dd),add=T,col="red",lwd=2)
lines(density(kpinet$fuelc),add=T,col="cyan",lwd=2,lty=2)
```

```
## Warning in plot.xy(xy.coords(x, y), type = type, ...): "add" is not a graphical
## parameter
```

## Histogram of kpinet\$fuelc



```
shapiro.test(kpinet$fuelc)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  kpinet$fuelc
## W = 0.95701, p-value = 7.594e-05
```

## Question 2

2. Which are the variables statistically associated with the target (fuelc)? Indicate the suitable measure of association and/or tests that support your answer.

Hem de respondre la pregunta mitjançant el mètode condes().

```
vars <- names(kpinet)[c(1:6,8:17,25)]; vars
```

```
## [1] "TD"      "GP"      "DP"      "PVD"     "TW"
## [6] "NS"      "density" "mflow"   "ttt.h"   "mtt.s.km"
## [11] "mdelay.s.km" "mspeed"  "ttdis"   "fuelc"   "co2"
## [16] "nox"     "thrputrate"
```

```
res.con <- condes(kpinet[,vars],which("fuelc"==colnames(kpinet[,vars])))
```

Les variables numèriques globals i relacionades positivament amb fuelc són co2, nox, ttt.h, ttdis, mflow, densitat, mtt.s.km i mdelay.s.km. mspeed està relacionada inversament.

```
res.con$quanti
```

```
##          correlation      p.value
## co2          0.9574843 3.312107e-87
## nox          0.9378166 1.688136e-74
```

```
## ttt.h          0.8980833 3.063657e-58
## ttdis          0.8695197 2.860851e-50
## mflow          0.7823336 2.633173e-34
## density        0.5482947 6.148990e-14
## mtt.s.km       0.1892277 1.655307e-02
## mdelay.s.km    0.1841043 1.978221e-02
## mspeed         -0.6013352 4.171215e-17
```

Les variables factorials que s'associen globalment a combustibles de significació alta a baixa són: penetració de la guia (GP), patró de demanda (DP), configuració del controlador (TD) i factor d'interval de finestra de temps (TW).

```
res.con$quali
```

```
##           R2          p.value
## GP 0.66951548 1.178435e-32
## DP 0.55134607 5.279693e-27
## TD 0.34626062 2.382023e-14
## TW 0.07297049 2.611225e-03
```

Les categories que mostren un efecte additiu positiu sobre la gran mitjana de combustible són: \* GP de 80, 90 i 100 \* TW de 1.5 min \* DP de 30% \* TD de 60-20-20.

En el costat negatiu: \* GP de 0, 10 i 50 \* TW de 3 min \* TD de 40-50-10 \* DP de 0%

```
res.con$category
```

```
##           Estimate      p.value
## DP=DP-30      1996.2021 1.959939e-14
## GP=GP-100     2493.6503 1.058729e-07
## GP=GP-90      2332.4238 6.959762e-07
## TD=TD-60-20-20 884.6062 7.497169e-04
## GP=GP-80      1238.3998 1.501337e-03
## TW=TW-1.5      696.4612 1.832814e-03
## TD=TD-40-40-20 486.5391 2.080387e-03
## TD=TD-20-70-10 576.5114 2.888558e-02
## DP=DP-10      -436.8683 1.305695e-02
## GP=GP-30      -1543.7091 5.868600e-03
## TW=TW-3        -644.9636 2.595581e-03
## GP=GP-0       -1822.8303 6.610523e-06
## GP=GP-50      -2521.5017 1.172978e-08
## TD=TD-40-50-10 -1947.6567 4.730290e-16
## DP=DP-0       -2342.3820 4.023510e-17
```

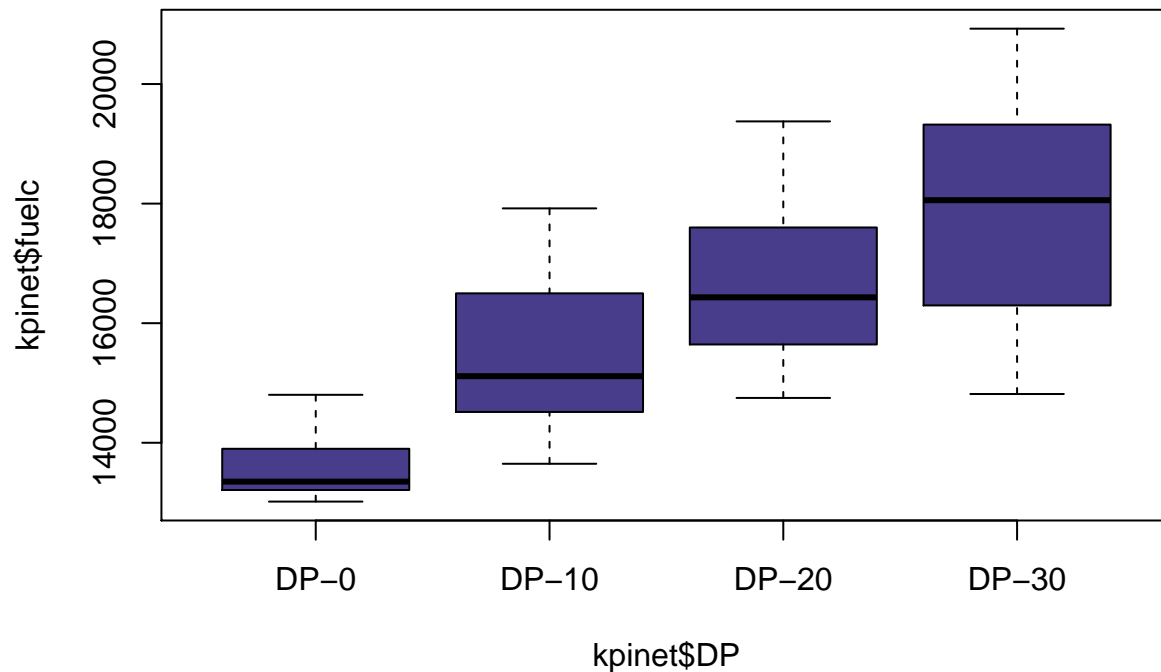
### Question 3

3. The average fuelc can be argued to be the same for all Demand Pattern levels (DP)? Which are the groups that show a significant greater average fuelc than the others?

Segons la prova no paramètrica de Kruskal sobre els mitjans de combustible segons els nivells de demanda (DP), es pot rebutjar clarament la hipòtesi nul·la de la mitjana uniforme.

Es mostra en un boxplot per fuelc segons els nivells de demanda (DP) i, a mesura que augmenta la demanda (number of trips), també augmenta el consum total de fuel.

```
Boxplot(kpinet$fuelc~kpinet$DP,col="darkslateblue")
```



```
kruskal.test(kpinet$fuelc~kpinet$DP)
```

```
##
## Kruskal-Wallis rank sum test
##
## data: kpinet$fuelc by kpinet$DP
## Kruskal-Wallis chi-squared = 94.008, df = 3, p-value < 2.2e-16
```

#### Question 4

4. Let us perform a one-way analysis of variance on TRB2018 dataset for target fuelc on Demand Pattern (DP) factor using the standard general linear model (*lm()* method). Assess the explicability of the model and interpret the model for prediction purposes writing the equation for each GP level.

El model que conté l'efecte brut del Demand Pattern (quantity of trips) explica el 55.13% de la variabilitat total del consum de combustible.

El consum de fuel (litres) a la demanda base es preveu que serà de 13579,5 litres, ja que la demanda augmenta en viatges de llarga distància un 10, 20 i 30%, el consum mitjà es preveu com  $(13579.5 + 1905.5 = 15485.0)$  litres,  $(13579.5 + 3125.4 = 16704.9)$  litres i  $(13579.5 + 4338.6 = 17918.1)$  litres, respectivament.

```
model_1 <- lm(
  fuelc~
  DP
  ,data=kpinet
); summary(model_1)
```

```
##
## Call:
## lm(formula = fuelc ~ DP, data = kpinet)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3103.0   -963.6   -229.6   1019.3   3007.8
```

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  13579.5      258.5  52.537 < 2e-16 ***
## DPDP-10      1905.5      341.9   5.573 1.08e-07 ***
## DPDP-20      3125.4      341.9   9.140 3.16e-16 ***
## DPDP-30      4338.6      327.0  13.270 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1416 on 156 degrees of freedom
## Multiple R-squared:  0.5513, Adjusted R-squared:  0.5427
## F-statistic: 63.9 on 3 and 156 DF, p-value: < 2.2e-16
tapply(kpinet$fuelc,kpinet$DP,mean)

##      DP-0      DP-10      DP-20      DP-30
## 13579.54 15485.05 16704.97 17918.12
```

## Question 5

5. Use an inferential method to quantify the probability of the null hypothesis of a negligible gross effect for demand pattern (DP) factor.

Clarament, una prova de Fisher és adequada per provar per calcular el valor de la hipòtesi. El valor P és molt petit (l'estadística de Fisher és de 63.9, molt gran) rebutjant així la hipòtesi nul·la: el patró de demanda (quantity of trips) afecta significativament el consum total de combustible de la network.

```
model_2 <- lm(
  fuelc~
  1
  ,data=kpinet
)
model_3 <- lm(
  fuelc~
  DP
  ,data=kpinet
)

anova(model_2,model_3)

## Analysis of Variance Table
##
## Model 1: fuelc ~ 1
## Model 2: fuelc ~ DP
##   Res.Df      RSS Df Sum of Sq    F    Pr(>F)
## 1     159 696912403
## 2     156 312672489   3 384239914 63.902 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## Question 6

6. Let us perform a two-way analysis of variance on TRB2018 dataset for target fuelc on Guidance Penetration (GP) and Demand Pattern (DP) factors using the standard general linear model (lm() method). Are interactions needed for GP and DP to explain the total fuel consumption in the network? Are net effects for

*both factors significant?*

Segons la producció d'Anova(model\_2), els efectes nets d'ambdós factors són significatius.

De totes maneres, donat que els models additius GP+DP i GP\*DP complets estan nested, es pot utilitzar una prova de Fisher per comprovar la necessitat d'interaccions: ja que el valor de p és inferior a 0,05 l'indica normal, ambdós models no són equivalents, per tant les interaccions (el poques que es puguin estimar) contribueixen a explicar el consum total de fuel.

```
model_4 <- lm(
  fuelc~
  GP+DP
  ,data=kpinet
)
model_5 <- lm(
  fuelc~
  GP*DP
  ,data=kpinet
)

anova(model_4,model_5)

## Analysis of Variance Table
##
## Model 1: fuelc ~ GP + DP
## Model 2: fuelc ~ GP * DP
##   Res.Df      RSS Df Sum of Sq    F    Pr(>F)
## 1     148 37025155
## 2     136 18597690 12  18427465 11.23 2.288e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Anova(model_4)

## Anova Table (Type II tests)
##
## Response: fuelc
##           Sum Sq Df F value    Pr(>F)
## GP          275647334    8  137.73 < 2.2e-16 ***
## DP          193293608    3  257.55 < 2.2e-16 ***
## Residuals   37025155 148
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## Question 7

7. Assess the explicability of the BEST model selected in question 6 and calculate the point and 95% confidence prediction interval for fuel consumption for an scenario with a guidance penetration of 80% and demand level of 30%. Try to explain the reason of warning messages appearing for some methods depending on the selected mode.

El disseny experimental no permet estimar les interaccions entre els factors GP i DP, de manera que no es poden determinar moltes de les variables fictícies. Per als nivells particulars de 80% GP i 20% DP, la predicció puntual és:  $13145.04 + 1490.20 + 2529.35 + 1964.15 = 19128.74$  litres.

L'interval de confiança del 95% per al consum total és de  $[18361.75 \ 19895.72]$ , en litres.

```
summary(model_5)
```

```
##
## Call:
## lm(formula = fuelc ~ GP * DP, data = kpinet)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1489.79  -177.73   13.64   174.16  1109.10
##
## Coefficients: (12 not defined because of singularities)
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    13145.04     116.94  112.409 < 2e-16 ***
## GPGP-10         478.00      202.54   2.360 0.019698 *
## GPGP-20        1189.67      165.38   7.194 3.83e-11 ***
## GPGP-30         987.19      233.88   4.221 4.42e-05 ***
## GPGP-50         372.26      150.97   2.466 0.014914 *
## GPGP-70        3172.85      202.54  15.665 < 2e-16 ***
## GPGP-80        1490.20      202.54   7.357 1.60e-11 ***
## GPGP-90        4633.45      202.54  22.876 < 2e-16 ***
## GPGP-100       4864.97      202.54  24.019 < 2e-16 ***
## DPDP-10        -346.56      286.44  -1.210 0.228428
## DPDP-20        1748.55      202.54   8.633 1.41e-14 ***
## DPDP-30        2529.35      165.38  15.294 < 2e-16 ***
## GPGP-10:DPDP-10 1286.17      330.75   3.889 0.000157 ***
## GPGP-20:DPDP-10      NA         NA      NA      NA
## GPGP-30:DPDP-10   452.35      369.79   1.223 0.223352
## GPGP-50:DPDP-10  1508.62      344.26   4.382 2.33e-05 ***
## GPGP-70:DPDP-10  -63.27      309.39  -0.205 0.838267
## GPGP-80:DPDP-10      NA         NA      NA      NA
## GPGP-90:DPDP-10  -749.18      330.75  -2.265 0.025089 *
## GPGP-100:DPDP-10      NA         NA      NA      NA
## GPGP-10:DPDP-20   293.63      309.39   0.949 0.344269
## GPGP-20:DPDP-20  -452.32      286.44  -1.579 0.116639
## GPGP-30:DPDP-20      NA         NA      NA      NA
## GPGP-50:DPDP-20      NA         NA      NA      NA
## GPGP-70:DPDP-20 -1136.12      309.39  -3.672 0.000345 ***
## GPGP-80:DPDP-20   994.39      309.39   3.214 0.001635 **
## GPGP-90:DPDP-20  -532.41      309.39  -1.721 0.087557 .
## GPGP-100:DPDP-20 -1492.47      309.39  -4.824 3.72e-06 ***
## GPGP-10:DPDP-30      NA         NA      NA      NA
## GPGP-20:DPDP-30      NA         NA      NA      NA
## GPGP-30:DPDP-30      NA         NA      NA      NA
## GPGP-50:DPDP-30      NA         NA      NA      NA
## GPGP-70:DPDP-30      NA         NA      NA      NA
## GPGP-80:DPDP-30  1964.15      261.48   7.512 6.99e-12 ***
## GPGP-90:DPDP-30      NA         NA      NA      NA
## GPGP-100:DPDP-30      NA         NA      NA      NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 369.8 on 136 degrees of freedom
## Multiple R-squared:  0.9733, Adjusted R-squared:  0.9688
## F-statistic: 215.7 on 23 and 136 DF,  p-value: < 2.2e-16
```



```

predict(model_5,newdata=data.frame(GP="GP-80",DP="DP-30"))

## Warning in predict.lm(model_5, newdata = data.frame(GP = "GP-80", DP =
## "DP-30")): prediction from a rank-deficient fit may be misleading
##          1
## 19128.73

predict(model_4,newdata=data.frame(GP="GP-80",DP="DP-30"))

##          1
## 18772.07

predict(model_5,newdata=data.frame(GP="GP-80",DP="DP-30"),interval="prediction")

## Warning in predict.lm(model_5, newdata = data.frame(GP = "GP-80", DP =
## "DP-30"), : prediction from a rank-deficient fit may be misleading
##          fit          lwr          upr
## 1 19128.73 18361.75 19895.72

predict(model_4,newdata=data.frame(GP="GP-80",DP="DP-30"),interval="prediction")

##          fit          lwr          upr
## 1 18772.07 17754.11 19790.02

```

## Question 8

8. Consider a general linear modeling for total fuel consumption when all numeric variables included in the list of KPIs are taken into account. Assess explicability, try to reduce the number of explanatory variables without losing too much explicability and discuss colinearity among explanatory variables.

Les variables explicatives es correlacionen entre elles.

L'explicabilitat segons el coeficient de determinació és del 99,93%, de manera que explica el 99,93% de la variabilitat total en combustible, però el model no és adequat com es pot veure amb vif().

El mètode step(model\_6) no ens ajuda a definir un model adequat, ja que no s'elimina cap variable.

S'ha de proposar un model que contingui 2 o 3 variables no correlacionades.

```

model_6 <- lm(
  fuelc~
  co2+
  nox+
  ttt.h+
  ttdis+
  mflow+
  density+
  mtt.s.km+
  mdelay.s.km+
  mspeed
, data=kpinet
); summary(model_6)

##
## Call:
## lm(formula = fuelc ~ co2 + nox + ttt.h + ttdis + mflow + density +
##      mtt.s.km + mdelay.s.km + mspeed, data = kpinet)
##

```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -161.269  -37.072    0.229   29.150  132.714
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.093e+04  1.919e+03   5.694 6.34e-08 ***
## co2          1.994e-04  3.838e-05   5.195 6.56e-07 ***
## nox          1.994e-02  1.600e-02   1.246  0.21454
## ttt.h        1.614e+00  7.433e-02  21.721 < 2e-16 ***
## ttdis        1.117e-01  9.255e-03  12.068 < 2e-16 ***
## mflow       -1.562e-01  1.349e-02 -11.583 < 2e-16 ***
## density     -3.061e+02  3.203e+01  -9.555 < 2e-16 ***
## mtt.s.km     -1.035e+02  1.979e+01  -5.231 5.58e-07 ***
## mdelay.s.km  9.930e+01  1.961e+01   5.065 1.18e-06 ***
## mspeed      -9.357e+01  3.532e+01  -2.649  0.00893 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 56.56 on 150 degrees of freedom
## Multiple R-squared:  0.9993, Adjusted R-squared:  0.9993
## F-statistic: 2.419e+04 on 9 and 150 DF,  p-value: < 2.2e-16
```

```
vif(model_6)
```

```
##           co2           nox           ttt.h           ttdis           mflow           density
## 1362.66468 1419.05991 100.13455 216.58627 36.40855 154.51691
## mtt.s.km mdelay.s.km mspeed
## 12746.72939 12705.61565 96.00476
```

```
step(model_6)
```

```
## Start: AIC=1300.94
## fuelc ~ co2 + nox + ttt.h + ttdis + mflow + density + mtt.s.km +
## mdelay.s.km + mspeed
##
##              Df Sum of Sq    RSS    AIC
## - nox          1      4969 484743 1300.6
## <none>          0      479774 1300.9
## - mspeed       1     22447 502221 1306.3
## - mdelay.s.km  1     82048 561822 1324.2
## - co2          1     86335 566108 1325.4
## - mtt.s.km     1     87521 567295 1325.8
## - density      1    291986 771759 1375.0
## - mflow        1    429113 908887 1401.2
## - ttdis        1    465835 945608 1407.5
## - ttt.h        1   1509042 1988816 1526.5
##
## Step: AIC=1300.59
## fuelc ~ co2 + ttt.h + ttdis + mflow + density + mtt.s.km + mdelay.s.km +
## mspeed
##
##              Df Sum of Sq    RSS    AIC
## <none>          0      484743 1300.6
## - mspeed       1     20803 505546 1305.3
```

```
## - mdelay.s.km 1 103909 588652 1329.7
## - mtt.s.km 1 110042 594785 1331.3
## - density 1 340603 825346 1383.7
## - mflow 1 491665 976408 1410.6
## - co2 1 509966 994709 1413.6
## - ttdis 1 519488 1004231 1415.1
## - ttt.h 1 1510949 1995692 1525.0

##
## Call:
## lm(formula = fuelc ~ co2 + ttt.h + ttdis + mflow + density +
## mtt.s.km + mdelay.s.km + mspeed, data = kpinet)
##
## Coefficients:
## (Intercept) co2 ttt.h ttdis mflow density
## 1.145e+04 2.409e-04 1.615e+00 1.145e-01 -1.608e-01 -2.861e+02
## mtt.s.km mdelay.s.km mspeed
## -1.109e+02 1.066e+02 -8.973e+01

names(kpinet)

## [1] "TD" "GP" "DP" "PVD" "TW"
## [6] "NS" "replica" "density" "mflow" "ttt.h"
## [11] "mtt.s.km" "mdelay.s.km" "mspeed" "ttdis" "fuelc"
## [16] "co2" "nox" "tvehin" "tvehout" "virwait"
## [21] "virlostin" "virlostout" "inputveh" "demand" "thruptrate"
## [26] "waitrate"

round(cor(kpinet[,c(15,16,17,10,14,9,8,11,12,13)]),dig=2)

## fuelc co2 nox ttt.h ttdis mflow density mtt.s.km mdelay.s.km
## fuelc 1.00 0.96 0.94 0.90 0.87 0.78 0.55 0.19 0.18
## co2 0.96 1.00 1.00 0.96 0.71 0.65 0.76 0.42 0.42
## nox 0.94 1.00 1.00 0.96 0.67 0.62 0.80 0.47 0.47
## ttt.h 0.90 0.96 0.96 1.00 0.59 0.62 0.82 0.59 0.59
## ttdis 0.87 0.71 0.67 0.59 1.00 0.91 0.10 -0.29 -0.29
## mflow 0.78 0.65 0.62 0.62 0.91 1.00 0.16 -0.13 -0.13
## density 0.55 0.76 0.80 0.82 0.10 0.16 1.00 0.89 0.88
## mtt.s.km 0.19 0.42 0.47 0.59 -0.29 -0.13 0.89 1.00 1.00
## mdelay.s.km 0.18 0.42 0.47 0.59 -0.29 -0.13 0.88 1.00 1.00
## mspeed -0.60 -0.78 -0.81 -0.86 -0.14 -0.18 -0.96 -0.88 -0.88
## mspeed
## fuelc -0.60
## co2 -0.78
## nox -0.81
## ttt.h -0.86
## ttdis -0.14
## mflow -0.18
## density -0.96
## mtt.s.km -0.88
## mdelay.s.km -0.88
## mspeed 1.00
```

## Question 9

9. Consider linear models including as explanatory numeric variables total travel distance (*ttdis*) and mean speed (*mspeed*) and the main effect for guidance penetration factor. Discuss the model, while justifying the choice (nor residuals or influent data analysis has to be considered).

El model especificat explica el 99,5% de la variabilitat del consum de fuel, per tant és aparentment un model molt bo. Tots els efectes nets són significatius i el model no es pot reduir mitjançant el mètode step(model\_7). Els vifs són inferiors a 3, de manera que no hi ha problemes de colinearitat.

```
model_7 <- lm(
  fuelc~
    ttdis+
    mspeed+
    GP
  ,data=kpinet
); summary(model_7)

##
## Call:
## lm(formula = fuelc ~ ttdis + mspeed + GP, data = kpinet)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -383.2  -104.3    -7.4   103.6   483.8
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.697e+04  4.458e+02  38.070 < 2e-16 ***
## ttdis         2.128e-01  3.431e-03  62.015 < 2e-16 ***
## mspeed       -8.553e+02  1.574e+01 -54.338 < 2e-16 ***
## GPGP-10       7.685e+01  5.390e+01   1.426   0.156
## GPGP-20       5.783e+01  5.966e+01   0.969   0.334
## GPGP-30       7.589e+01  7.122e+01   1.066   0.288
## GPGP-50       3.949e+02  6.776e+01   5.827 3.35e-08 ***
## GPGP-70       4.833e+02  7.349e+01   6.577 7.64e-10 ***
## GPGP-80       3.608e+02  7.697e+01   4.688 6.17e-06 ***
## GPGP-90       5.993e+02  7.975e+01   7.514 4.92e-12 ***
## GPGP-100      5.918e+02  7.415e+01   7.981 3.57e-13 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 153.3 on 149 degrees of freedom
## Multiple R-squared:  0.995, Adjusted R-squared:  0.9946
## F-statistic: 2952 on 10 and 149 DF, p-value: < 2.2e-16

vif(model_7)

##              GVIF Df GVIF^(1/(2*Df))
## ttdis  4.052151  1      2.012996
## mspeed 2.596127  1      1.611250
## GP     9.395873  8      1.150293

step(model_7)

## Start:  AIC=1620.89
## fuelc ~ ttdis + mspeed + GP
```

```
##
##           Df Sum of Sq      RSS      AIC
## <none>                3499920 1620.9
## - GP           8   4190700  7690620 1730.8
## - mspeed       1  69354224  72854144 2104.6
## - ttdis        1  90337813  93837733 2145.1

##
## Call:
## lm(formula = fuelc ~ ttdis + mspeed + GP, data = kpinet)
##
## Coefficients:
## (Intercept)          ttdis          mspeed      GPGP-10      GPGP-20      GPGP-30
## 16970.0053         0.2128       -855.2664       76.8523       57.8301       75.8852
##      GPGP-50      GPGP-70      GPGP-80      GPGP-90      GPGP-100
## 394.8798      483.2836      360.8376      599.2620      591.8391

Anova(model_7)

## Anova Table (Type II tests)
##
## Response: fuelc
##           Sum Sq Df F value    Pr(>F)
## ttdis      90337813  1 3845.897 < 2.2e-16 ***
## mspeed     69354224  1 2952.575 < 2.2e-16 ***
## GP         4190700   8   22.301 < 2.2e-16 ***
## Residuals  3499920 149
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

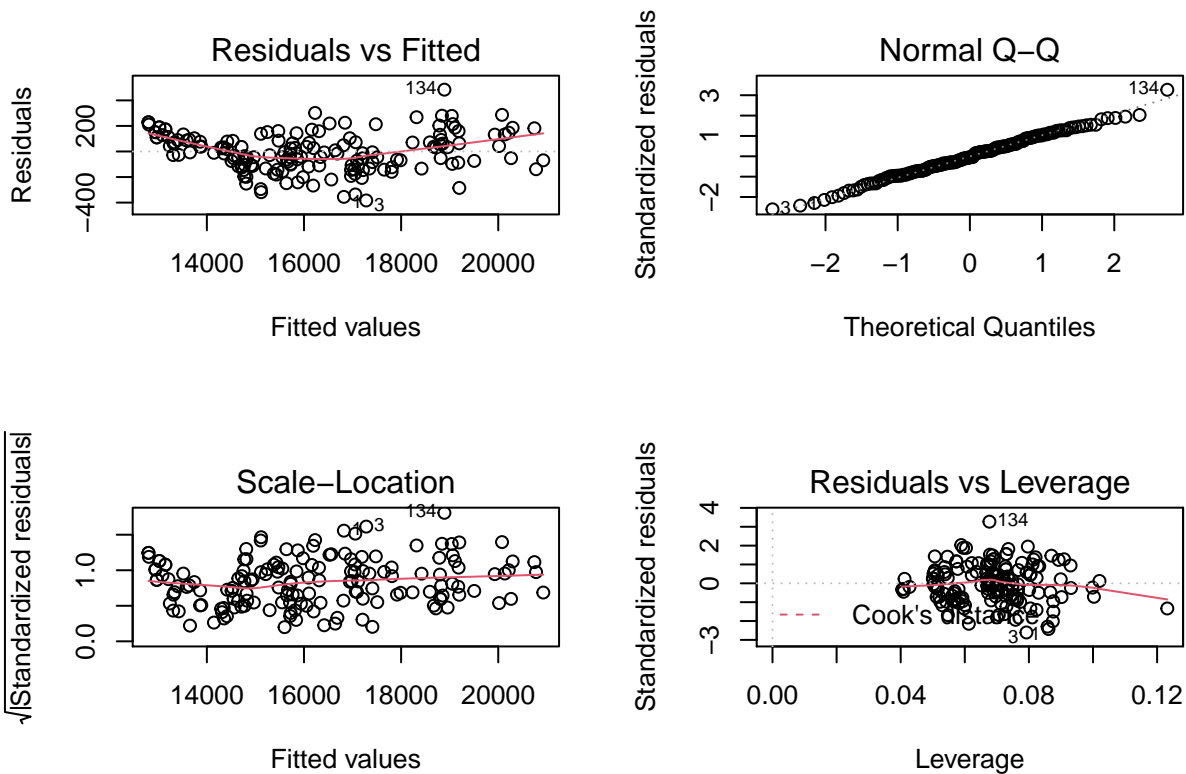
## Question 10

10. Consider the model for fuel consumption proposed in the previous question. Does the model fulfil the properties for linear models? Justify your answer using statistical arguments. Indicate the presence of lack of fit observations and influent data.

Sembla que es compleix la normalitat i homocedasticitat, però sembla que hi ha un patró sistemàtic en els residus (Residuals vs Fitted) que indica la necessitat d'una transformació no lineal o l'addició d'una nova variable (com es pot veure utilitzant `MarginalModelPlots(model_7)`, els perfils per a configuracions de velocitat baixa indiquen dos grups d'observacions).

La falta d'ajust és present a l'observació 134 (student residual > 3). El tall  $4/(n-p)$  per a la distància de Cooks és adequat a causa de la petita mida de la mostra: l'observació 134 és la més influent, seguida de 3, 1, 5 i 120.

```
par(mfrow=c(2,2))
plot(model_7)
```



```
par(mfrow=c(1,1))

residualPlots(model_7,id.n=5,id.method=cooks.distance(model_7),col="darkslateblue",pch=19)

## Warning in plot.window(...): "id.n" is not a graphical parameter
## Warning in plot.window(...): "id.method" is not a graphical parameter
## Warning in plot.xy(xy, type, ...): "id.n" is not a graphical parameter
## Warning in plot.xy(xy, type, ...): "id.method" is not a graphical parameter
## Warning in axis(side = side, at = at, labels = labels, ...): "id.n" is not a
## graphical parameter
## Warning in axis(side = side, at = at, labels = labels, ...): "id.method" is not
## a graphical parameter
## Warning in axis(side = side, at = at, labels = labels, ...): "id.n" is not a
## graphical parameter
## Warning in axis(side = side, at = at, labels = labels, ...): "id.method" is not
## a graphical parameter
## Warning in box(...): "id.n" is not a graphical parameter
## Warning in box(...): "id.method" is not a graphical parameter
## Warning in title(...): "id.n" is not a graphical parameter
## Warning in title(...): "id.method" is not a graphical parameter
## Warning in plot.xy(xy.coords(x, y), type = type, ...): "id.n" is not a graphical
## parameter
```

```

## Warning in plot.xy(xy.coords(x, y), type = type, ...): "id.method" is not a
## graphical parameter

## Warning in plot.window(...): "id.n" is not a graphical parameter

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## Warning in axis(side = side, at = at, labels = labels, ...): "id.n" is not a
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## graphical parameter

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## a graphical parameter

## Warning in box(...): "id.n" is not a graphical parameter

## Warning in box(...): "id.method" is not a graphical parameter

## Warning in title(...): "id.n" is not a graphical parameter

## Warning in title(...): "id.method" is not a graphical parameter

## Warning in plot.xy(xy.coords(x, y), type = type, ...): "id.n" is not a graphical
## parameter

## Warning in plot.xy(xy.coords(x, y), type = type, ...): "id.method" is not a
## graphical parameter

## Warning in bxp(list(stats = structure(c(-294.425425458011, -133.028332223498, : c("Duplicated arguments
## "Duplicated arguments \"12\" = 0.0149942076271822, \"13\" = 0.00387983948160642, \"14\" = 0.02728940
## "Duplicated arguments \"24\" = 0.00040915076248251, \"25\" = 0.000783560146544336, \"26\" = 0.002079
## "Duplicated arguments \"36\" = 0.0104728669028645, \"37\" = 0.00118003934645372, \"38\" = 0.00325578
## "Duplicated arguments \"48\" = 0.000565268256783413, \"49\" = 0.00139515144873673, \"50\" = 0.000755
## "Duplicated arguments \"60\" = 0.00621381386085154, \"61\" = 0.0147908331015293, \"62\" = 0.00187954
## "Duplicated arguments \"72\" = 0.00148888633029219, \"73\" = 0.00620028460948861, \"74\" = 0.0052042
## "Duplicated arguments \"84\" = 0.00461906823308417, \"85\" = 0.000796862580198926, \"86\" = 0.008041
## "Duplicated arguments \"96\" = 0.000577838577666001, \"97\" = 0.0001616418639045, \"98\" = 0.0048117
## "Duplicated arguments \"108\" = 0.00252180321736648, \"109\" = 0.00694156708948346, \"110\" = 0.0048
## "Duplicated arguments \"120\" = 0.0352383993666355, \"121\" = 1.19101427948458e-05, \"122\" = 0.0016
## "Duplicated arguments \"132\" = 0.022729700493275, \"133\" = 0.00546416963509447, \"134\" = 0.070526
## "Duplicated arguments \"144\" = 0.00213476467677683, \"145\" = 0.000317157561394311, \"146\" = 0.004
## "Duplicated arguments \"156\" = 0.00747893913491907, \"157\" = 0.0171489193339797, \"158\" = 0.00397

## Warning in plot.window(...): "id.n" is not a graphical parameter

## Warning in plot.window(...): "id.method" is not a graphical parameter

## Warning in plot.xy(xy, type, ...): "id.n" is not a graphical parameter

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## Warning in axis(side = side, at = at, labels = labels, ...): "id.n" is not a
## graphical parameter

```

```
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## graphical parameter

## Warning in axis(side = side, at = at, labels = labels, ...): "id.method" is not
## a graphical parameter

## Warning in box(...): "id.n" is not a graphical parameter

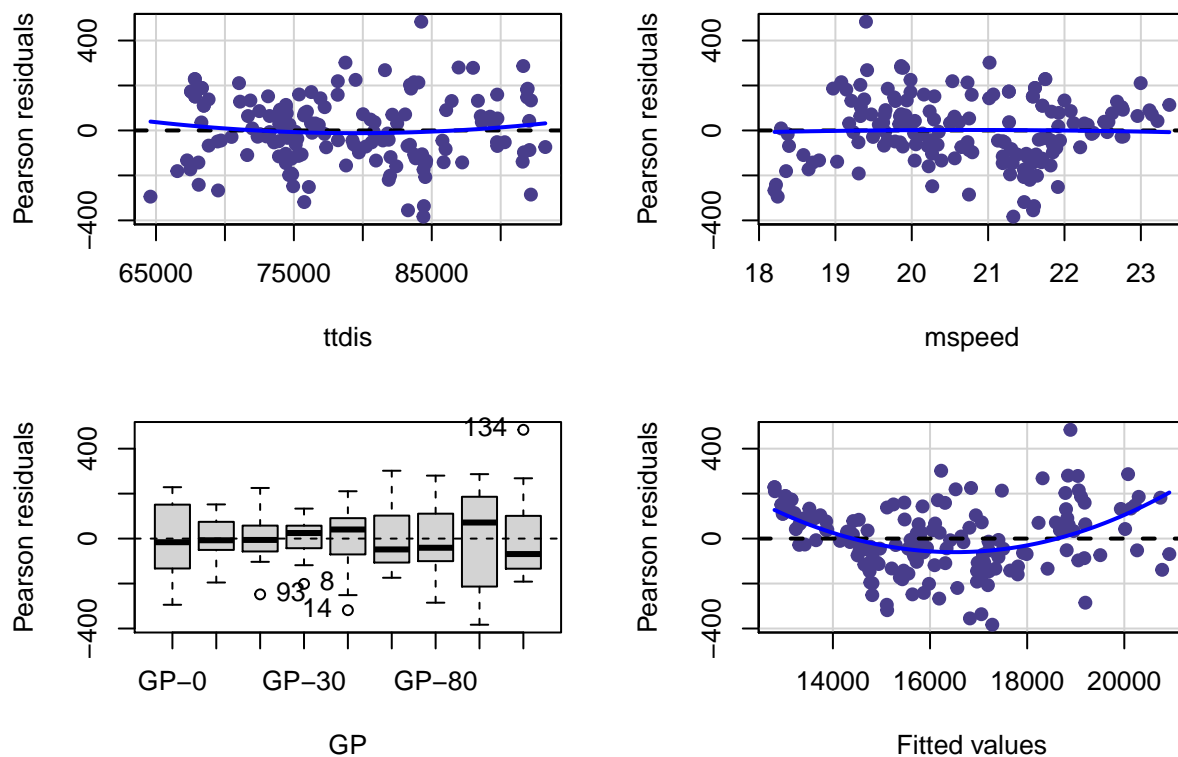
## Warning in box(...): "id.method" is not a graphical parameter

## Warning in title(...): "id.n" is not a graphical parameter

## Warning in title(...): "id.method" is not a graphical parameter

## Warning in plot.xy(xy.coords(x, y), type = type, ...): "id.n" is not a graphical
## parameter

## Warning in plot.xy(xy.coords(x, y), type = type, ...): "id.method" is not a
## graphical parameter
```



```
##          Test stat Pr(>|Test stat|)
## tdis      1.6111      0.1093
## mspeed    -0.2785      0.7810
## GP
## Tukey test  8.3676      <2e-16 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
marginalModelPlots(model_7,id.n=5,id.method=cooks.distance(model_7),col="darkslateblue",pch=19)
```

```
## Warning in plot.window(...): "id.n" is not a graphical parameter
```



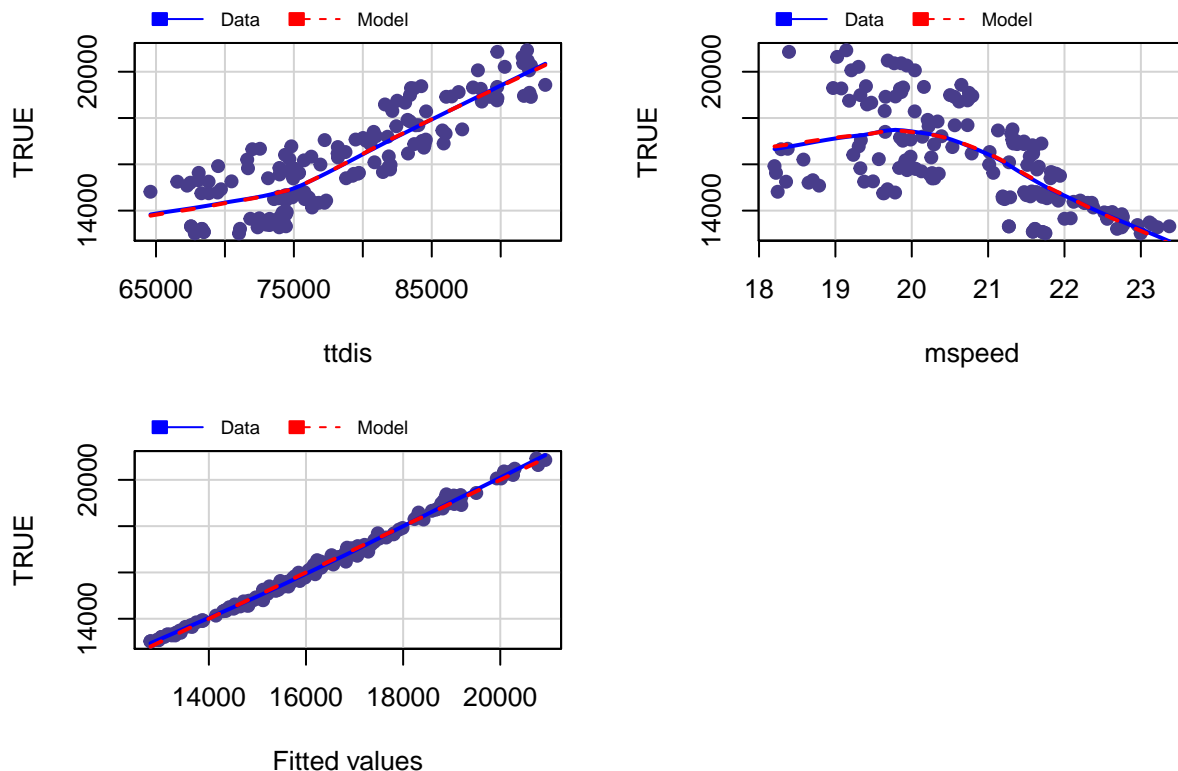
```

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## Warning in plot.xy(xy.coords(x, y), type = type, ...): "id.method" is not a
## graphical parameter
## Warning in mmps(...): Interactions and/or factors skipped
```

## Marginal Model Plots



```
influencePlot(model_7,id.n=5)
```

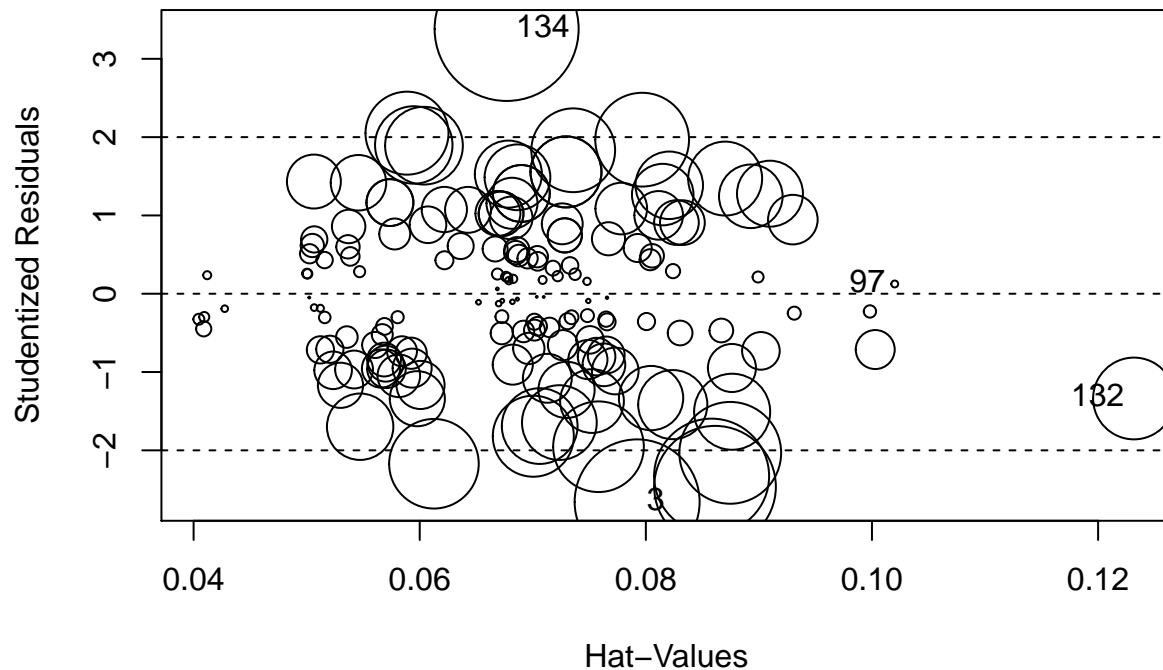
```
## Warning in plot.window(...): "id.n" is not a graphical parameter
## Warning in plot.xy(xy, type, ...): "id.n" is not a graphical parameter
## Warning in axis(side = side, at = at, labels = labels, ...): "id.n" is not a
## graphical parameter

## Warning in axis(side = side, at = at, labels = labels, ...): "id.n" is not a
## graphical parameter

## Warning in box(...): "id.n" is not a graphical parameter

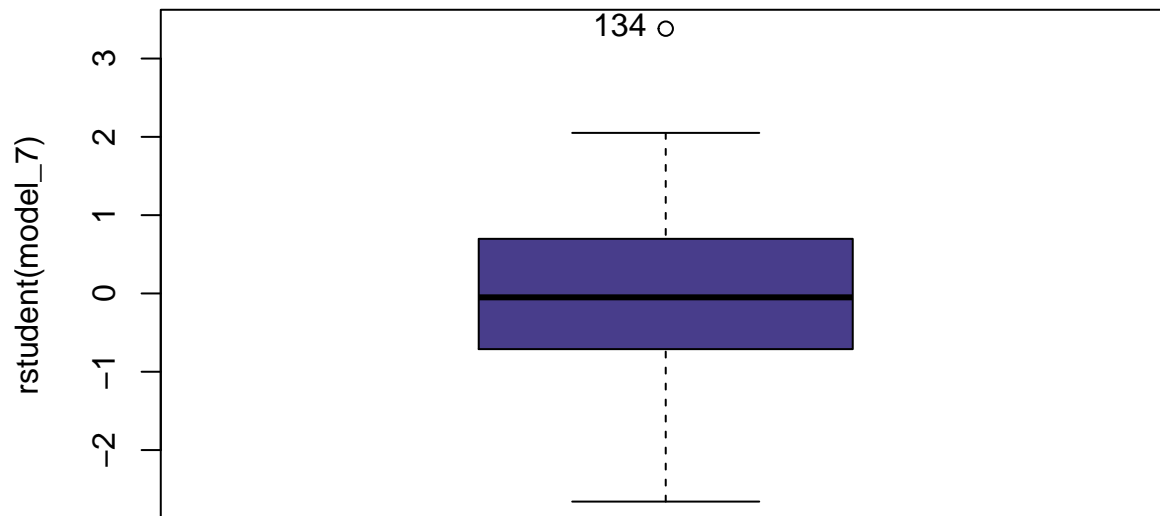
## Warning in title(...): "id.n" is not a graphical parameter

## Warning in plot.xy(xy.coords(x, y), type = type, ...): "id.n" is not a graphical
## parameter
```



```
##      StudRes      Hat      CookD
## 3  -2.6579649 0.07922927 0.0531022681
## 97   0.1246918 0.10201438 0.0001616419
## 132 -1.3376359 0.12317385 0.0227297005
## 134  3.3814196 0.06768643 0.0705261775
```

```
Boxplot(rstudent(model_7),col="darkslateblue") # Lack of fit: outlier in residuals
```



```
## [1] 134
```

```
kpinet[134,]
```

```
##          TD      GP      DP      PVD      TW      NS replica density mflow      ttt.h
## 134 TD-20-70-10 GP-100 DP-20 PVD-20 TW-6 NS-LI      R3  10.882 44819 5648.025
##      mtt.s.km mdelay.s.km mspeed      ttdis      fuelc      co2      nox tvehin
## 134 236.9667      163.4125 19.4004 84238.56 19375.19 41347221 101333.7  6297
##      tvehout virwait virlostin virlostout inputveh demand thrputrate waitrate
## 134  44819      963          2      236      45670 46633      96.11      2.07
```

```
Boxplot(cooks.distance(model_7),col="darkslateblue")
```

```
## [1] 134  3  1  5 120 90 28 14 135 59
```

```
abline(h=4/(nrow(kpinet)-length(coef(model_7))),col="red",lwd=2,lty=2)
```

