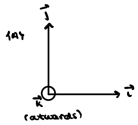
Exercici : (→ Invariància del dot product = scalar product de 2 vectors)

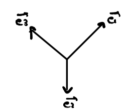


Note that: $\vec{i} \times \vec{j} = \vec{k}$
↑
vector product

- $\vec{v_1}$; $\vec{v_2}$ ⇒ $\vec{v_1} \cdot \vec{v_2}$ = resultat independent del sistema de referència on fem el càlcul
- Donats 3 vectors ($\vec{e_1}, \vec{e_2}, \vec{e_3}$) ortonormals:

$$\left.\begin{aligned} &\hookrightarrow {}^A\vec{e_1} = (0.5,\ \sqrt{3}/2,\ 0) \\ &\hookrightarrow {}^A\vec{e_2} = (-\sqrt{3}/2,\ 0.5,\ 0) \\ &\hookrightarrow {}^A\vec{e_3} = (0,\ 0,\ 1) \end{aligned}\right\} \rightsquigarrow \text{Donats en el ref Frame \{B\}}$$

{B}



Es demana :

1) Trobar ${}^A_B R$

2) Trobar $\vec{i}, \vec{j}, \vec{k}$ expressat en {B}

3) Comprovar que [$\vec{v_1} \cdot \vec{v_2}$ = resultat independent del sistema de referència on fem el càlcul] holds by, computing a) in {A} ⤳ $P = \vec{e_1} \cdot \vec{i}$ en {A}
⤷ $P' = \vec{e_1} \cdot \vec{i}$ en {B}

1)
$$\begin{array}{cc} {}^A_B R = & \begin{bmatrix} 0.5 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ & \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \vec{e_1} & \vec{e_2} & \vec{e_3} \end{array} \end{array} \rightsquigarrow {}^B_A R = \left({}^A_B R\right)^T = \begin{bmatrix} 0.5 & \sqrt{3}/2 & 0 \\ -\sqrt{3}/2 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2) 2 maneres

→ Manera 1

$$\left.\begin{aligned} \vec{e_1} &= 0.5\vec{i} + \sqrt{3}/2\,\vec{j} \\ \vec{e_2} &= -\sqrt{3}/2\,\vec{i} + 0.5\vec{j} \\ \vec{e_3} &= \vec{k} \end{aligned}\right\} \rightarrow \text{aïllem } \vec{i}, \vec{j}, \vec{k}$$

→ Manera 2

$${}^B_A R = \begin{bmatrix} 0.5 & \sqrt{3}/2 & 0 \\ -\sqrt{3}/2 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightsquigarrow \begin{aligned} {}^B\vec{i} &= 0.5\,\vec{e_1} - \sqrt{3}/2\,\vec{e_2} \\ {}^B\vec{j} &= \sqrt{3}/2\,\vec{e_1} + 0.5\,\vec{e_2} \\ {}^B\vec{k} &= \vec{e_3} \end{aligned}$$

↑ 1r vector de {A} expressat en {B}
↑ 2n vector de {A} expressat en {B}
↑ 3r vector de {A} expressat en {B}

Let's check it :

$$\vec{j} = (\sqrt{3}/2)\,(\underbrace{0.5\vec{i} + \sqrt{3}/2\,\vec{j}}_{\vec{e_1}}) + 0.5(\underbrace{-\sqrt{3}/2\,\vec{i} + 0.5\vec{j}}_{\vec{e_2}}) = \frac{\sqrt{3}}{4}\vec{i} + \frac{3}{4}\vec{j} - \frac{\sqrt{3}}{4}\vec{i} + \frac{1}{4}j = \vec{j} \quad //$$

3) en {A} :

$$P = {}^A\vec{e_1} \cdot {}^A\vec{i} = (0.5, \sqrt{3}/2, 0)\,(1, 0, 0) = 0.5$$

en {B}

$$P = {}^B\vec{e_1} \cdot {}^B\vec{i} = (1, 0, 0)(0.5, -\sqrt{3}/2, 0) = 0.5$$

$\rightsquigarrow$ Extensió:

$$\left.\begin{array}{l} {}^{A}\vec{V}_1 = (1, 2, 3) \\ {}^{A}\vec{V}_2 = (1, 3, 5) \end{array}\right\} \rightarrow P =$$

$\hookrightarrow$ En $\{A\}$

$$P = {}^{A}\vec{V}_1 \cdot {}^{A}\vec{V}_2 = 1 \cdot 1 + 2 \cdot 3 + 3 \cdot 5 = 22$$

$\hookrightarrow$ En $\{B\}$

$$ {}^{B}\vec{V}_1 = {}^{B}_{A}R \cdot {}^{A}\vec{V}_1 \rightsquigarrow = \left(\tfrac{1}{2} + \sqrt{3} \,,\; -\sqrt{3}/2 + 1 \,,\; 3 \right)$$

$$ {}^{B}\vec{V}_2 = {}^{B}_{A}R \cdot {}^{A}\vec{V}_2 \rightsquigarrow = \left(\tfrac{1}{2} + \tfrac{3\sqrt{3}}{2} \,,\; (3-\sqrt{3})/2 \,,\; 5 \right)$$

Llavors: ${}^{B}\vec{V}_1 \cdot {}^{B}\vec{V}_2 = \left(\tfrac{1}{2} + \sqrt{3}, -\sqrt{3}/2 + 1, 3 \right) \left(\tfrac{1}{2} + \tfrac{3\sqrt{3}}{2}, (3-\sqrt{3})/2, 5 \right) = 22$ ✓✓