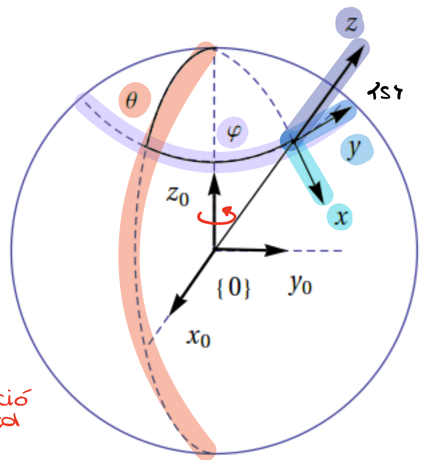
Problem 1

A ship has lost its power and is located adrift at an unknown position in the ocean. The crew intends to calculate its actual position finding the values of the angular coordinates (θ, φ) (see figure) describing their local system $S = (x, y, z)$. In order to do so they will be measuring, at different times, the angle formed by the position of the Sun in the sky and its local Zenith axis z (φ) with a sextant (an apparatus designed just for this purpose). The meridian intersecting the x_0 (θ) axis is the Greenwhich meridian of longitude 0.



12:00 (dia de interès) $\left\{ \begin{array}{l} n_{x0} = 0.918067 \\ n_{y0} = -0.0364722 \\ n_{z0} = 0.394744 \end{array} \right\} \vec{n}$

distància al sol — direcció al sol

$\vec{R}_{sun} = D \cdot \vec{n}$

$w = 15^\circ/\text{hora}$

t no [hora]

For this calculation the crew has a set of astronomical tables which provide the position of the Sun relative to the Earth system at noon time (12 h. 0 min. UTC time, which we use as our time origin) $\equiv \{0\}$ for each day of the year. The data for the Sun position the day of interest (in the $\{0\}$ system) is

$$(nx0 = 0.918067,\ ny0 = -0.0364722,\ nz0 = 0.394744)$$

The unit vector $\vec{n} = (nx0,\, ny0,\, nz0)$ describes the direction pointing to the Sun, so that $\vec{R}_{Sun} = D\,\vec{n}$ where D is the distance between the Sun and Earth centers. Whilst the position of the Sun is constant in the $\{0\}$ system, the Earth is rotating along the z_0 axis at a rate $w = 15\,Degrees/hour$, so that at a time of t (expressed in hours) after our time origin ($12\ h.\ UTC$) the Earth system $\{t\}$ is given by ${}^0_tR = R_z(w\,t)$. The translation movement of the Earth along its orbit around the Sun during the time the measurements are being made, as well as the Earth radius, are both perfectly negligible.

15°

a) Find the transformation matrix ${}^0_ST(t) = {}^0_SR(t)$ between the $\{0\}$ and $\{S\}$ systems (you can express the result as a composition of transformations).

b) In the $\{S\}$ system, the position of the Sun can be written as $\vec{R}_{Sun} = D\ {}^S\vec{n}$. Find the equation for ${}^Sn_z(t)$.

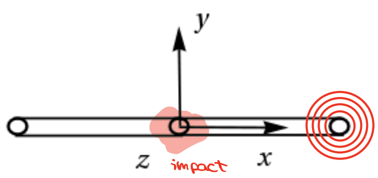
c) Since ${}^Sn_z(t)$ corresponds to the cosinus of the angle between the Sun position and the local z axis, it is expected that ${}^Sn_z(t)$ will be maximum at the time of closest approach of the direction to the Sun to the local z axis. From the measurements of the crew, it turns out that the maximum value ${}^Sn_z(max) = 0.98212$ is attained 55 min 42 seconds after 12 UTC $i.e.,\ t = 0.92833\,(hours)$. What are the values of (θ, φ)?

Note 1/2: the ship was navigating in an area delimited by $50^{\rm o} < \theta < 60^{\rm o}$.

Note 2/2: if you happen to find an equation of the kind $X = A\,Sin(\theta) + B\,Cos(\theta)$, the solution for θ is $\theta = arctan(\frac{A}{B}) - arccos(\frac{X}{C})$, with $C = \sqrt{A^2 + B^2}$. If you use this expression, check that the value you get for θ reproduces the X value in your equation when pluged into it (use at least 5 decimal places in your numerical values).

Problem 2

A bullet is shot against the bar shown in the figure, aiming at its most right point.



$M = 2.5kg$ $L = 0.75$ cm $m_b = 0.0125kg$

Data: ($M = 2.5$, $L = 0.75$, $m_b = 0.0125$) (masses in kg, lengths in cm)
Impact point: ($L/2$, 0, 0)
Bullet velocity: (v_x, v_y, $v_z = 0$)
Make all calculations in the CM reference frame shown in the figure.

P unidades de momento → $[m][v]$ → $kg \cdot m/s$

L unidades de momento angular → $[m][r][v] = kg \cdot m^2/s$
↳ radio?

a) Which are the values of the linear momentum $\vec{P} = (P_x,\, P_y,\, P_z)$ and angular momentum $\vec{L} = (L_x,\, L_y,\, L_z)$ of the bar after it has been hit by the bullet? (Assume that the bar is initially at rest, but otherwise free to move, all forces other than the impact ones are negligible and that the bullet has lost all its linear and angular momentum in the process, and express the results as a function of (v_x, v_y).

b) Assuming that the system is a single bar of mass M and length L , with $\boxed{I_{zz} = \frac{1}{12} M\, L^2}$ $\cdot\omega_z = L_z$ with the movement confined to the $\{x, y\}$ plane:

Find, immediately after the impact, the velocity of the center of mass $\vec{V}_{CM}$, the angular velocity $\vec{w}$, the location of the center of instantaneous rotation and the velocity of a generic point of the bar with coordinates (x, $y = 0$, $z = 0$). Which of all points of the bar has a minimum absolute velocity?

c) Let us consider the system as a two-bar system of lengths L/2 and masses M/2 each one, with a revolution joint (of axis z) at the origin, depicted as a circle in the figure. We describe the movement immediately after the impact of the bullet by means of $\{\vec{v}_{CM}(1), \vec{w}(1)\}$ (left bar) and $\{\vec{v}_{CM}(2), \vec{w}(2)\}$ (right bar). The movement is still confined to the $\{x, y\}$ plane. Find the expressions of the total linear momentum (P_x, P_y) and total angular momentum L_z as a function of $\{\vec{v}_{CM}(1), \vec{w}(1), \vec{v}_{CM}(2), \vec{w}(2)\}$.

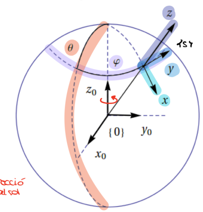
2

a) $${}^{0}_{S}T(t) = {}^{0}_{S}R(t)$$

$$\{0\} \xrightarrow[R_z(\omega t)]{} \{t\} \xrightarrow[R_z(\varphi)\cdot R_y(\theta)]{} \{S\}$$

$R_z(\omega t) \cdot R_z(\varphi)\cdot R_y(\theta)$

axis is the Greenwhich meridian of longitude 0.



12:00 (dia de interès) $\left\{ \begin{array}{l} n_{x0} = 0.918069 \\ n_{y0} = -0.0364722 \\ n_{z0} = 0.394744 \end{array} \right\} \vec{n}$

$\vec{R}_{sun} = \underset{\substack{\text{distància} \\ \text{al sol}}}{D} \cdot \underset{\substack{\text{direcció} \\ \text{al sol}}}{\vec{n}}$

$\omega = 15^\circ/\text{hora}$

t no [hora]

For this calculation the crew has a set of astronomical tables which provide the position of the

2)

a) $\vec{P}$, $\vec{L}$ = ?

producte vectorial

$$\vec{L} = m_b \cdot \left(\frac{L}{2}, 0, 0\right) \times (v_x, v_y, 0) = \left(0, 0, \frac{L \cdot v_y \cdot m_b}{2}\right)$$

$L_x = 0 \quad L_y = 0 \quad L_z = \frac{L}{2} \cdot v_y \cdot m_b$

$$\vec{P} = m_b \cdot (v_x, v_y, 0) = (m_b v_x, m_b v_y, 0)$$

momento linear

p unidades de momento

$\rightsquigarrow [m][v] \rightsquigarrow kg \cdot m/s$

$\vec{p} = m\vec{v}$

momento angular

L unidades de momento angular

$\rightsquigarrow [m][r][v] = kg \cdot m^2/s$

↳ radio?

$\vec{L} = \vec{r} \times m\vec{v} = \vec{r} \times \vec{p}$

↳ distance from reference

$\vec{r_c}$

donde $v = 0$

b) $\vec{V}_{CM}$, $\vec{\omega}$, centro rot instantánea, $V_P \rightsquigarrow P = (x, 0, 0)$, point w/ minimum absolute velocity

momento linear

✓ $\vec{V}_{CM} = \frac{\vec{P}}{M} = \frac{(m_b v_x, m_b v_y, 0)}{M} = (0.005 v_x, 0.005 v_y, 0)$

?? $\vec{\omega} = (0, 0, \omega_z)$ $\qquad \vec{L} = I \cdot \vec{\omega}$

$$L_z = \underbrace{\frac{1}{12} M L^2}_{I_{zz}} \omega_z \qquad \omega_z = \frac{L_z \cdot 12}{M L^2} = \frac{\frac{\overset{0.75}{-}}{2} \cdot v_y \cdot 0.0125 \cdot 12}{2{,}5 \cdot 0.75^2} = \frac{v_y}{25}$$

$\vec{r_c} \rightsquigarrow \vec{v_c} = 0$

$$\vec{v_c} = \vec{V}_{CM} + \vec{\omega} \times (\vec{r_c} - \vec{r}_{CM})$$

$$0 = (0.005 v_x, 0.005 v_y, 0) + \left(0, 0, \frac{v_y}{25}\right) \times [\vec{r_c} - (0, 0, 0)]$$

↓

$x_c = -1/8$

$y_c = v_x / 8 v_y$

$z_0 = 0$