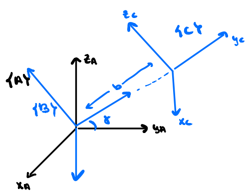
Problema 7



- $\{A\} \xrightarrow{R(x,\gamma)} \{B\}$
- $\{B\} \xrightarrow{T(y,b)} \{C\}$

x) Trobeu ${}^{A}_{C}T$

a) Càlcul directe (normalment impossible)

b) Càlcul segons metodologia habitual $\rightarrow$ composició d'operacions

a) $${}^{A}_{C}T = \left[\begin{array}{c|c} {}^{A}_{C}R & \overset{\text{origen de \{C\} segons \{A\}}}{P} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & c\gamma & -s\gamma & bc\gamma \\ 0 & s\gamma & c\gamma & bs\gamma \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

b) $\{A\} \xrightarrow[{}^{A}_{B}T]{} \{B\} \xrightarrow[{}^{B}_{C}T]{} \{C\}$

$${}^{A}_{C}T = {}^{A}_{B}T \cdot {}^{B}_{C}T = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & c\gamma & -s\gamma & 0 \\ 0 & s\gamma & c\gamma & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \cdot \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & c\gamma & -s\gamma & bc\gamma \\ 0 & s\gamma & c\gamma & bs\gamma \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$\Big\downarrow$ inversa

Trasposta R ?

Negativa P

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & c\gamma & s\gamma & -bc\gamma \\ 0 & -s\gamma & c\gamma & -bs\gamma \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

mas menos

Seria correcte si escribim P en {B}

Inversa CORRECTE

$R' = R^T$

$P' =$ P en base{B} $= \left({}^{B}_{A}R\right)\left({}^{A}_{\cdot}P\right) = -{}^{A}_{B}R^{T}\ {}^{A}_{\cdot}P$

$$\Rightarrow \quad {}^{A}_{C}T = \left[\begin{array}{c|c} {}^{A}_{C}R & P \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{c|c} {}^{A}_{C}R^{T} & -{}^{A}_{\cdot}R^{T} \cdot P \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$
{}^{A}_{C}T = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & c\gamma & -s\gamma & bc\gamma \\ 0 & s\gamma & c\gamma & bs\gamma \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad \rightsquigarrow \quad \left({}^{A}_{C}T\right)^{-1} = {}^{C}_{A}T = \left[\begin{array}{c|c} {}^{A}_{C}R^{T} & -{}^{A}_{C}R^{T} \cdot P \\ \hline 0 \; 0 \; 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & c\gamma & s\gamma & -b \\ 0 & -s\gamma & c\gamma & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$

$$
\underbrace{-\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & s\gamma \\ 0 & -s\gamma & c\gamma \end{bmatrix}}_{-{}^{A}_{C}R^{T}} \cdot \underbrace{\begin{vmatrix} 0 \\ bc\gamma \\ bs\gamma \end{vmatrix}}_{P} = \begin{bmatrix} 0 \\ -b \\ 0 \end{bmatrix}
$$