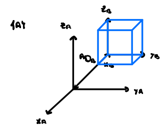
Problema 9

Tenim el sistema {B} fixat al cub

a) Trobar coords dels 8 vèrtexs del cub respecte {A} i {B}

b) Fem girar el cub, i {B} segons $R(y, 45°)$. Trobar les noves coordenades respecte {A} i {B}.



$^A O_B = (1, 1, 1)$

Trobarem $^A_B T = \left[\begin{array}{c|c} R & P \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$

Cas a, No hi ha rotació → $^A_B R = id$

$$^A_B T = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Coordenades respecte {B}

(0 0 0)
(0 0 1)
(0 1 0)
(0 1 1)
(1 0 0)
(1 0 1)
(1 1 0)
(1 1 1)

Coordenades respecte {A}

$\begin{bmatrix} ^A P \\ 1 \end{bmatrix} = {^A_B T} \cdot \begin{bmatrix} ^B P \\ 1 \end{bmatrix}$ → Generalitzat → $^B P = (b_1, b_2, b_3)$

$$^A P = \begin{vmatrix} \cdot \\ \cdot \\ \cdot \\ 1 \end{vmatrix} = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \cdot \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ 1 \end{vmatrix} = \begin{vmatrix} b_1+1 \\ b_2+1 \\ b_3+1 \\ 1 \end{vmatrix}$$

Cas b, ara tenim una rotació de 45° en y.

$$^A_B T = \left|\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right| \cdot \left[\begin{array}{ccc|c} c(45) & 0 & s(45) & 0 \\ 0 & 1 & 0 & 0 \\ -s(45) & 0 & c(45) & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} c(45) & 0 & s(45) & 1 \\ 0 & 1 & 0 & 1 \\ -s(45) & 0 & c(45) & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Coordenades respecte {B}

(0 0 0)
(0 0 1)
(0 1 0)
(0 1 1)
(1 0 0)
(1 0 1)
(1 1 0)
(1 1 1)

Coordenades respecte {A}

$\begin{bmatrix} ^A P \\ 1 \end{bmatrix} = {^A_B T} \cdot \begin{bmatrix} ^B P \\ 1 \end{bmatrix}$ → Generalitzat → $^B P = (b_1, b_2, b_3)$

$$^A P = \begin{vmatrix} \cdot \\ \cdot \\ \cdot \\ 1 \end{vmatrix} = \left[\begin{array}{ccc|c} c(45) & 0 & s(45) & 1 \\ 0 & 1 & 0 & 1 \\ -s(45) & 0 & c(45) & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \cdot \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ 1 \end{vmatrix} = \begin{vmatrix} 1 + {b_1}/{\sqrt{2}} + {b_3}/{\sqrt{2}} \\ 1 + b_2 \\ 1 - {b_1}/{\sqrt{2}} + {b_3}/{\sqrt{2}} \\ 1 \end{vmatrix}$$

Afegit → velocitat

Si la rotació no és estàtica → $R(t) = R[y, \omega t]$

constant, velocitat angular → si hi ha acceleració

$\omega = \frac{t}{\alpha}$ → $\alpha = \omega t$

$\omega = \frac{\Delta \alpha}{\Delta t}$

Si el cos gira simultàniament en tots tres eixos:

$$\left\{ \begin{array}{l} \omega_x = \frac{\Delta \alpha_x}{\Delta t} \\ \omega_y = \frac{\Delta \alpha_y}{\Delta t} \\ \omega_z = \frac{\Delta \alpha_z}{\Delta t} \end{array} \right\} \vec{\omega} = \{\omega_x, \omega_y, \omega_z\}$$

$$T[t] = \left[\begin{array}{ccc|c} c\omega t & 0 & s\omega t & 1 \\ 0 & 1 & 0 & 1 \\ -s\omega t & 0 & c\omega t & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$^A P = \begin{vmatrix} \cdot \\ \cdot \\ \cdot \\ 1 \end{vmatrix} = \left[\begin{array}{ccc|c} c\omega t & 0 & s\omega t & 1 \\ 0 & 1 & 0 & 1 \\ -s\omega t & 0 & c\omega t & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \cdot \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ 1 \end{vmatrix}$$

Com trobem velocitat de cada vèrtex en funció de w constant ?

a) Tradicional (poc pràctica)

b) Bona (sistemàtica)

a) $\vec{r}[t]$; $\vec{v}[t] = \dfrac{d\,\vec{r}[t]}{dt}$

$\vec{r}[t] = {}^{A}P[t]$ (3 primeres coords)

$\vec{r}[t] = \{1 + b1\,\mathrm{Cos}[t\,w] + b3\,\mathrm{Sin}[t\,w],\ 1 + b2,\ 1 + b3\,\mathrm{Cos}[t\,w] - b1\,\mathrm{Sin}[t\,w]\}$

$$\vec{v}[t] = \frac{d\,\vec{r}[t]}{dt} = \begin{pmatrix} b3\,w\,\mathrm{Cos}[t\,w] - b1\,w\,\mathrm{Sin}[t\,w] \\ 0 \\ -b1\,w\,\mathrm{Cos}[t\,w] - b3\,w\,\mathrm{Sin}[t\,w] \end{pmatrix} \begin{array}{l} \to x \\ \to y \rightsquigarrow v=0 \\ \to z \end{array}$$

b) Recordem que:

$${}^{A}P = \begin{vmatrix} \cdot \\ \cdot \\ \cdot \\ 1 \end{vmatrix} = \left[\begin{array}{ccc|c} c\,wt & 0 & s\,wt & 1 \\ 0 & 1 & 0 & 1 \\ -s\,wt & 0 & c\,wt & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \cdot \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ 1 \end{vmatrix}$$

$$\text{velocitat} = \frac{\partial}{\partial t}\left({}^{A}P[t]\right) = \frac{\partial}{\partial t}\left(\left[\begin{array}{ccc|c} c\,wt & 0 & s\,wt & 1 \\ 0 & 1 & 0 & 1 \\ -s\,wt & 0 & c\,wt & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \cdot \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ 1 \end{vmatrix} \right) = \frac{\partial}{\partial t}\left(\left[\begin{array}{ccc|c} c\,wt & 0 & s\,wt & 1 \\ 0 & 1 & 0 & 1 \\ -s\,wt & 0 & c\,wt & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \right) \cdot \begin{vmatrix} b_1 \\ b_2 \\ b_3 \\ 1 \end{vmatrix}$$

↳ derivada

$$\begin{vmatrix} -\omega\, s(wt) & 0 & \omega\, c(wt) & 0 \\ 0 & 0 & 0 & 0 \\ -\omega\, c(wt) & 0 & -\omega\, s(wt) & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

Per tant $\dfrac{d}{dt}\left({}^{A}P[t]\right) = \left(\dfrac{d}{dt}\, {}^{A}_{B}T[t]\right) \cdot {}^{B}P$

Si hi ha altres transformacions:

Per tant :

$$\frac{d}{dt}\left({}^{A}P[t]\right) = \left[\left(\frac{d}{dt} T_1[t]\right) . T_2[t] + T_1[t] . \left(\frac{d}{dt} T_2[t]\right) \right] . {}^{C}P$$

Notació : $\dfrac{d}{dt} x(t) = \dot{x}(t)$