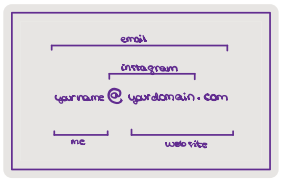
⇒ Incorporar a un sistema les lleis dinàmiques (⇒ ús del jacobià)

⇒ Ús de la física

⇝ $\Delta x = J \cdot \Delta\theta$ on $(\Delta\theta_j = \theta_j(t+\Delta t) - \theta_j(t))$



⇒ Equacions de Lagrange → $\left\{ \frac{d}{dt}\left(\frac{\partial K}{\partial \dot\theta_j}\right) - \frac{\partial K}{\partial \theta_j} = F_j \right.$

⇝ Sistemes amb N graus de llibertat ≡ θ_j ; $(x, y, z, \theta_x^1, \theta_y^1, \theta_z^1, \theta_x, \theta_y, \dots, \theta_N)$

⇝ Expressió de l'energia cinètica (K) : $K = \sum_{i=1}^{\#\text{masses}} \frac{1}{2} m_i v_i^2$

↳ $\vec{v_i}$ = velocitat de la massa m_i

$\left.\begin{array}{l} \vec{v_i} = \frac{d}{dt}\vec{r_i}(t) = \\ = \sum_{j=1}^{\text{Nde}} \underbrace{[\partial_j T(i,\theta)]}_{\text{Jacobià}} \dot\theta_j \end{array}\right\} \rightarrow \vec{v_i} = [\text{Jacobià}]\,[\dot\theta]$

$\left|\begin{matrix}\vdots\\ \vec{v_i}\\ \vdots\end{matrix}\right| = \left|\begin{matrix}\cdots\\ \partial_j T(i,\theta)\\ \cdots\end{matrix}\right| \left|\begin{matrix}\dot\theta_1\\ \vdots\\ \dot\theta_N\end{matrix}\right|$

on $\partial_j = \left[\frac{\partial}{\partial\theta_j} \overset{\text{com}}{\underset{\text{mi}}{}} T(\theta)\right]$

⇝ Com calculem el Jacobià (pràctica 4)

- Recordem:
$$X_\alpha = \left[\overset{\sigma}{\underset{\text{pinça}}{}} T\right]_\alpha$$
- Llista de variables externes (selecció d'elements que ens interessen) ≡ particionar valors (elements de la matriu $\underbrace{\overset{\sigma}{\underset{\text{pinça}}{}} T(\theta)}_{4\times N}$
- Definició del Jacobià $\Delta x_\alpha = \sum_j \partial_{\alpha j}\,[\Delta\theta_j]$

$\left|\begin{matrix}\vdots\\ \Delta x\\ \vdots\end{matrix}\right| = \left|\begin{matrix}\cdots\\ \text{Jacobià}\\ \cdots\end{matrix}\right| \left|\begin{matrix}\\ \Delta\theta\\ \\ \end{matrix}\right|$

- Càlcul del Jacobià $(\partial_{\alpha j})$

$$\left.\begin{aligned} \partial_{\alpha j} &= \frac{\partial x_\alpha}{\partial\theta_j} = \frac{\partial}{\partial\theta_j}\left(\overset{\sigma}{\underset{\text{pinça}}{}} T\right)_\alpha = \\ &= \partial_j \left(\overset{\sigma}{\underset{\text{i}_0}{}} T(i_0)\right)_\alpha = \\ &= [\partial_j T(i_0)]_\alpha \end{aligned}\right\} \text{Recepta: } {}^{\circ}T_{i_0} = \boxed{{}^{1}_{1}T(\theta_1)} \cdot \dots \cdot {}^{\sigma}_{4}T(\theta_4)\,{}^{\sigma}_{i_0}T$$

↳ $(R_x(\alpha)\,T_x(\alpha)\,T_z(\alpha)\,R_z(\theta_j))$

- Notes:

$${}^{\sigma}T(i) = T(\alpha_0)\cdot T_z(\theta_1)\cdot T(\alpha_1) \dots T(\alpha_{j-1})T_z(\theta_j)\,T(\alpha_j)$$

a) $T_z(\theta_j) = \left(\begin{array}{ccc|c} & R_z(\theta_j) & & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$ b) $T_z(\theta_j) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \theta_j = \theta_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$